CSCI 411 - Advanced Algorithms and Complexity
Assignment 5

September 22, 2022

Solutions to the written portion of this assignment should be submitted via PDF to Blackboard. Make sure to justify your answers. C++ code should be submitted on both Blackboard and turnin. Both parts of the assignment are due before **October 2nd at 11:59 pm**.

There will likely be time in class to discuss these problems in small groups and I highly encourage you to collaborate with one another outside of class. However, you must write up your own solutions **independently** of one another. Feel free to communicate via Discord and to post questions on the appropriate forum in Blackboard. Do not post solutions. Also, please include a list of the people you work with at the top of your submission.

Written Problems

1. (Adapted from exercise 6.2 from Sanjoy Dasgupta, et al., *Algorithms*, McGraw Hill, 2008)
   You are going on a long trip. You start on the road at mile post 0. Along the way there are n hotels, at mile posts $a_1 < a_2 < \cdots < a_n$, where each a_i is measured from the starting point. The only places you are allowed to stop are at these hotels, but you can choose which of the hotels you stop at. You must stop at the final hotel (at distance a_n), which is your destination.

   You'd ideally like to travel 200 miles a day, but this may not be possible (depending on the spacing of the hotels). If you travel x miles during a day, the *penalty* for that day is $(200 - x)^2$. You want to plan your trip so as to minimize the total penalty - that is, the sum, over all travel days, of the daily penalties.

   (a) (15 pts) Describe the optimal substructure of this problem. In particular, define the minimum total penalty in terms of penalties for shorter trips. Justify your answer.

   (b) (10 pts) Let the sequence of hotel locations be $A = [a_1, a_2, \ldots, a_n]$. Write pseudocode for a function `hotelSequence(A)` which returns the minimum total penalty.

   (c) (5 pts) Analyze the asymptotic run time of your algorithm.

2. Given an undirected graph $G = (V, E)$, a vertex cover of G is a subset of vertices $C \subseteq V$ such that, for every edge $\{u, v\} \in E$, either $u \in C$ or $v \in C$. It turns out that, while finding a vertex cover of minimum size is difficult for general graphs, it can be done efficiently for trees using dynamic programming.

   (a) (25 pts) Describe the optimal substructure of this problem. In particular, given a tree $T = (V, E)$, define the size of a minimum vertex cover in terms of smaller trees. Justify your answer.

   (b) (15 pts) Write pseudocode for a function `treeVC(T)` which returns a minimum vertex cover of T.

   (c) (5 pts) Analyze the asymptotic run time of your algorithm.

Coding Problem

(25 pts) Write a C++ implementation of the pseudocode you developed for problem (1b) and submit to Blackboard and to turnin as assignment_5.cpp. You may find the skeleton code in assignment_5_skeleton.cpp on Blackboard helpful.

- Input will come from cin
  - The first line will include two integers, n and m, separated by a space.
    - $4 \leq n \leq 1000$ is the total number of hotels.
    - $10 \leq m \leq 1000$ is the ideal number of miles traveled per day. In question 1, $m = 200$.
  - n lines follow.
  - Each line contains the distance of a particular hotel from the starting point.
  - You may assume that these distances are in sorted order.
- Print output to cout
  - Your output should consist of two lines.
  - The first line should be a space separated list of integers between 1 and n indicating the hotels visited. These should be in ascending order.
  - The second line should be the minimum total penalty associated with the hotels visited.

Examples

Example 1:

Input:
4 200
200
400
600
800

Expected output:
1 2 3 4

0

Example 2:

Input:
4 200
190
210
390
590

Expected output:
1 3 4
100

Example 3:

Input:
11 10
0
1
2
3
4
5
6
7
8
9
10

Expected output:
11
0