Chapter-1
(Matrices and Gaussian Elimination)

1.1 Introduction:

Aim: To solve system of equations using normal elimination method and to know about the number of solutions.

~~Outcome~~ Course outcome: Students will get idea about the Gauss elimation method from the normal elimination method.

Ex:-1

$$x + 2y = 3 \text{ ——— } (1)$$
$$4x + 5y = 6 \text{ ——— } (2)$$

Equation (2) minus 4× equation (1) gives

$$-3y = -6$$
$$\Rightarrow y = 2$$

Putting $y=2$ in eqn (1), we get

$$x + 4 = 3$$
$$\Rightarrow x = -1$$

So, the solution is $(-1, 2)$ and is unique.

Ex:-2 $x + 2y = 3$ ——— (1)

$$4x + 8y = 6 \text{ ——— } (2)$$

Equation (2) minus 4× equation (1) gives

$$0 = -6,$$

which is not possible i.e. impossible.

So, the system has no solution i.e. zero number of soln.

Ex:3 $x + 2y = 3$ —— (1)

$4x + 8y = 12$ —— (2)

Equation (2) minus $4 \times$ equation (1) gives

$0 = 0$,

which is an identity.

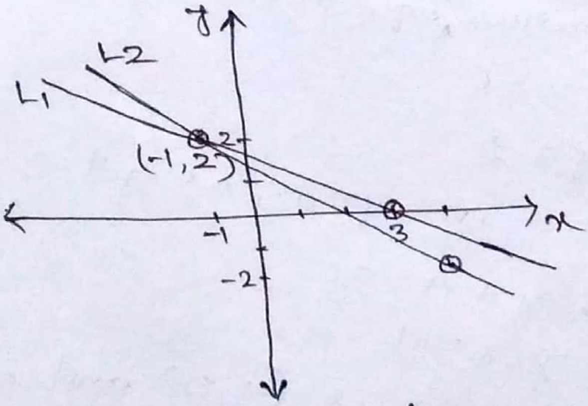
So, the system has infinite no. of solution i.e. more than one solution i.e. whole line of solution.

Graphical realization:

Graph of example-1 system:

$L_1: x + 2y = 3 \longrightarrow (3,0), (-1,2)$

$L_2: 4x + 5y = 6 \longrightarrow (-1,2), (4,-2)$

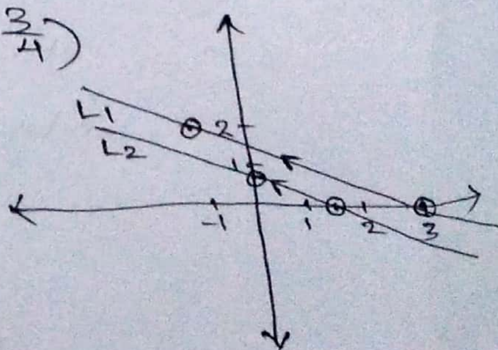


From the graph, it is clear that the two lines are intersecting. They intersect each other at $(-1,2)$, which is the solution of the system and is unique.

Graph of Ex-2 system:

$L_1: x + 2y = 3 \longrightarrow (3,0), (-1,2)$

$L_2: 4x + 8y = 6 \longrightarrow (\frac{3}{2},0), (0,\frac{3}{4})$



From, the graph, it is clear that the two lines are parallel and different.

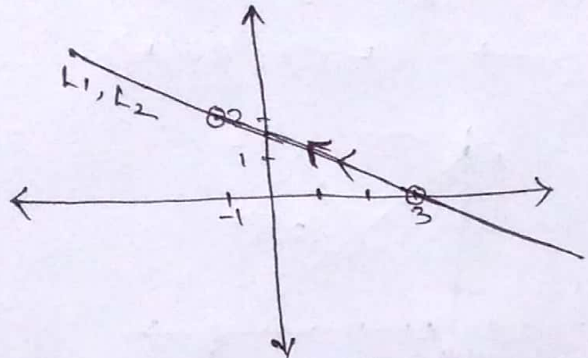
There is no point common between the two lines.

So, the system has no solution.

Graph of ~~solution~~ example-3 system:

$L_1 : x + 2y = 3 \longrightarrow (3,0), (-1,2)$

$L_2 : 4x + 8y = 12 \longrightarrow (3,0), (-1,2)$



From the graph, it is clear that the two lines are overlapping. More than one points are common between the two lines. So, the system has more than one solution i.e. no. of solutions.

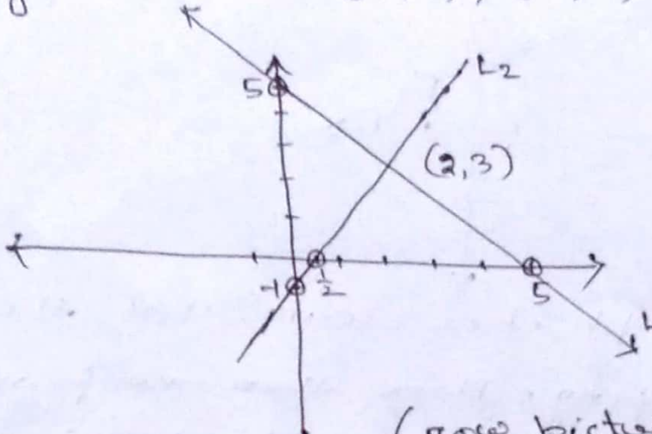
1.2 The Geometry of Linear Equations

Course Outcome: Students will have understanding about row picture, column picture, singular, non-singular, consistent and inconsistent systems.

Row Picture:

$L_1: 2x - y = 1 \longrightarrow (0,-1), (\frac{1}{2}, 0)$

$L_2: x + y = 5 \longrightarrow (5,0), (0,5)$



(row picture)

Adding both the equations of the system, we have

$$3x = 6$$
$$\Rightarrow x = 2$$
$$x = 2 \Rightarrow 4 - y = 1 \Rightarrow y = 3$$

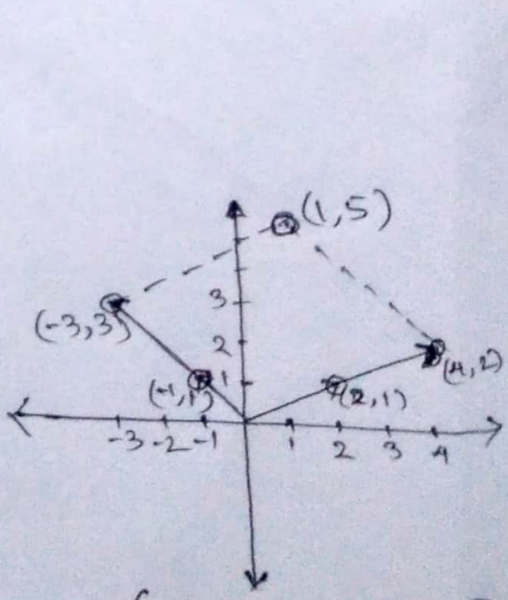
So, point of intersection of L_1 and L_2 is $(2,3)$.

Column Picture:

$$2x - y = 1$$
$$x + y = 5$$

$$\Rightarrow x\begin{bmatrix}2\\1\end{bmatrix} + y\begin{bmatrix}-1\\1\end{bmatrix} = \begin{bmatrix}1\\5\end{bmatrix}$$

$$\Rightarrow 2\begin{bmatrix}2\\1\end{bmatrix} + 3\begin{bmatrix}-1\\1\end{bmatrix} = \begin{bmatrix}1\\5\end{bmatrix}$$



(column picture)

Consistent system: A system is said to be consistent if it has at least one solution.

Inconsistent system: A system is said to be inconsistent if it has ~~at least~~ no solution.

Singular system: A system is said to be singular if determinant of the co-efficient matrix associated with the system is zero.

Nonsingular system: A system is said to be nonsingular if determinant of the coefficient matrix associated with the system is nonzero.

Ex: $x + 2y = 3$
$4x + 5y = 6$

The coefficient matrix is

$$A = \begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix}$$

$|A| = 5 - 8 = -3 \neq 0$

$\Rightarrow$ The system is non-singular.

Ex: $x + 2y = 3$
$4x + 8y = 6$

The coefficient matrix is

$$A = \begin{bmatrix} 1 & 2 \\ 4 & 8 \end{bmatrix}$$

$|A| = 8 - 8 = 0$

$\Rightarrow$ The system is singular.

Ex:- $x + 2y = 3$

$4x + 8y = 12$

The coefficient matrix is

$$A = \begin{bmatrix} 1 & 2 \\ 4 & 8 \end{bmatrix}$$

$|A| = 8 - 8 = 0$

$\Rightarrow$ The system is singular.

Notes:

1. System nonsingular $\Leftrightarrow$ unique sol^n.
2. System singular $\Leftrightarrow$ The system has either no solution or infinite no. of solution.

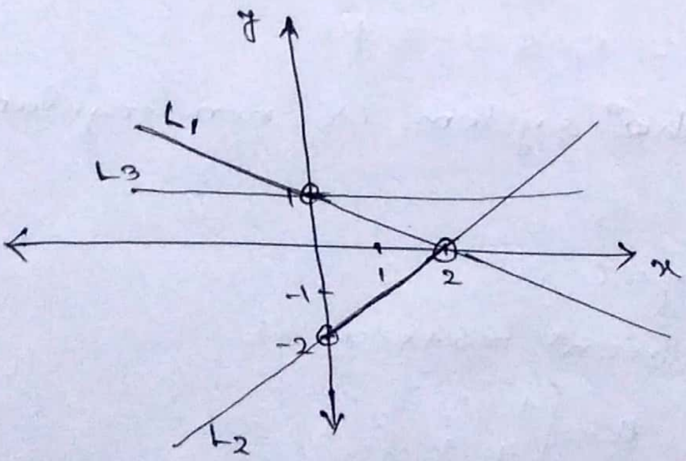
Problem Set 1.2

No. 2. Given:

$L_1: \; x + 2y = 2 \longrightarrow (2,0), (0,1)$

$L_2: \; x - y = 2 \longrightarrow (2,0), (0,-2)$

$L_3: \; y = 1$



From the graph it is clear that the three lines are not passing through any point. So, the system is not solvable.

If all the right hand sides are zero, then the lines will passes through origin and (0,0) is a solution of the new system.

Yes, the nonzero choice (2,2,0) of the right hand sides allows the three lines to intersect at the common point (2,0).

No. 7. Given:

$$u + v + w = 2 \quad \text{—— (1)}$$
$$u + 2v + 3w = 1 \quad \text{—— (2)}$$
$$v + 2w = 0 \quad \text{—— (3)}$$

Eq^n (1) − eq^n (2) + eq^n (3) gives

$$0 = 1,$$

which is impossible.

⇒ The system has no solution.

⇒ The system is singular.

If the right hand side '0' of equation (3) will be replaced by −1, then the new system will have solution.

$$\left.\begin{array}{r} u + v + w = 2 \\ u + 2v + 3v = 1 \\ v + 2w = -1 \end{array}\right] \text{ new system}$$

$$v + 2w = -1$$
$$\Rightarrow v = -1 - 2w$$

Let $w = 0$. Then $v = -1$.

$$u = 2 - v - w = 3.$$

So, $(3, -1, 0)$ is a solution.

No. 8. Point : $(0, y_1), (1, y_2), (2, y_3)$.

The three points will lie on a straight line if area of the triangle whose three vertices are the three points.

$$\Rightarrow \frac{1}{2}\begin{vmatrix} 0 & y_1 & 1 \\ 1 & y_2 & 1 \\ 2 & y_3 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 0 & y_1 & 1 \\ 1 & y_2 & 1 \\ 2 & y_3 & 1 \end{vmatrix} = 0$$

$$\Rightarrow -y_1(1-2)+1(y_3-2y_2)=0.$$

$$\Rightarrow \boxed{y_1 - 2y_2 + y_3 = 0},$$

is the required condition on y_1, y_2 and y_3.

No. 11

$$u\begin{bmatrix}1\\1\\0\end{bmatrix} + v\begin{bmatrix}1\\2\\1\end{bmatrix} + w\begin{bmatrix}1\\3\\2\end{bmatrix} = b \quad \text{—— ①}$$

Let $b=(0,0,0)$. Then

$$u+v+w=0$$
$$u+2v+3w=0$$
$$v+2w=0$$

$$v=-2w$$

Let $w=1$. Then $v=-2$.

$$u=-v-w=1.$$

So,
$$1\begin{bmatrix}1\\1\\0\end{bmatrix} - 2\begin{bmatrix}1\\2\\1\end{bmatrix} + 1\begin{bmatrix}1\\3\\2\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}1\\3\\2\end{bmatrix} = 2\begin{bmatrix}1\\2\\1\end{bmatrix} - \begin{bmatrix}1\\1\\0\end{bmatrix}$$

$\Rightarrow$ Col. 3 = 2 × Col. 2 − Col. 1

$\Rightarrow$ Three columns on the left of eqn ① lie on the same plane.

For $b=(0,0,0)$, $(u,v,w) = (c,-2c,c)$, $c \in \mathbb{R}$ are the solutions of the homogeneous system.