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Exercise 4.6

Saswat Mohanty
1941012407
CSE - D

Q2) Determine the mobile integers in

$$\overrightarrow{4}\ \overleftarrow{8}\ \overrightarrow{3}\ \overleftarrow{1}\ \overrightarrow{6}\ \overleftarrow{7}\ \overleftarrow{2}\ \overrightarrow{5}$$

(Ans) Mobile integer says that the integer K is mobile if its arrow points to the integer smaller than K.

so, the mobile integers are :-

3, 7 & 8

$$\left(\begin{array}{l} \overrightarrow{4}\ \overleftarrow{8} = 4<8 \\ \overrightarrow{3}\ \overleftarrow{1} = 1<3 \\ \overrightarrow{6}\ \overleftarrow{7} = 6<7 \end{array}\right)$$

Q4) Prove that in the algorithm of section 4.1, which generates directly the permutations of $\{1, 2, \ldots, n\}$ the directions of 1 and 2 never change.

(Ans) Here in permutation $\{1, 2, \ldots, n\}$ there is no integer m (A/q to theorem) with $1 > m$.

Therefore the direction of 1 will never change. Now, the direction of 2 will change if $2 > m$. That means $m = 1$. But $m = 1$ was detected as mobile integer, which cannot be true because 1 can never be mobile integers (as there is no integer smaller than 1 in $\{1, 2, \ldots, n\}$).

Hence, the direction of 1 and 2 never change.

Q5) Let $i_1, i_2, \ldots i_n$ be a permutation of $\{1, 2, \ldots, n\}$ with inversion sequence $b_1, b_2, \ldots, b_n$ and let $K = b_1 + b_2 + \ldots + b_n$. Show by induction that we cannot bring $i_1, i_2, \ldots, i_n$ to $12 \ldots n$ by fewer than K successive switches of adjacent terms?

(Ans) For $k=0$, the claim that we need at least k steps is false, therefore $k>0$.

The number of steps required to reach $12\ldots\ldots n$ is $(k-1)$

As the inversion starts at k and ends at 0 it follows that it must be equal to $(k-1)$ at some stage where one switch is required to reach to that permutation.

Then at least $(k-1)$ switches are required by the principle of mathematical induction

In this step, we have atleast $1+(k-1)=k$ switches are required which means that we cannot bring $i_1 i_2 \ldots\ldots i_n$ to $12\ldots\ldots n$ by ~~four~~ fewer than k successive switches of adj. terms.

Q6) Determine the inversion sequences of the following permutations of $\{1,2,\ldots\ldots,8\}$:

a) 35168274

b) 83476215

(Ans) a) The inversion sequence of permutation 35168274 is:

24040010

Here the no. in the inversion sequence denotes a number of integers that preceed 1 in the permutations & are greater than 1.

so, the numbers are 3 & 5.

Therefore, first no. in the sequence is 2.

similarly nos greater than 2 are 3, 5, 6 & 8. so, the second no. in the sequence is 4 & so on.

b) The inversion sequence of permutation 83476215 is:

6 5 1 1 3 2 1 0

Q7) construct the permutations of {1, 2, ..., 8} whose inversion sequences are:

a) 2, 5, 5, 0, 2, 1, 1, 0

b) 6, 6, 1, 4, 2, 1, 0, 0

(Ans) a) By Algorithm - I :-

$a_1 = (2+1) = 3$ i.e. 3^{rd} block $\rightarrow$ 1

$\boxed{4}$ $\boxed{8}$ $\boxed{1}$ $\boxed{6}$ $\boxed{5}$ $\boxed{7}$ $\boxed{2}$ $\boxed{3}$

$a_2 = (5+1) = 6$ i.e. 6^{th} block $\rightarrow$ 2

$a_3 = (5+1) = 6$ i.e. 6^{th} block $\rightarrow$ 3

$a_4 = (0+1) = 1$ i.e. 1^{st} block $\rightarrow$ 4

$a_5 = (2+1) = 3$ i.e. 3^{rd} block $\rightarrow$ 5

$a_6 = (1+1) = 2$ i.e. 2^{nd} block $\rightarrow$ 6

$a_7 = (1+1) = 2$ i.e. 2^{nd} block $\rightarrow$ 7

$a_8 = (0+1) = 1$ i.e. 1^{st} block $\rightarrow$ 8

$\therefore$ Required permutation is:

4 8 1 6 5 7 2 3

b) $\boxed{7}$ $\boxed{3}$ $\boxed{6}$ $\boxed{5}$ $\boxed{8}$ $\boxed{4}$ $\boxed{1}$ $\boxed{2}$

$a_1 = (6+1) = 7$ i.e. 7^{th} block $\rightarrow$ 1

$a_2 = (6+1) = 7$ i.e. 7^{th} block $\rightarrow$ 2

$a_3 = (1+1) = 2$ i.e. 2^{nd} block $\rightarrow$ 3

$a_4 = (4+1) = 5$ i.e. 5^{th} block $\rightarrow$ 4

$a_5 = (2+1) = 3$ i.e. 3^{rd} block $\rightarrow$ 5

$a_6 = (1+1) = 2$ i.e. 2^{nd} block $\rightarrow$ 6

$a_7 = (0+1) = 1$ i.e. 1^{st} block $\to 7$

$a_8 = (0+1) = 1$ i.e. 1^{st} block $\to 8$

Q8) How many permutations of $\{1, 2, 3, 4, 5, 6\}$ have

a) exactly 15 inversions?

b) exactly 14 inversions ?

c) exactly 13 inversions ?

(Ans) a) Exactly one permutation has 15 inversions

6 5 4 3 2 1

(6, 1), (6, 2) (6, 3) (6, 4) (6, 5)
(5, 1) (5, 2) (5, 3) (5, 4)
(4, 1) (4, 2) (4, 3)
(3, 1) (3, 2)
(2, 1)

b) 5 permutations that have 14 inversions

5 6 4 3 2 1

(6, 1) (6, 2) (6, 3) (6, 4)
(5, 1) (5, 2) (5, 3) (5, 4)
(4, 1) (4, 2) (4, 3)
(3, 1) (3, 2)
(2, 1)

6 4 5 3 2 1

(6, 1) (6, 2) (6, 3) (6, 4) (6, 5)
(5, 1) (5, 2) (5, 3)
(4, 1) (4, 2) (4, 3)
(3, 1) (3, 2)
(2, 1)

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(6,1) (6,2) (6,3) (6,4) (6,5)

(5,1) (5,2) (5,3) (5,4)

(4,1) (4,2)

(3,1) (3,2)

(2,1)

654321

(6,1) (6,2) (6,3) (6,4) (6,5)

(5,1) (5,2) (5,3) (5,4)

(4,1) (4,2) (4,3)

(3,1)

(2,1)

654312

(6,1) (6,2) (6,3) (6,4) (6,5)

(5,1) (5,2) (5,3) (5,4)

(4,1) (4,2) (4,3)

(3,1) (3,2)

c) 11 permutations that have 13 inversions

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(6,1) (6,2) (6,3) (6,4)

(5,1) (5,2) (5,3) (5,4)

(4,1) (4,2)

(3,1) (3,2)

(2,1)

564231

(6,1) (6,2) (6,3) (6,4)

(5,1) (5,2) (5,3) (5,4)

(4,1) (4,2) (4,3)

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(5,1) (5,2) (5,4)

(4,1) (4,2)

(3,1) (3,2)

(2,1)

643521

(6,1) (6,2) (6,3) (6,4) (6,5)

(5,1) (5,2)

(4,1) (4,2) (4,3)

(3,1) (3,2)

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645231

(6,1) (6,2) (6,3) (6,4) (6,5)

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(3,1)

(2,1)

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(3,1) (3,2)

(2,1)

654132

(6,1) (6,2) (6,3) (6,4) (6,5)

(5,1) (5,2) (5,3) (5,4)

(4,1) (4,2) (4,3)

(3,2)

654213

(6,1) (6,2) (6,3) (6,4) (6,5)

(5,1) (5,2) (5,3) (5,4)

(4,1) (4,2) (4,3)

(2,1)