Theory question

$Let\ S\ be\ the\ implicit\ surface\ defined\ by\ f(x) = 0.$

$Formally\ prove\ that\ the\ normal\ of\ S\ at\ point\ p \in S\ is\ proportional\ to\ \nabla f(p).$

- the surface where $f\ (p)\ =\ 0$ is a level surface.
- Every curve can be written as $p(t)$ for a parameter t, when $p(t) = (x(t), y(t), z(t))$, and if we require this curve to be on the $f\ (p) = 0\ then\ f\big(p(t)\big) = 0.$ we derive both sides and get:

$$0 = \frac{\partial}{\partial t} f\big(p(t)\big) = \frac{\partial f}{\partial x} * \frac{dx}{dt} + \frac{\partial f}{\partial y} * \frac{dx}{dt} + \frac{\partial f}{\partial z} * \frac{dx}{dt} = \nabla f \cdot p'(t)$$

- meaning that the gradient is perpendicular to the tangent (dot product 0), which means that the normal goes in the same direction as $\nabla f(p)$ ■