# MID SEMESTER EXAM, FEBRUARY -2018
# PROBABILITY AND STATISTICS (MTH 2002)

**Program: B.Tech.**
**Full Marks: 30**

**Semester: 4th**
**Time: 2 Hours**

| Subject/Course Learning Outcome | *Taxonomy Level | Ques. Nos. | Marks |
|---|---|---|---|
| Apply probability axioms to compute probability and conditional probability | L2,L3,L4, L4, | 1(c) 2(a,b,c) | 2*4 |
| Define random variables and compute probability distributions, joint & marginal distribution | L4,L4,L4, L4, L4, L4,L4 | 3(a), 3(b), 3(c), 4(a), 4(b), 4(c),5(c) | 2*7 |
| Compute expectation of random variables | L5,L4, | 5(a,b) | 2*2 |
| Estimate the variance | L4, L4 | 1(a,b) | 2*2 |

*Bloom's taxonomy levels: Knowledge (L1), Comprehension (L2), Application (L3), Analysis (L4), Evaluation (L5), Creation (L6)

**Answer all questions. Each question carries equal mark.**

1. (a) Compute the sample mean and sample standard deviation from the following data. 2

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 2.2 | 4.1 | 3.5 | 4.5 | 3.2 | 3.7 | 3.0 | 2.6 |
| 3.4 | 1.6 | 3.1 | 3.3 | 3.8 | 3.1 | 4.7 | 3.7 |
| 2.5 | 4.3 | 3.4 | 3.6 | 2.9 | 3.3 | 3.9 | 3.1 |
| 3.3 | 3.1 | 3.7 | 4.4 | 3.2 | 4.1 | 1.9 | 3.4 |
| 4.7 | 3.8 | 3.2 | 2.6 | 3.9 | 3.0 | 4.2 | 3.5 |

(b) Construct a relative frequency histogram of the data given in 1(a). 2

(c) Prove that $P(A' \cap B') = 1 + P(A \cap B) - P(A) - P(B)$ 2

2. (a) A random sample of 200 adults is classified below by sex and their level of education attained. If a person picked at random from this group, compute the probability that the 2

person is a male, given the person has a secondary education.

| Education | Male | Female |
|---|---|---|
| Elementary | 38 | 45 |
| Secondary | 28 | 50 |
| College | 22 | 17 |

(b) In a certain region of the country it is known from past experience that the probability of selecting an adult over 40 years of age with cancer is 0.05. If the probability of a doctor correctly diagnosing a person with cancer as having the disease is 0.78 and the probability of incorrectly diagnosing a person without cancer as having the disease is 0.06. Compute the probability that an adult over 40 years of age is diagnosed as having cancer. 2

(c) Referring to 2(b), compute the probability that a person diagnosed as having cancer actually has the disease. 2

3. (a) Determine the value $c$ so that the following function can serve as a probability distribution of the discrete random variable $X$: 2

$f(x) = c(x^2+4),\ x = 0,1,2,3$

(b) Suppose the random variable X having the density function 2

$$f(x) = \begin{cases} k(3-x^2), & -1 \le x \le 1 \\ 0 & , \text{Otherwise} \end{cases}$$

Evaluate **k.**

(c) Evaluate $P(|X| > 0.8)$ using the probability density function defined in 3(b). 2

4. (a) Consider the random variable X having the probability density 2

$$f(x) = \begin{cases} \frac{x^2}{3}, & -1 < x < 2 \\ 0 & , \text{Otherwise} \end{cases}$$

Compute cumulative distribution function F(x).

(b) The joint density function of the random variables X and Y is 2

$$f(x,y) = \begin{cases} 6x, & 0 < x < 1, 0 < y < 1-x \\ 0 & , \text{Otherwise} \end{cases}$$

Show that X and Y are not independent.

(c) Referring the above joint density function, calculate $P(X > 0.3 \mid Y = 0.5)$. 2

5. (a) Suppose that X and Y have the following joint probability function. Compute marginal distributions of X and Y. 2

| f(x,y) | | X | |
|---|---|---|---|
| | | 2 | 4 |
| Y | 1 | 0.10 | 0.15 |
| | 3 | 0.20 | 0.30 |
| | 5 | 0.10 | 0.15 |

Evaluate $\mu_x$ and $\mu_y$ for the above data.

(b) Compute $P(\mu - 2\sigma < X < \mu + 2\sigma)$, where X has density function $f(x) = \begin{cases} 6x(1-x), & 0<x<1 \\ 0 & , \text{otherwise} \end{cases}$ 2

(c) Compare the result of 5(b) with the probability bound computed by using Chebyshev's theorem. 2

*End of Questions*