Assignment - 5

1) partition (A, low, high){

```
pivot = A[high];
i = low - 1;
for(j = low → high - 1; j++){
  if(A[j] < pivot){
    i++;
    swap(A[i], A[j]);
  }
}
swap(A[i+1], A[high])
return(i+1);
}
```

Time complexity = $O(n^2)$

Space complexity = $O(\log n)$

2) $T(n) = T(\frac{n}{9}) + T(9n/10) + O(n)$

$T(n) = T(n/9) + T(9n/10) + n$

$T(n) = T(n/81) + 9(n/10) + n/9 + n$

$T(n) = (n/9^3) + T(n/9) + n/9^2 + n/9 + n$

$\vdots$

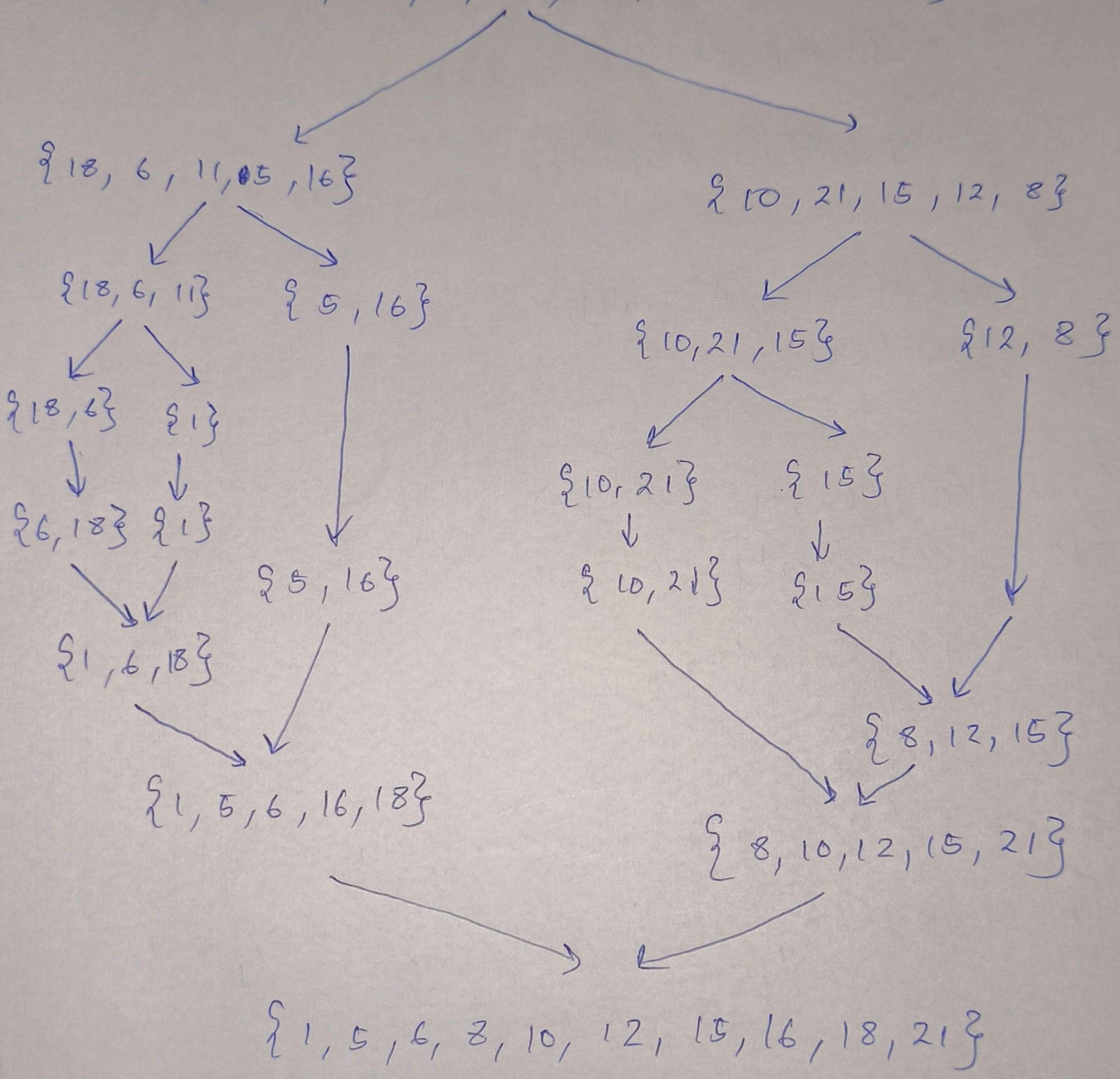
$T(n) = (n/9^k) + T\left(\frac{n}{9^{k-2} \times 10}\right) + \frac{n}{9^{k-1}} + \dots + n/9^2 + n/9 + n$

$T(1) = T(n/9^k)$

$K = \log_9 n$

In Quick sort pivot can be changed & implemented in different ways & since its inner loop can be implemented in many architectural structures

3) A = {18, 6, 11, 5, 16, 10, 21, 15, 12, 8}



4) Algorithm :-

(i) Find median using partition, we can take $p(n/2)$ as the one

(ii) Divide the array into 2 halves

(iii) Recursively find the smallest distance

(iv) Return the mind between the two point out of these

(v) The third point would be having more chance

As per the given data, C has a chance to win the price

5) (i) $T(n) = T(n-1) + n$

$T(n-1) = T(n-2) + n \ldots T(n-n) + T(n-n+1)$

$T(n) = T(n-2) + n + n$

$T(n) = T(n-n) + n + n + n + \ldots n = n(n)$

$= T(0) + n^2$

$= O(n^2)$

(ii) $T(n) = T(n/2) + n$

$T(n/2) = T(n/2^2) + n/2 \ldots T(n/2^k)$

$= T(n/2^{k+1}) + n/2^k$

$T(n) = T(n/2) + n$

$= T(n/2^2) + n/2 + n$

$T(n) = T(n/n) + n + n/2 + n/4 \cdots n/2^n$

$T(n) = 1 + n\left(\frac{1}{1-1/2}\right) = 1 + \frac{n}{2} = O(\log n)$

6)

$X = 10110101$

$Y = 11001100$

$XY = (X(L) * 2^{n/2} + X_R)(Y(L) * 2^{n/2} + Y_R)$

$= (1 * (10000) + 1)(1 * (10000) + 0)$

$= 2^8(1) + 2^4(0+1) + 0$

$= 100000000 + 10000$

$= 100010000$

$T(n) = 3T(n/2) + n$

$T(n/2) = 3T(n/2) + n$

$T(n) = 3^2 T(n/2^2) + (3n/2) + 3n$

$= 3^n T(1) + 3^{n-1}\frac{n}{2^n} + \ldots + \frac{3n}{2}$ *

$= 1 + 3n\frac{\left(\left(\frac{3}{2}\right)^n - 1\right)}{1/2}$

$= 1 + \frac{3}{2}(n^4)$

$= \log n^n$

$T(n) = O(n \log n)$

7)

7) (2) $A(x) = \{6, 3, 4, 5\}$

$B(x) = (249x + 4)x^3 + (103x + 3)x^2 + (29x + 2)x + (3x + 1)$

(22) $B(10) = (2494)10^3 + (1033)100 + (292)10 + 31$

$$
\begin{array}{r}
= \quad 2494000 \\
\quad\quad 103300 \\
\quad\quad\quad 2920 \\
+ \quad\quad\quad\quad 31 \\
\hline
2600251
\end{array}
$$

8) $\{P(10) = 2134, Q(10) = 352\}$

$P(10) = 20000 + 1000 + 30 + 4$

$= 10(2000 + 100) + 30 + 4$

$= 20000 + 1000 + 30 + 4$

$P(x) = 2x^4 + x^3 + 3x + 4 = (4, 3, 0, 1, 2)$

$Q(10) = 300 + 50 + 2$

$Q(x) = 3x^2 + 5x + 2 = (2, 5, 3)$