THE GEOMETRY OF LINEAR EQUATIONS

To solve a system of equations geometrically two methods are used.

(i) **Row picture method** :

1. Plot the straight lines corresponding to the given equations.
2. Find the points of intersection(s) if exist. The x-coordinate value of the point of intersection represents the value of x and y- coordinate value gives the value of y.
3. Here if the lines are intersecting then unique solution.
4. If they are parallel then no solution.
5. If they represent the same line then every point on the line is a solution of it.

(See the first figure)

(ii) **Column picture method** :

1. Write the given system of equations as a linear combinations of column vectors equal to the rhs vector.

$$\begin{bmatrix} a_{11} \\ a_{21} \end{bmatrix} x + \begin{bmatrix} a_{12} \\ a_{22} \end{bmatrix} y = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}.$$

2. Plot the points $P = (a_{11}, a_{21})$, $Q = (a_{12}, a_{22})$ in xy-plane.
3. Join each point with the origin O. Extend the lines.
4. Plot the point $B = (b_1, b_2)$. Draw a line from B to OP parallel to OQ and get the coordinates of point of intersection (h_1, h_2).

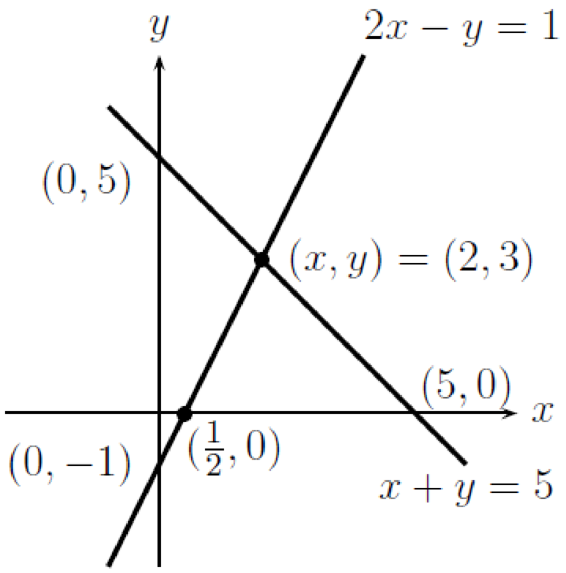
6. Also, draw a line from B to OQ parallel to OP and get the coordinates of point of intersection (k_1, k_2).

7. Find the values of x and y from the equations: $\begin{bmatrix} a_{11} \\ a_{21} \end{bmatrix} x = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}$ and $\begin{bmatrix} a_{12} \\ a_{22} \end{bmatrix} y = \begin{bmatrix} k_1 \\ k_2 \end{bmatrix}$.

(See the second figure)

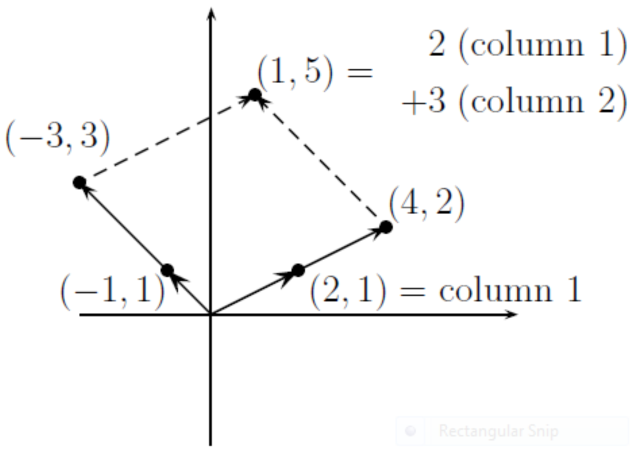
Consider a system of equations

$$\begin{aligned} 2x - y &= 1 \\ x + y &= 5. \end{aligned}$$



(a) Lines meet at $x = 2$, $y = 3$

Figure 1: **Row Picture Method**



(b) Columns combine with 2 and 3

Figure 2: **Column Picture Method**