Lecture-17

Poisson Distribution

Poisson Distribution:

The probability distribution of the Poisson random variable X, representing the number of outcomes occurring a given time interval t is

$$P(x;\lambda t) = \frac{e^{-\lambda t}\,(\lambda t)^x}{x!}, \qquad x = 0, 1, 2, ...$$

For $t = 1$

$$P(x;\lambda) = \frac{e^{-\lambda}\lambda^x}{x!}, \qquad x = 0, 1, 2, ...$$

The Poission random variable X, has the mean $\mu = \lambda$ and variance $\sigma^2 = \lambda$.

Note: For the purpose of minimizing calculation, refer Poisson distribution table in the problems.

(Q.58) A certain area of the eastern United States is, on average, hit by 6 hurricanes a year. Find the probability that in a given year that area will be hit by (a) fewer than 4 hurricanes; (b) anywhere from 6 to 8 hurricanes.

Ans:

The average number of hurricane hits in a year is 6 i.e. $\lambda = 6$

Let X : The number of hurricane hits in a year

a.

P(fewer than 4 hurricanes) means

$$P(X \leq 3) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)$$
$$= \tfrac{e^{-6}6^0}{0!} + \tfrac{e^{-6}6^1}{1!} + \tfrac{e^{-6}6^2}{2!} + \tfrac{e^{-6}6^3}{3!} = 0.1512$$

b.

P(anywhere from 6 to 8 hurricanes) means

$$P(6 \le X \le 8) = P(X=6) + P(X=7) + P(X=8)$$
$$= \tfrac{e^{-6}6^6}{6!} + \tfrac{e^{-6}6^7}{7!} + \tfrac{e^{-6}6^8}{8!} = 0.4015$$
$$Or$$
$$= F(8) - F(5) = 0.4015 \ (using\ table)$$

(Q.60) The average number of field mice per acre in a 5-acre wheat field is estimated to be 12. Find the probability that fewer than 7 field mice are found (a) on a given acre; (b) on 2 of the next 3 acres inspected.

Ans:

The average number of field mice per acre is 12 i.e. $\lambda = 12$

(a) Let X : The number of mice per acre

Hence,

$$P(X < 7) = P(X \le 6) = 0.0458 \ \ (using\ table)$$

(b) Let Y : The number of acres of land inspected.

Here Y follows binomial distribution.

Given $n = 3, y = 2$

Hence, $P(Y = 2) = \begin{pmatrix} 3 \\ 2 \end{pmatrix} p^2 q^{3-2}, \ \ with\ p = 0.0458$ (from part a), $q = 1 - p$

(Q.69) The probability that a person will die when he or she contracts a virus infection is 0.001. Of the next 4000 people infected, what is the mean number who will die?

Ans:

Given $p = 0.001, \ n = 4000$

Therefore, $\mu = np = 4000 \times 0.001 = 4$