PROBLEM SET 2.2 CONTINUED...

Excercise 2.2.44 Choose the number q so that (if possible) the ranks (a) 1 (b) 2 (c) 3

$$A = \begin{bmatrix} 6 & 4 & 2 \\ -3 & -2 & -1 \\ 9 & 6 & q \end{bmatrix}, C = \begin{bmatrix} 3 & 1 & 3 \\ q & 2 & q \end{bmatrix}$$

Solution (1) $A = \begin{bmatrix} 6 & 4 & 2 \\ -3 & -2 & -1 \\ 9 & 6 & q \end{bmatrix} \xrightarrow{R_2 \to 2R_2 + R_1, R_3 \to 6R_3 - 9R_1} \begin{bmatrix} 6 & 4 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 6q-18 \end{bmatrix}$

$\xrightarrow{R_2 \leftrightarrow R_3} \begin{bmatrix} 6 & 4 & 2 \\ 0 & 0 & 6q-18 \\ 0 & 0 & 0 \end{bmatrix}$

(a) rank of A is 1 if $q = 3$
(b) rank of A is 2 if $q \neq 3$
(c) rank of A is 3 not possible.

(2) $B = \begin{bmatrix} 3 & 1 & 3 \\ q & 2 & q \end{bmatrix} \xrightarrow{R_2 \to 3R_2 - qR_1} \begin{bmatrix} 3 & 1 & 3 \\ 0 & 6-q & 0 \end{bmatrix}$

(a) rank of B is 1 if $q = 6$
(b) rank of B is 2 if $q \neq 6$
(c) rank of B is 3 not possible.

Excercise 2.2.54 : True or False? (Give reason if true, or counterexample to show it is false.)
(a) A square matrix has no free variables.
(b) An invertible matrix has no free variables.
(c) An m by n matrix has no more than n pivot variables.
(d) An m by n matrix has no more than m pivot variables.
Solution :
(a) A square matrix has no free variables.
Ans. False , because $A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

(b) An invertible matrix has no free variables.
Ans. True , a matrix A is invertible if and only if its columns are linearly independent.so all column has pivot.so invertible matrix has no free variables.

(c) An m by n matrix has no more than n pivot variables.
Ans. True , Since n is number of columns. and each column has atmost 1 pivot. An m by n matrix has no more than n pivot variables.

(d) An m by n matrix has no more than m pivot variables.
Ans. True , Since m is number of rows. and each row has atmost 1 pivot.So An m by n matrix has no more than m pivot variables.

Excercise 2.2.59: The equation $x - 3y - z = 0$ determines a plane in R^3. What is the matrix A in this equation? Which are the free variables? The special solutions are(3,1,0) and (...), the parallel plane $x - 3y - z = 12$ contains the particular point (12,0,0) all points on this plane have the following form (fill in the first component).

Solution: Since $\begin{matrix} x - 3y - z = 0 \\ x = 3y + z, \quad y, z \in R \end{matrix}$ Consider

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3y + z \\ y \\ z \end{bmatrix} = y \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} + z \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

Hence special solutions are $\begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ We have to find a matrix A whose special solutions are $\begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

Consider $A \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} = 0$

$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$a_{11} + a_{13} = 0 \Rightarrow a_{13} = -a_{11}$$

$$a_{21} + a_{23} = 0 \Rightarrow a_{23} = -a_{21}$$

$$a_{31} + a_{33} = 0 \Rightarrow a_{33} = -a_{31}$$

So $A = \begin{bmatrix} a_{11} & -3a_{11} & -a_{11} \\ a_{21} & -3a_{21} & -a_{21} \\ a_{31} & -3a_{31} & -a_{31} \end{bmatrix}$

given parallel plane is $x - 3y - z = 12$

$$x = 3y + z + 12.$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 + 3y + z \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix} + y \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} + z \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$