Assignment

Q6) How many integers greater than 5400 have both of the following properties

a) The digits are distinct

b) the digits 2 and 7 do not occur

Ans) a) since the digits are distinct so we can form at most 8 digits

(i) <u>5-digit numbers :-</u>

since, the first digit cannot be zero, we have 7 options in 1^{st} digit. For the 100's-digit we have 6 options, 10's-digit 5 options and for units 4 option

so, $7 \cdot 7 \cdot 6 \cdot 5 \cdot 4 = 5880$

(ii) 6-digit numbers :-

$7 \times 7 \times 6 \times 5 \times 4 \times 3 = 17,640$

(iii) 7-digit numbers :-

$7 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 = 35,280$

(iv) 8-digits numbers :-

Same as 7-digit $= 35,280$

b) since 2 and 7 doesn't occur, and all digits are distinct, the numbers should have at most 8 digit and at least 4 digit

so, $3 \times 7 \times 6 \times 5 + 4 \times 6 \times 5$

$= 750$

so, total is $= 750 + 5880 + 17,640 + (2 \times 35280)$

$= 94830.$

Q21) How many permutations are there of the letters of the word ADDRESSES? How many 8-permutations are there of these nine letters?

(Ans) (a) Total letters in the word

ADDRESSES = 9

Total D's = 2

Total E's = 2

Total S's = 3

Total no. of permutations $= \dfrac{9!}{2!\,2!\,3!} = 15,120$

(b) when we eliminate letter 'A'

No. of permutations $= \dfrac{8!}{2!\,2!\,3!}$

$= 1680.$

when we eliminate 'D'

No. of permutations $= \frac{8!}{2!\,3!}$

$= 3360$

when we eliminate 'R'

No. of permutations $= \frac{8!}{2!\,2!\,3!}$

$= 1680$

when we eliminate 'E'

No. of permutations $= \frac{8!}{2!\,3!}$

$= 3360$

when we eliminate 'S'

No. of permulations $= \frac{8!}{2!\,2!\,2!}$

$= 5040$

$\therefore$ Total no. of permulations $= (2\times1680) + (2\times3360) + 5040$

$= 15120$

Q30) We are to seat 5 boys, 5 girls and 1 parent in a circular arrangement around a table. In how many ways can this be done if no boy is to sit next to a boy and no girl is to sit next to a girl? What is there are 2 parents?

(Ans) a) Total no. of arrangements $= 5! \times 5! = 14400$

i.e. 5 boys and 5 girls

since no boy is to sit next to a boy and no girl is to sit next to a girl, so

let girls be G_1, G_2, G_3, G_4, G_5 and boys be B_1, B_2, B_3, B_4, B_5 and let parent be P_1

So,

P_1

G_5 (1)

(5) B_1 B_5 (1)

(5) G_1 G_4 (2)

(4) B_2 B_4 (2)

G_2 (4) B_3 (3) G_3 (3)

So, total no. of arrangement is

$$2 \times 14400 = 28\,800$$

b) If P_1 is flanked by different genders, all we need to do now is to fit P_2 (let P_2 be 2nd parent) in any of the 11 gaps

So, $11 \cdot 2 \cdot (5!)^2$

Now, another possibility exists, with P_2 being flanked by identical genders, say GP_1G and BP_2B in any of 4 places

So, $2 \cdot 4 \cdot (5!)^2$

So, total no. of arrangement $= 11 \cdot 2 \cdot (5!)^2 + 2 \cdot 4 \cdot (5!)^2$

$= 43200$

Q34) Determine the numbers of 11-permutations of the multiset

$$S = \{3a, 3 \cdot b, 3 \cdot c, 3 \cdot d\}$$

(Ans) Here we will decrease the repetition number of the object a

Now, the multiset is $\{2 \cdot a, 3 \cdot b, 3 \cdot c, 3 \cdot d\}$

It has 11 elements

$$\text{So, } \frac{11!}{2!\,3!\,3!\,3!} = \frac{11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3!}{2!\,3!\,3!\,3!}$$

$= 92,400$

similarly, we will decrease the repetition no. of the object b, we will get

$\{3a, 2.b, 3.c, 3.d\}$

Like that we will have same results for multisets $\{3.a, 3.b, 2.c, 3.d\}$ and $\{3.a, 3.b, 3.c, 2.d\}$

so, total sum $= 4 \times 92400 = 369600$

Q38) How many integral solutions of

$$x_1 + x_2 + x_3 + x_4 = 30$$

satisfy $x_1 \geq 2, x_2 \geq 0, x_3 \geq -5$ and $x_4 \geq 8$?

Ans) Let us introduce the new variables

$y_1 = x_1 - 2,\ y_2 = x_2,\ y_3 = x_3 + 5,\ y_4 = x_4 - 8$

Now, the given equation becomes

$$y_1 + y_2 + y_3 + y_4 = x_1 + x_2 + x_3 + x_4 - 2 - 8 + 5$$

$$= 30 - 5$$

$$\Rightarrow y_1 + y_2 + y_3 + y_4 = 25$$

Here, $k = 4$ and $r = 25$

Now, the no. of non-negative solutions of the original eq^n is

$$\left({}^{r+k-1}C_r\right) = \left({}^{25+4-1}C_{25}\right)$$

$$= \left({}^{28}C_{25}\right)$$

$$= \frac{28!}{25!\ 3!}$$

$$= \frac{28 \times 27 \times 26 \times 25!}{25! \times 3 \times 2!}$$

$= 3276$ ways