END SEMESTER EXAMINATION, APRIL-2019
PROBABILITY & STATISTICS (MTH-2002)

Programme: B.Tech | **Semester : 4th**
Full Marks: 60 | **Time: 3 Hours**

Subject/Course Learning Outcome	*Taxonomy Level	Ques. Nos.	Marks
Apply probability axioms to compute probability and conditional probability	L3,L3,L4,L3	1(a), 2(a)3(a)	2*3
Define random variables and compute probability distributions, joint & marginal distribution	L4,L4,L3,L5, L5	1(b), 2(b,c), 3(b,c)	2*5
Compute expectation of random variables and their functions and compute moments and moment generating functions of a random variable	L3,L4,L4 L3,L4	4(a,b,c), 7(a),6(c)	2*5
Discuss discrete probability distribution viz: Binomial, Poisson & Hypergeometric and continuous probability distribution distributions viz: Uniform, Normal Gamma & Exponential	L3,L4, L4, L4	1(c),5(a, b,c), 6(a,b),	2*6
Estimate the population mean and variance of a normal distribution by point and interval estimation	L4	7(b,c) 8(b)	2*3
Infer about population parameter through hypothesis testing with the help of a random sample	L1,L4,L3,L4, L4, L3,L4,L4	8(a,c), 9(a,b,c),	2*5
Analyze linear regression and co-relation	L3,L5,L5	10(a,b,c)	2*3

*Bloom's taxonomy levels: Knowledge (L1), Comprehension (L2), Application (L3), Analysis (L4), Evaluation (L5), Creation (L6)

Answer all questions. Each question carries equal mark.

1 a) The probability that a regularly scheduled flight departs on time is $P(D)=0.83$; the probability that it arrives on time is $P(A)=0.82$ and the probability that it departs and arrives on time is $P(A\cap D)=0.78$. compute the probability that a plane arrives on time, given that it departed on time 2

b) In a certain region of the country it is known from past experience that the probability of selecting an adult over 40 years of age with cancer is 0.05. If the probability that the doctor correctly diagnosing a person with cancer as having the disease is 0.78 and the probability that incorrectly diagnosing a person without cancer as having the disease is 0.06, obtain the probability that a person diagnosed as having cancer actually has the disease. 2

c) A shipment of 7 television sets contains 2 defective sets. A hotel makes a random purchase of 3 of the sets. If x is the number of defective sets purchased by the hotel, compute the probability distribution of X. 2

2 a) Suppose that the random variable X having probability density function $f(x)=\begin{cases}\frac{x^2}{3}, & -1<x<2\\ 0, & \text{elsewhere}\end{cases}$ 2

Compute the cumulative distribution function of X.

b) If the joint probability distribution of X & Y is given by $f(x,y)=c(x+y)$, for $x=1,2,3; y=1,3$. Find the value of 'c'. 2

c) Form the data given in (b), evaluate $P(X>2\,|\,Y=1)$. 2

3 a) Evaluate the Value of 'c' for which $f(x)=cx,\ x=1,2,3$ will be a valid probability function. 2

b) Suppose that the random variable X has probability density function is $f(x)=\begin{cases}2(1-x), & 0<x<1\\ 0, & \text{elsewhere}\end{cases}$. Calculate the variance of X. 2

c) A random variable X has a mean $\mu = 10$ and variance $\sigma^2 = 4$. Using Chebyshev's theorem, compute the bounds of P $(4 < x < 16)$. 2

4 a) The probability that a patient recovers from a delicate heart operation is 0.9. Find the probability that at least 5 of the next 7 patients having this operation survives. 2

b) Find the probability that a person flipping a coin gets 2

(i) The third head on the seventh flip.

(ii) The first head on the fourth flip.

c) An urn contains 3 green balls, 3 blue balls and 4 red balls. If 5 balls are selected randomly from the urn, compute the probability that one green ball and 1 red ball are selected. 2

5 a) The average number of field mice per acre in a 5-acre wheat field is estimated to be 12. Using Poisson distribution Compute the probability that fewer than 7 field mice are found on a given acre. 2

b) Let X be a random variable with probability distribution function 2

$f(x) = \frac{x}{6}$, for $x = 1,2,3$.

Calculate the expected value of $y(X) = 4X - 3$.

c) Suppose X follows a continuous uniform distribution from 1 to 5. Determine the conditional probability $P(X > 2.5 \mid X \le 4)$. 2

6 a) A certain type of storage battery lasts, on average, 3.0 years with a standard deviation of 0.5 year. Assuming that battery life is normally distributed, find the probability that a given battery will last less than 2.3 years. 2

b) Given the normally distributed random variable 'X' with mean 30 and standard deviation 6, calculate the value of 'k' that has 80% of the normal curve area to the right. 2

c) A process yields 10% defective items. If 100 items are randomly selected, calculate the probability that the number of defectives doesn't exceed 13 by using the normal approximation to the Binomial distribution. 2

7 a) Let the random variable X, has a Gamma distribution with $\alpha = 2$ and $\beta = 1$. Evaluate $P(1.8 < X < 2.4)$. 2

b) A random variable X has the Poission distribution with mean μ, Compute the moment generating function for X. 2

c) Let X be a continuous random variable with probability distribution 2

$$f(x) = \begin{cases} 2(1-x), & 0 < x < 1, \\ 0, & elsewhere \end{cases}.$$

Calculate the probability distribution of the random variable Y=2X-3.

8 a) Compute the maximum likelihood estimator for 'p' of binomial distribution from the sample of observations $X_1, X_2, X_3, \ldots X_N$. 2

b) The contents of seven similar containers of sulphuric acid are 9.8, 10.2, 10.4, 9.8, 10.0, 10.2 and 9.6 liters. Find a 95% confidence interval for the mean contents of all such containers, assuming an approximately normal distribution. 2

c) A random sample of 10 chocolate energy bars of a certain brand has, on average, 230 calories per bar with a standard deviation of 15 calories. Construct a 99% confidence interval for the variance (σ^2). Assume that the distribution of the calorie content is approximately normal.

9 a) An electrical firm manufactures light bulbs that have a lifetime that is approximately normally distributed with a mean of 800 hours and a standard deviation of 40 hours. Test the hypothesis that $\mu = 800$ hours against the alternative, $\mu \neq 800$ hours, if a random sample of 30 bulbs has an average life of 788 hours. Use 0.05 level of significance. 2

b) The content of containers of a particular lubricant is known to be normally distributed with a variance of 0.03 liter. Test the hypothesis that $\sigma^2 = 0.03$ against $\sigma^2 \neq 0.03$ for the random sample of 10 containers has a standard deviation $s = 0.29$. Use a 0.01 level of significance. 2

c) A die is tossed 180 times with the following results: 2

x	1	2	3	4	5	6
f	28	36	36	30	27	23

Is this a balanced die? Use a 0.01 level of significance.

10 The grades of a class of 6 students on a midterm report (x) and on the final examination (y) are as follows:

x	65	66	67	67	68	69
y	67	68	65	68	72	72

a) Estimate the linear regression line. 2

b) Estimate the final examination grade of a student who received a grade of 85 on the midterm report. 2

c) Estimate the correlation coefficient. 2

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