Assignment - 3

Q) How many seven digits number are there such that the digits are distinct integers taken from $\{1, 2, \ldots, 9\}$ and such that the digits 5 and 6 do not appear consecutively in either order?

Sol:- We have to find 7 permutations of the set $\{1, 2, \ldots, 9\}$
And we ~~can~~ have 4 cases to ~~find~~ divide these 7 permutations

case - I :- neither 5 nor 6 appears as a digit

In this case the permutations are the 7-permutations of the set $\{1, 2, 3, 4, 7, 8, 9\}$.

$\therefore$ The numbers is - $P(7,7) = 7! = 5040$

case - II :- 5 appears, but 6 does not appear as a digit

The permutations for this can be counted as follows:

The digit equal to 5 can be any of the seven digits.

The remaining six digits are a 6-permutation of $\{1, 2, 3, 4, 7, 8, 9\}$.

Hence the total numbers $= 7 \times P(7,6)$

$= 7 \times 7!$

$= 35,280$

case III :- 6 appears, but 5 does not appear as a digit

The permutation for this can be counted as follows:

The digit equal to 6 can be any of the seven digits.

The remaining six digits are a 6-permutation of $\{1, 2, 3, 4, 7, 8, 9\}$

Hence the total numbers $= 7 \times P(7,6)$

$= 7 \times 7!$

$= 35,280$

case IV :- both 5 & 6 appear as a digit.

To find permutations for this case we again have to partition this case into 3 different subcases :-

a) First digit equal to 5 & so second digit not equal to 6

$\boxed{5}\ \boxed{\neq 6}\ \boxed{\ }\ \boxed{\ }\ \boxed{\ }\ \boxed{\ }\ \boxed{\ }$

There are five places for 6.
The other five digits constitute a ~~5~~ 5- permutation of the 7 digits $\{1, 2, 3, 4, 7, 8, 9\}$.

Hence, there are $= 5 \times P(7,5)$

$$= 5 \times \frac{7!}{2!} = 12,600$$

b) Last digit equal to 5, & so next to last digit not equal to 6

$\boxed{\ }\ \boxed{\ }\ \boxed{\ }\ \boxed{\ }\ \boxed{\ }\ \boxed{\neq 6}\ \boxed{5}$

~~The~~ In this case also the permutations will be same as the one above.

So, there are : $5 \times P(7,5) = 12,600$

c) A digit other than the first or last is equal to 5

$\boxed{\ }\ \boxed{\ }\ \boxed{\neq 6}\ \boxed{5}\ \boxed{\neq 6}\ \boxed{\ }\ \boxed{\ }$

The place occupied by 5 is any one of the five interior places

The place for the 6 can then be choosen in four ways.

The remaining five digits constitute a ~~5-permutato~~ 5-permutation of the seven digits $\{1, 2, 3, 4, 7, 8, 9\}$

Hence, there are $= 5 \times 4 \times P(7,5)$

$= 50400$

Thus ~~by rotation~~ there are

$$2(12,600)+50,400 = 75,600$$

~~are~~ numbers for the 4^{th} case

Therefore,

by addition principle, the total number of 7-digits numbers from 4 different cases are :

$$5040 + 2(35,280) + 75,600 = 151,200$$

Q) How many different 5 digit ~~nos~~ nos can be constructed out of the digits 1, 1, 1, 3, 8 ?

Sol:- Digits given : 1, 1, 1, 3, 8

we need to construct 5 digit no. of with three 1s, one 3 and one 8.

Total no. of different 5 digit nos are $= 5!$

But, as 1 appears three times. So, total no. of distinct 5 digit nos $= \frac{5!}{3!} = 5 \times 4 = 20$