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CISP - 440

Assignment 5

10/18/2018

Part 0 - Relations.

Description:

The goal for this assignment was to perform a multitude of problems related to relations. In these problems we were to practice our skills in relation operations and perform the operations on the given sets of relations.

Problem 2:

Write the relation as a set of ordered pairs.

2.

a	3
b	1
b	4
c	1

This operation results in the following answer.

R = {(a, 3) (b, 1) (b, 4) (c, 1)};

Problem 6:

Write the relation as a table.

R = {(Roger, Music) (Pat, History) (Ben, Math) (Pat, PolySci)};

This operation results in the following table.

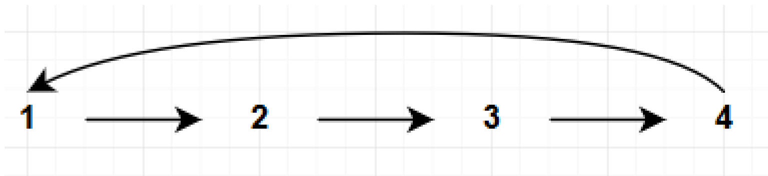
Roger	**Music**
Pat	**History**
Ben	**Math**
Pat	**PolySci**

Problem 11:

Draw the digraph of the relation.

R = {(1, 2) (2, 3) (3, 4) (4, 1)} on {1, 2, 3, 4}

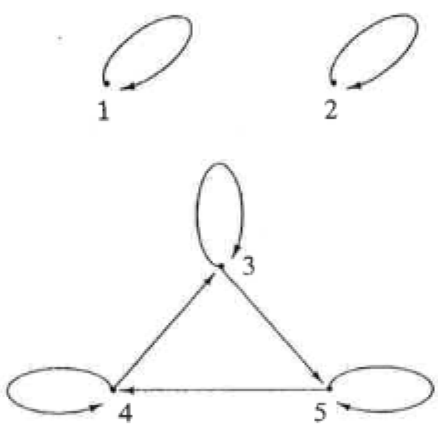
This operation would result in the following digraph.



Problem 14:

Write the relation as a set of ordered pairs.

14.



This digraph would result in the following set of ordered pairs.

R = {(1, 1) (2, 2) (3, 3) (4, 4) (5, 5) (4,3) (3, 5) (5, 4)};

Problem 18(3):

Find the inverse (as a set of ordered pairs) of the relation in exercise 3.

The relation in exercise 3 is as follows.

3.

Sally	Math
Ruth	Physics
Sam	Econ

The non-inverse relation in ordered pairs form would be as follows.

R = {(Sally, Math) (Ruth, Physics) (Sam, Econ)}

Thus the inverse would be as follows.

R-inverse = {(Math, Sally) (Physics, Ruth) (Econ, Sam)}

Problem 27:

Is the relation of Exercise 25 reflexive, symmetric, transitive, antisymmetric, and/or a partial order.

The relation in exercise 25 is as follows, $(X, Y) \in R$ if $X + Y <= 6$

R = {(1, 1) (2, 2) (3, 3) (1, 2) (1, 3) (1, 4) (1, 5) (2, 1) (2, 3) (2, 4) (3, 1) (3, 2) (4, 1) (4, 2) (5, 1)};

This relation has the following properties.

Symmetric and Transitive

Problem 33:

Determine whether each relation defined on the set of positive integer is reflexive, symmetric, antisymmetric, transitive, and/or a partial order.

The relation is as follows $(x, y) \in R$ if 3 divides $x - y$.

This relation has the following properties. (as long as negative divisions are allowed)

Symmetric and Transitive

Problem 39:

Give an example of a relation on {1, 2, 3, 4} that is reflexive, not symmetric or transitive.

The following relation would fulfill the above criteria.

R = {(1, 1) (2, 2) (3, 3) (4, 4) (1, 4)}

Part 1 - Equivalence Relations.

Description:

The goal for this section of the assignment was to perform a multitude of problems related to equivalence relations. In these problems we were to practice our skills in relation operations and perform the operations on the given sets of relations.

Problem 2:

Determine whether the following relation is an equivalence relation on {1, 2, 3, 4, 5}. If the relation is, list the equivalence classes.

R = {(1, 1) (2, 2) (3, 3) (4, 4) (5, 5) (1, 3) (3, 1) (3, 4) (4, 3)}

The following relation is symmetric, reflexive, but **not transitive**. Thus it cannot be an equivalence relation. Need a (4, 1) & (1, 4) to make it an ER.

Problem 6:

Determine whether the following relation is an equivalence relation on {1, 2, 3, 4, 5}. If the relation is, list the equivalence classes.

R = {(x, y) | 4 divides x - y}

Since this relation is **not reflexive** it cannot be an equivalence relation.

Problem 8:

Determine whether the following relation is an equivalence relation on {1, 2, 3, 4, 5}. If the relation is, list the equivalence classes.

R = {(x, y) | x divides 2 - y}

Since this relation is **not reflexive** it cannot be an equivalence relation.

Problem 10:

Determine whether the given relation is an equivalence relation on the set of all people.

R = {(x, y) | x and y have, at some time, lived in the same country}

If X and X have lived in the same country then X and X have lived in the same country. Thus this is reflexive.

If X and Y have lived in the same country then Y and X have lived in the same country. Thus this is symmetric.

If X and Y have lived in the same country and Y and Z have lived in the same country then X and Z have lived in the same country. Thus it is transitive.

Thus this relation is an Equivalence Relation

Problem 14:

Determine whether the given relation is an equivalence relation on the set of all people.

R = {(x, y) | x and y have the same hair color}

If X and X have the same hair color then X and X have the same hair color. Thus this is reflexive.

If X and Y have the same hair color then Y and X have the same hair color. Thus this is symmetric.

If X and Y have the same hair color and Y and Z have the same hair color then X and Z have the same hair color. Thus it is transitive.

Thus this relation is an Equivalence Relation

Problem 17:

List the members of the equivalence relations on {1, 2, 3, 4} by the given partition. Also find the equivalence classes [1], [2], [3], and [4].

ER = {{1}, {2}, {3}, {4}}

This would represent the following relation.

R = {(1, 1) (2, 2) (3, 3) (4, 4)}

This relation represents the following equivalence classes.
[1] = { 1 }

[2] = { 2 }
[3] = { 3 }
[4] = { 4 }

Problem 20:

List the members of the equivalence relations on {1, 2, 3, 4} by the given partition. Also find the equivalence classes [1], [2], [3], and [4].

ER = {{1}, {2, 4}, {3}}

This would represent the following relation.

R = {(1, 1) (2, 2) (3, 3) (4, 4) (2, 4) (4, 2)}

This relation represents the following equivalence classes.
[1] = { 1 }
[2] = { 2 , 4 }
[3] = { 3 }
[4] = { 4, 2 }

Problem 21:

Let X = {1, 2, 3, 4, 5}, Y = {3, 4} and C = {1, 3}. Define the relation R on $\mathcal{P}$(X), the set of all subsets of X, as ArB if and only if A union Y = B union Y.

Since relation A and B must equal each other when united with Y we know this is **reflexive** since for example, (1, 1) would be the same if united separately to Y. Resulting in { 1, 3, 4} in both cases.

Since elements that already exist in Y are able to pass the test the relation would include (3, 4) and (4, 3)... This results in **symmetry**.

Since only elements like those stated above can pass the resulting relation would be **transitive**.

Thus, this relation would be an **Equivalence Relation**

Problem 28:

By listing ordered pairs, give an example of an equivalence relation on {1, 2, 3, 4, 5, 6} having exactly four equivalence classes.

The following relation would fulfill the above requirements.

R = {(1, 1) (2, 2) (3, 3) (4, 4) (5, 5) (6, 6) (1, 2) (2, 1) (5, 6) (6, 5)}

Problem 29:

How many equivalence relations are there on the set {1, 2, 3}

The possible relations are as follows…
{ {1}, {2}, {3} }
{ {1, 2, 3} }
{ {1, 2}, {3} }
{ {1, 3}, {2} }
{ {2, 3}, {1} }

Thus, there are 5 possible relations.

Part 2 - Matrices of Relations.

Description:

The goal for this section of the assignment was to perform a multitude of problems related to matrices and relations. In these problems we were to practice our skills in relation and matrix operations and perform the operations on the given sets of matrices and/or relations.

Problem 3:

Find the matrix of the relations from X to Y relative to the ordering given.

R = {(x, a) (x, c) (y, a) (y, b) (z, d)}

Ordering of X: x, y, z
Ordering of Y: a, b, c, d

Matrix of Relations

	a	**b**	**c**	**d**
x	1	0	1	0
y	1	1	0	0
z	0	0	0	1

Problem 5:

Find the matrix of relation R on X relative to the ordering given.

R = {(1, 2) (2, 3) (3, 4) (4, 5)}

Ordering of X: 5, 3, 1, 2, 4

Matrix of Relations

	5	**3**	**1**	**2**	**4**
5	0	0	0	0	0
3	0	0	0	0	1
1	0	0	0	1	0
2	0	1	0	0	0
4	1	0	0	0	0

Problem 9:

Write the relation R, given by the matrix as a set of ordered pairs.

9.
$$\begin{array}{cc} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \end{matrix} & \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{pmatrix} \end{array}$$

The relation is as follows.

R = {(1, 1) (1, 3) (2, 2) (2, 3) (2, 4)}

Problem 12:

Tell whether the relation of exercise 10 is reflexive, symmetric, transitive, antisymmetric, a partial order, and/or an equivalence relation.

The matrix is as follows.

10.
$$\begin{array}{c|cccc} & w & x & y & z \\ \hline w & & & & \\ x & & & & \\ y & & & & \\ z & & & & \end{array} \quad \begin{array}{c} \\ w \\ x \\ y \\ z \end{array}\begin{array}{c} w \quad x \quad y \quad z \\ \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$

This relation is **Symmetric**, **Transitive**.

Problem 17:

Find the matrix A1 of the relation R1 (relative to the given orderings).

R1 = {(x, y) | x divides y}
R1 is from X to Y
X and Y: 2, 3, 4, 5

	2	**3**	**4**	**5**
2	1	0	0	0
3	0	1	0	0
4	0	0	1	0
5	0	0	0	1

Find the matrix A2 of the relation R2 (relative to the given orderings).

R2 = {(y, z) | y = z + 1}
R2 is from Y to Z
Z: 1, 2, 3, 4

	1	**2**	**3**	**4**
2	1	0	0	0

3	0	1	0	0
4	0	0	1	0
5	0	0	0	1

Find the matrix product of A1 and A2

Resulting product

	1	**2**	**3**	**4**
2	1	0	0	0
3	0	1	0	0
4	0	0	1	0
5	0	0	0	1

Use the result to find the matrix of the relation R2 o R1

Matrix of Relation

	1	**2**	**3**	**4**
1	1	0	0	0
2	0	1	0	0
3	0	0	1	0
4	0	0	0	1

Use the result to find the relation as a set of ordered pairs.

R = {(1, 1) (2, 2) (3, 3) (4, 4)

Conclusion

This assignment was quick and painless. I mostly encountered difficulty from problem 17 in section 2. Some of my answers I am unsure about. However, I believe I was/am on the right track. Looking forward to the implementation of this subject!