ε - closure

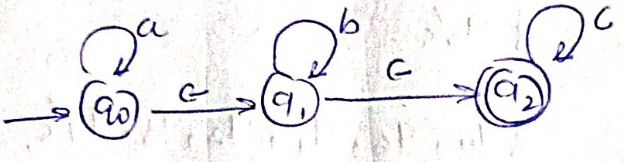
* **ε - closure** :- of a given state A means a set of states which can be reached from that state A with only ε (null) move including the state A it self.

(i) ε - closure (P) = P where, $P \in Q$.

(ii) If $\delta(P, \epsilon) = q$ and $\delta(q, \epsilon) = r$

$\therefore$ ε- closure (P) = {P, q, r}.

(P1)

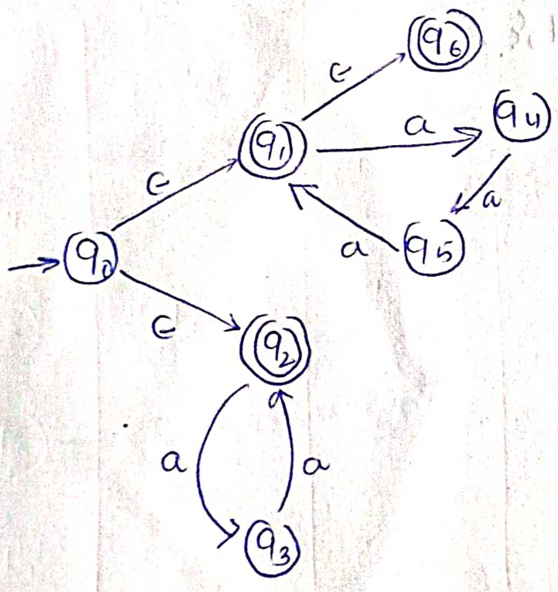


ε- closure (q_0) = {q_0, q_1, q_2}

ε- closure (q_1) = {q_1, q_2}

ε- closure (q_2) = {q_2}.

(P2)



ε - closure (q_0) = {q_0, q_1, q_2, q_6}.

ε- closure (q_1) = {q_1, q_6}

ε- closure (q_2) = {q_2}

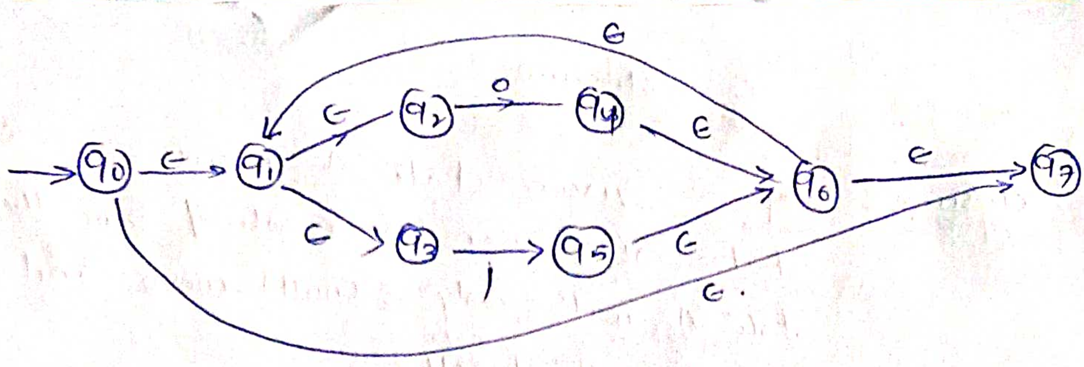
ε- closure (q_3) = {q_3}

ε- closure (q_4) = {q_4}

ε- closure (q_5) = {q_5}

ε- closure (q_6) = {q_6}.

(P3)



ϵ - closure (q_0) = $\{q_0, q_1, q_2, q_3, q_7\}$.

ϵ- closure (q_1) = $\{q_1, q_2, q_3\}$.

ϵ- closure (q_2) = $\{q_2\}$

ϵ- closure (q_3) = $\{q_3\}$

ϵ- closure (q_4) = $\{q_4, q_6, q_1, q_2, q_3, q_7\}$.

ϵ- closure (q_5) = $\{q_5, q_6, q_1, q_2, q_3, q_7\}$.

ϵ- closure (q_6) = $\{q_6, q_1, q_2, q_3, q_7\}$.

ϵ- closure (q_7) = $\{q_7\}$.

Conversion of ε-NFA to NFA

↳ Indirect method for conversion.
i.e. ε-NFA ⇒ NFA ⇒ DFA.

* The process of conversion of ε-NFA to NFA is called as Thomson Construction.

* Important points :-
  (i) No change in the initial state.
  (ii) No change in the total no of states.
  (iii) May be changed in the final states.

* Algorithm :- Let $M = (Q, \Sigma, \delta, q_0, F)$ be the ε-NFA
  ↳ $M' = (Q', \Sigma, \delta', q_0', F')$ be the NFA.

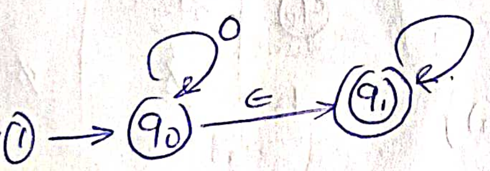
(S1) Initial state in NFA :- $q_0' = q_0$

(S2) Construction of δ' :-

$$\delta'(q_i, x) = \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_i), x))$$

(S3) Final state :- Every state who's ε-closure contain final state of ε-NFA is final state in NFA.

Ex: (1)



∴ ε-closure $(q_0) = \{q_0, q_1\}$.
ε-closure $(q_1) = \{q_1\}$.

δ	0	1	ε*
→q_0	$\{q_0\}$	ϕ	$\{q_0, q_1\}$
(q_1)	ϕ	$\{q_1\}$	$\{q_1\}$

→ (q_0')

$$\therefore \delta'(q_0', 0) = \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_0), 0))$$
$$= \epsilon\text{-closure}(\delta((q_0, q_1), 0))$$
$$= \epsilon\text{-closure}(\delta(q_0, 0) \cup \delta(q_1, 0))$$
$$= \epsilon\text{-closure}(q_0 \cup \phi) = \epsilon\text{-closure}(q_0)$$
$$= \{q_0, q_1\}$$

$\delta'(q_0, 1) = \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_0), 1))$

$= \epsilon\text{-closure}(\delta((q_0, q_1), 1))$

$= \epsilon\text{-closure}(\delta(q_0, 1) \cup \delta(q_1, 1))$

$= \epsilon\text{-closure}(\phi \cup \{q_1)$

$= \epsilon\text{-closure}(q_1) = \{q_1\}$.

$\delta'(q_1, 0) = \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_1), 0))$

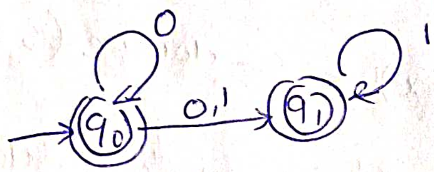
$= \epsilon\text{-closure}(\delta(q_1, 0))$

$= \epsilon\text{-closure}(\phi) = \phi$

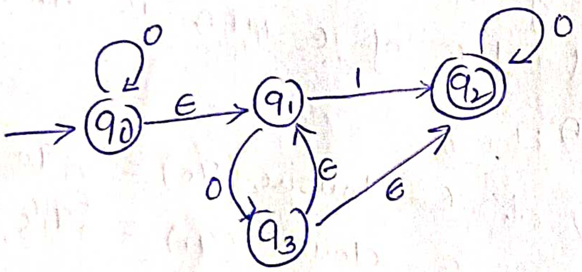
$\delta'(q_1, 1) = \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_1), 1))$

$= \epsilon\text{-closure}(\delta(q_1, 1))$

$= \epsilon\text{-closure}(q_1). = \{q_1\}$



Ex: (2)



δ	0	1	ϵ^*
$\to q_0$	$\{q_0\}$	ϕ	$\{q_0, q_1\}$
q_1	$\{q_3\}$	$\{q_2\}$	$\{q_1\}$
q_2	$\{q_2\}$	ϕ	$\{q_2\}$
q_3	ϕ	ϕ	$\{q_1 q_2 q_3\}$

$\delta'(q_0, 0) = \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_0), 0))$

$= \epsilon\text{-closure}(\delta((q_0, q_1), 0))$

$= \epsilon\text{-closure}(\delta(q_0, 0) \cup \delta(q_1, 0))$

$= \epsilon\text{-closure}(q_0 \cup q_3)$

$= (\{q_0, q_1\} \cup \{q_1, q_2, q_3\})$

$= \{q_0, q_1, q_2, q_3\}$

$\epsilon\text{-closure}(q_0) = \{q_0, q_1\}$.

$\epsilon\text{-closure}(q_1) = \{q_1\}$

$\epsilon\text{-closure}(q_2) = \{q_2\}$

$\epsilon\text{-closure}(q_3) = \{q_1 q_2 q_3$

③

$\delta'(q_0, 1) = \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_0), 1))$

$= \epsilon\text{-closure}(\delta((q_0, q_1), 1))$

$= \epsilon\text{-closure}(\delta(q_0, 1) \cup \delta(q_1, 1))$

$= \epsilon\text{-closure}(\phi \cup q_2)$

$= \epsilon\text{-closure}(q_2) = \{q_2\}.$

$\delta'(q_1, 0) = \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_1), 0))$

$= \epsilon\text{-closure}(\delta(q_1, 0)) = \epsilon\text{-closure}(q_3) = \{q_1, q_2, q_3\}$

$\delta'(q_1, 1) = \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_1), 1)) = \epsilon\text{-closure}(\delta(q_1, 1))$

$= \epsilon\text{-closure}(q_2) = \{q_2\}.$

$\delta'(q_2, 0) = \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_2), 0)) = \epsilon\text{-closure}(\delta(q_2, 0))$

$= \epsilon\text{-closure}(q_2) = \{q_2\}.$

$\delta'(q_2, 1) = \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_2), 1)) = \epsilon\text{-closure}(\delta(q_2, 1))$

$= \epsilon\text{-closure}(\phi) = \phi.$

$\delta'(q_3, 0) = \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_3), 0)) = \epsilon\text{-closure}(\delta((q_1, q_2, q_3), 0)$

$= \epsilon\text{-closure}(\delta(q_1, 0) \cup \delta(q_2, 0) \cup \delta(q_3, 0))$

$= \epsilon\text{-closure}(\cancel{\phi} \{q_3\} \cup \{q_2\} \cup \phi)$

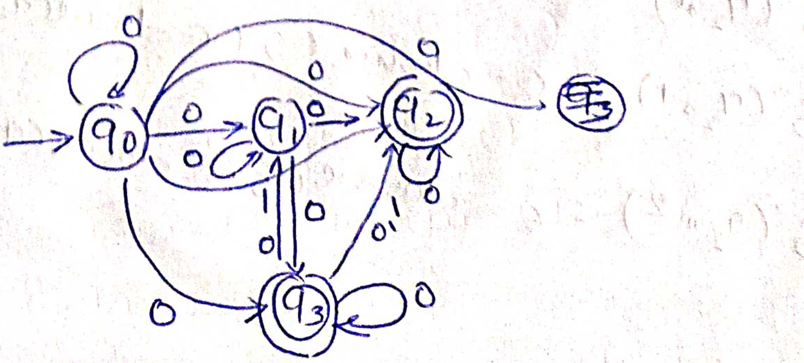
$= \{q_1, q_2, q_3\}.$

$\delta'(q_3, 1) = \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_3), 1)) = \epsilon\text{-closure}(\delta(q_1, q_2, q_3$

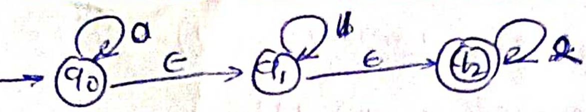
$= \epsilon\text{-closure}(\delta(q_1, 1) \cup \delta(q_2, 1) \cup \delta(q_3, 1))$

$= \epsilon\text{-closure}(\{q_2\} \cup \phi \cup \phi)$

$= \{q_2\}.$



Ex sn 3



δ	0	1	2	ϵ^*
$\rightarrow q_0$	$\{q_0\}$	ϕ	ϕ	$\{q_0, q_1, q_2\}$
q_1	ϕ	$\{q_1\}$	ϕ	$\{q_1, q_2\}$
q_2	ϕ	ϕ	$\{q_2\}$	$\{q_2\}$

ϵ-closure $(q_0) = \{q_0, q_1, q_2\}$
ϵ-closure $(q_1) = \{q_1, q_2\}$
ϵ-closure $(q_2) = \{q_2\}$

$\delta'(q_0, 0) = \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_0), 0)) = \epsilon\text{-closure}(\delta((q_0 q_1 q_2), 0))$
$= \epsilon\text{-closure}(\delta(q_0, 0) \cup \delta(q_1, 0) \cup \delta(q_2, 0))$
$= \epsilon\text{-closure}(\{q_0\} \cup \phi \cup \phi) = \epsilon\text{-closure}(q_0) = \{q_0, q_1, q_2\}.$

$\delta'(q_0, 1) = \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_0), 1)) = \epsilon\text{-closure}(\delta((q_0, q_1, q_2), 1))$
$= \epsilon\text{-closure}(\delta(q_0, 1) \cup \delta(q_1, 1) \cup \delta(q_2, 1)) = \epsilon\text{-closure}(q_1)$
$= \{q_1, q_2\}$

$\delta'(q_0, 2) = \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_0), 2))$
$= \epsilon\text{-closure}(\delta((q_0, q_1, q_2), 2)) = \epsilon\text{-closure}(\delta(q_0, 2) \cup \delta(q_1, 2) \cup \delta(q_2, 2)\}$
$= \epsilon\text{-closure}(\phi \cup \phi \cup q_2) = \{q_2\}.$

$\delta'(q_1, 0) = \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_1), 0)) = \epsilon\text{-closure}(\delta(q_1, 0) \cup \delta(q_2, 0)\}$
$= \epsilon\text{-closure}(\phi) = \phi$

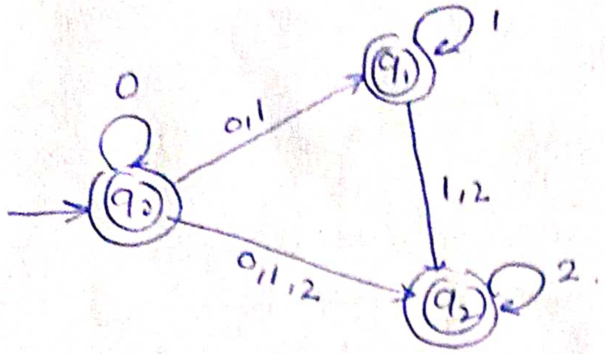
$\delta'(q_1, 1) = \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_1), 1)) = \epsilon\text{-closure}(\delta(q_1, 1) \cup \delta(q_2, 1)$
$= \epsilon\text{-closure}(\{q_1\} \cup \phi) = \{q_1, q_2\}$

$\delta'(q_1, 2) = \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_1), 1)) = \epsilon\text{-closure}(\delta(q_1, 2) \cup \delta(q_2, 2)$
$= \epsilon\text{-closure}(\phi \cup \{q_2\}) = \{q_2\}.$

$\delta'(q_2, 0) = \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_2), 0)) = \epsilon\text{-closure}(\delta(q_2, 0)) = \phi$

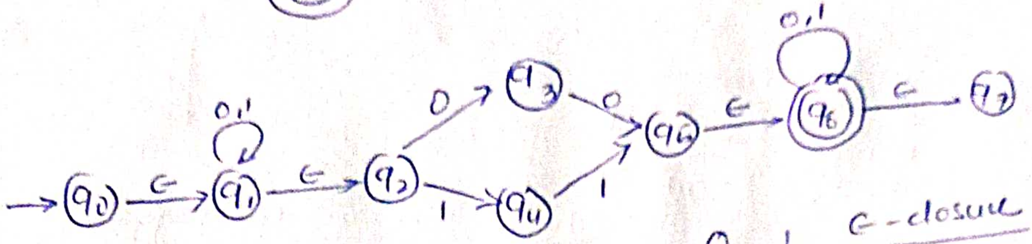
$\delta'(q_2, 1) = \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_2), 1)) = \epsilon\text{-closure}(\delta(q_2, 1)) = \phi$

$\delta'(q_2, 2) = \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_2), 2)) = \epsilon\text{-closure}(\delta(q_2, 2)) = \{q_2\}$



Equivalent NFA

Ex:- 4 :-



Q	ϵ-closure
q_0	$\{q_0, q_1, q_2\}$
q_1	$\{q_1, q_2\}$
q_2	$\{q_2\}$
q_3	$\{q_3\}$
q_4	$\{q_4\}$
q_5	$\{q_5, q_6, q_7\}$
q_6	$\{q_6, q_7\}$
q_7	$\{q_7\}$.

δ'	0	1
$\rightarrow q_0$	$\{q_1, q_2, q_3\}$	$\{q_1, q_2, q_4\}$
q_1	$\{q_1, q_2, q_3\}$	$\{q_1, q_2, q_4\}$
q_2	$\{q_3\}$	$\{q_4\}$
q_3	$\{q_5, q_6, q_7\}$	ϕ
q_4	ϕ	$\{q_5, q_6, q_7\}$
q_5	$\{q_6, q_7\}$	$\{q_6, q_7\}$
q_6	$\{q_6, q_7\}$	$\{q_6, q_7\}$.
q_7	ϕ	ϕ

$\delta'(q_0, 0) = \epsilon\text{-cl}(\delta(q_0, 0) \cup \delta(q_1, 0) \cup \delta(q_2, 0))$

$= \epsilon\text{-cl}(\phi \cup \{q_1\} \cup \{q_3\})$

$= \{q_1, q_2, q_3\}$

$\delta'(q_0, 1) = \epsilon\text{-cl}(\delta(q_0, q_1, q_2), 1)$

$= \epsilon\text{-cl}(\phi \cup \{q_1\} \cup \{q_4\})$

$= \{q_1, q_2, q_4\}$.

$\delta'(q_1, 0) = \epsilon\text{-cl}(\delta(q_1, q_2), 0)$

$= \epsilon\text{-cl}(\{q_1\} \cup \{q_3\})$

$= \{q_1, q_2, q_3\}$

$\delta'(q_1, 1) = \epsilon\text{-cl}(\delta(q_1, q_2), 1)$

$= \epsilon\text{-cl}(\{q_1\} \cup \{q_4\})$

$= \{q_1, q_2, q_4\}$.

Conversion of ϵ-NFA to DFA

(Direct).

- Method of conversion :- [Indirect :- ϵ-NFA to NFA to DFA]

Step ① :- Consider $M = \{Q, \Sigma, \delta, q_0, F\}$ is a NFA with ϵ. We have to convert this NFA with ϵ to equivalent DFA $M' = \{Q', \Sigma', \delta', q_0', F'\}$

- First we need to obtain ϵ-closure of q_0.

Let. ϵ-closure(q_0)

$\epsilon\text{-closure}(q_0) = \{P_1, P_2, P_3, \ldots, P_n\}$ then.

$[P_1, P_2, P_3, \ldots, P_n]$ becomes the start state of DFA M'.

- Now $[P_1, P_2, P_3, \ldots, P_n] \in Q'$

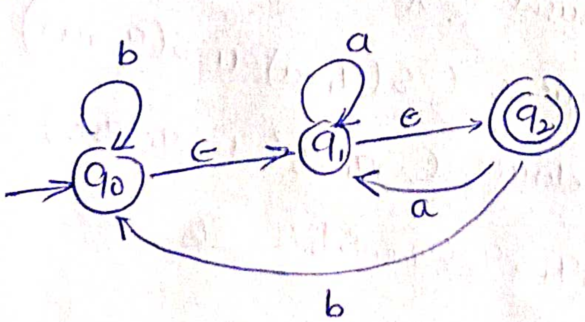
Step ② :- Now we will obtain δ' transition on $[P_1, P_2, \ldots P_n]$ for each input.

$\therefore \delta'([P_1, P_2, P_3, \ldots, P_n], a) = \epsilon\text{-closure}(\delta(P_1, a) \cup \ldots \cup \delta(P_n, a))$

Where $a \in \Sigma$.

Step ③ :- The states containing final state in P_i is a final state in DFA.

Example :- Convert the following NFA to DFA.



• First we need to find ε-clousre of all states as :-

i) ε-clousre(q_0) = $\{q_0, q_1, q_2\}$

ii) ε-clousre(q_1) = $\{q_1, q_2\}$

iii) ε-clousre(q_2) = $\{q_2\}$

Let us start from the ε-clousre(q_0)

$\therefore$ ε-clousre(q_0) = $\{q_0, q_1, q_2\}$, we will call this as state A.

$$\therefore \delta'(A, a) = \varepsilon\text{-clousre}(\delta(A, a))$$
$$= \varepsilon\text{-clousre}(\delta((q_0, q_1, q_2), a))$$
$$= \varepsilon\text{-clousre}(\delta(q_0, a) \cup \delta(q_1, a) \cup \delta(q_2, a))$$
$$= \varepsilon\text{-clousre}(\phi \cup \{q_1\} \cup \{q_1, q_2\})$$
$$= \varepsilon\text{-clousre}(q_1 \cup q_2)$$
$$= \{q_1, q_2\}$$ let us call it state B.

$$\therefore \delta'(A, b) = \varepsilon\text{-clousre}(\delta(A, b))$$
$$= \varepsilon\text{-clousre}(\delta((q_0, q_1, q_2), b))$$
$$= \varepsilon\text{-clousre}(\delta(q_0, b) \cup \delta(q_1, b) \cup \delta(q_2, b))$$
$$= \varepsilon\text{-clousre}(\{q_0, q_1, q_2\} \cup \{\} \cup \{q_0, q_1, q_2\})$$
$$= \cancel{\varepsilon\text{-clousre}(q_0 \cup q_1 \cup q_2)}$$
$$= \{q_0, q_1, q_2\}$$ i.e. state A.

$$\therefore \delta'(B, a) = \varepsilon\text{-clousre}(\delta(B, a))$$
$$= \varepsilon\text{-clousre}(\delta((q_1, q_2), a))$$
$$= \varepsilon\text{-clousre}(\delta(q_1, a) \cup \delta(q_2, a))$$
$$= \varepsilon\text{-clousre}(\{q_1, q_2\} \cup \{q_1, q_2\})$$
$$= \{q_1, q_2\}$$ i.e. state B.

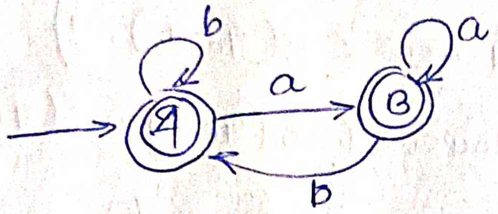
$\delta'(B, b) = \epsilon\text{-closure}(\delta(B, b))$

$= \epsilon\text{-closure}(\delta((q_1, q_2), b))$

$= \epsilon\text{-closure}(\delta(q_1, b) \cup \delta(q_2, b))$

$= \epsilon\text{-closure}(\{\} \cup \{q_0, q_1, q_2\})$

$= \{q_0, q_1, q_2\}$ i.e. state A.

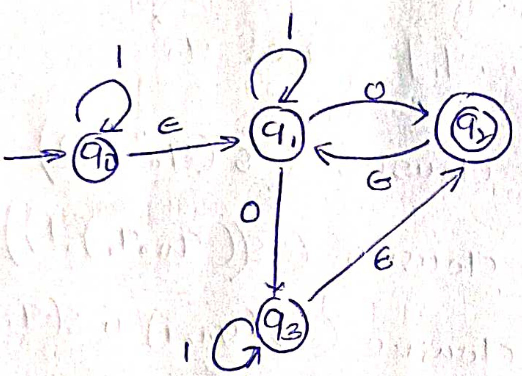


$\therefore F' = \{A, B\}$.

$A = \{q_0, q_1, q_2\}$

$B = \{q_1, q_2\}$.

(P1) :-



Convert it to DFA.

State	0	1	ϵ
$\rightarrow q_0$	ϕ	q_0	q_0, q_1
q_1	q_2, q_3	q_1	q_1
q_2	ϕ	ϕ	q_1, q_2
q_3	ϕ	q_3	q_1, q_2, q_3

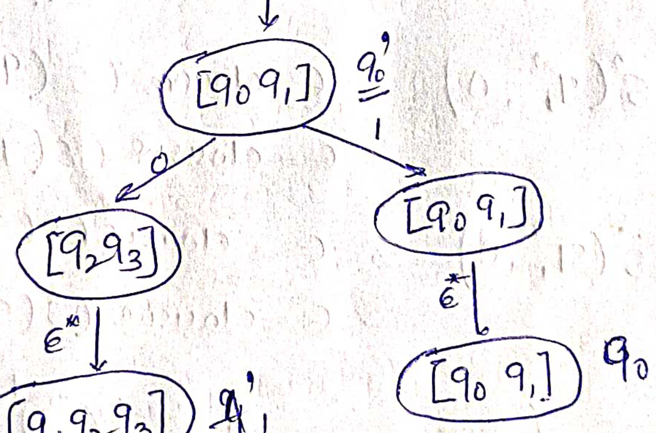
$\epsilon\text{-closure}(q_0) = \{q_0, q_1\}$

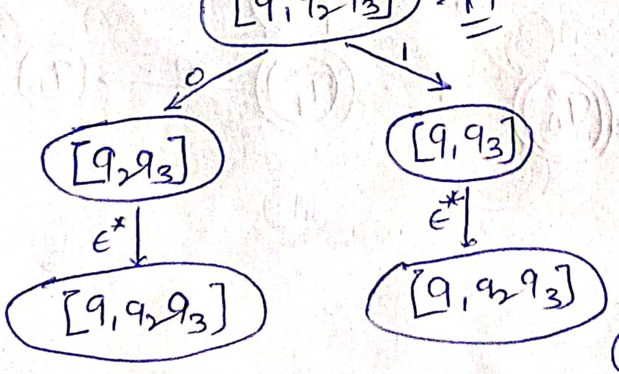
$\therefore q_0' = [q_0 q_1]$

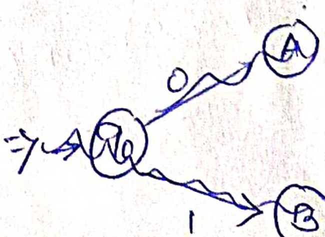
$\epsilon\text{-closure}(q_1) = \{q_1\}$

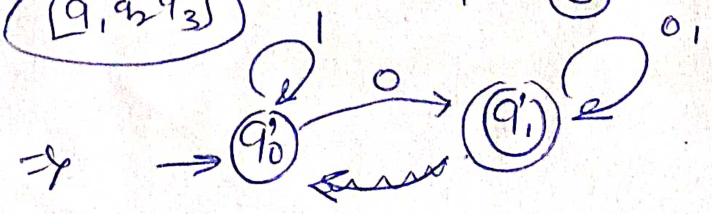
$\epsilon\text{-closure}(q_2) = \{q_1, q_2\}$

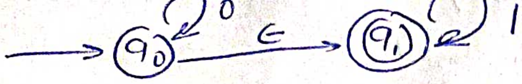
$\epsilon\text{-closure}(q_3) = \{q_1, q_2, q_3\}$









(P3) 

Sol^n:- Initial state $q_0' = \epsilon - \text{clousre}(q_0)$

$= \{q_0, q_1\}$.

$\therefore q_0' = [q_0, q_1]$

$\therefore \delta'(q_0', 0) = \epsilon - \text{clousre}(\delta(q_0', 0))$

$= \epsilon - \text{clousre}(\delta((q_0, q_1), 0))$

$= \epsilon - \text{clousre}(\delta(q_0, 0) \cup \delta(q_1, 0))$

$= \epsilon - \text{clousre}(q_0 \cup \phi)$

$= [q_0, q_1]$. i.e. $\underline{\underline{q_0'}}$

$\therefore \delta'(q_0', 1) = \epsilon - \text{clousre}(\delta(q_0', 1))$

$= \epsilon - \text{clousre}(\delta((q_0, q_1), 1))$

$= \epsilon - \text{clousre}(\delta(q_0, 1) \cup \delta(q_1, 1))$

$= \epsilon - \text{clousre}(\phi \cup q_1)$

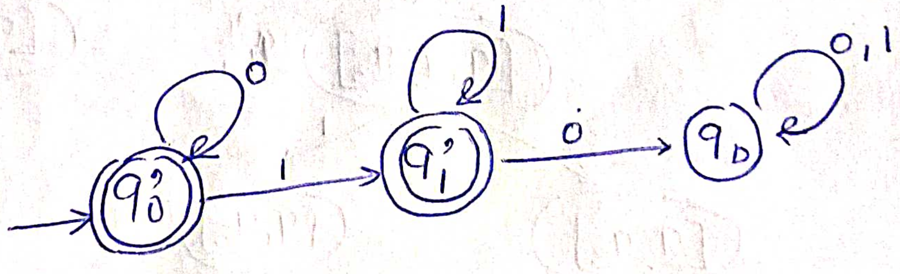
$= [q_1]$ i.e. q_1'

$\therefore \delta'(q_1', 0) = \epsilon - \text{clousre}(\delta(q_1', 0))$

$= \epsilon - \text{clousre}(\delta(q_1, 0)) = \phi$

$\therefore \delta'(q_1', 0) = \epsilon - \text{clousre}(\delta(q_1', 1))$

$= \epsilon - \text{clousre}(\delta(q_1, 1)) = [q_1]$ i.e. q_1'.



DFA Minimization :-

→ Partitioning Method :- (Equivalence Theorem).

1. Let M be a DFA = {$Q, \Sigma, \delta, q_0, F$} that recognize a Language L. Then the minimized DFA M = {$Q', \Sigma, q_0, \delta', F'$} can be constructed as follows:

(S1) Divide set Q into two sets, one set will contain all final states and another set will contain all non-final states. Called this partition as P_0.

(S2) Initialize $K = 1$.

(S3) Find P_K partitioning of the the different sets of P_{K-1}. In each set of P_{K-1}, take all the possible pair of states. If two states of a set are distinguishable split the set into different sets in P_K.

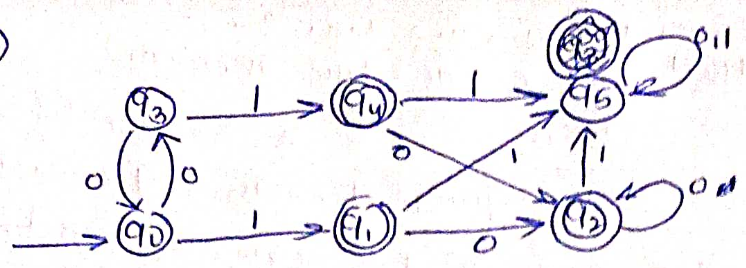
(S4) Stop when $P_K = P_{K-1}$ (No changes in the partition).

(S5) All sets of one set are merged into one. Number of states in the minimized DFA will be equal to No. of sets in P_K.

→ How to find that two states in a set are distinguishable or not?

Solⁿ → Two states (q_i, q_j) are distinguishable in the partition if for any input symbol a, $\delta(q_i, a)$ and $\delta(q_j, a)$ are in different sets in partition P_{K-1}.

Example:- 1



Sol^n:-

~~Set~~ Step: 1 - Partition P_0 will have two sets in.
$\{q_1, q_2, q_4\}$ and $\{q_0, q_3, q_5\}$.

Step: 2 - Calculate P_1, we will check for the new possible partition.

(i) for set 1:- $\delta(q_1, 0) = q_2$ $\delta(q_2, 0) = q_2$ &] so q_1 and q_2 are non-distinguishable
$\delta(q_1, 1) = q_5$ $\delta(q_2, 1) = q_5$

$\delta(q_1, 0) = q_2$ $\delta(q_4, 0) = q_2$] so q_1 and q_4 are non-distinguishable.
$\delta(q_1, 1) = q_5$ $\delta(q_4, 1) = q_5$

$\therefore$ q_1, q_2 and q_4 are non-distinguishable.

(ii) for set-2:- $\delta(q_0, 0) = q_3$. $\delta(q_3, 0) = q_0$] for 0 the moves are on the same set &
$\delta(q_0, 1) = q_1$ $\delta(q_3, 1) = q_4$] for 1 also moves are in the one set.

$\therefore$ q_0 and q_3 are non-distinguishable.

$\delta(q_0, 0) = q_3$ $\delta(q_5, 0) = q_5$ → move to set non final states.
$\delta(q_0, 1) = q_1$ $\delta(q_5, 1) = q_5$ → Different set moves.

$\therefore$ q_0 and q_5 are distinguishable.

$\therefore$ set $\{q_0, q_3, q_5\}$ will be partitioned into $\{q_0, q_3\}$ and $\{q_5\}$.

$\therefore P_1 = \{\{q_1, q_2, q_4\}, \{q_0, q_3\}, \{q_5\}\}$.

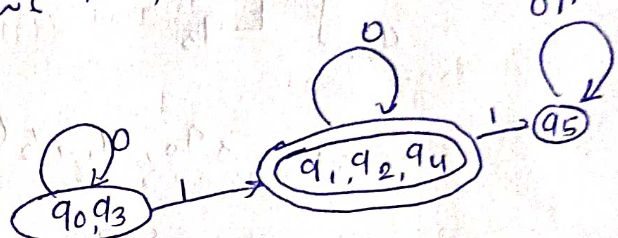
for set 1 :- q_1 and q_2 are non distingshible.
q_2 and q_4 are non distinguishable.
$\therefore$ q_1, q_2 and q_4 are non distinguishable.

for set 2 :- q_0 and q_3 are non distinguishable.

for set 3 :- set has only one states, so can not be partitioned further

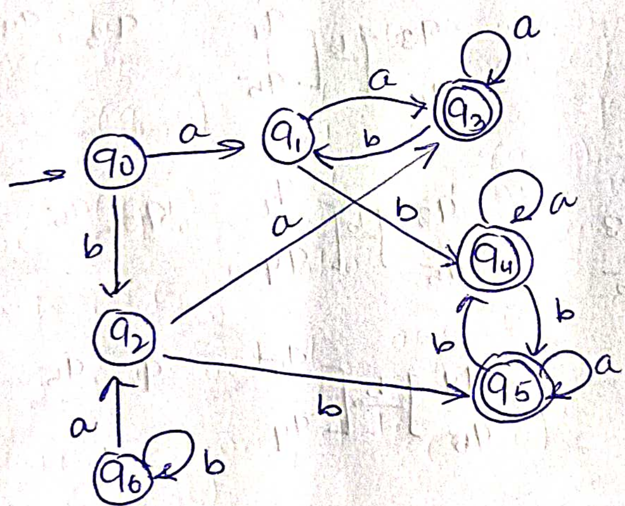
$\therefore$ $P2 = \{\{q_1, q_2, q_4\}, \{q_0, q_3\}, \{q_5\}\}$

A :- $\underline{P_1 = P_2}$, so this is final partitioned.



$\therefore$ Minized DFA ←

Example ② :-



δ	a	b
→ q_0	q_1	q_2
q_1	q_3	q_4
q_2	q_3	q_5
q_3	q_3	q_1
q_4	q_4	q_5
q_5	q_5	q_4
q_6	q_2	q_6

(i) Remove unreachable state. i.e. q_6

(ii) Remove Dead state – No dead state.

(iii) (a) Find the partition. (Initial).

Non-Final

Final.

$P_0 = \{\{q_0, q_1, q_2\}, \{q_3, q_4, q_5\}\}$

(b) Find the next partition P_1. for set $\{q_0, q_1, q_2\}$.

$\left.\begin{matrix} q_0 \\ q_1 \end{matrix}\right] \xrightarrow{a} \left[\begin{matrix} q_1 \\ q_3 \end{matrix}\right.$ $\quad$ $\left.\begin{matrix} q_0 \\ q_1 \end{matrix}\right] \xrightarrow{b} \left[\begin{matrix} q_2 \\ q_4 \end{matrix}\right.$ $\quad$ q_0, q_1 are distinguishable.

$\left.\begin{matrix} q_0 \\ q_2 \end{matrix}\right] \xrightarrow{a} \left[\begin{matrix} q_1 \\ q_3 \end{matrix}\right.$ $\quad$ $\left.\begin{matrix} q_0 \\ q_2 \end{matrix}\right] \xrightarrow{b} \left[\begin{matrix} q_2 \\ q_5 \end{matrix}\right.$ $\quad$ q_0, q_2 are distinguishable.

$\left.\begin{matrix} q_1 \\ q_2 \end{matrix}\right] \xrightarrow{a} \left[\begin{matrix} q_3 \\ q_3 \end{matrix}\right.$ $\quad$ $\left.\begin{matrix} q_1 \\ q_2 \end{matrix}\right] \xrightarrow{b} \left[\begin{matrix} q_4 \\ q_5 \end{matrix}\right.$ $\quad$ q_1, q_2 are non-distinguishable.

$\therefore \{q_0, q_1, q_2\} \rightarrow \{\{q_0\}, \{q_1, q_2\}\}$

for set $\{q_3, q_4, q_5\}$

$\left.\begin{matrix} q_3 \\ q_4 \end{matrix}\right] \xrightarrow{a} \left[\begin{matrix} q_3 \\ q_4 \end{matrix}\right.$ $\quad$ $\left.\begin{matrix} q_3 \\ q_4 \end{matrix}\right] \xrightarrow{b} \left[\begin{matrix} q_1 \\ q_5 \end{matrix}\right.$ $\quad$ q_3, q_4 are distinguishable.

$\left.\begin{matrix} q_3 \\ q_5 \end{matrix}\right] \xrightarrow{a} \left[\begin{matrix} q_3 \\ q_5 \end{matrix}\right.$ $\quad$ $\left.\begin{matrix} q_3 \\ q_5 \end{matrix}\right] \xrightarrow{b} \left[\begin{matrix} q_1 \\ q_4 \end{matrix}\right.$ $\quad$ q_3, q_5 are distinguishable.

$\left.\begin{matrix} q_4 \\ q_5 \end{matrix}\right] \xrightarrow{a} \left[\begin{matrix} q_4 \\ q_5 \end{matrix}\right.$ $\quad$ $\left.\begin{matrix} q_4 \\ q_5 \end{matrix}\right] \xrightarrow{b} \left[\begin{matrix} q_5 \\ q_4 \end{matrix}\right.$ $\quad$ q_4, q_5 are non-distinguishable

$\therefore \{q_3, q_4, q_5\} \longrightarrow \{\{q_3\}, \{q_4, q_5\}\}$.

$\therefore P_1 = \{\{q_0\}, \{q_1, q_2\}, \{q_3\}, \{q_4, q_5\}\}$

(c) Find Partition P_2 for $\{q_1, q_2\}$.

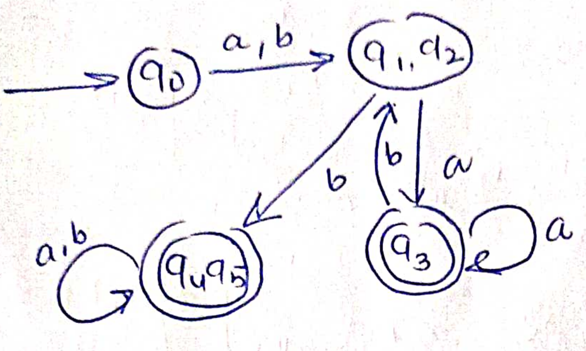
$\left.\begin{matrix} q_1 \\ q_2 \end{matrix}\right] \xrightarrow{a} \left[\begin{matrix} q_3 \\ q_3 \end{matrix}\right.$ $\quad$ $\left.\begin{matrix} q_1 \\ q_2 \end{matrix}\right] \xrightarrow{b} \left[\begin{matrix} q_4 \\ q_5 \end{matrix}\right.$ $\quad$ $\therefore$ non-distinguishable.

for $\{q_4, q_5\}$. $\quad$ $\left.\begin{matrix} q_4 \\ q_5 \end{matrix}\right] \xrightarrow{a} \left[\begin{matrix} q_4 \\ q_5 \end{matrix}\right.$ $\quad$ $\left.\begin{matrix} q_4 \\ q_5 \end{matrix}\right] \xrightarrow{b} \left[\begin{matrix} q_5 \\ q_4 \end{matrix}\right.$ $\quad$ $\therefore$ non-distinguishable.

$\therefore P_2 = \{\{q_0\}, \{q_1, q_2\}, \{q_3\}, \{q_4, q_5\}\}$.

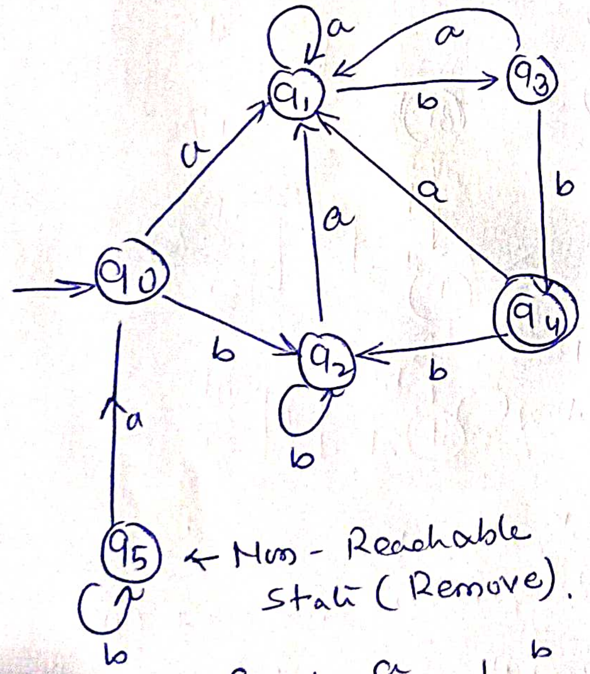
$\therefore P_2 = P_1$ so find Partition.

There fore Minimized DFA.



DFA Minimization

→ Table Filling Method.



← Non-Reachable State (Remove).

q_0					
q_1	X				
q_2	=	X			
q_3	X	X	X		
q_4	X	X	X	X	
	q_0	q_1	q_2	q_3	q_4

S	a	b
→ q_0	q_1	q_2
q_1	q_1	q_3
q_2	q_1	q_2
q_3	q_1	q_4
q_4	q_1	q_4

→ Pair
- → Both non-F → No mark
- → Both F → No Mark
- → Different → Mark

$q_0 q_1 \xrightarrow{a} q_1$

$q_0 q_1 \xrightarrow{b} q_2 q_3$.

$q_0 q_3 \xrightarrow{a} q_1$

$q_0 q_3 \xrightarrow{b} q_2 q_4$

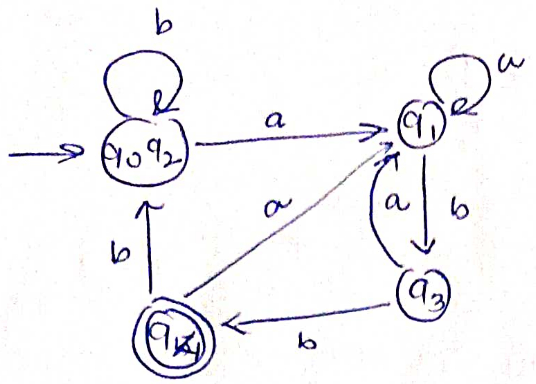
$q_1 q_2 \xrightarrow{a} q_1 q_1$

$q_1 q_2 \xrightarrow{b} q_3 q_2$

$q_1 q_3 \xrightarrow{a} q_1$

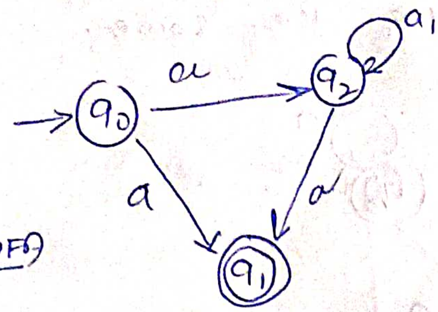
$q_1 q_3 \xrightarrow{b} q_3 q_4$

$\therefore$ q_0 and q_2 are equivalent.



TOC Problems

(Q1)

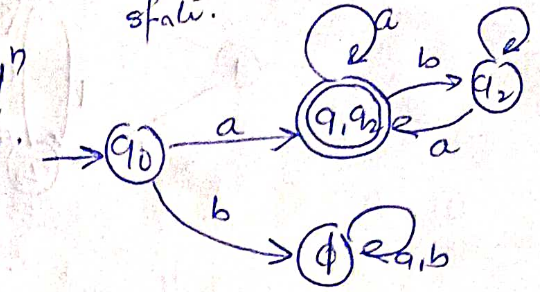


NFA

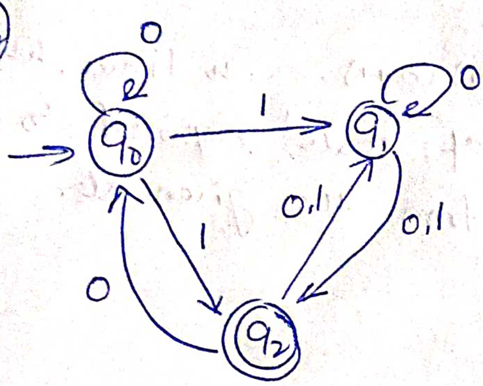
(i) How many Final state.

(ii) How many transitions are involved with final state.

Sol^n =



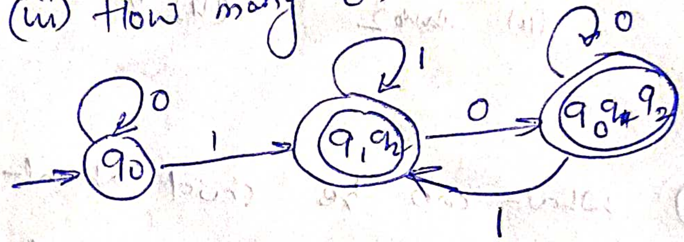
(Q2)



(i) How many Final state

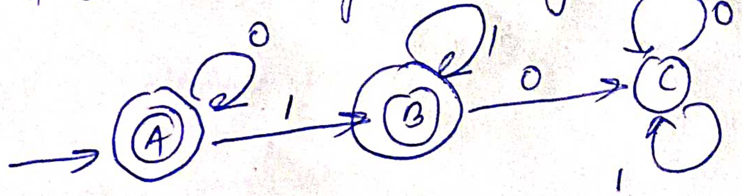
(ii) How many transition involve in ~~the~~ with the final state.

(iii) How many unreachable state.



(3) A FSA can ~~add~~ be designed to add two integers of any arbitrary length. (NO)

(4) RE of the L recognized by the FSA.



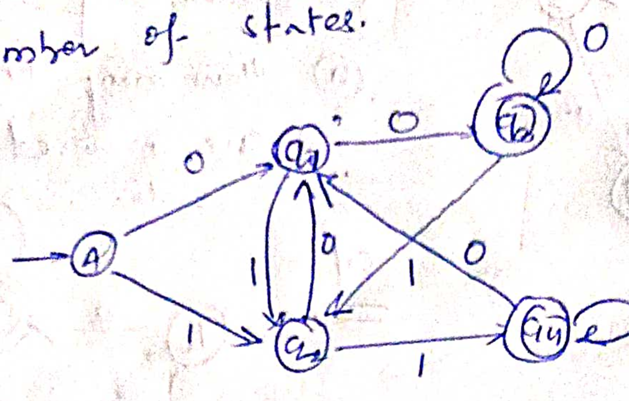
RE of A + RE of B.

$0^* + 0^* 1 1^*$

$0^*(\epsilon + 11^*)$

$\underline{0^* 1^*}$

(5) Let L be the set of all binary strings whose last two symbols are the same. The number of states.



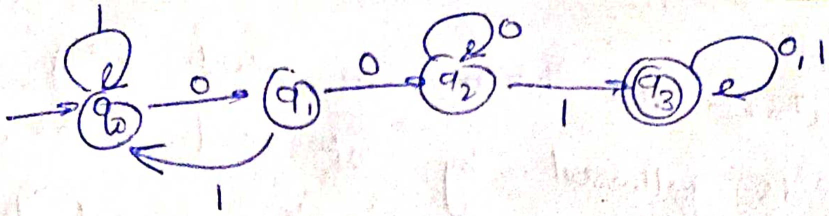
(6) Consider the RE (0+1) (0+1) (0+1) .... n times. What will be the minimum no of state required to accept the L generated from the given RE.

(i) n
(ii) ~~n~~+2
(iii) 2^n
(iv) n+1 ✓

(7) What can be said about a regular Language L over {a} whose minimal FSA has two states.

(i) can be $\{a^n \mid n \text{ is odd}\}$
(ii) can be $\{a^n \mid n \text{ is even}\}$
(iii) Can be $\{a^n \mid \geq 0\}$
(iv) Either L can be $\{a^n \mid n \text{ is even}\}$ or L can be $\{a^n \mid n \text{ is odd}\}$

(8)



Let S denotes the 7 bit binary string in which the first, the fourth, and the last bits are 1. The number of strings in S that are acceptable by M is.

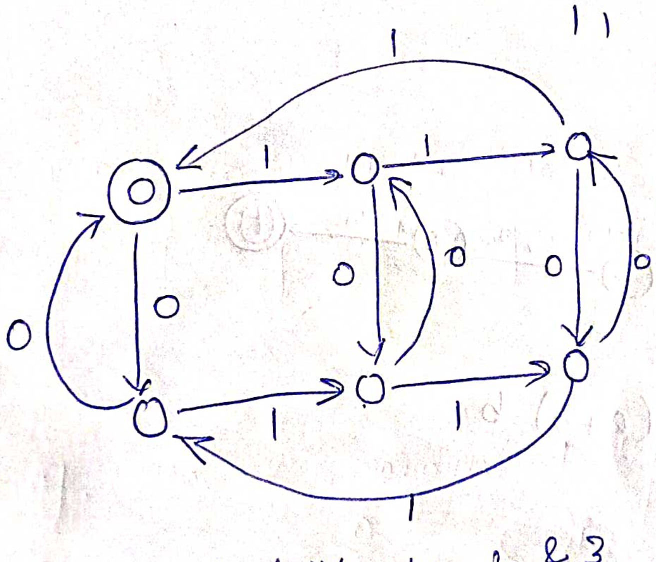
(i) 1 (ii) 5 (iii) 8 (iv) 7.

Sol^n: 1 _ _ 1 _ _ 1

$2^4 = \underline{16}$.

00	00
00	01
00	10
00	11
01	00
10	00
11	00.

(9)



M accepts all the binary string in which no of 1's and 0's are respectively

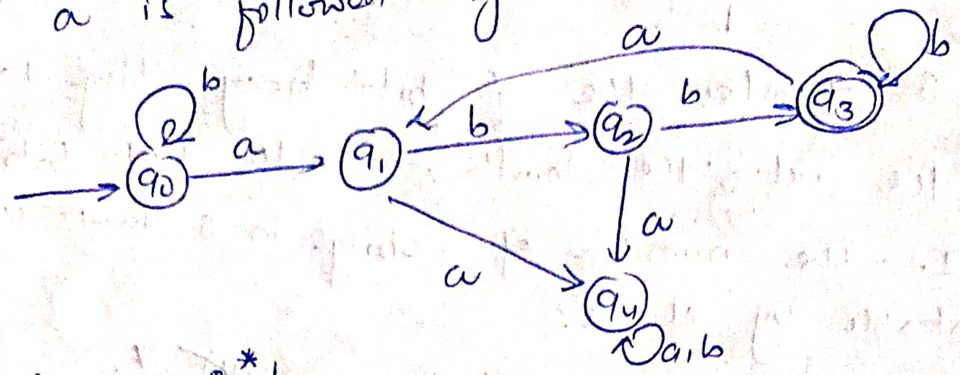
(i) divisible by 2 & 3.

(ii) odd and even

(iii) even and odd

(iv) divisible by 3 & 2

(10) Construct the DFA in which every a is followed by at least 2 b's.

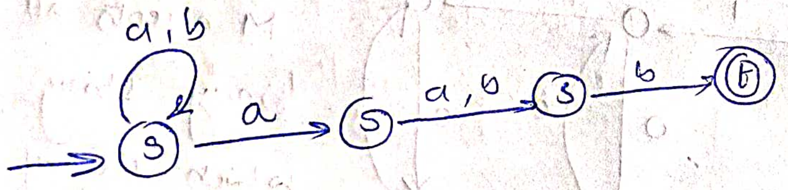


(i) $\{w \in \{a,b\}^* \mid$ every a is followed by at b's$\}$

(ii) $\{w \in \{a,b\}^* \mid$ every a is followed by at least 2 b's$\}$

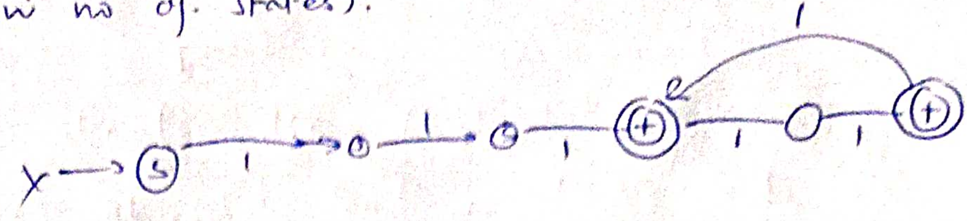
(iii) $\{w \in \{a,b\}^* \mid w$ contains abb$\}$

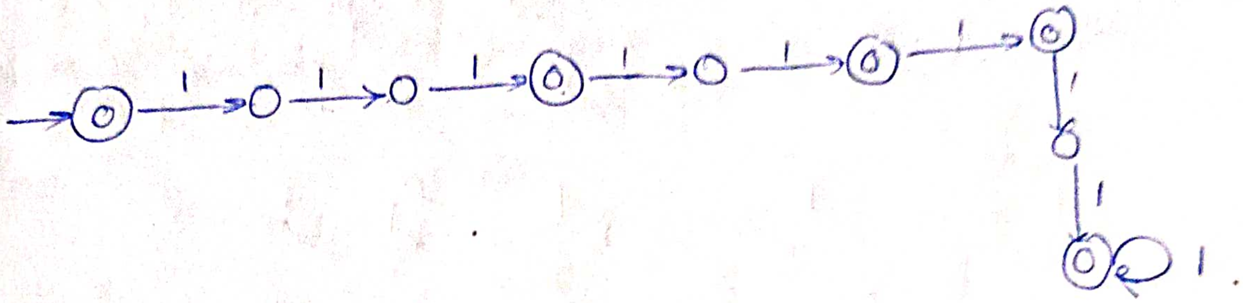
(11) Which RE best describes the language accepted by the n-DFA.



$(a+b)^* a (a+b) b$

(12) Consider the Language $L = \{111 + 11111\}^*$ (mini no of states).





(13) RE $\Sigma^* 0011 \Sigma^*$ where $\Sigma = \{0,1\}$. what is the minimum no of states required DFA that recognize L.

$$(0+1)^* 0011 (0+1)^* \rightarrow \underline{\underline{4+1}}$$