Sequences and Tuples

Set

1. Set is enclosed with { } a pair of braces.
2. Order & repeatation doesn't matter.

Sequence

1. Sequence is enclosed within a pair of parenthesis ( )
2. Order & repeatation matters

- A sequence of objects is a list of these objects in some order.
- We usually designate a sequence by writing the list within parentheses.
- For example, the sequence 7, 21, 57 would be written (7, 21, 57).

$\{7,21,57\}=\{57,7,21\}$
Order not matter

$(7,21,57) \neq (57,7,21)$
order matters

- The order doesn't matter in a set, but in a sequence it does.
  - Hence (7, 21, 57) is not the same as (57, 7, 21).
- Similarly, repetition does matter in a sequence, but it doesn't matter in a set.
  - Thus (7, 7, 21, 57) is different from both of the other sequences, whereas the set 7, 21, 57 is identical to the set 7,7, 21, 57 .

$\{7,7,21,57\} = \{7,21,57\}$
Repetation not matter

$(7,7,21,57) \neq (7,21,57)$
Repetation matters

Sequences and Tuples (Cont.)

3 tuple ← (7, 21, 57) → no of ele = 3 so finite sequence
4 tuple ← (1, 5, 9, 13) → " " " = 4 " " "
(1, 2, 3, 4, ...) → " " " : Not countable so infinite sequence

- As with sets, sequences may be finite or infinite.
- Finite sequences often are called tuples.
- A sequence with k elements is a k-tuple.
- Thus (7, 21, 57) is a 3-tuple.
- A 2-tuple is also called an ordered pair.
  - 2-tuple ← (0, 1), (a, b) and (21, 57) are some examples of ordered pair.
  - The set of all ordered pairs whose elements are 0's and 1's is (0, 0), (0, 1), (1, 0), (1, 1).

Sequences and Tuples (Cont.)

- A sequence is an ordered list whose elements are all of the same type, similar to an array.
- In contrast to arrays (and other standard container types), however, sequences are immutable, similar to strings in Java and other languages.
- There are operators to determine the length of a sequence s, to retrieve a single element, to select arbitrary subsequences, and to concatenate two sequences etc.

Sequences and Tuples (Cont.)

- Tuples are also useful in programming languages as well.
- A function receives several parameters, then it is same as to receiving a tuple of values as a single parameter.
- Similarly, a function could easily return multiple values by returning a tuple of values.

Functions and Relations

input $\xrightarrow{\text{Function}}$ Output

- Functions are central to mathematics.
- A function is an object that sets up an input–output relationship.
- A function takes an input and produces an output.
- In every function, the same input always produces the same output.
- If f is a function whose output value is b when the input value is a, we write, f(a) = b.

function name   input   output

abs functn returns the +ve value

let
Case 1 $x = 5$ i.e. $x \geq 0$ $abs(x) = (x) = 5$

Case 2 $x = -5$ i.e. $x < 0$ $abs(x) = -(x) = -(-5) = 5$

- A function also is called a mapping, and, if $f(a) = b$, we say that f maps a to b.
- For example, the absolute value function abs takes a number x as input and returns x if x is positive and x if x is negative.
- Thus $abs(2) = abs(-2) = 2$.

$$abs(x) = \begin{cases} (x) & \text{if } x \geq 0 \quad \text{case 1} \\ -(x) & \text{if } x < 0 \quad \text{case 2} \end{cases}$$

↳ abs

Domain

$abs : R \to R^+$

set of Real nos

Co-domain

Set of +ve Real nos

Range

Functions and Relations (cont.)

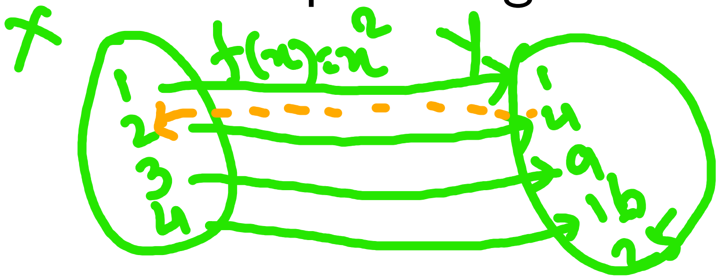
$f : A \to B$ Let $A = \{1, 2, 3, 4, 5\}$ Range

$f(x) = x^2$ Domain $B = \{1^2, 2^2, 3^2, 4^2, 5^2\} = \{1, 4, 9, 16, 25\}$

- Lets f is a function defined from the set 'A' to the set 'B' is $f : A \to B$, where the set A is called the domain of the function and the set B is called the co-domain of the function. co-domain $f : A \to N$
- The range of the function is subset of the co-domain set B.
- The elements in A which are mapped to the elements in B are called the pre-image while the elements in B which are having pre-image in A are called the image.

codomain Range Range codomain

- If every element of B has a pre-image in A then the function is called **onto function**.
- If one or more elements of B does not have pre-image in A then the function is called **into function**.

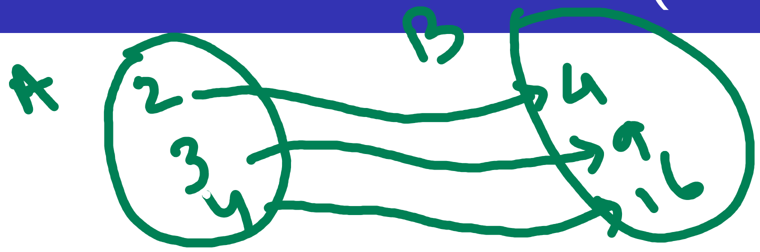


4 is image of 2 2 is pre image of 4

16 is " of 4 4 " " of 16

9 " " " 3 3 " " " 9

$f(x)=x^2$



For 4 preimage is 2
9 " " 3
16 " " 4

So onto

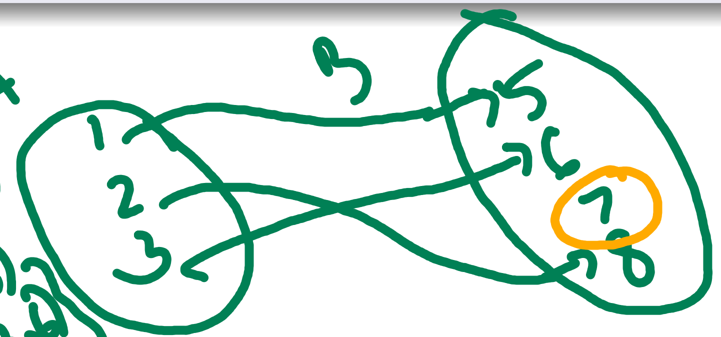
Example 1.8:

Consider the function $f : A \to B$ where $A = \{2, 3, 4\}$ and $B = \{4, 9, 16\}$ and $f(x) = x^2$ then 2 have an image 4, 3 have an image 9 and 4 have an image 16 and the function here is the onto function.

Example 1.9:

Consider the function $f : A \to B$ where $A = \{1, 2, 3\}$ and $B = \{5.6, 7, 8\}$ and $f(x) = \{(1, 5), (2, 8), (3, 6)\}$ then there exists an element 7 in B having no pre-image in A. Therefore, f is into function.

$f(x)=\{(1,5)$
$(2,8)$
$(3,6)\}$



So into

$\{0,1\} \times \{0,1\} = \{(0,0),(0,1),(1,0),(1,1)\}$ → 2 tuple

A   A

$f : A \times A \to B$

- If the domain of a function is the Cartesian product of k number of sets i.e. $A_1 A_2 \ldots A_k$ then the function is called a **k-ary function**
- If k is 1,the function is called a **unary function**.
- If k is 2, f is a **binary function**.

Example 2.0: $C = \{(2,5),(2,9),(4,5),(4,9)\}$

Let a function $f : A \times B \to C$ where $A = \{2,4\}$ and $B = \{5,9\}$ and $C = \{7,9,11,13\}$ and $f(x,y) = x + y$. Here f is n example of binary function.

$A \times B \times C \to R$ is a ternary funcn

$A \times B \times C \times \cdots \times$ kth set → (k-ary func)