PROBLEM SET 1.2

Q. Explain why the given system is singular by finding a combination of the three equations that adds up to 0=1. What value should replace the last 0 on the r.h.s. to allow the equations to have solutions and what is one of the solutions?

$$\begin{aligned} u + v + w &= 2 \\ u + 2v + 3w &= 1 \\ v + 2w &= 0. \end{aligned}$$

Ans. Here in the left hand side $Row1 + Row3 = Row2$ but not in right hand side. Hence the system is singular but no solution exists. If the last 0 is replaced by -1 then l.h.s. and r.h.s. both satisfy the condition $Row1 + Row3 = Row2$.

Hence solution exists and $\begin{bmatrix} 3+w \\ -1-2w \\ w \end{bmatrix}$ is a general solution. For every value of w it gives a solution of

$$\begin{aligned} u + v + w &= 2 \\ u + 2v + 3w &= 1 \\ v + 2w &= -1. \end{aligned}$$

In patricular, $\begin{bmatrix} 4 \\ -3 \\ 1 \end{bmatrix}$ is a solution of it.

Q. Under what condition on y_1, y_2, y_3 do the points $(0, y_1)$, $(1, y_2)$, $(2, y_3)$ lie on a straight line?

Ans. The points $(0, y_1)$, $(1, y_2)$, $(2, y_3)$ lie on a straight line means the slopes of the line joining the points $(0, y_1)$ and $(1, y_2)$ and the points $(1, y_2)$ and $(2, y_3)$ are equal. That is,

$$\frac{y_2 - y_1}{1 - 0} = \frac{y_3 - y_2}{2 - 1}$$

which implies that $y_1 - 2y_2 + y_3 = 0$. Hence for $y_1 - 2y_2 + y_3 = 0$ the points $(0, y_1)$, $(1, y_2)$, $(2, y_3)$ lie on a straight line.

Q. The column picture form of a system is

$$u \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + v \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} + w \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} = b.$$

Show that the three columns on the left lie in the same plane by expressing the third column as a combination of the first two. What are all the solutions (u, v, w) if b is the zero vector $(0, 0, 0)$?

Ans. Here it can be observed that $2C_2 - C_1 = C_3$ that is, $2 \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$. Hence the system of the equations

$$u \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + v \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} + w \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

has infinitely many solutions. For every value of w the vector $(w, -2w, w)$ represents a solution of it. All solutions of it is represented by the set $\{(w, -2w, w) \in R^3 : w \in R\}$.

Q. Sketch these three lines and decide if the equations are solvable :

$$\begin{aligned} x + 2y &= 2 \\ x - y &= 2 \\ y &= 1. \end{aligned}$$

What happens if all the right-hand sides are zero? Is there any nonzero choice of right hand sides that allows the three lines to intersect at the same point ?

Ans. The first figure represents the straight lines represented in the question.

$$\begin{aligned} x + 2y &= 2 \\ x - y &= 2 \\ y &= 1. \end{aligned}$$

Which shows that there exist no point common to all straight lines. Hence no solution exists.

The second figure gives the straight lines with r.h.s. vector 0, that is

$$\begin{aligned} x + 2y &= 0 \\ x - y &= 0 \\ y &= 0. \end{aligned}$$

Hence $x = 0$ and $y = 0$ is a solution of it.

The third figure gives the graph of

$$\begin{aligned} x + 2y &= 2 \\ x - y &= 2 \\ y &= 0. \end{aligned}$$

With r.h.s. vector $\begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}$ and $x = 2$, $y = 0$ satisfies all the equations. Hence it has a solution at $(2, 0)$.

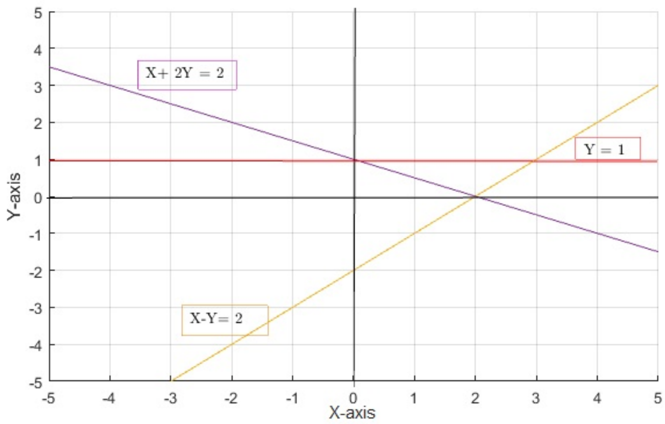

FIG.1

Figure 1: **Solution does not exist**

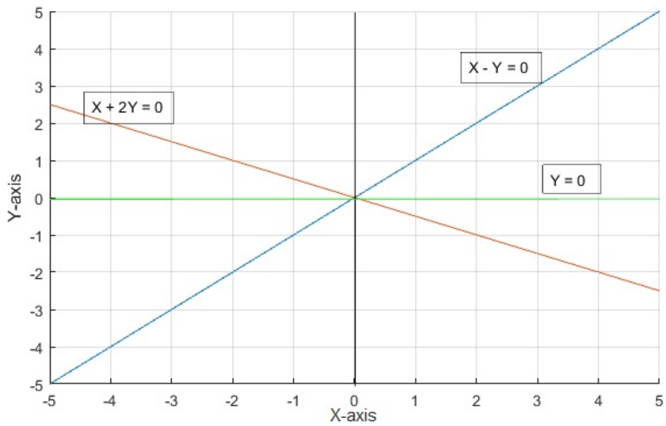

FIG. 2

Figure 2: **Solution exist and is the zero solution**

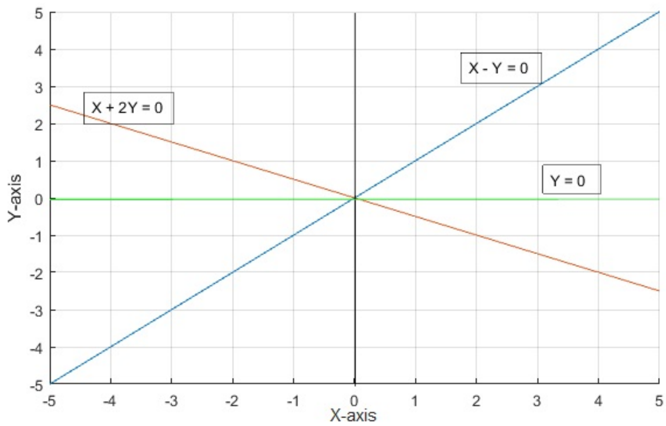

FIG. 2

Figure 3: **Nonzero solution exists**