① Let $\Sigma = \{v, w, x, y\}$ and $A = \bigcup_{n=1}^{4} \Sigma^n$. How many strings in A have xy as a proper prefix?

Ans) Here $\Sigma = \{v, w, x, y\}$

$\Rightarrow |\Sigma| = 4$

Since we are fixing to string in two place.

So to have xy as a proper prefix we have atmost 3 length string and 4 length string.

So total no. of string with xy as a proper prefix is

$\bigcup_{n=3}^{4} |\Sigma|^{n-2}$

For $n=3$ we have $(4)^{3-2} = (4)^1 = 4$ | x | y | 4 |

for $n=4$ we have $(4)^{4-2} = (4)^2 = 16$ | x | y | 4 | 4 |

So total no. of string with xy as a proper prefix is $4+16 = 20$ strings.

② Let Σ be an alphabet. Let $x_i \in \Sigma$ for $1 \le i \le 100$ (where $x_i \ne x_j$ $\forall$ $1 \le i \le j \le 100$). How many nonempty substrings are there for the string $s = x_1 x_2 \cdots x_{100}$?

Ans) Given an alphabet $x_i \in \Sigma$ for $1 \le i \le 100$ ($\because x_i \ne x_j$)

$\forall$ $1 \le i \le j \le 100$

string $s = x_1 x_2 \cdots x_{100}$

Nonempty strings:-

string with length 1 = $x_1, x_2, x_3, \cdots, x_{100} = 100$

string with length 2 = $x_1x_2, x_2x_3, \cdots, x_{99}x_{100} = 99$

⋮

string with length 100 = $x_1x_2x_3 \cdots x_{100} = 1$

So total number of nonempty substrings $= 100 + 99 + \cdots + 1 = \sum_{i=1}^{100} \frac{100 \times 101}{2} = 5050$

$\therefore$ There are total 5050 non empty substrings present.

③ For each $\Sigma = \{0,1\}$ determine whether the string 00010 is in each of the following languages (taken from Σ^*).

ⓐ $\{0,1\}^*$ ⓑ $\{000, 101\}\{10, 11\}$ ⓒ $\{00\}\{0\}^*\{10\}$ ⓓ $\{000\}^*\{1\}^*\{0\}$

ⓔ $\{00\}^*\{10\}^*$ ⓕ $\{0\}^*\{1\}^*\{0\}^*$

Ans) Given string = 00010

(a) $\{0,1\}^*$ is the lang. which include all the strings of 0s and 1s.
So, yes 00010 is accepted.

(b) $\{000, 101\}\{10, 11\}$.
This lang. is divided into 2 parts $\{000, 101\}\{10, 11\}$.
The first part $\{000\}$ can indicate to the string containing 000 and 11 will be indicating to the end part.
So $000 \in \{000, 101\}$ and $11 \in \{10, 11\}$.
So yes 00010 is accepted.

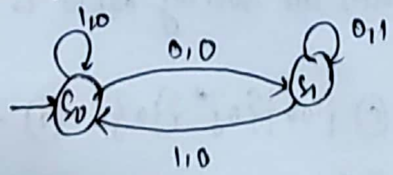
(c) $\{00\}\{0\}^*\{10\}$
This lang can accept the string 00010 as it is a concatenation of 00, 1, 0.

(d) $\{000\}^*\{1\}^*\{0\}$
This lang is divided into 3 parts $\{000\}^*\{1\}^*\{0\}$.
So this lang can accept the string 00010. Yes

(e) $\{00\}^*\{10\}^*$
This lang is divided into 2 parts $\{00\}^*\{10\}^*$.
So $00010 \notin \{00\}\{10\}$. Since there is no way to obtain odd number of starting zeros in either of the languages. No.

(f) $\{0\}^*\{1\}^*\{0\}^*$.
This lang is divided into 3 parts $\{0\}^*\{1\}^*\{0\}^*$.
This lang is divided into 3 parts $\{0\}^*\{1\}^*\{0\}^*$.
So the given string 00010 can be accepted by this lang.
As $\{000\} \in \{0\}^*$
$10 \in \{1\}^*$
$0 \in \{0\}^*$ Yes

(4) Machine M has $I = \{0,1\} = O$ and is determined by the diagram shown in this figure.



ⓐ Describe in words what is the finite state machine does?

ⓐ The machine recognises (with an output of 1) every 0 (in an input string s) that is preceeded by another 0.

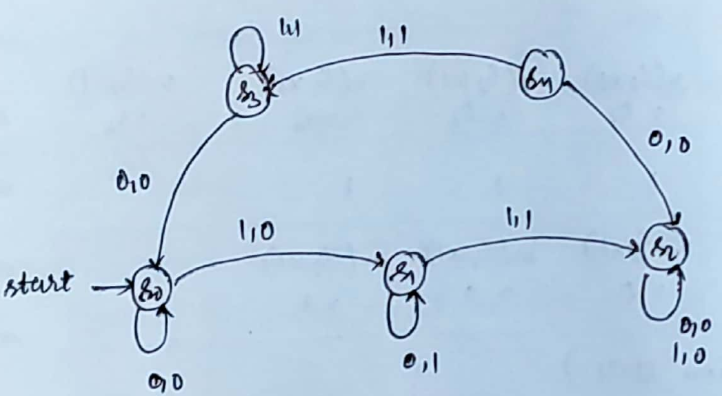
ⓑ What must state s_1 remember?

ⓑ state s_1 remember that atleast one 0 has been supplied for an input string s.

ⓒ Find two languages $A, B \subseteq I^*$ such that every $x \in AB$, $w(s_0, x)$ has 1 suffix.

ⓒ $A = \{1\}^*$, $B = \{00\}$

Q⑤ A finite state machine $M = (S, I, O, v, w)$ has $I = O = \{0,1\}$ and is determined by the state diagram :-



ⓐ Determine the output string for the input string 110111, string at s_0, what is the last transition state.

Given i/p string = 110111

ⓐ

state	s_0	$v(s_0 \times 1) = s_1$	$v(s_1 \times 1) = s_2$	$v(s_2 \times 0) = s_2$	$v(s_2 \times 1) = s_2$	$v(s_2 \times 1) = s_2$	$v(s_2 \times 1) = s_2$
input	1	1	0	1	1	1	
output	$w(s_0 \times 1) = 0$	$w(s_1 \times 1) = 1$	$w(s_2 \times 0) = 0$	$w(s_2 \times 1) = 0$	$w(s_2 \times 1) = 0$	$w(s_2 \times 1) = 0$	

∴ o/p = 010000 ; s_2 (last transistion state)

ⓑ Answer part ⓐ for the same string but with s_1 as the starting state. what about s_2 and s_3 as the starting state.

(b) Here i/p string = 110111

s_1 as the starting state :-

state	s_1	$v(s_1 \times 1) = s_2$	$v(s_2 \times 1) = s_2$	$v(s_2 \times 0) = 0$	$v(s_2 \times 1) = s_2$	$v(s_2 \times 1) = s_2$	$v(s_2 \times 1) = s_2$
input	1	1	0	1	1	1	
output	$w(s_1 \times 1) = 1$	$w(s_2 \times 1) = 0$	$w(s_2 \times 0) = 0$	$w(s_2 \times 1) = 0$	$w(s_2 \times 1) = 0$	$w(s_2 \times 1) = 0$	

$\therefore$ o/p = 100000 ; s_2 (last transistion state)

s_2 as the starting state :-

state	s_2	$v(s_2 \times 1) = s_2$	$v(s_2 \times 1) = s_2$	$v(s_2 \times 0) = s_2$	$v(s_2 \times 1) = s_2$	$v(s_2 \times 1) = s_2$	$v(s_2 \times 1) = s_2$
input	1	1	0	1	1	1	
output	$w(s_2 \times 1) = 0$	$w(s_2 \times 1) = 0$	$w(s_2 \times 0) = 0$	$w(s_2 \times 1) = 0$	$w(s_2 \times 1) = 0$	$w(s_2 \times 1) = 0$	

$\therefore$ o/p = 000000 ; s_2 (last transisistion state.)

s_3 as the starting state :-

state	s_3	$v(s_3 \times 1) = s_3$	$v(s_3 \times 1) = s_3$	$v(s_3 \times 0) = s_0$	$v(s_0 \times 1) = s_1$	$v(s_1 \times 1) = s_2$	$v(s_2 \times 1) = s_2$
input	1	1	0	1	1	1	
output	$w(s_3 \times 1) = 1$	$w(s_3 \times 1) = 1$	$w(s_3 \times 0) = 0$	$w(s_0 \times 1) = 0$	$w(s_1 \times 1) = 1$	$w(s_2 \times 1) = 0$	

$\therefore$ o/p = 110010 ; s_2 (last transistion state)

(c) Find the state table for this machine.

(c) State table for the above machine :-

	v		w	
	0	1	0	1
s_0	s_0	s_1	0	0
s_1	s_1	s_2	1	1
s_2	s_2	s_2	0	0
s_3	s_0	s_3	0	1
s_4	s_2	s_3	0	1

(d) In which state should be start so that the input string 10010 produces the output 10000?

Ans) If we start from the state s_1 it will give the output as 10000.

(e) Determine an input string $x \in I^*$ of minimal length, such that $v(s_4, x) = s_1$. Is x unique.

Ans) To reach the state s_1, starting from s_4 is only through the state s_3 and s_0.
so the required input is 1, 0, 1.

Yes, x is unique.

14/01/23

① ⓐ Rephrases the definations for the reflexive, symmetric, transitive and antisymmetric properties of a Relation R, using quantifiers.

ⓐ For reflexive property :- $\forall a \in A, (a,a) \in R$.

ⓑ For symmetric property :- $\forall a,b \in A, (a,b) \in R \Rightarrow (b,a) \in R$.

ⓒ For transistive property :-

$\forall a,b,c \in A$

$\Rightarrow (a,b) \in R \wedge (b,c) \in R$

$\Rightarrow (a,c) \in R$.

ⓓ For antisymmetric properties :-

$\forall a,b \in A$

$(a,b) \in R \wedge (b,a) \in R$

$\Rightarrow a = b$.

ⓑ Use the resnet of part ⓐ to specify when a relation R (on a set A) is (i) not reflexive, (ii) not symmetric, (iii) not transitive (iv) not antisymmetric.

Ans) (i) not reflexive :-

$\exists a \in A, (a,a) \notin R$

(ii) not symmetric :-

$\exists a,b \in A$

$(a,b) \in R \wedge (b,a) \notin R$

(iii) not transitive :-

$\exists a,b,c \in A$

$(a,b) \in R \wedge (b,c) \in R \wedge (a,c) \notin R$.

(iv) not antisymmetric :-

$\exists a,b \in A$

$(a,b) \in R \wedge (b,a) \in R \wedge a \neq b$.

② For each of the following statements about relations on a set A, where $|A| = n$, determine whether the statement is true or false. If it is false, give a counter example.

(a) If R is a relation on A and $|R| \geq n$, then R is reflexive.

(a) This statement is false.

Counter example:

$A = \{a, b, c\} \Rightarrow n = 3$

$R = \{(a,a), (b,b), (c,b), (a,b), (b,c), (c,a)\}$

In this case $|R| = 6$, but if only $(a,b), (b,c), (c,a)$ will be present in this relation it violates the reflexive cond^n and also this relation violates the reflexive cond^n.

(b) If R_1 and R_2 are relations on A and $R_2 \supseteq R_1$ then R_1 reflexive (Symmetric, antisymmetric, transitive) $\Rightarrow R_2$ reflexive (Symmetric, antisymmetric, transitive).

This condition is true for reflexive. Let $A = \{1, 2\}$, $R_1 =$, $R_2 = \{(1,1), (2,2)\}$ $R = R_2$

For symmetric it is false. but $R_2 \supseteq R_1$

Counter example:

$A = \{1, 2\}$, $R_1 = \{(1,1)\} \Rightarrow$ symmetric.

$R_2 = \{(1,2), (1,1)\} \Rightarrow$ not symmetric. $R_2 \supseteq R_1$ but R_2 is not symmetric.

For transitive it is also false.

$A = \{1, 2\}$

$R_1 = \{(1,1), (2,2)\} \Rightarrow$ transitive.

$R_2 = \{(1,2), (1,1), (2,2)\} \Rightarrow$ is not transitive. $R_2 \supseteq R_1$ but R_2 is not transitive.

For antisymmetric it also false.

$A = \{1, 2\}$

$R_1 = \{(1,2)\} \Rightarrow$ antisymmetric

$R_2 = \{(1,2), (2,1)\} \Rightarrow$ not antisymmetric. $R_2 \supseteq R_1$ but R_2 is not antisymmetric.

(c) If R_1 and R_2 are relations on A and $R_2 \supseteq R_1$, then R_2 reflexive (Symmetric, antisymmetric, transitive) $\Rightarrow R_1$ reflexive (Symmetric, antisymmetric, transitive.)

Ans) 2 For reflexive false.

counter example:- $A = \{1,2\}$

$R_1 = \{(1,1)\}$, $R_2 = \{(1,1),(2,2)\}$, $R_2 \supseteq R_1$

but R_1 is not reflexive.

For symmetric false.

counter example:- $A = \{1,2\}$

$R_1 = \{(1,2)\}$, $R_2 = \{(1,2),(2,1)\}$, $R_2 \supseteq R_1$

but R_1 is not symmetric

for antisymmetric True. — $R_1 = \{(1,2)\}$, $R_2 = \{(1,2),(2,1),(1,1),(2,2)\}$

$R_2 \supseteq R_1$ & both antisymmetric.

For transitive false.

counter example:- $A = \{1,2\}$

$R_1 = \{(1,2),(2,1)\}$

$R_2 = \{(1,1),(1,2),(2,1),(2,2)\}$ $R_2 \supseteq R_1$ but R_1 is not transitive.

(d) If R is an equivalence relation on A, then $n \leq |R| \leq n^2$

Ans) true.

(3) If $A = \{w,x,y,z\}$ determine the number of relations on A that are (a) reflexive (b) symmetric (c) reflexive and symmetric (d) reflexive and contains (x,y) (e) symmetric and contains (x,y) (f) antisymmetric (g) antisymmetric contains (x,y) (h) symmetric and antisymmetric (i) reflexive, symmetric and antisymmetric.

Ans (3) (a) Given $A = \{w,x,y,z\}$ $n=4$

No. of reflexive relation $= 2^{n^2-n} = 2^{16-4} = 2^{12}$

(b) No. of symmetric relation :-

$n^2+n = 16+4 = 20$

$(n^2+n)/2 = 20/2 = 10$

$2^{\frac{1}{2}(n^2+n)} = 2^{10}$

(c) No. of reflexive and symmetric $= 2^{\frac{1}{2}(n^2-n)}$

$\Rightarrow (n^2-n) = 16-4 = 12$ $\quad (n^2-n)/2 = 12/2 = 6$ $\Rightarrow 2^{\frac{1}{2}(n^2-n)} = 2^6$

(d) reflexive and contains $(x,y) = 2^{11}$ [reflexive not containting $(x,y) = 2^{11}$] ∴ containting $(x,y) = 2^{12} - 2^{11} = 2^{11}(2-1)$ $= 2^{11}$

(e) Symmetric and contains $(x,y) = (2^4)(2^5) = 2^9$

same

(f) antisymmetric $\quad 2^n \cdot 3^{(n^2-n)/2}$

$n^2 - n = 4^2 - 4 = 16 - 4 = 12$

$(n^2-n)/2 = 12/2 = 6$

so $\quad 2^n \cdot 3^{(n^2-n)/2}$

$\Rightarrow 2^4 \cdot 3^6$

(g) antisymmetric contains(x,y) $= 2^4 \cdot 3^5$

(h) symmetric and antisymmetric $= 2^4$

(i) reflexive, symmetric, antisymmetric $= 1$.

Q(4) let A be a set with $|A| = n$, and let R be a relation on A that is antisymmetric. What is the maximum value of $|R|$? How many antisymmetric relations can have this size?

Ans) There are n ordered pairs of the form $(x,x) \in A$. For each of the $(n^2-n)/2$ sets $\{(x,y),(y,x)\}$ of ordered pairs are there $x,y \in A$, $x \neq y$, one element is choosen.

$\boxed{\text{So the max}^m \text{ value} = n + \frac{(n^2-n)}{2} = \frac{(n^2+n)}{2}}$

So the no. of antisymmetric relation with this size $= 2^{(n^2-n)/2}$

Q(5) with $A = \{1,2,3,4\}$, let $R = \{(1,1),(1,2),(2,3),(3,3),(3,4),(4,4)\}$ be a relation on A. find 2 relations S, T on A where $S \neq T$ but $R \circ S = R \circ T$
$= \{(1,1),(1,2),(1,4)\}$

Ans) $S = \{(1,1),(1,2),(2,4)\}$

$R \cdot S = \{(1,1),(1,2),(1,4)\}$

$T = \{(1,1),(1,2),(1,4)\}$

$R \cdot T = \{(1,1),(1,2),(1,4)\}$ So $R \cdot S = R \cdot T$

Q⑥ let A be a set with $|A| = n$, and let R be an equivalence relation on A with $|R| = r$ where $r-n$ always even.

Ans) $r-n$ counts the elements in R of the form (a,b) and $a \neq b$. Since R is Symmetric $r-n$ is even.

Q⑦ Determine how many integer solution there are to $x_1+x_2+x_3+x_4 = 19$ if

ⓐ $0 \le x_i, \forall \ 1 \le i \le 4$

ⓑ $0 \le x_i < 8 \ \forall \ 1 \le i \le 4$

ⓒ $0 \le x_1 \le 5,\ 0 \le x_2 \le 6,\ 3 \le x_3 \le 7,\ 3 \le x_4 \le 8$.

⑦ⓐ Given $x_1+x_2+x_3+x_4 = 19$

ⓐ $0 \le x_i, \ \forall \ 1 \le i \le 4$ Here $r = 19$, $k = 4$

using inclusion - enclusion principle:-

$$\Rightarrow \binom{19+4-1}{19} = \binom{19+3}{19} = \binom{22}{19}$$

$$\Rightarrow {}^{22}C_{19} = \frac{22!}{19!(22-19)!} = \frac{22 \times 21 \times 20 \times 19!}{19! \times 3!} = \frac{22 \times \cancel{21}^{7} \times \cancel{20}^{10} \times \cancel{19!}}{\cancel{19!} \times \cancel{3} \times \cancel{2} \times 1}$$

$\Rightarrow 1540 = N$

ⓑ $0 \le x_i < 8 \ \forall \ 1 \le i \le 4$

$N(C_1) = \quad x_1+x_2+x_3+x_4 = 19$ --- ①

let $y_1 = x_1 - 8$ --- ⓩ $\Rightarrow x_1 = y+8$ --- ②

so from eqn ① and ②

$y_1 + 8 + x_2 + x_3 + x_4 = 19$

$y_1 + x_2 + x_3 + x_4 = 19 - 8$ Here $r = 11$, $k = 4$

$\Rightarrow y_1 + \cancel{x_2} + \cancel{y} \ x_3 + x_4 = 11$

so $N(C_1) = \binom{11+4-1}{C_{11}} = \left({}^{14}C_{11}\right)$

$$\binom{14}{C_{11}} = \frac{14!}{11!(14-11)!}$$

$$\Rightarrow \frac{14\times13\times12\times11!}{11!\times3!} = 364$$

So $(N(c_2) = N(c_3) = N(c_4) = N(c_1) = 364$.

Again $N(c_1 c_2)$:-

$y_1 = x_1 - 8$

$\Rightarrow x_1 = y_1 + 8$ --- (3)

$y_2 = x_2 - 8$

$\Rightarrow x_2 = y_2 + 8$ — (4)

Putting eqn (3) (4) in eqn (1) we have :-

$y_1 + 8 + y_2 + 8 + x_3 + x_4 = 19$

$y_1 + y_2 + x_3 + x_4 = 19 - 16$

$y_1 + y_2 + x_3 + x_4 = 3$ Here $n=3$, $k=4$

$$N(c_1 c_2) = \binom{3+4-1}{C_3} = \binom{6}{C_3}$$

$$\binom{6}{C_3} = \frac{6!}{3!\,3!} = 20$$

So $N(c_1 c_3) = N(c_1 c_4) = N(c_2 c_3) = N(c_2 c_4) = N(c_3 c_4) = N(c_1 c_2) = 20$

$\therefore N(\bar{c}_1 \bar{c}_2 \bar{c}_3 \bar{c}_4) = N - 4N(c_1) + 6N(c_1 c_2)$

$\Rightarrow 1540 - 1456 - 120$

$= 204$

(c) Given $0 \le x_1 \le 5$, $0 \le x_2 \le 6$, $3 \le x_3 \le 7$, $3 \le x_4 \le 8$

$\Rightarrow 0 \le x_1 \le 5$, $0 \le x_2 \le 6$, $0 \le x_3 - 3 \le 4$, $0 \le x_4 - 3 \le 5$

$\Rightarrow 0 \le y_1 \le 5$, $0 \le y_2 \le 6$, $0 \le y_3 \le 4$, $0 \le y_4 \le 5$

where $x_1 = y_1$, $x_2 = y_2$, $x_3 = y_3+3$, $x_4 = y_4+3$

so now the condn becomes:-

$y_1+y_2+y_3+y_4 = 19-6 = 13$

Here $r = 13$

$\Rightarrow y_1+y_2+y_3+y_4 = 13$

$|S| = N = \left({}^{13+4-1}C_{13} \right) = \left({}^{16}C_{13} \right) = 560$

Now we have $0 \le y_1 \le 5$, $0 \le y_2 \le 6$, $0 \le y_3 \le 4$, $0 \le y_4 \le 5$

$N(C_1) = y_1+6+y_2+y_3+y_4 = 13$

Here $r = 7$

$\Rightarrow y_1+y_2+y_3+y_4 = 13-6 = 7$

$\Rightarrow N(C_1) = \left({}^{7+4-1}C_7 \right) = \left({}^{10}C_7 \right) = 120 = N(C_4)$

Again $y_1+y_2+7+y_3+y_4 = 13$

Here $r = 6$

$\Rightarrow y_1+y_2+y_3+y_4 = 6$

$\Rightarrow N(C_2) = \left({}^{6+4-1}C_6 \right) = \left({}^{9}C_6 \right) = 84$

Again $y_1+y_2+y_3+5+y_4 = 13$

$\Rightarrow y_1+y_2+y_3+y_4 = 8$

$N(C_3) = \left({}^{8+3}C_8 \right) = \left({}^{11}C_8 \right) = 165$

Similarly $N(C_1C_2) = 1$

Again C_1C_3 $y_1+y_2+y_3+y_4 = 13$

$\Rightarrow y_1+6+y_2+y_3+5+y_4$

$\Rightarrow y_1+y_2+y_3+y_4 = 2$

$N(C_1C_3) = \left({}^{2+3}C_2 \right) = \left({}^{5}C_2 \right) = 10$

Again $y_1 + y_2 + y_3 + y_4 = 13$

$\Rightarrow y_1 + 6 + y_2 + y_3 + y_4 + 6 = 13$

$\Rightarrow y_1 + y_2 + y_3 + y_4 = 13 - 12 = 1$

$N(c_1 c_4) = \binom{1+3}{c_1} = ({}^4c_1) = 4$

Similarly $N(c_2 c_3) = ({}^4c_1) = 4$

$N(c_2 c_4) = 1, \quad N(c_3 c_4) = {}^5c_2 = 10$

Now $N(\bar{c_1}\,\bar{c_2}\,\bar{c_3}\,\bar{c_4}) = N - [2 \times 120 + 84 + 165] + 2[1 + 4 + 10]$

$= 560 - 489 + 30 = 101$ (Ans)

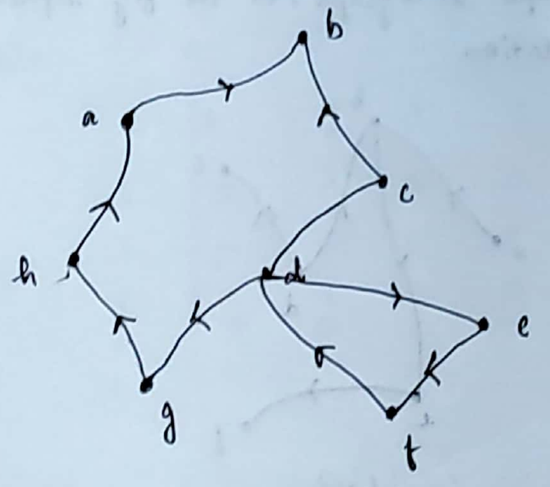
Q(8) For the directed graph $G = (V, E)$ classify each of the following statements are true or false.

(a) vertex c is the origin of two edges in G.

(b) Vertex g is adjacent to vertex h.

(c) There is a directed path in G from d to b.

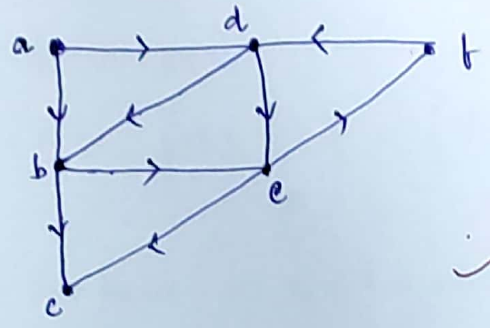
(d) There are two directed cycles in G.



Ans (a) True (b) True (c) True (d) false.

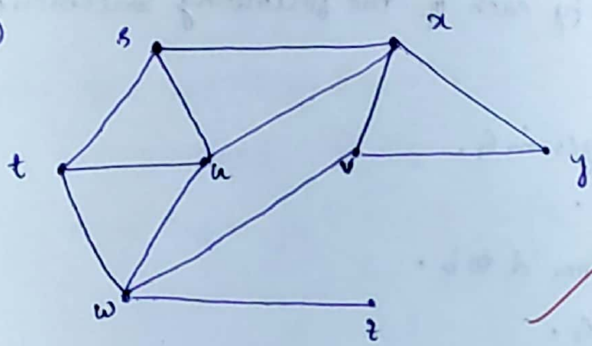
(9) (a) Draw the digraph $G_1 = (V_1, E_1)$ where $V_1 = \{a, b, c, d, e, f\}$ and $E_1 = \{(a,b), (a,d), (b,c), (b,e), (d,b), (d,e), (e,c), (e,f), (f,d)\}$

(a)

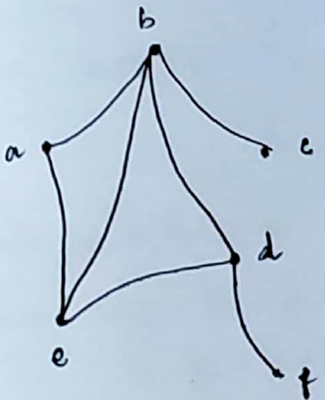


(b) Draw the undirected graph $G_2 = (V_2, E_2)$ where $V_2 = \{s, t, u, v, w, x, y, z\}$ and $E_2 = \{(s,t), (s,u), (s,x), (t,u), (t,w), (u,w), (u,x), (v,w), (v,x), (v,y), (w,z), (x,y)\}$

Ans (b)

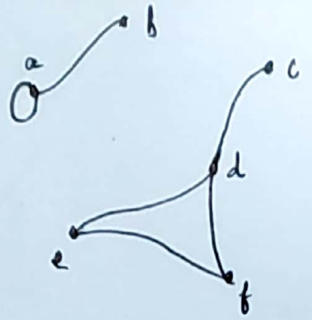


(10) For $A = \{a, b, c, d, e, f\}$, each graph or digraph, in the fig represents a relation R on A. Determine the relation

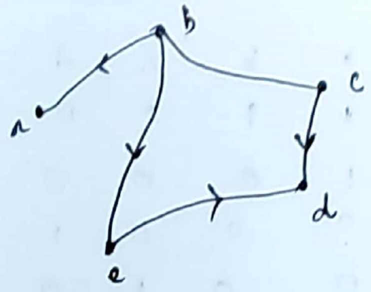


(i)

(ii)



(iii)



(iv)

Ans (10) (i) $R = \{(a,b), (b,a), (b,c), (c,b), (a,e), (e,a), (e,d), (d,e), (b,e), (e,b), (b,d), (d,b), (d,f), (f,d)\}$

$$M(R) = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \\ f \end{array} \begin{array}{c} \begin{array}{cccccc} a & b & c & d & e & f \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

(ii) $R = \{(a,b), (b,e), (e,f), (d,b), (d,c)\}$

$$M(R) = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \\ f \end{array} \begin{array}{c} \begin{array}{cccccc} a & b & c & d & e & f \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

(iii) $R = \{(a,a), (a,b), (b,a), (e,d), (d,e), (c,d), (d,c), (e,f), (f,e), (f,d), (d,f)\}$

$M(R) =$

$$\begin{array}{c} \\ a \\ b \\ c \\ d \\ e \\ f \end{array} \begin{array}{c} \begin{array}{cccccc} a & b & c & d & e & f \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

(iv) $R = \{(b,a), (b,c), (c,b), (b,e), (c,d), (e,d)\}$

$M(R) =$

$$\begin{array}{c} \\ a \\ b \\ c \\ d \\ e \\ f \end{array} \begin{array}{c} \begin{array}{cccccc} a & b & c & d & e & f \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

28/10/23