

MID-SEMESTER EXAMINATION, February-2018 APPLIED LINEAR ALGEBRA (MTH-3003)

Programme: B.Tech

Full Marks: 30

Semester: 4th

Time: 2 Hours

Subject/Course Learning Outcome	*Taxonomy Level	Ques. Nos.	Marks
Concept of row picture to understand the geometrical meaning of the solution of the system of equations, Gaussian elimination method and singular system.	L3, L3, L5	1. a, b, c	2, 2, 2
Explains to understand the concept of matrix construction and matrix multiplication. Also, explains the role of elementary matrices to convert a matrix into upper triangular form.	L3, L4, L5	2. a, b, c	2, 2, 2
Explains the concept of triangular factorization, matrix inverse using Gauss Jordan method,	L3, L3, L3	3. a, b, c	2, 2, 2
Explains the concept vector space, subspaces, column space and nullspace, echelon form to find the rank.	L2, L2, L3	4. a, b, c	2, 2, 2
Explains the concept of reduced row echelon form of matrices, linear independence and dependence of vectors, basis and dimension.	L2, L3, L3	5. a, b, c	2, 2, 2

*Bloom's taxonomy levels: Knowledge (L1), Comprehension (L2), Application (L3), Analysis (L4), Evaluation (L5), Creation (L6)

Answer all questions. Each question carries equal mark.

1. (a) Sketch the row picture and decide the number of [2]
solutions for the following system.

$$2x - y = 1$$

$$x + y = 5$$
- (b) Apply Gaussian elimination to solve the following system. [2]

$$u + v + w = 2$$

$$2u + 3w = 5$$

$$3u + v + 4w = 6$$

- (c) Write the values of α for which the elimination breaks down (a) permanently (b) temporarily. [2]

$$\alpha x + 3y = -3$$

$$4x + 6y = 6$$

2. (a) Decide the value of α for which the system is singular [2]
and the value of β for which the system has infinitely
many solutions.

$$x + 4y - 2z = 1$$

$$x + 7y - 6z = 6$$

$$3y + \alpha z = \beta$$

- (b) Write the three elementary matrices that put the [2]
following matrix into upper triangular form.

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 2 \\ 3 & 4 & 5 \end{bmatrix}.$$

- (c) Define skew-symmetric matrix and give examples of 2nd [2]
and 3rd order.

3. (a) If $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, find an example with $AB = AC$ but $B \neq C$. [2]

- (b) Write the LU and LDU factorization of the following [2]
matrix.

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}.$$

- (c) Use Gauss-Jordan method to find the inverse of the [2]
following matrix.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}.$$

4. (a) Verify whether the following set is a subspace of \mathbb{R}^2 or not. [2]

$$V = \{(b_1, b_2) : b_1 > 0, b_2 > 0 \text{ and } b_1 \in \mathbb{R}, b_2 \in \mathbb{R}\}$$

- (b) Describe the column space and the nullspace of the matrix [2]

$$A = \begin{bmatrix} 1 & 0 \\ 2 & 0 \\ 0 & 0 \end{bmatrix}.$$

- (c) Compute the rank of the matrix. [2]

$$A = \begin{bmatrix} 1 & 3 & 3 & 2 \\ 2 & 6 & 9 & 7 \\ -1 & -3 & 3 & 4 \end{bmatrix}.$$

5. (a) Find the reduced row echelon form of the 3×4 matrix A with $a_{ij} = (-1)^{i+j}$. [2]

- (b) Decide the dependence or independence of the vectors $(1, 2, 3)$, $(2, 3, 1)$, $(3, 2, 1)$. [2]

- (c) Write the dimension of the plane $x + 2y - 3z - t = 0$ in \mathbb{R}^4 . [2]
Then find three independent vectors on the plane.

End of Questions