

Prove that  $f + g$  is  $\Theta(h)$ .

The proof can be divided into two subproofs:

Proof 1:

$$\begin{aligned} f + g &= \Theta(h) + O(h) \\ &= O(h) + O(h) \\ &= O(h) \end{aligned}$$

Proof 2:

$$\begin{aligned} f + g &= \Theta(h) + O(h) \\ &= \Omega(h) + O(h) \\ &= \Omega(h) \end{aligned}$$

Since  $f + g$  is both  $O(h)$  and  $\Omega(h)$  then it proves  $f + g$  is  $\Theta(h)$ .