Lecture 1
Chapter 1: Introduction to Statistics and Data Analysis

Measures of Location: The Sample Mean and Median

The aim of this lecture is to explain the following concepts :

- Measures of Location.
- The Sample Mean and Median.
- The Sample Range and Sample Standard Deviation.
- Histogram.

Definition 1 *Suppose that the observations in a sample are* $x_1, x_2, ..., x_n$*. The **sample mean**, denoted by* $\bar{x}$*, is*

$$\bar{x} = \sum_{i=1}^{n} \frac{x_1 + x_2 + ... + x_n}{n}$$

Definition 2 *Given that the observations in a sample are* $x_1, x_2, ..., x_n$*, arranged in increasing order of magnitude, the **sample median** is*

$$\bar{x} = \begin{cases} x_{(n+1)/2}, & if\ n\ is\ odd \\ \frac{1}{2}(x_{n/2} + x_{n/2+1}), & if\ n\ is\ even. \end{cases}$$

Definition 3 *A **trimmed mean** is computed by "trimming away" a certain percent of both the largest and the smallest set of values.*
For example, the 10% trimmed mean is found by eliminating the largest 10% and smallest 10% and computing the average of the remaining values.

Definition 4 *The **sample variance**, denoted by s^2, is given by*

$$s^2 = \sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{n-1}$$

*The **sample standard deviation**, denoted by s, is the positive square root of s^2, that is,*

$$s = \sqrt{s^2}$$

Example 1 *: An engineer is interested in testing the "bias" in a pH meter. Data are collected on the meter by measuring the pH of a neutral substance (pH = 7.0). A sample of size 10 is taken, with results given by*
7.07 7.00 7.10 6.97 7.00 7.03 7.01 7.01 6.98 7.08.
Find sample variance and standard deviation.

Solution : *The sample mean $\bar{x}$ is given by*

$$\bar{x} = \frac{7.07 + 7.00 + 7.10 + ... + 7.08}{10} = 7.0250.$$

The sample variance s^2 is given by

$$s^2 = \frac{1}{9}[(7.07{-}7.025)^2{+}(7.00{-}7.025)^2{+}(7.10{-}7.025)^2{+}....{+}(7.08{-}7.025)^2] = 0.001939.$$

As a result, the sample standard deviation is given by

$$s = \sqrt{0.001939} = 0.044.$$

So the sample standard deviation is 0.0440 with n-1 = 9 degrees of freedom.

Exercises :

1. The following measurements were recorded for the drying time, in hours, of a certain brand of latex paint.
3.4 2.5 4.8 2.9 3.6
2.8 3.3 5.6 3.7 2.8
4.4 4.0 5.2 3.0 4.8
Assume that the measurements are a simple random sample.
(a) What is the sample size for the above sample?
(b) Calculate the sample mean for these data.
(c) Calculate the sample median.
(d) Compute the 20% trimmed mean for the above data set.
Solution :

(a) sample size = 15.

(b) $\bar{x} = \frac{1}{15}(3.4 + 2.5 + 4.8 + ... + 4.8) = 3.787$

(c) Sample median is the 8th value, after the data is sorted from smallest to largest = 3.6.

(d) After trimming total 40% of the data (20% highest and 20% lowest), the data becomes:
2.9 3.0 3.3 3.4 3.6
3.7 4.0 4.4 4.8.
So. the trimmed mean is

$$\bar{x}_{tr20} = \frac{1}{9}(2.9 + 3.0 + ... + 4.8) = 3.678.$$

2. According to the journal Chemical Engineering, an important property of a fiber is its water absorbency. A random sample of 20 pieces of cotton fiber was taken and the absorbency on each piece was measured. The following are the absorbency values:
18.71 21.41 20.72 21.81 19.29 22.43 20.17
23.71 19.44 20.50 18.92 20.33 23.00 22.85
19.25 21.77 22.11 19.77 18.04 21.12
(a) Calculate the sample mean and median for the above sample values.
(b) Compute the 10% trimmed mean.
Solution :
Given sample size = 20.
(a) Mean=20.768 and Median=20.610.
(b) $\bar{x}_{tr10} = 20.743$.

7. The following measurements were recorded for the drying time, in hours, of a certain brand of latex paint.
3.4 2.5 4.8 2.9 3.6
2.8 3.3 5.6 3.7 2.8
4.4 4.0 5.2 3.0 4.8
Compute the sample variance and sample standard deviation.
Solution : The sample variance s^2 is given by

$$s^2 = \frac{1}{15-1}[(3.4–3.787)^2+(2.5–3.787)^2+(4.8–3.787)^2+.....+(4.8–3.787)^2] = 0.94284.$$

As a result, the sample standard deviation is given by

$$s = \sqrt{0.9428} = 0.971.$$

8. Compute the sample variance and standard deviation for the water absorbency data of the above Q. No. 2.

Solution : The sample variance s^2 is given by

$$s^2 = \frac{1}{20-1}[(18.71-20.768)^2+(21.41-20.768)^2+....+(21.12-20.768)^2] = 0.94284.$$

As a result, the sample standard deviation is given by

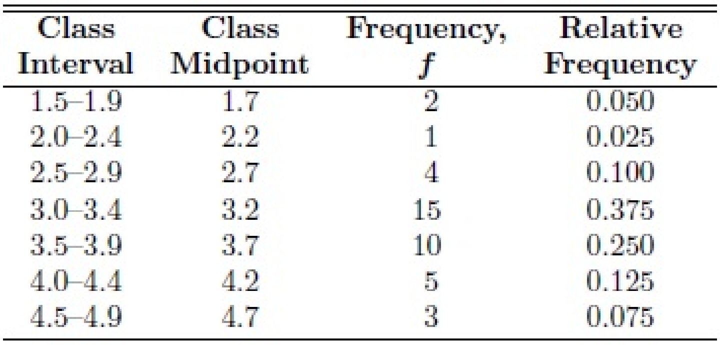
$$s = \sqrt{2.5345} = 1.592.$$

Histogram :

A table listing relative frequencies is called a **relative frequency distribution**.

The information provided by a relative frequency distribution in tabular form is easier to grasp if presented **graphically**.

Using the midpoint of each interval and the corresponding relative frequency, we construct a **relative frequency histogram**.



Class Interval	Class Midpoint	Frequency, f	Relative Frequency
1.5–1.9	1.7	2	0.050
2.0–2.4	2.2	1	0.025
2.5–2.9	2.7	4	0.100
3.0–3.4	3.2	15	0.375
3.5–3.9	3.7	10	0.250
4.0–4.4	4.2	5	0.125
4.5–4.9	4.7	3	0.075

Figure 1: Relative Frequency Distribution of Battery Life

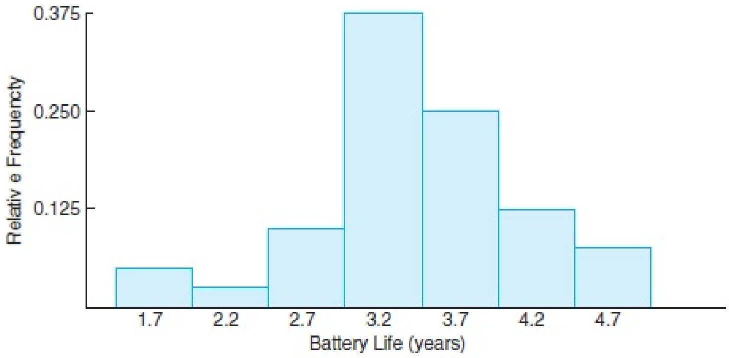


Figure 2: Relative frequency histogram