LECTURE-32 and 33

ONE AND TWO-SAMPLE TESTS OF HYPOTHESES

General Concepts(CH-10.1, 10.2, 10.3)

Hypothesis: A statement regarding a parameter of a distribution is called hypothesis.

Statistical Hypothesis: A statistical hypothesis is an assertion in one or more population.

Null Hypothesis: A hypothesis whose truth value is tested is a null hypothesis.

Alternative Hypothesis: A hypothesis which is true when the null hypothesis is rejected i.e any hypothesis which is complimentary to the null hypothesis is called an alternative hypothesis. It is denoted by H_1 i.e $H = H_1$

Eample:- Mean height of all students is $\mu = 5'9''$ not accepted.

The hypothesis is tested using sample.

Then $\mu \neq 5'9''$ or $\mu < 5'9''$ or $\mu > 5'9''$

There are two hypothesis

(i) Null hypothesis (H_0)

(ii) Alternative hypothesis (H_1)

Null hypothesis is formed for rejection.

Once the hypothesis are formed, test statistics are raised to test the hypothesis.

(H_0) accepted or rejected $\Rightarrow$ (H_1) rejected or accepted.

Critical Value(Significant Value): The value of the test static which separate the rejection region and acceptance region is called critical value. It is denoted by C.

Types of Test:

(1) $H \neq H_0$ (Two-sided test/Two tail test)

(2) $H > H_0$ (Right-sided test/One-sided test)

(3) $H < H_0$ (Left-sided test/One-sided test)

Left Sided Test (L.S.T.): Suppose we want 35 kg. wheat.

If it is >35 ⇒ accept

<35 ⇒ reject

but if it is 34.99kg. then we can accept, like this, up to what limit we can tolerate. See figure:-1

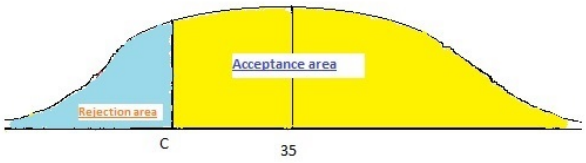

Figure 1: Left Sided Test

Where C is the critical point i.e tolerance limit.

Right Sided Test (R.S.T.):

Eample:- Suppose our budget is 10,000 i.e Null Hypothesis(NH)

If it is >10,000 ⇒ reject

<10,000 ⇒ accept. See figure:-2

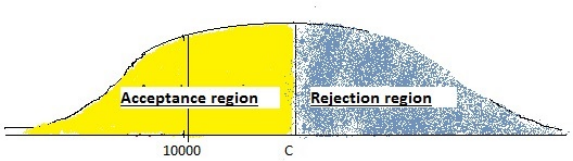

Figure 2: Right Sided Test

But if it is 10,005/10,010/10,015 ⇒ accept.

i.e upto a tolerance limit we can accept but after that we cann't tolerate.

i.e C lies in the right side.

Two Sided Test (T.S.T.):

Eample:- Suppose one pin has to fit a hole (*depends on diameter*). If the diameter of pin is very much more or very less, then we reject but small deviation is acceptable.

Eample:-Suppose your shirt size is 40, then 44 size or 36 size not manageable. But 41 or 39 is manageable, if $C_1 < 41, 39 < C_2$ i.e shirt size is manageable if $C_1 < shirtsize < C_2$. Otherwise rejected. See figure:-3

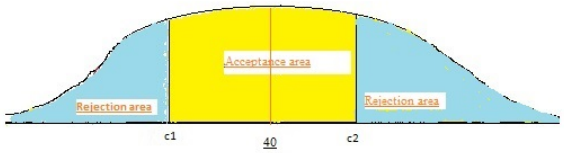

Figure 3: Two Sided Test

From previous chapter we got an idea that sample result will give the population result.

Whether it is perfect or not we are testing i.e testing of hypothesis.

i.e. we have to take the decision whether we accept or reject.

Testing of Hypothesis:

Procedure:

(i) Set up a null hypothesis i.e $H = H_0$ which is to be tested.

(ii) Set up an alternative hypothesis i.e $H = H_1$ against null hypothesis.

i.e $H \neq H_0$ or $H > H_0$ or $H < H_0$.

(iii) Choose a significance level $\alpha(5\ \%\ , 1\ \%\ , .1\ \%\ , ...)$.

(iv) Use an appropriate random variable and determine the observed value i.e $\hat{H}$.

(v) Find the tabular value (critical value) i.e C from the corresponding table with given level of significance depends on alternative hypothesis.

Significance level α: The probability of the value of the variate falling in the rejection region.

i.e the percentage of tolerance limit of error.

Table 1: Decision Table

	H_0 is true	H_1 is true
Do not reject H_0	Correct decision	Type II error
Reject H_0	Type I error	Correct decision

Error:

Type-I error: When a null hypothesis is true but we reject.

P(reject null hypothesis when it is true)=α

Type-II error: When a null hypothesis is false but we accept it.

P(accept a null hypothesis when it is false)=β

Book Questions

10.2 A sociologist is concerned about the effectiveness of a training course designed to get more drivers to use seat belts in automobiles.

(a) What hypothesis is she testing if she commits a type I error by erroneously concluding that the training course is ineffective?

Ans: (a) The training course is effective.

(b) What hypothesis is she testing if she commits a type II error by erroneously concluding that the training course is effective?

Ans: (b) The training course is effective.

10.3 A large manufacturing firm is being charged with discrimination in its hiring practices.

(a) What hypothesis is being tested if a jury commits a type I error by finding the firm guilty?

Ans: (a) The firm is not guilty.

(b) What hypothesis is being tested if a jury commits a type II error by finding the firm guilty?

Ans: (b) The firm is guilty.

10.4 A fabric manufacturer believes that the proportion of orders for raw material arriving late is p = 0.6. If a random sample of 10 orders shows that 3 or fewer arrived late, the hypothesis that p = 0.6 should be rejected in favor of the alternative p ¡ 0.6. Use the binomial distribution.

(a) Find the probability of committing a type I error if the true proportion is p = 0.6.

Ans: (a) $\alpha = P(X \leq 5|(p = 0.6)) + P(X \geq 13|(p = 0.6)) = 0.0338 + (1 - 0.9729) = 0.0609$

(b) Find the probability of committing a type II error for the alternatives p = 0.3, p = 0.4,

and p = 0.5.

Ans: (a) $\beta = P(6 \leq X \leq 12|(p = 0.5)) = 0.9963 - 0.1509 = 0.8454$

$\beta = P(6 \leq X \leq 12|(p = 0.7)) = 0.8732 - 0.0037 = 0.8695$

Chapter-10.3:

Power of Test: $\eta = 1 - \beta$ is called Power of Test (It is the probability of rejection of null hypothesis given that a specific alternative hypothesis is true).

Some Results from Confidence Interval:

Standardized R.V.

$$Z = \frac{X - \mu}{\sigma} \tag{1}$$

Case-I:The R.V.

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \tag{2}$$

has a normal distribution.

Case-II:The R.V.

$$T = \frac{\bar{X} - \mu}{\frac{s}{\sqrt{n}}} \tag{3}$$

has a t-distribution with $n - 1$ degrees of freedom.

Case-III:The R.V.

$$Y = (n - 1)\frac{S^2}{\sigma^2} \tag{4}$$

has a Chi-square distribution with $n - 1$ degrees of freedom.