



# CSE 3131: ALGORITHM DESIGN 1

## ASSIGNMENT 5:

Submission due date: 19/01/2023

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- Assignment scores/markings depend on neatness and clarity.
  - Write your answers with enough detail about your approach and concepts used, so that the grader will be able to understand it easily. You should ALWAYS prove the correctness of your algorithms either directly or by referring to a proof in the book.
  - The marking would be out of 100.
  - You are allowed to use only those concepts which are covered in the lecture class till date.
  - Plagiarized assignments will be given a zero mark.
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CO5: to understand various types and aspects of *Divide and Conquer*.

### Divide and Conquer:

Merge sort  
 Counting Inversions  
 Quick Sort  
 Closest pair of points  
 Fast Integer Multiplication  
 Polynomial and FFT

Sl.No.	Question	PO	Level
1.	Write the pseudocode for a partition algorithm that returns the median of $n$ numbers in the given array as the pivot element for sorting the given array in descending order. Compare the worst-case time and space (input, auxiliary and stack spaces) complexity of the QUICKSORT algorithm with this new partition procedure to that of the traditional implementation of QUICKSORT with the partition procedure that selects the last element as the pivot. Sort the array $A = \{8, 26, 7, 18, 6, 12, 2, 5, 22, 10\}$ using this new method. Count the number of swapping and comparison operations carried out in the sorting process.	PO2, PO3,PO4	L3, L5
2.	Assuming an equal probability for the size of each of the sub-problems produced by the PARTITION procedure, derive the average-case time complexity for QUICKSORT algorithm. Compute the worst-case space complexity. Though the worst case time complexity of QUICKSORT is same as that of the Bubble, Selection and Insertion sort, why it is considered a superior algorithm for sorting large input instances as compared to the other three?	PO2, PO3,PO4	L4
3.	Modify the MERGESORT algorithm to sort the given list of numbers in descending order instead of ascending order. Using this modified algorithm, sort the given array $A = \{18, 6, 11, 5, 16, 10, 21, 15, 12, 8\}$ showing each of the intermediate steps. Discuss the amount of input space, auxiliary space and environmental stack space used by the MERGESORT algorithm.	PO2, PO3,PO4	L3
4.	Three friends A, B and C are participating in an online competition where the participants have to arrange a set of products in increasing order of their personal preferences. The person whose preference list will have the highest degree of dissimilarity with the preference lists of any of the other participants will win the prize. If the preference lists of the three friends A, B and C are given as follows, who has a chance to win the prize? Give an $O(n \lg n)$ algorithm to solve the problem. Show the intermediate steps of the solution.	PO2, PO3,PO4	L2, L4,L5

	<table><tr><td></td><td><math>P_1</math></td><td><math>P_2</math></td><td><math>P_3</math></td><td><math>P_4</math></td><td><math>P_5</math></td><td><math>P_6</math></td><td><math>P_7</math></td><td><math>P_8</math></td><td><math>P_9</math></td><td><math>P_{10}</math></td><td><math>P_{11}</math></td><td><math>P_{12}</math></td></tr><tr><td>A</td><td>7</td><td>10</td><td>12</td><td>8</td><td>2</td><td>1</td><td>6</td><td>3</td><td>5</td><td>4</td><td>11</td><td>9</td></tr><tr><td>B</td><td>3</td><td>1</td><td>7</td><td>11</td><td>8</td><td>2</td><td>6</td><td>9</td><td>5</td><td>12</td><td>4</td><td>10</td></tr><tr><td>C</td><td>6</td><td>2</td><td>4</td><td>11</td><td>10</td><td>12</td><td>3</td><td>8</td><td>1</td><td>5</td><td>9</td><td>7</td></tr></table>		$P_1$	$P_2$	$P_3$	$P_4$	$P_5$	$P_6$	$P_7$	$P_8$	$P_9$	$P_{10}$	$P_{11}$	$P_{12}$	A	7	10	12	8	2	1	6	3	5	4	11	9	B	3	1	7	11	8	2	6	9	5	12	4	10	C	6	2	4	11	10	12	3	8	1	5	9	7		
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5.	<p>Formulate the recurrence relation for the divide and conquer solution for Closest pair of points problem using:</p> <ul style="list-style-type: none"><li>i. Without using MERGE operation to sort the points occurring in the <math>2\delta</math>-strip.</li><li>ii. Using MERGE operation to sort the points occurring in the <math>2\delta</math>-strip.</li></ul> <p>Solve the two recurrences using recursion tree method.</p>	PO2, PO3,PO4	L3																																																				
6.	<p>Given two integers <math>a</math> and <math>b</math> in their binary representations as <math>a = 10110101</math>, <math>b = 11001100</math>. Find the product <math>a*b</math> using Karatsuba algorithm for integer multiplication showing each of the steps of division and combining the intermediate solutions. Formulate the recurrence relation for the algorithm and compute the value of time complexity. Discuss the amount of input space, auxiliary space and environmental stack space required by the algorithm.</p>	PO2, PO3,PO4	L5																																																				
7.	<p>Given two polynomials, <math>A(x) = 5x^3 + 4x^2 + 3x + 6</math> and <math>B(x) = \{(1, 3), (2, 29), (3, 103), (4, 249)\}</math> one in co-efficient representation and the other in sample representation.</p> <ul style="list-style-type: none"><li>i. Convert the coefficient representation of <math>A(x)</math> using Discrete Fourier Transform to perform a polynomial multiplication <math>A(x) \cdot B(x)</math>.</li><li>ii. Convert the point value representation of <math>B(x)</math> using Inverse Discrete Fourier Transform to evaluate <math>B(10)</math>.</li></ul>	PO2, PO3,PO4	L2, L4,L5																																																				
8.	<p>Multiply two polynomials <math>P(x)</math> and <math>Q(x)</math> which are evaluated at <math>x = 10</math> as two integer number <math>P(10) = 21034</math> and <math>Q(10) = 352</math> in <math>O(n \lg n)</math> using the Fast Fourier Transform. Show each of the steps of division and combining the intermediate solutions. Find the convolution of the co-efficient vectors of <math>P(x)</math> and <math>Q(x)</math> and verify that the result is same as that of the multiplication operation. (Hint: Convert <math>P</math> and <math>Q</math> into two polynomial functions, using the formula: If <math>D(10) = d_3d_2d_1d_0</math> where <math>d_i</math> is a digit in the decimal number <math>D(10)</math>, then <math>D(x) = d_0 \cdot x^0 + d_1 \cdot x^1 + d_2 \cdot x^2 + d_3 \cdot x^3</math>.)</p>	PO2, PO3,PO4	L2,L5																																																				

### Submission and Grading:

Submit the hard copy of your assignment by the due date, i.e. 19.01.2023.

Part of your assignment grade comes from its "external correctness." This is based on correct output on various sample inputs.

The rest of your assignment's score comes from "internal correctness." Internal correctness includes:

1. Use of methods to minimize the number of steps.
2. Appropriate use of rules, axioms, and suitable diagrams to enhance readability of your responses.

Send a zip folder (name of the zip folder must be your registration number\_AD1) containing the code and output file/screenshot of each program implementation mentioned to the official email id of your AD1 class teacher. On the top of each program, you must mention your full name, registration number, title of the program and date.