

Regression-II

Multiple Linear Regression

Introduction to Multiple Linear Regression

- Multiple Linear Regression (MLR) models the relationship between a dependent variable and two or more independent variables.

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_n x_n + \epsilon$$

- y : Dependent variable
- x_1, x_2, \dots, x_n : Independent variables
- β_0 : Intercept
- $\beta_1, \beta_2, \dots, \beta_n$: Coefficients
- ϵ : Error term

Problem Statement

- Predict sales (y) based on TV advertising (x_1) and Radio advertising (x_2).

Data

Observation	TV Advertising (x_1)	Radio Advertising (x_2)	Sales (y)
1	150	22	240
2	160	28	260
3	170	26	275
4	180	32	290
5	190	34	310

Step 1: Calculate Means

$$\bar{x}_1 = \frac{1}{n} \sum_{i=1}^n x_{1i}, \quad \bar{x}_2 = \frac{1}{n} \sum_{i=1}^n x_{2i}, \quad \bar{y} = \frac{1}{n} \sum_{i=1}^n y_i$$

Step 2: Calculate Sum of Squares

$$SS_{x_1x_1} = \sum_{i=1}^n (x_{1i} - \bar{x}_1)^2, \quad SS_{x_2x_2} = \sum_{i=1}^n (x_{2i} - \bar{x}_2)^2$$

$$SS_{x_1x_2} = \sum_{i=1}^n (x_{1i} - \bar{x}_1)(x_{2i} - \bar{x}_2)$$

$$SS_{x_1y} = \sum_{i=1}^n (x_{1i} - \bar{x}_1)(y_i - \bar{y}), \quad SS_{x_2y} = \sum_{i=1}^n (x_{2i} - \bar{x}_2)(y_i - \bar{y})$$

Step 3: Calculate Coefficients

$$\beta_1 = \frac{SS_{x_2x_2}SS_{x_1y} - SS_{x_1x_2}SS_{x_2y}}{SS_{x_1x_1}SS_{x_2x_2} - (SS_{x_1x_2})^2}$$

$$\beta_2 = \frac{SS_{x_1x_1}SS_{x_2y} - SS_{x_1x_2}SS_{x_1y}}{SS_{x_1x_1}SS_{x_2x_2} - (SS_{x_1x_2})^2}$$

$$\beta_0 = \bar{y} - \beta_1\bar{x}_1 - \beta_2\bar{x}_2$$

Final Regression Equation

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2$$