

Proof

Chapter 2

A 2-dimensional simplex is a triangle

A simplex C is the convex hull $\text{conv}(S)$ of a set S of $k + 1$ affinely independent vectors v_0, \dots, v_k . The dimension of a convex set is the dimension of its affine hull. Thus if C is 2-dimensional then its affine hull is 2-dimensional.

Now suppose that the simplex is not a triangle, and the simplex has more than 3 sides (lets assume it has 4 sides). Then $k > 2$ since the number of vectors p in S needed to create a shape with at least 4 sides is $p > 3$ and $k = p - 1$. If $k > 2$ then the dimension of our simplex is also greater than 2, since the dimension of a set of 4 affinely independent points is 3.