Number System: NUMBER-BASE CONVERSIONS

Lecture-2

By
Bhagyalaxmi Behera
Asst. Professor (Dept. of ECE)

Number Systems

A **number system** is defined as a **system** of expressing **numbers**. It is the mathematical notation for representing **numbers** of a given set by using digits or other symbols in a consistent manner.

- Decimal Number System
  - Uses 10 digits from 0 to 9= $\{ 0, 1, 2, 3, \ldots, 9 \}$
  - Example-35,64,135,2345 etc

- Binary Number System
  - Uses two digits, 0 and 1.
  - Also called base 2 number system
  - Example-100,011,101,1100,10 etc

Number Systems

- Octal Number System:
  - Uses eight digits, 0,1,2,3,4,5,6,7.
  - Also called base 8 number system
  - Example-35,64 ,71,135 etc

- Hexadecimal Number System:
  - Uses 10 digits and 6 letters, 0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F.
  - Letters represents numbers starting from 10.

    A = 10, B = 11, C = 12, D = 13, E = 14, F = 15.
  - Also called base 16 number system.
  - Example-135,782,18D,6FC etc

Decimal Number System

- Decimal number system, symbols = { 0, 1, 2, 3, …, 9 }
- Example: $(7594)_{10} = (7x10^3) + (5x10^2) + (9x10^1) + (4x10^0)$

A decimal number such as 7,594 represents a quantity equal to 7 thousands, plus 5 hundreds, plus 9 tens, plus 4 units. The thousands, hundreds, etc., are powers of 10 implied by the position of the coefficients (symbols) in the number.

- In general, $(a_n a_{n-1} \ldots a_0)_{10} = (a_n \text{ x } 10^n) + (a_{n-1} \text{ x } 10^{n-1}) + \ldots + (a_0 \text{ x } 10^0)$
- $(2.75)_{10} = (2 \text{ x } 10^0) + (7 \text{ x } 10^{-1}) + (5 \text{ x } 10^{-2})$
- In general, $(a_n a_{n-1} \ldots a_0 \,.\, f_1 f_2 \ldots f_m)_{10} = (a_n \text{ x } 10^n) + (a_{n-1} \text{x} 10^{n-1}) + \ldots + (a_0 \text{ x } 10^0) + (f_1 \text{ x } 10^{-1}) + (f_2 \text{ x } 10^{-2}) + \ldots + (f_m \text{ x } 10^{-m})$

NUMBER-BASE CONVERSIONS

- Representations of a number in a different radix are said to be equivalent if they have the same decimal representation. For example, $(0011)_8$ and $(1001)_2$ are equivalent—both have decimal value 9.

- The conversion of a number in base *r to decimal is done by* expanding the number in a power series and adding all the terms as shown previously.

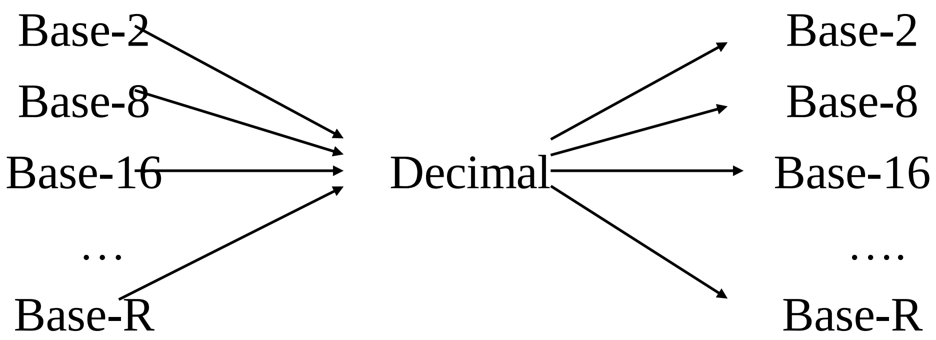
- We now present a general procedure for the reverse operation of converting a decimal number to a number in base *r.*

- *If the number includes a radix point, it is necessary to* separate the number into an integer part and a fraction part, since each part must be converted differently.

- The conversion of a decimal integer to a number in base *r is done* by dividing the number and all successive quotients by *r and accumulating the remainders.*

Conversion between Decimal and other Bases

- Decimal to base-R
  - whole numbers: repeated division-by-R
  - fractions: repeated multiplication-by-R



Decimal-to-Binary Conversion

- *Repeated Division-by-2 Method* (for whole numbers)
- *Repeated Multiplication-by-2 Method* (for fractions)

Repeated Division-by-2 Method

- To convert a whole number to binary, use successive division by 2 until the quotient is 0. The remainders form the answer, with the first remainder as the *least significant bit (LSB)* and the last as the *most significant bit (MSB)*.

$(43)_{10} = (101011)_2$

2	43		
2	21	rem 1	← LSB
2	10	rem 1	
2	5	rem 0	
2	2	rem 1	
2	1	rem 0	
	0	rem 1	← MSB

Decimal to Binary Conversion

Divide by 2 Process

Decimal # $13 \div 2 = 6$ remainder 1

$6 \div 2 = 3$ remainder 0

$3 \div 2 = 1$ remainder 1

$1 \div 2 = 0$ remainder 1

Divide-by-2 Process Stops When Quotient Reaches 0

1 1 0 1

Repeated Multiplication-by-2 Method

To convert decimal fractions to binary, repeated multiplication by 2 is used. The process is continued until the fraction becomes 0 or until the number of digits has sufficient accuracy.

The coefficients of the binary number are obtained from the integers as follows: The first integer is written as the MSB, and the last as the LSB.

	Coefficient	
$0.3125 \times 2 = 0.625$	0	←MSB
$0.625 \times 2 = 1.25$	1	
$0.25 \times 2 = 0.50$	0	
$0.5 \times 2 = 1.00$	1	←LSB

$$(0.3125)_{10} = (.0101)_2$$

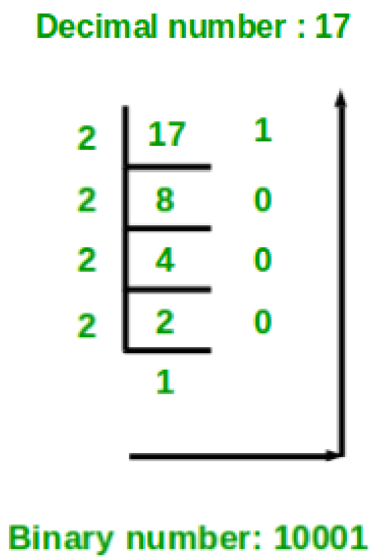
Decimal-to-Binary Conversion

Convert $(0.6875)_{10}$ **to Binary**

	integer		*fraction*	*coefficient*
$0.6875 \times 2 =$	1	+	0.3750	$a_{-1} = 1$
$0.3750 \times 2 =$	0	+	0.7500	$a_{-2} = 0$
$0.7500 \times 2 =$	1	+	0.5000	$a_{-3} = 1$
$0.5000 \times 2 =$	1	+	0.0000	$a_{-4} = 1$

$$(0.6875)_{10} = (0.a_{-1}a_{-2}a_{-3}a_{-4})_2 = (0.1011)_2$$

Decimal to Binary Conversion of 17.65



$$0.65 * 2 = 1.3 \longrightarrow 1$$

$$0.3 * 2 = 0.6 \longrightarrow 0$$

$$0.6 * 2 = 1.2 \longrightarrow 1$$

$$0.2 * 2 = 0.4 \longrightarrow 0$$

$$0.4 * 2 = 0.8 \longrightarrow 0$$

$$(17.65)_{10} = (10001.10100)_2$$

Practice

Convert the following decimal numbers into binary:

Decimal $(11)_{10}$

Decimal $(4)_{10}$

Decimal $(17)_{10}$

Decimal $(47.2)_{10}$

Binary to Decimal Conversion

$$10110_2 = 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 = 22_{10}$$

$(1010.011)_2 = (?)_{10}$

Place Value

- ***Example* - Place value in binary system:**

	2^3	2^2	2^1	2^0
Place Value	8	4	2	1
Binary Number	1	1	0	0

RESULT: $(1100)_2$ = decimal 8 + 4 + 0 + 0 = $(12)_{10}$

Binary to Decimal Conversion

Convert Binary Number 110011 to a Decimal Number:

	2^5	2^4	2^3	2^2	2^1	2^0
Binary	1	1	0	0	1	1

Decimal $32 + 16 + 0 + 0 + 2 + 1 = 51$

1-Bit Binary Numbers	2-Bit Binary Numbers	3-Bit Binary Numbers	4-Bit Binary Numbers	Decimal Equivalents
0	00	000	0000	0
1	01	001	0001	1
	10	010	0010	2
	11	011	0011	3
		100	0100	4
		101	0101	5
		110	0110	6
		111	0111	7
			1000	8
			1001	9
			1010	10
			1011	11
			1100	12
			1101	13
			1110	14
			1111	15

Hexadecimal Number System

Uses 16 symbols - Base 16 System
0-9, A, B, C, D, E, F

Decimal	Binary	Hexadecimal
1	0001	1
9	1001	9
10	1010	A
15	1111	F
16	10000	10

Decimal to Hexadecimal Conversion

Divide by 16 Process

Decimal # $47 \div 16 = 2$ remainder 15

Divide-by-16 Process
Stops When
Quotient Reaches 0

0 remainder 2

2 F

Find the Hex equivalent for the Decimal 3509

Divisor	Quotient	Remainder	
16	3509	5	LSD
16	219	11	
16	13	13	MSD
	0		

MSD - most significant digit
LSD - least significant digit

For Hex value 13 = D, 11 = B & 5 = 5
Therefore, the equivalent Hex number for decimal 3509 is DB5

Hexadecimal to Decimal Conversion

Convert hexadecimal number 2DB to a decimal number

Place Value	256	16	1
	16^2	16^1	16^0
Hexadecimal	2	D	B
	(256 x 2)	(16 x 13)	(1 x 11)
Decimal	512 +	208 +	11 = 731

$$2ED_{16} = 2 \times 16^2 + E \times 16^1 + D \times 16^0 = 749_{10}$$

two two hundred fifty six's	fourteen sixteens	thirteen ones

Practice

Convert Hexadecimal number A6 to Binary

A6 = 1010 0110 (Binary)

Convert Hexadecimal number 16 to Decimal

16 = 22 (Decimal)

Convert Decimal 63 to Hexadecimal

63 = 3F (Hexadecimal)

Hexadecimal Digit	Decimal Equivalent	Binary Equivalent
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Octal Numbers

Uses 8 symbols - Base 8 System
0, 1, 2, 3, 4, 5, 6, 7

Decimal	Binary	Octal
1	001	1
6	110	6
7	111	7
8	001 000	10
9	001 001	11

Octal ⟺ Decimal conversion

To Convert Decimal To Octal

		Remainder
8	123	3
8	15	7
	1	

$123_{10} = 173_8$

Octal Number to Decimal

$$2754_8 = \begin{array}{lr} 2 \times 8^3 & 1024 \\ 7 \times 8^2 & 448 \\ 5 \times 8^1 & 40 \\ 4 \times 8^0 & 4 \\ \hline & 1516_{10} \end{array}$$

$$(630.4)_8 = 6 \times 8^2 + 3 \times 8 + 4 \times 8^{-1} = (408.5)_{10}$$

Decimal to Octal Conversion

Convert $(0.513)_{10}$ to octal.

$$0.513 \times 8 = 4.104$$

$$0.104 \times 8 = 0.832$$

$$0.832 \times 8 = 6.656$$

$$0.656 \times 8 = 5.248$$

$$0.248 \times 8 = 1.984$$

$$0.984 \times 8 = 7.872$$

$$(0.513)_{10} = (0.406517\ldots)_8$$

Binary-Octal/Hexadecimal Conversion

- Binary $\rightarrow$ Octal: Partition in groups of 3

  $(10\ 111\ 011\ 001\ .\ 101\ 110)_2 = (2731.56)_8$

- Octal $\rightarrow$ Binary: reverse

  $(2731.56)_8 = (10\ 111\ 011\ 001\ .\ 101\ 110)_2$

- Binary $\rightarrow$ Hexadecimal: Partition in groups of 4

  $(101\ 1101\ 1001\ .\ 1011\ 1000)_2 = (5D9.B8)_{16}$

- Hexadecimal $\rightarrow$ Binary: reverse

  $(5D9.B8)_{16} = (101\ 1101\ 1001\ .\ 1011\ 1000)_2$

Hexadecimal and Binary Conversions

- Hexadecimal to Binary Conversion

Hexadecimal	C	3
	↓	↓
Binary	1100	0011

- Binary to Hexadecimal Conversion

Binary	1110	1010
	↓	↓
Hexadecimal	E	A

Binary-Octal/Hexadecimal Conversion

$$(\underbrace{10}_{2}\,\underbrace{110}_{6}\,\underbrace{001}_{1}\,\underbrace{101}_{5}\,\underbrace{011}_{3}\;\cdot\;\underbrace{111}_{7}\,\underbrace{100}_{4}\,\underbrace{000}_{0}\,\underbrace{110}_{6})_2 = (26153.7406)_8$$

$$(\underbrace{10}_{2}\,\underbrace{1100}_{C}\,\underbrace{0110}_{6}\,\underbrace{1011}_{B}\;\cdot\;\underbrace{1111}_{F}\,\underbrace{0010}_{2})_2 = (2C6B.F2)_{16}$$

$$(673.124)_8 = (\underbrace{110}_{6}\,\underbrace{111}_{7}\,\underbrace{011}_{3}\;\cdot\;\underbrace{001}_{1}\,\underbrace{010}_{2}\,\underbrace{100}_{4})_2$$

$$(306.\,D)_{16} = (\underbrace{0011}_{3}\,\underbrace{0000}_{0}\,\underbrace{0110}_{6}\;\cdot\;\underbrace{1101}_{D})_2$$

Base-R to Decimal Conversion

- $(1101.101)_2 = 1\times2^3 + 1\times2^2 + 1\times2^0 + 1\times2^{-1} + 1\times2^{-3}$
  $= 8 + 4 + 1 + 0.5 + 0.125$
  $= (13.625)_{10}$
- $(572.6)_8 = 5\times8^2 + 7\times8^1 + 2\times8^0 + 6\times8^{-1}$
  $= 320 + 56 + 2 + 0.75 = (378.75)_{10}$
- $(2A.8)_{16} = 2\times16^1 + 10\times16^0 + 8\times16^{-1}$
  $= 32 + 10 + 0.5 = (42.5)_{10}$
- $(341.24)_5 = 3\times5^2 + 4\times5^1 + 1\times5^0 + 2\times5^{-1} + 4\times5^{-2}$
  $= 75 + 20 + 1 + 0.4 + 0.16 = (96.56)_{10}$

Numbers with Different Bases

Decimal (base 10)	Binary (base 2)	Octal (base 8)	Hexadecimal (base 16)
00	0000	00	0
01	0001	01	1
02	0010	02	2
03	0011	03	3
04	0100	04	4
05	0101	05	5
06	0110	06	6
07	0111	07	7
08	1000	10	8
09	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

Problems

1. Convert the following numbers with the indicated bases to decimal:

(a) $(4310)_5$ (b) $(198)_{12}$
(c) $(435)_8$ (d) $(345)_6$

2. Convert the hexadecimal number 64CD to binary, and then convert it from binary to octal.

3. Express the following numbers in decimal:

(a) $(10110.0101)_2$ (b) $(16.5)_{16}$

(c) $(26.24)_8$ (d) $(DADA.B)_{16}$

(e) $(1010.1101)_2$

4. Convert the following binary numbers to hexadecimal and to decimal:

(a) 1.10010 (b) 110.010.

5. Convert the decimal number 431 to binary in two ways:

(a) convert directly to binary;

(b) convert first to hexadecimal and then from hexadecimal to binary.

Which method is faster?

6. Determine the base of the numbers in each case for the following operations to be correct:
(a)14/2 = 5 (b) 54/4 = 13 (c) 24+17=40

Solution:

The base of the numbers in each case for the following operations to be correct:

14/2 = 5;

Find decimal equivalent

$14=1 \times r^1 + 4 \times r^0 = r + 4$

$2=2 \times r^0=2$

$5=5 \times r^0=5$

$(4+r)/2=5$

Solving this equation, we get r=6, base 6

$54/4 = 13$;

Find decimal equivalent

$54=5 \times r^1 + 4 \times r^0 = 5r + 4$

$4=4 \times r^0=4$

$13=1 \times r^1 + 3 \times r^0 = r + 3$

$(5r+4)/4= r + 3$

Solving this equation, we get r=8, base 8

$24+17=40$;

Find decimal equivalent

$24=2 \times r^1 + 4 \times r^0 = 2r + 4$

$17=1 \times r^1 + 7 \times r^0 = r + 7$

$40=4 \times r^1 + 0 \times r^0 = 4r + 0$

$(2r + 4) + (r + 7) = 4r$

Solving this equation, we get r=11, base 11