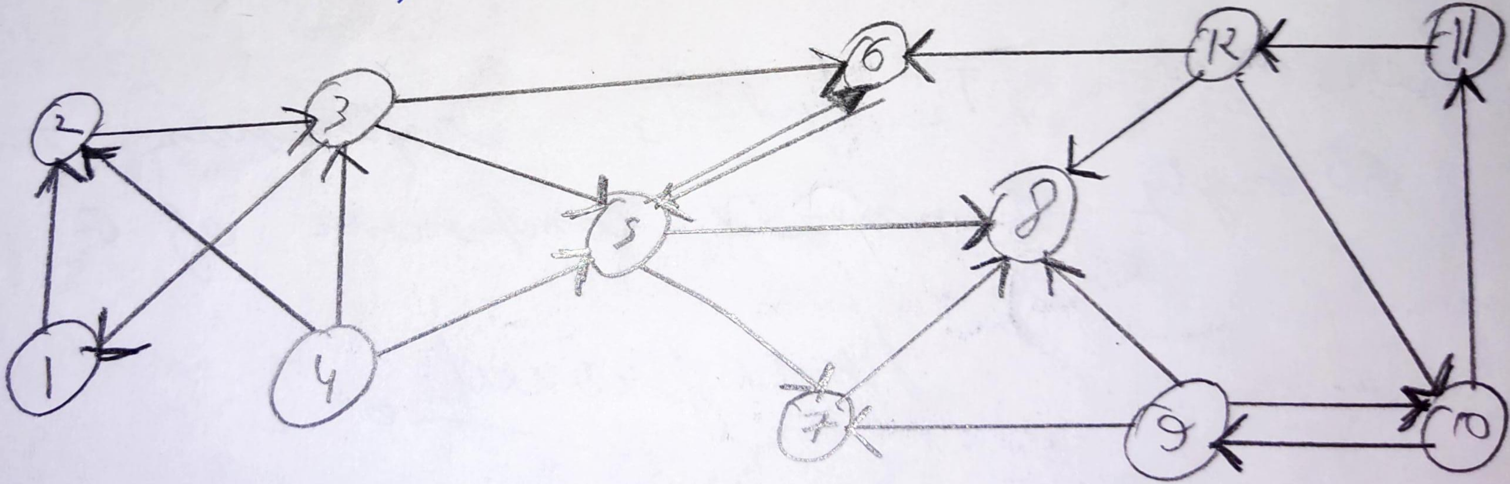
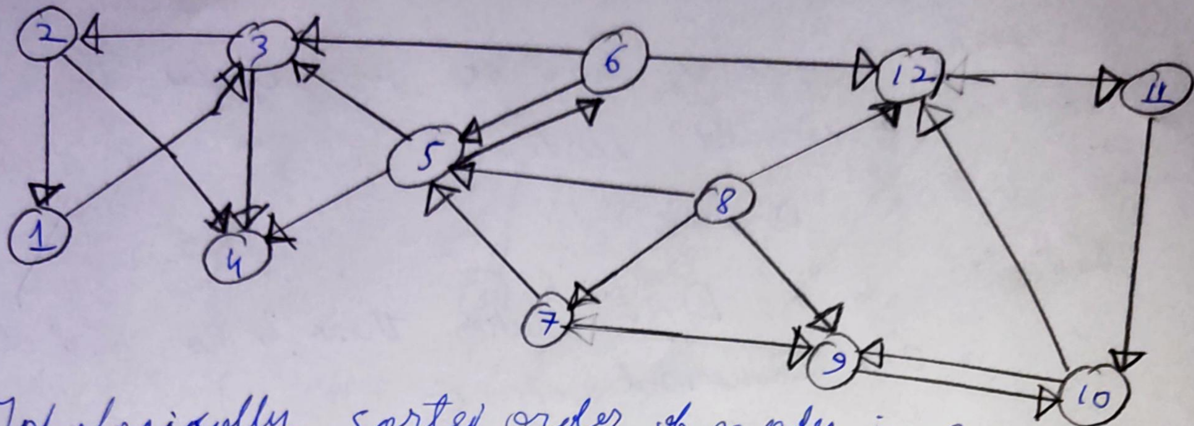
(7)



G

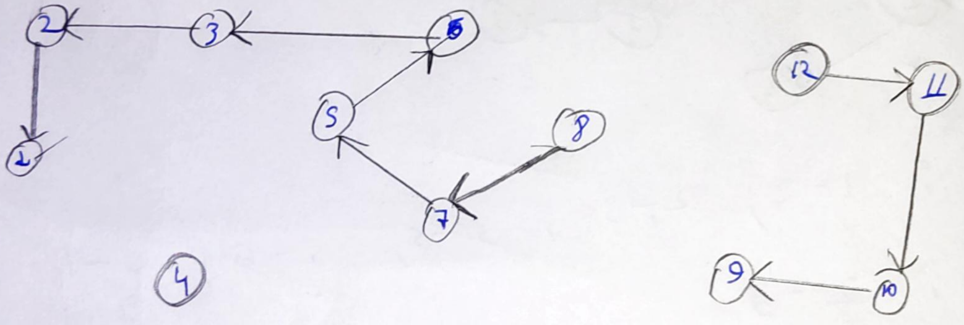
Tronspose of G is



Topologically sorted order of nodes in G

1 → 2 → 3 → 6 → 5 → 7 → 8 → 9 → 10 → 11 → 12 → 4

let this ~~the~~ be the order of elements in A

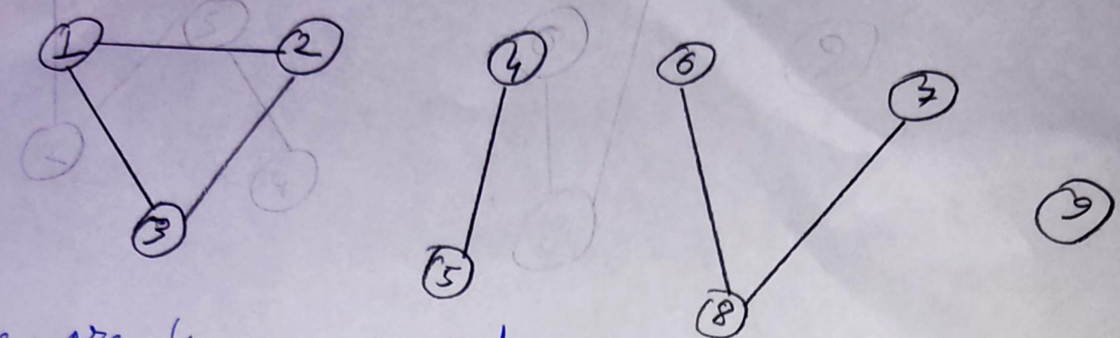


let there be T_1, T_2 and T_3 as the three trees

here T_1, T_2 and T_3 represents the strongly connected components of G

hence proved.

(8) From the given adjacency matrix graph G is



There are 4 connected components

Edges = [[1,2],[2,3],[1,3],[4,5] [6,8],[7,8]]

array = [1 , 2 , 3 , 4 , 5 , 6 , 7 , 8 , 9]
1 2 3 4 5 6 7 8 9

Edge 1-2 :

array = [2 , 2 , 3 , 4 , 5 , 6 , 7 , 8 , 9]
1 2 3 4 5 6 7 8 9

Edge 2-3 :

array = [2 , 3 , 3 , 4 , 5 , 6 , 7 , 8 , 9]
1 2 3 4 5 6 7 8 9

Edge 1-3 :

array = [2 , 3 , 3 , 4 , 5 , 6 , 7 , 8 , 9]
1 2 2 4 5 6 7 8 9

Edge 4-5 :

array = [2 , 3 , 3 , 5 , 5 , 6 , 7 , 8 , 9]
1 2 3 4 5 6 7 8 9

edge 6-8 :

array = [2 , 3 , 3 , 5 , 5 , 8 , 7 , 8 , 9]
1 2 3 4 5 6 7 8 9

Edge 7-8 :

array = [2 , 3 , 3 , 5 , 5 , 8 , 8 , 8 , 9]
1 2 3 4 5 6 7 8 9

unique parents = [3, 5, 8, 9]

$\therefore$ our Graph G has 4 components.

(9) i) let Adj [1 .. |V|] be a new adjacency list of the transposed G^T

for each vertex $u \in G.V$
  for each vertex $v \in Adj[u]$
    Insert (Adj'[v], u)

Time Complexity : $O(|E| + |V|)$

2)

(10) $BB^T(i,j) = \sum_{e \in E} b_{ie} b^T_{ej}$

$= \sum_{e \in E} b_{ie} b_{je}$

if $i = j$

then $b_{ie} b_{je} = 1$ whenever i enters or leaves vertex i, and 0 otherwise

if $i \neq j$

then $b_{ie} b_{je} = -1$ when $e = (i, j)$ or $e = (j, i)$ or 0 otherwise

Thus

$$BB^T(i,j) = \begin{cases} \text{indegree}(i) + \text{outdegree}(i) & \text{if } i = j \\ -(\text{nb of edges connecting } i \text{ and } j) & \text{if } i \neq j \end{cases}$$