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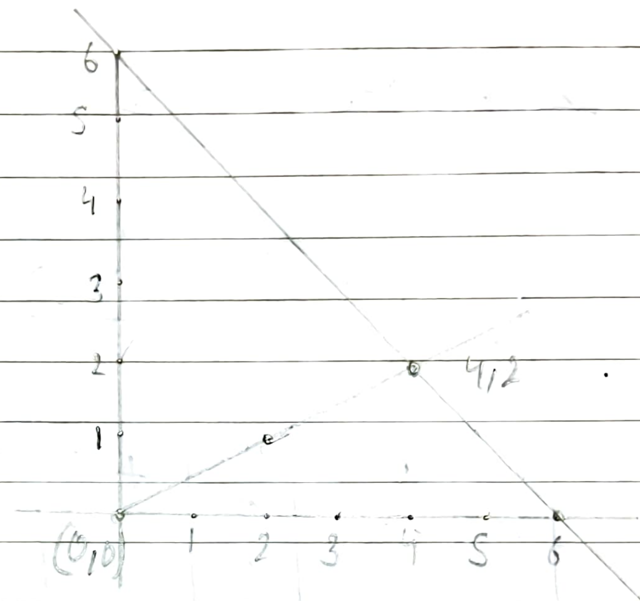
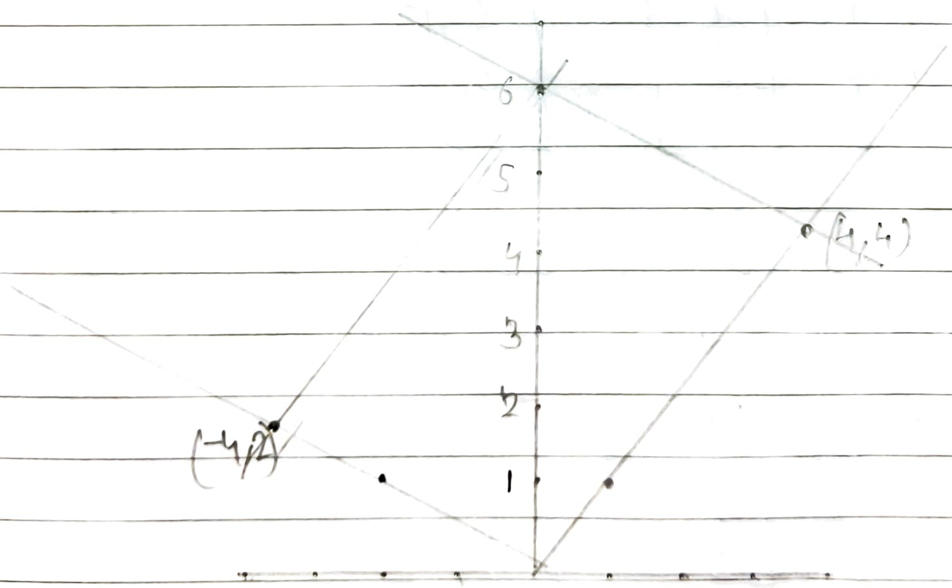
15) Draw the two picture in two planes for equation

$x-2y=0, \; x+y=6$

$x=0, y=0$	$x=2, y=1$
$x=0, y=6$	$y=0, x=6$



$$x\begin{bmatrix}1\\1\end{bmatrix} + y\begin{bmatrix}-2\\1\end{bmatrix} = \begin{bmatrix}0\\6\end{bmatrix}$$



$x\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}4\\4\end{bmatrix} \Rightarrow x=4$, $\quad y\begin{bmatrix}-2\\1\end{bmatrix} = \begin{bmatrix}-4\\2\end{bmatrix} \Rightarrow y=2$

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17) For being singular column three should be 0 therefore $\underline{z = 0}$

Let the given columns be x, y, z

$x = 2,\ y = 3,\ z = 5$

$x_1 = 4,\ y_1 = 6,\ z = c$

$$\frac{x}{x_1} = \frac{2}{4} = \frac{y}{y_1} = \frac{3}{6} = \frac{z}{z_1} = \frac{5}{c}$$

$$\frac{x}{x_1} = \frac{1}{2} = \frac{5}{c} \Rightarrow \underline{c = 10}$$

$$x\begin{bmatrix}1\\1\\2\end{bmatrix} + y\begin{bmatrix}1\\2\\3\end{bmatrix} + z\begin{bmatrix}1\\1\\2\end{bmatrix} = \begin{bmatrix}2\\3\\5\end{bmatrix}$$

First case for being singular $z = 0, x = 1, y = 1$

$$\begin{bmatrix}1\\1\\2\end{bmatrix} + \begin{bmatrix}1\\2\\3\end{bmatrix} = \begin{bmatrix}2\\3\\5\end{bmatrix}$$

second case as the column of x & z are equal therefore now $x = 0, y = 1, z = 1$

$$\begin{bmatrix}1\\2\\3\end{bmatrix} + \begin{bmatrix}1\\1\\2\end{bmatrix} = \begin{bmatrix}2\\3\\5\end{bmatrix}$$

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18) $7w = 7$

$w = 1$

$6u + 7v + 8 = 8 \Rightarrow 6u + 7v = 0$ — (I)

$4u + 5v + 9 = 9 \Rightarrow 4u + 5v = 0$ — (II)

multiply both (I) & (II) by 4 & 6 respectively

$$\left.\begin{aligned} 24u + 28v &= 0 \\ 24u + 30v &= 0 \end{aligned}\right\} \text{subtracting (II) from (I)}$$

~~$v = 0$ & $u = 0$ as subtra~~

$2v = 0 \Rightarrow v = 0$

$24u = 0 \Rightarrow u = 0$

$u = 0, v = 0, w = 1$

22) $x + y + z = 2$

$x + 2y + z = 3$

$2x + 3y + 2z = 5$

$$x\begin{bmatrix}1\\1\\2\end{bmatrix} + y\begin{bmatrix}1\\2\\3\end{bmatrix} + z\begin{bmatrix}1\\1\\2\end{bmatrix} = \begin{bmatrix}2\\3\\5\end{bmatrix}$$

The solution has infinetly many solⁿ therefore

$2z = 0 \Rightarrow z = 0$

~~$x + y + 0$~~ $\left.\begin{aligned} x + y &= 2 \\ x + 2y &= 3 \end{aligned}\right\}$ subtracting the eqⁿ

$y = 1$

solⁿ (1)

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$$x\begin{bmatrix}1\\1\\2\end{bmatrix}+1\begin{bmatrix}1\\2\\3\end{bmatrix}+0\begin{bmatrix}1\\1\\2\end{bmatrix}=\begin{bmatrix}2\\3\\5\end{bmatrix}$$

$x=1$

$\boxed{x=1,\ y=1\ \&\ z=0}$

Sol^n 2

as column of x & z are same. therefore $x=0$

$\boxed{x=0,\ y=1\ \&\ z=1}$

Sol^n 3

$$\frac{1}{2}\begin{bmatrix}1\\1\\2\end{bmatrix}+1\begin{bmatrix}1\\2\\3\end{bmatrix}+\frac{1}{2}\begin{bmatrix}1\\1\\2\end{bmatrix}=\begin{bmatrix}2\\3\\5\end{bmatrix}$$

$$\boxed{x=\frac{1}{2},\ y=1,\ z=\frac{1}{2}}$$

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1·3

9) $Kx + 3y = 6$
$3x + Ky = -6$

The given system of eq^n can be written as $AX = b$

$A = \begin{bmatrix} K & 3 \\ 3 & K \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \end{bmatrix}$ $b = \begin{bmatrix} 6 \\ -6 \end{bmatrix}$

argumented matrix

$[A|b] = \left[\begin{array}{cc|c} k & 3 & 6 \\ 3 & k & -6 \end{array}\right]$

for break down to occur if zero appears in pivot point.

1) $K = 0$

$[A|b] = \left[\begin{array}{cc|c} 0 & 3 & 6 \\ 3 & 0 & -6 \end{array}\right]$ ~~temporary~~ breakdown & ~~1~~ sol^n

$R_2 \leftrightarrow R_1$

$\left[\begin{array}{cc|c} 3 & 0 & -6 \\ 0 & 3 & 6 \end{array}\right]$

$3y = 6 \Rightarrow y = 2$
$3x = -6 \Rightarrow x = -2$

~~2) $K \neq 0$~~
~~$[A|b] = \left[\begin{array}{cc|c} K & 3 & 6 \\ 3 & k & -6 \end{array}\right]$ R_1~~
~~$R_2 \to R_2 -$~~

2) $K \neq 0$
$k - 3 = 0$
$k = 3$ — (i) $\Rightarrow$ $\left[\begin{array}{cc|c} 3 & 3 & 6 \\ 3 & 3 & -6 \end{array}\right]$ $\begin{array}{l} R_1 \Rightarrow \\ R_2 \to R_2 - R_1 \end{array}$ $\begin{bmatrix} 3 & 3 & 6 \\ 0 & 0 & -12 \end{bmatrix}$
~~$K + 3 = 0$~~
~~$K = -3$ — (ii)~~

permanent breakdown & No sol^n

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$k+3=0$

$k=-3 \Rightarrow \left[\begin{array}{cc|c} -3 & 3 & 6 \\ 3 & -3 & -6 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \to R_2+R_1 \end{array} \Rightarrow \left[\begin{array}{cc|c} -3 & 3 & 6 \\ 0 & 0 & 0 \end{array}\right]$

permanent breakdown & ∞ Sol^n

10

$x+by=0$

$x-2y-z=0$

$y+z=0$

eq^n can be written in

$\left[\begin{array}{ccc|c} 1 & b & 0 & 0 \\ 1 & -2 & -1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \to R_2 - R_1 \\ R_3 \end{array}$

$\left[\begin{array}{ccc|c} 1 & b & 0 & 0 \\ 1 & -2-b & -1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \end{array}$ if $-2-b=0$ then no pivot point $b=-2$

To make singular

$\left[\begin{array}{ccc|c} 1 & b & 0 & 0 \\ 1 & -2 & -1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \to R_2 - R_1 \\ R_3 \end{array} \left[\begin{array}{ccc|c} 1 & b & 0 & 0 \\ 0 & -2-b & -1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right]$

$-2-b=1 \quad b=-1$ new matrix $\left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \end{array}$

$R_3 \to R_3 + R_2$

$\left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 1 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$a_{33}=0$ so singular

non zero singular

$-y-z=0 \Rightarrow y=-z$

$x-y=0 \Rightarrow x=y$

$(1, 1, -1)$

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12) $2x + 3y + z = 8$

$4x + 7y + 5z = 20$

$-2y + 2z = 0$

Eq^n can be written inform of $Ax = b$

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 4 & 7 & 5 \\ 0 & -2 & 2 \end{bmatrix} \quad x = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad b = \begin{bmatrix} 8 \\ 20 \\ 0 \end{bmatrix}$$

Argumented matrix $[A|b]$

$$\left[\begin{array}{ccc|c} 2 & 3 & 1 & 8 \\ 4 & 7 & 5 & 20 \\ 0 & -2 & 2 & 0 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \to R_2 - 2R_1 \\ R_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 2 & 3 & 1 & 8 \\ 0 & 1 & 3 & 4 \\ 0 & -2 & 2 & 0 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \to R_3 + 2R_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 2 & 3 & 1 & 8 \\ 0 & 1 & 3 & 4 \\ 0 & 0 & 8 & 8 \end{array}\right]$$

Crossponding eq^n are

$8z = 8 \Rightarrow z = 1$

$y + 3z = 4 \Rightarrow y = 1$

$2x + 3y + z = 8 \Rightarrow x = 2$

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26) a) If the third equation starts with a zero coefficient then no multiple of equation 1 will be subtracted from equation 3 → True as it's already 0 therefore no need of row operation

b) If the third equation has zero as its second coefficient then no multiple of eqn 2 will be subtracted from eqn 3 → True as it's already 0 below the pivot element. so no need of row operation

c) If the third eqn contains 0u & 0v then no multiple of eqn 1 or eqn 2 will be subtracted from 3 → True reason same as stated in (b)

32)

a)
$$u+v+w=6$$
$$u+2v+2w=11$$
$$2u+3v-4w=3$$

$Ax=b$

$A=\begin{bmatrix}1&1&1\\1&2&2\\2&3&-4\end{bmatrix}$ $x=\begin{bmatrix}x\\y\\z\end{bmatrix}$ $B=\begin{bmatrix}6\\4\\3\end{bmatrix}$

$$[A|b]=\left[\begin{array}{ccc|c}1&1&1&6\\1&2&2&4\\2&3&-4&3\end{array}\right]\begin{array}{l}R_1\\R_2\to R_2-R_1\\R_3\to R_3-2R_1\end{array}$$

$$\left[\begin{array}{ccc|c}1&1&1&6\\0&1&1&5\\0&1&-6&-9\end{array}\right]\begin{array}{l}R_1\\R_2\\R_3\to R_3-R_2\end{array}\qquad\left[\begin{array}{ccc|c}1&1&1&6\\0&1&1&5\\0&0&-7&-14\end{array}\right]$$

$$u+v+w=6 \Rightarrow u=1$$
$$v+w=5 \Rightarrow v=3$$
$$-7w=-4 \Rightarrow w=2$$

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32 b) $u+v+w=7$
$u+2v+2w=10$
$2u+3v-4w=3$

Eq^n can be written in form of
$AX=b$

$$A=\begin{bmatrix}1&1&1\\1&2&2\\2&3&-4\end{bmatrix}\quad X=\begin{bmatrix}u\\v\\w\end{bmatrix}\quad b=\begin{bmatrix}7\\10\\3\end{bmatrix}$$

Argumented matrix $[A|b]$

$$\left[\begin{array}{ccc|c}1&1&1&7\\1&2&2&10\\2&3&-4&3\end{array}\right]\begin{array}{l}R_1\\R_2\to R_2-R_1\\R_3\to R_3-2R_1\end{array}$$

$$\left[\begin{array}{ccc|c}1&1&1&7\\0&1&1&3\\0&1&-6&-11\end{array}\right]\begin{array}{l}R_1\\R_2\\R_3\end{array}$$

$R_3\to R_3-R_2$

$$\left[\begin{array}{ccc|c}1&1&1&7\\0&1&1&3\\0&0&-7&-14\end{array}\right]$$

Crossponding eq^{n}s are

$-7w=-14 \Rightarrow w=2$
$v+w=3 \Rightarrow v=1$
$u+v+w=7 \Rightarrow u=4$

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11)

a) True: Scalar products (row in A) × (first column in B) and (row in A) × (third column in B) will be equal

b) False

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & 2 & 2 \\ 0 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 2 \\ 2 & 4 & 4 \\ 4 & 6 & 6 \end{bmatrix}$$

c) True

The scalar products (first row in A) × (a column in B) and (third row in A) × (a column in B) will be equal

d) False

$(AB)^2 = ABAB$

If $AB = BA$ then the statement would be true but its is not always the case.

Take $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

$AB = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ $(AB)^2 = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$

$A^2B^2 = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}$

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28) This 4 by 4 matrix needs which elimination matrices E_{21} and E_{32} and E_{43}?

$$A = \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} \xrightarrow{R_2 = R_2 + (1/2)R_1} \begin{bmatrix} 2 & -1 & 0 & 0 \\ 0 & 3/2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} \xrightarrow{R_3 = R_3 + (\frac{2}{3})}$$

$$E_{21} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1/2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad E_{32} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 2/3 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 & 0 & 0 \\ 0 & 3/2 & -1 & 0 \\ 0 & 0 & 4/3 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} \rightarrow R_4 = R_4 + \frac{3}{4} R_3 \qquad \begin{bmatrix} 2 & -1 & 0 & 0 \\ 0 & 3/2 & -1 & 0 \\ 0 & 0 & 4/3 & -1 \\ 0 & 0 & 0 & 5/4 \end{bmatrix}$$

$$E_{43} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 3/4 & 1 \end{bmatrix}$$

50) $$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\begin{bmatrix} a+b & a+b \\ c+d & c+d \end{bmatrix} = \begin{bmatrix} a+c & b+d \\ a+c & b+d \end{bmatrix}$$

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9) $A = \begin{bmatrix} 2 & 1 \\ 8 & 7 \end{bmatrix} \begin{matrix} R_1 \\ R_2 \to R_2 - 4R_1 \end{matrix}$

$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$

$A = L\ U$

$L = \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix}$

$A = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix} \begin{matrix} R_1 \\ R_2 \to R_2 - 1/3 R_1 \\ R_3 \to R_3 - 1/3 R_1 \end{matrix}$

$\begin{bmatrix} 3 & 1 & 1 \\ 0 & 8/3 & 2/3 \\ 0 & 2/3 & 8/3 \end{bmatrix} \begin{matrix} R_1 \\ R_2 \\ R_3 = R_3 - 1/4 R_2 \end{matrix}$

$U = \begin{bmatrix} 3 & 1 & 1 \\ 0 & 8/3 & 2/3 \\ 0 & 0 & 5/2 \end{bmatrix}$

$\begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1/3 & 1 & 0 \\ 1/3 & 1/4 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 & 1 \\ 0 & 8/3 & 2/3 \\ 0 & 0 & 5/2 \end{bmatrix}$

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$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 4 & 4 \\ 1 & 4 & 8 \end{bmatrix} \begin{array}{l} R_1 \\ R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{array}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 3 & 3 \\ 0 & 3 & 7 \end{bmatrix} \begin{array}{l} R_1 \\ R_2 \\ R_3 = R_3 - R_2 \end{array}$$

$$U = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 3 & 3 \\ 0 & 0 & 4 \end{bmatrix}$$

$A = LU$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 4 & 4 \\ 1 & 4 & 8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 3 & 3 \\ 0 & 0 & 4 \end{bmatrix}$$

27) $$A = \begin{bmatrix} a & a & a & a \\ a & b & b & b \\ a & b & c & c \\ a & b & c & d \end{bmatrix} \begin{array}{l} R_1 \\ R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \\ R_4 \to R_4 - R_1 \end{array}$$

$$\begin{bmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & b-a & c-a & c-a \\ 0 & b-a & c-a & d-a \end{bmatrix} \begin{array}{l} R_1 \\ R_2 \\ R_3 \to R_3 - R_2 \\ R_4 \to R_4 - R_2 \end{array}$$

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$$\begin{bmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & 0 & c-b & c-b \\ 0 & 0 & c-b & d-b \end{bmatrix} \begin{array}{l} R_1 \\ R_2 \\ R_3 \\ R_4 \to R_4 - R_3 \end{array}$$

$$U = \begin{bmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & 0 & c-b & c-b \\ 0 & 0 & 0 & d-c \end{bmatrix}$$

i) $a \neq 0$
ii) $b \neq a$
iii) $c \neq b$
iv) $d \neq c$

$$\begin{bmatrix} a & a & a & a \\ a & b & b & b \\ a & b & c & c \\ a & b & c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & 0 & c-b & c-b \\ 0 & 0 & 0 & d-c \end{bmatrix}$$

30) a) $A = \begin{bmatrix} 2 & 4 \\ 4 & 11 \end{bmatrix} \begin{array}{l} R_1 \\ R_2 \to R_2 - 2R_1 \end{array} \begin{bmatrix} 2 & 4 \\ 0 & 3 \end{bmatrix}$

$L = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \quad U = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = L^T$

b) $A = \begin{bmatrix} 1 & 4 & 0 \\ 4 & 12 & 4 \\ 0 & 4 & 0 \end{bmatrix} \begin{array}{l} R_1 \\ R_2 \to R_2 - 4R_1 \\ R_3 \end{array} \begin{bmatrix} 1 & 4 & 0 \\ 0 & -4 & 4 \\ 0 & 4 & 0 \end{bmatrix} \begin{array}{l} R_1 \\ R_2 \\ R_3 = R_3 + R_2 \end{array}$

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$$\begin{bmatrix} 1 & 4 & 0 \\ 0 & -4 & 4 \\ 0 & 0 & 4 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & 4 \end{bmatrix} \quad U = \begin{bmatrix} 1 & 4 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} = L^T$$

32) $A = \begin{bmatrix} 2 & 4 & 8 \\ 0 & 3 & 9 \\ 0 & 0 & 7 \end{bmatrix}$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 7 \end{bmatrix}$$

$A = LDU$

$$U = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

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40) $A = \begin{bmatrix} 0 & 0 & 6 \\ 1 & 2 & 3 \\ 0 & 4 & 5 \end{bmatrix} \begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix} \quad R_2 \longleftrightarrow R_1 \quad \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 6 \\ 0 & 4 & 5 \end{bmatrix} \begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix} \quad R_2 \longrightarrow R_3 \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix} = U$

$P_{23} P_{12} A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 6 \\ 1 & 2 & 3 \\ 0 & 4 & 5 \end{bmatrix}$

$P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

$P_1 A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 6 \\ 1 & 2 & 3 \\ 0 & 4 & 5 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 6 \\ 0 & 4 & 5 \\ 1 & 2 & 3 \end{bmatrix}$

$P_1 A P_2 = \begin{bmatrix} 0 & 0 & 6 \\ 0 & 4 & 5 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 \\ 5 & 4 & 0 \\ 3 & 2 & 1 \end{bmatrix}$

$P_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \quad P_2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

41) $P_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \quad P_2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

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$$P_1 P_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$P_2 P_1 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$P_1 P_2 \neq P_2 P_1$$

$$P_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \qquad P_4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$P_3 P_4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P_4 P_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P_3 P_4 = P_4 P_3$$

2) $(A^2)^{-1} = B$

$A^2B = I = BA^2$

$A^2B = I$

$(AA)B = I$

$A(AB) = I$

$AA^{-1} = I$ - (1)

AB is the right inverse of A

$BA^2 = I$

$BAA = I$

$BAAA^{-1} = A^{-1}$ (multiplying A^{-1} both side)

~~$BA = AA^{-1}$~~

$BA = A^{-1}$

$ABA = AA^{-1}$

$(AB)A = I$

AB is left inverse of A

$(AB)A = I = A(AB)$

$A^{-1} = AB$

4) $$A_1 = \begin{array}{c c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} & \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 3 & 0 & 0 \\ 4 & 0 & 0 & 0 \end{bmatrix} & \begin{array}{c} R_1 \\ R_2 \\ R_3 \\ R_4 \end{array} \end{array}$$

$R_1 \leftrightarrow R_4$ & $C_2 \leftrightarrow C_3$

$$A_1'' = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} \quad A_1^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 0 \\ 0 & 0 & 1/3 & 0 \\ 0 & 0 & 0 & 1/4 \end{bmatrix}$$

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$$A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1/2 & 1 & 0 & 0 \\ 0 & -2/3 & 1 & 0 \\ 0 & 0 & -3/4 & 1 \end{bmatrix}$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1/2 & 1 & 0 & 0 \\ 0 & -2/3 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -3/4 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \to R_3 + 2/3\, R_2 \\ R_4 \end{array}$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1/2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1/3 & 2/3 & 1 & 0 \\ 0 & 0 & -3/4 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \\ R_4 \to R_4 + 4/3\, R_3 \end{array}$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1/2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1/3 & 2/3 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1/4 & 1/2 & 3/4 & 1 \end{array}\right]$$

A_2^{-1}

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5) $A = \begin{bmatrix} 1 & 1 \\ 3 & 3 \end{bmatrix}$

$|A| = 3 - 3 = 0$ $|A| = 0$ hence no inverse exist

$$\begin{bmatrix} 1 & 1 \\ 3 & 3 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$a + c = 1$	$b + d = 0$
$3a + 3c = 0$	$3b + 3d = 1$
$3(a+c) = 0$	$3(b+d) = 1$
$3 \times 1 = 0$	$3(0) = 1$

A has no inverse

12) $A = \begin{bmatrix} 1 & 3 & 5 \\ 3 & 12 & 18 \\ 5 & 18 & 30 \end{bmatrix}$ $\begin{matrix} R_1 \\ R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 - 5R_1 \end{matrix}$

$\cancel{\begin{bmatrix} 1 & 3 & 5 \\ 0 & 9 & 13 \\ 0 & 1 & \end{bmatrix}}$ $\begin{bmatrix} 1 & 3 & 5 \\ 0 & 3 & 3 \\ 0 & 3 & 5 \end{bmatrix}$ $\begin{matrix} R_1 \\ R_2 \\ R_3 \to R_3 - R_2 \end{matrix}$

$U = \begin{bmatrix} 1 & 3 & 5 \\ 0 & 3 & 3 \\ 0 & 0 & 2 \end{bmatrix}$

$L = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 5 & 1 & 1 \end{bmatrix}$ $D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ $L^T = \begin{bmatrix} 1 & 3 & 5 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

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$$A = \begin{bmatrix} a & b \\ b & d \end{bmatrix} \begin{array}{l} R_1 \\ R_2 \to R_2 - b/a\,R_1 \end{array}$$

$$U = \begin{bmatrix} a & b \\ 0 & d - b^2/a \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 \\ b/a & 1 \end{bmatrix} \quad D = \begin{bmatrix} a & 0 \\ 0 & d - b^2/a \end{bmatrix} \quad L^T = \begin{bmatrix} 1 & b/a \\ 0 & 1 \end{bmatrix}$$

37) $$[A \ I] = \left[\begin{array}{ccc|ccc} 2 & 1 & 0 & 1 & 0 & 0 \\ 1 & 2 & 2 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \to R_2 - 1/2\,R_1 \\ R_3 \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 2 & 1 & 0 & 1 & 0 & 0 \\ 0 & 3/2 & 1 & -1/2 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \to R_3 - \frac{2}{3} R_2 \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 2 & 1 & 0 & 1 & 0 & 0 \\ 0 & 3/2 & 1 & -1/2 & 1 & 0 \\ 0 & 0 & 4/3 & 1/3 & -2/3 & 1 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \to R_2 - 3/4\,R_3 \\ R_3 \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 2 & 1 & 0 & 1 & 0 & 0 \\ 0 & 3/2 & 0 & -3/4 & 6/4 & -3/4 \\ 0 & 0 & 4/3 & 1/3 & -2/3 & 1 \end{array}\right] \begin{array}{l} R_1 \to R_1 - 2/3\,R_2 \\ R_2 \\ R_3 \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 2 & 0 & 0 & 6/4 & 1 & 2/4 \\ 0 & 3/2 & 0 & -3/4 & 6/4 & -3/4 \\ 0 & 0 & 4/3 & 1/3 & -2/3 & 1 \end{array}\right]$$

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$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3/4 & 1/2 & 1/4 \\ 0 & 1 & 0 & -1/2 & 1 & -1/2 \\ 0 & 0 & 1 & 1/4 & -1/2 & 3/4 \end{array}\right]$$

$[I \; A^{-1}]$

52) $(AB)^T = \begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$ | $A^T = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ $B^T = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$

$$B^T A^T = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix} = (AB)^T$$

$$A^T B^T = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 2 \\ 3 & 1 \end{bmatrix} \neq (AB)^T$$

Verified

Given $AB = BA$ — (1)

L.H.S $\cdot B^T A^T$

$= (AB)^T$

$= (BA)^T$ (using (1))

$= A^T B^T$

$=$ R.H.S

$B^T A^T = A^T B^T$ (proved)

54) a) $[0 \; 1] \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = [0 \; 1] \begin{bmatrix} 2 \\ 5 \end{bmatrix}$

$= 5$

b) $x^T A = [0 \; 1] \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = [4 \; 5 \; 6]$

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c) $A_y = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$

58) a) $(A^2 - B^2)^T$

$= (A^2)^T - (B^2)^T$

$= (AA)^T - (BB)^T$

$A^T A^T - B^T B^T$

$= (A^T)^2 - (B^T)^2$

$= A^2 - B^2$

Yes

b) $((A+B)(A-B))^T$

$= (A+B)^T (A+B)^T$

$= (A^T - B^T)(A^T + B^T)$

$(A-B)(A+B)$

$\neq (A+B)(A-B)$

No

c) $(ABA)^T$

$A^T B^T A^T$

$= ABA$

Yes

d) $(ABAB)^T$

$= B^T A^T B^T A^T$

$= BABA$

$\neq ABAB$

No