LECTURE-34

ONE AND TWO-SAMPLE TESTS OF HYPOTHESES

CH-10.4: Test concerning a single mean using single sample:

Let the population is mean μ we want to test

$H_0 : \mu = \mu_0$

$H_1 : \mu \neq \mu_0$

Case-I: The sample is drawn from the normal population where population variance is known.

$Z = \frac{X-\mu}{\frac{\sigma}{\sqrt{n}}}$

Test concerning a single mean using single sample:-

(i) Variance is known $Z = \frac{X-\mu}{\frac{\sigma}{\sqrt{n}}}$

(ii) Variance is not known $T = \frac{X-\mu}{\frac{s}{\sqrt{n}}}$

Types of Test:

(i) $H_0 : \mu = \mu_1$

$H_1 : \mu > \mu_1$

(ii) $H_0 : \mu = \mu_1$

$H_1 : \mu < \mu_1$

(iii) $H_0 : \mu = \mu_1$

$H_1 : \mu \neq \mu_1$

For (i), if $H_0 : \mu = \mu_1$

$H_1 : \mu > \mu_1$

Then test statistic $z = \frac{x-\mu}{\frac{\sigma}{\sqrt{n}}}$ is normally distributed $n(0, 1)$

Let α be the level of significance. Take $\alpha = 0.01$.

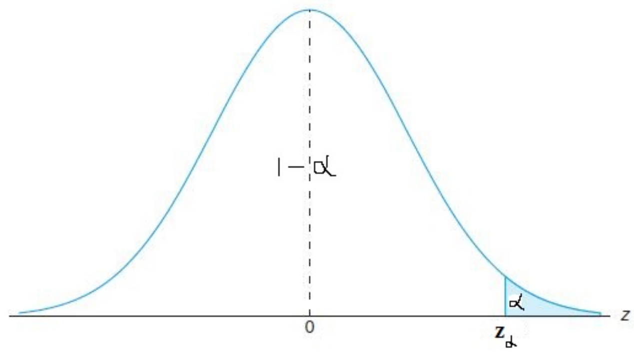

Figure 1: Right Sided Test

Now the value of z is calculated using the sample observation $z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$

From the normal distribution table find z_α such that $P(Z > z_\alpha) = \alpha$

If $z < z_\alpha$ then the H_0 is accepted.

If $z \geq z_\alpha$ then the H_0 is rejected.

(ii) If $H_0 : \mu = \mu_1$

$H_1 : \mu < \mu_1$

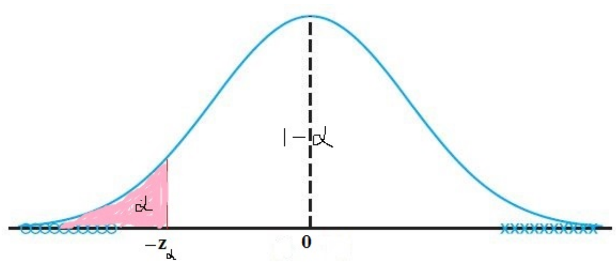

Figure 2: Left Sided Test

Then test statistic $z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$

From the normal distribution table find z_α such that $P(Z > z_\alpha) = \alpha$

If $z \leq -z_\alpha$ then the H_0 is rejected.

If $z > -z_\alpha$ then the H_0 is accepted at α level of significance.

(iii) If $H_0 : \mu = \mu_1$

$H_1 : \mu \neq \mu_1$

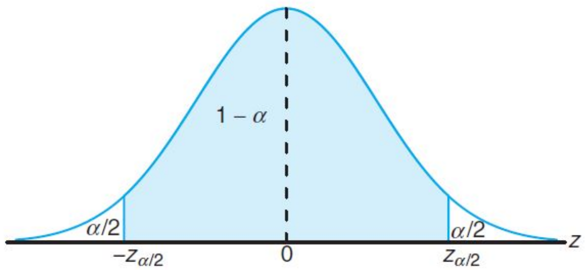

Figure 3: Two Sided Test

Then observation value is $z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$

If $-z_{\frac{\alpha}{2}} < z < z_{\frac{\alpha}{2}}$ then the H_0 is accepted.

If $z > z_{\frac{\alpha}{2}}$ or If $z < -z_{\frac{\alpha}{2}}$ then the H_0 is rejected.

(i) **<u>When variance is known</u>**:, Test statistic is $z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$

Example 10.3: A random sample of 100 recorded deaths in the United States during the past year showed an average life span of 71.8 years. Assuming a population standard deviation of 8.9 years, does this seem to indicate that the mean life span today is greater than 70 years? Use a 0.05 level of significance.

Ans: Given $n = 100$

$\bar{x} = 71.8$

$\sigma = 8.9$

Mean life span is greater than 70 year.

$H_0 : \mu = 70$

$H_1 : \mu > 70$ (It is a right tail test)

$\alpha = 0.05$

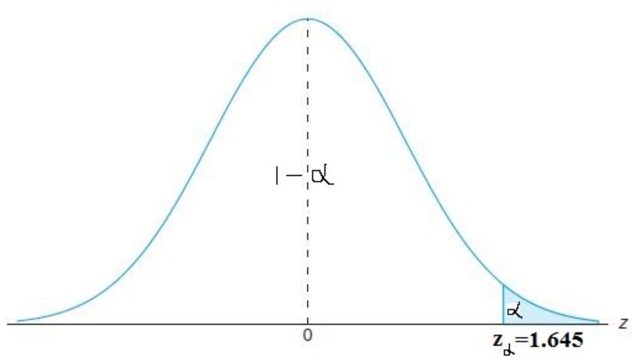

Figure 4: Right Sided Test

$z_\alpha = 1.645$

$z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} = \frac{71.8-70}{\frac{8.9}{\sqrt{100}}} = 2.02$

Since computed value of z > tabulated value $\Rightarrow$ The statistic fall in the rejection region or critical region. So, H_0 is rejected at 0.05 level of significance. Hence, average life span is greater than 70 year.

Example 10.4: A manufacturer of sports equipment has developed a new synthetic fishing line that the company claims has a mean breaking strength of 8 kilograms with a standard deviation of 0.5 kilogram. Test the hypothesis that $\mu = 8$ kilograms against the alternative that $\mu \neq 8$ kilograms if a random sample of 50 lines is tested and found to have a mean breaking strength of 7.8 kilograms. Use a 0.01 level of significance.

Ans: Given, significance level = α

$\bar{x} = 8$ kilogram

$\mu = 8$, $\mu \neq 8$, $\mu \longrightarrow$ mean

(n) no. of samples=50,

($\bar{x}$) avg strength = 7.8 kg,

(σ) standard deviation = 0.5 kg

sol^n: $H_0 : \mu = 8$

$H_1 : \mu \neq 8$

This is a two tail test.

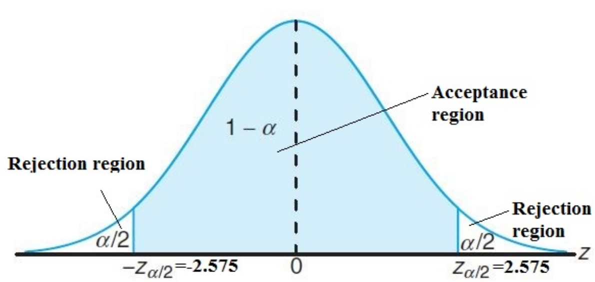

Figure 5: Two Sided Test

$P(Z < -z_{\frac{\alpha}{2}}) = \frac{\alpha}{2} = \frac{0.01}{2} = 0.005$

From the table, -$z_{\frac{\alpha}{2}} = -2.575$

$z_{\frac{\alpha}{2}} = 2.575$

Which are two critical values

$z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} = \frac{7.8-8}{\frac{0.5}{\sqrt{50}}} = -2.85$

Computed values of $z = -2.85 <$ tabulated value (-2.575)

So, H_0 is rejected at 0.01 level of significance.

<u>Problem-10.20</u>: A random sample of 64 bags of white cheddar popcorn weighed, on average, 5.23 ounces with a standard deviation of 0.24 ounce. Test the hypothesis that $\mu = 5.5$ ounces against the alternative hypothesis, $\mu < 5.5$ ounces, at the 0.05 level of significance.

Ans: Given $n = 64$, $\bar{x} = 5.23$, $\mu = 5.5$, $\sigma = 0.24$, $\alpha = 0.05$

$H_0 : \mu = 5.5$

$H_1 : \mu < 5.5$

Tabulated value is $z_\alpha = -1.645$ $(From\ \ Normal\ \ distribution\ \ table\ \ \frac{-1.65-1.64}{2} = -1.645)$

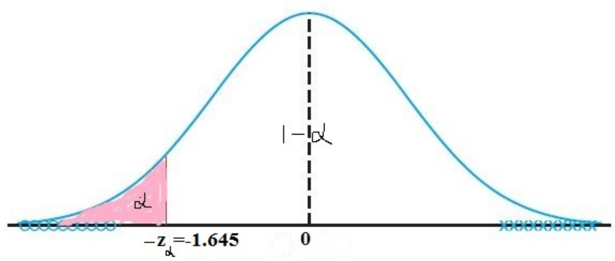

Figure 6: Left Sided Test

Computed value is $z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} = \frac{5.23-5.5}{\frac{0.24}{\sqrt{64}}} \frac{-0.27}{0.03} = -9$

Since computed value $<$ tabulated value, so H_0 is rejected.