

## 2.4 The Execution of Commands

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# Execution of Commands

## Definition (execution relation)

A relation

$$\langle c, \sigma \rangle \rightarrow \sigma'$$

means that execution of command  $c$  in state  $\sigma$  terminates in final state  $\sigma'$ .

## Example

$$\langle X := 5, \sigma \rangle \rightarrow \sigma'$$

# Replacement of Location

## Notation

Let  $\sigma$  be a state. Let  $m \in \mathbf{N}$ . Let  $X \in \mathbf{Loc}$ . A new state which obtained from  $\sigma$  by replacing its contents in  $X$  by  $m$  is denoted as follows;

$$\sigma[m/X].$$

And we have

$$\sigma[m/X](Y) = \begin{cases} m & \text{if } Y = X \\ \sigma(Y) & \text{if } Y \neq X \end{cases}$$

## Example (Quiz)

- $(\sigma[5/X])[3/Y](X) = ?$
- $\sigma[2/Y](X) = ?$

# Rules for Commands 1

## Definition

Atomic commands

$$\frac{}{\langle \mathbf{skip}, \sigma \rangle \rightarrow \sigma} \quad \frac{\langle a, \sigma \rangle \rightarrow m}{\langle X := a, \sigma \rangle \rightarrow \sigma[m/X]}$$

Sequencing

$$\frac{\langle c_0, \sigma \rangle \rightarrow \sigma'' \quad \langle c_1, \sigma'' \rangle \rightarrow \sigma'}{\langle c_0; c_1, \sigma \rangle \rightarrow \sigma'}$$

Conditionals

$$\frac{\langle b, \sigma \rangle \rightarrow \mathbf{true} \quad \langle c_0, \sigma \rangle \rightarrow \sigma'}{\langle \mathbf{if } b \mathbf{ then } c_0 \mathbf{ else } c_1, \sigma \rangle \rightarrow \sigma'} \quad \frac{\langle b, \sigma \rangle \rightarrow \mathbf{false} \quad \langle c_1, \sigma \rangle \rightarrow \sigma'}{\langle \mathbf{if } b \mathbf{ then } c_0 \mathbf{ else } c_1, \sigma \rangle \rightarrow \sigma'}$$

## Rules for Commands 2

### Definition

While-loops

$$\frac{\langle b, \sigma \rangle \rightarrow \mathbf{false}}{\langle \mathbf{while } b \text{ do } c, \sigma \rangle \rightarrow \sigma}$$
$$\frac{\langle b, \sigma \rangle \rightarrow \mathbf{true} \quad \langle c, \sigma \rangle \rightarrow \sigma'' \quad \langle \mathbf{while } b \text{ do } c, \sigma'' \rangle \rightarrow \sigma'}{\langle \mathbf{while } b \text{ do } c, \sigma \rangle \rightarrow \sigma'}$$

# Equivalence Relation on Commands

## Definition (equivalence relation $\sim$ on commands)

Let  $c_0$  and  $c_1$  be a command, a equivalence relation  $\sim$  on commands is defined as follows;

$$c_0 \sim c_1 := \forall \sigma, \sigma' \in \Sigma. \langle c_0, \sigma \rangle \rightarrow \sigma' \iff \langle c_1, \sigma \rangle \rightarrow \sigma'.$$

## Example

- $X := 5 \sim X := 2 + 3$  ?
- $\text{if true then } X := 5 \text{ else } X := 0 \sim \text{if false then } X := 0 \text{ else } X := 5$  ?
- $X := 5; \text{skip} \sim X := 5; \text{while false do } X := 0$  ?