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Exercise-3.4

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Q5) Show that if $n+1$ distinct integers are choosen from the set $\{1, 2, \ldots\ldots, 3n\}$, then there are always two which differ by at most 2.

(Ans) Divide the given set $\{1, 2, \ldots\ldots, 3n\}$ in n subsets containing three elements.

So $\{1, 2, 3\}$, $\{4, 5, 6\}$, $\ldots\ldots$, $\{3n-2, 3n-1, 3n\}$

Now, we can choose $(n+1)$ elements from this subsets.

From Theorem 3.1.1 it follows that two of the elements will belong to the same subset, & these two elements will always differ by at most 2, because we have choosen the subsets in that way.

Q9) In a room, there are 10 people, none of whom are older than 60 (ages are given in whole number only) but each of whom is at least 1 year old. Prove that we can always find two groups of people

(with no common person) the sum of whose age is the same.
Can 10 be replaced by a smaller number?

(Ans) i) As each person can be either selected in a group or not, and this can be done in 2^{10} ways i.e. $2^{10} = 1023$ ways, with a total of 10 people

Age of people varies from 1 to 60, so,

<u>Maximum age of a group :-</u>

$$60+59+58+57+56+55+54+53+52+51$$
$$= 555$$

<u>Minimum age of a group :-</u>

$$1+2+3+4+5+6+7+8+9+10 = 55$$

so the number of possible age sums is :-

$$555-55 = 501$$

According to theorem 3.1.1 we have that from 1023 ways & 501 possible age sums those must be two age groups whose sum is the same.

ii) we are given to replace 10 by a smaller number, so let's take 8 (you can take any smaller number but not equal to 10).

Here $2^8 = 255$ ways

<u>Maximum age of a group :-</u>

$$60+59+58+57+56+55+54+53 = 452$$

<u>Minimum age of a group :-</u>

$$1+2+3+4+5+6+7+8 = \cancel{30}\ 36$$

so, no. of possible age sums is :-

$$452-36+1 = 417$$

According to theorem 3.1.1 we have that from 255 ways and 417 possible age sums, there are must be two age groups whose sum is the same

(Q) A student has 37 days to prepare for an examination. From past experience she knows that she will require no more than 60 hours of study. She also wishes to study at least 1 hour per day. Show that no matter how she schedules her study time, there is a succession of days during which she will have studied exactly 13 hours.

(Ans) Let n be the no. of hours a student studies on the i^{th} day

As she has 37 days to prepare, then the range of index is $1 \le i \le 37$

Let S_i be the no. of hours the student studies for the i^{th} days,

so; $S_i = n_1 + n_2 + \dots + n_i$

As she wishes to study at least one hour per day & because of her past experience (i.e. 60 hrs)

$1 \le S_1 < S_2 < S_3 < \dots < S_{37} \le 60$ / +13

$14 \le S_1 + 13 < S_2 + 13 < \dots < S_{37} + 13 \le 73$

observe set A:

$A = \{S_1, S_2, S_3, \dots, S_{37}, S_1 + 13, S_2 + 13, \dots, S_{37} + 13\}$

There are exactly 74 elements in this set & all of them have to take values from 1 to 73. From theorem 3.1.1 we have that at least two elements have the same values, Lets split set A into 2 sets, set B and C

so, $B = \{S_1, S_2, \dots, S_{37}\}$

and $C = \{S_1 + 13, S_2 + 13, \dots, S_{37} + 13\}$

Here one of the elements is in set B & the other is set C and as their value is equal, we have:

$$S_i = S_j + 13$$

$$n_1 + n_2 + \dots + n_j + n_{j+1}, \dots + n_i = n_1 + n_2 + \dots + n_j + 13$$

$$n_{j+1} + n_{j+2} + \dots + n_i = 13$$

Therefore we have proven that no matter how the student schedules her study time, there is a succession of days during which she will have studied exactly 13 hrs.