PROBABILITY AND STATISTICS (MTH 2002)
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SECTION → CSIT - D / 2041010

5 (a) Given, $f(x) = c(x^2+4)$ $x = 0, 1, 2, 3$

We know, $\sum_{x=0}^{3} f(x) = 1$

$c(4) + c(5) + c(8) + c(3) = 1$

$\Rightarrow 30c = 1$

Hence, $c = 1/30$

(b) Given, $f(x) = c\, {}^2C_x \; {}^3C_{3-x}$, for $x = 0, 1, 2$

$c(1+6+3) = 1$

$\Rightarrow c = 1/10$

6 (a) $f(y) \geq 0$ and $\int_0^1 5(1-y)^4 dy$

$= \left[-(1-y)^5\right]\Big|_0^1$

$= \left[-(0)+1\right] = 1$

$\therefore$ It is a density function.

(b) $P(Y<0.1) = \int_{-\infty}^{0.1} 5(1-y)^4 dy = \int_0^{0.1} 5(1-y)^4 dy$

$= \left(-(1-y)^5\right)\Big|_0^{0.1}$

$= (-0.9)^5 + 1^5$

$= 0.40951$

(c) $P(Y>0.5) = 1 - P(Y \le 0.5)$

$$= 1 - \int_0^{0.5} 5(1-y)^4 \, dy$$

$$= 1 - \int_0^{0.5} 5(1-y)^4 \, dy$$

$$= 1 - \left(-(1-y)^5\right)\Big|_0^{0.5}$$

$$= 1 - \left(-0.5^5 + 1^5\right)$$

$$= 0.03125$$

7 (a) $1 = K\int_{30}^{50}\int_{30}^{50}(x^2+y^2)\,dx\,dy = K(50-30)\left(\int_{30}^{50} x^2\,dx + \int_{30}^{50} y^2\,dy\right)$

$$= \frac{392K}{3}\cdot 10^4 \Rightarrow K = \frac{3}{392}\cdot 10^{-4}$$

(b) $P(30 \le X \le 40,\ 40 \le Y \le 50) = \frac{3}{392}\cdot 10^{-4}\int_{30}^{40}\int_{40}^{50}(x^2+y^2)\,dy\,dx$

$$= \frac{3}{392}\cdot 10^{-3}\left(\int_{30}^{40} x^2\,dx + \int_{40}^{50} y^2\,dy\right)$$

$$= \frac{3}{392}\cdot 10^{-3}\left(\frac{40^3-30^3}{3} + \frac{50^3-40^3}{3}\right) = \frac{49}{196}$$

(c) $P(30 \le X \le 40,\ 30 \le Y \le 40) = \frac{3}{392}\cdot 10^{-4}\int_{30}^{40}\int_{30}^{40}(x^2+y^2)\,dx\,dy$

$$= 2\cdot\frac{3}{392}\cdot 10^{-4}(40-30)\int_{30}^{40} x^2\,dx = \frac{3}{196}\cdot 10^{-3}\,\frac{40^3-30^3}{3}$$

$$= 37/196$$

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(8) $f(x) = {}^3C_x \left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{(3-x)}$, $x = 0, 1, 2, 3$

Mean →

$E(X) = \sum_{x=0}^{3} x f(x) = 0(27/64) + 1(27/64)$
$+ 2(9/64) + 3(1/64)$
$= 3/4$

Expected value of random variable →

$E(X^2) = 0(27/64) + 1(27/64) + 4(9/64)$
$+ 9(1/64)$

$= 9/8$

(9) $Y = 3X - 2$; $f(x) = (1/4)\, e^{-x/4}$

Mean → $\mu_Y = E(3X-2) \Rightarrow \frac{1}{4} \int_0^\infty (3x-2) e^{-x/4}\, dx$

$= 10$

Variance → $\sigma_Y^2 = E\{[(3x-2) - 10]^2\}$

$= \frac{9}{4} \int_0^\infty (x-4)^2 e^{-x/4}\, dx$

$= 144$

(10) (a) $E(Y) = \int_0^\infty y\, e^{-y/4}\, dy = 4$

(b) $E(Y^2) = \int_0^\infty y^2 e^{-y/4}\, dy = 32$

$Var(Y) = 32 - 4^2 = 16$