Machine Learning: Exercise Set I

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1 Probabilities (Theoretical)

1.1 Couples

For couples that have exactly 2 kids we have the following cases, assuming binary genders: DD, DS, SD, SS where D is for Daughter and S is for Son.

Assuming the genders are equi-probable when giving birth it we can say that each case is also equally probable on a single couple. Hence the cases of a single couple having 2 kids with different genders are DS and SD, $\frac{2}{4}$ of the total cases, so the probability is 0.5.

Respectively, given that the gender of a couple's kids is an independent event from what another couple's kids' genders are, for 2 couples (A, B) the probability of both of them having different gender kids is $P(A_{DS} \cup A_{SD}) * P(B_{DS} \cup B_{SD}) = 0.5^2 = 0.25$

Finally, the probability for all n couples ($A_i,\ i \in [0, n-1]$) to have 2 kids with different genders is:

$$\prod_{i=0}^{n-1} P(A_{DS}^{(i)} \cup A_{SD}^{(i)}) = (0.5)^n,\ n \in \mathbb{N}$$

1.2 Fair Coin

Coin flips are also an independent from each other. Given the coin is fair, the probability of getting Heads once is 0.5. Following the previous logic for repeating the same experiment n times independently, the probability for getting n times heads is $(0.5)^n,\ n \in \mathbb{N}$

1.3 Magic Beads

We define the following events: R, G, B representing the event of a random picked bead being Red, Green, Blue respectively and H for the bead being hollow. Given that, we are looking for $P(H)$. Using the law of total probability, we can calculate it at follows:

$$P(H) = P(H \cap R) + P(H \cap G) + P(H \cap B)$$

So we just need to find a way to calculate one of those 3 terms and the others will follow. Thankfully, uncle Bayes is here to help! ☺

Starting with Reds, using the chain rule, the probability of a random picked bead being hollow and red is:

$$P(H \cap R) = P(H|R)P(R) = 0.5 \cdot 0.3 = 0.15$$

Respectively:

$$P(H \cap G) = P(H|G)P(G) = 0.66 \cdot 0.5 = 0.33$$

$$P(H \cap B) = P(H|B)P(B) = 0.66 \cdot 0.3 = 0.132$$

Finally:

$$P(H) = 0.15 + 0.33 + 0.132 = 0.612$$

2 Bayesian Theorem (Programming)

2.1 Theoretical Estimation

We define the following events: L for the event of receiving a photon package, D for the detector reporting a detection. We are looking for $P(L|D)$. Uncle Bayes, I summon thee! ☺

$$P(L|D) = \frac{P(D|L) \cdot P(L)}{P(D|L) \cdot P(L) + P(D|\neg L) \cdot P(\neg L)}$$

We know that $P(D|\neg L) = 0.1$, $P(D|L) = 0.85$ and $P(L) = 1e - 7$, which implies $P(\neg L) = 1 - 1e - 7 = 0.9999999$. So:

$$\implies P(L|D) = \frac{0.85 \cdot 10^{-7}}{0.85 \cdot 10^{-7} + 0.1 \cdot 0.9999999} \approx 8.49999362500478e - 07$$

2.2 Package Energy

The code is pretty much self explanatory, so I'll focus on the results on this report.

After creating a histogram with the total package energy on our experiments, we can see that even though we uniformly sample the energies on our experiments, the resulting distribution follows a normal (Gaussian) distribution. This is due to the Central Limit Theorem as the experiment results are independent and identically distributed random variables.

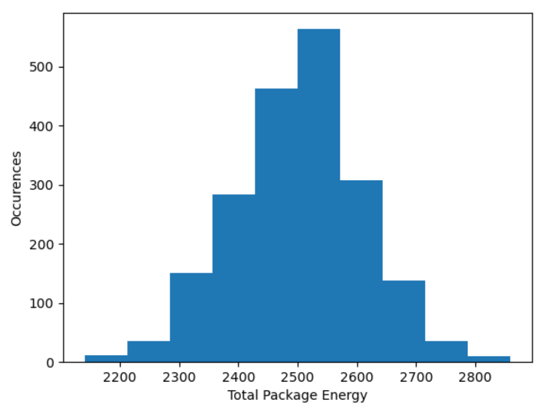

Figure 1: The histogram of the various total photon packages energies resulted from our experiment

2.3 Stochastic Emissions

This distribution also follows a Normal Distribution (cut in half due to the rejected negative samples, and x axis is scaled down to the 10^{-6}). This experiment makes more sense as it takes in factor the environmental factors (atmospheric interactions in this case) for the stochastic emissions of the star.

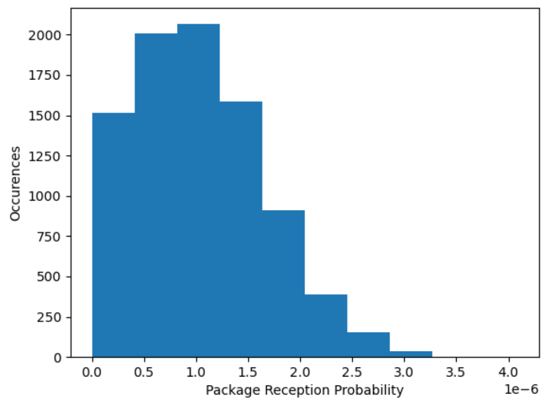

Figure 2: The histogram of the various reception probabilities w.r.t. the various sampled photon probabilities

3 Appendix

Figure 3: Of course, I didn't forget to put labels on my **axes**.