## Lecture-15

## Multinomial Distribution

**Multinomial distribution:**

Consider a trial which results k outcomes, $E_1$, $E_2$, ....$E_k$ with probabilitites $p_1$, $p_2$, ...., $p_k$ respectively such that

$$\sum_{i=1}^{k} p_i = 1$$

Let

$X_1$ = no. of times $E_1$ occurs in n independent trials

$X_2$ = no. of times $E_2$ occurs in n independent trials

. . . . . . ..

$X_k$ = no. of times $E_k$ occurs in n independent trials

Now,

$$P(X_1 = x_1, X_2 = x_2, ........X_k = x_k) = f(x_1, x_2, ....x_k; p_1, p_2, .....p_k)$$

$$= \begin{pmatrix} n \\ x_1, x_2, ....x_k \end{pmatrix} p_1{}^{x_1} p_2{}^{x_2} ..... p_k{}^{x_k}$$

With $\sum_{i=1}^{n} x_i = n$, $\quad \sum_{i=1}^{n} p_i = 1$

**(Q.19)** As a student drives to school, he encounters a trafic signal. This trafic signal stays green for 35 seconds, yellow for 5 seconds, and red for 60 seconds. Assume that the student goes to school each weekday between 8:00 and 8:30 a.m. Let $X_1$ be the number of times he encounters a green light, $X_2$ be the number of times he encounters a yellow light, and $X_3$ be the number of times he encounters a red light. Find the joint distribution of $X_1$, $X_2$, and $X_3$

**Ans:**

Let

$X_1$ = no. of times he encounters a green light

$X_2$ = no. of times he encounters a yellow light

$X_3$ = no. of times he encounters a red light

Given $p_1 = 0.35, p_2 = 0.05, p_3 = 0.60$

Therefore,

$$P(X_1 = x_1, X_2 = x_2, X_3 = x_3) = f(x_1, x_2, .x_3; n, 0.35, 0.05, 0.60)$$
$$= \begin{pmatrix} n \\ x_1, x_2, ....x_k \end{pmatrix} (0.35)^{x_1} (0.05)^{x_2} (0.60)^{x_3}$$

Where $x_1 + x_2 + x_3 = n$

**(Q.22)** According to a genetics theory, a certain cross of guinea pigs will result in red, black, and white ofspring in the ratio 8:4:4. Find the probability that among 8 ofspring, 5 will be red, 2 black, and 1 white.

**Ans:**

Let

$X_1$ = no. of red guinea pigs

$X_2$ = no. of black guinea pigs

$X_3$ = no. of white guinea pigs

It is given that the ratio of red, black, and white guinea pigs is 8:4:4

Hence,

P (guinea pig is red) = 8/16 =0.5

P (guinea pig is black) = 4/16 =0.25

P (guinea pig is white) = 4/16 =0.25

$$P(X_1 = 5, X_2 = 2, X_3 = 1) = f(5, 2, .1; 8, 0.5, 0.25, 0.25)$$
$$= \begin{pmatrix} 8 \\ 5, 2, 1 \end{pmatrix} (0.5)^5 (0.25)^2 (0.25)^1$$
$$= 21/256$$