The Formal Semantics of Programming Languages / Chapter1-3

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month DD, YYYY

Diagonal Argument

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- If $y \in Y$ then $y \notin Y$, because $y \notin \theta(y)$.

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In either case, we have contradiction. $\square$

Why it's Called "Diagonal" Arugument

consider following table where ith row and jth column is placed 1 if $x_i \in \theta(x_j)$ and 0 otherwise

$$
\begin{array}{c|cccccc}
 & \theta(x_0) & \theta(x_1) & \theta(x_2) & \cdots & \theta(x_j) & \cdots \\
\hline
x_0 & 0 & 1 & 1 & \cdots & 1 & \cdots \\
x_1 & 1 & 1 & 0 & \cdots & 0 & \cdots \\
x_2 & 0 & 0 & 1 & \cdots & 1 & \cdots \\
\vdots & \vdots & \vdots & \vdots & & \vdots & \\
x_i & 0 & 1 & 0 & \cdots & 1 & \cdots \\
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$$Y = \{x_0\}$$

Direct and Inverse Image of Relation

Definition (direct image)