The Formal Semantics of Programming Languages / Chapter1-3

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Diagonal Argument

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Suppose $Y = \{x \in X \mid x \notin \theta(x)\}$. Y is a subset of X and therefore in correspondence with a $y \in X$.

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- If $y \in Y$ then $y \notin Y$, because $y \notin \theta(y)$.

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- If $y \in Y$ then $y \notin Y$, because $y \notin \theta(y)$.
- If $y \notin Y$ then $y \in \theta(y)$, so $y \in Y$.

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- If $y \notin Y$ then $y \in \theta(y)$, so $y \in Y$.

In either case, we have contradiction. $\square$