2.4 The Execution of Commands

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June 15, 2023

Execution of Commands

Definition (execution relation)

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$$\langle c, \sigma \rangle \to \sigma'$$

means that execution of command c in state σ terminates in final state σ'.

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- $\langle X := 5, \sigma \rangle \to \sigma'$
- $\langle \mathbf{skip}, \sigma \rangle \to \sigma$

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Example

- $\langle X := 5, \sigma \rangle \to \sigma'$
- $\langle \mathbf{skip}, \sigma \rangle \to \sigma$
- $\langle \mathbf{while\ false\ do}\ X := 5, \sigma \rangle \to \sigma$

Replacement of Location

Notation

Let σ be a state. Let $m \in \mathbf{N}$. Let $X \in \mathbf{Loc}$. A new state which obteined from σ by replacing its contents in X by m is denoted as follows;

$$\sigma[m/X].$$

And we have

$$\sigma[m/X](Y) = \begin{cases} m & \text{if} \quad Y = X \\ \sigma(Y) & \text{if} \quad Y \neq X \end{cases}$$

Example (Quiz)

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- $\sigma[2/Y](X) =$?

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- $(\sigma[5/X])[3/Y](X) = 5$
- $\sigma[2/Y](X) = \sigma(X)$

Rules for Commands 1

Definition (commands execution)

Atomic commands

$$\frac{}{\langle \mathbf{skip}, \sigma\rangle \to \sigma} \qquad \frac{\langle a, \sigma\rangle \to m}{\langle X := a, \sigma\rangle \to \sigma[m/X]}$$

Sequencing

$$\frac{\langle c_0, \sigma\rangle \to \sigma'' \quad \langle c_1, \sigma''\rangle \to \sigma'}{\langle c_0; c_1, \sigma\rangle \to \sigma'}$$

Conditionals

$$\frac{\langle b, \sigma\rangle \to \mathbf{true} \quad \langle c_0, \sigma\rangle \to \sigma'}{\langle \mathbf{if}\ b\ \mathbf{then}\ c_0\ \mathbf{else}\ c_1, \sigma\rangle \to \sigma'} \qquad \frac{\langle b, \sigma\rangle \to \mathbf{false} \quad \langle c_1, \sigma\rangle \to \sigma'}{\langle \mathbf{if}\ b\ \mathbf{then}\ c_0\ \mathbf{else}\ c_1, \sigma\rangle \to \sigma'}$$

Rules for Commands 2

Definition (commands execution)

While-loops

$$\frac{\langle b, \sigma\rangle \to \mathbf{false}}{\langle \mathbf{while}\ b\ \mathbf{do}\ c, \sigma\rangle \to \sigma}$$

$$\frac{\langle b, \sigma\rangle \to \mathbf{true} \quad \langle c, \sigma\rangle \to \sigma'' \quad \langle \mathbf{while}\ b\ \mathbf{do}\ c, \sigma''\rangle \to \sigma'}{\langle \mathbf{while}\ b\ \mathbf{do}\ c, \sigma\rangle \to \sigma'}$$

Example

A deriving tree of $\langle X := 2; \textbf{if } X \leq 3 \textbf{ then } X := 0 \textbf{ else } X := 1, \sigma\rangle \to \sigma[0/X]$ is

$$\dfrac{\dfrac{\langle 2, \sigma\rangle \to 2}{\langle X := 2, \sigma\rangle \to \sigma[2/X]} \qquad \dfrac{\dfrac{\dfrac{}{\langle X, \sigma[2/X]\rangle \to 2} \quad \dfrac{}{\langle 3, \sigma[2/X]\rangle \to 3}}{\langle X \leq 3, \sigma[2/X]\rangle \to \textbf{true}} \quad \dfrac{\langle 0, \sigma[2/X]\rangle \to 0}{\langle X := 0, \sigma[2/X]\rangle \to \sigma[0/X]}}{\langle \textbf{if } X \leq 3 \textbf{ then } X := 0 \textbf{ else } X := 1, \sigma[2/X]\rangle \to \sigma[0/X]}}{\langle X := 2; \textbf{if } X \leq 3 \textbf{ then } X := 0 \textbf{ else } X := 1, \sigma\rangle \to \sigma[0/X]}.$$

Equivalence Relation on Commands

Definition (equivalence relation $\sim$ on commands)

Let c_0 and c_1 be a command, a equivalence relation $\sim$ on commands is defined as follows;

$$c_0 \sim c_1 := \forall \sigma, \sigma' \in \Sigma. \langle c_0, \sigma \rangle \to \sigma' \iff \langle c_1, \sigma \rangle \to \sigma'.$$

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- does $X := 5; \mathbf{skip} \sim X := 5; \mathbf{while\ false\ do}\ X := 0$ hold? yes