ODD SEMESTER EXAMINATION, NOVEMBER-2016
PROBABILITY & STATISTICS (MTH-2002)

Programme: B.Tech **Semester : 3rd**
Full Marks: 60 **Time: 3 Hours**

Subject/Course Learning Outcome	*Taxonomy Level	Ques. Nos.	Marks
Apply probability axioms to compute probability and conditional probability	L3,L3,L4,L3	1(a,b,c), 2(a)	2*4
Define random variables and compute probability distributions, joint & marginal distribution	L4,L4,L1,L5, L5	2(b,c)3(a, b,c)	2*5
Compute expectation of random variables and their functions and compute moments and moment generating functions of a random variable	L4,L1	4(a),6(a)	2*2
Discuss discrete probability distribution viz: Binomial, Poisson & Hypergeometric and continuous probability distribution distributions viz: Uniform, Normal Gamma & Exponential	L4,L5,L3, L4,L5,L3,L4, L1	4(b,c), 5(a,b,c), 6(b,c),7(a)	2*8
Estimate the population mean and variance of a normal distribution by point and interval estimation	L3, L5	7(b,c)	2*2
Infer about population parameter through hypothesis testing with the help of a random sample	L2,L4,L4, L3,L4,L4	8(a,b,c), 10(a,b,c),	2*6
Analyze linear regression and co-relation	L3,L5,L5	9(a,b,c)	2*3

*Bloom's taxonomy levels: Knowledge (L1), Comprehension (L2), Application (L3), Analysis (L4), Evaluation (L5), Creation (L6)

Answer all questions. Each question carries equal mark.

1	a)	List the elements of the following sample space S. $S = \{x : 2x - 4 \geq 0 \text{ and } x < 1\}$	2
	b)	If 3 books are picked at random from a shelf containing 7 mathematics, 3 physics and 2 chemistry books. Calculate the probability that 2 mathematics books are selected ?	2
	c)	The Police in a city plan to enforce speed limits by using radar traps at location L_1, L_2, L_3 and L_4 which will be operated 40%, 30%, 20% & 30% of the time respectively. If a person who is speeding on his way to work has probabilities 0.2, 0.1, 0.5 & 0.2, respectively, of passing through these locations, Compute the probability that he will receive a speeding ticket.	2

2	a)	With reference to 1(c), Calculate the probability that the person passed through the radar trap located at L_2, given he already got a speeding ticket.	2
	b)	If random variable 'X' has the probability distribution given by $$f(x)=\begin{cases}\frac{3}{16}\left(3-x^2\right), & -1\le x\le 1\\ 0, & elswhere\end{cases}$$ then derive the cumulative distribution function of X.	2
	c)	With reference 2(b), compute the probability that the random variable X is less than ½.	2
3	a)	If the joint probability distribution of X & Y is given by $f(x,y)=c(x+y),\ x=0,1,2,3;\ y=0,1,2,$ find the value of 'c'.	2
	b)	With reference to 3(a), evaluate the marginal distributions of the random variables X & Y.	2
	c)	From 3(a), calculate P $(X>2,\ Y\le 1)$	2
4	a)	A coin is biased such that a head is three time as likely to occur as a tail. Calculate the expected number of tails when this coin is tossed twice.	2
	b)	Suppose X is a random variable with probability density function given by $$f(x)=\begin{cases}\frac{8}{x^3}, & x>2\\ 0, & \text{otherwise}\end{cases}$$ Compute the variance of $Z = 2X + 1$.	2
	c)	A random variable X has a mean $\mu = 20$ and variance $\sigma^2 = 9$. Using Chebyshev's theorem, find $P(10 < X < 22)$.	2
5	a)	On average, the author of a text book makes two typing errors per page. Compute the probability that on the next page the author will make 4 or more errors.	2
	b)	Given the normally distribution random variable 'X' with mean 30 and standard deviation 6. Calculate the value of 'k' that has 80% of the normal curve area to the left and compute P $(X \le 22)$.	2

	c)	A process yields 10% defective items. If 100 items are randomly selected. Calculate the probability that the number of defectives is less than 8 by using the normal approximation to the Binomial distribution.	2
6	a)	Let X be a random variable with probability distribution $$f(x)=\begin{cases}\frac{1+x}{2}, & -1<x<1\\ 0, & otherwise.\end{cases}$$ Find the probability distribution of $Y=X^2$	2
	b)	Find the moment generating function for poison distribution.	2
	c)	Compute the probability that a random sample of 25 observations from a normal population with variance 6 will have sample variance S^2 between 3.462 to 10.745.	2
7	a)	State type - I and type-II error.	2
	b)	Find the maximum likelihood estimator for 'μ' of normal population from the sample of observations $x_1, x_2, \ldots x_n$	2
	c)	A random sample of 64 bags of white cheddar popcorn weighed on average 5.23 ounces with a standard deviation of 0.24 ounce. Test the hypothesis that μ=5.5 against the alternative hypothesis $\mu < 5.5$ at 0.05 level of significance.	2
8	a)	A new process for cement manufacturing results on mean compressive strength 5000kg per square cm, with standard deviation 120 kg. To test hypothesis $\mu = 5000$ against the alternative $\mu < 5000$ a random sample of 50 pieces of cement is tested. The critical region is defined to be $\bar{x} < 4970$. Find type – I error.	2
	b)	A soft-drink machine is said to be out of control if the variance of the contents exceeds 1.15 deciliters. A random sample of 25 drinks from this machine has a variance 2.03 deciliters. Use $\alpha = 0.05$ and explain the critical region (Assume normal distribution).	2
	c)	With reference to 8(b), test whether the machine is out of control.	2

9	a)	A study of the amount of rainfall and the quantity of air pollution removed, produced the following data:	2

Daily rainfall (in 0.01 cm) x	4.3	4.5	5.9	5.6	6.1	5.2	3.8
Particulate Removed ($\mu\ g/m^3$) y	126	121	116	118	114	118	132

Calculate the correlation co-efficient.

	b)	Deduce the regression line of Y on x, from 9(a).	2
	c)	Estimate the amount of particulate removed when daily rainfall is 4.8 units, from 9(b).	2
10	a)	A coin is thrown until a head occurs and the number of tosses (X) recorded. After 256 tosses the following result is obtained.	2

(x)	1	2	3	4	5	6	7	8
Frequency (f)	136	60	34	12	9	1	3	1

Find the expected frequencies for the values of X.

	b)	With reference to 10(a), test the hypothesis that observed distribution of X may be fitted by geometric distribution. ($\alpha = 0.05$)	2
	c)	In an experiment, to study the dependence of hypertension on smoking habits, the following data were taken on 180 individuals.	2

	Non smokers	Moderate smokers	Heavy smokers
Hypertension	21	36	30
No hypertension	48	26	19

Test the hypothesis, that the presence or absences of hypertension is independent of smoking habits.