Lecture 29

One-Sample Estimation Problems (Contd..)

In the previous lecture, we have constructed a $100(1-\alpha)\%$ CI for the normal mean μ when the variance σ^2 is known. This section refers to the estimation of a normal mean and variance when the variance σ^2.

1 Estimating The Mean μ Of A Normal Population When σ^2 Is Unknown

Applying the same argument as above, from the graph of t distribution (see figure below) we have

$$P(-t_{\alpha/2} < T < t_{\alpha/2}) = 1 - \alpha \tag{1}$$

where

$$T = \sqrt{n}\frac{(\overline{X} - \mu)}{s} \sim t_{\alpha, n-1}.$$

So, from eq(1), we have

$$\begin{aligned} & P\left(- t_{\alpha/2} < \sqrt{n}\frac{(\overline{X} - \mu)}{s} < t_{\alpha/2} \right) = 1 - \alpha \\ \Rightarrow \quad & P\left(\overline{X} - t_{\alpha/2}\frac{s}{\sqrt{n}} < \mu < \overline{X} + t_{\alpha/2}\frac{s}{\sqrt{n}} \right) = 1 - \alpha t_{\alpha/2} \end{aligned}$$

So a $100(1-\alpha)\%$ CI for μ is given by $\left(\overline{X} - t_{\alpha/2}\frac{s}{\sqrt{n}}, \overline{X} + t_{\alpha/2}\frac{s}{\sqrt{n}} \right)$

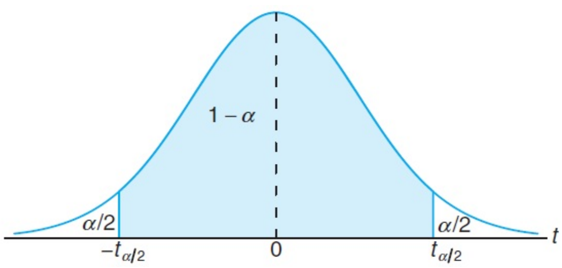

Figure 1: $P(-t_{\alpha/2} < T < t_{\alpha/2}) = 1 - \alpha$

Now we consider some examples.

Example 1 *The contents of several similar containers of Sulphuric acid are* $9.8, 10.2, 10.4, 9.8, 10.0, 10.2$ *and* 9.6 *litres. Find a* 95% *CI for the mean contents of all such containers assuming normality.*

Answer:- σ is not known. $\overline{X} = 10.0, s = 0.283, t_{0.005} = 2.447$ at $v = n - 1 = 6$ degrees of freedom. Hence, the $100(1 - \alpha)\%$ C.I is thus given by

$$10.0 - 2.447\Big(\frac{0.283}{\sqrt{7}}\Big) < \mu < 10.0 + 2.447\Big(\frac{0.283}{\sqrt{7}}\Big) \quad \Rightarrow \quad (9.74 < \mu < 10.26)$$

Example 2 *SAT mathematics scores of a of* 500 *PG students in Odisha show* $\overline{X} = 501, s = 112$. *Find a* 99% *confidence interval.*

Answer:- Proceeding as above the $100(1-\alpha)\%$ CI for μ is given by $488.1 < \mu < 513.9$.
Assignments:-Q-$4 - 7$.

2 Estimating The Varience σ^2 Of A Normal Population When Mean μ May Be Known Or Unknown

We know that $\chi^2 = \frac{(n-1)S^2}{\chi^2} \sim \chi^2_{n-1}$, from the graph of χ^2 distribution, we have

$$P(\chi^2_{1-\alpha/2} < \sigma^2 < \chi^2_{\alpha/2}) = 1 - \alpha \tag{2}$$

Substituting the values of χ^2 and on further simplification we have

$$P\left(\frac{(n-1)S^2}{\chi^2_{\alpha/2}} < \chi^2 < \frac{(n-1)S^2}{\chi^2_{1-\alpha/2}}\right) = 1 - \alpha.$$

So the $100(1 - \alpha)\%$ CI for σ^2 is thus given by

$$\left(\frac{(n-1)S^2}{\chi^2_{\alpha/2}}, \frac{(n-1)S^2}{\chi^2_{1-\alpha/2}}\right) \tag{3}$$

Example 3 *The following are the weights in decagrams, of* 10 *packages of grass seed distributed by a certain company,* 46.4, 46.1, 45.8, 47, 46.1, 45.9, 45.8, 46.9, 45.2 *and* 46. *Find a* 95% *CI for the variance of weights of all such packages of hrass seed distributed by the company. Assume normality.*

Answer:- Here we see that the observed sample varience

$$s^2 = \frac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{n-1} = \frac{1}{n(n-1)}\left[n\sum_{i=1}^{n} x_i^2 - \Big(\sum_{i=1}^{n} x_i\Big)^2\right] = 0.286.$$

From χ^2 distribution table, we have $\chi^2_{0.025,9} = 19.023$ and $\chi^2_{0.975,9} = 2.700$. So the CI for σ^2 can be estimated as

$$0.135 < \sigma^2 < 0.953.$$

Example 4 *Q-72 A random sample of 20 students yielded a mean of* $\overline{x} = 72$ *and a variance of* $s^2 = 16$ *for scores on a college placement test in mathematics. Assuming the scores to be normally distributed, construct a* 98% *confidence interval for* σ^2.

Answer:-Here $s^2 = 16$ with $v = 19$ DF. It is known $\chi^2_{0.01} = 36.191$ and $\chi^2_{0.099} = 7.633$. Hence substituting all the values in equation (3), the CI for σ^2 is thus estimated as

$$8.400 < \sigma^2 < 39.827.$$

Students are advised to practice the following assignment problems.
Assignments:-Q- 73, 77.