Sub — ADM

Ch-2

Permutations and Combinations

In this chapter, we explore four general principle of counting and some of the counting formulas with application to different problems.

Partition of a set

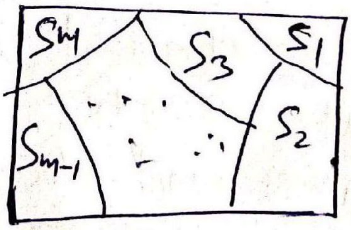
Let S be a given set. A partition of S is a collection of subsets $S_1, S_2, S_3, \ldots, S_n$ of S such that each element of S is exactly one of those subsets:

(i) $S = S_1 \cup S_2 \cup \cdots \cup S_n$

$= \bigcup_{i=1}^{n} S_i$

(ii) $S_i \cap S_j = \phi, \; i \neq j$

Here the sets $S_1, S_2, \ldots, S_n$ are pairwise disjoint sets and their union is S and Intersection is empty.



Basic counting principle

1. Addition principle

Let $S_1, S_2, \ldots, S_n$ form a partition on S, Then the number of objects in S can be determined by finding the number of objects in each of the parts and adding those elements i.e. $|S| = |S_1| + |S_2| + \cdots + |S_m|$

Ex:- suppose a student is to allowed to take either a mathematics or a biology course but not both. If there are 4 mathematics courses and three biology courses then in how many ways is this possible?

Ans:- $|S| = |S_1| + |S_2| = 4 + 3 = 7$

Multiplication Rule

Let 'S' be a set of ordered pairs (a, b) where $a \in S_1$ with $|S_1| = p$, $b \in S_2$, $|S_2| = q$

Then the size of 'S' will be

$$|S| = |S_1|\,|S_2| = pq.$$

Ex:- How many two digited numbers are possible?

Soln $\boxed{9 \,|\, 10}$, $|S| = |S_1||S_2| = 9 \times 10 = 90$.

Ex:- A student is to take two courses. The first meets at any of 3 hrs in the morning and the 2nd at any one 4 hrs in the afternoon. How many number of schedules are possible for the student?

Soln $3 \times 4 = 12$.

Ex:- Chalk comes in three different lengths, eight different colors and four different diameters. How many different kinds of chalk are there?

Solⁿ We have to choose a length, color and a diameter to select a chalk. So by multiplication principle, the different kinds of chalk possible are $3 \times 8 \times 4 = 96$.

Ex: How many ways a man, woman, boy and girl can be selected from five men, six women, two boys and four girls?

Solⁿ Required ways $= 5 \times 6 \times 2 \times 4 = 240$.

Ex: Determine the number of positive integers that are factors of $3^4 \times 5^2 \times 11^7 \times 13^8$.

Solⁿ By fundamental thⁿ of arithmetic

If $x = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_n^{\alpha_n}$

where $p_1, p_2, \ldots, p_n$ are primes, $\alpha_1, \alpha_2, \ldots, \alpha_n \in \mathbb{Z}^+$.

then the number of factors of x are $(\alpha_1+1)(\alpha_2+1) \cdots (\alpha_n+1)$.

Hence by multiplication principle the number of factors $= 5 \times 3 \times 8 \times 9$
$= 1080$ Ans

Ex: How many two digit numbers have distinct and non zero digits?

Solⁿ

T	U
8	9

$8 \times 9 = 72$

Subtraction Principle

Let A be the set and U be a larger set containing A. Then

$\overline{A} = U \setminus A = \{x \in U : x \notin A\}$

i.e. $|\bar{A}| = |U| - |A|$ or $|A| = |U| - |\bar{A}|$.

Ex: Computer passwords are to consist of a string of six symbols taken from the digits 0, 1, 2, --- 9 and the lowercase letters a, b, c, ---, z. How many computer passwords have a repeated symbol?

Sol^n Let A be the set of computer passwords with a repeated symbol. Let U be the set of all computer passwords. $\bar{A}$ be the set of computer passwords with no repeated symbol.

$$|U| = 36^6$$

$$|\bar{A}| = 36 \cdot 35 \cdot 34 \cdot 33 \cdot 32 \cdot 31$$

$$|A| = |U| - |\bar{A}| = 36^6 - 36 \cdot 35 \cdot 34 \cdot 33 \cdot 32 \cdot 31$$

$$= 774,372,096 \text{ Ans}$$

Ex: How many three digited numbers are possible such that at least one of their digit is repeated?

Sol^n $|U| = 9 \times 10^2 = 900$ (Total no. of 3 digited numbers)

Let A be the set of all 3 digit nos such that none of their digit is repeated.

So $|A| = 9 \times 9 \times 8 = 648$

Hence $|\bar{A}| = |U| - |A| = 900 - 648 = 252$ Ans

Division Principle

Let S be a finite set that is partitioned into K parts in such a way that each part contains the same number of objects.

Then the number of parts in the partition is given by $K = \frac{|S|}{\text{Number of objects in a part}}$

Ex: There are 740 pigeons in a collection of pigeonholes. If each pigeonhole contains 5 pigeons, the number of pigeonholes equals to $\frac{740}{5} = 148$.

Ex: How many 3 digit even numbers are possible?

Soln S_1 = Set of 3 digit even numbers that ends with 0 only.

S_2 = set of all 3 digit even numbers that ends with 2, 4, 6, 8.

Then $|S_1| = 9 \times 10 \times 1 = 90$ | 9 | 10 | 1 |

$|S_2| = 9 \times 10 \times 4 = 360$ | 9 | 10 | 4 |

Required three digit even numbers
$= 90 + 360 = 450$ Ans

Ex: How many 5 digit numbers can be formed out of the digits 0, 1, 2, 3, 4 and 5 such that the number is divisible by 3 and no digit can be repeated more than once.

Soln Case-I when '0' is excluded

$|S_1| = 5 \times 4 \times 3 \times 2 \times 1 = 120$

Case - II '3' is excluded (0, 1, 2, 4, 5)

using the digits 0, 1, 2, 4, 5

Number of 5 digit numbers $= 4 \times 4 \times 3 \times 2 \times 1$ $= 96$

4	4	3	2	1

No other choice is possible.

Hence the total number of 5 digit numbers which is divisible by 3 using the digits 0, 1, 2, 3, 4, 5 is

$120 + 96 = 216$ Ans

Ex: How many odd numbers between 1000 and 9999 have distinct digits?

Solⁿ The number between 1000 and 9999 is a four digited number and odd. So its unit digit can be any of the 1, 3, 5, 7, 9 i.e. 4 ways. Thousand place can be filled by 8 ways (since 0 excluds and one out of 1, 3, 5, 7, 9 is placed in unit place).

For hundredth place, It can be filled by 8 ways (since 8 is included) and tenth place can by 7 ways. since the digits are distinct. Hence the required odd numbers formed $= 5 \times 8 \times 8 \times 7 = 2240$.

8	8	7	5

Ex: How many different five-digit numbers can be constructed out of the digits 1, 1, 1, 3, 8? Ans: $5 \times 4 = 20$