Problem Set 6.3

1) $A = \begin{bmatrix} 1 & 4 \\ 2 & 8 \end{bmatrix}$

a)

$|A - \lambda I| = 0$

$\Rightarrow \begin{vmatrix} 1-\lambda & 4 \\ 2 & 8-\lambda \end{vmatrix} = 0$

$\Rightarrow (1-\lambda)(8-\lambda) - 8 = 0$

$\Rightarrow \lambda^2 - 9\lambda + \cancel{8} - \cancel{8} = 0$

$\Rightarrow \lambda(\lambda - 9) = 0$

$\therefore \lambda_1 = 9 \mid \lambda_2 = 0$

$AA^T = \begin{bmatrix} 1 & 4 \\ 2 & 8 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 4 & 8 \end{bmatrix} = \begin{bmatrix} 17 & 34 \\ 34 & 68 \end{bmatrix}$

$= 17\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$

$= 17B$

To find eigenvalue of B,

$\begin{vmatrix} 1-\lambda & 2 \\ 2 & 4-\lambda \end{vmatrix} = 0$

$\Rightarrow \lambda^2 - 5\lambda + \cancel{4} - \cancel{4} = 0$

$\Rightarrow \lambda(\lambda - 5) = 0$

$\Rightarrow \lambda_1 = 5 \mid \lambda_2 = 0$

$\therefore$ Eigenvalues of $AA^T = 17B$,

$\lambda_1' = 85 \mid \lambda_2' = 0$

$\therefore \sigma_1^2 = 85 \mid \therefore \sigma_2^2 = 0$

For $\lambda_1' = 85$,

$\begin{bmatrix} -68 & 34 \\ 34 & -17 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\Rightarrow x_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

Unit eigenvector: $u_1 = \begin{bmatrix} 1/\sqrt{5} \\ 2/\sqrt{5} \end{bmatrix}$

For $\lambda_2' = 0$,

$\begin{bmatrix} 17 & 34 \\ 34 & 68 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\Rightarrow x_2 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

Unit eigenvector:

$u_2 = \begin{bmatrix} 2/\sqrt{5} \\ -1/\sqrt{5} \end{bmatrix}$

b)

$Av_1 = \sigma_1 u_1$

$\Rightarrow \begin{bmatrix} 1 & 4 \\ 2 & 8 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \sqrt{85}\begin{bmatrix} 1/\sqrt{5} \\ 2/\sqrt{5} \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & 4 \\ 2 & 8 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} \sqrt{17} \\ 2\sqrt{17} \end{bmatrix}$

$\Rightarrow y + 4z = \sqrt{17}$

$v_1 = \begin{bmatrix} 1/\sqrt{17} \\ 4/\sqrt{17} \end{bmatrix}$

$Av_2 = \sigma_2 u_2$

$\Rightarrow \begin{bmatrix} 1 & 4 \\ 2 & 8 \end{bmatrix}\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\Rightarrow y + 4z = 0$

$\therefore v_2 = \begin{bmatrix} -4/\sqrt{17} \\ 1/\sqrt{17} \end{bmatrix}$

c) Basis for $C(A)$: $\left\{\begin{bmatrix}1\\2\end{bmatrix}\right\}$

Orthonormal basis: $\left\{\begin{bmatrix}1/\sqrt{5}\\2/\sqrt{5}\end{bmatrix}\right\} = u_1$

Basis for $N(A)$: $\left\{\begin{bmatrix}-4\\1\end{bmatrix}\right\}$

Orthonormal basis: $\left\{\begin{bmatrix}-4/\sqrt{17}\\1/\sqrt{17}\end{bmatrix}\right\} = v_2$

Basis for $C(A^T)$: $\left\{\begin{bmatrix}1\\4\end{bmatrix}\right\}$

Orthonormal basis: $\left\{\begin{bmatrix}1/\sqrt{17}\\4/\sqrt{17}\end{bmatrix}\right\} = v_1$

Basis for $N(A^T)$: $\left\{\begin{bmatrix}+2\\-1\end{bmatrix}\right\}$

Orthonormal basis: $\left\{\begin{bmatrix}+2/\sqrt{5}\\-1/\sqrt{5}\end{bmatrix}\right\} = u_2$

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2) $A = \begin{bmatrix}1 & 4\\2 & 8\end{bmatrix}$

$A^T A = \begin{bmatrix}1 & 2\\4 & 8\end{bmatrix}\begin{bmatrix}1 & 4\\2 & 8\end{bmatrix} = \begin{bmatrix}5 & 20\\20 & 80\end{bmatrix}$

$\lambda_1 + \lambda_2 = 85 = \text{trace}(A^T A)$

$\lambda_1 \lambda_2 = 0 = \det(A^T A)$

$\therefore \lambda_1 = \sigma_1^2 = 85 \ ; \ \lambda_2 = \sigma_2^2 = 0$

$\begin{bmatrix}-80 & 20\\20 & -5\end{bmatrix}\begin{bmatrix}y\\z\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$

$\Rightarrow x_1 = \begin{bmatrix}1\\4\end{bmatrix}$

$\therefore$ Unit eigenvector:

$v_1 = \begin{bmatrix}1/\sqrt{17}\\4/\sqrt{17}\end{bmatrix}$

$\begin{bmatrix}5 & 20\\20 & 80\end{bmatrix}\begin{bmatrix}y\\z\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$

$\Rightarrow x_2 = \begin{bmatrix}-4\\1\end{bmatrix}$

$\therefore$ Unit eigenvector:

$v_2 = \begin{bmatrix}-4/\sqrt{17}\\1/\sqrt{17}\end{bmatrix}$.

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$$\begin{bmatrix} 1/\sqrt{5} & 2/\sqrt{5} \\ 2/\sqrt{5} & -1/\sqrt{5} \end{bmatrix} \begin{bmatrix} \sqrt{85} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1/\sqrt{17} & -4/\sqrt{17} \\ 4/\sqrt{17} & 1/\sqrt{17} \end{bmatrix}^T$$

$$= \begin{bmatrix} \sqrt{17} & 0 \\ 2\sqrt{17} & 0 \end{bmatrix} \begin{bmatrix} 1/\sqrt{17} & 4/\sqrt{17} \\ -4/\sqrt{17} & 1/\sqrt{17} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 4 \\ 2 & 8 \end{bmatrix}$$

$$= A$$

$\therefore$ $A = U\Sigma V^T$ (Verified)

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3) Given, $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

$$AA^T = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$A^TA = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

For, AA^T:

For eigenvalues:

$$|AA^T - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (2-\lambda)^2 - 1 = 0$$

$$\Rightarrow \lambda^2 - 4\lambda + 3 = 0$$

$$\Rightarrow (\lambda-3)(\lambda-1) = 0$$

$$\therefore \lambda_1 = \sigma_1^2 = 3 \quad | \quad \lambda_2 = \sigma_2^2 = 1$$

For eigenvectors:

For $\sigma_1^2 = 3$

$$\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow x_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$\therefore$ Unit eigenvectors: $u_1 = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$

For $\sigma_2^2 = 1$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\therefore \Rightarrow x_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$\therefore$ Unit eigenvectors: $u_2 = \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}$

$$\therefore U = \begin{bmatrix} \frac{1}{\sqrt{2}} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$$

For A^TA:

For eigenvalues:

$$|A^TA - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} 1-\lambda & 1 & 0 \\ 1 & 2-\lambda & 1 \\ 0 & 1 & 1-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (1-\lambda)\left[(2-\lambda)(1-\lambda) - 1\right] - 1(1-\lambda) = 0$$

$$\Rightarrow (1-\lambda)\left[\lambda^2 - 3\lambda + 1\right] - 1(1-\lambda) = 0$$

$$\Rightarrow (1-\lambda)\left[\lambda^2 - 3\lambda + \cancel{1} - \cancel{1}\right] = 0$$

$$\Rightarrow \lambda(\lambda-3)(1-\lambda) = 0$$

$$\therefore \lambda_1 = \sigma_1^2 = 3 \;\Big|\; \lambda_2 = \sigma_2^2 = 1 \;\Big|\; \lambda_3 = \sigma_3^2 = 0$$

For, eigenvectors:

For $\sigma_1^2 = 3$

$$\begin{bmatrix} -2 & 1 & 0 \\ 1 & -1 & 1 \\ 0 & 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow x_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

Unit eigenvectors:

$$v_1 = \begin{bmatrix} 1/\sqrt{6} \\ 2/\sqrt{6} \\ 1/\sqrt{6} \end{bmatrix}$$

For, $\sigma_2^2 = 1$

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow x_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

Unit eigenvector:

$$v_2 = \begin{bmatrix} 1/\sqrt{2} \\ 0 \\ -1/\sqrt{2} \end{bmatrix}$$

For $\sigma_3^2 = 0$

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow x_3 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

Unit eigenvector:

$$v_3 = \begin{bmatrix} 1/\sqrt{3} \\ -1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}$$

$$\therefore V = \begin{bmatrix} 1/\sqrt{6} & 1/\sqrt{2} & 1/\sqrt{3} \\ 2/\sqrt{6} & 0 & -1/\sqrt{3} \\ 1/\sqrt{6} & -1/\sqrt{2} & 1/\sqrt{3} \end{bmatrix}$$

$$\therefore U\Sigma V^T = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}\begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1/\sqrt{6} & 2/\sqrt{6} & 1/\sqrt{6} \\ 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 1/\sqrt{3} & -1/\sqrt{3} & 1/\sqrt{3} \end{bmatrix}$$

$$= \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}\begin{bmatrix} 1/\sqrt{2} & 2/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 0 & -1/\sqrt{2} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$= A \quad \text{(Verified)}$$

4) Given $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

$\therefore A^T A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

$\therefore |A^T A - \lambda I| = 0$

$\Rightarrow \begin{vmatrix} 2-\lambda & 1 \\ 1 & 1-\lambda \end{vmatrix} = 0$

$\Rightarrow (2-\lambda)(1-\lambda) - 1 = 0$

$\Rightarrow \lambda^2 - 3\lambda + 1 = 0$

$\Rightarrow \lambda = \dfrac{3 \pm \sqrt{9-4}}{2}$

$\therefore \lambda_1 = \sigma_1^2 = \dfrac{3+\sqrt{5}}{2} = \left(\dfrac{1+\sqrt{5}}{2}\right)^2$ | $\lambda_2 = \sigma_2^2 = \dfrac{3-\sqrt{5}}{2} = \left(\dfrac{1-\sqrt{5}}{2}\right)^2$

Solve the rest part.

14) Given

$$A = [1 \ 1 \ 1 \ 1]$$

$$AA^T = [1 \ 1 \ 1 \ 1]\begin{bmatrix}1\\1\\1\\1\end{bmatrix} = [4]$$

$\therefore \lambda_1 = \sigma_1^2 = 4$

$\therefore u_1 = x_1 = 1$

$\therefore A = U \Sigma V^T$

$= [1][2 \ 0 \ 0 \ 0]\begin{bmatrix}1/2\\1/2\\1/2\\1/2\end{bmatrix}$

$\therefore A = U \Sigma V^T$

$= [u_1][2 \ 0 \ 0 \ 0][v_1 \ v_2]$

$$A^TA = \begin{bmatrix}1\\1\\1\\1\end{bmatrix}[1 \ 1 \ 1 \ 1] = \begin{bmatrix}1&1&1&1\\1&1&1&1\\1&1&1&1\\1&1&1&1\end{bmatrix}$$

$\lambda_1 = \sigma_1^2 = 4 \mid \lambda_2 = \lambda_3 = \lambda_4 = 0$

For $\lambda_1 = 4$

$$\therefore \begin{bmatrix}-3&1&1&1\\1&-3&1&1\\1&1&-3&1\\1&1&1&-3\end{bmatrix}\begin{bmatrix}u\\v\\w\\x\end{bmatrix} = \begin{bmatrix}0\\0\\0\\0\end{bmatrix}$$

$$\therefore x_1 = \begin{bmatrix}1\\1\\1\\1\end{bmatrix} \mid \therefore v_1 = \begin{bmatrix}1/2\\1/2\\1/2\\1/2\end{bmatrix}$$

For $\lambda_2 = \lambda_3 = \lambda_4 = 0$

$$\therefore \begin{bmatrix}1&1&1&1\\1&1&1&1\\1&1&1&1\\1&1&1&1\end{bmatrix}\begin{bmatrix}u\\v\\w\\x\end{bmatrix} = \begin{bmatrix}0\\0\\0\\0\end{bmatrix}$$

$$\therefore x_2 = \begin{bmatrix}1\\-1\\0\\0\end{bmatrix} \quad \therefore v_2 = \begin{bmatrix}1/\sqrt{2}\\-1/\sqrt{2}\\0\\0\end{bmatrix}$$

$$x_3 = \begin{bmatrix}1\\0\\-1\\0\end{bmatrix} \quad \therefore v_3 = \begin{bmatrix}1/\sqrt{2}\\0\\-1/\sqrt{2}\\0\end{bmatrix}$$

$$x_4 = \begin{bmatrix}1\\0\\0\\-1\end{bmatrix} \quad \therefore v_4 = \begin{bmatrix}1/\sqrt{2}\\0\\0\\-1/\sqrt{2}\end{bmatrix}$$

$\therefore A = U \Sigma V^T$

$\Rightarrow A = [u_1][2 \ 0 \ 0 \ 0][v_1 \ v_2 \ v_3 \ v_4]^T$

$\therefore A^+ = V \Sigma^+ U^T$

$$= \begin{bmatrix}1/2 & 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2}\\1/2 & -1/\sqrt{2} & 0 & 0\\1/2 & 0 & -1/\sqrt{2} & 0\\1/2 & 0 & 0 & -1/\sqrt{2}\end{bmatrix}\begin{bmatrix}1/2\\0\\0\\0\end{bmatrix}[1] = \begin{bmatrix}1/4\\1/4\\1/4\\1/4\end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

$$AA^T = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$$

$\lambda_1 = \sigma_1^2 = 2$

$\lambda_2 = \sigma_2^2 = 0$

For $\lambda_1 = 2$,

$$\begin{bmatrix} 0 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \quad x_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = u_1$$

For $\lambda_2 = 0$,

$$\begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\therefore x_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = u_2$$

$$A^TA = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$\therefore \lambda_1 = \sigma_1^2 = 2 \quad | \quad \lambda_2 = \sigma_2^2 = 0$

For $\lambda_1 = 2$,

$$\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow x_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\therefore v_1 = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

For $\lambda_2 = 0$,

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\therefore x_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\therefore v_2 = \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}$$

$$\therefore C = U\Sigma V^T$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} \sqrt{2} & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$$

$$\therefore C^+ = V\Sigma^+U^T$$

$$= \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}\begin{bmatrix} 1/\sqrt{2} & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow C^+ = \begin{bmatrix} 1/2 & 0 \\ 1/2 & 0 \end{bmatrix} \text{ (Ans)}$$

~~$B = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$~~

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$\therefore B = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

$BB^T = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Eigenvalue:

$\lambda_1 = 1 = \sigma_1^2$

Eigenvector:

$x_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = u_1$

$\lambda_2 = 1 = \sigma_2^2$

Eigenvector:

$x_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = u_2$

$B^T B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Eigenvalue:

$\lambda_1' = 1 = \sigma_1^2$

Eigenvector:

$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\therefore x_1' = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = v_1$

$\lambda_2' = 1 = \sigma_2^2$

$x_2' = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = v_2$

$\lambda_3' = 0$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\Rightarrow x_3' = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = v_3$

$\therefore \cancel{\underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{U}\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}}_{\Sigma}\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{V^T}}$

$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

$= B$

$B^+ = V\Sigma^+U^T$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 0 \end{bmatrix}$ (Ans)

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