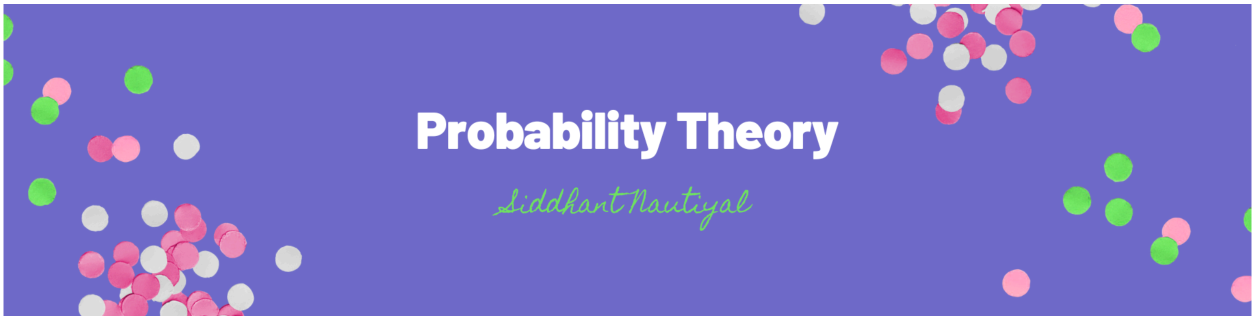

Probability Theory

Probability Theory for Machine Learning and Business Statistics.

Probability

- Probability is the **likelihood** of an event occurring, that ranges from 0 (*impossible*) to 1 (*definite*).
- When a **trial** is performed multiple times under **identical circumstances**, the ratio of favorable outcomes to all possible outcomes is called the probability of an **event**.

$$P(e) = FavorableOutcomes/AllPossibleOutcomes$$

Trial and Event

- A trial is a **single instance** of a **random experiment** where an outcome is observed.
- For example: Flip a fair coin or roll a die.
- An event is an **outcome** or **set of outcomes** that occur as a result of a trial in a random experiment.
- For example: getting a head when a coin is flipped (single outcome), and getting an even number when a die is rolled (a set of outcomes).
- A trial is a process and an event is the outcome of that process.

Random Experiment and Independent Event

- A random experiment is a **series of trials** where the outcome of each trial is independent.
- For example: Flip a fair coin two times.
- An independent event is the **outcome of trials** in a random experiment.
- For example: Get simultaneous heads when two fair coins are flipped.
- Mathematically, two events A and B are said to be independent of each other if any of the following conditions hold:

$$P(A/B) = P(A) or P(B/A) = P(B)$$

- The probability of events A and B happening can be obtained with the multiplication rule of independent events.

$$P(AandB) = P(A) * P(B)$$

- P (2 heads when flipping two fair coins) = P(head for coin 1) * P(head for coin 2)
- P (2 heads when flipping two fair coins) = 1/2 ∗ 1/2 = 1/4
- Alternatively, the probability of this independent event can be obtained with its **sample space**.
- P (head for both coins) = {(H, H), (H, T), (T, T), (T, H)} which is 1/4.

Sample Space

- Sample Space **(S)**, is the set of all possible outcomes of a random experiment.
- Getting an even number when rolling a die, S = {2,3,6}
- Getting a prime number when rolling a die, S = {2,3,5}

Addition Rule for Probability

Addition Rule for Mutually Exclusive Events

- When two events A and B are **mutually exclusive**, the probability that A or B will occur is the sum of the probability of each event.

$$P(AUB) = P(A) + P(B)$$

For example - The probability of getting 3 or 4 while rolling a die:

- P (3 U 4) = P(3) + P(4), 1/6 + 1/6, that is 1/3.

General Rule of Addition

- When two events A and B are not mutually exclusive, the probability that A or B will occur is the sum of the probability of each event minus the intersection of A and B.

$$P(AUB) = P(A) + P(B) - P(A \cap B)$$

For example - The probability of drawing a heart or ace card from a deck of cards:

- P (Heart or Ace) = P(Heart) + P (Ace) - P (Heart∩Ace)
- P (Hear or Ace) = 13/52 + 4/52 - 1/52, that is 4/13.

Cumulative Probability

Cumulative probability is the likelihood that the value of a random variable is within a given range.

For example - Consider a random experiment where two fair coins are flipped. The probability of getting 1 or fewer heads in this experiment could be obtained with cumulative probability:

- P (H ≤ 1) = P (H=0) + P (H=1), S = { (H, H), (H, T), (T, H), (T, T) }
- P (H ≤ 1) = 1/4 + 2/4, that is 3/4.

Conditional Probability

- **Conditional probability** is the probability of an event occurring given that another event has already occurred.
- The conditional probability of event A given event B could be obtained with:

$$P(A|B) = P(B \cap A)/P(B)$$

For example - Consider a sack containing 6 red balls and 4 black balls. Two balls are picked from the sack blindly, without replacement. The probability of both balls being red could be obtained with:

- P (A and B) = P (B and A) / P (B)
- P (B and A) = P (B) * P(A and B)
- P (Both Red Balls) = 4/10 ∗ 3/9, that is 12/90.

Another example - The probability of drawing a spade, given that it is an ace:

- P (S | A) = P (A | S) / P (A)
- P (S | A) * P (A) = P (A | S)
- 1/52 ∗ 52/4, that is 1/4.

Bayes' Theorem

- Bayes' Theorem extends condition probability by inverting the probabilities.
- Conditional probability determines the probability that A given B has occurred.
- Bayes' theorem determines the probability of B given A when A given B is known.

$$P(A|B) = P(B|A) * P(A)/P(B)$$

Consider the same example - The probability of drawing a spade, given that it is an ace:

- P (S | A) = P (A | S) * P (S) / P(A)

- $P(A \mid S) = 1/13$, $P(S) = 13/52$ that is $1/4$, and $P(A) = 4/52$ that is $1/13$.
- $P(S \mid A) = 1/13 * 1/4 \; / \; 1/13$, that is $1/4$.