My Notebook - OneNote for Windows 10

Q.6 A random variable X has a mean $\mu = 10$ and a variance $\sigma^2 = 4$. Using Chebyshev's theorem, the value of constant c such that $P(|X - 10| \geq c) \leq 0.01$ is:

(A) 10

(B) 100

(C) 20

(D) None

$\sigma = 2$

$P(|X-\mu| \geq k\sigma) \leq \frac{1}{k^2}$

$k\sigma = c$

$\frac{1}{k^2} = 0.01 \Rightarrow k^2 = 100 \Rightarrow k = 10$

$c = k\sigma = 10 \times 2 = 20$

Q.7 If X and Y are independent random variables with variances $\sigma_X^2 = 5$ and $\sigma_Y^2 = 3$, the variance of the random variable $Z = -2X + 4Y - 3$ is:

(A) 8

(B) 2

(C) 68

(D) None

$Z = -2X + 4Y - 3$

$Var(Z) = Var(-2X + 4Y - 3) = Var(-2X + 4Y)$

$= (-2)^2 Var(X) + 4^2 Var(Y)$

$= 4 \times \sigma_X^2 + 16 \times \sigma_Y^2$

$= 4 \times 5 + 16 \times 3 = 20 + 48 = \underline{68}$

Q.8 The probability of getting a total of 5 or 10 when a pair of fair dice is tossed is:

(A) $\frac{1}{2}$

(B) $\frac{1}{18}$

(C) 7

5 ← (1,4), (4,3), (3,2)

10 ← (4,6), (5,5), (6,4)

$\frac{7}{6 \times 6} = \frac{7}{36}$

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✓ ~~(C) $\frac{7}{36}$~~

(D) None

$5 \leftarrow (1,4)\ (2,3)\ (3,2)\ (4,1)$

$10 \leftarrow (4,6)\ (5,5)\ (6,4)$

$$\frac{7}{6 \times 6} = \frac{7}{36}$$

Q.9 If 2 books are picked at random from a shelf containing 5 novels, 3 books of poems, and a dictionary, then the probability that the dictionary is not selected is:

(a) $\frac{2}{9}$

✓ ~~(b) $\frac{7}{9}$~~

(c) $\frac{1}{3}$

(d) None

$5 + 3 + 1 = 9$

$$P = \frac{\binom{5}{1}\binom{3}{1} + \binom{5}{2}\binom{3}{0} + \binom{5}{0}\binom{3}{2}}{\binom{9}{2}}$$

$$\binom{9}{2} = \frac{9 \times \cancel{8}^{4}}{\cancel{2 \times 1}} = 36$$

$$= \frac{5 \times 3 + 10 \times 1 + 1 \times 3}{36} = \frac{28}{36} = \frac{7}{9}$$

Q.10 One bag contains 3 white balls and 2 black balls, and a second bag contains 2 white balls and 3 black balls. One ball is drawn from the first bag and placed unseen in the second bag. The probability that a ball now drawn from the second bag is black is:

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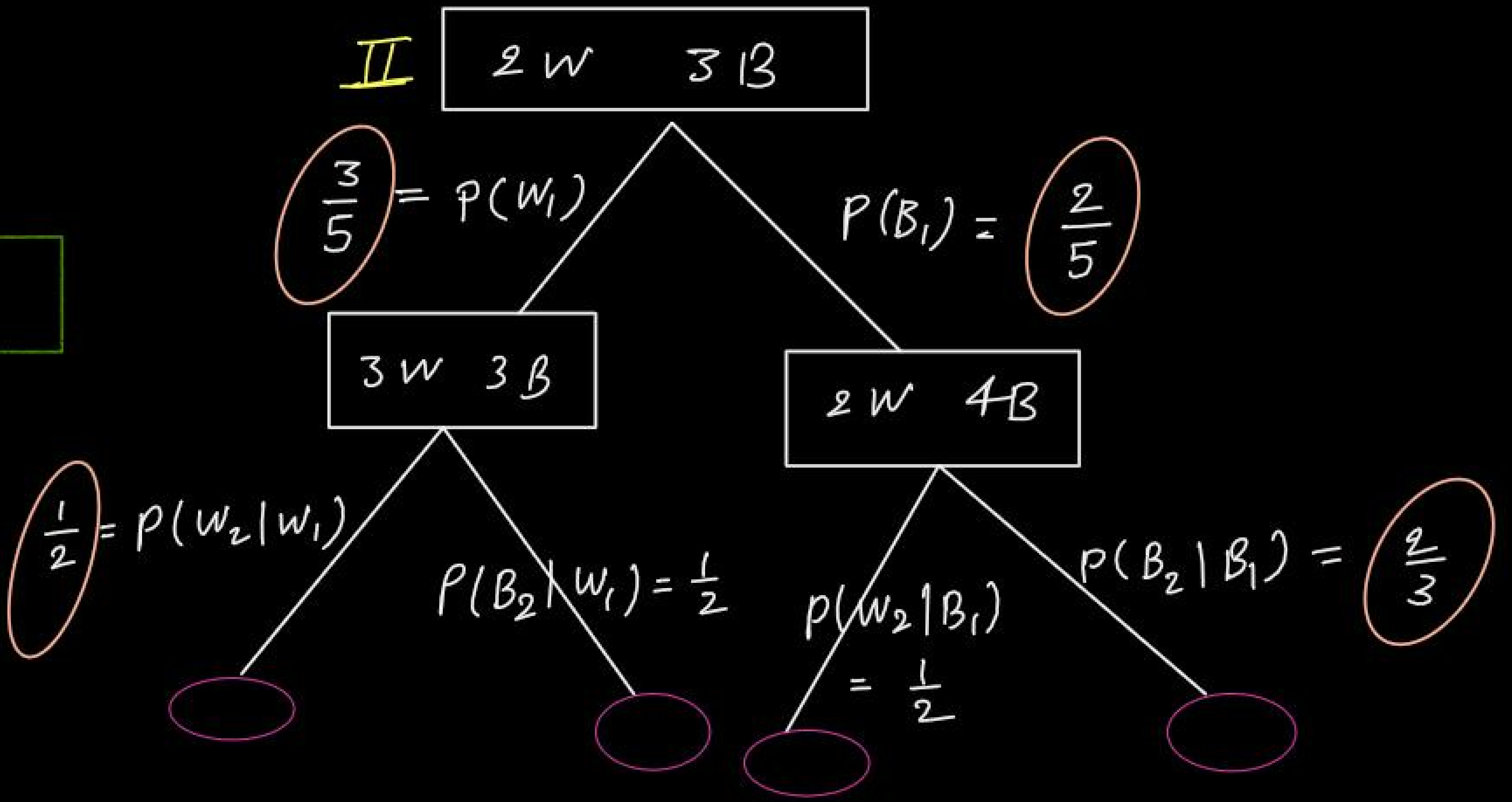
(A) 0.6

(B) ~~0.57~~

(C) 0.1

(D) None

I $\boxed{3W \quad 2B}$



$$P(B_2) = \frac{3}{5} \times \frac{1}{2} + \frac{4}{5} \times \frac{4}{3}$$

$$= \frac{3}{10} + \frac{4}{15} = \frac{9+8}{30} = \frac{17}{30}$$

Q.11 For any constant k, E(k) is:

(A) 0

(B) ~~k~~

(C) k^2

(D) 1

$$E(k) = \sum k f(x)$$

$$= k\left(\sum f(x)\right) \rightarrow \text{total prob}$$

$$= k \times 1$$

$$= k$$

Q.12 A manufacturing firm employs three analytical plans for the design and development of a particular product. For cost reasons, all three are used at varying times. In fact, plans 1, 2, and 3 are used for 40%, 10%, and 50% of the products, respectively. The defect rate is different for the three procedures as follows: P(D|P1)=0.01, P(D|P2)= 0.03, P(D|P3)=0.02, where P(D|Pj) is the probability of a defective product, given plan j. If a random product was observed and found to be defective, what is the probability that plan 2 was used and thus responsible?

Q.13 The probability that a patient recovers from a delicate heart operation is 0.8. What is the probability that none of the next 3 patients who have this operation survive ?

(a) 0.8

(b) 0.2

(c) 0.512

(d) 0.008

P(none of the next 3 patients survive)

$= (1-0.8)(1-0.8)(1-0.8)$

$= 0.008$

$= \frac{0.10 \times 0.03}{0.017}$

$= \frac{3}{17}$

Q.14 Let X be a continuous random variable with pdf f(x) and cdf F(x). Which of the following is NOT necessarily true?

(A) $F(-\infty) = 0$ ✓

(B) $F(\infty) = 1$ ✓

(C) $f(0) = 0$ ✗

(D) $P(X=0) = 0$ ✓

$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}, \quad -\infty < x < \infty$

$f(0) = \frac{1}{\sqrt{2\pi}} e^{-0} = \frac{1}{\sqrt{2\pi}} \neq 0$

$P(X=0) = 0$

Q.15 A certain form of cancer is known to be found in women over 60 with probability 0.07. A blood test exists for the detection of the disease, but the test is not infallible. In fact, it is known that 10% of the time the test gives a false negative (i.e., the test incorrectly gives a negative result) and 5% of the time the test gives a false positive (i.e., incorrectly gives a positive result). If a woman over 60 is known to have taken the test and received a favorable (i.e., negative) result, the probability that she has the disease is

$P(C) = 0.07$

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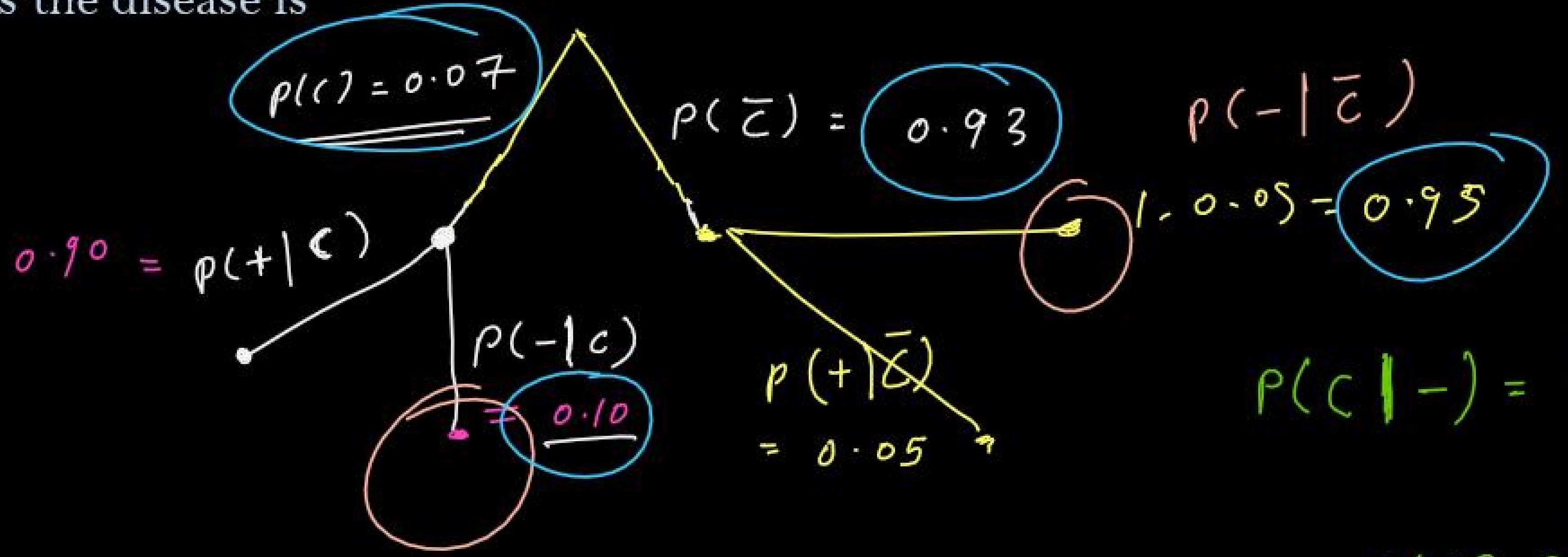
(A) 0.001

(B) 0.005

(C) 0.007

(D) None



$$P(c \mid -) = \frac{P(c \cap -)}{P(-)}$$

$$= \frac{p(c)\, p(-\mid c)}{p(-)}$$

$$= \frac{0.07 \times 0.10}{0.07 \times 0.10 + 0.93 \times 0.95}$$

$$= \frac{0.007}{0.8905}$$

$$= 0.00786$$

Q.16 If cumulative distribution function of a random variable is given by $F(0) = \frac{1}{16}$, $F(1) = \frac{5}{16}$, $F(3) = \frac{11}{16}, F(4) = \frac{15}{16}, F(5) = 1$, *then* $P(X \geq 2)$ *is*

(A) $\frac{15}{16}$

(B) $\frac{1}{16}$

(C) $\frac{7}{16}$

(D) $\frac{11}{15}$

$$P(X \geq 2) = 1 - P(X < 2)$$

$$= 1 - P(X \leq 1)$$

$$= 1 - F(1)$$

$$= 1 - \frac{1}{16} = \frac{15}{16}$$

Q.17 The value c so that each of the function

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$$f(x) = c\binom{100}{x}\binom{400}{400-x}, x = 0,1,2,\dots,100$$

can serve as a probability distribution of the discrete random variable X is

(a) $\binom{100}{2}^{-1}$

(b) $\binom{500}{100}^{-1}$

(c) $\binom{400}{100}^{-1}$

(d) None

100 | 400

$\binom{500}{400} =$

$\sum f(x) = 1$

$$c\sum_{x=0}^{100}\binom{100}{x}\binom{400}{400-x} = 1$$

$$\frac{1}{c} = \sum_{x=0}^{100}\binom{100}{x}\binom{400}{400-x}$$

$$= \binom{100}{0}\binom{400}{400} + \binom{100}{1}\binom{400}{399} + \dots + \binom{100}{100}\binom{400}{300}$$

$=$

Q.18 On a laboratory assignment, if equipment is working, the density function of the observed outcome, X , is

$$f(x) = \begin{cases} 2(1-x), & 0 < x < 1 \\ 0, & elsewhere \end{cases}$$

Given that X ≥ 0.5, the probability that X will be less than 0.75 is

(A) 0.25

(B) 0.50

(C) 0.75

(D) 1

$$P(\underbrace{X < 0.75}_{A} \mid \underbrace{X \ge 0.5}_{B}) = \frac{P(X < 0.75 \cap X \ge 0.5)}{P(X \ge 0.5)}$$

$$= \frac{P(0.5 \le X < 0.75)}{P(X \ge 0.5)} = \frac{\int_{0.5}^{0.75} 2(1-x)\,dx}{\dots}$$

(C) 0.75

(D) 1

$$= \frac{P(0.5 \le X < 0.75)}{P(X \ge 0.5)} = \frac{\int_{0.5}^{0.75} 2(1-x)\,dx}{\int_{0.5}^{1} 2(1-x)\,dx}$$

Q.19 Let A, B, and C be three events with P(A) = 0.1, P(B) = 0.7, and P(C) = 0.5. If A and B are disjoint, then $P(A\cap B \cap C)$ *is*:

(a) 1

(b) 0.8

(c) 0.3

(d) 0

$A \cap B = \phi$

$A \cap B \cap C = \phi$

$P(A \cap B \cap C) = 0$

Q.20 To find out the prevalence of a virus in a city's population of size 1,00,000, a blood test was carried out on 200 randomly selected citizens. If the test returned 8 positive results, the distribution of number of affected persons in a random sample of size 500 from the population can be approximately be taken as

(a) Poisson(40)

(b) Poisson(20)

(c) Poisson(8)

(d) Poisson(16/5)

$\hat{p} = \frac{8}{200} =$

$X \sim P(\lambda)$

$P(20)$

$200 \rightarrow 8$

$1 \rightarrow \frac{8}{200}$

$500 \rightarrow \frac{\cancel{500} \times 8}{\cancel{200}} = 5 \times 4 = 20$

λ

Q.21 A traffic control engineer reports that 75% of the vehicles passing through a checkpoint are from within the state. The probability that fewer than 4 of the next 9 vehicles are from out of state is

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(a) 0.0571

(b) 0.0142

(c) 0.0101

(d) None

$P(X < 4)$ $\qquad X \sim B(n=9,\ p=0.25)$

$= P(X \le 3)$

$= \sum_{x=0}^{3} b(x;\ n=9,\ p=0.25)) = 0.8343$

Q.22 The probability that a person will die when he or she contracts a virus infection is 0.002. Of the next 10,000 people infected, the mean number who will die is

(A) 100

(B) 20

(C) 50

(D) None

$n = 10{,}000$

$p = 0.02$

$X \sim B(n, p)$

$E(X) = np = 10{,}000 \times 0.002$

$= 10 \times 2 = 20$