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CISP - 440

Assignment 15

12/13/2018

## Part 0 - Languages and Grammars.

### Description:

The goal for this assignment is to demonstrate my knowledge on languages and grammars. To do this, I was to perform a number of practice problems defined by the professor. Note, each question will be listed below the problem.

### Problem 2:

Determine whether the given grammar is context-sensitive, context-free, regular, or none of these. Give all characterizations that apply.

$T = \{a, b, c\}$

$N = \{\sigma, A, B\}$

#### Productions:

$\sigma \rightarrow AB$	$AB \rightarrow BA$	$A \rightarrow \sigma A$
$B \rightarrow Bb$	$A \rightarrow a$	$B \rightarrow b$

Start =  $\sigma$

This grammar defines a **context free** language. This is because it fulfills the three requirements for such a language. It has choice, a "B" can either become a "b" or a "Bb." It has Recursion, "B" goes to "Bb." And, it has a base case. Eventually the grammar would terminate because no non-terminals would be left.

### Problem 3:

Determine whether the given grammar is context-sensitive, context-free, regular, or none of these. Give all characterizations that apply.

$T = \{a, b\}$

$N = \{\sigma, A, B\}$

#### Productions:

$\sigma \rightarrow A$	$\sigma \rightarrow AAB$	$Aa \rightarrow ABa$
$A \rightarrow aa$	$Bb \rightarrow ABb$	$AB \rightarrow ABB$ $B \rightarrow b$

Start =  $\sigma$

This grammar is **context sensitive**. This is because some substitutions rely on what is around to apply. For example “Aa” to “ABa” requires an “Aa” to be there (context) before the “B” can be wedged in between.

### Problem 8:

Show that the given string “a” is in L(G) for the given grammar G by giving the derivation of “a.”

G = problem 2

“a” = abab

Q -> AB -> QAB -> ABAB -> aBAB -> abAB -> abaB -> abab

**Note:** The underlined portion is where the sub is being performed.

### Problem 11:

Show that the given string “a” is in L(G) for the given grammar G by giving the derivation of “a.”

T = {a, b}

N = {S, A, B}

#### Productions:

S -> bS          S -> aA          S -> a

A -> aS          A -> bB

B -> bA          B -> aS          B -> b

Start = S

“a” = abaabbabba

S -> aA -> abB -> abaS -> abaaA -> abaabB -> abaabbA -> abaabba -> abaabbabS -> abaabbabb -> abaabbabb -> abaabbaba

**Note:** The underlined portion is where the sub is being performed.

## Problem 12:

Write the grammars of Examples 10.3.4 and 10.3.9 and Exercises 1-4 and 6 in BNF.

### 10.3.4

$\langle O' \rangle ::= ba \mid a \langle S \rangle$

$\langle S \rangle ::= b \langle S \rangle \mid b$

### 10.3.9

$\langle O' \rangle ::= a \langle A \rangle \langle B \rangle \mid a \langle B \rangle$

$\langle A \rangle ::= a \langle A \rangle \langle C \rangle \mid a \langle C \rangle$

$\langle B \rangle ::= \langle D \rangle c$

$\langle D \rangle ::= b$

$\langle C \rangle \langle D \rangle ::= \langle C \rangle \langle E \rangle$

$\langle C \rangle \langle E \rangle ::= \langle D \rangle \langle E \rangle$

$\langle D \rangle \langle E \rangle ::= \langle D \rangle \langle C \rangle$

$\langle C \rangle c ::= \langle D \rangle cc$

### Exercise 1

$\langle O' \rangle ::= b \langle O' \rangle \mid a \langle A \rangle \mid b$

$\langle A \rangle ::= a \langle O' \rangle \mid b \langle A \rangle \mid a$

### Exercise 2

$\langle O' \rangle ::= \langle A \rangle \langle B \rangle$

$\langle A \rangle ::= a \langle A \rangle \mid a$

$\langle B \rangle ::= \langle B \rangle b \mid b$

$\langle A \rangle \langle B \rangle ::= \langle B \rangle \langle A \rangle$

### Exercise 3

$\langle O' \rangle ::= \langle A \rangle \mid \langle A \rangle \langle A \rangle \langle A \rangle \langle B \rangle$

$\langle A \rangle ::= aa$

$\langle B \rangle ::= b$

$\langle A \rangle a ::= \langle A \rangle \langle B \rangle a$

$\langle B \rangle b ::= \langle A \rangle \langle B \rangle b$

$\langle A \rangle \langle B \rangle ::= \langle A \rangle \langle B \rangle \langle B \rangle$

### Exercise 4

$\langle O' \rangle ::= \langle B \rangle \langle A \rangle \langle B \rangle \mid \langle A \rangle \langle B \rangle \langle A \rangle$

$\langle A \rangle ::= \langle A \rangle \langle B \rangle \mid a \langle A \rangle \mid a \langle B \rangle$

$\langle B \rangle ::= \langle B \rangle \langle A \rangle \mid b$

**Exercise 6**

$$\langle O' \rangle ::= \langle A \rangle \langle A \rangle \langle O' \rangle$$

$$\langle A \rangle \langle A \rangle ::= \langle B \rangle$$

$$\langle B \rangle ::= b \langle B \rangle$$

$$\langle A \rangle ::= a$$
**Problem 16:**

Write a grammar that generates the strings having the given property.

**Property:** Strings over  $\{a, b\}$  ending with “ba.”

$$T = \{a, b\}$$

$$N = \{S, A, B\}$$
**Productions:**

$$S \rightarrow ABS \quad S \rightarrow BAS \quad S \rightarrow ba$$

$$A \rightarrow aA \quad A \rightarrow aB \quad A \rightarrow a$$

$$B \rightarrow bB \quad B \rightarrow bA \quad B \rightarrow b$$

Starting symbol = S

The above grammar is capable of only making strings that end with “ba.”

**Problem 17:**

Write a grammar that generates the strings having the given property.

**Property:** Strings over  $\{a, b\}$  containing “ba.”

$$T = \{a, b\} \text{ and } N = \{S, A, B\}$$
**Productions:**

$$S \rightarrow AsB \quad S \rightarrow BsA \quad S \rightarrow ba$$

$$A \rightarrow aA \quad A \rightarrow aB \quad A \rightarrow a$$

$$B \rightarrow bB \quad B \rightarrow bA \quad B \rightarrow b$$

Starting symbol = S

The above grammar is capable of only making strings that contain “ba.”

**Problem 19:**

Write a grammar that generates the strings having the given property.

**Property:** Integers with no leading 0's.

$T = \{-\text{integers}, +\text{integers}, 0\}$

$N = \{S, A, B\}$

**Productions:**

$S \rightarrow SAB \quad S \rightarrow SBA \quad S \rightarrow -\text{integers} \quad S \rightarrow +\text{integers}$

$A \rightarrow +\text{integers} \quad A \rightarrow 0$

$B \rightarrow +\text{integers} \quad B \rightarrow 0$

Starting symbol = S

The above grammar is capable of making positive or negative integers that do NOT start with 0.

**Problem 21:**

Write a grammar that generates the strings having the given property.

**Property:** Exponential numbers (numbers including floating point)

$T = \{-\text{integers}, +\text{integers}, 0, \text{point}, \text{exponent}\}$

$N = \{S, A, B\}$

Starting symbol = S

**Productions:**

$S \rightarrow A \text{ point } B \quad S \rightarrow A \text{ exponent } A \quad S \rightarrow A \text{ point } B \text{ exponent } A$

$A \rightarrow -\text{integers} \quad A \rightarrow +\text{integers} \quad A \rightarrow 0$

$B \rightarrow +\text{integers} \quad B \rightarrow 0$

This above grammar is capable of producing a float, or exponential number that is either negative or positive and is raised to some negative or positive power. Note, it is not possible to create a number such as 9.-12 since a negative sign is not allowed past the point.

**Problem 28:**

Each grammar is proposed as generating the set  $L$  of strings over  $\{a, b\}$  that contain equal numbers of  $a$ 's and  $b$ 's. If the grammar generates  $L$ , prove that it does so. If the grammar does not generate  $L$ , give a counterexample and prove that your counterexample is correct. In each grammar the starting symbol is  $S$ .

$$S \rightarrow abS \mid baS \mid aSb \mid bSa \mid \lambda$$

This grammar is only capable of producing a set of strings that contain an equal number of  $a$ 's and  $b$ 's. This is because for every substitution of  $S$  one " $a$ " and one " $b$ " are introduced. Never is a letter added by itself. Thus, the string is always balanced.

**Conclusion**

This assignment was a little bit confusing. Primarily because I was slightly confused on how to write such grammars. It would've been helpful in class to do more examples closer to that which we see in the book. Otherwise, I found this homework to be interesting. The idea of grammars defined by certain logical rules intrigues me. Looking forward to our next and last assignment!