

Numerical on Physical Layer
Computer Networking(CSE3034)

1. A noiseless 4-kHz channel is sampled every 1 msec. What is the maximum data rate?

Sol:

1. The key word here is “noiseless”. With a normal 4 KHz channel, Shannon limit would not allow this. For the 4 KHz channel we can make 8000 samples/sec. In this case if each sample is 1024 bits this channel can send 8.2 Mbps.
2. Television channels are 6 MHz wide. How many bits/sec can be sent if four-level digital signals are used? Assume a noiseless channel.

Sol:

2. Using the Nyquist theorem, which is "Max. data rate = $2B \log_2 V$ bits/sec", we can sample = $2 (6\text{MHz}) \log_2 (4) = 24$ million times/sec. Therefore, using four level signals total data rate will be of 24 Mbps.
3. If a binary signal is sent over a 3-kHz channel whose signal-to-noise ratio is 20 dB, what is the maximum achievable data rate?

Sol:

3. Signal-to-Noise ratio (S/N) = 20 dB, which implies that, $10 \log_{10}(S/N) = 20$
 $\Rightarrow \log_{10}(S/N) = 2$
 $\Rightarrow S/N = 10^2 = 100.$

Now, from Shannon's theorem we know,

Max. data rate = $B \log_2(1 + S/N)$ bits/sec

= $(3000 \text{ Hz}) \log_2(1 + 100)$ bits/sec

= $(3000 \text{ Hz}) \log_2(101)$ bits/sec

= $(3000) (6.643)$ bits/sec

= 19.92 kbps.

4. What signal-to-noise ratio is needed to put a T1 carrier on a 50-kHz line?

Sol:

4. $B = 50,000 \text{ Hz}$.

Now based on, Shannon's theorem, $B \log_2 (1 + S/N) \text{ bits/sec} = T1\text{'s data-rate}$.

$$50,000 \log_2(1 + S/N) = 1.544 \times 10^6 \log_2(1 + S/N) = 30.88$$

$$S/N = (2^{30.88}) - 1$$

$$\text{In dB, } S/N = 10 \log_{10} (S/N) = 10 \log_{10} ((2^{30.88}) - 1) = 92.95 \text{ dB}.$$

Therefore, the signal-to-noise ratio needs to be 92.95dB.

5. It is desired to send a sequence of computer screen images over an optical fiber. The screen is 480×640 pixels, each pixel being 24 bits. There are 60 screen images per second. How much bandwidth is needed, and how many microns of wavelength are needed for this band at 1.30 microns?

Sol:

5. Bandwidth needed is $480 * 640 * 24 * 60 = 442\,368\,000 \text{ bits}$

6. Radio antennas often work best when the diameter of the antenna is equal to the wavelength of the radio wave. Reasonable antennas range from 1 cm to 5 meters in diameter. What frequency range does this cover?

Sol:

6. $\text{Freq} = C/W$ where C: speed of light. W: wavelength. \ Convert 1 cm to m $\rightarrow 1 \text{ cm} = 0.01 \text{ m}$

$$\text{For diameter of 1 cm: } \text{Freq} = (3 \times 10^8) / 0.01 \text{ } \text{Freq} = 3 \times 10^{10} = 30$$

GHz

$$\text{For diameter of 5 m: } \text{Freq} = (3 \times 10^8) / 5 \text{ } \text{Freq} = 6 \times 10^7 = 60 \text{ MHz}$$

The cover range is from 60 MHz to 30 GHz.

7. A modem constellation diagram similar to Fig. 2-25 has data points at the following coordinates: (1, 1), (1, -1), (-1, 1), and (-1, -1). How many bps can a modem with these parameters achieve at 1200 baud?

Answer:

7. QPSK encodes 2 bits/symbol. $\text{rate} = \text{baud} * \text{bits/symbol} = 1200 * 2 = 2400 \text{ bps}$

8. A modem constellation diagram similar to Fig. 2-25 has data points at (0, 1) and (0, 2). Does the modem use phase modulation or amplitude modulation?

Answer:

This is amplitude modulation because both points are the same angle from the positive x axis but are different distances away from origin (0,0).

9. How many frequencies does a full-duplex QAM-64 modem use?

Answer:

9. Two, one for upstream and one for downstream. The modulation scheme itself just uses amplitude and phase. The frequency is not modulated.

10. Ten signals, each requiring 4000 Hz, are multiplexed on to a single channel using FDM. How much minimum bandwidth is required for the multiplexed channel? Assume that the guard bands are 400 Hz wide.

Answer:

10. There are ten 4000 Hz signals.

Therefore, we need nine guard bands to avoid any interference.

Altogether, the minimum bandwidth required is $(4000 \times 10 + 400 \times 9)$ or, 43,600 Hz.

11. Why has the PCM sampling time been set at 125 μ sec?

Answer:

11. A sampling time of 125 μ sec (micro-sec) corresponds to 8000 samples per second, because, in 1 second (or, in 106 μ sec) we sample $(106/125)$ or, 8000. According to the Nyquist theorem (which is "Max. data rate = $2B \log_2 V$ bits/sec"), this is the sampling frequency needed to capture all the information in a ($B=$) 4-kHz channel, such as a telephone channel (Actually the nominal bandwidth is less, but the cutoff is not sharp.). Note:(assuming two signal level or, possible symbol),
Max. data rate = $2B \log_2 V$ bits/sec = $2 \times 4,000 \times \log_2(2)$ bits/sec = $2 \times 4000 \times 1$ = 8,000.

12. What is the percent overhead on a T1 carrier; that is, what percent of the 1.544 Mbps are not delivered to the end user?

Answer:

12. With a modern T1 line, the end users get $8 \times 24 = 192$ of the 193 bits in a frame.
The overhead is therefore $1 / 193 = 0.5\%$. Therefore, at least 0.5% of the 1.544 Mbps are not delivered to the end user.

13. Compare the maximum data rate of a noiseless 4-kHz channel using
(a) Analog encoding (e.g., QPSK) with 2 bits per sample.
(b) The T1 PCM system.

Answer:

13. In both cases 8000 samples/sec are possible. With dibit encoding, two bits are sent per sample. With T1, 7 bits are sent per period. The respective data rates are 16 kbps and 56 kbps.

14. If a binary signal is sent over a 3kHz bandwidth channel whose signal to noise ratio is 20dB, what is the maximum achievable data rate?

Answer:

14. From Shannon's theorem:

$$\text{Max Data Rate} = W \log_2(1+S/N)$$

Note that the signal to noise ratio (SNR) given here is a power ratio, yet we are given the SNR in decibels. We therefore need to convert back to a power ratio:

$$\text{SNR in Db} = 10 \log_2(1+S/N)$$

$$S/N=100$$

$$\text{Max Data Rate} = W \log_2(1+S/N) = 20 \text{ kbps}$$

The Nyquist limit for binary signalling over a 3kHz channel is

$$\text{Max Data Rate} = 2W \log_2 M = 6 \text{ kbps}$$

Therefore, the maximum achievable data rate is 6kbps. (To achieve higher rates than this (up to the Shannon limit), one would have to use a different signalling method.)

15. The loss in a cable is usually defined in decibels per kilometer (dB/km). If the signal at the beginning of a cable with -0.3 dB/km has a power of 2 mW, what is the power of the signal at 5 km?

Answer:

15. The loss in the cable in decibels is $5 \times (-0.3) = -1.5$ dB.

We can calculate the power as

$$dB = 10 \log_{10} \frac{P_2}{P_1} = -1.5$$

Thus $P_2 = 1.4 \text{ mW}$

16. A TV channel has a bandwidth of 6 MHz. If we send a digital signal using one channel, what are the data rates if we use one harmonic, three harmonics, and five harmonics?

Solution:

16. Using the first harmonic, data rate = $2 \times 6 \text{ MHz} = 12 \text{ Mbps}$

Using three harmonics, data rate = $(2 \times 6 \text{ MHz}) / 3 = 4 \text{ Mbps}$

Using five harmonics, data rate = $(2 \times 6 \text{ MHz}) / 5 = 2.4 \text{ Mbps}$

17. A signal travels from point A to point B. At point A, the signal power is 100 W. At point B, the power is 90 W. What is the attenuation in decibels?

Solution:

$$17. \text{ dB} = 10 \log_{10} (90 / 100) = -0.46 \text{ dB}$$

18. If the bandwidth of the channel is 5 Kbps, how long does it take to send a frame of 100,000 bits out of this device?

Answer:

$$18. 100,000 \text{ bits} / 5 \text{ Kbps} = 20 \text{ s}$$

19. A file contains 2 million bytes. How long does it take to download this file using a 56-Kbps channel? 1-Mbps channel?

Answer:

19. The file contains $2,000,000 \times 8 = 16,000,000$ bits. With a 56-Kbps channel, it takes $16,000,000 / 56,000 = 289 \text{ s}$. With a 1-Mbps channel, it takes 16 s

20. A signal with 200 milliwatts power passes through 10 devices, each with an average noise of 2 microwatts. What is the SNR? What is the SNR_{dB}?

Answer:

$$20. \text{ We have } \text{SNR} = (200 \text{ mW}) / (10 \times 2 \times \mu\text{W}) = 10,000$$

$$\text{We then have } \text{SNR}_{\text{dB}} = 10 \log_{10} \text{SNR} = 40$$

21. Calculate the baud rate for the given bit rate and type of modulation.

- a. 2000 bps, FSK
- b. 4000 bps, ASK
- c. 6000 bps, QPSK
- d. 36,000 bps, 64-QAM

Answer:

21. We use the formula

$S = (1/r) \times N$, but first we need to calculate the value of r for each case.

- a. $r = \log_2 2 = 1 \rightarrow S = (1/1) \times (2000 \text{ bps}) = 2000 \text{ baud}$
- b. $r = \log_2 2 = 1 \rightarrow S = (1/1) \times (4000 \text{ bps}) = 4000 \text{ baud}$
- c. $r = \log_2 4 = 2 \rightarrow S = (1/2) \times (6000 \text{ bps}) = 3000 \text{ baud}$
- d. $r = \log_2 64 = 6 \rightarrow S = (1/6) \times (36,000 \text{ bps}) = 6000 \text{ baud}$

22. Calculate the bit rate for the given baud rate and type of modulation.

- a. 1000 baud, FSK
- b. 1000 baud, ASK
- c. 1000 baud, BPSK
- d. 1000 baud, 16-QAM

Answer:

22. We use the formula $N = r \times S$, but first we need to calculate the value of r for each case.

- a. $r = \log_2 2 = 1 \rightarrow N = (1) \times (1000 \text{ baud}) = 1000 \text{ bps}$
- b. $r = \log_2 2 = 1 \rightarrow N = (1) \times (1000 \text{ baud}) = 1000 \text{ bps}$
- c. $r = \log_2 2 = 1 \rightarrow N = (1) \times (1000 \text{ baud}) = 1000 \text{ bps}$
- d. $r = \log_2 16 = 4 \rightarrow N = (4) \times (1000 \text{ baud}) = 4000 \text{ bps}$

23. A cable company uses one of the cable TV channels (with a bandwidth of 6 MHz) to provide digital communication for each resident. What is the available data rate for each resident if the company uses a 64-QAM technique?

Answer:

23. 36Mbps

24. We have 14 sources, each creating 500 8-bit characters per second. Since only some of these sources are active at any moment, we use statistical TDM to combine these sources using character interleaving. Each frame carries 6 slots at a time, but we need to add 4-bit addresses to each slot.

Answer the following questions:

- a. What is the size of an output frame in bits?
- b. What is the output frame rate?
- c. What is the duration of an output frame?
- d. What is the output data rate?

Answer:

$$24. \text{ Frame size} = (\# \text{ of slots}) \times (\text{character size} + \text{slot address}) = 6 \times (8 \text{ bits} + 4 \text{ bits}) = 72 \text{ bits}$$

We can assume that we have only 6 input lines. Each frame needs to carry one character from each of these lines. This means that the link needs to send 500 frames/s

$$\text{Frame duration} = 1 / (\text{frame rate}) = 1 / 500 = 2 \text{ ms}$$

$$\text{Data rate} = (500 \text{ frames/s}) \times (72 \text{ bits/frame}) = 36 \text{ Kbps}$$

25. Two channels, one with a bit rate of 190 kbps and another with a bit rate of 180 kbps, are to be multiplexed using pulse-stuffing TDM with no synchronization bits. Answer the following questions:

- What is the size of a frame in bits?
- What is the frame rate?
- What is the duration of a frame?
- What is the data rate?

Answer:

25. We need to add extra bits to the second source to make both bit rates = 190 Kbps. Now we have two sources, each of 190 Kbps. Since the data unit was not specified, assume that it is one bit. Frame size = 2 bits.

$$\text{Frame rate} = 190 \text{ k frames/s}$$

$$\text{Frame duration} = 1 / \text{frame rate} = 1 / 190 \text{ k} = 5.26 \mu\text{s}$$

$$\text{Data rate} = 190 \text{ k} \times 2 = 380 \text{ kbps}$$