

# Title

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**Abstract.** In the article we present

## 1 Introduction

The task of pattern recognition is a classical machine learning problem. On the input we pass a training dataset, consisting of labelled patterns belonging to  $c$  classes. In the process we expect to form a model that would be able to assign correct labels to new cases (new observations).

It is important to have in mind that patterns in their original form are some sort of signal, for instance images or voice recordings. Due to the fact that the original patterns are often collected using some signal-acquiring devices, we may encounter patterns that do not belong to any of proper classes. An example of such situation may happen, when the device that we use to acquire data has been automatically reset due to power outage and poor default calibration distorts the segmentation process. Another scenario may happen when we collect data in a noisy (out-of-lab) environment and apart from proper patterns there is a lot of unexpected residual elements. The problem with garbage patterns is that we cannot predict their character and therefore we cannot include information about them in the model training process.

The motivation of our study is to provide algorithmic approaches to distinguish proper patterns, that we call **native patterns** from garbage and erroneous patterns, that we call **foreign patterns**. The task that we describe we call **foreign patterns rejection**. The design assumption is to provide methods based on native patterns only. In this way the framework that we propose is truly versatile and it can be adapted to any pattern recognition problem in an uncertain environment, where foreign patterns may appear.

We propose several classification with foreign elements rejection scenarios based on well-known combinations of machine learning algorithms. The novelty of the contribution presented in this paper is not in the methods that we use, but in what we achieve with them. [In particular, the objective of this paper is to discuss ...](#)

The proposed methods are applied in a case study of handwritten digits recognition.

The remainder of this paper is organized as follows. Section 2 presents the background knowledge on foreign elements detection present in the literature.

Section 3 presents the backbone algorithms well-known in the literature that we use to construct our models. Section 4 presents the proposed approach. In Section 5 we discuss a series of experiments based on images of handwritten digits. Lastly, Section 6 concludes the paper and highlights future research directions.

## 2 Literature Review

Data collection and processing is a vital study problem across multiple domains of science. Along with a substantial automation of data acquiring we meet with difficulties that appear due to poor data quality. The research we present in this paper has been motivated by the issue of contaminated datasets, that apart from proper patterns contain garbage.

Let us start our discussion with review of relevant literature positions in machine learning that tackle the issue of contaminated datasets.

First and foremost let us discuss outlier detection methods. Outliers are native patterns that substantially differ from the remainder of class elements. Detection of outliers is an important domain of machine learning, as many learning methods are very likely to fail, when they process such data. Even though outliers are proper native elements, they are often removed from the training dataset in order to construct a good model. In consequence, it is very likely that appearance of outliers in a testing set would result in their misclassification. However, this is a relatively small price to pay for a model that is well-fitted to model majority of the data. We may distinguish the following groups of methods dealing with outliers:

- statistical,
- model-based.

The first group constitutes of methods based on sample statistics. The second group consists of approaches with a stage of model building.

A very intuitive way to detect outliers is to visualize the data we process, for instance with histograms or box plots. Automating this process typically relies on calculating sample statistics and detecting those observations that differ remarkably from the mean or median. This idea could be briefly summarized by saying that outliers are those elements that fall far from the majority of the data. This straightforward method works fairly well in practice. Model designer is responsible for determining a cutoff threshold to distinguish between outlying and regular observation. There is a wide variety of papers relating to this idea, where various decision rules or tests for outliers are discussed. For instance, we may mention Z-score that is defined as:

$$Z_i = \frac{x_i - \bar{x}}{sd} \quad (1)$$

where  $\bar{x}$  is sample mean and  $sd$  is standard deviation. In [9] authors propose a modified Z-score based on median:

$$MZ_i = \frac{0.6745(x_i - \tilde{x})}{MAD} \quad (2)$$

where  $\tilde{x}$  is the median and  $MAD$  is the median absolute deviation. The authors recommend to set the cutoff point to 3.5.

It is also worth to mention formal outlier tests: Grubbs' Test, Tietjen-Moore Test, and Generalized ESD Test. Named tests assume that there are no outliers in the data set. However, the alternative hypotheses differ. Grubbs' Test assumes that there is exactly one outlier in the data set, Tietjen-Moore Test assumes that there are exactly  $k$  outliers in the data set, while Generalized ESD Test assumes that there are up to  $r$  outliers in the data set. In the subsequent steps a simple statistics are computed. It has to be emphasized though that they are also based on simple notions: sample mean, maximal distance from the mean, or standard deviation.

The second group of outlier detection methods is based on various models. For instance, one may adapt clustering algorithms for outlier detection. A good examples are soft clustering approaches, for instance Gaussian Mixture Models (GMMs) and fuzzy c-means. Those methods express cluster membership as a number from the  $[0, 1]$  interval. An element, whose dominating membership cannot be easily decided could be regarded as outliers. The limitation of these approaches is that the quality of outliers removal is determined by the quality of internal data structure detection. If an algorithm fails to detect proper clusters, it is likely to reject many proper patterns in process.

Discussion on approaches related to foreign elements rejection should also mention one-class classification methods. Especially, there are two noteworthy examples: centroid-based methods and One-Class Support Vector Machine.

Centroid-based methods rely on distinguishing cluster centres (centroids). Region reserved for proper elements is usually defined by the distance between centre and the furthest proper pattern.

One-Class SVM has been introduced in [13]. While "regular" SVM algorithm forms hyperplane separating two classes, the One-Class SVM separates data points from the entire feature space. Notably, the One-Class SVM provides a soft decision rules, as there is a  $\nu$  parameter determining the fraction for outliers.

It is worth to emphasize the analogy between outlier detection and foreign elements rejection. However, the motivation for those two problems remains different.

When it comes to the study on foreign elements rejection, there is relatively few papers to review. This issue, in spite of its importance, remains somehow neglected. Among noteworthy studies one may mention rank-based methods, for instance ones described in [2, 4, 6, 7, 12, 16]. In a nutshell, mentioned papers propose to attach confidence scores along with class labels. Rejection occurs when none of native class labels was assigned with a satisfying confidence.

In our paper we assume an approach that in its background resembles more model-based outlier detection techniques than reported in the literature foreign elements rejection schemes. The objective of our study is to form a model that would be able to distinguish between the region reserved for native patterns and the remainder of the feature space.

### 3 Preliminaries - Backbone Algorithms

The proposed approach builds upon several existing algorithms. In order to provide a self-contained description of our methods in this section we present backbone literature algorithms applied in our method. In what follows we present the Minimum Volume Enclosing Ellipsoid (MVEE) algorithm and a suite of 3 classification methods: Random Forests (RF), Support Vector Machines (SVM), and K-Nearest Neighbors algorithm (K-NN). Listed methods are employed in various configurations to native elements classification with foreign elements rejection. Our approach, based on those algorithms, is discussed in Section 4.

#### 3.1 Ellipsoids for Foreign Elements Rejection

In computational geometry, the smallest enclosing box problem is that of finding the oriented minimum bounding box enclosing a set of points. In many cases there's a need for computing convex hull of set of points and testing inclusions of other points in the same space. This can become very complex problem especially in higher dimensions, so more often an approximation of the hull is used. This helps to reduce time needed for computations, since most of the methods have lower construction and inclusion-testing complexities. Some of such approaches include using figures like hypercubes, diamonds, spheres or ellipsoids to successfully enclose given set of points.

When comparing highlights and drawbacks of each method, from computational complexity, ease of testing point inclusion and algorithm implementation point of view, ellipsoids seem to be a reasonable choice. Constructed ellipsoid is superior to the minimal cuboid in many ways. It is unique, better approximation of the object it contains and if  $E(S)$  is the bounding ellipsoid for a point set  $S$  with convex hull  $C(S)$  in dimension  $d$ , then:

$$\frac{1}{d}E(S) \subseteq C(S) \subseteq E(S)$$

where scaling is with respect to the center of  $E(S)$ .

Opis o bardzo nierównym poziomie szczegółowości: przeskok z 'bounding box' do 'hull', brakuje określenia parametru  $d$ .

Adaptation of the smallest enclosing box problem to foreign elements rejection/native elements identification seems to be a very natural approach. The justification is fairly simple: if we enclose elements belonging to native classes, using for instance ellipsoids, formed geometrical model will discriminate a region of features space reserved for native patterns between a region where we may encounter foreign patterns. With this premise in mind, let us present a detailed description of the MVEE algorithm.

**MVEE** problem is solved by several known algorithms that can be categorized as first-order, second-order interior-point or combination of the two. For small dimensions  $d$ , the MVEE problem can be solved in  $O(d^{O(d)}m)$  operations using

randomized or deterministic algorithms[15]. In this paper the algorithm based on Khachiyan solution is used.

An ellipsoid in center form is given by

$$E = \{x \in \mathbb{R}^n | (x - c)^T A (x - c) \leq 1\}$$

where  $c \in \mathbb{R}^n$  is the center of the ellipse E and  $A \in \mathbb{S}_{++}^n$ . Points lying inside the ellipsoid satisfy

$$(x_i - c)^T A (x_i - c) \leq 1$$

However this is not a convex optimization problem and the equations have to be changed. It turns out that even then, the solution is not easily obtainable so the dual problem has to be found. For more precise and in depth solution description see [15].

**Tutaj dodalbym opis parametrow 'tolerance' i 'acceptance'**

The pseudocode for the MVEE algorithm used in this paper is as follows:

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#### Algorithm 1.1: MVEE

---

```

1  input :
2      P – row-ordered matrix of m n-dimensional points
3      tolerance – solution error value with respect to optimal value
4
5  output :
6      c – center point of found ellipsoid
7      A – matrix %TODO Add description
8
9  begin
10     N ← number of points in P
11     d ← number of dimensions for points in P
12
13     Q ←  $\begin{pmatrix} P_{1,1} & P_{1,2} & \dots & P_{1,n} & 1 \\ P_{2,1} & P_{2,2} & \dots & P_{2,n} & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ P_{m,1} & P_{m,2} & \dots & P_{m,n} & 1 \end{pmatrix}$ 
14
15     error ← 1
16
17     u ←  $[\frac{1}{N}, \frac{1}{N}, \dots, \frac{1}{N}]$ , |u| = N
18
19     while error > tolerance
20         X ← Q *  $\begin{pmatrix} u_1 & 0 & \dots & 0 \\ 0 & u_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & u_n \end{pmatrix}$  * QT
21
22         M ← diag(QT * X-1 * Q)
23
24         jdx ← Mj :  $\forall_{0 \leq i \leq |M|} M_j \geq M_i$ 
25
26         step_size ←  $\frac{M[jdx] - d - 1.0}{(d+1) * (M[jdx] - 1.0)}$ 
27
28         new_u ← (1 - step_size) * u
29
30         new_u[jdx] ← new_u[jdx] + step_size
31
32         error ← ||new_u - u||

```

```

33
34   u ← new_u
35   done
36
37   c ← u * P
      (PT *  $\begin{pmatrix} u_1 & 0 & \dots & 0 \\ 0 & u_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & u_n \end{pmatrix} * P - c^T * c)^{-1}$ 
38   A ←  $\frac{\quad}{d}$ 
39 end

```

---

For a detailed pseudo-code of the MVEE algorithm one may consult [xx] and [yy]. - prosze dodac uzyte odwolania

### 3.2 Native Elements Classification

The task of native elements classification relies on forming a model based on a labelled training dataset that assigns proper class labels to new problem instances. In a conventional scenario the training dataset consists of  $m$  instances of  $n$ -dimensional feature vectors. Feature vectors are numerical descriptors of actual patterns. There is a multitude of classification algorithms, among which we have selected 3 different ones that are applied in our methods. It is necessary to emphasize that if someone would like to adapt our method to their own domain, those algorithms could be substituted with some other classification tools that may be more efficient in that domain. Without further ado let us move towards a brief description of the methods that we apply in our study.

**Support Vector Machines** are a set of supervised learning methods used for classification, regression and outliers detection. [The SVM algorithm relies on a construction of hyperplane with a maximal margin that is separating instances of two classes, \[5\]](#). SVMs are effective in high dimensional spaces, memory efficient and quite versatile with many kernel functions that can be specified for the decision function. Although in some cases, where number of features is much greater than the number of samples, this method can give poor results, and is not cost-efficient when calculating probability estimates, it is well suited for problem presented in this paper. Given a set of training examples, each marked for belonging to one of two categories, an SVM training algorithm builds a model that assigns new examples into one category or the other. For multi-class classification "one-against-one" approach is used. For  $n$  classes  $n * (n - 1) / 2$  classifiers are constructed, each trains data from two different classes. The order for classes 0 to  $n$  is "0 vs 1", "0 vs 2", ... "0 vs  $n$ ", "1 vs 2", ... "n - 1 vs  $n$ ".

**Random Forests** is a popular ensemble method. [The main principle behind ensemble methods, in general, is that a group of "weak learners" can come together to form a "strong learner". In the Random Forest algorithm the weak learners are decision trees. After a relatively large number of trees is generated,](#)

they vote for the most popular class. Random Forests join few important benefits: (a) they are relatively prone to the influence of outliers, (b) they have an embedded ability of feature selection, (c) they are prone to missing values, and (d) they are prone to overfitting, [3].

**K-Nearest Neighbors** is an example of a "lazy classifier", where the entire training dataset is a model. There is no typical model building phase, hence the name. Class membership is determined based on class labels encountered in K closest observations in the training dataset, [1]. In a typical application, the only choices the model designer has to make are selection of K and distance metrics. Both are often extracted with a help of a supervised learning procedures.

## 4 Methodology

In this section let us present our methods for foreign elements rejection.

### 4.1 Internal Rejecting

Internal Rejecting w znaczeniu adaptacji klasyfikatora podstawowego (to znaczy, konstruowanego na klasach native) do odrzucania. Np. (a) jeli klasyfikator podstawowy jest zbudowany z binarnych klasyfikatorow klasa kontra klasa, to prubujemy odrzuca, gdy roznica miedzy maksymalna liczba zgloszen i druga liczba zgloszen nie jest odpowiednio duza (z testow wynika, ze to nie jest dobry pomysl, ale warto o tym napisac), (a) jesli klasyfikator podstawowy jest zbudowany z binarnych klasyfikatorow, to probujemy odrzucac, gdy liczba zgloszen wygrywajacej klasy nie jest odpowiednio duza (pomysl Piotra, nie wiem, jak to wygladalo na testach, ale te warto o tym napisac), (c) jesli klasyfikator podstawowy jest zbudowany z binarnych klasyfikatorow klasa kontra pozostale, to odrzucamy, gdy zaden sie nie zglosi, tutaj trzeba jeszcze zastanowic sie nad kolejnoscia zastosowanie klasyfikatorow jesli chcemy liczyc CC. Warto to wszystko przedstawic, nawet jesli nie wyniki nie sa zachecajace

Pewnie tez nalezy dodac schemat - pseudokod? opisujacy internal rejecting.

### 4.2 External Global and Local Rejecting

External Rejecting za pomoca elipsoid, konstruujemy elisoidy zawierajace poszczególne klasy natives, dwie metody (a) Global (jesli element nie nalezy do zadnej, to jest odrzucany, wpp jest poddawany klasyfikacji za pomoca podstawowego klasyfikatora) (b) Local (element jest poddawany klasyfikacji za pomoca podstawowego klasyfikatora, a nastepnie jest sprawdzana jego przynaleznosc do elipsoidy odpowiadajacej klasie wskazanej dla tego elementu)

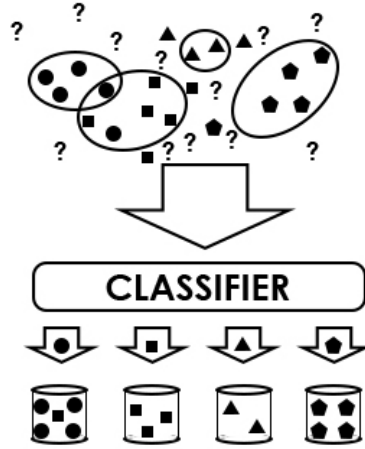


Fig. 1: Tutaj obrazki z klasami symboli - kolka, kwadraty, trojkaty - i opis koncepcji klasyfikacji oraz odrzucania na dwóch poziomach

### 4.3 Quality Evaluation

In order to evaluate the quality of proposed methods the patterns from the following groups are counted:

- CC (Correctly Classified) - correctly classified patterns, i.e. native patterns classified as native ones with the correct class,
- TP (True Positives) - native patterns classified as native (no matter, into which native class),
- FN (False Negatives) - native patterns incorrectly classified as foreign,
- FP (False Positives) - foreign patterns incorrectly classified as native,
- TN (True Negatives) - foreign patterns correctly classified as foreign.

These numbers are then used to form measures reflecting specific aspects of classification and rejection, cf. Table 1. Notions that we use are well-known in the domain of pattern recognition. We have included a detailed description of those measures in our previous paper [8].

- *Strict Accuracy* measures classifier's performance. It is the ratio of the number of all *correctly classified* patterns to the number of all patterns being processed.



Table 1: Quality measures for classification and rejection

$$\begin{aligned}
\text{Native Precision} &= \frac{TP}{TP+FP} & \text{Accuracy} &= \frac{TP+TN}{TP+FN+FP+TN} \\
\text{Foreign Precision} &= \frac{TN}{TN+FN} & \text{Strict Accuracy} &= \frac{CC+TN}{TP+FN+FP+TN} \\
\text{Native Sensitivity} &= \frac{TP}{TP+FN} & \text{Fine Accuracy} &= \frac{CC}{TP} \\
\text{Foreign Sensitivity} &= \frac{TN}{TN+FP} & \text{Strict Native Sensitivity} &= \frac{CC}{TP+FN} \\
\text{F-measure} &= 2 \cdot \frac{\text{Precision} \cdot \text{Sensitivity}}{\text{Precision} + \text{Sensitivity}}
\end{aligned}$$

- *Accuracy* is a "softer" characteristic derived from the Strict Accuracy. Accuracy describes the ability to distinguish between native and foreign patterns. The difference is that we do not require that the native patterns are labelled with their proper class label.
- *Native Precision* is the ratio of the number of not rejected native patterns to the number of all not rejected patterns (i.e. all not rejected native and foreign ones). The higher the value of this measure, the better ability to distinguish foreign elements from native ones. Native Precision does not evaluate how effective identification of native elements is.
- *Native Sensitivity* is the ratio of the number of not rejected native patterns to all native ones. The higher the value of Native Sensitivity, the more effective identification of native elements. Unlike the Native Precision, this measure does not evaluate the effectiveness of separation between native and foreign elements.
- *Strict Native Sensitivity* takes only correctly classified native patterns and does not consider native patterns, which are not rejected and assigned to incorrect classes, unlike Native Sensitivity, where all not rejected native patterns are taken into account.
- *Fine Accuracy* is the ratio of the number of native patterns classified to correct classes, i.e. assigned to their respective classes, to the number of all native patterns not rejected. This measure conveys how precise is correct classification of not rejected patterns.
- *Foreign Precision* corresponds to Native Precision.
- *Foreign Sensitivity* corresponds to Native Sensitivity.
- Precision and Sensitivity are complementary and there exists yet another characteristic that combines them: the *F-measure*. It is there to express the balance between precision and sensitivity since these two measures affect each other. Increasing sensitivity can cause a drop in precision since, along with correctly classified elements, there might be more incorrectly classified.

## 5 Experiments

In this section we move towards description of a series of experiments where we apply rejection strategies discussed theoretically in Sections 4.1 and 4.2. In what follows we describe the datasets, experiments' settings and the results.

### 5.1 Datasets

The Experiments' section is devoted to a study of handwritten digits recognition.

Foreign patterns are usually of unexpected shapes and origin. In this study as foreign patterns we assume a dataset of handwritten letters from the Latin alphabet. The justification to assume such foreign dataset for testing purposes is that appearance of other real symbols, not belonging to any proper class, is a common issue in a character recognition problem.

We would like to stress again, that foreign patterns do not participate in model building phase. The entire scheme is based on native patterns only. Handwritten letters are used only for rejection mechanisms quality evaluation.

Samples of processed patterns are displayed in Figure 2.

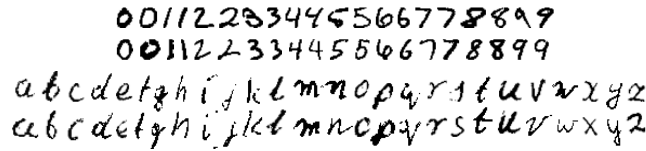


Fig. 2: Top: samples of handwritten digits - native patterns in our study. Bottom: samples of handwritten letters - foreign patterns in our study.

The training dataset was made of 10,000 handwritten digits with approximately 1,000 observations in each class taken from publicly available MNIST database, [10]. We split each class in proportion 7:3 and in a process we got two sets. First one was made of 6,999 observations and we used it for training purposes. The second set was the test set and it contained 3,001 observations. We have constructed the foreign dataset by ourselves and it contained 26,000 handwritten letters, 1,000 letters in each class. xxx different people have had participated in the creation of this dataset.

All patterns were normalized and feature vectors comprising of 106 numerical features were created. Feature vectors described not only but also: projections, histograms of projections, transitions, offsets, minimum/maximum values, raw moments, central moments, etc. Subsequently, we have applied best first search method to select relevant features out of the entire feature set.

Best first search is one of popular wrapper feature selection methods. Wrapper methods select multiple subsets of features, for all those subsets they train a classifier (in general: a model) and they select this subset for which numerical

quality was the best. In our particular case as quality evaluator we used a 10-class SVM. A commonly known advantage of wrapper feature selection methods is that the subsets they select usually provide satisfying performance. The disadvantage is their computational complexity. The best first search for the optimal feature subset has been performed using FSelector R package, [11]. The final feature vector contained 24 features. **We normalized those features ...?**

## 5.2 Experimental Settings

Tutaj należy przedstawić (a) wybór środowiska programistycznego, spis użytych bibliotecznych metod z referencjami, (b) co napisaliśmy sami, (c) parametry . Należy dać opis testów przeprowadzonych w celu wyboru 'dobrych' parametrów klasyfikatora. Do tekstu objaśniającego dodałabym pewnie jakieś wykresy. Potem można powiedzieć, że parametry tol/acc w MVEE mają wpływ na wynik i dlaczego (nawiązać do objaśnienia tych parametrów), ale szczegóły parametrów z MVEE można opisać w osobnej sekcji.

## 5.3 Concept of Classification with Rejection

Table 2: **Tutaj Confusion Table, np. test set z pliku "Ellipsoids results new format ratio 7/3.xls" przesłanego 11 marca dla TOL=0.2 ACC=0.001. Załączam przykładową tabelkę, gdzie wyniki są w procentach, ale u nas niech będą liczby bezwzględne. Pominęte są zera dla większej czytelności, niewypełnione foreign.**

	0	1	2	3	4	5	6	7	8	9	<i>f</i>
0	99.29		0.20		0.10		0.20		0.10	0.10	
1		99.03	0.26	0.09			0.35	0.26			
2	0.19		95.45	1.74	0.39	1.07	0.29	0.19	0.39	0.29	
3		0.10	0.69	95.74		1.29		0.59	0.79	0.79	
4			1.32	0.10	95.72	0.31	0.10	0.20	0.20	2.04	
5			1.68	2.69		93.83	0.34	0.22	0.78	0.45	
6			0.21		0.10	0.42	98.54		0.73		
7		0.10	0.19	0.49	0.58	0.39		94.94		3.31	
8	0.41		0.92	1.13	0.21	1.13	0.21		95.89	0.10	
9	0.10		0.20	0.69	1.59	0.50		1.29	0.40	95.24	
<i>f</i>											

Ja bym tutaj powiedziała że tabelka confusion table stanowi podstawę do obliczania miar przedstawionych w subsekcji "Quality Evaluation" i na podstawie takich danych będziemy prowadzić dyskusję w dalszej części pracy. Należy objasnić zawartość tabelki. Należy napisać, że interesuje nas a) klasyfikacja i jej jakość, b) odrzucanie i jego jakość c) wpływ odrzucania na klasyfikację.

## 5.4 Experimental Settings

Tutaj należy przedstawić (a) wybór środowiska programistycznego, spis użytych bibliotecznych metod z referencjami, (b) co napisaliśmy sami, (c) parametry metod. Należy dać opis testów przeprowadzonych w celu wyboru 'dobrych' parametrów klasyfikatorów: svm, rf, knn. Do tekstu objaśniającego dodałbym pewnie jakieś wykresy. Potem można powiedzieć, że parametry tol/acc w MVEE mają wpływ na wynik i dlaczego (nawiązać do objaśnienia tych parametrów), ale szczegóły parametrów z MVEE można opisać w osobnej sekcji. Potem powiedzieć, że parametry tol/acc w MVEE mają wpływ na wynik i dlaczego (nawiązać do objaśnienia tych parametrów), ale bez szczegółów. Należy podać parametry MVEE użyte w eksperymentach pokazanych w tym artykule i powiedzieć że zostały dobrane metoda ręcznej walidacji.

## 5.5 Classification Quality

Table 3: Results for classification with rejection on train and test sets of native patterns in comparison with classification results without rejection mechanism. RF - results for random forest, SVM - results for Support Vector Machines, .... W poniższej tabelce będą dane dotyczące odrzucania tylko na natives za pomocą elipsoid i na dwóch poziomach (global i local). Czy jest sens dla train set? Czy nie będzie 100%? Na poziomie local chyba nie będzie 100%, dlatego jest sens. Jako ostatnie dodane jest internal rejecting - czy damy te wyniki?

	no rejection	with rejection
Basic Classifier	RF   SVM   KNN	RF   SVM   KNN
external global rejection		
Data Set	Native Patterns, Train/Test?? Set	
Fine Accuracy		
Strict Native Sensitivity		
Native Sensitivity		
external local rejection		
Data Set	Native Patterns, Train/Test?? Set	
Fine Accuracy		
Strict Native Sensitivity		
Native Sensitivity		—   —
internal rejection		
Data Set	Native Patterns, Train/Test?? Set	
Fine Accuracy		
Strict Native Sensitivity		
Native Sensitivity		—   —

Adding a rejection mechanism, ideally, may be seen as a method for improvement of classification rates. It would be perceived as a positive side of the rejection mechanism, if it would be able to reject those native patterns, which would

be incorrectly classified when there is no rejection mechanism at all. Trained models only partially fulfill this wish. Conducted tests showed that: TUTAJ OPIS WPLYWU ODRZUCANIA NA KLASYFIKACJE, NALEZY POWIEDZIEC KTORA METODA JEST Z TEJ PERSPEKTYWY NAJLEPSZA, ewentualnie ilustracja (wykres)?

W tej tabelce jest tylko External Rejecting? Warto zastanowic sie nad dodaniem Internal rejecting lub uzasadnieniem rezygnacji z tych wynikow.

Adding a rejection mechanism, ideally, may be seen as a method for improvement of classification rates. It would be perceived as a positive side of the rejection mechanism, if it would be able to reject those native patterns, which would be incorrectly classified when there is no rejection mechanism at all. Trained models only partially fulfill this wish. Conducted tests showed that: TUTAJ SLOWNY OPIS WPLYWU ODRZUCANIA NA KLASYFIKACJE, NALEZY POWIEDZIEC KTORA METODA JEST Z TEJ PERSPEKTYWY NAJLEPSZA

## 5.6 Rejection Quality

Table 4: Results of classification with rejection on the set of native patterns supplemented with different sets of semi-synthetic foreign patterns.... Moze na razie wstrzymajmy si z semi-synthetic, poniewaz wymgaloby to rozszerzenia zakresu obliczen. Spróbujmy uporządkowac i ewentualnie uzupełnic otrzymane dane bazujac na dotychczasowych danych: cyfrach i literach, tak uzupełnic, by uzyskac mozliwie najwiecej wnioskow.

Basic Classifier	RF SVM KNN	RF SVM KNN
Data Set	xxx	x x
Strict Accuracy		
Accuracy		
Native Precision		
Native Sensitivity		
Foreign Precision		
Foreign Sensitivity		
Native F-measure		
Foreign F-measure		
Data Set	yyy	zzz
Strict Accuracy		
Accuracy		
Native Precision		
Native Sensitivity		
Foreign Precision		
Foreign Sensitivity		
Native F-measure		
Foreign F-measure		

TODO: na razie wlasciwe wyniki sa w 2 tabelkach, czesciowo sie powtarzaja, jak tabelki zostana uzupelnione bedziemy myslec o atrakcyjniejszym przedstawieniu wynikow (wykresy)

## 5.7 MVEE Parameters and their Influence on Rejection Quality

## 6 Conclusion

Proposed ...

In future ....

Let us conclude this paper by saying that various adaptations of the idea of foreign elements rejection have a vital role in modern machine learning. It is needless to mention areas such as text mining, fraud detection, or medical diagnosis systems where we deal with various reincarnations of the foreign elements. In this perspective we believe that the study in this direction is worth further efforts.

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