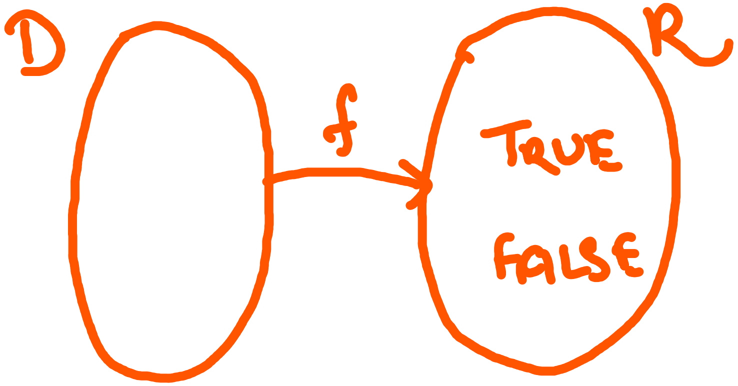
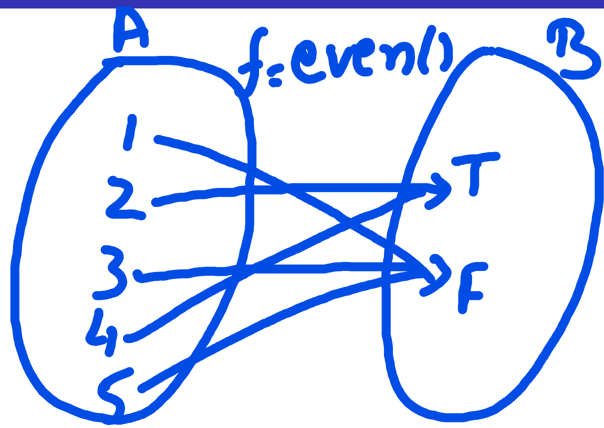
Functions and Relations (cont.)

$f: D \to R$





- A **predicate** or **property** is a function whose range is *TRUE*, *FALSE*.
  - Considering as example, the even be a property that checks a number is even or odd. It is *TRUE* if the input is an even number and *FALSE* if the input is an odd number. Thus $even(2) = TRUE$ and $even(3) = FALSE$.

Functions and Relations (cont.)

A **relation** in mathematics defines the relationship between two different sets of information. If two sets are considered, the relation between them will be established if there is a connection between the elements of two or more non-empty sets.

- In the morning assembly at schools, students are supposed to stand in a queue in ascending order of the heights of all the students. This defines an ordered relation between the students and their heights.

- A special type of binary relation, called an equivalence relation, captures the notion of two objects being equal in some feature.
- A binary relation R is an equivalence relation if R satisfies three conditions:
  1. R is **reflexive relation** i.e x, xRx.
  2. R is **symmetric relation** i.e if xRy then $yRx \forall x, yA$.
  3. R is **transitive relation** i.e if xRy and yRz then $xRz \forall x, y, z \in A$.

Example 1.9: Define an equivalence relation on the set of natural numbers, written as $\equiv_7$ → – is multiple of 7

.

For i, j $\in$ N , let $i \equiv_7 j$, if $i - j$ is a multiple of 7. This is an equivalence relation because it satisfies the following three conditions.

1. First, it is reflexive, as $i - i = 0$, which is a multiple of 7. xRx
2. Second, it is symmetric, as $i - j$ is a multiple of 7 if $j - i$ is a multiple of 7. $i - j = 7k \Rightarrow j - i = -7k = 7(-k)$ is a multiple of 7
   $xRy \Rightarrow yRx$
3. Third, it is transitive, as whenever $i - j$ is a multiple of 7 and $j - k$ is a multiple of 7, then $i - k = (i - j) + (j - k)$ is the sum of two multiples of 7 and hence a multiple of 7, too.

$xRy, yRz \Rightarrow xRz$

$iRj \quad jRk \quad iRk$

$$\left.\begin{aligned} i - j &= 7p \\ j - k &= 7q \end{aligned}\right\} \text{Adding}$$

$i - j + j - k = 7p + 7q \Rightarrow i - k = 7(p+q) = 7r$ is a multiple of 7.

Graphs

The edges have no direction,

- An undirected graph, or simply a graph $G(V, E)$, is a set of points with lines connecting some of the points.
- The points are called nodes or vertices (V), and the lines are called edges (E), as shown in the following figure.

$E = \{(P,Q), (P,R), (P,T), (S,T), (R,S), (R,T)\}$

$V = \{P, Q, R, S, T\}$

Degree of $P = 3$
" " $Q = 1$
" " $R = 3$
" " $S = 2$
" " $T = 3$

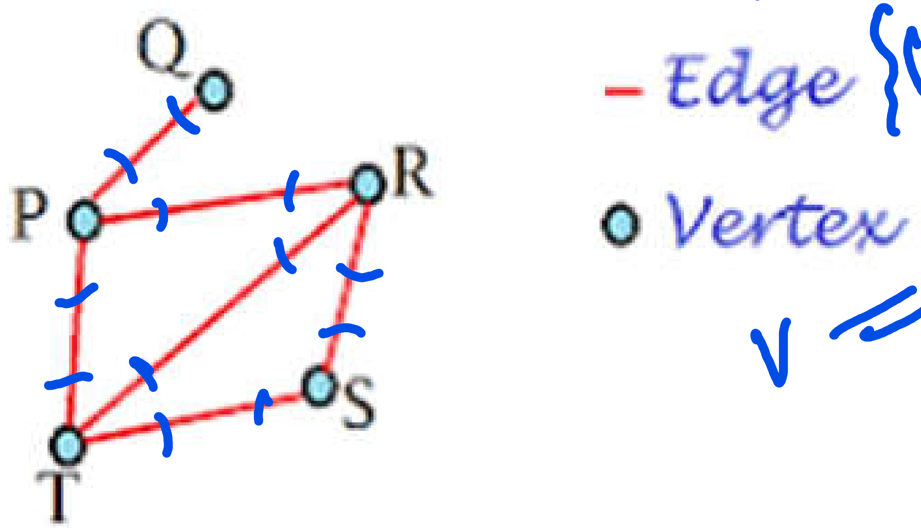


Figure: Diagram showing an undirected graph.

Graphs (cont..)

- The number of edges at a particular node is the **degree** of that node.
- In the above figure the nodes P, Q, R, S and T have the degree 3, 1, 3, 2, and 3 respectively.
- No more than one edge is allowed between any two nodes. An edge from a node to itself is allowed and is called a **self-loop**.

Labeled Graph

- The nodes and/or edges of a graph are labeled, which then is called a **labeled graph**. The Figure depicts a graph whose nodes and edges are labeled.

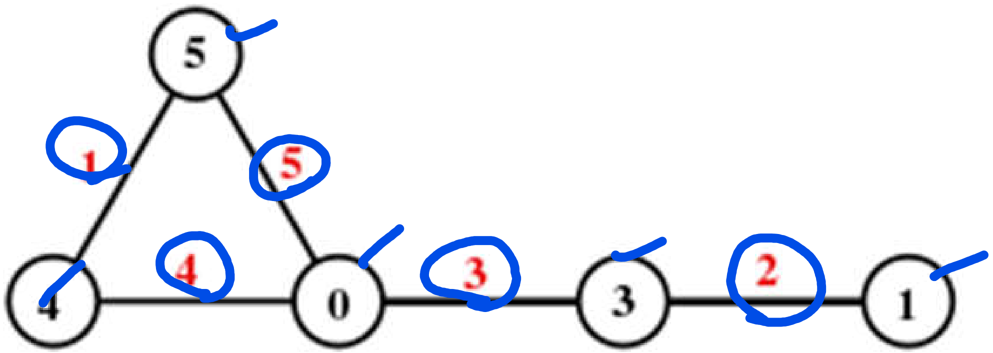
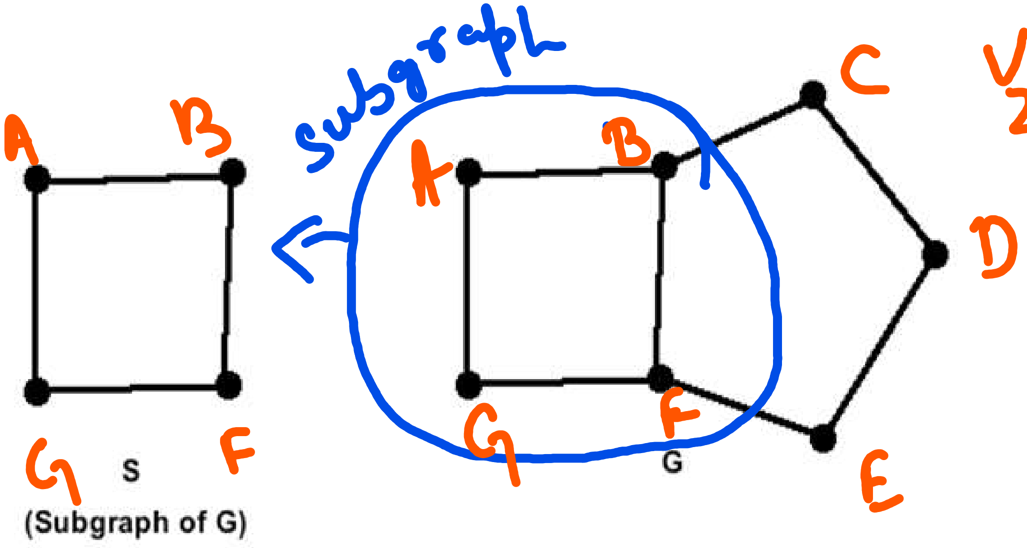


Figure: Diagram showing a labeled graph.

Subgraph

- A **subgraph** S of a graph G is a graph whose vertex set $V(S)$ is a subset of the vertex set $V(G)$ that is $V(S) \subseteq V(G)$, and whose edge set $E(S)$, is a subset of the edge set $E(G)$, that is $E(S) \subseteq E(G)$. Generally, a subgraph is a graph within a larger graph. For example, in this following Figure S is a subgraph of a graph G.

$S = (\underbrace{\{A,B,G,F\}}_{V_2}, \underbrace{\{(A,B),(B,F),(F,G),(G,A)\}}_{E_2})$



$V_2 \subset V_1$

$E_2 \subset E_1$

$\Downarrow$

$S \subset G$

Figure: Diagram showing a subgraph.

$G = (\underbrace{\{A,B,C,D,E,F,G\}}_{V_1}, \overbrace{\{(A,B),(B,C),(B,F),(C,D),(D,E),(E,F),(F,G),(G,A)\}}^{E_1})$

Graph (cont.)

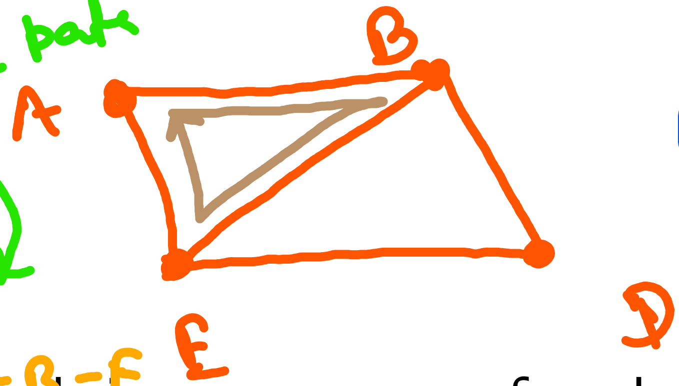
Path from A to E : A-B-E

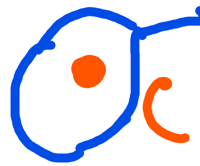
or A-B-D-E

or A-B-E-D-B-E

etc.

Simple path

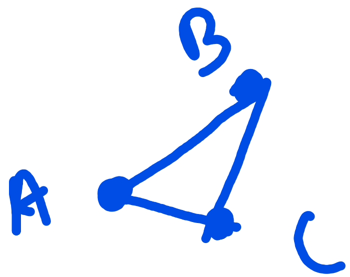




Isolated vertex

Not Connected Graph.

- A **path** in a graph is a sequence of nodes connected by edges.
- A **simple path** is a path that doesn't repeat any nodes.
- A graph is **connected** if every two nodes have a path between them.
- A path is a **cycle** if it starts and ends in the same node. A simple cycle is one that contains at least three nodes and repeats only the first and last nodes.



=> Connected Graph

Path is a cycle

A - B - E - A

A-B-D-E-A

Path from a node to itself.

Tree

- A graph is a **tree** if it is connected and has no simple cycles.
- A tree may contain a specially designated node called the **root**.
- The nodes of degree 1 in a tree, other than the root, are called the **leaves** of the tree.
- Structure of a tree is shown in this Figure.

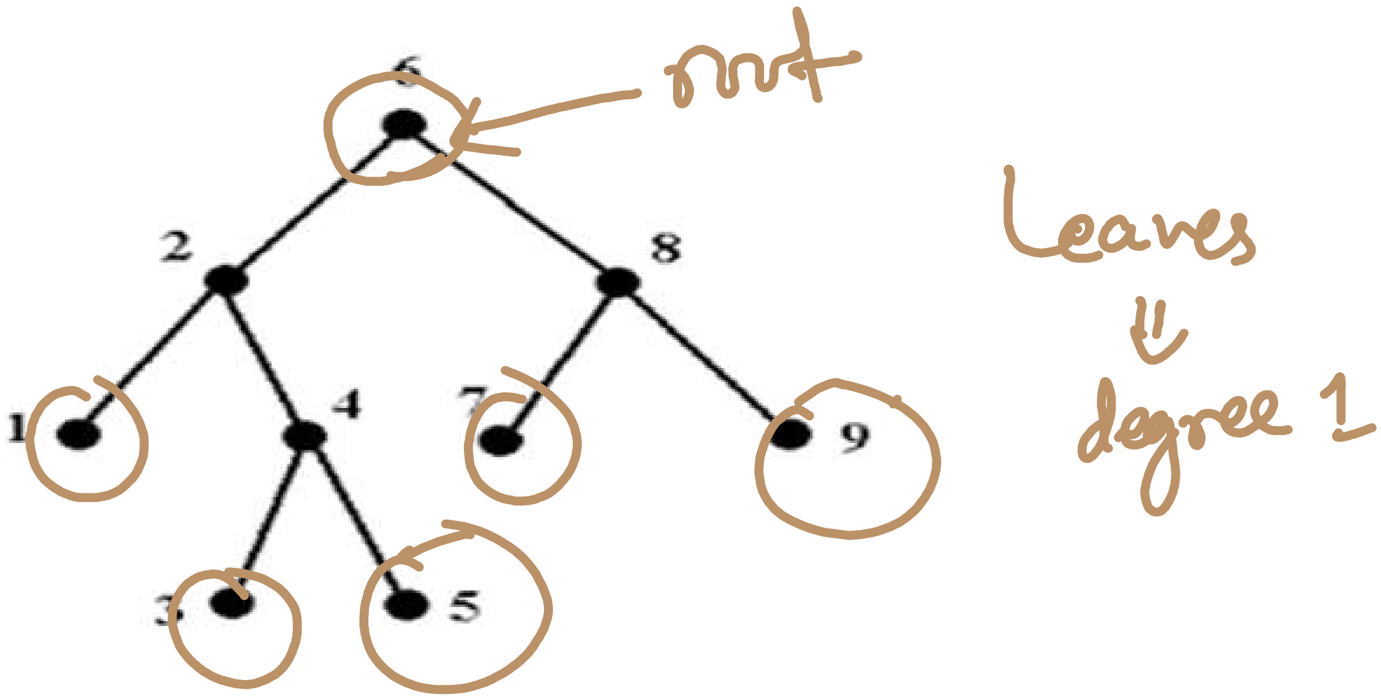


Figure: Diagram showing a tree.

Directed graph

- A **directed graph** has arrows instead of lines, as shown in below Figure.
- The number of arrows pointing from a particular node is the **outdegree** of that node, and the number of arrows pointing to a particular node is the **indegree**.

Indegree of 1 = 0
" of 2 = 1
" " 3 = 2
" " 4 = 3
" " 5 = 1
" " 6 = 3
" " 7 = 2

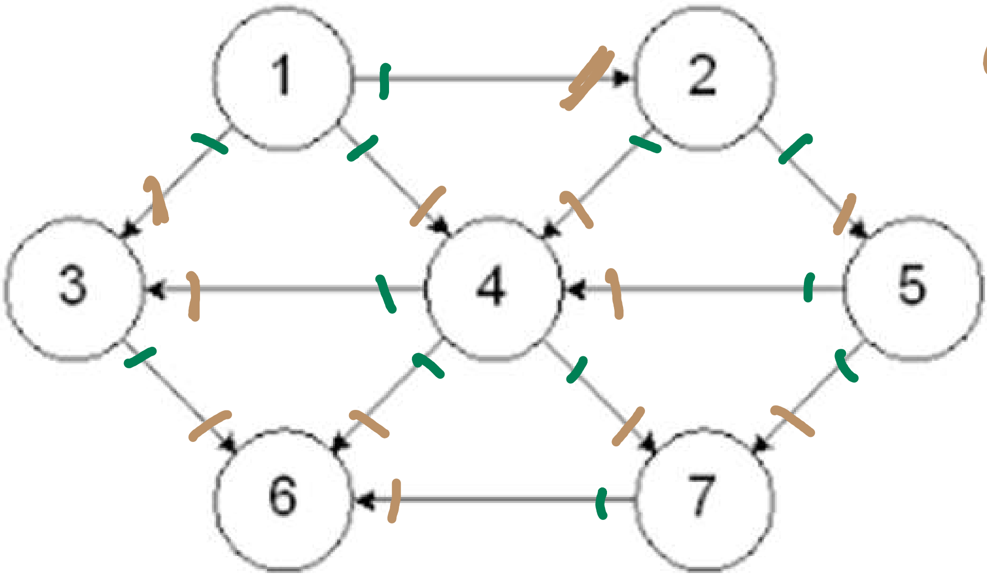


Figure: Diagram showing a directed graph.

Outdegree of 1 = 3
" " 2 = 2
" " 3 = 1
" " 4 = 3
" " 5 = 2
" " 6 = 0
" " 7 = 1

Strongly connected graph

A path in which all the arrows point in the same direction as its steps is called a directed path. A directed graph is **strongly connected** if a directed path connects every two nodes.

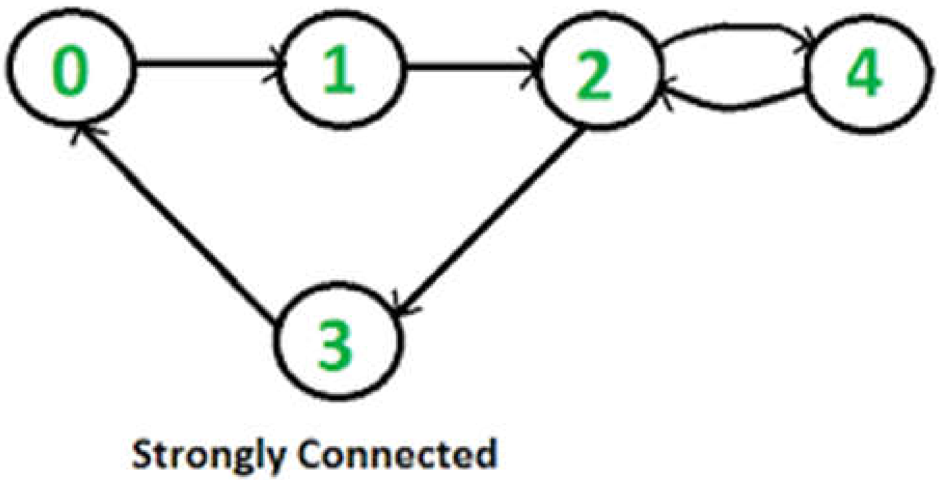


Figure: Diagram showing a strongly connected graph.