PS Assignment - 1

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Q.1) A ~~two~~ town has two fire engines operating independently. The probability that a specific engine is available when needed is 0.96. compute the probability that neither is available when needed.

(Ans) Let A and B represent the availability of each fire engine.

$$P(A' \cap B') = P(A')P(B')$$

$$= [1-P(A)][1-P(B)] \quad [\text{As } P(A') = 1-P(A)]$$

$$= (1-0.96)(1-0.96) \quad [\text{As } P(A) = P(B) = 0.96]$$

$$= (0.04)(0.04)$$

$$= 0.0016$$

Q2) consider the density function,

$$f(x) = \begin{cases} k\sqrt{x}, & 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$$

a) evaluate k,

b) also find the cumulative distribution function $F(x)$.

(Ans) a)

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\Rightarrow \int_0^1 k\sqrt{x}\,dx = 1$$

$$\Rightarrow k\left[\frac{x^{3/2}}{3/2}\right]_0^1 = 1$$

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$$\Rightarrow \frac{2k}{3} = 1$$

$$\Rightarrow \boxed{k = \frac{3}{2}}$$

b) $$F(x) = \int_{-\infty}^{x} f(t)\,dt$$

$$= \int_{-\infty}^{0} 0 \cdot dt + \int_{0}^{x} \frac{3}{2}\sqrt{t}\,dt \qquad \left[\text{As } k = \frac{3}{2}\right]$$

$$= \int_{0}^{x} \frac{3}{2} t^{1/2}\,dt$$

$$= \frac{3}{2}\left[\frac{t^{3/2}}{3/2}\right]_0^x$$

$$= \left[t^{3/2}\right]_0^x = x^{3/2}$$

So, $$F(x) = \begin{cases} 0, & x < 0 \\ x^{3/2}, & 0 \leq x < 1 \\ 1, & x \geq 1 \end{cases}$$

Q3) The random variable X and Y have the following probability distribution.

$f(x,y)$		$x=1$	$x=2$	$x=3$
y	1	0.05	0.05	0.10
	2	0.05	0.10	0.35
	3	0.00	0.20	0.10

Evaluate marginal distribution of X and Y

(Ans) Marginal distribution of X :-

we know, $g(x) = \sum_y f(x,y)$

so, $g(1) = \sum_y f(1, y)$

$= f(1,1) + f(1,2) + f(1,3)$

$= 0.05 + 0.05 + 0.00$

$= 0.1$

$g(2) = \sum_y f(2, y)$

$= f(2,1) + f(2,2) + f(2,3)$

$= 0.05 + 0.10 + 0.20$

$= 0.35$

$g(3) = \sum_y f(3, y)$

$= f(3,1) + f(3,2) + f(3,3)$

$= 0.10 + 0.35 + 0.10$

$= 0.55$

Marginal distribution of Y :-

we know, $h(y) = \sum_y f(x, y)$

so, $h(1) = \sum_y f(x, 1)$

$= f(1,1) + f(2,1) + f(3,1)$

$= 0.05 + 0.05 + 0.10$

$= 0.20$

$h(2) = \sum_y f(x, 2)$

$= f(1,2) + f(2,2) + f(3,2)$

$= 0.05 + 0.10 + 0.35$

$= 0.50$

$h(5) = \sum_y f(x, 3)$

$= f(1,3) + f(2,3) + f(3,3) = 0.00 + 0.20 + 0.10$

$= 0.30$

Q4) Let X be a random variable with density function

$$f(x) = \begin{cases} 1/3 , & -1 < x < 2 \\ 0 , & \text{elsewhere} \end{cases}$$

Find the expected value of $g(X) = 4X + 3$

Sol:- since, X is a continuous random variable, we will use the formula

$$\mu_g(X) = E(g(X)) = \int_{\infty}^{\infty} g(x) f(x)\, dx$$

Now,

$$\mu_g(x) = E(g(X))$$

$$= \int_{-\infty}^{-1} g(x)\,(0)\,dx + \int_{-1}^{2} \left(\tfrac{1}{3}\right) g(x)\,dx + \int_{2}^{\infty} g(x)\,(0)\,dx$$

$$= 0 + \int_{-1}^{2} \frac{1}{3}(4x+3) + 0$$

$$= \frac{1}{3}\left[\frac{4x^2}{2} + 3x\right]_{-1}^{2}$$

$$= \frac{1}{3}\left[2x^2 + 3x\right]_{-1}^{2}$$

$$= \frac{1}{3}\left[14 - (-1)\right] = \frac{1}{3} \times 15 = 5$$

Here the expected value of $g(X) = 4X + 3$, is 5.

Q5) In a certain assembly plant, three machines, B_1, B_2 & B_3 make 30%, 45% and 25% respectively of the products. It is known from past experience that 2%, 3% and 2% of the products made by each machine, respectively, are defective. Now, suppose that a finished product is randomly selected. What is the probability that it is defective?

(Ans) A : product is defective

B_1 : product is made by machine B_1

B_2 : product is made by machine B_2

B_3 : product is made by machine B_3

ATQ, $P(A) = P(B_1)P(A|B_1) + P(B_2)P(A|B_2) + P(B_3)P(A|B_3)$

$= (0.3)(0.002) + (0.45)(0.03) + (0.25)(0.02)$

$= 0.006 + 0.0135 + 0.005$

$= 0.0245$