o Regular Expression & Regular Languages :-

(i) Regular Expression

(ii) Regular Set

(iii) Regular Language

(iv) Identities of REs.

* Finite representation of Languages :-

→ The main issue in the TOC is the representation of languages by finite specifications.

→ We are unable to represent all languages finitely.

→ For the set Σ^* of strings over an alphabet Σ is countably infinite, so the number of possible representation of a Language is countably infinite.

→ The set of all possible languages over an given alphabet Σ i.e, 2^{Σ^*} (power set of Σ^*) is uncountably infinite.

→ Thus, we will try to find some finite representation of some of the Languages.

○ Regular Expression :— (RE)

→ RE are one way to represent languages. All the REs will be accepted by the automata.

→ Few defination (Informal) :—

(i) The language accepted by finite automata can be easily described by simple expression called regular expression. It is the most effective way to represent any language.

(ii) A regular expression can also be described as a sequence of pattern that defines a string.

→ Defination (Formal) :—

→ The regular expression over Σ^* are all strings over the alphabet $\Sigma^* \cup \{(,), \phi, *\}$ that can be obtained as follows:

(i) ϕ is a RE

(ii) ϵ is a RE

(iii) for $\forall a \in \Sigma$, a is a RE

(iv) if a is RE, then (a) is a RE.

(v) If a and b are two RE, then (ab) is a RE

(vi) If a and b are two RE, then $(a \cup b)$ is a RE

(vii) If a is a RE, then a^* is a RE.

○ For instance :— In RE.

(i) x^* means zero or more occurrance of x as $\{\epsilon, x, xx, \ldots\}$

(ii) x^+ means one or more occurrance of x as $\{x, xx, \ldots\}$

~~Operations over RL~~

* Regular Set :-

- Any set representation of RE is called Regular Set.
- If a and $b \in \Sigma$, then

(i) (a) denote the set $\{a\}$.

(ii) $(a+b)$ or $((a)+(b))$ or $((a)\cup(b))$ or $(a \cup b)$ denotes the set $\{a, b\}$.

(iii) (ab) or $(a \circ b)$ denotes the set $\{ab\}$.

(iv) (a^*) denotes the set $\{\varepsilon, a, aa, aaa, \ldots\}$

(v) $((a+b)^*)$ or $((a \cup b)^*)$ denotes the set $\{a, b\}^*$.

- Few examples of RE :- Let $\Sigma = \{0, 1\}$

(1) (0) is RE over Σ and RS $= \{0\}$

(ii) (0^*) is RE over Σ and RS $= \{\epsilon, 0, 00, 000, \ldots\}$

(iii) (1) is RE over Σ and RS $= \{1\}$

(iv) (1^*) is a RE over Σ and RS $= \{\epsilon, 1, 11, 111, \ldots\}$

(v) $(1(0)^*)$ is a RE over Σ i.e., $(10^*) = \{1, 10, 100, 1000, \ldots\}$

(vi) $(0(1)^*)$ is a RE over Σ i.e. $(01^*) = \{0, 01, 011, 0111, \ldots\}$

(vii) Thus $((1(0)^*) \cup (0(1)^*))$ is a RE over Σ i.e.,

$(10^* \cup 01^*) \approx (10^* + 01^*) = \{\epsilon, 1, 10, 100, 1000, \ldots, 0, 01, 011, 0111, \ldots\}$

$\therefore (1(0)^* \cup (0(1)^*)$ can be written as:- $(\{1\} \circ \{0^*\}) \cup (\{0\} \circ \{1^*\})$ or $\underline{10^* \cup 01^*}$

- Properties of Regular Set –

(P1) The union of two Regular set is regular.

Proof :– Let us take two regular Expression.

$RE_1 = a(aa)^*$

$RE_2 = (aa)^*$

So, $L_1 = \{a, aaa, aaaaa, \ldots\}$

$L_2 = \{\epsilon, aa, aaaa, \ldots\}$

$\therefore L_1 \cup L_2 = \{\epsilon, a, aa, aaa, aaaa, \ldots\}$

(string of all possible lengths including null)

$\therefore RE_1 \cup RE_2 = a^*$ (which is a regular expression)

$RE(L_1 \cap L_2) = a^*$

(P2) Intersection of two Regular set is Regular.

proof :– Let us take two Regular expression

$RE_1 = a(a^*)$ and $RE_2 = (aa^*)$

So, $L_1 = \{a, aa, aaa, \ldots\}$

$L_2 = \{\epsilon, aa, aaaa, \ldots\}$

$L_1 \cap L_2 = \{aa, aaaa, \ldots\}$

$RE(L_1 \cap L_2) = aa(aa)^*$ which is a regular expression itself.

(P3) The compliment of Regular set is regular.

Proof :- Let us take $RE = (aa)^*$

So, $L = \{\epsilon, aa, aaaa, \ldots\}$

$\therefore$ compliment of L is the set of string that is not in L.

$L^c = \{a, aaa, aaaaa, \ldots\}$

$RE(L^c) = a(aa)^*$ which is a regular expression

(P4) The difference of two regular set is regular.

proof :- Let us take two REs -

$RE_1 = a(a^*)$ and $RE_2 = (aa)^*$

So, $L_1 = \{a, aa, aaa, \ldots\}$

$L_2 = \{\epsilon, aa, aaaa, \ldots\}$

$L_1 - L_2 = \{a, aaa, aaaaa, \ldots\}$

$RE(L_1 - L_2) = a(aa)^*$ which is a regular expression

(P5) The reversal of two regular set is regular.

proof :- we have to show that L^R is also regular if L is RE.

Let ~~L = {01, 10}~~, $RE_1 = 01^*$

So, $L_1 = \{0, 01, 0111, \ldots\}$

$L_1^R = \{0, 10, 1110, \ldots\}$

$RE(L_1^R) = 1^*0$ i.e. Regular expression

(P6) The concatenation of two Regular ... is regular.

Proof :- Let two REs are.

$RE_1 = (0+1)^* 0$ and

$RE_2 = 01(0+1)^*$

So, $L_1 = \{0, 00, 01, 000, 001, \ldots\}$

$L_2 = \{01, 010, 011, 0100, \ldots\}$

$\therefore L_1 \cdot L_2 = \{001, 0010, 0011, 0001, \ldots\}$

(Set of strings containing 001 as sub string)

i.e. $RE = \underline{(0+1)^* 001 (0+1)^*}$

(Rules)

- Identities related to REs :-

Given R, P, L, Q as regular expression, the following identities hold:-

1. $\phi^* = \epsilon$
2. $\epsilon^* = \epsilon$
3. $RR^* = R^*R$
4. $(R^*)^* = R^*$
5. $R^* R^* = R^*$
6. ~~$(PQ)^*P = P(QP)$~~
6. $(a+b)^* = (a^* b^*)^* = (a^* + b^*)^*$
7. $R + \phi = \phi + R = R$
8. $R\epsilon = \epsilon R = R$
9. $R + R = R$
10. $L(M+N) = LM + LN$.
11. $(M+N)L = ML + NL$
12. $\epsilon + RR^* = \epsilon + R^*R = R^*$
13.

Statement :-

- Let P and Q be two regular expression.
- If P does not contain null string, then

$R = Q + RP$ has a unique solution

that is, $R = QP^*$.

Proof :- $R = Q + RP$

$R = Q + (Q + RP)P$

$= Q + QP + RPP$

$\Rightarrow Q + QP + (Q + RP)PP$

$\Rightarrow Q + QP + QPP + RPPP$

So, $\Rightarrow Q + QP + QP^2 + QP^3 + QP^4 + \dots$

So $\Rightarrow Q\{\varepsilon + P + P^2 + P^3 + P^4 + \dots\}$

So, $\Rightarrow QP^*$

$R = QP^*$ is unique sol^n.

o Examples on RE :-

Eg① Write a RE for the Language accepting all combination of a's over $\Sigma = \{a\}$.

Sol^n = $\{\varepsilon, a, aa, aaa, aaaa \dots\} \Rightarrow a^*$

Language represented by a RE :-

→ If α is RE, then $L(\alpha)$ is the language represented by α. The function L is defined as follows:

(i) $L(\phi) = \phi$

(ii) $L(a) = \{a\}$ for each symbol $a \in \Sigma$.

(iii) If a and b are two REs, then $L((ab)) = L(a) \cdot L(b)$

→ i.e., string in $L(ab)$ is the string from $L(a)$ followed by string of $L(b)$.

(iv) If a and b are two REs, then.

$$L((a \cup b)) = L(a) \cup L(b)$$

→ i.e., a string in $L((a \cup b))$ or $L((a+b))$ is string from $L(a)$ or $L(b)$.

(v) If a is an RE, then $L(a^*) = L(a)^*$

→ i.e., a string in $L(a^*)$ is a string obtained by concatinating n elements of some $n \geq 0$.

Problems :-

P① Describe the following set by RE.

(a) $\{101\}$ (b) $\{abba\}$ (c) $\{01, 10\}$

(d) $\{\epsilon, ab\}$ (e) $\{abb, a, b, bba\}$

(f) $\{\epsilon, 0, 00, 000, \ldots\}$ (g) $\{1, 11, 111, \ldots\}$

sol^n:- ⓐ $\{101\}$

→ The RE (1) or 1 represents RS $\{1\}$

→ The RE (0) or 0 represents RS $\{0\}$

→ ∴ The RE ((1)(0)(1)) or (101) or 101 represent the RS $\{101\}$.

ⓑ $\{abba\}$

→ RE (ab) ⇒ RS = $\{ab\}$

→ RE (ba) ⇒ RS = $\{ba\}$

→ RE ((ab)∘(ba)) or (abba) ⇒ RS = $\{abba\}$.

ⓒ $\{01, 10\}$

→ RE (01) ⟹ RS = $\{01\}$

→ RE (10) ⟹ RS = $\{10\}$

→ RE (01 ∪ 10) or (01 + 10) ⇒ RS = $\{01, 10\}$.

ⓓ $\{\epsilon, ab\}$

→ RE (ϵ + ab) or (ϵ + ab) or (ϵ ∪ ab) ⇒ RS = $\{\epsilon, ab\}$.

ⓔ $\{abb, a, b, bba\}$

→ RE (abb ∪ a ∪ b ∪ bba) or (abb + a + b + bba) ⇒ RS = $\{abb, a, b, bba\}$

ⓕ $\{\epsilon, 0, 00, \ldots\}$

→ RE (0^*) or 0^*

ⓖ $\{1, 11, 111, \ldots\}$

→ RE ($1 \cdot 1^*$) or 11^*

o Identities of REs:-

→ Two REs P and Q are equivalent (i.e., $P = Q$) if P and Q represent the same Language.

→ Assume P, Q, and R are REs.

* For Union (∪) operation:-

I_1: $P \cup (Q \cup R) = (P \cup Q) \cup R$ [Associativity]

I_2: $P \cup \phi = \phi \cup P = P$ [Identity]

I_3: $P \cup Q = Q \cup P$ [Commutativity]

I_4: $P \cup P = P$ [Idempotence]

I_5: $(P \cup Q)R = PR \cup QR$ [Distributive]

~~I_6: $(P \circ Q) \cup R = (PR) \cup (P \cup R) \circ (Q \cup R)$~~

I_6: $R(P \cup Q) = RP \cup RQ$ [Distributive]

* For Concatination (∘) operation:-

I_7: $P(QR) = (PQ)R$ [Associativity]

I_8: $\epsilon R = R\epsilon = R$ [Identity]

I_9: $PQ \neq QP$ [Non-Commutativity]

* I_{10}: $P\phi = \phi P = \phi$ (

I_{11}: $P^+ = PP^* = P^*P$

I_{12}: If $P_1 \subseteq P_2$ and $P_3 \subseteq P_4$, then $P_1 P_3 \subseteq P_2 P_4$.

* For Closure (*) operations:-

I_{13} : $\epsilon^* = \epsilon$

I_{14} : $\phi^* = \epsilon$

I_{15} : $P^* P^* = P^*$

I_{16} : $P^* P = P P^*$

I_{17} : $(P^*)^* = P^*$

I_{18} : $\epsilon \cup PP^* = \epsilon \cup P^* P = P^*$

I_{19} : $((PQ)^*)P = P((QP)^*)$

I_{20} : $(P \cup Q)^* = (P^* Q^*)^* = (P^* \cup Q^*)^*$

Eg: ① Suppose $\Sigma = \{0, 1\}$, what is $L(1(0^*)) \cup L(0(1^*))$?

$\therefore L(1(0^*)) \cup L(0(1^*)) = (L(1) \circ L(0^*)) \cup (L(0) \circ L(1^*))$

$= (\{1\} \circ \{0^*\}) \cup (\{0\} \circ \{1^*\})$

$= (\{1\}\{0^*\}) \cup (\{0\} \cdot \{1^*\})$

$= 10^* + 01^*$

$= \{w \in \{0,1\}^* :$ w start with 0 or 1$\}$

Eg: ② Suppose $\Sigma = \{a, b\}$, what is $L(((a \cup b)^*)a)$?

$\therefore L(((a \cup b)^*)a) = L((a \cup b)^*) \circ L(a)$

$= (L(a) \cup L(b)^*) \circ L(a)$

$= (\{a\} \cup \{b\})^* \{a\}$

$= (a+b)^* a$

$= \{w \in \{a,b\}^* :$ w ends with a$\}$

Eg:③ what language represented by the RE.

$$(((a^*a)b)\cup b)$$

$\therefore$

$$\mathcal{L}(((a^*a)b)\cup b) = \mathcal{L}((a^*a)b) \cup \mathcal{L}(b)$$
$$= (\mathcal{L}(a^*a) \circ \mathcal{L}(b)) \cup \mathcal{L}(b)$$
$$= ((\mathcal{L}(a)^* \circ \mathcal{L}(a))\mathcal{L}(b)) \cup \mathcal{L}(b)$$
$$= ((\{a\}^* \circ \{a\}) \circ \{b\}) \cup \{b\}$$
$$= ((\{a\}^*\{a\})\{b\}) \cup \{b\}$$
$$= \{w \in \{a,b\}^* : w \text{ zero or more a's followed by a single b}\}$$

Eg:-④ Rewrite the REs as a simpler expression representing the same set.

ⓐ $\phi^* \cup a^* \cup b^* \cup (a\cup b)^*$

$\Rightarrow$ • $\phi^* = \phi^0 \cup \phi^1 \cup \phi^2 \cup \phi^3 \cup \dots = \{\epsilon\}$.

• $a^* \subseteq (a\cup b)^*$ and $b^* \subseteq (a\cup b)^*$

$\therefore \quad a^* \cup b^* \subseteq (a\cup b)^*$

• Therefore, $\phi^* \cup a^* \cup b^* \cup (a\cup b)^* = (a\cup b)^*$

ⓑ $(((a^*b^*)^*(b^*a^*)^*)^*$

$\Rightarrow$ Since, $(a^*b^*)^* = (a\cup b)^*$

$\therefore ((a^*b^*)^*\}(b^*a^*)^*)^* = ((a\cup b)^*)((a\cup b)^*))^*$

$\therefore ((a\cup b)^*(a\cup b)^*)^* = ((a\cup b)^*)^* = \underline{(a\cup b)^*}$.

ⓒ $(a^*b)^* \cup (b^*a)^*$

$\Rightarrow \therefore a^* = \{\epsilon, a, aa, \ldots\}$

$\therefore a^*b = \{b, ab, aab, \ldots\}$ i.e., all string of a ending with b.

$\therefore (a^*b)^* \Rightarrow$ would give all possible strings over $\{a,b\}$ i.e., $\{a,b\}^*$

$\therefore (b^*a)^* \Rightarrow$ would give all possible strings over $\{a,b\}$ i.e., $\{a,b\}^*$

$\therefore (a^*b)^* \cup (b^*a)^* = (a \cup b)^*$

ⓓ $(a \cup b)^* a (a \cup b)^*$

$\Rightarrow$ all possible string over $\{a,b\}$ to the left, followed by an a, followed by all possible strings over $\{a,b\}$ to the right.

Eg:- ⑤ Let $\Sigma = \{a,b\}$, write the RE for following set.

ⓐ All strings in Σ^* with no more than three a's.

$\therefore \Rightarrow b^* \Rightarrow$ gives all possible string without a's.

$\Rightarrow b^*ab^* \Rightarrow$ gives all possible string over $\{a,b\}$ with exactly one a's.

$\Rightarrow b^*ab^*ab^* \Rightarrow$ gives exactly two a's

$\Rightarrow b^*ab^*ab^*ab^* \Rightarrow$ will have exactly three a's.

$\therefore$ RE.

$$b^* \cup b^*ab^* \cup b^*ab^*ab^* \cup b^*ab^*ab^*ab^*$$

or

$$b^* + b^*ab^* + b^*ab^*ab^* + b^*ab^*ab^*ab^*$$

gives no more than three a's.

ⓑ All strings in Σ^* with number of a's divisible by 3.

⇒ The a's must come in a group of (0, 3, 6, ...)

∴ $(b^*ab^*ab^*ab^*)$ will give three a's.

∴ $(b^*ab^*ab^*a)^*b^*$ will give a's in multiple of 3.

ⓒ All strings in Σ^* with exactly one occurrence of Substring aaa.

⇒ All String must follow. $\underbrace{x}_{\text{can be empty}}$ aaa $\underbrace{y}_{\text{can be empty.}}$ form

⇒ if ~~x is~~ non-empty ∴ ~~it~~ x should ~~start with b and~~ end with b. and y should start with b.

∴ $X = (x_1 b)^*$ ∴ $x^* aaa y^* = (x_1 b)^* aaa(b y_1)^*$

$Y = (b y_1, \cancel{b})^*$

⇒ ∴ x_1 and y_1 can have one a's, two a's, or any no of b's.

∴ $x_1 = a \cup aa \cup b^*$ and $y_1 = a \cup aa \cup b^*$

∴ $xaaay = \underline{((a \cup aa \cup b^*)b)^* aaa(b(a \cup aa \cup b^*))^*}$

More Examples on REs :-

- Assume that $\Sigma = \{a, b\}$, then answer the following :-

Eg. ① :- What is the RE for :-

$L = \{$ Set of all strings of length two $\}$

Sol^n :- $L = \{$ set of all strings of length two $\}$

i.e, $L = \{aa, ab, ba, bb\}$.

$\therefore (a+b)(a+b) \approx (a \cup b)(a \cup b)$.

Eg. ② :- ~~What~~ $L = \{$ set of all strings of length at least two $\}$.

i.e., $L = \{aa, ab, ba, bb, aaa, aab, \ldots\}$

$\therefore (a \cup b)(a \cup b) \Rightarrow$ gives string of length two.

$\therefore (a \cup b)(a \cup b)(a \cup b)^*$ or $(a \cup b)^*(a \cup b)(a \cup b)$

will give the string of at least length 2.

Eg :- ③ $L = \{$ set of all strings of length at most two $\}$.

i.e., $L = \{\epsilon, a, b, aa, ab, ba, bb\}$

$\therefore (a \cup b \cup \epsilon)(a \cup b \cup \epsilon)$ will give the string of length at most two.

Eg :- ④ $L = \{$ set of all strings of even length $\}$.

i.e. $L = \{\epsilon, aa, ab, ba, bb, aaaa, aaab, \ldots\}$

$\therefore (a \cup b)(a \cup b) \Rightarrow$ gives string of length two

$\therefore ((a \cup b)(a \cup b))^* \Rightarrow$ even length strings.

Eg:-⑤ $L = \{$set of all strings of odd length$\}$

i.e., $L = \{a, b, aaa, aab, \ldots\}$

$\therefore ((a \cup b)(a \cup b))^* \Rightarrow$ gives even length string

$\therefore (a \cup b)((a \cup b)(a \cup b))^*$ or $((a \cup b)(a \cup b))^*(a \cup b) \Rightarrow$ gives odd length string.

Eg:-⑥ $L = \{$set of all strings of length divisible by three$\}$

i.e., $\{\epsilon, aaa, aab, abb, \ldots, aaaaaa, aaaaab, \ldots\}$

$\therefore ((a \cup b)(a \cup b)(a \cup b))^*$

Eg:-⑦ $L = \{w \in \{a,b\} : |w| \cong 2 \bmod 3\}$

i.e., $L = \{aa, ab, ba, bb, aaaaa, aaaab, \ldots\}$

$\therefore ((a \cup b)(a \cup b)(a \cup b))^* \Rightarrow$ gives strings of length divisible by 3.

$\therefore ((a \cup b)(a \cup b)(a \cup b))^*(a \cup b)(a \cup b) \Rightarrow$ gives strings of length 2 mod 3.

Eg:-⑧ $L = \{$set of all strings containing exactly two a's$\}$

i.e., $L = \{aa, aab, aba, baa, aabb, \ldots\}$

$\therefore \underset{\sim}{b^*}\, a\, \underset{\sim}{b^*}\, a\, \underset{\sim}{b^*} \Rightarrow \underline{b^* a b^* a b^*}$

Eg:-⑨ $L = \{$set of all strings containing at least two a's$\}$

i.e., $L = \{aa, aab, baa, aabb, bbaa, abab, abba, \ldots\}$

$\therefore b^* a b^* a (a \cup b)^*$

Eg:-(10) L = {set of strings containing at most two a's}

i.e., L = {ε, b, a, aa, bb, ba, ab, aba, abb, bbb, ...}

$\therefore$ $\cancel{b^*(a\cup b)^* b^* (a\cup b)^* b^*}$

$\therefore$ $b^* (a \cup \epsilon)\, b^* (a \cup \epsilon)\, b^*$

Eg:- (11) L = {set of all strings containing even no of a's}

i.e., L = {ε, b, bb, aa, aab, aba, ...}

$\therefore$ $(b^* a b^* a b^*)^* \cup (bb^*)$

Eg:- (12) L = {set of all strings starting with a}

i.e. L = {a, aa, ab, aba, abb, ...}

$\therefore$ $a(a \cup b)^*$

Eg:- (13) L = {set of all strings ends with a}

i.e., L = {a, aa, ba, baa, aba, ...}

$\therefore$ $(a \cup b)^* a$

Eg:- (14) L = {set of all strings containing at least 1 a}

i.e., L = {a, aa, ab, aba, aaa, ...}

$\therefore$ $(a \cup b)^* a (a \cup b)^*$

Eg:- 15 $L = \{$ set of all strings, starting and ending with different symbols. $\}$

i.e. $L = \{ab, ba, aab, abb, baa, bba, \ldots\}$

$\therefore \; (a(a \cup b)^* b) \cup (b(a \cup b)^* a)$

Eg:- 16 $L = \{$ set of all strings, starting & ending with same symbols $\}$

i.e. $L = \{\epsilon, a, b, aa, bb, aba, bab, abba, baab, \ldots\}$

$\therefore \; (a(a \cup b)^* a) \cup (b(a \cup b)^* b) \cup a \cup b \cup \epsilon$

(P1) starting and ending with same symbol.

$$(a(a+b)^*b)+(b(a+b)^*a)+a+b+\epsilon$$

(P2) two conseentive a's and b's.

$$(a+b)^*(aa(a+b)^*bb+bb(a+b)^*aa)(a+b)^*$$

(P3) even no of a's.

$$\cancel{(b^*ab^*aa)^*b^*}$$

$$(b^*ab^*ab^*)^*+(bb^*)$$

(P4) no pair of conseentive 0's.

$$(0+01)^*(0+\epsilon)$$

(P5) two conseentive 0's and 1's.

$$(0+1)^*(00(0+1)^*11+11(0+1)^*00)(0+1)^*$$

* Problems related to REs :-

1. $L = \{a\}$ REs $\Rightarrow (a)$
2. $L = \{ab\}$ REs $\Rightarrow (ab)$
3. $L = \{a, b\}$ REs $\Rightarrow (a+b)$
4. $L = \{aaa\}$ REs $\Rightarrow (aaa)$
5. $L = \{$ set of string contains any no of a's $\}$ REs $= a^*$
6. $L = \{$ set of string contain any combination of a and b $\}$

   REs $= (a+b)^*$
7. $L = \{$ set of all strings containing exactly 2 a's $\}$

   REs $= b^* a b^* a b^*$
8. $L = \{$ set of all strings containing at least two a's $\}$

   REs $= b^* a b^* a b^* (a \cup b)^* \rightarrow b^* a b^* a (a \cup b)^*$
9. $L = \{$ set of all string containing at most two a's $\}$

   REs $= b^*(a+\epsilon) b^* (a+\epsilon) b^*$
10. $L = \{$ set of all strings containing even no of a's $\}$

    REs $=$ ~~$b^*(a+\epsilon)$~~

    $L = \{\epsilon, b, bb, aa, aabb, \ldots\}$

    REs $= (b^* a b^* a b^*)^* \cup (bb^*)$

(2)

(11) α = {set of all strings starting with a's}

REs = $a(a+b)^*$

(12) α = {set of all strings ending with a's}

REs = $(a+b)^*a$

(13) α = {set of all strings contaiing at least 1 a}

RES. ⇒ $(a+b)^* a (a+b)^*$

(14) α = {set of all strings starting with a}

REs ⇒ $a(a+b)^*a$

(15) α = {set of all strings starting & ending with same symbol}

REs ⇒ $(a(a+b)^*a) + (b(a+b)^*b)$

(16) α = {set of all strings statting and ending with different symbol}

REs ⇒ $(a(a+b)^*b) + (b(a+b)^*a)$.

(17) α = {$w = (0+1)^*$ | w has even no of 1's}

REs ⇒ $0^*(10^*10^*)^*$

(18) Let ⇒ $r = a(a+b)^*$, $s = aa^*b$ and $t = a^*b$. are three REs.

∴ (i) $L(s) \subseteq L(r)$ and $L(s) \subseteq L(t)$

(ii) $L(r) \subseteq L(s)$ and $L(s) \subseteq L(t)$

only (i) is true.

(19) $L = \{ w \in \{0,1\}^* \mid w$ has no pair of consecutive zeros$\}$

∴ REs ⇒ $(0+01)^*(0+\underset{\epsilon}{\lambda})$.

(20) $L = \{$ set of all string over $\{0,1\}$ containing at least one 0 and one 1 $\}$

RES ⇒ $[(0+1)^* 0 (0+1)^* 1 (0+1)^*] + [(0+1)^* 1 (0+1)^* 0 (0+1)^*]$

(21) $\alpha = \{$ set of all strings that donot contain substring 01 $\}$

REs ⇒ $\{\epsilon, 0, 1, 00, 11, 10, 100, 1100, \ldots\}$

REs ⇒ $1^* 0^*$

(22) $\alpha = \{$ set of all strings in which every zero is followed by at least two 1's $\}$

REs ⇒ $(011+1)^*$

(23) if L is a Regular Language over $\Sigma = \{a,b\}$, which one of the following is not regular.

(i) $L.L^R = \{ xy \mid x \in L, y^R \in L \}$

(ii) $\{ ww^R \mid w \in L \}$ ✓

(iii) Prefix $(L) = \{ x \in \Sigma^* \mid \exists x \in \Sigma^*$ such that $xy \in L \}$

(iv) Suffix $(L) = \{ y \in \Sigma^* \mid \exists x \in \Sigma^*$ such that $xy \in L \}$

(0) ④

(Q4) α = { set of all strings containing two consecutive 0's and 1's }

RE $\Rightarrow$ $(0+1)^*(00(0+1)^*11 + 11(0+1)^*00)(0+1)^*$

(Q5) How many strings of length less than 4 contains the language described by the RE $(x+y)^* y (a+ab)^*$

Sol^n:- Length 1 $\Rightarrow$ $y \rightarrow 1$

Length 2 $\Rightarrow$ $xy, yy, ya \Rightarrow 3$

Length 3 $\Rightarrow$ $xxy, xyy, yxy, yyy, xya, yya, yab, yaa,$ $\rightarrow 8$.

Total $\Rightarrow$ $1+3+8 = 12$.