LECTURE - 10

CHEPTER-4

4.1 Mean of random variable

If two coins are tossed 16 times and X is the number of heads that occur per toss, then the values of X are 0, 1, and 2. Suppose that the experiment yields no heads, one head, and two heads a total of 4, 7, and 5 times, respectively. The average number of heads per toss of the two coins is then

$$\frac{(0)(4)+(1)(7)+(2)(5)}{16} = 1.06$$

This can be written as

$$(0)(\tfrac{4}{16}) + (1)((\tfrac{7}{16}) + (2)((\tfrac{5}{16}) = 1.06$$

Here $\frac{4}{16}, \frac{7}{16}, and \frac{5}{16}$ are probabilities of getting 0 head, one head, two heads in tossing of two coin respectively . This average value is the mean of the random variable X or the mean ofthe probability distribution of X and write it as μ_x or simply as μ.

It is also common among statisticians to refer to this mean as the mathematical expectation, or the expected value of the random variable X, and denote it as E(X).

Definition 4.1

Mathematical Expectation E(X)

Let X be a random variable with probability distribution f(x). The mean, or expected value of X is

$$\mu = \mathrm{E(X)} = \sum_x xf(x)$$

if X is discrete

$$\int_{-\infty}^{\infty} xf(x)dx$$

if X is continuous.

Example-4.1

A lot containing 7 components is sampled by a quality inspector; the lot contains 4 good components and 3 defective components. A sample of 3 is taken by the inspector. Find the expected value of the number of good components in this sample.

Solution : Let X represent the number of good components in the sample. The probability distribution of X is

$$f(x) = \frac{\binom{4}{x}\binom{3}{3-x}}{\binom{7}{3}}, \text{ x=0,1,2,3}$$

So $f(0) = \frac{1}{35}, f(1) = \frac{12}{35}, f(2) = \frac{18}{35} and\ f(3) = \frac{4}{35}$

Therefore $E(X) = (0)(\frac{1}{35}) + (1)(\frac{12}{35}) + (2)(\frac{18}{35}) + (3)(\frac{4}{35}) = \frac{12}{7}$

Exercise-4.4
A coin is biased such that a head is three times as likely to occur as a tail. Find the expected number of tails when this coin is tossed twice.

Solution: Let X denotes the number of tails. So X takes the values 0,1,2. Here a head is three times as likely to occur as a tail.
So p(H)=$\frac{3}{4}$ and p(T)=$\frac{1}{4}$.

Now f(0)=$\frac{9}{16}$, f(1)=$\frac{6}{16}$ and f(2)=$\frac{1}{16}$.

Therefore E(X)=(0)($\frac{9}{16}$)+(1)($\frac{6}{16}$)+(2)($\frac{1}{16}$)=$\frac{1}{2}$

Exercise-4.7
By investing in a particular stock, a person can make a profit in one year of \$4000 with probability 0.3 or take a loss of \$1000 with probability 0.7. What is this person's expected gain?
solution:
Let the profit variable is X

The person's expected gain
$E(X) = \sum_x xf(x) = (4000)(0.3) + (1000)(0.7) = \$1900$

Theorem-4.1
Let X be a random variable with probability distribution f(x). The expected value of the random variable g(X) is

$$\mu_g(X) = \text{E[g(X)]} = \sum_x g(x)f(x)$$

if X is discrete, and

$$\mu_g(X) = \text{E[g(X)]} = \int_{-\infty}^{\infty} g(x)f(x)dx$$

if X is continuous.

Example-4.4
Suppose that the number of cars X that pass through a car wash between

4:00 P.M. and 5:00 P.M. on any sunny Friday has the following probability distribution:

X	4	5	6	7	8	9
P(X=x)	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{6}$

Let g(X)=2X−1 represent the amount of money, in dollars, paid to the attendant by the manager. Find the attendant's expected earnings for this particular time period.

Solution : The attendant can expect to receive
E(g(X))=E(2X-1)=$\sum_{x=4}^{9}(2x-1)f(x)$

$$=(7)(\tfrac{1}{12} + (9)(\tfrac{1}{12}) + (11)(\tfrac{1}{4}) + (13)(\tfrac{1}{4}) + (15)(\tfrac{1}{6}) + (17)(\tfrac{1}{6})$$

$$= \$12.67$$

Exercise-4.12

If a dealer's profit, in units of $5000, on a newautomobile can be looked upon as a random variable X having the density function

$$\text{f(x)=}\begin{cases} 2(1-x) & 0 \le x \le 1 \\ 0 & elsewhere \end{cases}$$

Find the average profit per automobile.

Solution : E(X)=$\int_0^1 xf(x)dx = x^2 - \frac{2x^3}{3}\Big|_0^1 = \frac{1}{3}$

The average profit per automobile($\frac{1}{3}$)(5000)=$ $\frac{5000}{3}$

Definition 4.2

Let X and Y be random variables with joint probability distribution f(x, y). The mean, or expected value, of the random variable g(X, Y) is

$$\mu_{g(X,Y)} = E[g(X,Y)] = \sum_x \sum_y g(x,y)f(x,y)$$

if X and Y are discrete and

$$\mu_{g(X,Y)} = E[g(X,Y)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y)f(x,y)dxdy$$

if X and Y are continuous

Exercise-4.10

Two tire-quality experts examine stacks of tires and assign a quality rating to each tire on a 3-point scale. Let X denote the rating given by expert A and Y denote the rating given by B. The following table gives the joint distribution for X and Y .

	f(x,y)	y 1	2	3
	1	0.10	0.05	0.02
x	2	0.10	0.35	0.05
	3	0.10	0.35	0.05

Solution :

	f(x,y)	y 1	2	3	row total g(x)
	1	0.10	0.05	0.02	0.17
x	2	0.10	0.35	0.05	0.50
	3	0.03	0.10	0.20	0.33
column total	h(y)	0.23	0.50	0.27	1

$\mu_X = \sum_x xg(x) = (1)(0.17) + (2)(0.50) + (3)(0.33) = 2.16$
$\mu_Y = \sum_y yh(y) = (1)(0.23) + (2)(0.50) + (3)(0.27) = 2.04$

Exercise-4.20
A continuous random variable X has the density function

$$f(x)=\begin{cases} e^{-x} & x > 1 \\ 0 & elsewhere \end{cases}$$

Find the expected value of g(X) $= e^{\frac{2X}{3}}$
Solution :
E(g(X))$=\int_1^\infty g(x)f(x)dx$

$$= \int_1^\infty (e^{\frac{2x}{3}})(e^{-x})dx = \int_1^\infty e^{\frac{-x}{3}} dx = -3(e^{\frac{-x}{3}})\Big|_1^\infty$$

$$= 3e^{\frac{-1}{3}}$$

Exercise-4.23
Suppose that X and Y have the following joint probability function:

	f(x,y)	x 2	4
	1	0.10	0.15
y	3	0.20	0.30
	5	0.10	0.15

(a) Find the expected value of g(X, Y) = XY^2.
(b) Find μ_X and μ_Y.
Solution :

	f(x,y)	x		Row total h(y)
		2	4	
	1	0.10	0.15	0.25
y	3	0.20	0.30	050
	5	0.10	0.15	0.25
Column total	g(x)	0.40	0.60	1

(a) E(g(X,Y))=$\sum_x \sum_y g(x,y)f(x,y) = \sum_x \sum_y xy^2 f(x,y)$

=(2)(1)(0.10)+(4)(1)(0.15)+(2)(9)(.20)+(4)(9)(0.30)+(2)(25)(0.10)

+(4)(25)(0.15)

=0.20+0.60+3.60+10.80+5.00+15.00=35.20

(b)

$(\mu_X = \sum_x xg(x) = (2)(0.40) + (4)(0.60) = 3.20$

$\mu_Y = \sum_y yh(y) = (1)(0.25) + (3)(0.50) + (5)(0.25) = 3$

*****Completed*****