Lecture 12
Fuzzy Logic and Expert Systems

CS 180 – Intelligent Systems

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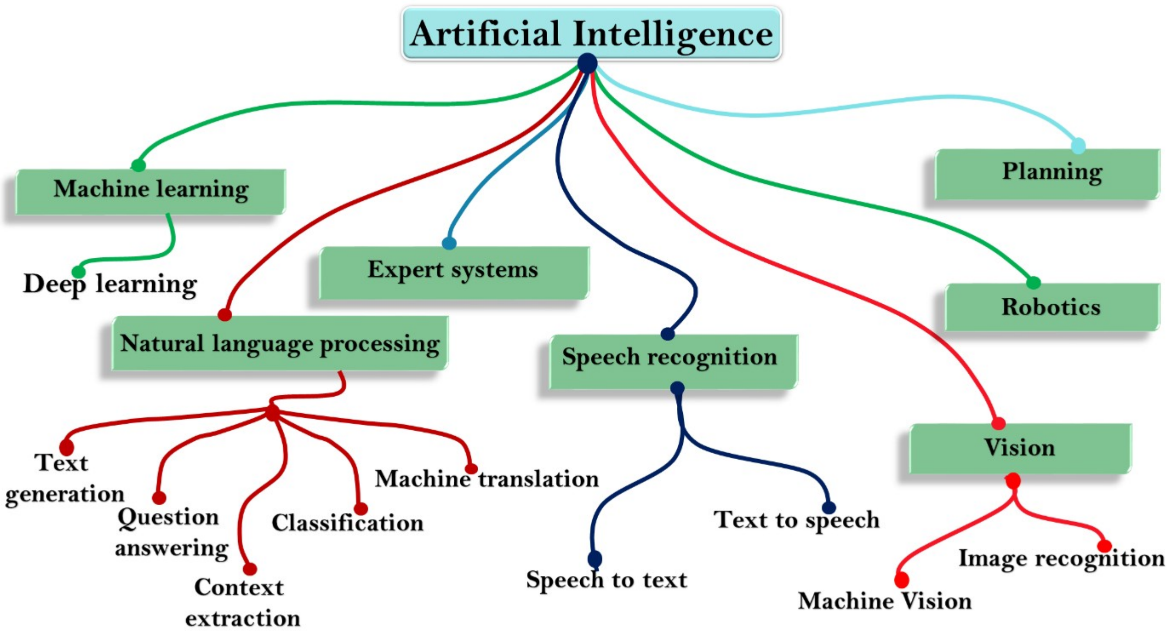
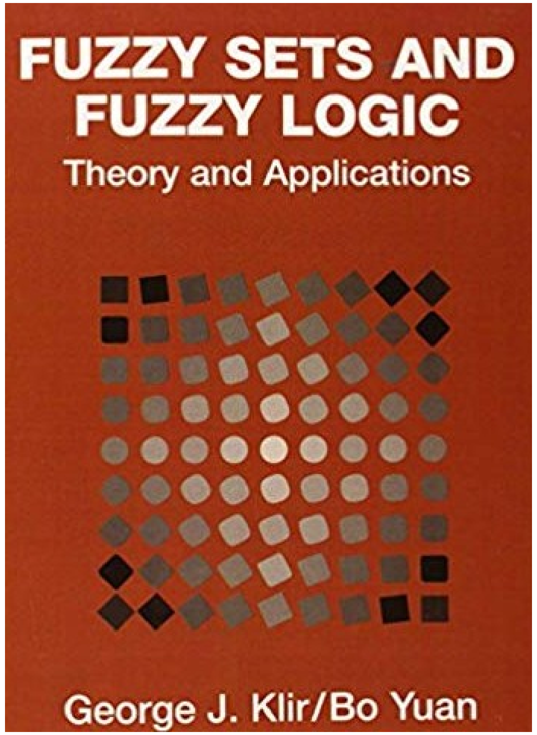
Three AI areas

- Knowledge Learning
  - Machine learning/Neural networks
    - **No closed-form** math expression
- Optimization
  - Search
  - Adversarial search
  - Genetic algorithms
- Knowledge Reasoning
  - Bayesian inference
  - Fuzzy logic



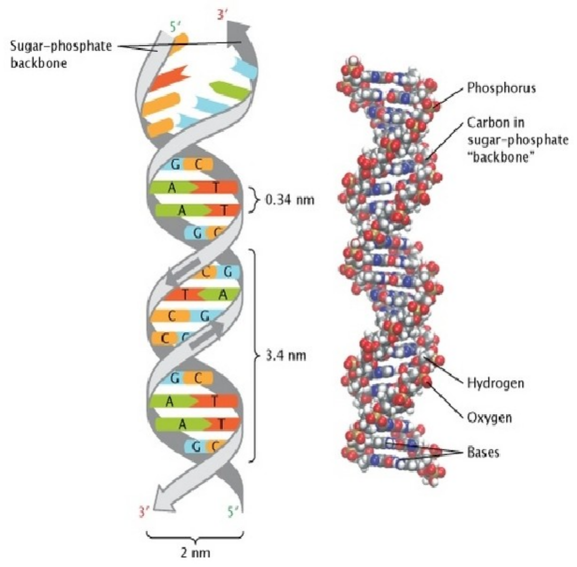
What is a FUZZY LOGIC?

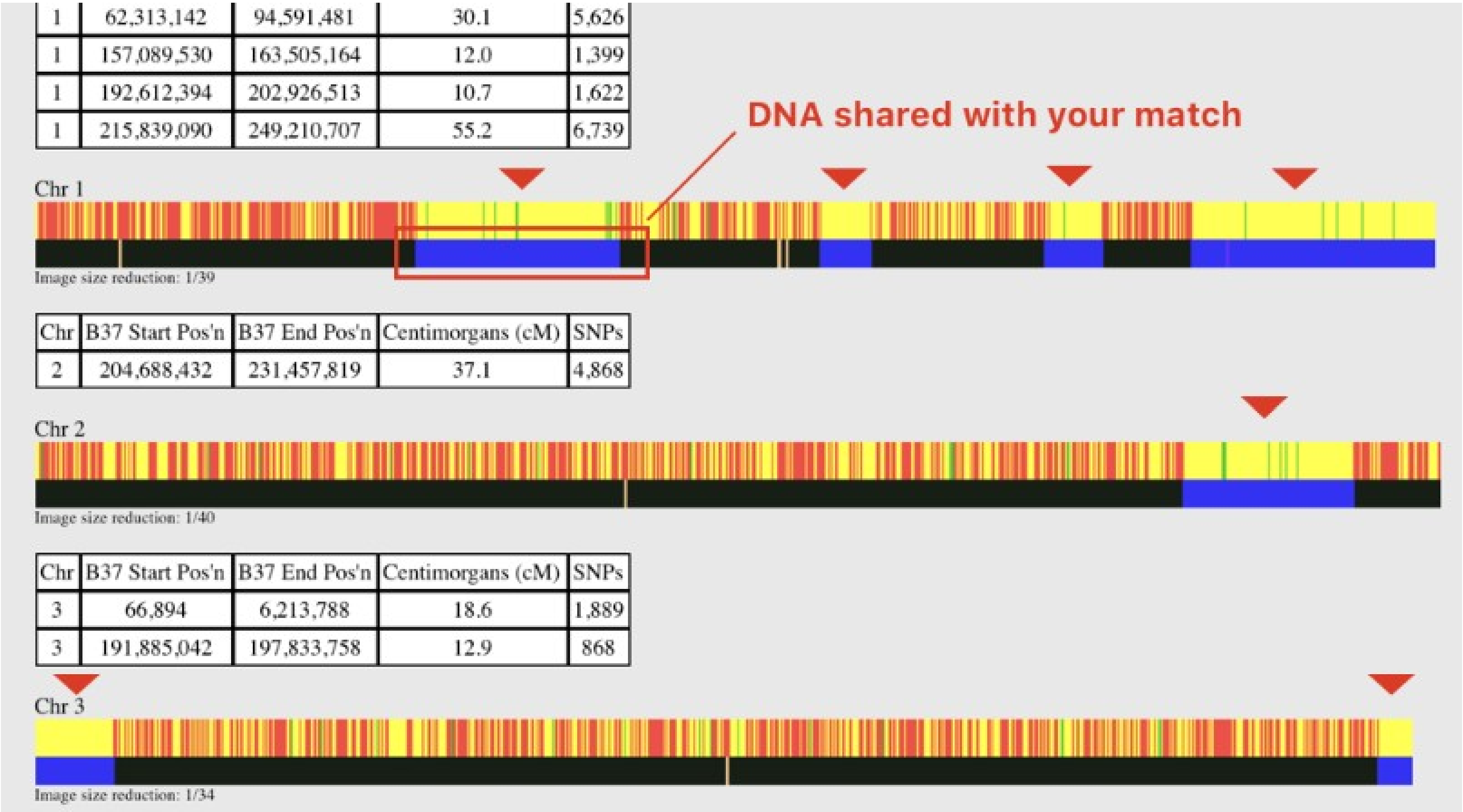
Fuzzy Logic is an approach to computing based on "degrees of truth" rather than the usual "true or false" (1 or 0) boolean (binary) logic, on which the modern computers are based.



DNA match/search using fuzzy Logic

To find approximate matches for a search key





1	62,313,142	94,591,481	30.1	5,626
1	157,089,530	163,505,164	12.0	1,399
1	192,612,394	202,926,513	10.7	1,622
1	215,839,090	249,210,707	55.2	6,739

Chr	B37 Start Pos'n	B37 End Pos'n	Centimorgans (cM)	SNPs
2	204,688,432	231,457,819	37.1	4,868

Chr	B37 Start Pos'n	B37 End Pos'n	Centimorgans (cM)	SNPs
3	66,894	6,213,788	18.6	1,889
3	191,885,042	197,833,758	12.9	868

Text match/search using fuzzy Logic

To find approximate matches (slang, misspelling, abbreviation on social media) for a search key

Other fuzzy logic applications

- To match customers to identify the buying behavior
- For customer address matching
- To match file paths
- To detect plagiarism (text re-use)
- For spam filtering

Fuzzy Sets (Fuzzy Logic) vs Crisp Sets (Boolean Logic)

1. Crisp Set ☾ Classical sets which have distinct objects.
   For e.g.,
   A = {apples, oranges, mangoes}
   B = {2, 4, 6, 8}
   (each element has a membership degree of 1)

2. Fuzzy Set ☾ The membership degree can be **0<μ(x) < 1**

$$A = \left\{ \frac{0.4}{train} + \frac{0.6}{car} + \frac{0.7}{cycle} \right\}$$

- where — is a separator, and + is the logic OR
- **train** has a membership degree of **0.4**

- If the membership degree is 0, it means that the element is not associated with set A.
- If the membership degree is 1, it means that the element is strongly associated with

FUZZY THEORY

Boolean Logic (Crisp Sets)

Is Bob honest?

True (1)

False (0)

Membership value

$\mu_A(x) = 1$, iif $x \varepsilon A$

$= 0$, iif $x \not\varepsilon A$

Fuzzy Logic (Fuzzy Sets)

Is Bob honest?

- Very honest **(1)**
- Honest **(0.8)**
- Honest (sometimes) **(0.5)**
- dishonest **(0)**

(membership value/degree)

Membership value

$\mu_A(x) \; \varepsilon \; [0, 1]$

Definition of Fuzzy Sets

- A fuzzy set A is defined by using

  $$A=\{\, x \mid \mu(x) \,\}$$

  where $\mu(x)$ is a **membership function**.

  - **The range of $\mu(x)$ is [0,1]. We cannot say clearly if x is in A or not when $0<\mu(x) < 1$.**

- A fuzzy set A is described as follows:

  $$A = \mu_A(x_1)/x_1 + \mu_A(x_2)/x_2 + \ldots + \mu_A(x_N)/x_N$$

  where / is a separator, and + is the logic OR.

Operations on fuzzy sets

Meaning	Set notation	Membership function
Equivalence	$A = B$	$\mu_A(x) = \mu_B(x)$
Implication	$A \subseteq B$	$\mu_A(x) \leq \mu_B(x)$
Complement (negation)	$\overline{A}$	$\mu_{\overline{A}}(x) = 1 - \mu_A(x)$
Union	$A \cup B$	$\mu_{A \cup B}(x) = \max\{\mu_A(x), \mu_B(x)\}$
Intersection	$A \cap B$	$\mu_{A \cap B}(x) = \min\{\mu_A(x), \mu_B(x)\}$

Operations of fuzzy sets

Meaning	Set notation	Membership function
Difference	$A - B \ or \ (A \cap \overline{B})$	$\mu_{A\text{-}B}(x) = \min\{ \mu_A(x), \neg\mu_B(x)\}$
Sum	$A + B$	$\mu_{A+B}(x) = \mu_A(x) + \mu_B(x) - (\mu_A(x) \cdot \mu_B(x))$
Product	$A \cdot B$	$\mu_{A \cdot B}(x) = \mu_A(x) \cdot \mu_B(x)$
Bounded sum	$A \oplus B$	$\mu_{A \oplus B}(x) = \min\{1, \mu_A(x) + \mu_B(x)\}$
Bounded difference	$A \ominus B$	$\mu_{A \ominus B}(x) = \max\{0, \mu_A(x) - \mu_B(x)\}$

Union

Q1) $$\underset{\sim}{B}_1 = \left\{ \frac{1}{1.0} + \frac{0.75}{1.5} + \frac{0.3}{2.0} + \frac{0.15}{2.5} + \frac{0}{3.0} \right\}$$

$$\underset{\sim}{B}_2 = \left\{ \frac{1}{1.0} + \frac{0.6}{1.5} + \frac{0.2}{2.0} + \frac{0.1}{2.5} + \frac{0}{3.0} \right\}$$

Sol^n:

$$\underset{\sim}{B}_1 \cup \underset{\sim}{B}_2 = \max\left[\mu_A(x), \mu_B(x)\right]$$ UNION

$$\underset{\sim}{B}_1 \cup \underset{\sim}{B}_2 = \left\{ \frac{1}{1.0} + \frac{0.75}{1.5} + \frac{0.3}{2.0} + \frac{0.15}{2.5} + \frac{0}{3.0} \right\}$$

Intersection

Q1)

$$\underset{\sim}{B}_1 = \left\{ \frac{1}{1.0} + \frac{0.15}{1.5} + \frac{0.3}{2.0} + \frac{0.15}{2.5} + \frac{0}{3.0} \right\}$$

$$\underset{\sim}{B}_2 = \left\{ \frac{1}{1.0} + \frac{0.6}{1.5} + \frac{0.2}{2.0} + \frac{0.1}{2.5} + \frac{0}{3.0} \right\}$$

Sol^n:

INTERSECTION

$$\underset{\sim}{B}_1 \cap \underset{\sim}{B}_2 = \min\left[\mu_A(x), \mu_B(x)\right]$$

$$\underset{\sim}{B}_1 \cap \underset{\sim}{B}_2 = \left\{ \frac{1}{1.0} + \frac{0.15}{1.5} + \frac{0.2}{2.0} + \frac{0.1}{2.5} + \frac{0}{3.0} \right\}$$

Complement

Q1)

$$\underset{\sim}{B_1} = \left\{ \frac{1}{1.0} + \frac{0.75}{1.5} + \frac{0.3}{2.0} + \frac{0.15}{2.5} + \frac{0}{3.0} \right\}$$

$$\underset{\sim}{B_2} = \left\{ \frac{1}{1.0} + \frac{0.6}{1.5} + \frac{0.2}{2.0} + \frac{0.1}{2.5} + \frac{0}{3.0} \right\}$$

Sol^n: COMPLIMENT

$$\overline{\underset{\sim}{B_1}} = \boxed{1 - \mu_B(x)}$$

$$= \left\{ \frac{0}{1.0} + \frac{0.85}{1.5} + \frac{0.7}{2.0} + \frac{0.85}{2.5} + \frac{1}{3.0} \right\}$$

Difference

Q1)

$$\underset{\sim}{B_1} = \left\{ \frac{1}{1.0} + \frac{0.15}{1.5} + \frac{0.3}{2.0} + \frac{0.15}{2.5} + \frac{0}{3.0} \right\}$$

$$\underset{\sim}{B_2} = \left\{ \frac{1}{1.0} + \frac{0.6}{1.5} + \frac{0.2}{2.0} + \frac{0.1}{2.5} + \frac{0}{3.0} \right\}$$

Sol^n: DIFFERENCE

$$\boxed{\underset{\sim}{B_1} \mid \underset{\sim}{B_2} = \underset{\sim}{B_1} \cap \overline{\underset{\sim}{B_2}}} \qquad \boxed{\underset{\sim}{B_2} \mid \underset{\sim}{B_1} = \underset{\sim}{B_2} \cap \overline{\underset{\sim}{B_1}}}$$

$$\overline{\underset{\sim}{B_2}} = \left\{ \frac{0}{1.0} + \frac{0.4}{1.5} + \frac{0.8}{2.0} \quad \frac{0.9}{2.5} \quad \frac{1}{3.0} \right\}$$

$$\underset{\sim}{B_1} \mid \underset{\sim}{B_2} = \left\{ \frac{0}{1.0} + \frac{0.15}{1.5} + \frac{0.3}{2.0} + \frac{0.15}{2.5} + \frac{0}{3.0} \right\}$$

Sum

Q1) $\underset{\sim}{A} = \left\{ \frac{1}{1} + \frac{0.5}{1.5} + \frac{0.3}{2.0} + \frac{0.4}{2.5} \right\}$

$\underset{\sim}{B} = \left\{ \frac{0.4}{1} + \frac{0.2}{1.5} + \frac{0.7}{2.0} + \frac{0.1}{2.5} \right\}$

Sol^n: Algebraic Sum

$$\mu_{A+B}(x) = \left[\mu_A(x) + \mu_B(x)\right] - \left[\mu_A(x) \cdot \mu_B(x)\right]$$

$1.4 - 0.4 = 1$

$0.7 - 0.1 = 0.6$

$1.0 - 0.21 = 0.79$

$0.5 - 0.04 = 0.46$

$$= \left\{ \frac{1}{1} + \frac{0.6}{1.5} + \frac{0.79}{2.0} \quad \frac{0.46}{2.5} \right\}$$

Product

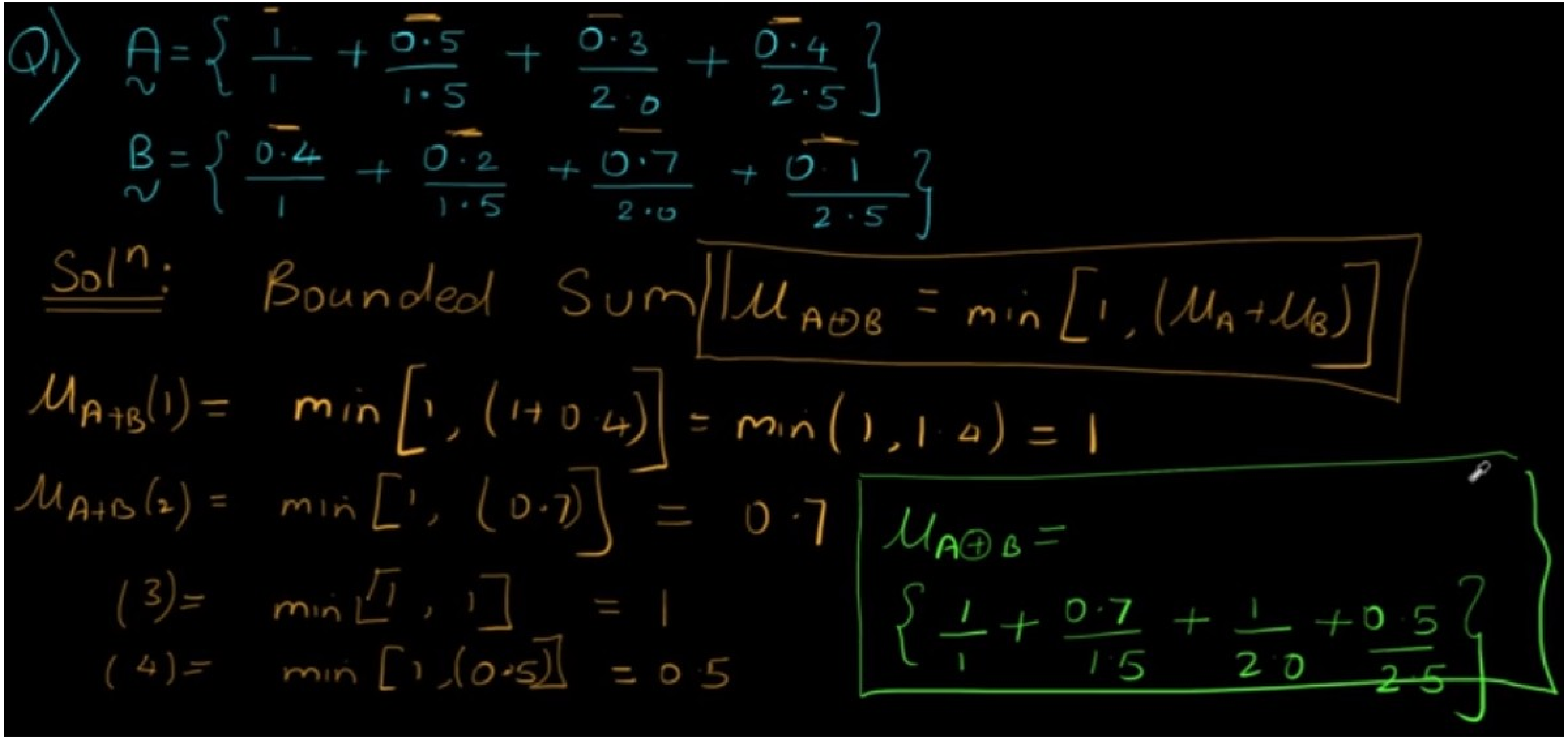
Q1) $\underset{\sim}{A} = \left\{ \frac{1}{1} + \frac{0.5}{1.5} + \frac{0.3}{2.0} + \frac{0.4}{2.5} \right\}$

$\underset{\sim}{B} = \left\{ \frac{0.4}{1} + \frac{0.2}{1.5} + \frac{0.7}{2.0} + \frac{0.1}{2.5} \right\}$

Sol^n: Algebraic Product $-$ $\boxed{\mu_{A.B} = \mu_A(x) \cdot \mu_B(x)}$

$$\mu_{A.B} = \left\{ \frac{0.4}{1} + \frac{0.1}{1.5} + \frac{0.21}{2.0} + \frac{0.04}{2.5} \right\}$$

Bounded Sum


Q1) A = { 1/1 + 0.5/1.5 + 0.3/2.0 + 0.4/2.5 }
B = { 0.4/1 + 0.2/1.5 + 0.7/2.0 + 0.1/2.5 }
Soln: Bounded Sum
μA⊕B = min [1, (μA + μB)]
μA+B(1) = min [1, (1+0.4)] = min (1, 1.4) = 1
μA+B(2) = min [1, (0.7)] = 0.7
(3) = min [1, 1] = 1
(4) = min [1, (0.5)] = 0.5
μA⊕B =
{ 1/1 + 0.7/1.5 + 1/2.0 + 0.5/2.5 }

Bounded Difference

Q1) $\underset{\sim}{A} = \left\{ \frac{1}{1} + \frac{0.5}{1.5} + \frac{0.3}{2.0} + \frac{0.4}{2.5} \right\}$

$\underset{\sim}{B} = \left\{ \frac{0.4}{1} + \frac{0.2}{1.5} + \frac{0.7}{2.0} + \frac{0.1}{2.5} \right\}$

Sol^n: Bounded Difference $\mu_{A \odot B}(x) = \max\left[0, (\mu_A(x) - \mu_B(x))\right]$

$\mu_{A \odot B}(1) = \max[0, 0.6] = 0.6$

$(2) = \max[0, 0.3] = 0.3$

$(3) = \max[0, -0.4] = 0$

$(4) = \max[0, 0.3] = 0.3$

$$\boxed{\mu_{A \odot B} = \left\{ \frac{0.6}{1} + \frac{0.3}{1.5} + \frac{0}{2.0} + \frac{0.3}{2.5} \right\}}$$

Practice

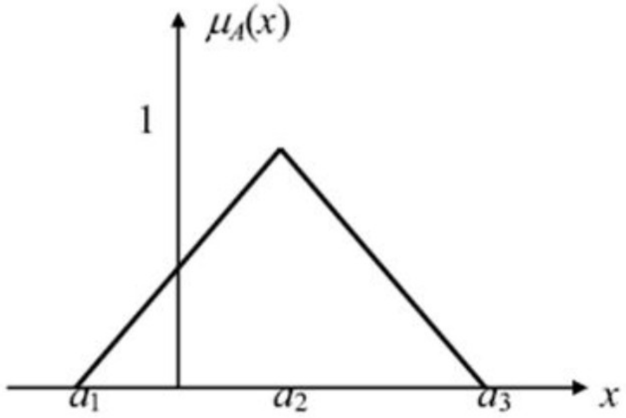
- All the people: **X={Alice, Bob, Charles, Eric, William}**
- Fuzzy sets A="young persons" and B="tall persons" are defined by
  - A = 0.4/Alice + 0.6/Bob + 0.8/Charles +1.0/Eric + 0.9/William
  - B = 0.3/ Alice + 0.5/Bob +0.9/Charles + 0.6/Eric + 1.0/William
- Who are the young or tall persons?
  - A∪B = 0.4/Alice + 0.6/Bob + 0.9/Charles + 1.0/Eric + 1.0/William
- Who are the young and tall persons?
  - A∩B =0.3/ Alice +0.5/Bob +0.8/ Charles + 0.6/ Eric + 0.9/William
- Who are tall and old persons?
  - B - A = B ∩ $\bar{A}$
  - $\bar{A}$ = 0.6/Alice + 0.4/Bob + 0.2/Charles +0/Eric + 0.1/William
  - B - A = 0.3/Alice + 0.4/Bob + 0.2/Charles +0/Eric + 0.1/William

Fuzzy Numbers

- Fuzzy numbers are fuzzy sets defined on real numbers
- Given A = $\mu_A(x_1)/x_1 + \mu_A(x_2)/x_2 + \ldots + \mu_A(x_N)/x_N$, if $x_1, x_2, \ldots x_N$ are real numbers and **$0 < \mu_A(x_i) < 1$**, the A is a fuzzy number.
- On the other hand, if for any x_N , **$\mu(x_N)$** = 0 or 1, then A is a crisp number or crisp set

One example fuzzy number

$$\mu_{(A)}(x)=\begin{cases}0, & x<a_1\\ \dfrac{x-a_1}{a_2-a_1}, & a_1\le x\le a_2\\ \dfrac{a_3-x}{a_3-a_2}, & a_2\le x\le a_3\\ 0, & x>a_3\end{cases}$$

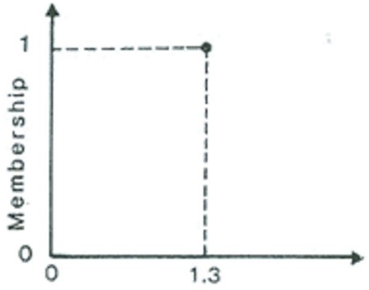


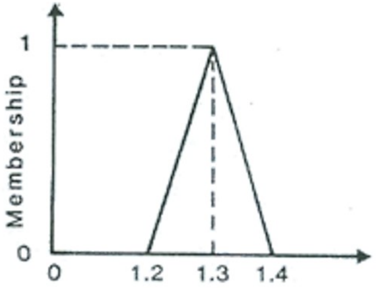
fuzzy numbers vs crisp numbers

Crisp

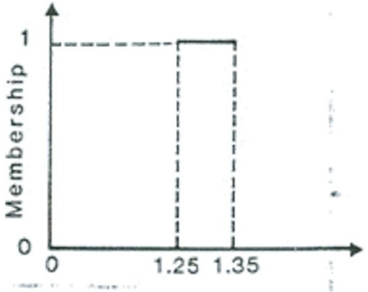
fuzzy

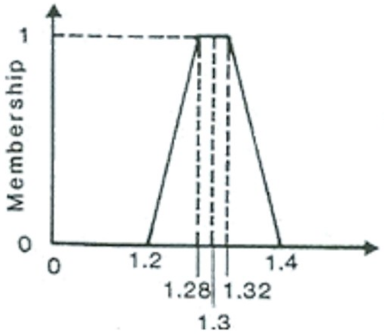
An crisp member





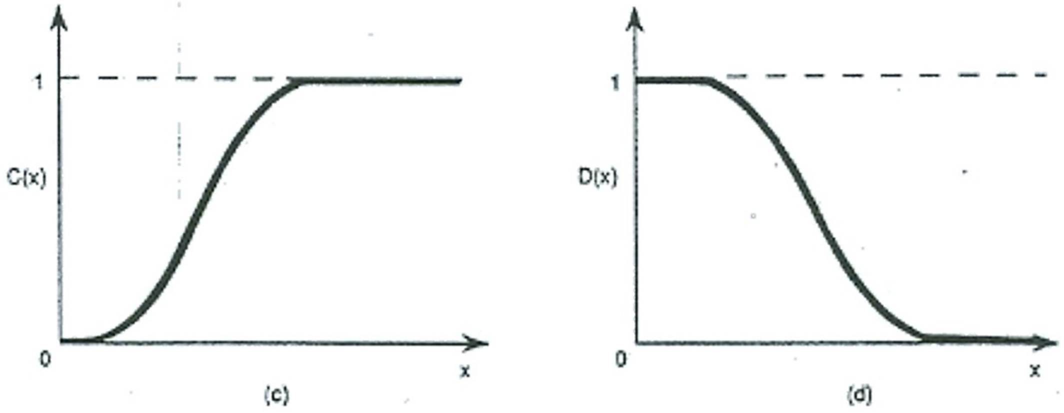
A crisp number set





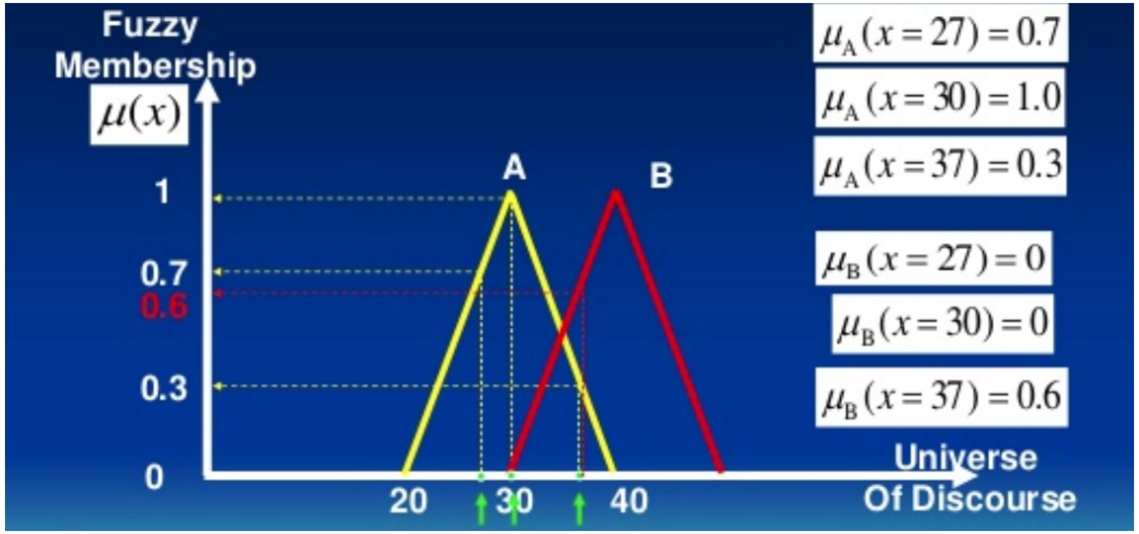
Cases of fuzzy numbers

- Membership functions of fuzzy numbers can be unsymmetric

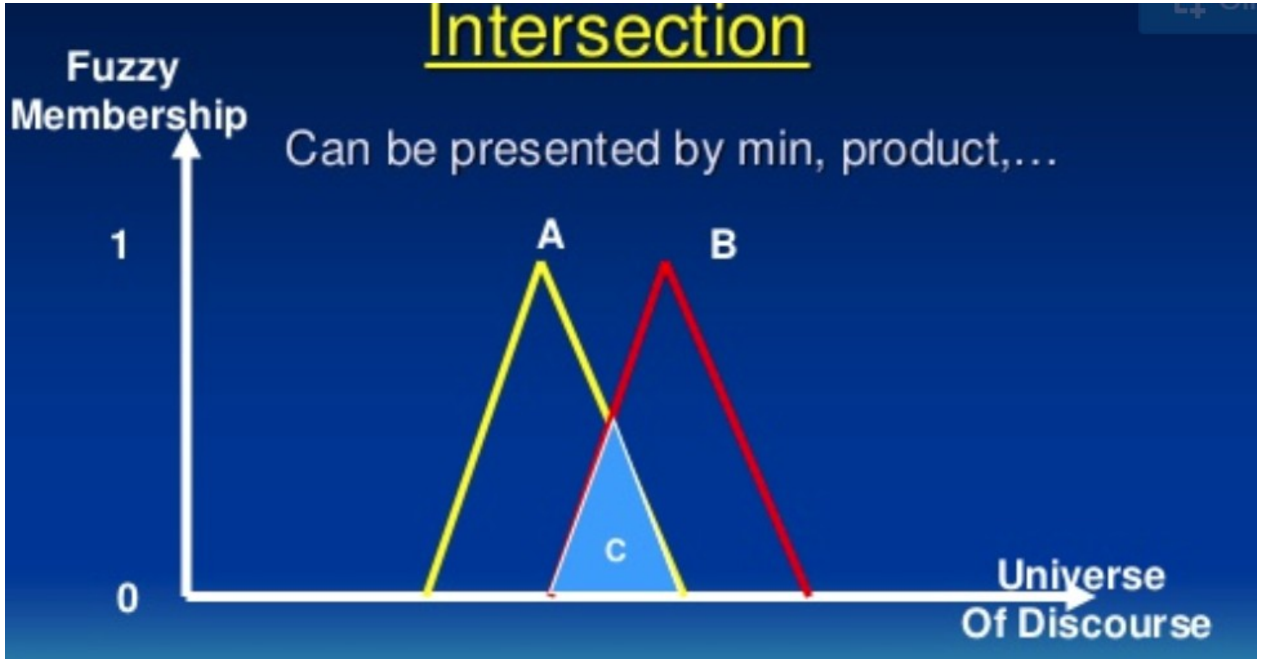


large number small number

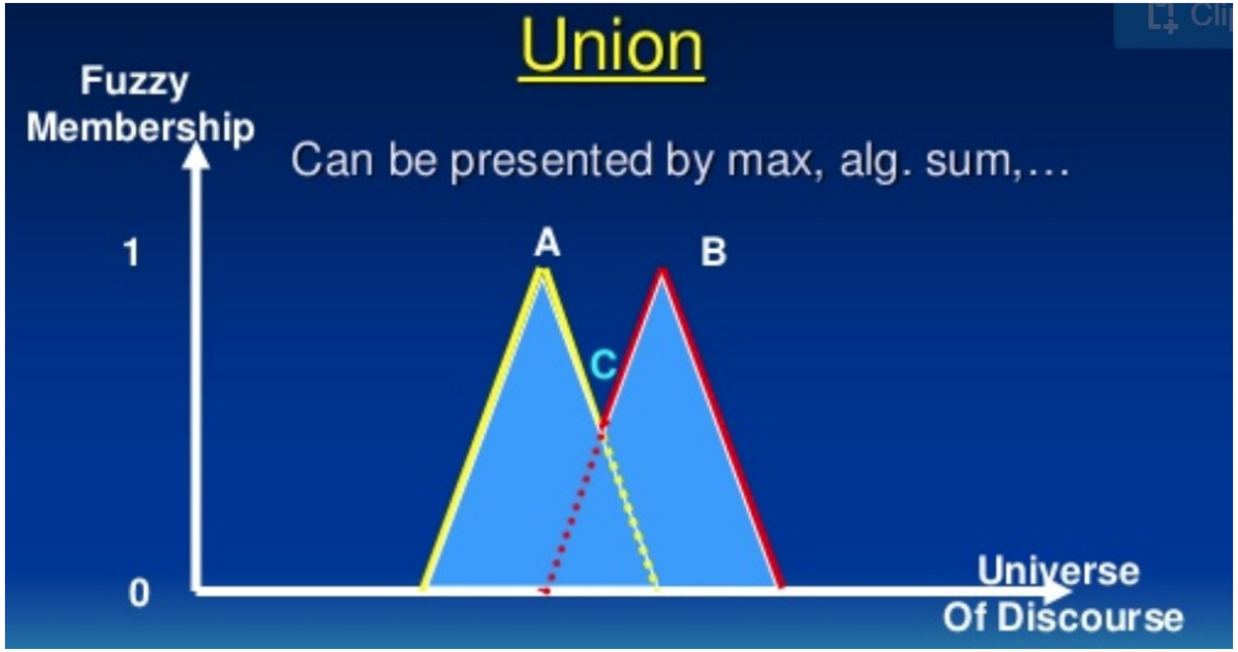
Two fuzzy numbers (each X has a membership value for each fuzzy number)



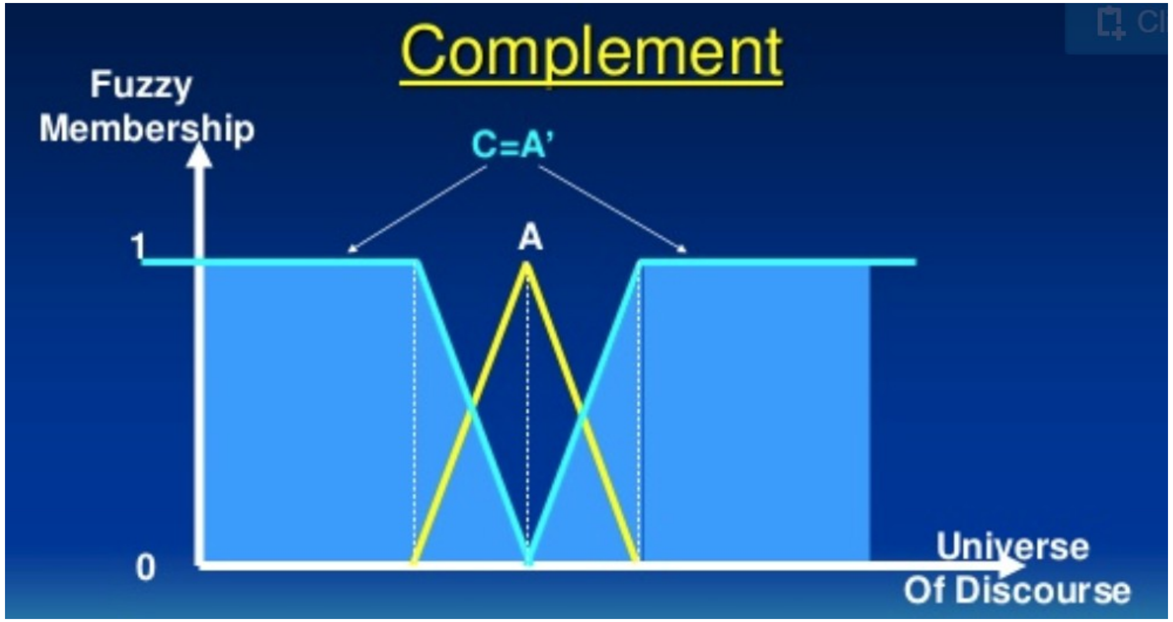
Intersection of fuzzy numbers



Union of fuzzy numbers



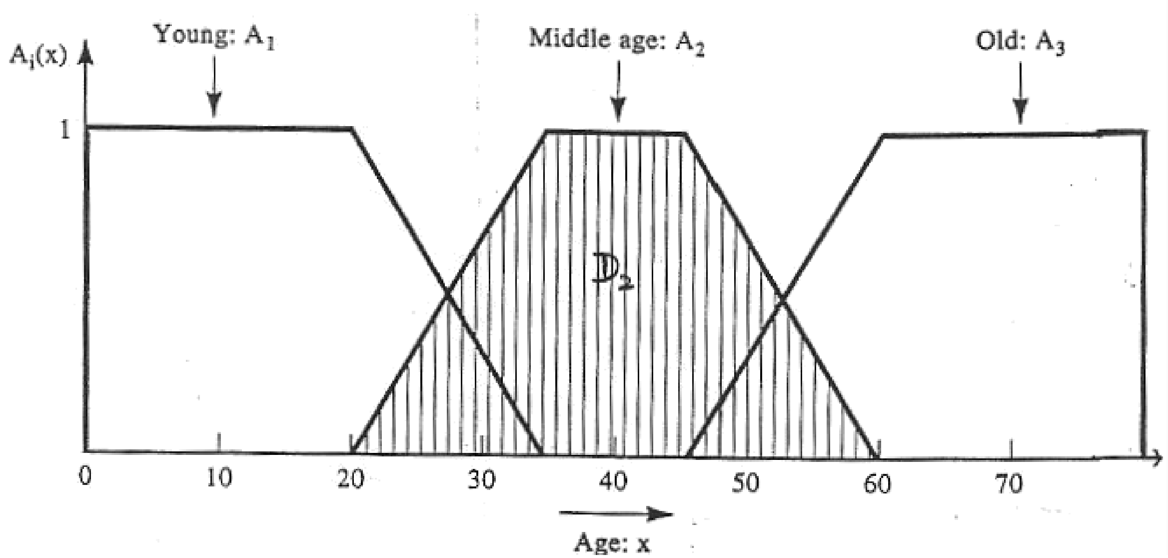
Negation (Complement) of fuzzy number

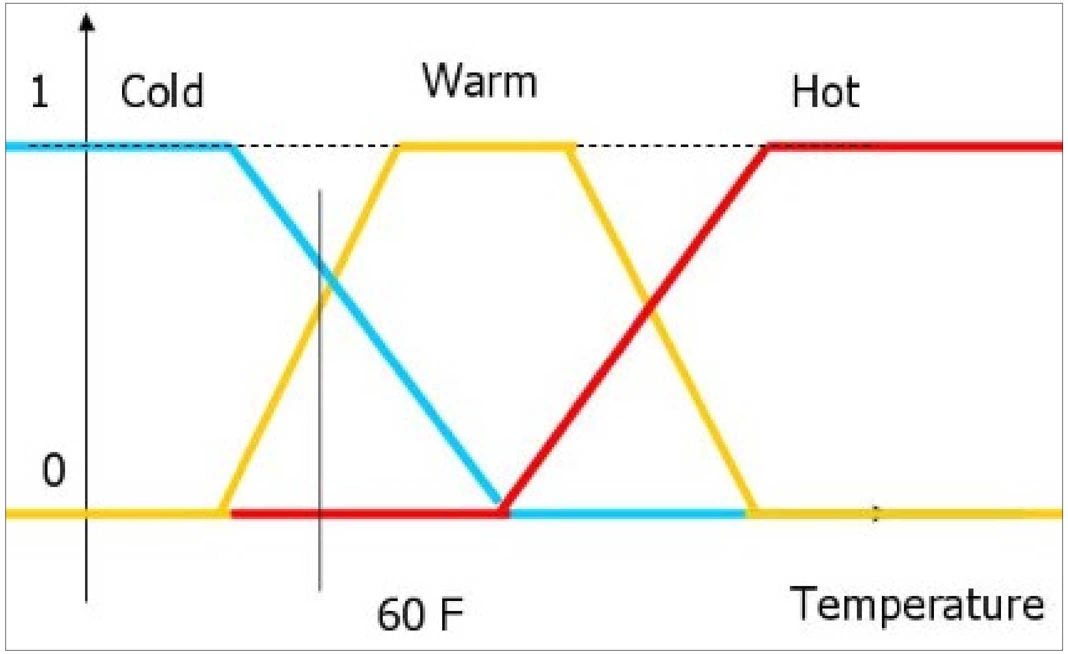


In Artificial Intelligence, the ultimate question is:

MACHINES THINK LIKE HUMAN

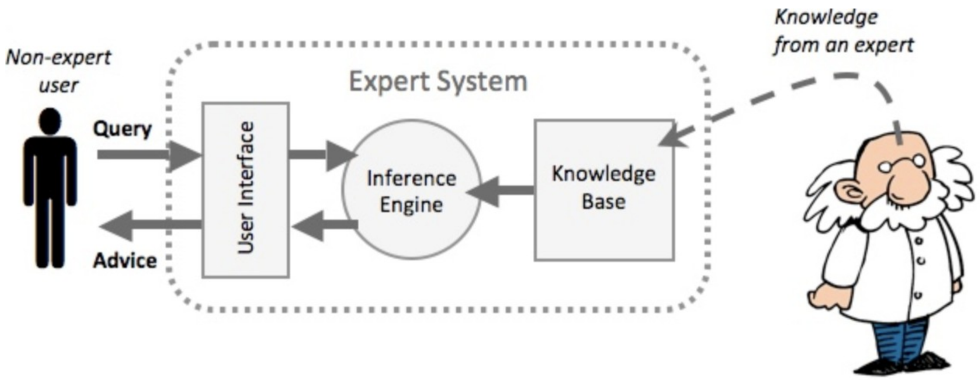
Humans think and reason in a way similar to fuzzy logic/numbers





Expert Systems and Fuzzy inference

- An expert system is an AI that solves problems by reasoning. An expert system has two components: a knowledge base and an inference engine



- Most of the inference engines used in experts systems were built on fuzzy inference techniques

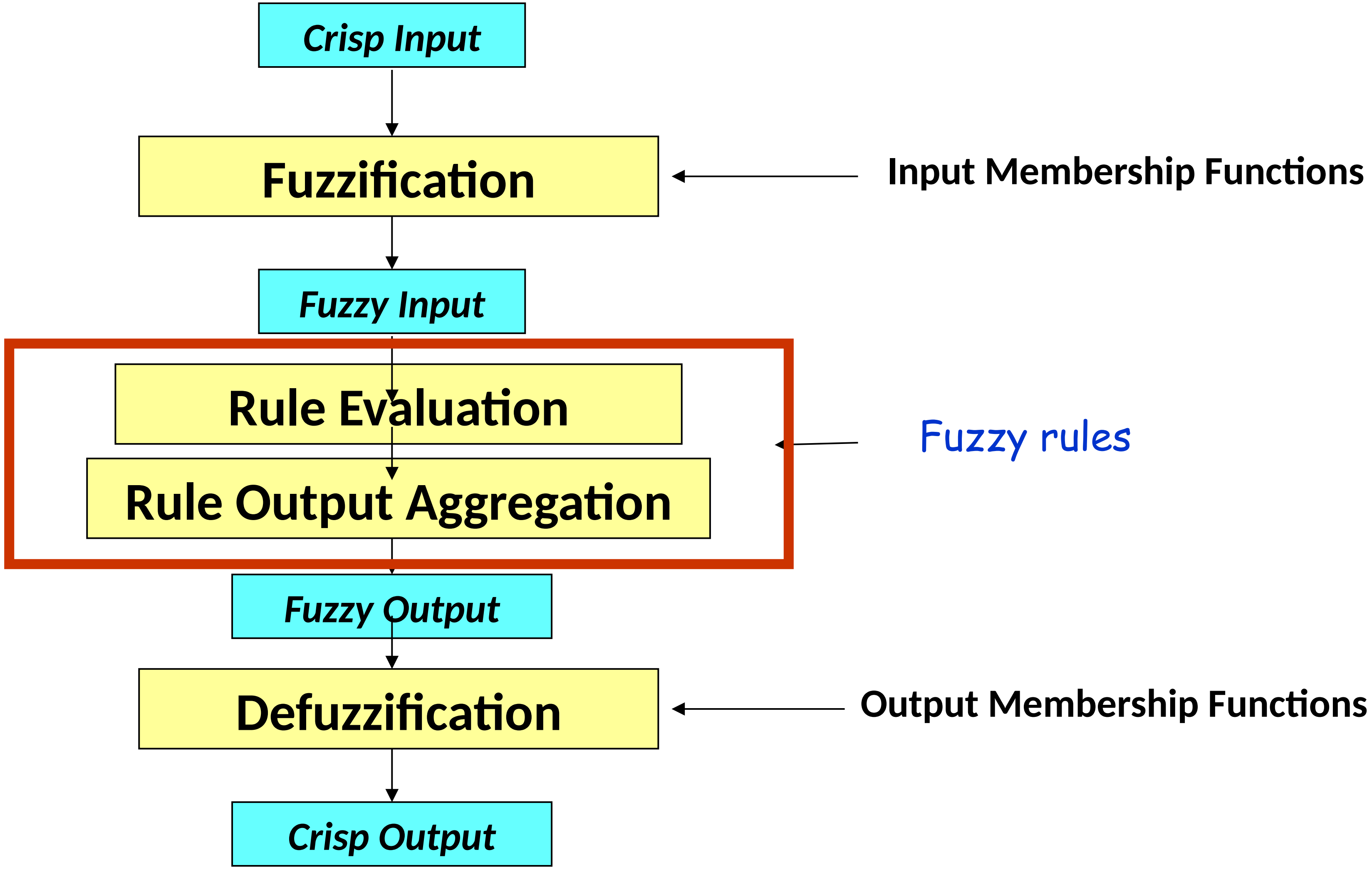
- The most used fuzzy inference technique is the so-called Mamdani method.

Fuzzy inference (Mamdani method)

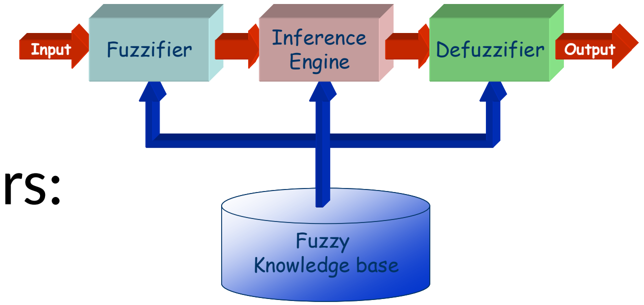
The fuzzy inference process is performed in four steps:

1. Fuzzification of the input variables,
2. Evaluation of fuzzy rules (i.e., rules on fuzzy numbers);
3. Aggregation of the rule outputs
4. Defuzzification.

Operation of Fuzzy System



Two (crisp) input variables:



X is on a scale of 0 to 10, described using three fuzzy numbers:

A1 : *project_funding* is *inadequate*
A2: *project_funding* is *marginal*
A3: *project_funding* is *adequate*

Y is on a scale of 0 to 10, described using two fuzzy numbers:

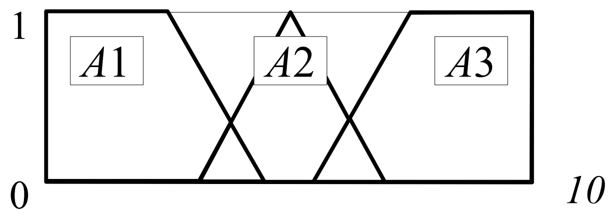
B1: *project_staffing* is *small*
B2: *project_staffing* is *large*

Output (crisp) variable: Z = ? Z is on a scale of 0 to 100

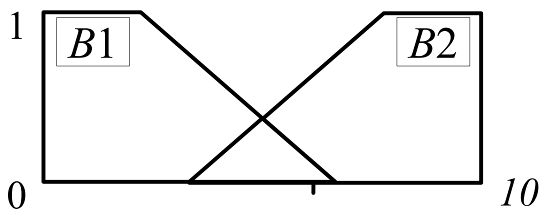
C1: *risk* is *low*
C2: *risk* is *normal*
C3: *risk* is *high*

Step 1: Let's define fuzzy numbers

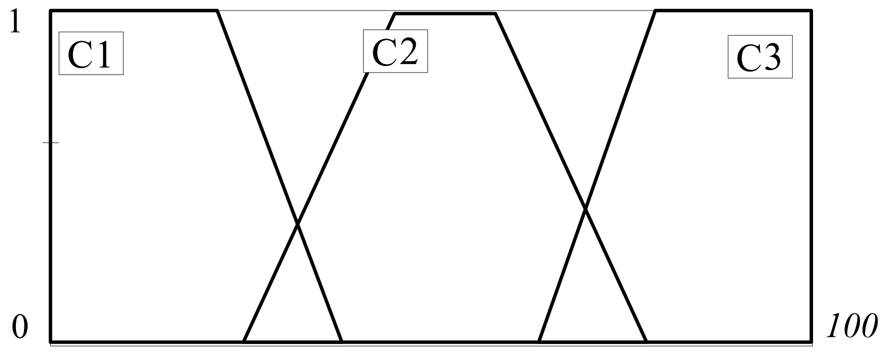
A1 : *project_funding* is *inadequate*
A2: *project_funding* is *marginal*
A3: *project_funding* is *adequate*

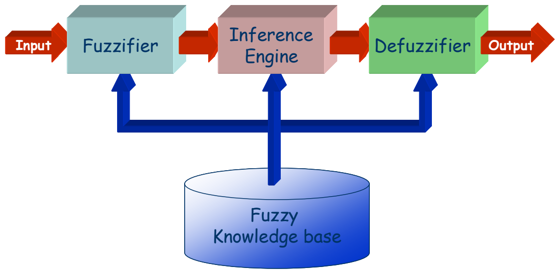


B1: *project_staffing* is *small*
B2: *project_staffing* is *large*



C1: *risk* is *low*
C2: *risk* is *normal*
C3: *risk* is *high*





Step 2: Let's define fuzzy rules on fuzzy numbers:

Rule: 1
IF x is $A3$
OR y is $B1$
THEN z is $C1$

Rule: 1
IF *project_funding* is *adequate*
OR *project_staffing* is *small*
THEN *risk* is *low*

Rule: 2
IF x is $A2$
AND y is $B2$
THEN z is $C2$

Rule: 2
IF *project_funding* is *marginal*
AND *project_staffing* is *large*
THEN *risk* is *normal*

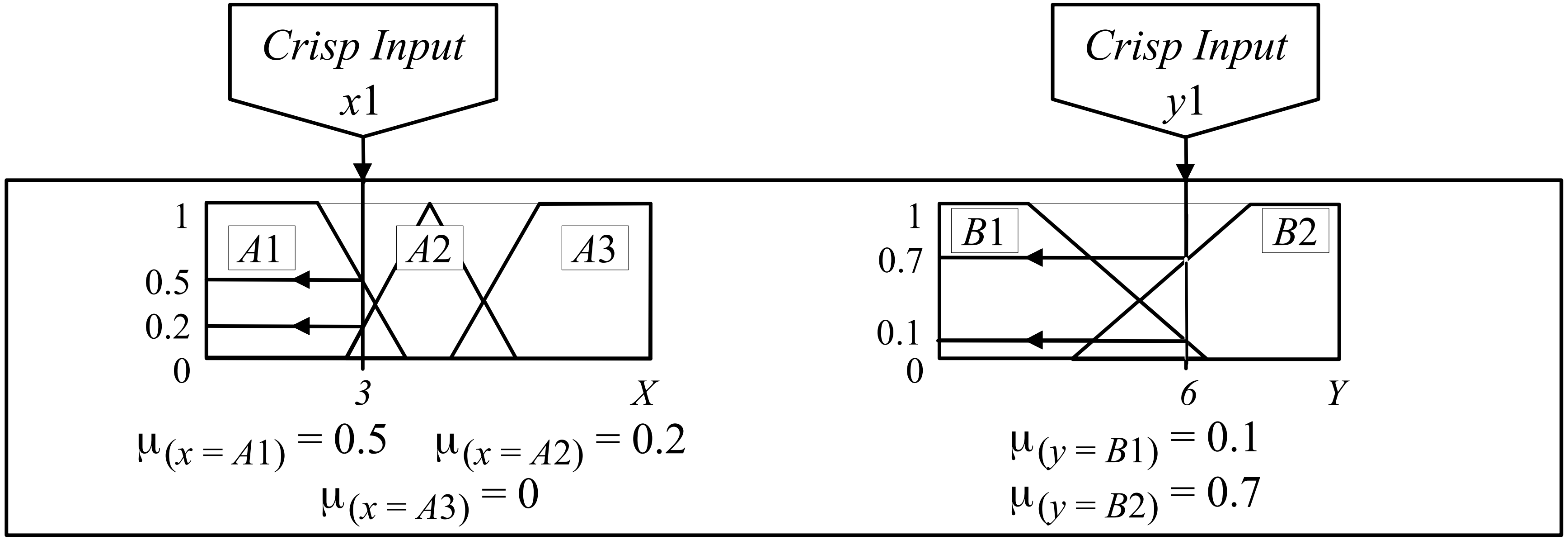
Rule: 3
IF x is $A1$
THEN z is $C3$

Rule: 3
IF *project_funding* is *inadequate*
THEN *risk* is *high*

Step 3: Fuzzification

- Take the crisp inputs, *X* and *Y* (*project funding* and *project staffing*)
- Determine the degree to which these inputs belong to each fuzzy number.
- The output is called fuzzy input

Suppose X (*project funding*) = 3 and Y (*project staffing*) = 6



project funding *project staffing*

- **Fuzzy input:** $\mu_{(x=A1)} = 0.5$ $\mu_{(x=A2)} = 0.2$ and $\mu_{(x=A3)} = 0$, $\mu_{(y=B1)} = 0.1$ and $\mu_{(y=B2)} = 0.7$

Step 4: Rule Evaluation

- Apply the fuzzy input to the fuzzy rules.
- To evaluate the disjunction of the rule inputs, we use the **OR fuzzy operation**.

$$\mu_{A \cup B}(x) = max\ [\mu_A(x), \mu_B(x)]$$

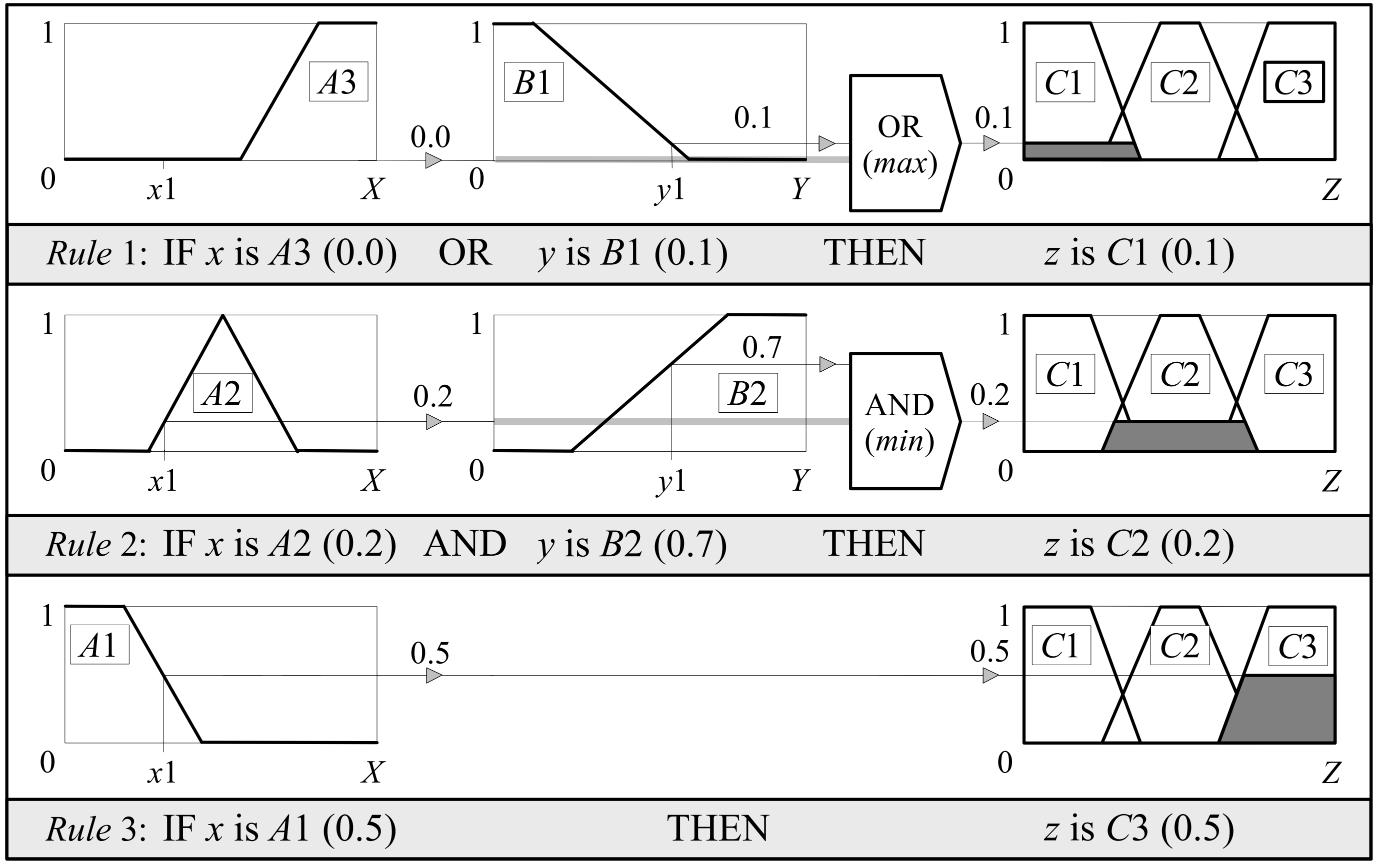
- To evaluate the conjunction of the rule inputs, we apply the **AND fuzzy operation** :

$$\mu_{A \cap B}(x) = min\ [\mu_A(x), \mu_B(x)]$$

Mamdani-style rule evaluation

Fuzzified inputs:

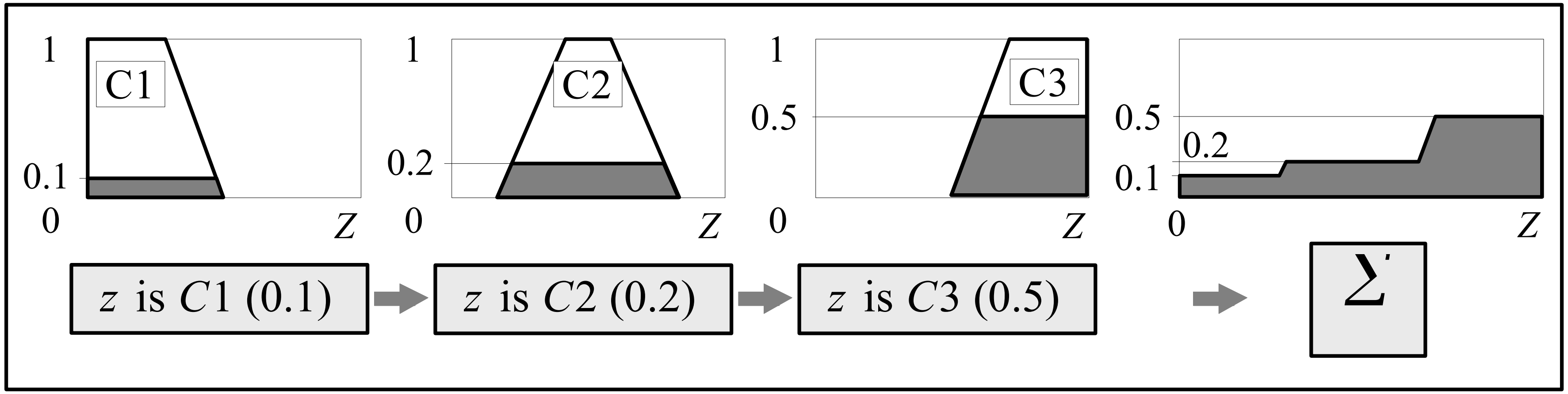
- $\mu_{(x=A1)} = 0.5$
- $\mu_{(x=A2)} = 0.2$
- $\mu_{(x=A3)} = 0$,
- $\mu_{(y=B1)} = 0.1$
- $\mu_{(y=B2)} = 0.7$



Fuzzy output: *z* is C1 (0.1), *z* is C2 (0.2), and *z* is C3 (0.5),

Step 5: Aggregation of the Rule Outputs

- We combine fuzzy outputs into **a single fuzzy number**.



Step 6: Defuzzification

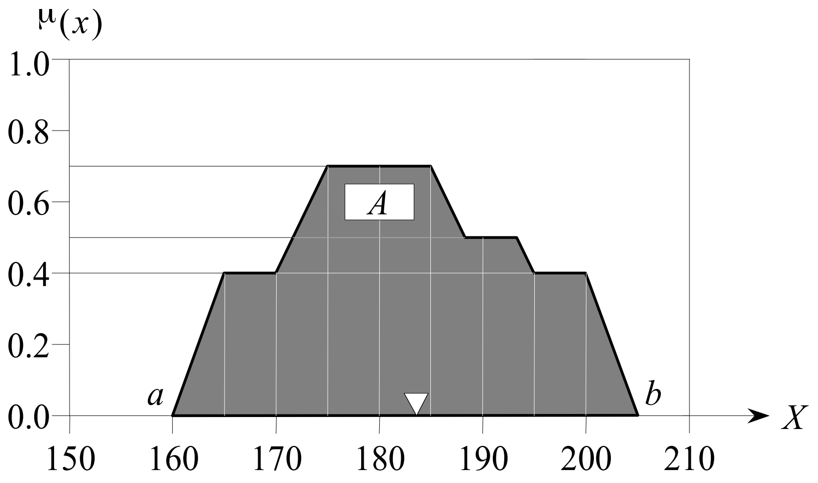
- The defuzzification process converts the aggregated fuzzy number to a single (crisp) number.

- In other words, here we convert the fuzzy output to the crisp output for final presentation

- Most popular defuzzification method is the **centroid defuzzification.**

- Centroid defuzzification method finds a point representing the center of gravity of the fuzzy number, *A*, on the interval, *ab*.

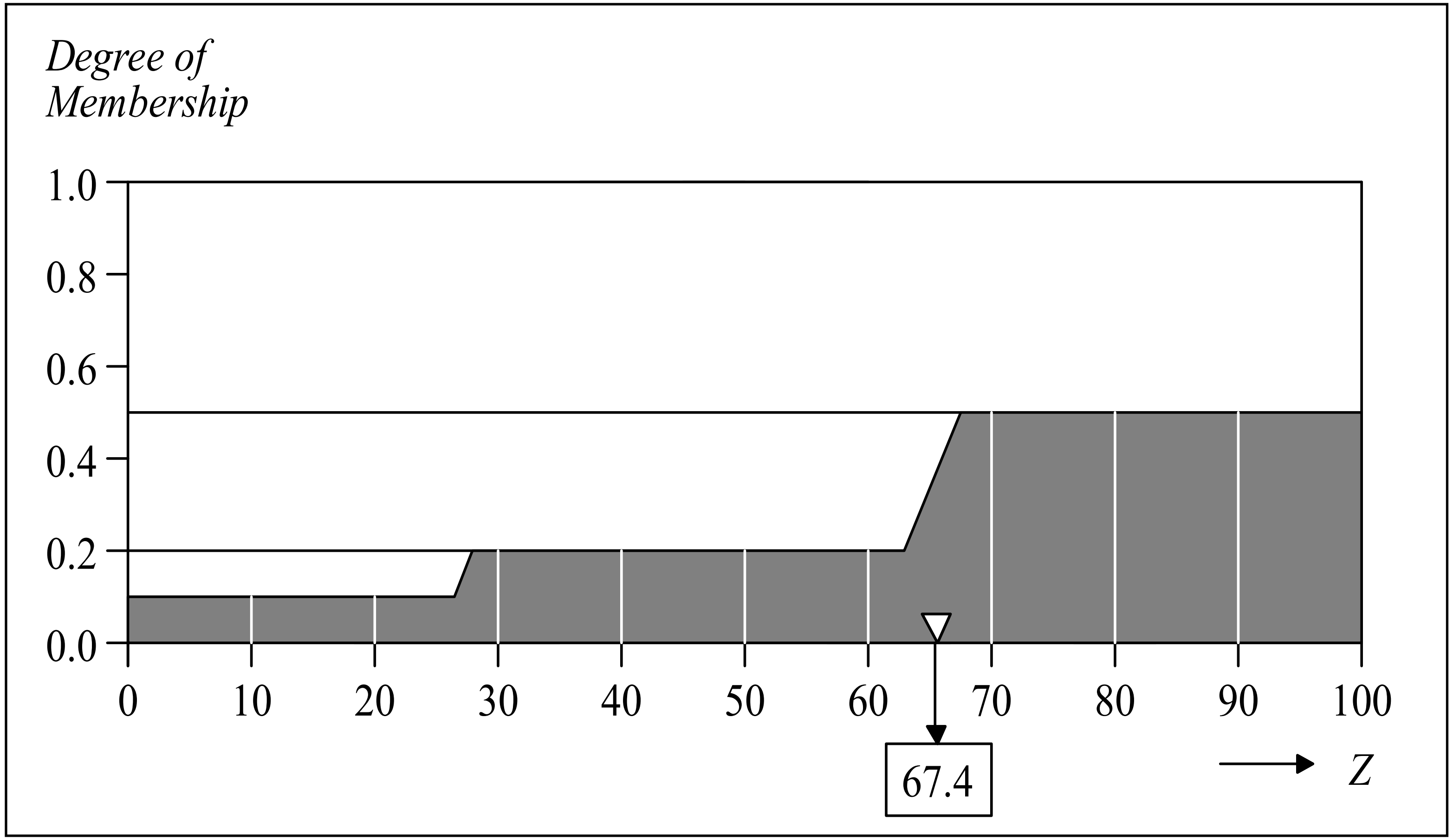
$$COG = \frac{\int_a^b \mu_A(x)\, x\,dx}{\int_a^b \mu_A(x)\,dx}$$



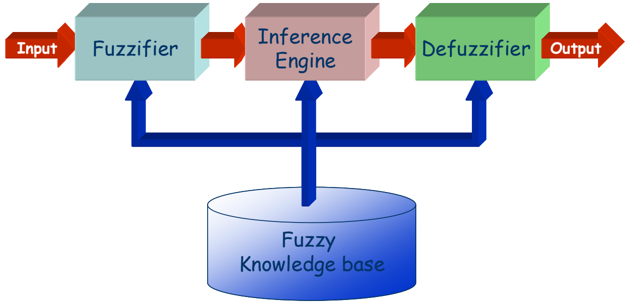
- ***A reasonable estimate*** can be obtained by calculating **weighed sum** over a sample of points.

Centre of gravity (COG):

$$COG = \frac{(0+10+20)\times 0.1 + (30+40+50+60)\times 0.2 + (70+80+90+100)\times 0.5}{0.1+0.1+0.1+0.2+0.2+0.2+0.2+0.5+0.5+0.5+0.5} = 67.4$$



Result



Two input variables:

project_funding = 3, on a scale of 0 to 10

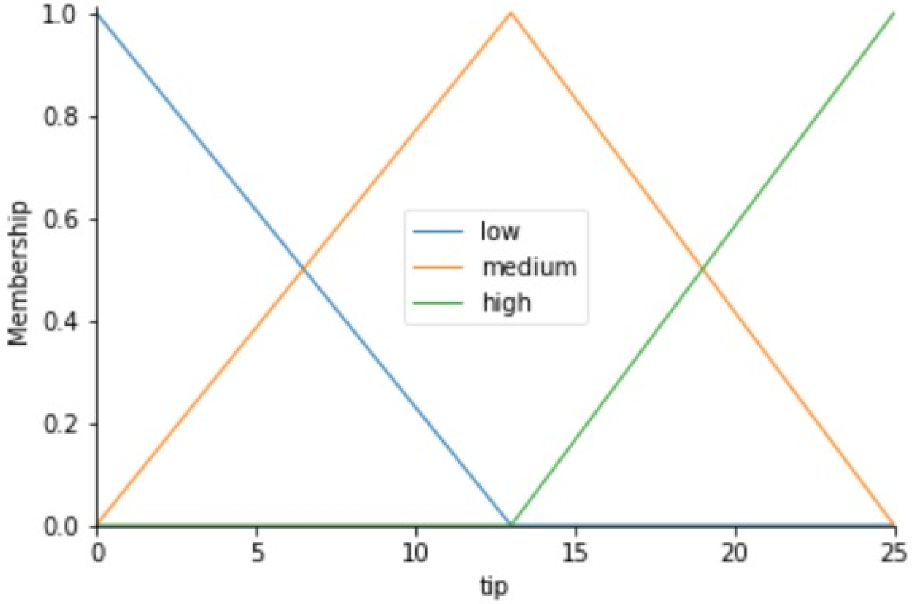
project_staffing = 6, on a scale of 0 to 10

Output :

risk = 67.4 on a scale of 0 to 100

Lab 14: Fuzzy Control Systems using Fuzzy Inference

- Scikit-fuzzy
- Tipping problem



Install Scikit-fuzzy for Lab 14

Anaconda Prompt (Anaconda3)

```
(base) C:\Users\chenh>pip install scikit-fuzzy
```