Lecture 7
Informed Search

CS 180 – Intelligent Systems

Dr. Victor Chen

Spring 2021

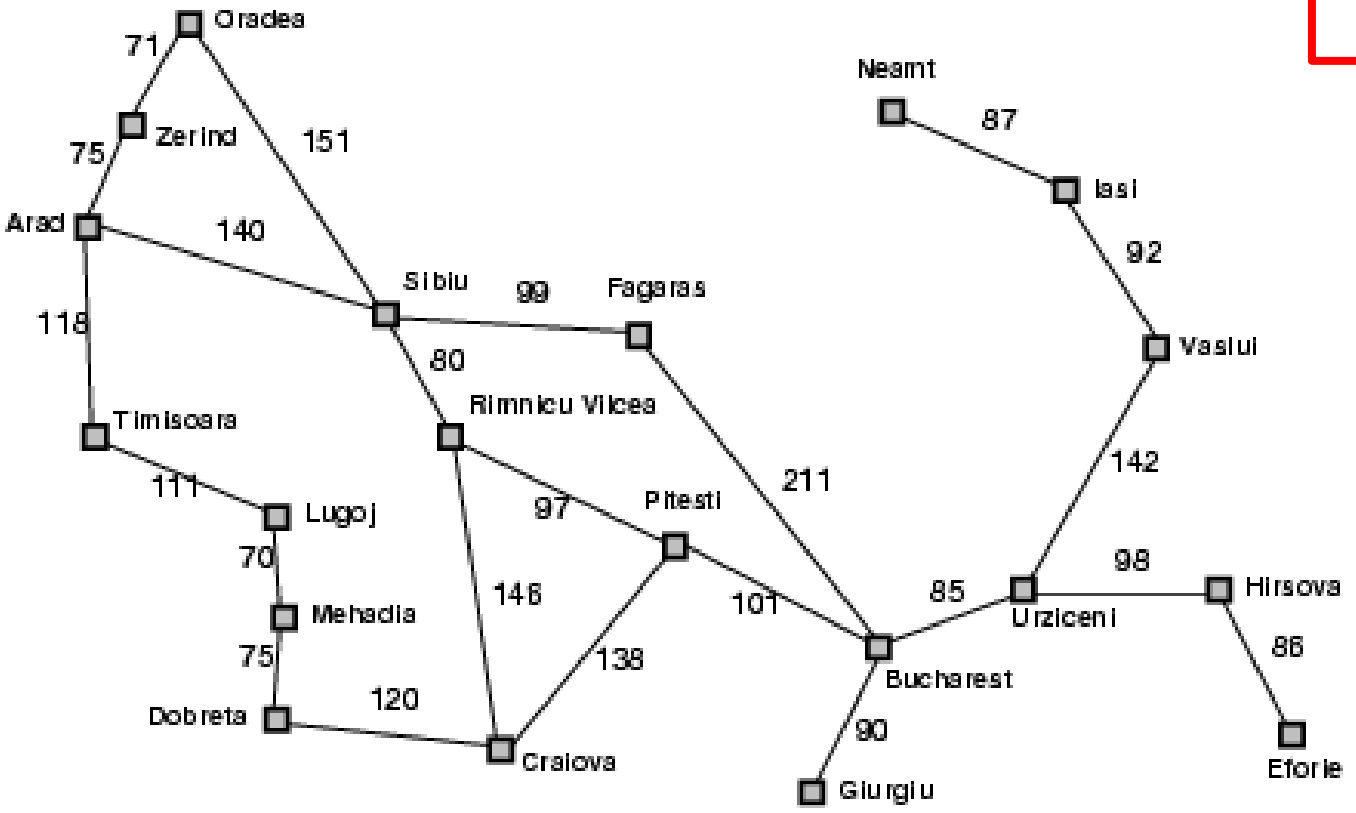
Informed Search

Informed (heuristic) Search

Idea: Use a ***heuristic function*** $h(n)$ to rank nodes in the *frontier* and expand the node with the lowest $h(n)$ value

- Greedy search (Best-first search)
- A* search

Heuristic for the Romania problem



heuristic function h(n)

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Eforie	161
Fagaras	176
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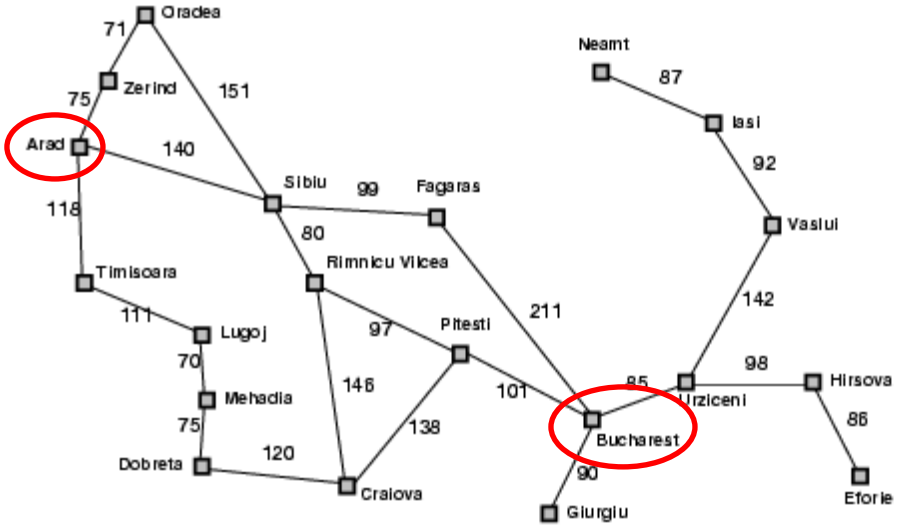
Zerind	374

Greedy best-first search example



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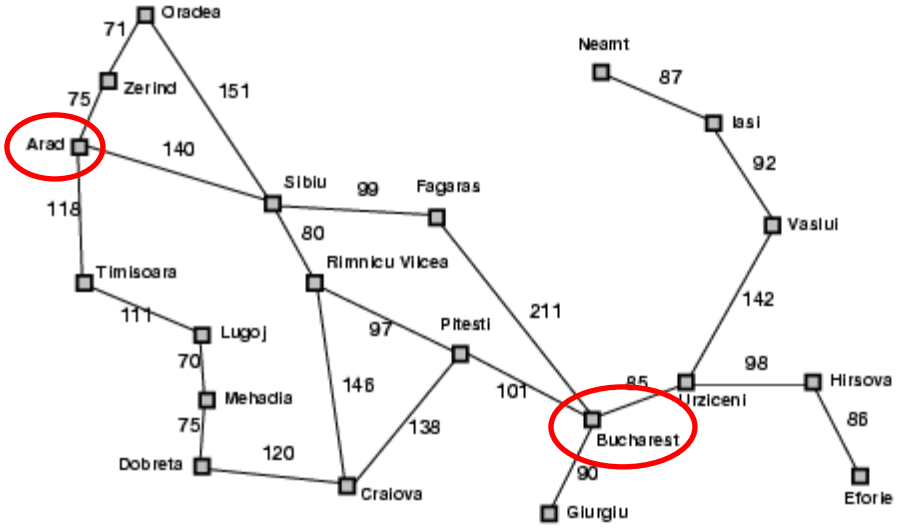

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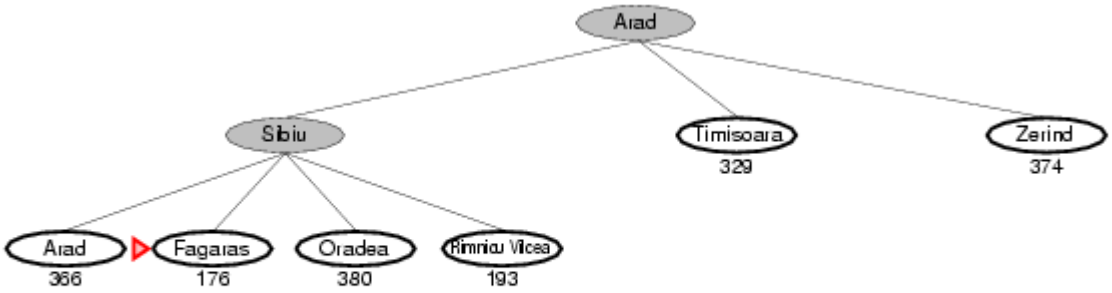

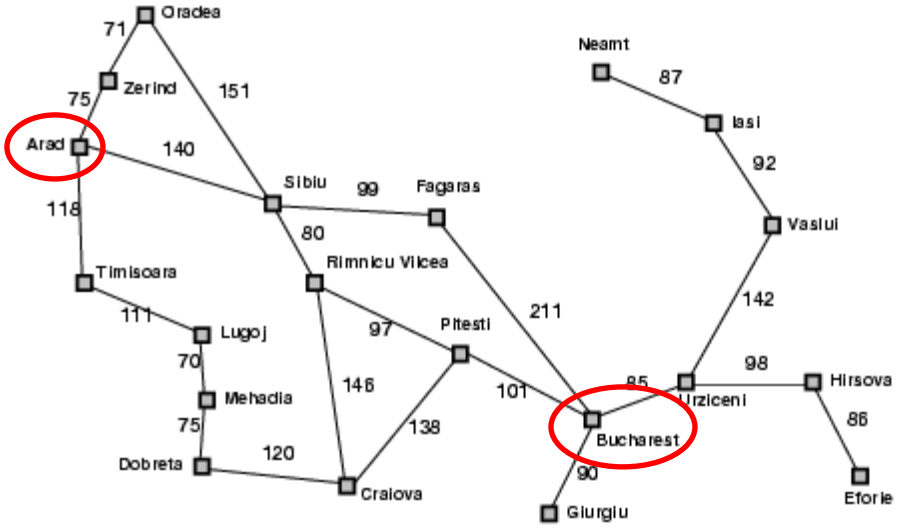

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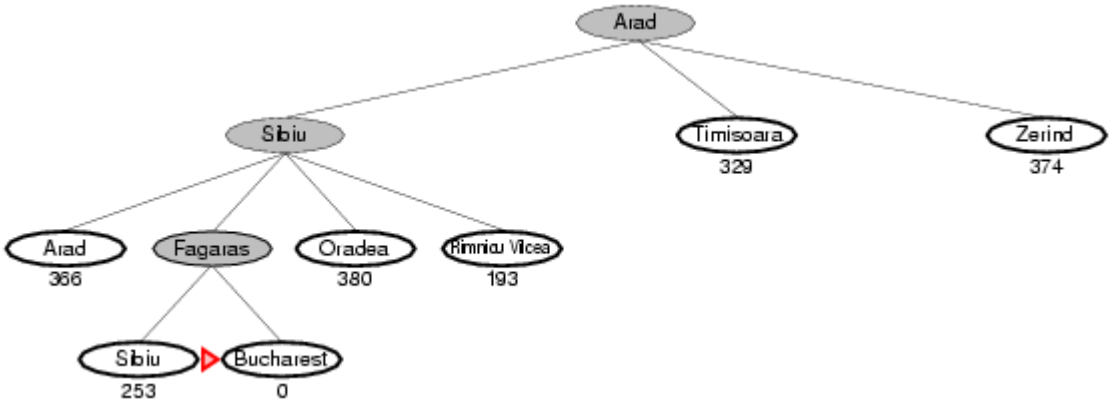

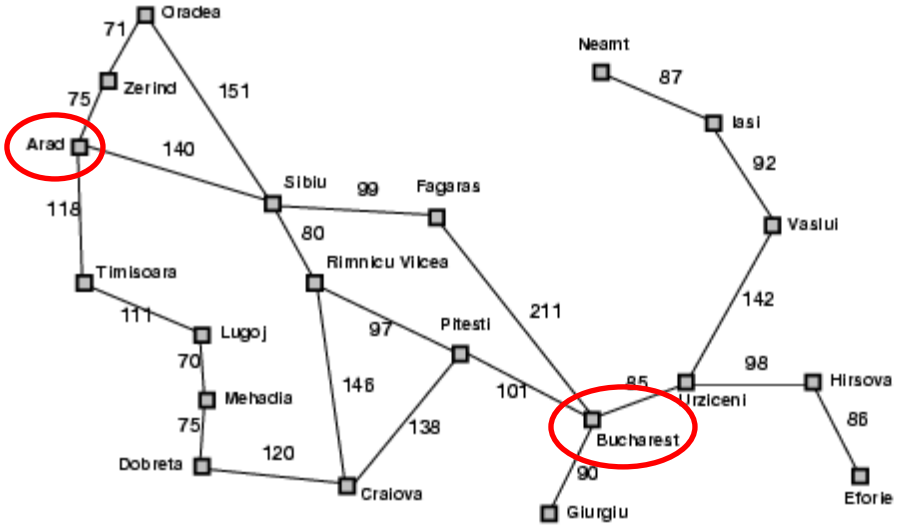

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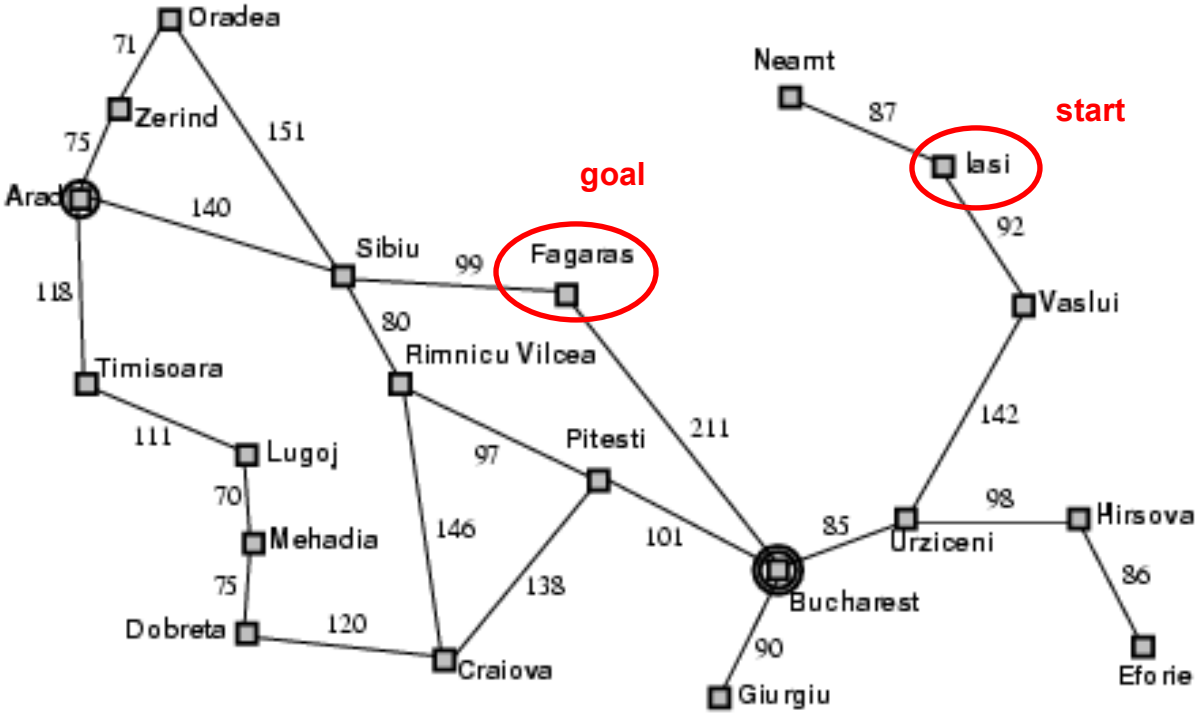
Properties of greedy search

Complete?

No – can get stuck in loops.



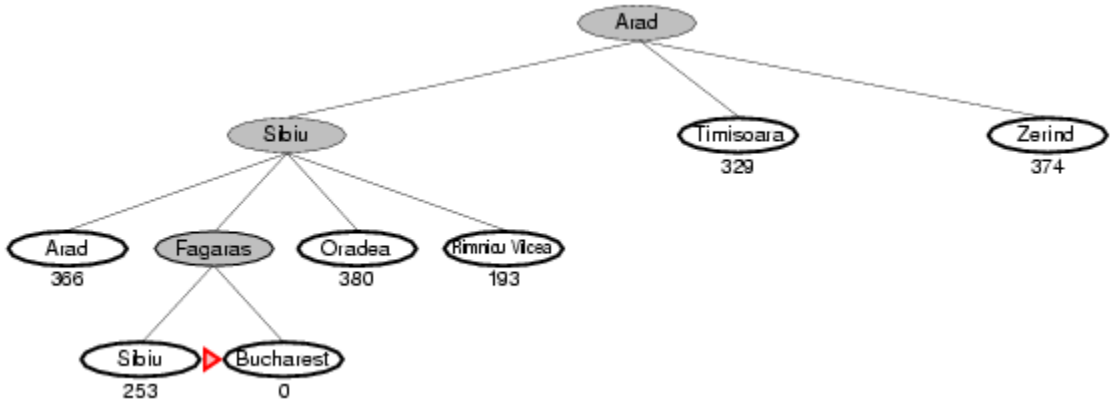
Properties of greedy best-first search

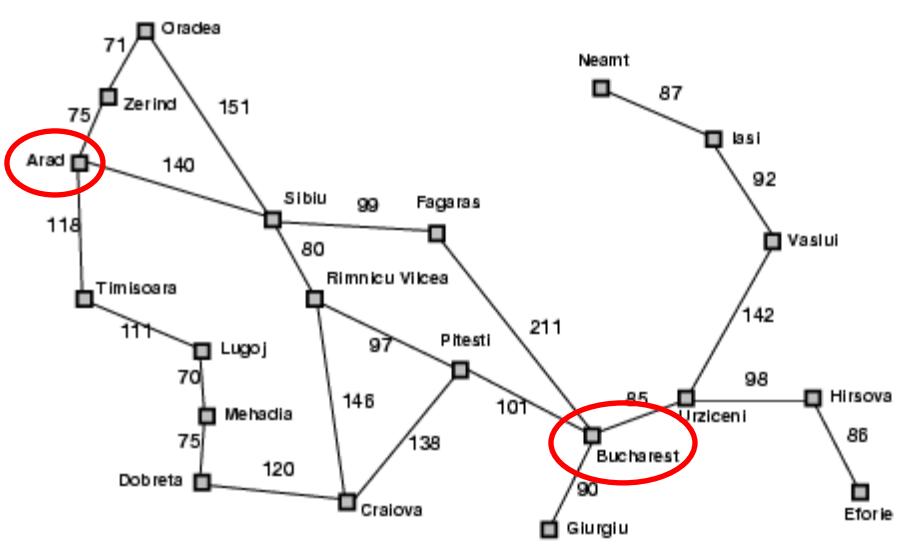
Complete?

No – can get stuck in loops

Optimal?

No





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Properties of greedy best-first search

Complete?

No – can get stuck in loops

Optimal?

No

Time cost?

Worst case: $O(b^m)$ where b is the branching factor and m is the maximum path length

Space cost?

Worst case: $O(b^m)$

How can we fix the greedy search?

In the Romania traveling problem, greedy search only considers the **remaining distance**

Idea: Keep track of the distance already traveled in addition to the distance remaining.

A* search

Idea: Use **evaluation function** $f(n)$ to estimate the path cost from *start node* to goal and expand the node with the lowest $f(n)$ value

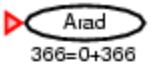
$f(n)$ is to estimate path cost from start node to goal:

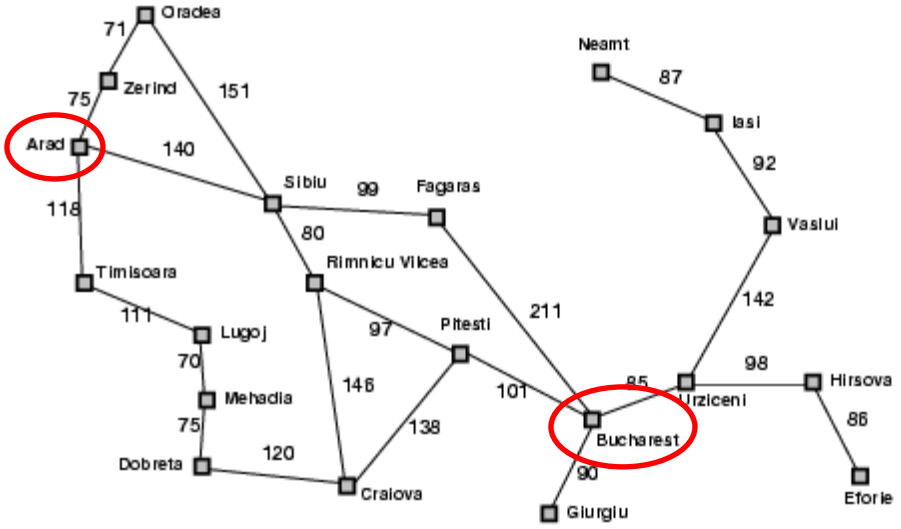
$$f(n) = g(n) + h(n)$$

$g(n)$: cost from start node to node n

$h(n)$: estimated cost from node n to goal (heuristic function)

A* search example (distance already traveled + distance remaining)





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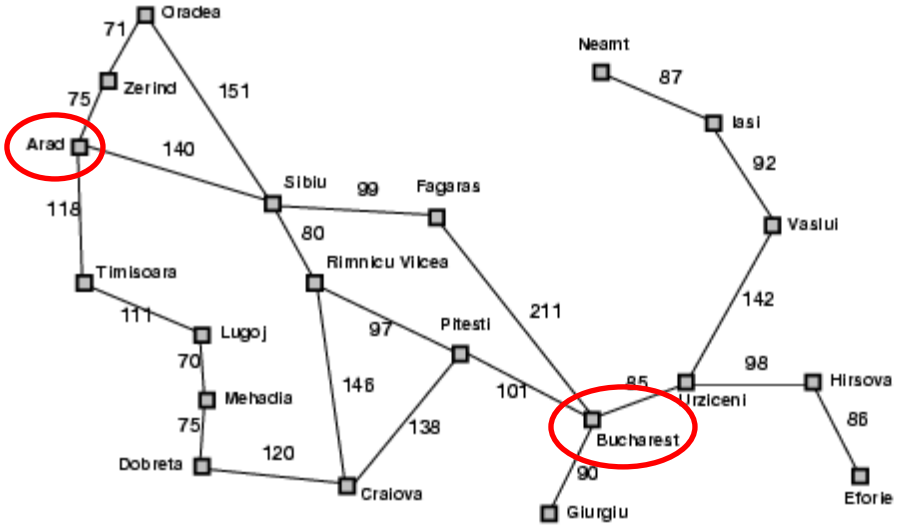
A* search example



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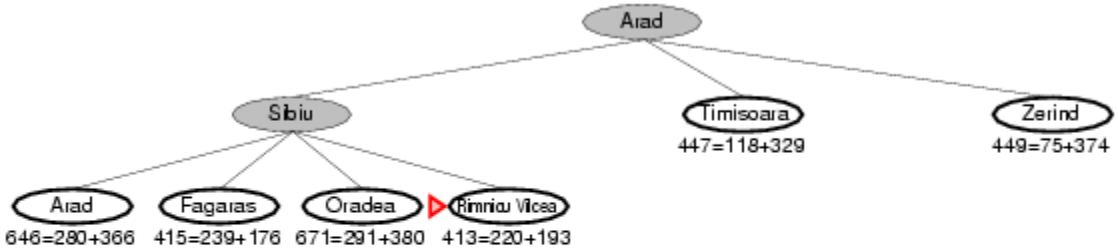

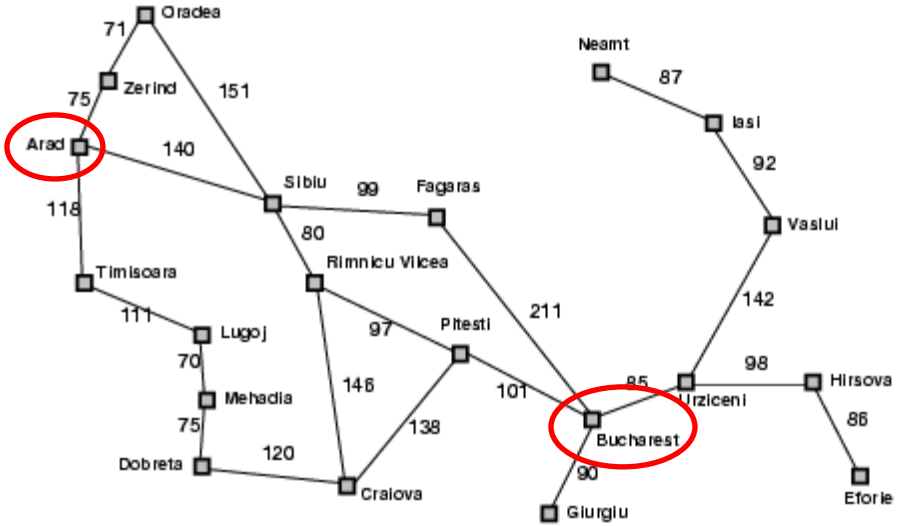

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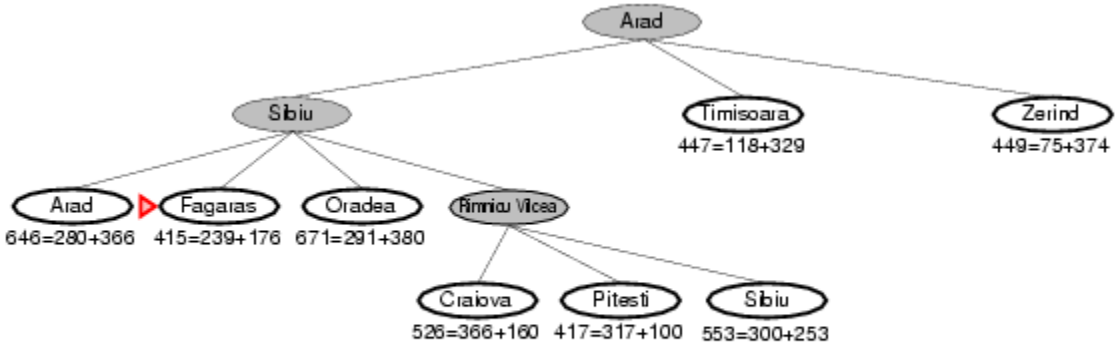

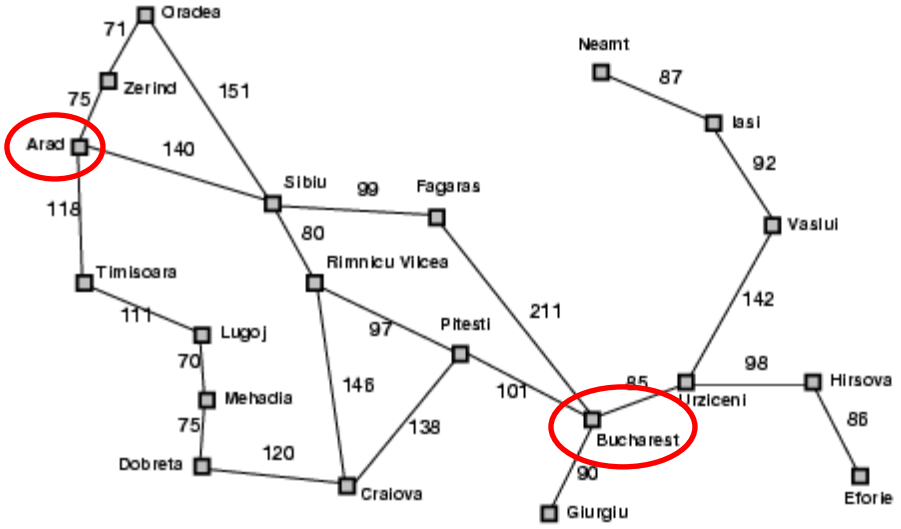

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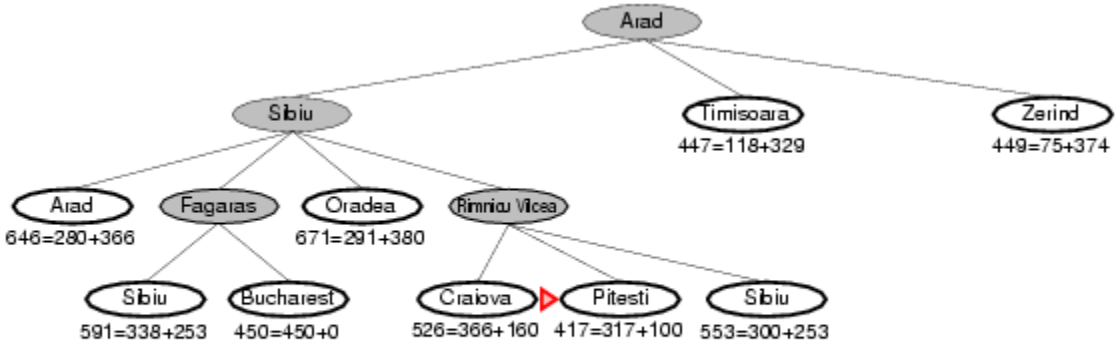

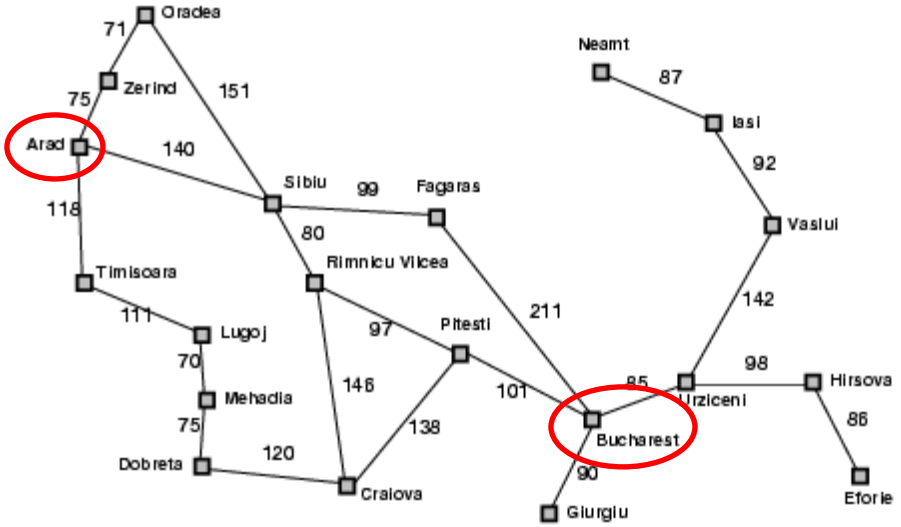

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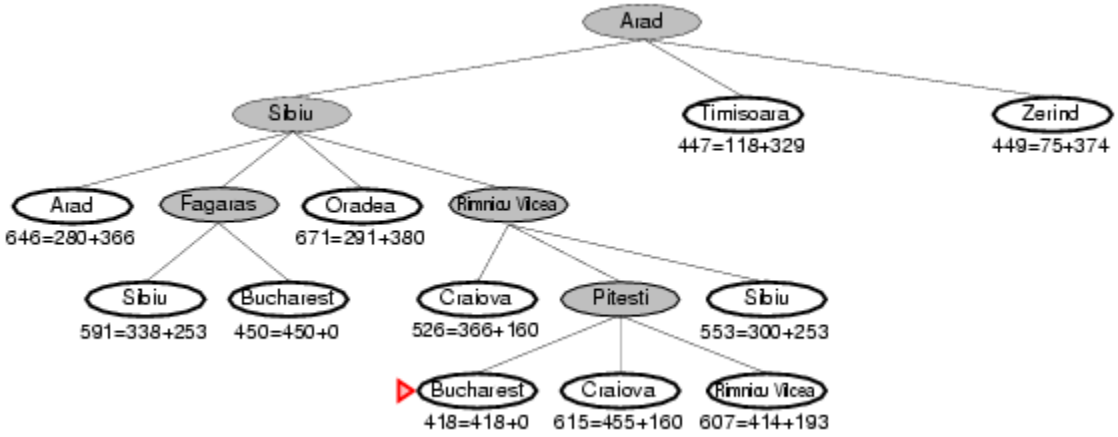

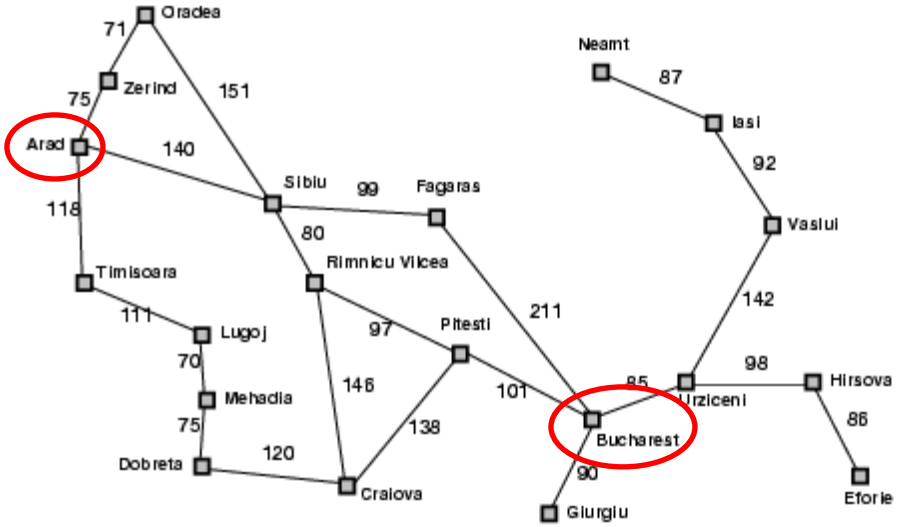

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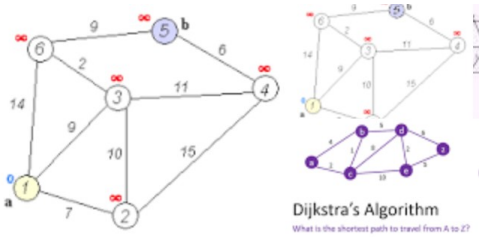
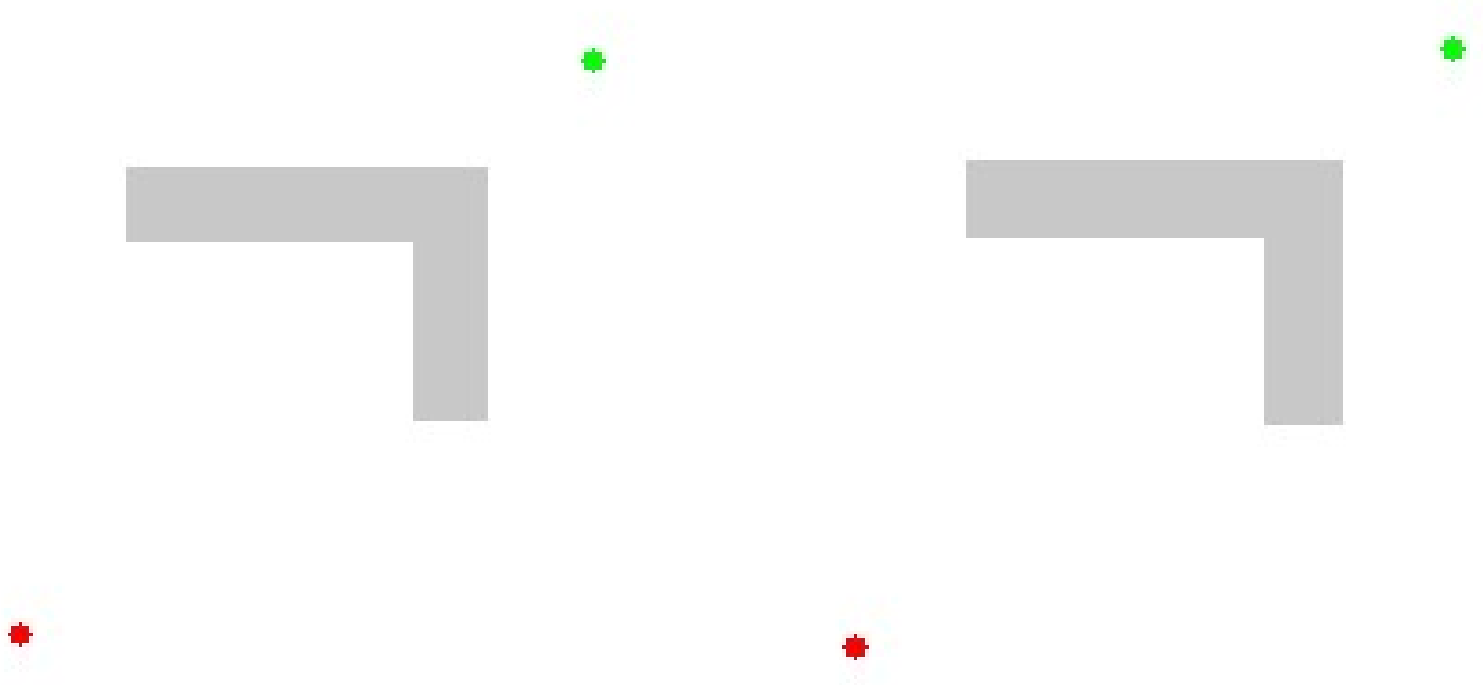
Dijkstra's vs Greedy vs A*

- Dijkstra's algorithm order all the nodes on their **g(n)** values (distance already traveled).
- Greedy algorithm order all the nodes on their **h(n)** values (distance remaining).
- A* balances Dijkstra's algorithm with greedy search by ordering all the nodes using **f(n) = g(n) + h(n),** which is the estimated path cost from **start node** through **n *to goal***.



Dijkstra's algorithm

Dijkstra's vs. A* search



Source: Wikipedia

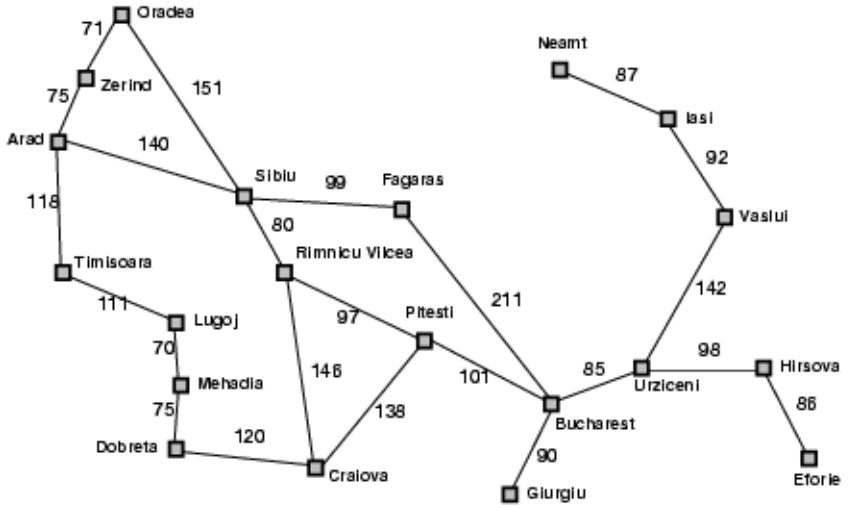
Is A* optimal?

- Is A* is guaranteed to provide the shortest path?

- **Theorem:** If $h(n)$ is admissible, then A* is optimal

Admissible heuristic

- A heuristic $h(n)$ is admissible if for every node n, $h(n) \leq h^*(n)$, where $h(n)$ is the estimated cost from n to goal while $h^*(n)$ is the true cost from n to goal.
- Is the distance heuristic we used before in Romanian traveling problem is admissible?



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- Yes. Because straight line distance never overestimates the

How to tell which heuristic is better?

If h_1 and h_2 are both admissible heuristics and $h_2(n) \geq h_1(n)$ for all n, then h_2 dominates h_1

Using $h_2(n)$ will lead to expanding fewer nodes than $h_1(n)$.

Properties of A*

Complete?

Yes – Will always find a solution if one exists

Optimal?

- Yes If $h(n)$ is admissible

Time/space cost?

- A* search expands every node with $h(n) \leq C^* - g(n)$
  - C^* is the path cost of the optimal solution
  - $g(n)$: cost from start node to node n
  - *A higher (dominating)* $h(n)$ will have a higher pruning power

Search strategies

Algorithm	Complete?	Optimal?	Time complexity	Space complexity
Greedy	No	No	Worst case: $O(b^m)$ where b is branching factor and m is maximum path length	
A*	Yes	Yes (if heuristic is admissible)	Number of nodes with $h(n) \leq C^* - g(n)$ $h(n)$: estimated cost from n to goal $g(n)$: cost from start node to n C^* is the path cost of the optimal solution	