Linear Algebra

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1 Introduction

Linear Algebra is a part of Mathematics and applied in every field of life. With the help of Linear Algebra we can convert our real life problems into various models involving, which are easy to deal with. One can visualize the objects in higher dimensions to get more information. It helps in solving Differential Equations, game developing, Machine Learning, Data Mining, Image Processing, Traffic Controlling, Electrical Circuit problems, Genetics, Cryptography, various economic model and several other fields. With the help of linear transformation concept one can study the properties of different entities in different spaces. Also a complicated geometrical problem can be studied by converting it into a simple algebraic problem. Hence Linear Algebra can be considered as a bridge between Geometry and Algebra.

2 System of Linear Equations

Definition : A linear system of equations is formed when two or more linear equations involving two or more unknowns considered together to represent a problem. For example:

$$\begin{gathered} a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n = b_1 \\ \mathrm{a}_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n = b_2 \\ ...\ ...\ ... \\ \mathrm{a}_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n = b_m \end{gathered}$$

Which can also be written as $Ax = b$. Where $A = \begin{bmatrix} a_{11} & a_{12} & ... & a_{1n} \\ a_{21} & a_{22} & ... & a_{2n} \\ ... & ... & ... & ... \\ a_{m1} & a_{m2} & ... & a_{mn} \end{bmatrix}$

is a $m \times n$ matrix having the coefficients of ith unknown as ith column elements and is said to be *coefficient matrix*. $x = \begin{bmatrix} x_1 \\ x_2 \\ . \\ . \\ . \\ x_n \end{bmatrix}$ is a vector of unknowns said to be *solution vector* and

$b = \begin{bmatrix} b_1 \\ b_2 \\ \cdot \\ \cdot \\ \cdot \\ b_m \end{bmatrix}$ is said to be the *righthand side vector or nonhomogeneous vector.*

The above system of equations can also be represented as

$$\begin{bmatrix} a_{11} \\ a_{21} \\ \cdot \\ \cdot \\ \cdot \\ a_{m1} \end{bmatrix} x_1 + \begin{bmatrix} a_{12} \\ a_{22} \\ \cdot \\ \cdot \\ \cdot \\ a_{m2} \end{bmatrix} x_2 + \cdots + \begin{bmatrix} a_{1n} \\ a_{2n} \\ \cdot \\ \cdot \\ \cdot \\ a_{mn} \end{bmatrix} x_n = \begin{bmatrix} b_1 \\ b_2 \\ \cdot \\ \cdot \\ \cdot \\ b_m \end{bmatrix}.$$

The values of $x_1, x_2, \cdots, x_n$ for which the given equations are satisfied form a vector $x = \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{bmatrix}$ is a *solution* of this system of equations.

A system of equations is said to *singular* if the corresponding coefficient matrix is singular. A matrix is said to be *singular* iff its rows or columns are linearly related to each other. That is a row (or column) can be obtained from the addition of scalar multiples of other rows(or columns).

The system of equations

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 &= b_1 \\ a_{21}x_1 + a_{22}x_2 &= b_2 \end{aligned}$$

is singular if $\frac{a_{11}}{a_{21}} = \frac{a_{12}}{a_{22}}$. Otherwise it is said to *nonsingular*. A nonsingular system of equations has a **unique** solution.

A singular system of equations has either infinitely many solutions or no solutions. Hence for the above system of equations

1. if $\frac{a_{11}}{a_{21}} \neq \frac{a_{12}}{a_{22}}$, then it has **unique** solution.
2. if $\frac{a_{11}}{a_{21}} = \frac{a_{12}}{a_{22}} \neq \frac{b_1}{b_2}$, then it has **no** solution.
3. if $\frac{a_{11}}{a_{21}} = \frac{a_{12}}{a_{22}} = \frac{b_1}{b_2}$, then it has **infinitely many** solutions.

The following figures give an pictorial representation of the three cases discussed here.

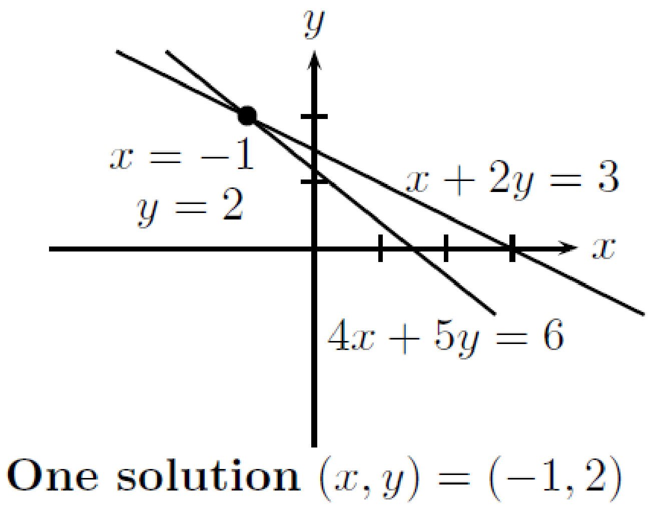

Figure 1: **One Solution**

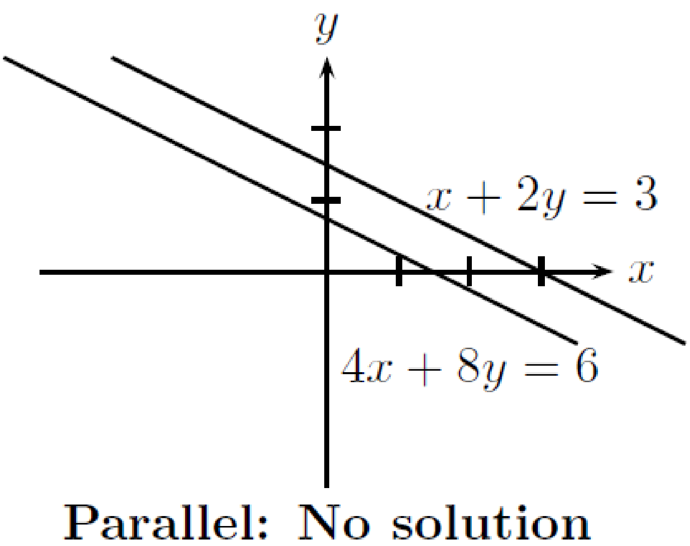

Figure 2: **No Solution**

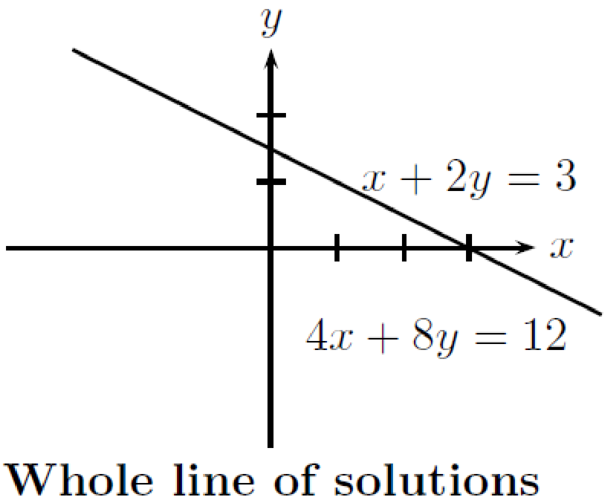

Figure 3: **whole lines of Solution**