P.S. QUIZ

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Date ______
Page ______

① Given X is a random variable having Poisson Distribution.

$$p(x, \mu) = \frac{e^{-\mu}\mu^{x}}{x!} \quad \text{for } x = 0, 1, 2$$

So, moment generating func of X is given by,

$$M_x(t) = E(e^{tx})$$

$$= \sum_0^\infty e^{tx} f(x)$$

$$= \sum_0^\infty e^{tx} \frac{e^{-\mu}\mu^{x}}{x!}$$

$$= e^{-\mu} \sum_0^\infty \frac{(\mu e^t)^x}{x!}$$

$$= e^{-\mu}\left(1 + \frac{\mu e^t}{1!} + \frac{(\mu e^t)^2}{2!} + \frac{(\mu e^t)^3}{3!} + \dots\right)$$

$$= e^{-\mu} e^{\mu e^t} = e^{\mu(e^t - 1)}$$

$$M_x(t) = e^{\mu(e^t - 1)}$$

$$\frac{dM_x(t)}{dt} = e^{\mu(e^t - 1)}\mu e^t$$

$$\frac{d^2 M_x(t)}{dt} = e^{\mu(e^t - 1)}(\mu e^t)^2 + e^{\mu(e^t - 1)}\mu e^t$$

Mean of random varible of X is

$$E(X) = \frac{dM_x(t)}{dt}\Big|_{t=0}$$

$$= e^{\mu(e^t-1)} \mu e^t \Big|_{t=0} = \mu$$

$$E(X^2) = \frac{d^2 M_x(t)}{dt}\Big|_{t=0} = \mu^2 + \mu$$

Variance of r.v. X is

$$V(X) = E(X^2) - (E(X))^2$$

$$= \mu^2 + \mu - \mu^2 = \mu$$

Ans 2 $\bar{x} = 11.69mg$

$$s^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}$$

$$= \frac{1}{n(n-1)} \left[n \sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i \right)^2 \right]$$

$$= 10.776mg.$$

Ans 3 We have to find $P(\mu_{\bar{x}} - 1.9\sigma_{\bar{x}} < \bar{X} < \mu_{\bar{x}} - 0.4\sigma_{\bar{x}})$

Substituting the value of $\mu_{\bar{x}} = 50$ and $\sigma_{\bar{x}} = 5/4$, then we have

$$P(\mu_{\bar{x}} - 1.9\sigma_{\bar{x}} < \bar{X} < \mu_{\bar{x}} - 0.4\sigma_{\bar{x}}) =$$
$$P(-1.9 < Z < -0.4) = P(Z < -0.4) - P(Z < -1.9)$$

Using standard normal tables we get the required probability $= 0.3446 - 0.0287$

~~Ans 4 (a) Using the~~ $= 0.3159$

Ans 4 (a) Using the table for $\alpha = 0.025$ and $v = 15$ yields.

$$\chi^2_{0.025} = 27.488$$

(b) Using the table for $\alpha = 0.01$ and $v = 7$ yields.

$$\chi^2_{0.01} = 18.475$$

(c) Using the table for $\alpha = 0.05$ and $v = 24$ yields.

$$\chi^2_{0.05} = 36.415$$

Ans 5. (a) greater than 9.1

Here we want to find
$P(S^2 > 9.1)$

First we ~~need~~ must change the probability to that involving the chi-squared distr. That is,

$$\chi^2 = \frac{(n-1)S^2}{\sigma^2}$$

has chi-squared distr with $v = (n-1)$.

Thus,

$$P(S^2 > 9.1) = P\left(\frac{(n-1)S^2}{\sigma^2} > \frac{(n-1)9.1}{\sigma^2}\right)$$

$$= P\left(\frac{24}{6}S^2 > \frac{24}{6}9.1\right)$$

$$= P(\chi^2 > 36.4)$$

Looking up 36.4 in the chi-squared table for $v = 24$ yields,

$$\chi^2_{0.05} = 36.415 \approx 36.4 \text{ for } v = 25$$

Thus, $P(S^2 > 9.1) = P(\chi^2 \geq 36.4) = 0.05$

(b) B/w 3.462 and 10.745

$$P(3.462 < S^2 < 10.745) = P(S^2 > 3.462) - P(S^2 > 10.745)$$

Date
Page

Now we change both probabilities to X^2 statistics using the technique from part a with $n-1 = 24$ and $\sigma^2 = 6$.

$$P(3.462 < S^2 < 36.4) = P(S^2 > 3.462) - P(S^2 > 36.4)$$

$$= P\left(\frac{24}{6}S^2 > \frac{24}{6}3.462\right) - P\left(\frac{24}{6}S^2 > \frac{24}{6}10.745\right)$$

$$= P(X^2 > 13.848) - P(X^2 > 42.980)$$

We look up these values in the chi-squared table for $v = 24$ to find

$$\chi^2_{0.95} = 13.848 \ \& \ \chi^2_{0.01} = 42.980$$
for $v = 24$.

$$\therefore P(3.462 < S^2 < 36.4) = 0.95 - 0.01$$
$$= 0.94$$

(6) $\bar{X} = 10.0$, $s = 0.283$, $t_{0.005} = 2.447$
at $v = n-1 = 6$ degree of freedom.

Hence, the $100(1-\alpha)\%$ C.I. is given by

$$10 - 2.447\left(\frac{0.283}{\sqrt{7}}\right) < \mu < 10.0 + 2.447\left(\frac{0.283}{\sqrt{7}}\right)$$

$$\Rightarrow 9.74 < \mu < 10.26$$

Ans 7. Observed Sample Variance,

$$S^2 = \sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{n-1}$$

$$= \frac{1}{n(n-1)} \left[n \sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i \right)^2 \right]$$

$$= 0.286$$

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