Ex: 4.4

(2) Area of the parallelogram $= \begin{vmatrix} 2 & 2 \\ -1 & 3 \end{vmatrix}$

$= 6 - 2 = 4$

$\therefore$ area $(ABC) = \frac{1}{2} \det(A)$

$= \frac{1}{2} \times 4 = 2$

(6) area $(ABC) = \frac{1}{2} \begin{vmatrix} 2 & 2 & 1 \\ -1 & 3 & 1 \\ 1 & -4 & 1 \end{vmatrix}$

$= \frac{1}{2} \begin{vmatrix} 1 & 6 & 0 \\ -2 & 7 & 0 \\ 1 & -4 & 1 \end{vmatrix} \quad \left(\begin{matrix} R_2 \to R_2 - R_3 \\ R_1 \to R_1 - R_3 \end{matrix} \right)$

$= \frac{1}{2} \begin{vmatrix} 1 & 6 \\ -2 & 7 \end{vmatrix}$

$= \frac{19}{2}$

(5) Use the cofactor matrix C to invert these symmetric matrices:-

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

Ans:- $\det(A) = 2\begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} - (-1)\begin{vmatrix} -1 & -1 \\ 0 & 2 \end{vmatrix} + 0\begin{vmatrix} -1 & 2 \\ 0 & -1 \end{vmatrix}$

$= 2(4-1) + 1(-2+0) + 0 = 4$

Cofactors:-

$C_{11} = (-1)^{1+1}\begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} = 3$

$C_{12} = 2, \quad C_{13} = 1$

$C_{21} = 2, \quad C_{22} = 4, \quad C_{23} = 2$ (Same process as 'C_{11}')

$C_{31} = 1, \quad C_{32} = 2, \quad C_{33} = 3$

$\therefore C = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 3 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} C^T = \frac{1}{4}\begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 3 \end{bmatrix}$

(For 'B' same process as 'A')

(7) Find x, y, z by Cramer's Rule:

$ax + by = 1$
$cx + dy = 0$ and

$x + 4y - z = 1$
$x + y + z = 0$
$2x + 3z = 0$

Ans:- $ax + by = 1$
$cx + dy = 0$

$x = \frac{\det B_1}{\det A} = \frac{d}{ad - bc}$

$y = \frac{\det B_2}{\det A} = \frac{-c}{ad - bc}$

$x + 4y - z = 1$
$x + y + z = 0$
$2x + 3z = 0$

$\Rightarrow \begin{bmatrix} 1 & 4 & -1 \\ 1 & 1 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$x = \frac{\det B_1}{\det A} = \frac{3}{1} = 3$

$y = \frac{\det B_2}{\det A} = \frac{-1}{1} = -1$

$z = \det B_3 / \det A = \frac{-2}{1} = -2$

(14)

(a) $2x_1 + 5x_2 = 1$
$x_1 + 4x_2 = 2$

$\Rightarrow \begin{bmatrix} 2 & 5 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

~~det A~~

$x_1 = \dfrac{\det B_1}{\det A} = \dfrac{\begin{vmatrix} 1 & 5 \\ 2 & 4 \end{vmatrix}}{3} = \dfrac{-6}{3} = -2$

$x_2 = \dfrac{\det B_2}{\det A} = \dfrac{\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix}}{3} = \dfrac{3}{3} = 1$

(b) Same process as (a).

(27) The ||gm with sides (2, 1) and (2, 3) has the same area as the parallelogram with sides (2, 2) and (1, 3). Find those areas from 2 by 2 determinants and say why they must be equal.

Ans: Sides of ||gm (1) $\Rightarrow$
$a = (2,1)$ & $b = (2,3)$

area (||gm (1)) $= \begin{vmatrix} 2 & 1 \\ 2 & 3 \end{vmatrix} = 4$

Sides of ||gm (2)
$a' = (2,2)$ and $b' = (1,3)$

area (parallelogram (2)) $= \begin{vmatrix} 2 & 2 \\ 1 & 3 \end{vmatrix} = 6-2 = 4$

They are equal because $A_2^T = A_1$

(29) A box has edges from (0,0,0) to (3,1,1), (1,3,1) and (1,1,3). Find its volume and also find the area of each ||gm face.

Ans: $A = (0,0,0)$, $B = (3,1,1)$, $C(1,3,1)$ and $D = (1,1,3)$

$AB = B - A \Rightarrow l = (3,1,1)$ | $AC = C - A \Rightarrow b = (1,3,1)$ | $AD = D - A \Rightarrow h = (1,1,3)$

$V = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix}$

$|V| = 3\begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} + 1\begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix}$

$= 3 \times 8 - 1(3-1) + 1(1-3)$

$= 24 - 2 - 2 = 20$ cubic units.

$$\text{Area}(ABED) = \|AB \times AD\|$$

So, $AB \times AD = \begin{vmatrix} i & j & k \\ 3 & 1 & 1 \\ 1 & 1 & 3 \end{vmatrix}$

$$= 2i - 8j + 2k$$

$$= \sqrt{2^2 + 8^2 + 2^2} = \sqrt{72} = 6\sqrt{2} \text{ squnits}$$

$$\text{Area}(ABHC) = \|AB \times AC\|$$

$$= \sqrt{(-2)^2 + 2^2 + 8^2} = 6\sqrt{2} \text{ sq units}$$

$$\text{Area}(ADGC) = \|AD \times AC\|$$

$$= \sqrt{(-8)^2 + (-2)^2 + (2)^2}$$

$$= 6\sqrt{2} \text{ squnits.}$$