<u>CISP 440 - Final Exam Guide</u>

Conversion Between Bases:

To convert between any base we can use the mod div algorithm to convert to any base from base 10. The algorithm is as follows. Divide original number by desired base. Mod original number by desired base. Divide the quotient by the desired base. Mod the quotient by the desired base… The answer is the remainders read backwards.

So 7 in **two's complement** notation is 00000111, just as it is as an unsigned integer. And -7 in **two's complement** notation is 11111001. So, yes, **positive** integers in **two's complement** notation are represented the same way they are with unsigned integers (assuming it is a valid integer for the **number** of bits being used)

<u>Example: Convert 365 to base 16</u>

365 / 16 = 22
365 % 16 = **<u>13 = D in Hex</u>**

22 / 16 = 1
22 % 16 = **<u>6</u>**

1 / 16 = 0
1 % 16 = **<u>1</u>**

Thus the answer is: **16D**

For negative numbers you must convert using the 2’s complement. To do this take the number in its positive form and convert binary. Then perform the 2’s complement operation by inverting all of the bits and adding 1.

<u>Example: Convert -10 to binary</u>

10 = 1010 in binary.

0000 1010 = 1111 0101 (invert) = 1111 0110 (add one)

<u>Example: Convert 12 to binary 2’s complement</u>

12 = 0000 1100 (done)

Example: Convert a negative 8 bit binary number to a decimal number

1111 0101 = 0000 1010 (invert) = 0000 1011 (add one) = -11

Example: Convert a positive 8 bit binary number to a decimal number

0010 1111 = 47 (done)

Overflow is detected in UNSIGNED arithmetic operations when a carry results in the resulting number being larger than the word type.

Overflow is detected in SIGNED arithmetic operations when a carry is larger than the word or the sign bit is flipped to a false value.

Binary Float Operations:

Floats are stored in binary in three parts. One being the sign, the other the mantissa, and lastly the exponent. The mantissa is the part of the number after the decimal point and the exponent is the power it is raised to with a certain excess.

1	0.5	0.25	0.125	.0625	.03125	...	...	...	...
2^0	2^-1	2^-2	2^-3	2^-4	2^-5	2^-6	2^-7	2^-8	2^-9
0.	1	1	1	1	1	1	1	1	1

Convert the given binary float to its decimal form in excess 4

Sign	Mantissa	Exponent
1	0110	110

The resulting number in binary is: 1.0110 * 2^2 which equals 101.10 which equals -5.5

Convert 12.125 to floating point in excess 4

12 = 1100

0.125 = 0.01

1100.01 = 12.125

1.10001 * 2^-3 = 12.125

Sign	Mantissa	Exponent
0	1000	111

Set Operations:

Many operations can be performed on sets. These vastly vary and can include many different operations. Bur in brief the operations that can be performed are as follows.

- **Complement - A’:** This represents all items in the universe which are not in the set.
- **Union - ∪:** Contains all elements of both sets. Dupes don’t matter.
- **Intersection - ∩:** Contains all the common elements of both sets.
- **Cardinality - |A|:** Contains the number of elements in the set.
- **Difference A - B:** Removes all the common elements from the leading set. A-B != B-A

The max number of subsets of a given set is 2 raised to the cardinality of the set in question. For example, if a set has 3 elements the max number of subsets would equal 2^3 = 8.

The Powerset is merely the set of all subsets. For example say {a, b} is a set with 4 possible subsets. Thus the powerset = { (), (a), (b), (a,b)}.

Cartesian product is a set of all possible combinations of size 2 elements.
Cartesian product of two sets is as follows. Say set A = {1, 2} and set B = {3, 4}....

AxB = {(1, 3), (1, 4), (2, 3), (2, 4)};

BxA = {(3, 1), (3, 2), (4, 1), (4, 2)};

Relations:

A relation is essentially a set with a few unique properties. For example, a relation is a subset of some cartesian product.

A relation is REFLEXIVE if all elements are comparable (related) to themselves. For example, For every A, B there must be an AA and a BB.

A relation is SYMMETRIC if for every (a,b) there is a (b, a) in the relation.

A relation is TRANSITIVE whenever (a,b) and (b, c) there is a (a, c).

A relation is ANTISYMMETRIC if for every (a,b) there is NO (b, a) in the relation.

If a relation is Reflexive, Symmetric, and Transitive it is considered an equivalence relation that has equivalence classes.
A relation is a Partial Order Relation if it is Reflexive, Antisymmetric, and Transitive.

Equivalence classes are sets that an Equivalence Relation can be split into. These sets that are not linked together form the equivalence classes.

A relation can be put into a matrix via the following method. For example let's say…

A = {(A, A), (B, B) (A, B), (B, A)};

The matrix would be as follows. (Domain = X Range = Y)

	A	**B**
A	1	1
B	1	1

To determine reflexivity in a matrix just make sure the diagonal from top right to bottom left is filled.

To determine symmetry just verify that for every (A, B) there is a (B, A).

To determine transitivity just square the matrix. If it is non-zero in the original matrix, it must also be non-zero in the resultant matrix. If so, it is transitive. Note, when two matrices of unequal height and width are multiplied the resultant will be the smallest height and width large.

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \times \begin{bmatrix} j & k & l \\ m & n & o \\ p & q & r \end{bmatrix} =$$

$$\begin{bmatrix} aj + bm + cp & ak + bn + cq & al + bo + cr \\ dj + em + fp & dk + en + fq & dl + eo + fr \\ gj + hm + ip & gk + hn + iq & gl + ho + ir \end{bmatrix}$$

Want to see another example? Here it is for the 1st row and **2nd column**:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \times \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix} = \begin{bmatrix} 58 & 64 \\ & \end{bmatrix}$$

$$(1, 2, 3) \bullet (8, 10, 12) = 1\times8 + 2\times10 + 3\times12$$
$$= 64$$

We can do the same thing for the **2nd row** and **1st column**:

$$(4, 5, 6) \bullet (7, 9, 11) = 4\times7 + 5\times9 + 6\times11$$
$$= 139$$

And for the **2nd row** and **2nd column**:

$$(4, 5, 6) \bullet (8, 10, 12) = 4\times8 + 5\times10 + 6\times12$$
$$= 154$$

And we get:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \times \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix} = \begin{bmatrix} 58 & 64 \\ 139 & 154 \end{bmatrix} \checkmark$$

DONE!

The composition of a relation is as follows: Do the first one then go to the second for the output. For example R1 = {(1, 1), (2,2)}; R2 = {(1,4), (2,3)} R1 o R2 would be…

R1oR2 = {(1, 4), (2, 3)};

Functions:

A function is a relation with certain properties. If a function has other certain properties. The inverse is also a function.

A function must be robust meaning it can handle the entire domain (all x values). Every element of set A occurs as a first element of some pair (a,).

A function must be deterministic, meaning you get the same answer every time. No element of set A occurs more than once.

A discrete function is where the points on the graph are points and not connected. Each with their own values. A continuous function is over an interval of time and is connected.

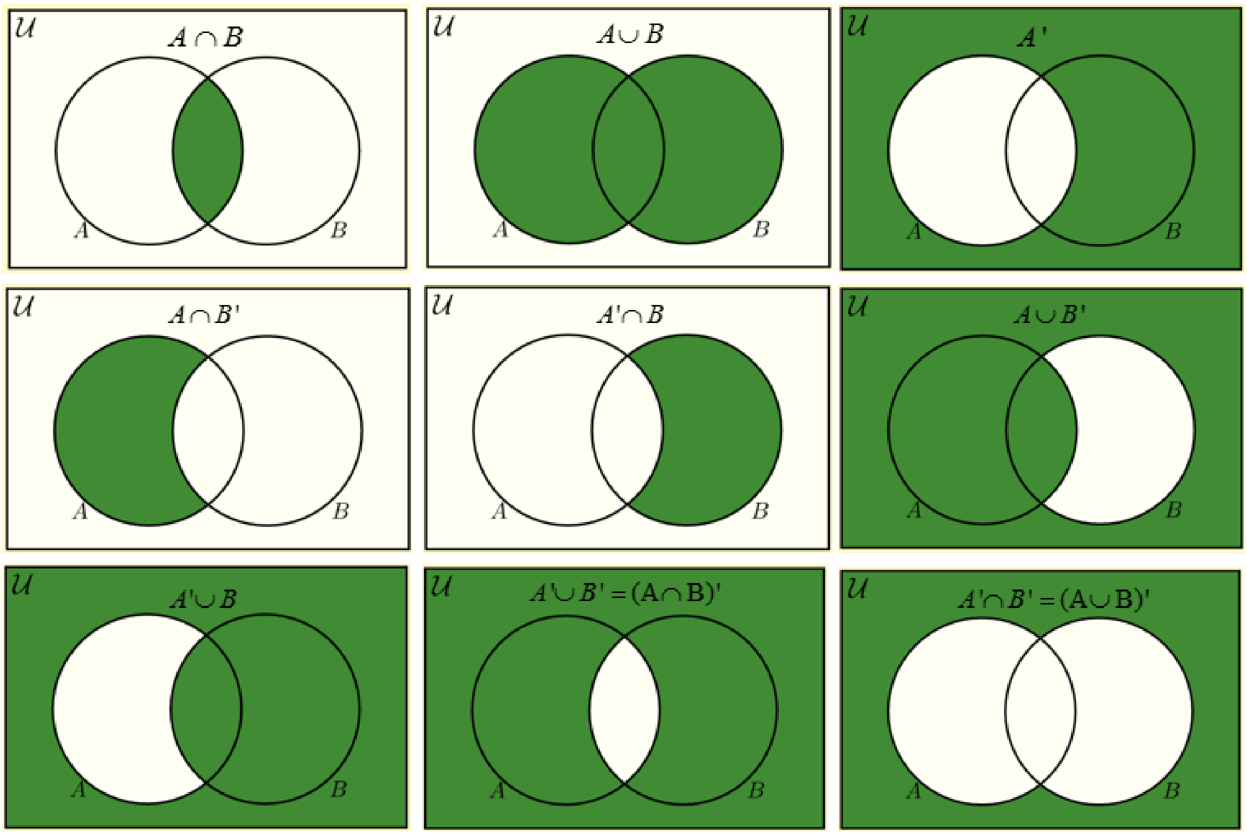
<u>If a function is 1:1 and Onto then its inverse is also a function.</u>

A = (1, 2, 3, 4) F: A->A

F = ({1,1), (2,4), (3, 2), (4, 1)} Not 1 to 1 since two numbers head to 1

F = ({1,1), (2,4), (3, 2), (4, 1)} Not onto since 1, 4, and 2 do not cover the entire range of 1-4.

The composition of two functions is for all intents and purposes the same process as the composition of two relations. Since a function is a relation that only demonstrates certain properties.



𝒰
A ∩ B
A
B
𝒰
A ∪ B
A
B
𝒰
A'
A
B
𝒰
A ∩ B'
A
B
𝒰
A' ∩ B
A
B
𝒰
A ∪ B'
A
B
𝒰
A' ∪ B
A
B
𝒰
A' ∪ B' = (A ∩ B)'
A
B
𝒰
A' ∩ B' = (A ∪ B)'
A
B