Machine Learning: Exercise Set IV

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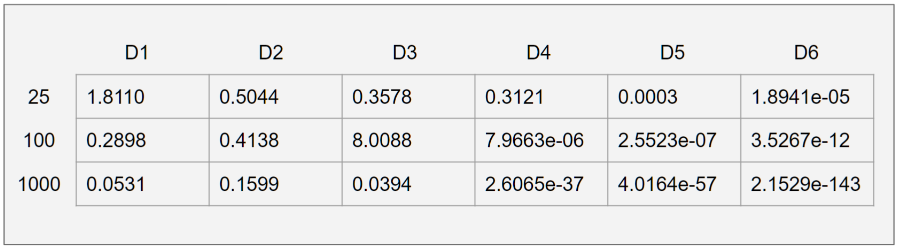
1 Hypothesis Testing

Starting off with the implementation details, I ran the experiments for 100 repeats and 100 permutations as I didn't have enough time for more. I'll provide all the requested plots below ($6 \cdot 3 = 18$, for each dataset and sample size), which are all labeled and describe what they represent. I also did not tamper with the histogram bins. Finally, I did not use any library functions and I implemented the CDF myself, with the formula provided by the recitation document.

1.1 Original p-value

We have the p-values table for each dataset and sample size:



	D1	D2	D3	D4	D5	D6
25	1.8110	0.5044	0.3578	0.3121	0.0003	1.8941e-05
100	0.2898	0.4138	8.0088	7.9663e-06	2.5523e-07	3.5267e-12
1000	0.0531	0.1599	0.0394	2.6065e-37	4.0164e-57	2.1529e-143

Figure 1: p-values from Chi square statistic and CDF for each dataset and sample size

If we set as acceptance threshold 0.05, we can reject null hypothesis for all cases with p-value < 0.05 and accept the others ((D3, 1000), (D4, 100), (D4, 1000), D5 and D6 columns will be rejected)

Note that the results vary a lot for different runs due to the random sampling.

1.2 Permutation Testing

The permutation testing proceedure is executed as follows:

```
1. for each dataset and for each sample size
   1.1. for N repeats
        1.1.1. random sample the dataset using the provided joint probability distributions and calculate chi test statistic $t_o$.
        1.1.2. for K permutations
               1.1.2.1. Shuffle X column, compute $X^2$ statistic and store the result
        1.1.3. for all permutation results, calculate the percentage of permutations where $|t_b| \geq |t_o|$, where $|t_b|$ the permutation Chi statistic test result and $|t_o|$ the original Chi statistic test
   1.2. plot p-value over N repeats
```

All tests were made with $N = 100$ and $K = 100$ as they were taking too long to produce. In general, we can see that for the cases we rejected the null hypothesis we usually have more $\hat{p}$-values near 0 (larger bins). The results vary a lot whenever we repeat the test, but they usually follow a similar distribution. Those distributions usually follow an increasing or decreasing pattern, except on the clearly null hypothesis rejection cases such as dataset 5 and 6. There are also some cases where they approximate a uniform distribution, which suggests that there is a high uncertainty for that case and generated dataset.

We note that as the dataset size increases, the distribution changes totally. The results would probably be more robust if they were done with more permutations and repeats, but as they require quite some running time I wasn't able to complete them in time. Also, for smaller datasets I get more nan values in T as there is a smaller probability to sample all combinations of X, Y.

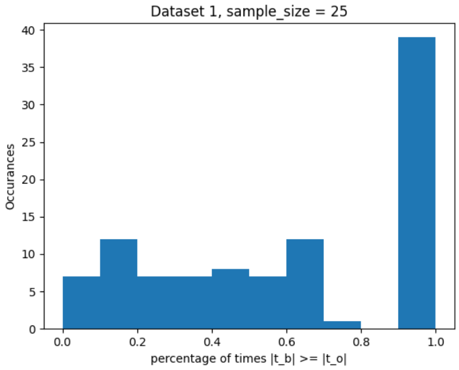


Figure 2: p-values distribution for dataset 1 and 25 samples

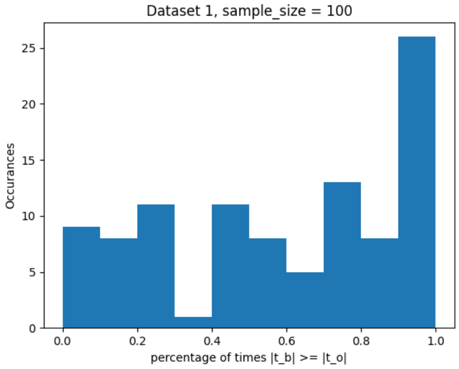


Figure 3: p-values distribution for dataset 1 and 100 samples

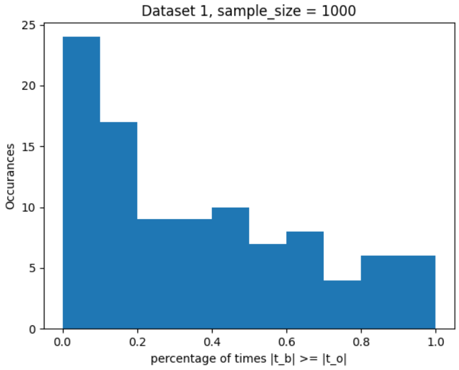


Figure 4: p-values distribution for dataset 1 and 1000 samples

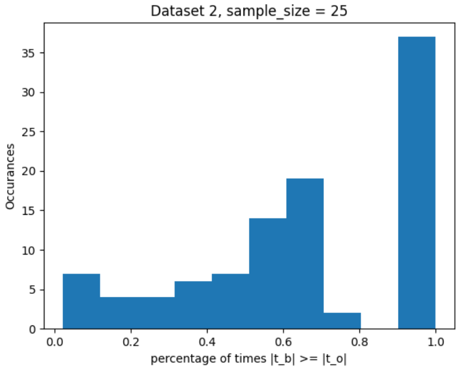


Figure 5: p-values distribution for dataset 2 and 25 samples

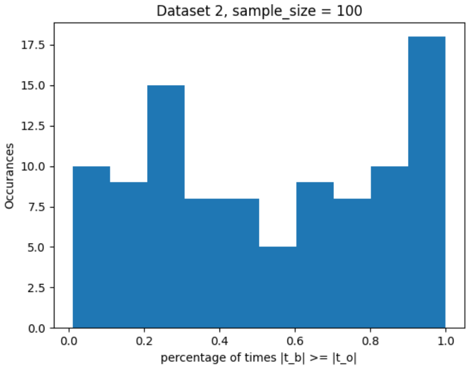


Figure 6: p-values distribution for dataset 2 and 100 samples

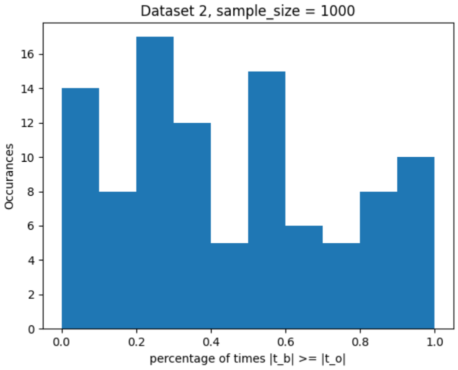


Figure 7: p-values distribution for dataset 2 and 1000 samples

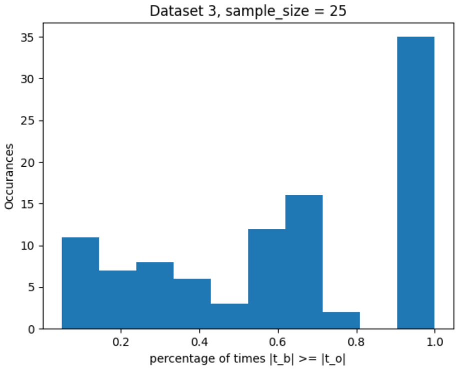


Figure 8: p-values distribution for dataset 3 and 25 samples

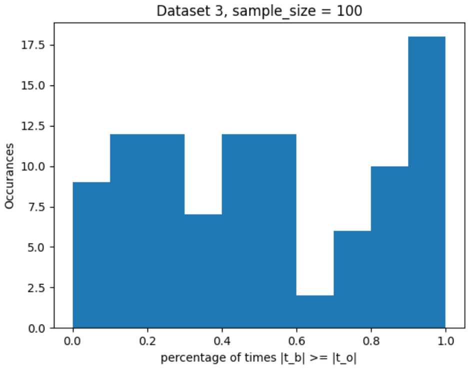


Figure 9: p-values distribution for dataset 3 and 100 samples

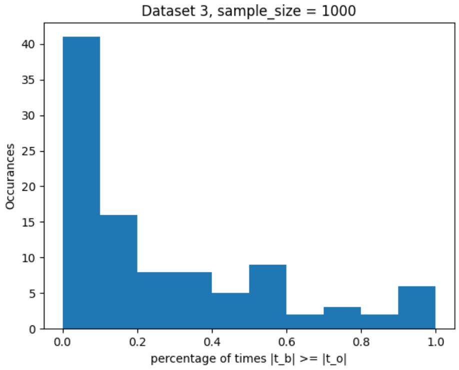


Figure 10: p-values distribution for dataset 3 and 1000 samples

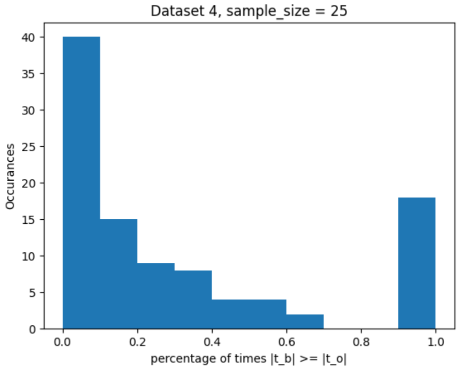


Figure 11: p-values distribution for dataset 4 and 25 samples

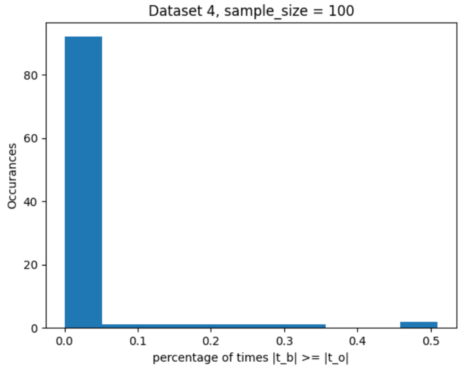


Figure 12: p-values distribution for dataset 4 and 100 samples

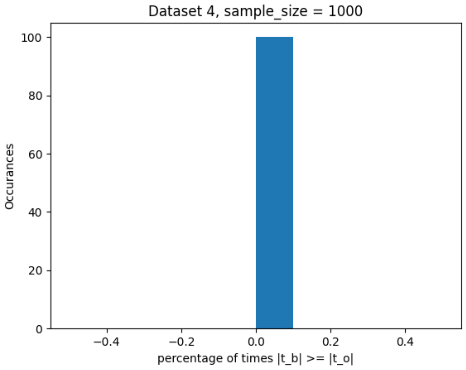


Figure 13: p-values distribution for dataset 4 and 1000 samples

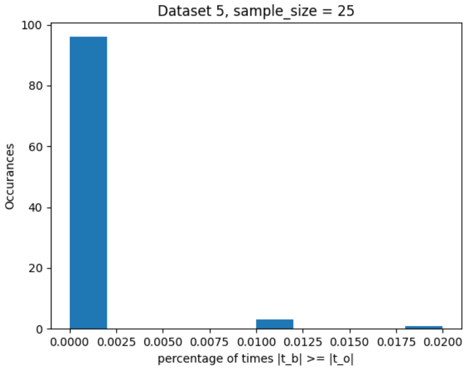


Figure 14: p-values distribution for dataset 5 and 25 samples

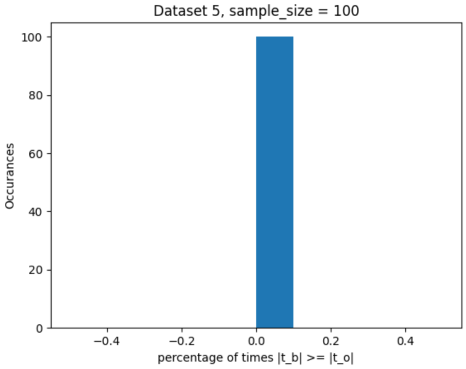


Figure 15: p-values distribution for dataset 5 and 100 samples

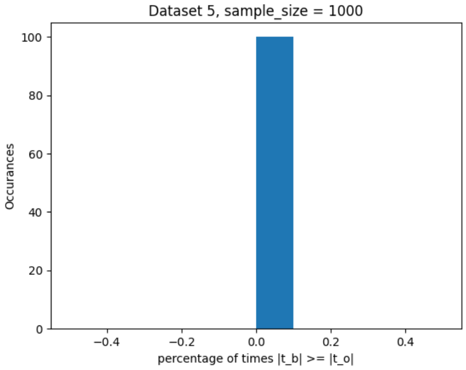


Figure 16: p-values distribution for dataset 5 and 1000 samples

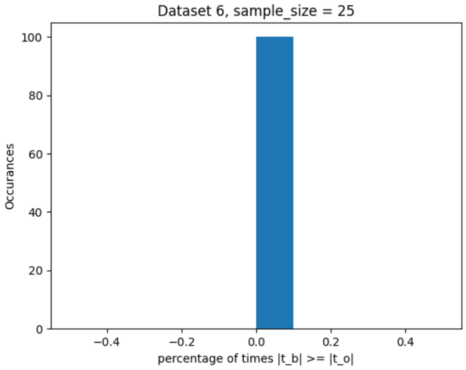


Figure 17: p-values distribution for dataset 6 and 25 samples

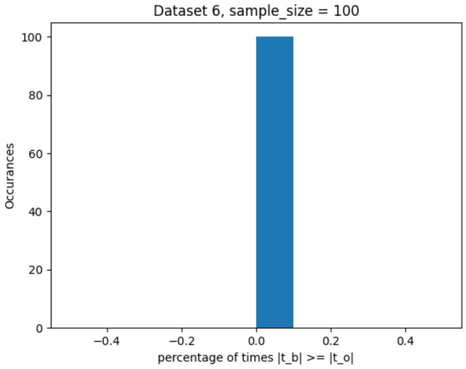


Figure 18: p-values distribution for dataset 6 and 100 samples

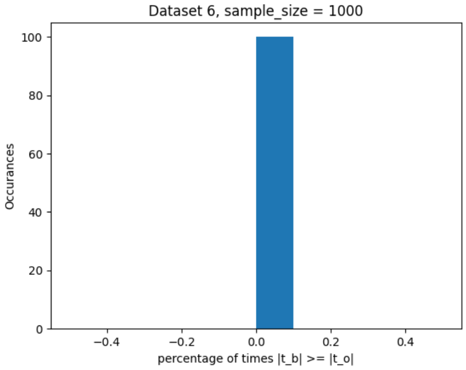


Figure 19: p-values distribution for dataset 6 and 1000 samples