LECTURE-35

ONE- AND TWO-SAMPLE TESTS OF HYPOTHESES

Rest of CH-10.4 and 10.5:

(ii) **When variance is unknown**:

In this case population variance is estimate as sample variance s^2. Test statistic is $t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}$ with (n-1) degrees of freedom.

Types of Test:

For (i) $H_0 : \mu = \mu_1$

$H_1 : \mu > \mu_1$

This is right tail test for α level of significance.

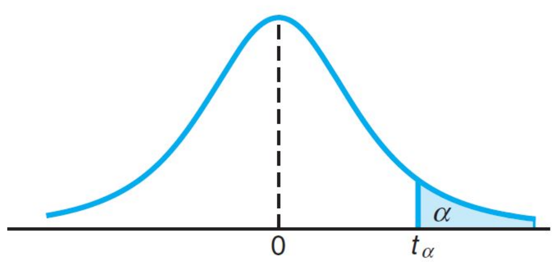

Figure 1: Right Sided Test

Find the tabular value of t_α from t-distribution table with $(n-1)$ degrees of freedom.

Find the computed value of t by using $t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}$ at $\alpha = 0.1(say)$.

If computed value $<$ tabulated value then accept H_0.

If computed value $\geq$ tabulated value then reject H_0.

For (ii) $H_0 : \mu = \mu_1$

$H_1 : \mu < \mu_1$

This is left tail test for α level of significance.

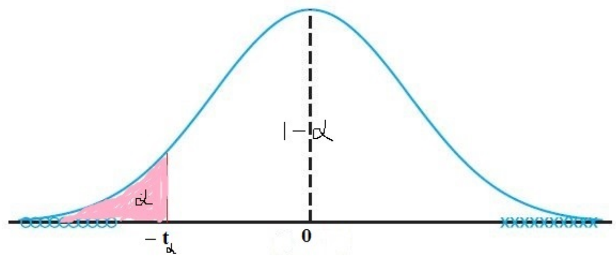

Figure 2: Left Sided Test

Find the tabular value of $-t_\alpha$ from t-distribution table with $(n-1)$ degrees of freedom.

Find the computed value of t by using $t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}$ at $\alpha = 0.1(say)$.

If computed value $(t) >$ tabulated value $(-t_\alpha)$ then, accept H_0.

If computed value $\leq$ tabulated value then reject H_0.

Example 10.5: The Edison Electric Institute has published figures on the number of kilowatt hours used annually by various home appliances. It is claimed that a vacuum cleaner uses an average of 46 kilowatt hours per year. If a random sample of 12 homes included in a planned study indicates that vacuum cleaners use an average of 42 kilowatt hours per year with a standard deviation of 11.9 kilowatt hours, does this suggest at the 0.05 level of significance that vacuum cleaners use, on average, less than 46 kilowatt hours annually? Assume the population of kilowatt hours to be normal.

Ans: Given $H_0 : \mu = 46$

$H_1 : \mu < 46$

level of significance $\alpha = 0.05$

$n = 12$, $s = 11.9$, $\bar{x} = 42$

This is a left tail test.

Since variance σ^2 is unknown, so test statistic is

$t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}$

which follows t-distribution with (n-1) degrees of freedom.

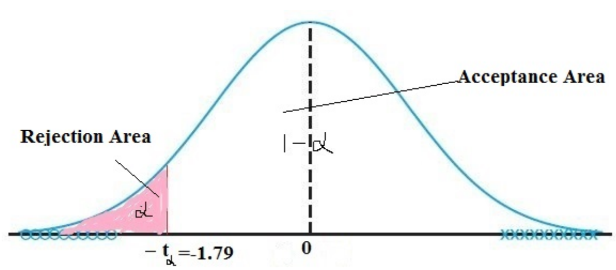

Figure 3: Left Sided Test

The tabulated value of t_α at $12-1=11$ degrees of freedom is 1.79 (from t-distribution table).

So the tabulated value of $-t_\alpha$ at $12-1=11$ degrees of freedom is -1.79.

Now $t=\frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}=\frac{42-46}{\frac{11.9}{\sqrt{12}}}=-1.16$

Since computed value $>$ tabulated value, so H_0 is accepted.

10.29:

Past experience indicates that the time required for high school seniors to complete a standardized test is a normal random variable with a mean of 35 minutes. If a random sample of 20 high school seniors took an average of 33.1 minutes to complete this test with a standard deviation of 4.3 minutes, test the hypothesis, at the 0.05 level of significance, that $\mu=35$ minutes against the alternative that $\mu<35$ minutes.

Ans: Given $\mu=35$, $n=20$, $\bar{x}=33.1$, $s=4.3$, $\alpha=0.5$, $\mu<35$

$H_0:\mu=35$

$H_1:\mu<35$

$d.\ f.\ =n-1=20-1=19$

The tabulated value of t_α at 19 $d.f.=1.729$.

The tabulated value of $-t_\alpha$ at 19 $d.f.=-1.729$.

$$t=\frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}=\frac{33.1-35}{\frac{4.3}{\sqrt{20}}}=\frac{-1.9}{(0.96)^2}=-1.89$$

Since computed value $<$ tabulated value, so H_0 is rejected.

It takes less than 35 minutes on the average to the take test.

For (iii) $H_0:\mu=\mu_1$

$H_1:\mu\neq\mu_1$

This is two tail test for α level of significance i.e total area is $\frac{\alpha}{2}+\frac{\alpha}{2}=\alpha$

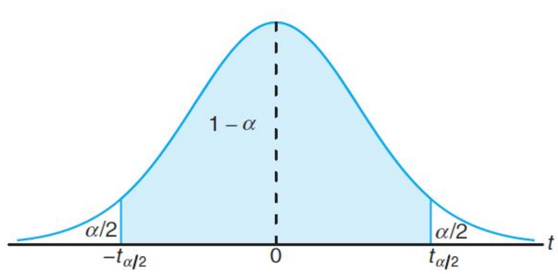

Figure 4: Two Sided Test

Find the tabular value of $-t_{\frac{\alpha}{2}}$ and $t_{\frac{\alpha}{2}}$ from t-distribution table with $(n-1)$ degrees of freedom. If computed value lies between $-t_{\frac{\alpha}{2}}$ and $t_{\frac{\alpha}{2}}$, then H_0 is accepted.

Book Questions

10.21: An electrical firm manufactures light bulbs that have a lifetime that is approximately normally distributed with a mean of 800 hours and a standard deviation of 40 hours. Test the hypothesis that $\mu = 800$ hours against the alternative, $\mu = 800$ hours, if a random sample of 30 bulbs has an average life of 788 hours. Use a P-value in your answer.

Home Tax

10.23: Test the hypothesis that the average content of containers of a particular lubricant is 10 liters if the contents of a random sample of 10 containers are 10.2, 9.7, 10.1, 10.3, 10.1, 9.8, 9.9, 10.4, 10.3, and 9.8 liters. Use a 0.01 level of significance and assume that the distribution of contents is normal.

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CH-10.5: Two Samples: Test on Two Means:

Test of Hypothesis concerning difference of mean of two population:

1. Population variances are known.

Test statistic is

$$z = \frac{(\bar{x_1} - \bar{x_2}) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

2. Population variances are unknown but equal.

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$

We reject H_0 at significance level α when the computed t-statistic

$$t = \frac{(\bar{x_1} - \bar{x_2}) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}}$$

Where estimate of population variance s_p^2 is

$$s_p^2 = \frac{s_1^2(n_1-1)+s_2^2(n_2-1)}{n_1+n_2-2}$$

exceeds $t_{\frac{\alpha}{2},\ n_1+n_2-2}$ or is less than $-t_{\frac{\alpha}{2},\ n_1+n_2-2}$.

Example 10.6:

An experiment was performed to compare the abrasive wear of two different laminated materials. Twelve pieces of material 1 were tested by exposing each piece to a machine measuring wear. Ten pieces of material 2 were similarly tested. In each case, the depth of wear was observed. The samples of material 1 gave an average (coded) wear of 85 units with a sample standard deviation of 4, while the samples of material 2 gave an average of 81 with a sample standard deviation of 5. Can we conclude at the 0.05 level of significance that the abrasive wear of material 1 exceeds that of material 2 by more than 2 units? Assume the populations to be approximately normal with equal variances.

Ans: Given two populations P_1 and P_2

Table 1: Given

Population	P_1	P_2
sample size	$n_1 = 12$	$n_2 = 10$
sample mean	$\bar{x_1} = 85$	$\bar{x_2} = 81$
sample standard deviation	$s_1 = 4$	$s_2 = 5$

$\alpha = 0.05$

$H_0 : \mu_1 - \mu_2 = 2$

$H_1 : \mu_1 - \mu_2 > 2$

As population variance are unknown but equal, we are t-statistic

$$t = \frac{(\bar{x_1}-\bar{x_2})-(\mu_1-\mu_2)}{\sqrt{\frac{s_p^2}{n_1}+\frac{s_p^2}{n_2}}}$$

$$s_p = \sqrt{\frac{s_1^2(n_1-1)+s_2^2(n_2-1)}{n_1+n_2-2}}$$

$$s_p = \sqrt{\frac{16 \times 11 + 25 \times 9}{12 + 10 - 2}} = 4.47$$

T-statistic follows t-distribution with $12 + 10 - 2 = 20$ d.f.

Computed value of t-statistic

$$t = \frac{(85 - 81) - 2}{4.47\sqrt{\frac{1}{12} + \frac{1}{10}}} = 1.04$$

This is a right tail test.

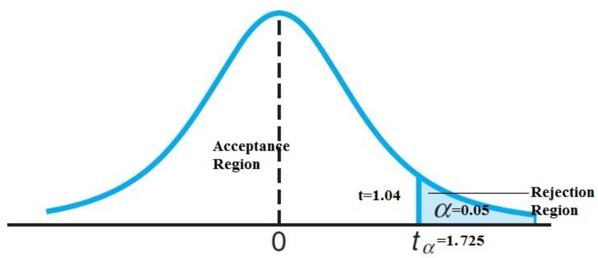

Figure 5: Right Sided Test

From the table t-value for 20 $d.f. = 1.725$

The computed value 1.04 is in acceptance region.

Hence H_0 is accepted at 0.05 level of significance.