Matrices

WaterExecution

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Tutorial 1 – Matrices

1a. $\begin{bmatrix} 2 & -3 & 1 & 7 \end{bmatrix}$

Ans: 1 x 4 (row x column)

1b. $\begin{bmatrix} 1 \\ 2 \\ 5 \\ 6 \end{bmatrix}$

Ans: 4 x 1 (row x column)

1c. $\begin{bmatrix} 2 & -1 & 3 \\ 0 & 3 & 1 \end{bmatrix}$

Ans: 2 x 3 (row x column)

1d. $\begin{bmatrix} 1 & 6 \\ -1 & -3 \\ 2 & 0 \end{bmatrix}$

Ans: 3 x 2 (row x column)

2. $\begin{bmatrix} 1 & 6 & 2 \\ -1 & -3 & 5 \\ 2 & 0 & 7 \end{bmatrix}$

Ans:

$$a_{2\,2} = -3$$

$$a_{2\,3} = 5$$

$$a_{3\,2} = 0$$

3.

Ans: $a^T = \begin{bmatrix} 2 \\ -3 \\ 1 \\ 7 \end{bmatrix}$ $b^T = \begin{bmatrix} 1 & 2 & 5 & 6 \end{bmatrix}$ $c^T = \begin{bmatrix} 2 & -1 & 3 \\ 0 & 3 & 1 \end{bmatrix}$ $d^T = \begin{bmatrix} 1 & -1 & 2 \\ 6 & -3 & 0 \\ 2 & 5 & 7 \end{bmatrix}$

Tutorial 1 − Matrices

5.

$$\begin{bmatrix} 1 & 0 & 2k+3 \\ 0 & k & 2 \\ k^2 & 2 & 3 \end{bmatrix}$$

A symmetric matrix is a matrix which when transposed has the same value as its original value.

Ans:

$$k^2 = 2k + 3(m_{3\,1} = m_{1\,3})$$

$$k^2 - 2k - 3 = 0$$

$$(k+1)(k-3) = 0$$

$$k \;=\; 3\,or\;-1$$

Factorising quadratics:(https://thirdspacelearning.com/gcse-maths/algebra/factorising-quadratics/)

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9.
A is a 2x3 matrix and B is a 3x2 matrix.
Which of the following is possible?

$$AB,\ BA,\ AB^T,\ BA^T,\ A^TB^T,\ B^TA^T,\ A^2,\ B^2$$

Column = Row
Ans:
AB ✓(2x3,3x2 = 2x2)
BA ✓(3x2,2x3 = 2x2)
AB^T (2x3,2x3)
BA^T (3x2,3x2)
A^TB^T ✓(3x2,2x3 = 2x2)
B^TA^T ✓(2x3,3x2 = 2x2)
A^2 (2x3,2x3)
B^2 (3x2,3x2)

10.
2x3,B = 2x4
Ans: B is a 3x4 matrix

11.
Check conformability before calculating.
a✓b✓c✓d✓e (2x4, 4x1, 2x4) f✓
Matrix 11e is not conformable for multiplication.

17.
A= $\begin{bmatrix} k & k \\ -2 & k \\ 1 & 1 \end{bmatrix}$

$A^TA = \begin{bmatrix} k & -2 & 1 \\ k & k & 1 \end{bmatrix} \begin{bmatrix} k & k \\ -2 & k \\ 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} k^2+4+1 & k^2-2k+1 \\ k^2-2k+1 & k^2+k^2+1 \end{bmatrix} = \begin{bmatrix} x & 0 \\ 0 & y \end{bmatrix}$(Diagonal matrix)

$k^2 - 2k + 1 = 0$
(k-1)(k-1)=0
k = 1//

Tutorial 1 – Matrices

18b.
Why $A^2 - B^2 \neq (A+B)(A-B)$?
Ans:
$(A+B)(A-B) = A^2 - AB + BA - B^2$
Since $AB \neq BA$, $A^2 - B^2 \neq A^2 - AB + BA - B^2$

19a.
Ans:

x $\begin{bmatrix}1\\2\end{bmatrix}$ +y $\begin{bmatrix}3\\1\end{bmatrix}$ $= \begin{bmatrix}x+3y\\2x+y\end{bmatrix}$
$\begin{bmatrix}x+3y\\2x+y\end{bmatrix} =$ A $\begin{bmatrix}x\\y\end{bmatrix}$
$\begin{bmatrix}1 & 3\\2 & 1\end{bmatrix} \begin{bmatrix}x\\y\end{bmatrix} =$ A $\begin{bmatrix}x\\y\end{bmatrix}$
A= $\begin{bmatrix}1 & 3\\2 & 1\end{bmatrix}$

23.
a)$AB = mI$
b)$A^2 + mA + nI = 0$
c)$A + B = nAB$
When multiple matrix position matters! $AB \neq BA$ if multiplying on left, do it on left for ALL.
Ans:
a)
$A^{-1}AB = mA^{-1}$ (Times A^{-1} on both sides)
$mA^{-1} = B$
$A^{-1} = \frac{1}{m}B$

b)
$A^2 + mA + = -nI$
$A^2A^{-1} + mAA^{-1} + = -nA^{-1}$ (Times A^{-1} on both sides)
$A + mI = -nA^{-1}$
$A^{-1} = -\frac{1}{n}(A + mI)$

c)
$AB^{-1} + BB^{-1} = nABB^{-1}$
$AB^{-1} + I = nA$
$A^{-1}AB^{-1} + A^{-1} = nA^{-1}A$
$B^{-1} + A^{-1} = nI$
$A^{-1} = nI - B^{-1}$

Lesson Learnt

- Order of matrices (row x column)
- Subscript notation of matrices ($M_{3\,1}$ = Row 3 Column 1)
- Transposing matrices
- A symmetric matrix is a matrix which when transposed has the same value as its original value
- Factorising quadratics:(https://thirdspacelearning.com/gcse-maths/algebra/factorising-quadratics/)
- Multiplication Conformability (Column = Row)
- Check conformability before calculating
- Give reason if not conformable (not conformable for multiplication)
- What a diagonal matrix is: $\begin{bmatrix} x & 0 \\ 0 & y \end{bmatrix}$
- $AB \neq BA$, position matters