Chapter-4

4. $P(X=0) = 3P(X=1)$

$P(X=0) + P(X=1) = 1$

$\Rightarrow 3P(X=1) + P(X=1) = 1$

$\Rightarrow P(X=1) = 0.25$ $\qquad \Rightarrow P(X=0) = 0.75.$

value of T can be 0, 1, 2,

$$P(T=0) = P(X=0, Y=0)$$
$$= P(X=0)P(Y=0)$$
$$= 0.75 \cdot 0.75$$
$$= 9/16$$

$$P(T=1) = P(X=1, Y=0) + P(X=0, Y=1)$$
$$= P(X=1)P(Y=0) + P(X=0)P(Y=1)$$
$$= 0.25 \cdot 0.75 + 0.75 \cdot 0.25$$
$$= 3/8$$

$$P(T=2) = P(X=1, Y=1)$$
$$= P(X=1)P(Y=1)$$
$$= 0.25 \cdot 0.25$$
$$= 1/16.$$

Expected number of trails

$$E(T) = \sum_t t\, f(t)$$
$$= \sum_{t=0}^{2} t \cdot P(T=t)$$
$$= 0 \cdot \frac{9}{16} + 1 \cdot \frac{3}{8} + 2 \cdot \frac{1}{16}$$
$$= 1/2$$

X represents the person's gain in a year.

$P(X=4000) = 0.3$ & $P(X=-1000) = 0.7$

$$E(X) = \sum_x x f(x) = \sum_x x \cdot P(X=x) = 4000 \cdot 0.3 + (-1000) \cdot 0.7$$
$$= 500$$

10. Marginal distribution of x

$g(1) = (0.1 + 0.05 + 0.02) = 0.17$

$g(2) = (0.1 + 0.35 + 0.05) = 0.5$

$g(3) = (0.03 + 0.1 + 0.2) = 0.33$

Now, $\mu_x = \sum x \, g(x)$

$\mu_x = [1 \times 0.17 + 2 \times 0.5 + 3 \times 0.33] = 2.16$

Marginal distribution of y,

$h(1) = (0.1 + 0.1 + 0.03) = 0.23$

$h(2) = (0.05 + 0.35 + 0.1) = 0.5$

$h(3) = (0.02 + 0.05 + 0.2) = 0.27$

now, $\mu_y = \sum_y y \, h(y)$

$\mu_y = (1 \times 0.23 + 2 \times 0.5 + 3 \times 0.27) = 2.04$

12. Given $F(x) = \begin{cases} 2(1-x) & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$

Expected number of automobiles sold by the

$E(x) = \int_x x F(x)\,dx = \int_0^1 2x(1-x)\,dx$

$E(x) = \left[x^2 - \frac{2x^3}{3} \right]_0^1 = 1/3$

Average profit made by dealer per autom is \$ 5000.

So, his profit on expected sales of autom

is $\to [1/3 \times 5000]$

$= \$ 1667.67$

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15.
$$F(x,y) = \begin{cases} 1/\pi a^2 & x^2+y^2 \le a^2 \\ & -a \le x, y \le a \\ 0 & \text{elsewhere} \end{cases}$$

Mean, $\mu_x = \int_x \int_y x\, F(x,y)\, dx\, dy = \frac{1}{\pi a^2} \int_{-a}^{a} \int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} x\, dx\, dy$

$$\mu_x = \int_{-a}^{a} \frac{x}{\pi a^2} 2\left[\sqrt{a^2-x^2}\right] dx$$

$$a^2 - x^2 = t^2$$

$$\Rightarrow -2x\,dx = 2t\,dt$$

$$\mu_x = \frac{-1}{\pi a^2} \int 2t^2\, dt \Rightarrow \frac{-2}{\pi a^2} \left. \frac{(a^2-x^2)^{3/2}}{3} \right|_{-a}^{a}$$

$$\mu_x = 0$$

20. For a continious random variable x, with $F(x)$ expected value of random variable $g(x)$ is

$$E\, g(x) = \int_x g(x) F(x).dx$$

$$E[g(x)] = \int_0^\infty e^{2x/3} e^{-x}\, dx$$

$$E[g(x)] = \int_0^\infty e^{-x/3}\, dx = -3 \left. e^{-x/3} \right]_0^\infty$$

$$E[g(x)] = 3 \qquad (\because e^{-\infty} = 0 \ \& \ e^0 = 1)$$

23. For a discrete random variable x,

$$E[g(x,y)] = \sum_x \sum_y g(x,y) F(x,y).$$

a) Given $g(x,y) = x y^2$

$F(x)$		2	4
	1	0.1	0.15
y	3	0.2	0.3
	5	0.1	0.15

So, $E[g(x,y)] = \sum_x \sum_y g(xy^2) f(x,y) \quad \forall \quad x = 2,4$
$y = 1,3,5$

now,

$g(2,1) = 2 \quad ; \quad g(4,3) = 36$

$g(4,1) = 4 \quad ; \quad g(2,5) = 50$

$g(2,3) = 18 \quad ; \quad g(4,5) = 100$

$$E[g(x,y)] = [2\times 0.1 + 4\times 0.15 + 18\times 0.2 + 0.3\times 36 + 0.1\times 50 + 0.15\times 100$$

$$E[g(x,y)] = 35.2$$

(b). Marginal distributions for discrete x,

$g(x)$ = Add the coloumn for corresponding 'x'.

$h(y)$ = Add the row demands for correspondi

now,

$g(2) = 0.4 \quad , \quad g(4) = 0.6$

$h(1) = 0.25 \quad , \quad h(3) = 0.5 \quad , \quad h(5) = 0.25$

Now, $\mu_x = E(x) = \sum x\, g(x)$

$\mu_x = [2\times 0.4 + 4\times 0.6] = 3.2$

& similarly $\mu_y = [1\times 0.25 + 3\times 0.5 + 5\times 0.25]$
$= 3$

26. $f(x,y) = \begin{cases} 4xy & 0 < x, y < 1 \\ 0 & \text{elsewhere} \end{cases}$

expected value of $z = \sqrt{x^2+y^2}$

$$E(z) = \int_{x,y} z \cdot f(x,y)\, dx\, dy = \int_0^1 \int_0^1 4xy \sqrt{x^2+y^2}\, dx\, dy$$

using substitution method, Put $x^2+y^2 = t^2$

so that $2x\, dx = 2t\, dt$

$$\Rightarrow E(z) = \int_0^t \int_t 4t\, y \cdot t\, dt\, dy$$

$$E(z) = \int_0^1 4y \cdot \left[t^3/3 \right]_t dy \Rightarrow \int_0^1 \frac{4y}{3} \left(\sqrt{x^2+y^2}\right)^3 \Big|_0^1 dy$$

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$$E(z) = \frac{4}{3}\int_0^1 y\,[(1+y^2)^{3/2} - y^3]\,dy$$

Again use substitution, put $(1+y^2) = k^2$

so that $y\,dy = k\,dk$

$$E(z) = \frac{4}{3}\left[\int_k k^4\,dk - \int_0^1 y^4\,dy\right]$$

$$E(z) = \frac{4}{3}\left[\frac{(1+y^2)^{5/3}}{5} - \frac{y^5}{5}\right]_0^1 \Rightarrow \frac{4}{3}\left[\frac{(2^{5/2}-1)-(1-0)}{5}\right]$$

$$E(z) = \frac{8}{15}(2^{3/2}-1) \Rightarrow 0.9752$$

34. Mean $\mu = \left[\sum_x x\,F(x)\right] = [-2\times 0.3 + 3\times 0.2 + 5\times 0.5]$

$= 2.5$

Std. variation, $\sigma = \sqrt{var(x)}$

$\sigma = \sqrt{E(x^2) - \mu^2}$

Let's find $E(x^2) = \sum_x x^2 F(x)$

$E(x^2) = (2)^2\,0.3 + (3)^2\,0.2 + (5)^2\,0.5$

$E(x^2) = 15.5$

$\therefore \sigma = \sqrt{15.5 - (2.5)^2} = 3.04$

35. $E(x) = \sum_x x\,F(x) = [2\times 0.01 + 3\times 0.25 + 4\times 0.4 + 0.3\times 5 + 6\times 0.04]$

$E(x) = 4.11$

and $E(x^2) = [2^2\times 0.01 + 3^2\times 0.25 + 4^2\times 0.4 + 5^2\times 0.3 + 6^2\times 0.04]$

$E(x^2) = 17.63$

Therefore $\sigma^2 = E(x^2) - (E(x)^2)$

$\sigma^2 = (17.63) - (4.11)^2 = 0.74$

50. [illegible]

$$E(x) = 2\int_0^1 x(1-x)\,dx = 2\left(\frac{x^2}{2} - \frac{x^3}{3}\right)\Big|_0^1 = 1/3$$

$$E(x^2) = 2\int_0^1 x^2(1-x)\,dx = 2\left(x^3/3 - x^4/4\right)\Big|_0^1 = 1/6$$

$$Var(x) = 1/6 - (1/3)^2 = 1/18$$

$$\sigma = \sqrt{1/18} = 0.2357$$

52. Since $F_x(x) = 2(1-x)$, $0 < x < 1$

$$F_y(y) = 2y, \quad 0 < y < 1$$

$$E(x) = 1/3, \quad E(y) = 2/3$$

$$Var(x) = Var(y) = 1/18$$

$$E(xy) = 1/4$$

$$\rho_{xy} = 1/2$$

57. $E(x) = -3 \times 1/6 + 6 \times 1/2 + 9 \times 1/3 = 11/2$

$$E(x^2) = (-3)^2 \times 1/6 + 6^2 \times 1/2 + 9^2 \times 1/3 = 93/2$$

$$E(2x+1)^2 = 4E(x^2) + 4E(x) + 1$$

$$= 4 \times \frac{93}{2} + 4 \times \frac{11}{2} + 1$$

$$= 209.$$

58. Since $E(x) = \int_0^1 x^2\,dx + \int_0^1 x(2-x)\,dx = 1$

$$E(x^2) = \int_0^1 x^3\, 2dx + \int_1^2 x^2(2-x)\,dx = 7/6$$

$$E(y) = 60E(x^2) + 39E(x)$$

$$= 60 \times 7/6 + 39 \times 1$$

$$= 109.$$

60. $E(x) = 2 \times 0.4 + 4 \times 0.6 = 3.20$

$$E(y) = 1 \times 0.25 + 3 \times 0.5 + 5 \times 0.25 = 3$$

a) $E(2x - 3y) = 2 \times 3.20 - 3 \times 3 = -2.60$

b) $E(xy) = 3.20 \times 3 = 9.60$

64. $E(x) = E(xy) = \int_{0}^{1}\int_{2}^{\infty} 16\, xy\, (y/x^3)\, dx\, dy$

$= 8/3$

75. $\mu = 900$ hr, $\sigma = 50$ hr

Solving $\mu - K\sigma = 700$

$K = 4$

So, by chebyshev's theorem

$P(\mu - 4\sigma < x < \mu + 4\sigma) \geq 1 - 1/4^2$

$= 0.9375$

$P(700 < x < 1100) \geq 0.9375$

Therefore $P(x \leq 700) \leq 0.03125$

77. a) $P(|x-10| \geq 3) = 1 - (|x-10| < 3)$

$= 1 - P[10 - (3/2) \times 2 < x < 10 + 3/2 \times 2]$

$\leq 1 - \left[1 - \frac{1}{(3/2)^2}\right]$

$= 4/9$

b) $P(|x - 10| < 3) = 1 - P(|x - 10| \geq 3) \geq 1 - 4/9 = 5/9$

c) $P(5 < x < 15) = P[10 - 5/2 \times 2 < x < 10 + 5/2 \times 2]$

$\geq 1 - \frac{1}{(5/2)^2}$

$= \frac{21}{25}.$

Chapter-5

9. $n = 15$, $P = 0.25$

a) $P(3 \leq x \leq 6) = P(x \leq 6) - P(x \leq 2)$

$= 0.7472 - 0.1178$

$= 0.6294$

b) $P(x \leq 5) = 0.0386$

c) $P(x \geq 8) = 1 - P(x \leq 7)$

$= 1 - 0.2763$

$= 0.7237$

11. From table, $n = 5$, $P = 0.7$

$P(x \geq 3) = 1 - P(x \leq 2)$

$= 1 - 0.1631$

$= 0.8369$

15. $P = 0.4$, $n = 5$

a) $P(x = 0) = 0.0778$

b) $P(x < 2) = P(x \leq 1) = 0.3370$

c) $P(x > 3) = 1 - P(x \leq 3) = 1 - 0.9130$

$= 0.0870$

16. probability of 2 or more of 4 engines operating when $P = 0.6$, $P(x \geq 2) = 1 - P(x \leq 1)$

$= 0.8208$

Probability of 1 or more of 2 engines operating when $P = 0.6$

$P(x \geq 1) = 1 - P(x \leq 0)$

$= 0.8400$

27. using multinomial distribution we have

$$\binom{8}{5,2,1}(1/2)^5 (1/4)^2 (1/4) = 21/256$$

31. $$h(x;6,3,4) = \frac{\binom{4}{x}\binom{2}{3-x}}{\binom{6}{3}}, \text{ for } x = 1,2,3.$$

$$P(2 \le x \le 3) = h(2;6,3,4) + h(3;6,3,4) = \frac{4}{5}$$

32. (a). Probability that all 4 fire

$$= h(4;10,4,7) = 1/6$$

(b). Probability that at most 2 will not fire $= \sum_{x=0}^{2} h(x;10,4,3) = 29/30.$

43. a) The extension of the hypergeometri distribution gives a probability

$$\frac{\binom{2}{1}\binom{3}{1}\binom{5}{1}\binom{2}{1}}{\binom{12}{4}} = \frac{4}{33}.$$

b) using the extension of the hyper-geometric distribution, we have

$$\frac{\binom{2}{1}\binom{3}{1}\binom{2}{2}}{\binom{12}{4}} + \frac{\binom{2}{2}\binom{3}{1}\binom{2}{1}}{\binom{12}{4}} + \frac{\binom{2}{1}\binom{3}{2}}{\binom{12}{4}}$$

$$= \frac{8}{165}$$

44. $\dfrac{\binom{2}{2}\binom{4}{1}\binom{3}{2}}{\binom{9}{5}} + \dfrac{\binom{2}{3}\binom{4}{2}\binom{3}{1}}{\binom{9}{5}} + \dfrac{\binom{2}{2}\binom{4}{3}\binom{3}{0}}{\binom{9}{5}}$

$= \dfrac{17}{63}$

47. a) $\dfrac{\binom{3}{0}\binom{17}{5}}{\binom{20}{5}} = 0.3991$

b) $\dfrac{\binom{3}{2}\binom{17}{3}}{\binom{20}{5}} = 0.1316$

49. using negative binomial distribution required probability is

$b*(10; 5, 0.3) = \binom{9}{4}(0.3)^5(0.7)^5$

$= 0.0515$

50. $b*(7; 3, 1/2) = \binom{6}{2}(1/2)^2$

$= 0.1172$ (negative binomial distribution).

$g(4; 1/2) = 1/2 \times (1/2)^3 = 1/16$ (geometric distribution).

51. probability that all coins turn up the same is 1/4. using geometric distribution

$P = 3/4$, $q = 1/4$

$P(x<4) = \sum_{x=1}^{3} g(x; 3/4) = \sum_{x=1}^{3} (3/4)(1/4)^{x-1}$

$= \dfrac{63}{64}$

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60. a). using poisson distribution $\mu = 12$,

$P(x < 7) = P(x \leq 6) = 0.0458$

2

b) using binomial distribution $P = 0.0458$

$b(2; 3, 0.0458) = \binom{3}{2} (0.0458)^2 (0.9542)$

$= 0.0060$

69. $\mu = 4000 \times 0.001 = 4$

70. $\mu = 1$, $\sigma^2 = 0.99$

Chapter - 6

2. $P(X > 2.5 \mid X \leq 4) = \dfrac{P(2.5 < X < 4)}{P(X \leq 4)} = \dfrac{4-2.5}{4-1} = 1/2$

4. (a). $P(X > 7) = \dfrac{10-7}{10} = 0.3$

(b). $P(2 < X < 7) = \dfrac{7-2}{10} = 0.5$

7. (a). Since $P(Z > k) = 0.2946$, then $P(Z < k) = 0.7054$, we find $k = 0.54$.

(b). $k = -1.72$.

(c). The area to the left of $z = -0.93$ is found from Table A.3 to be 0.1762. Therefore, the total area to the left of k is $0.1762 + 0.7235 = 0.8997$, and hence $k = 1.28$.

10. $z_1 = [(\mu - 3\sigma) - \mu]/\sigma = -3$, $z_2 = [(\mu + 3\sigma) - \mu]/\sigma = 3$;

$P(\mu - 3\sigma < z < \mu + 3\sigma) = P(-3 < z < 3)$

$= 0.9987 - 0.0013$

$= 0.9974$.

15. (a) $z = (30 - 24)/3.8 = 1.58$;

$P(X > 30) = P(Z > 1.58) = 0.0571$

(b). $z = (15 - 24)/3.8 = -2.37$; $P(X > 15) = P(Z > -2.37) = 0.9911$. He is late 99.11% of the time (c). $z = (25 - 24)/3.8 = 0.26$;

$P(x > 25) = P(z > 0.26) = 0.3974.$

(d). $z = 1.04$, $x = (3.8)(1.04) + 24 = 27.952$ minutes

(e). Using the binomial distribution with $p = 0.0571$, we get

$b(2; 3, 0.0571) = \binom{3}{2}(0.0571)^2(0.9429)$

$= 0.0092.$

22. a). $x_1 = \mu + 1.3\sigma$, $x_2 = \mu - 1.3\sigma$

$z_1 = 1.3$, $z_2 = 1.3$, $P(x > \mu + 1.3\sigma) + P(x < 1.3\sigma$

$= P(z > 1.3) + P(z < -1.3)$

$= 2P(z < -1.3) = 0.1936$

b) $x_1 = \mu + 0.52\sigma$, $x_2 = \mu - 0.52\sigma$

$z_1 = 0.52$, $z_2 = 0.52$

$P(\mu - 0.52\sigma < x < \mu + 0.52\sigma) = P(-0.52 < z <$

$= 0.6985 - 0.3015$

$= 0.3970$

26. $\mu = nP = 100 \times 0.1 = 10$ $\sigma = \sqrt{(100) \times 0.1 \times 0.9} = 3$

a) $z = (13.5 - 10)/3 = 1.17$, $P(x > 13.5) = P(z > 1.17)$

$= 0.1210.$

b) $z = (7.5 - 10)/3 = -0.83$, $P(x < 7.5)$

$= P(z < -0.83)$

$= 0.02033$

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29. $\mu = 1000 \times 0.2 = 200,$

$\sigma = \sqrt{1000 \times 0.2 \times 0.8} = 12.649$

a) $z_1 = (169.5 - 200)/12.649 = -2.41$

$z_2 = (185.5 - 200)/12.649 = -1.15$

$P(169.5 < x < 185.5) = P(-2.41 < z < -1.15)$

$= 0.1251 - 0.0080$

$= 0.1171$

b) $z_1 = (209.5 - 200)/12.649 = 0.75$

$z_2 = (225.5 - 200)/12.0$

$= 2.02$

$P(209.5 < x < 225.5) = P(0.75 < z < 2.02)$

$= 0.9783 - 0.7734$

$= 0.2049$

41. $P(1.8 < x < 2.4) = \int_{1.8}^{2.4} x e^{-x}\, dx = \left[-x e^{-x} - e^{-x}\right]\Big|_{1.8}^{2.4}$

$= 2.8 e^{-1.8} - 3.4 e^{-2.4}$

$= 0.1545$

47. a) $E(x) = \int_0^{\infty} x^2 e^{-x^2/2}\, dx = -x e^{-x^2/2}\Big|_0^{\infty} + \int_0^{\infty} e^{-x^2/2}\, dx$

$= 0 + \sqrt{2\pi} \cdot \frac{1}{\sqrt{2\pi}} \int_0^{\infty} e^{-x^2/2}\, dx$

$= \frac{\sqrt{2\pi}}{2} = \frac{\sqrt{\pi}}{2} = 1.25$

b) $P(x > 2) = \int_2^{\infty} x e^{-x^2/2}\, dx$

$= -e^{-x^2/2}\Big|_2^{\infty} = e^{-2} = 0.1353$

54. $\alpha\beta = 10$, $\sigma = \sqrt{\alpha\beta^2} = \sqrt{50} = 7.07$

a) using integration by parts

$$P(x \le 50) = \frac{1}{\beta^{\alpha} \cdot \Gamma(\alpha)} \int_0^{50} x^{\alpha-1} e^{-x/\beta} dx$$

$$= \frac{1}{25} \int_0^{50} x e^{-x/5} dx$$

$$= 0.9995$$

b) $$P(x < 10) = \frac{1}{\beta^{\alpha} \Gamma(\alpha)} = \int_0^{10} x^{\alpha-1} e^{x/\beta} dx$$

using incompleme gamma with

$y = x/\beta$

$$P(x < 10) = P(y < 2) = \int_0^2 y e^{-y} dy$$

$$= 0.5940$$

Chapter - 7

2. $y = x^2$, $x = 0, 1, 2, 3$, we obtain $x = \sqrt{y}$

$$g(y) = F(\sqrt{y}) = \binom{3}{\sqrt{y}} \left(\frac{2}{5}\right)^{\sqrt{y}} \left(\frac{3}{5}\right)^{3-\sqrt{y}},$$

For $y = 0, 1, 4, 9$

3. inverse Function of $y_1 = x_1 + x_2$

$y_2 = x_1 - x_2$

$x_1 = (y_1 + y_2)/2$, $x_2 = (y_1 - y_2)/2$

$$g(y_1, y_2) = \binom{2}{\frac{y_1+y_2}{2}, \frac{y_1-y_2}{2}, 2-y_1} \left(\frac{1}{4}\right)^{(y_1+y_2)/2} \times \left(\frac{1}{3}\right)^{(y_1 - y_2/2)} \times \left(\frac{5}{12}\right)^{2-y_1}$$

where $y_1 = 0, 1, 2$, $y_2 = -2, -1, 0, 1, 2$

$y_2 \le y_1$, $y_1 + y_2 = 0, 2, 4$

5. inverse Function of $y = -2 \ln x$ is given by $x = e^{-y/2}$ From which we obtain

$$|J| = |-e^{-y/2}/2| = e^{-y/2}/2$$

now, $g(y) = F(e^{y/2})\,|J| = e^{-y/2}/2$, $y > 0$

which is a chi-squared distribution with 2 degrees of freedom.

8. a) inverse of $y = x^2$ is $x = \sqrt{y}$, $0 < y < 1$

$|J| = 1/2\sqrt{y}$

$$g(y) = F(\sqrt{y})\,|J| = 2(1-\sqrt{y})/2\sqrt{y} = y^{-1/2} - 1, \; 0 < y < 1$$

b) $P(y < 0.1) = \int_0^{0.1} (y^{-1/2} - 1)\,dy$

$= (2y^{1/2} - y)\Big|_0^{0.1}$

$= 0.5324.$

10. a). Let $w = x$, inverse function of $z = x + y$

and $w = x$, $x = w$, $y = z - w$

$0 < w < z$, $0 < z < 1$

$$J = \begin{vmatrix} \frac{\partial x}{\partial w} & \frac{\partial x}{\partial z} \\ \frac{\partial y}{\partial w} & \frac{\partial y}{\partial z} \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ -1 & 1 \end{vmatrix} = 1$$

$g(w, z) = F(w, z - w)\,|J| = 24w(z - w)$, $0 < w < z$
$0 < z < 1$

marginal distribution of z is

$F_1(z) = \int_0^1 24(z - w)\,w\,dw = 4z^2 \quad (0 < z < 1)$

b) $P(1/2 < z < 3/4) = 4\int_{1/2}^{3/4} z^3\,dz = 65/256$

17. moment generating function of x is

$M_x(t) = E(e^{tx}) = \frac{1}{k}\sum_{x=1}^{k} e^{tx}$

$$= \frac{e^t(1 - e^{kt})}{k(1 - e^t)}$$

by summing geometric series

of k terms.

17. The moment generating function of a poisson random variable is

$$M_x(t) = E(e^{tx}) = \sum_{x=0}^{\infty} \frac{e^{tx} e^{-\mu} \mu^x}{x!}$$

$$= e^{-\mu} \sum_{x=0}^{\infty} \frac{(\mu e^t)^x}{x!}$$

$$\mu = M'_x(0) = \mu e^{\mu(e^t-1)+1} \Big|_{t=0}$$

$$= \mu$$

$$\mu' = M''_x(0) = \mu e^{\mu(e^t-1)+t} (\mu e^t + 1) \Big|_{t=0}$$

$$= \mu(\mu+1)$$

$$\sigma^2 = \mu_2' - \mu^2 = \mu(\mu+1) - \mu^2 = \mu$$

1. $M_x(t) = e^{4(e^t-1)}$ we obtain $\mu = 4$,

$\sigma^2 = 4$, $\sigma = 2$

$$P(\mu - 2\sigma < x < \mu + 2\sigma)$$

$$= P(0 < x < 8) = \sum_{x=1}^{3} P(x:4)$$

$$= 0.9489 - 0.0183$$

$$= 0.9306.$$

Chapter-8

2. $\bar{x} = 8.6$ minutes

$\tilde{x} = 9.5$ minutes.

mode are 5 & 10 min.

3. $\bar{x} = 3.2$ sec

$\tilde{x} = 3.1$ sec

5. a) $\bar{x} = 2.4$

b) $\tilde{x} = 2$

c) $= m = 3$

7. $\bar{x} = 53.75$

modes are 75 & 100

10. a) Range $= 4.3 - 2.3 = 2.0$

b) mean $\Rightarrow \bar{x} = \dfrac{2.5 + 3.6 + \ldots + 3.4}{9}$

$= 3.2$

$$s^2 = \frac{(2.5 - 3.2)^2 + (3.6 - 3.2)^2 + \ldots + (3.4 - 3.2)^2}{8}$$

$= 0.4975.$

12, a) $\bar{x} = 11.69$ milligrams.

b) $$s^2 = \frac{\sum_{i=1}^{n} x^2 - \left(\sum_{i=1}^{n} x_i\right)^2}{n(n-1)}$$

$$= \frac{8 \times 1168.21 - 93.5^2}{8 \times 7}$$

$= 10.776.$

17. $z_1 = -1.9$ $z_2 = -0.4$, hence

$P(\mu_{\bar{x}} - 1.9\sigma_{\bar{x}} < \bar{x} < \mu_{\bar{x}} - 0.4\sigma_{\bar{x}})$

$= P(-1.9 < z < -0.4)$

$= 0.3446 - 0.0287$

$= 0.3159$

20. $n = 54$, $\mu_{\bar{x}} = 4$, $\sigma^2_{\bar{x}} = \sigma^2/n = 8/3 / 54 = 4/81$

$\sigma_{\bar{x}} = 2/9$

$z_1 = (4.15 - 4)/(2/9) = 0.68,$

$z_2 = (4.35 - 4)(2/9) = 1.58$

$P(4.15 < \bar{x} < 4.35) = P(0.68 < z < 1.58)$

$= 0.9429 - 0.7517$

$= 0.1912$

23. a) $\mu = \sum x F(x) = 4 \times 0.2 + 5 \times 0.4 + 6 \times 0.3 + 7 \times 0.1 = 5.3$

$\sigma^2 = \sum (x - \mu)^2 F(x) = (4 - 5.3)^2 * 0.2 + (5 - 5.3)^2 \times 0.4 + (7.53)^2 \times 0.1 = 0.81$

$+ (6 - 5.3)^2 \times 0.3$

b) $n = 36$, $\mu_{\bar{x}} = \mu = 5.3$, $\sigma^2_{\bar{x}} = \sigma^2/n = 0.81/36$

$= 0.0225$

c) $n = 36$, $\mu_{\bar{x}} = 5.3$, $\sigma_{\bar{x}} = 0.9/6 = 0.15$

$z = (5.5 - 5.3)/0.15$

$= 1.3$

$P(\bar{x} < 5.5) = P(z < 1.33) = 0.9082$

24. $n = 36$, $\mu_{\bar{x}} = 40$, $\sigma_{\bar{x}} = 2/6 = 1/3,$

$z = (40.5 - 40)(1/3) = 1.5$

$P\left(\sum_{i=1}^{36} x_i > 1458\right) = P(\bar{x} > 40.5) = P(z > 1.5)$
$= 1 - 0.9332$
$= 0.0668$

26. $n = 64$, $\mu_{\bar{x}} = 3.2$, $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 1.6/8 = 0.2$

a) $z = (2.7 - 3.2)/0.2 = -2.5$, $P(\bar{x} < 2.7)$
$= P(z < -2.5)$
$= 0.0062$

b) $z = (3.5 - 3.2)/0.2 = 1.5$, $P(\bar{x} > 3.5) = P(z > 1.5)$
$= 1 - 0.9332$
$= 0.0668$

c) $z_1 = (3.2 - 3.2)/0.2 = 0$, $z_2 = (3.4 - 3.2)/0.2 = 1.0$
$P(3.2 < \bar{x} < 3.4) = P(0 < z < 1.0)$
$= 0.8413 - 0.5000$
$= 0.3413$..

37. a) 27.488
b) 18.475
c) 36.415

40. a) $\chi^2_\alpha = \chi^2_{0.99} = 0.297$
b) $\chi^2_\alpha = \chi^2_{0.025} = 32.852$
c) $\chi^2_{0.05} = 37.652$

$\alpha = 0.05 - 0.045 = 0.005$
$\chi^2_\alpha = \chi^2_{0.005} = 46.928$

41. a) $P(s^2 > 9.1) = P\left(\frac{(n-1)s^2}{\sigma^2} > \frac{(24)(9.1)}{6}\right)$

$= P(\chi^2 > 36.4) = 0.05$

b) $P(3.462 < s^2 < 10.745) = P\left(\frac{24 \times 3.462}{6} < \frac{(n-1)s^2}{\sigma^2} < 24 \times 10.7/8\right)$

$= P(13.848 < x^2 < 42.980)$

$= 0.95 - 0.01$

$= 0.94$

45. a) $P(T < 2.365) = 1 - 0.025 = 0.975$

b) $P(T > 1.318) = 0.10$

c) $P(T < 2.179) = 1 - 0.025 = 0.975$

$P(T > 1.356) = 0.10$

$P(T < -1.356) = P(T > 1.356) = 0.10$

$P(-1.356 < T < 2.179) = 0.975 - 0.010$

$= 0.875$

d) $P(T > -2.567) = 1 - P(T > 2.567)$

$= 1 - 0.01$

$= 0.99$

49. $t = (24 - 20)/(4.1/3) = 2.927$ with 8 degree of freedom

$t_{0.01} = 2.896$

no, $\mu > 20$

50. $\bar{x} = 0.45$, $s^2 = 0.0336$

$t = (0.475 - 0.5)/0.0648 = -0.39$

$P(\bar{x} < 0.475) = P(T < 0.39) \approx 0.35$