Lecture 3
2.4 Probability of an Event
2.5 Additive Rules

The aim of this lecture is to explain the following concepts :

- Probability of an Event.
- Additive Rules.

2.4 Probability of an Event :

Definition 1 *The probability of an event A is the sum of the weights of all sample points in A. Therefore,* $0 \leq P(A) \leq 1, P(\phi) = 0, and P(S) = 1$. *Furthermore, if* $A_1, A_2, A_3, ...$ *is a sequence of mutually exclusive events, then* $P(A_1 \cup A_2 \cup A_3 \cup ......) = P(A_1) + P(A_2) + P(A_3) + ...$.

Example 1 ***:*** *A coin is tossed twice. What is the probability that at least 1 head occurs ?*
Solution : *The sample space for this experiment is* $S = \{HH, HT, TH, TT\}$. *If the coin is balanced, each of these outcomes is equally likely to occur. If A represents the event of at least 1 head occurring, then* $A = \{HH, HT, TH\}$ *and* $P(A) = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$

Note : If an experiment can result in any one of N different equally likely outcomes, and if exactly n of these outcomes correspond to event A, then the probability of event A is $P(A) = \frac{n}{N}$.

2.5 Additive Rules :

Theorem 0.1 *If A and B are two events, then*

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

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Corollary 0.1 *If A and B are mutually exclusive, then*

$$P(A \cup B) = P(A) + P(B)$$

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Corollary 0.2 *If $A_1, A_2, ..., A_n$ are mutually exclusive, then*

$$P(A_1 \cup A_2 \cup .... \cup A_n) = P(A_1) + P(A_2) + ... + P(A_n)$$

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Corollary 0.3 *If $A_1, A_2, ..., A_n$ is a partition of sample space S, then*

$$P(A_1 \cup A_2 \cup .... \cup A_n) = P(A_1) + P(A_2) + ... + P(A_n) = P(S) = 1$$

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Theorem 0.2 *For three events A, B, and C, then*

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

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Theorem 0.3 *If A and $A^{'}$ are complementary events, then*

$$P(A) + P(A^{'}) = 1$$

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Exercises :

50. An experiment involves tossing a pair of dice, one green and one red, and recording the numbers that come up. If x equals the outcome on the green die and y the outcome on the red die, Assuming that all elements of S are equally likely to occur. Let A be the event A that the sum is greater than 8; B be the event that a 2 occurs on either die; and C be the event that a number greater than 4 comes up on the green die. Find

(a) the probability of event A.

(b) the probability of event C.

(c) the probability of event $A \cap C$.

Solution :

(a) $P(A) = \frac{5}{18}$.

(b) $P(C) = \frac{1}{3}$.

(c) $P(A \cap C) = \frac{7}{36}$.

53. The probability that an American industry will locate in Shanghai, China, is 0.7, the probability that it will locate in Beijing, China, is 0.4, and the probability that it will locate in either Shanghai or Beijing or both is 0.8. What is the probability that the industry will locate

(a) in both cities?

(b) in neither city?

Solution : Consider the events
S: industry will locate in Shanghai.
B: industry will locate in Beijing.

(a) $P(S \cap B) = P(S) + P(B) - P(S \cup B) = 0.7 + 0.4 - 0.8 = 0.3$.

(b) $P(S' \cap B') = 1 - P(S \cup B) = 1 - 0.8 = 0.2$

58. A pair of fair dice is tossed. Find the probability of getting

(a) a total of 8.

(b) at most a total of 5.

Solution :

(a) Here $|S| = 36$
Let A be the event of obtaining a total of 8 = {(2,6), (3,5), (4,4), (5,3), (6,2)}. i.e. $|A| = 5$
Hence the probability of obtaining a total of 8 is P(A)= $\frac{5}{36}$.

(b) Ten of the 36 elements total at most 5. Hence the probability of obtaining a total of at most is $\frac{10}{36} = \frac{5}{18}$.

59. In a poker hand consisting of 5 cards, find the probability of holding

(a) 3 aces.

(b) 4 hearts and 1 club.

Solution :

(a) $P(3\ aces) = \frac{\binom{4}{3}\binom{48}{2}}{\binom{52}{5}} = \frac{94}{54145}$

(b) $P(4hearts and 1club) = \frac{\binom{13}{4}\binom{13}{1}}{\binom{52}{5}} = \frac{143}{39984}$

65. Let A be the event that the component fails a particular test and B be the event that the component displays strain but does not actually fail. Event A occurs with probability 0.20, and event B occurs with probability 0.35.

(a) What is the probability that the component does not fail the test?

(b) What is the probability that the component works perfectly well (i.e., neither displays strain nor fails the test)?

(c) What is the probability that the component either fails or shows strain in the test?

Solution : P(A) = 0.2 and P(B) = 0.35
(a) $P(A') = 1 - 0.2 = 0.8$
(b) $P(A' \cap B') = 1 - P(A \cup B) = 1 - 0.2 - 0.35 = 0.45$
(c) $P(A \cup B) = 0.2 + 0.35 = 0.55$.

68. Interest centers around the nature of an oven purchased at a particular department store. It can be either a gas or an electric oven. Consider the decisions made by six distinct customers.

(a) Suppose that the probability is 0.40 that at most two of these individuals purchase an electric oven. What is the probability that at least three purchase the electric oven?

(b) Suppose it is known that the probability that all six purchase the electric oven is 0.007 while 0.104 is the probability that all six purchase the gas oven. What is the probability that at least one of each type is purchased?

Solution : (a) 1-0.40 = 0.60.
(b) The probability that all six purchasing the electric oven or all six purchasing the gas oven is 0.007 + 0.104 = 0.111.
So the probability that at least one of each type is purchased is 1-0.111 = 0.889.

72. Prove that

$$P(A^{'} \cap B^{'}) = 1 + P(A \cap B) - P(A) - P(B)$$

. **Solution :**
$P(A^{'} \cap B^{'})$
$= 1 - P(A \cup B)$
$= 1 - (P(A) + P(B) - P(A \cap B))$
$= 1 + P(A \cap B) - P(A) - P(B)$