Lecture-14

Binomial Distribution

Bernoulli trails:

A Series of trails that satisfies the following assumptions is known as Bernoulli trail.

1. There are only two possible outcomes for each trail (success and failure)
2. The probability of success is same for each trail
3. The outcomes from different trails are independent

Example-1:

Tossing a Coin 100 times is a Bernoulli trail.

There are only 2 outcomes, Head and Tail. Here getting Head can be considered as success and Tail as failure. Probability of getting Head or success remains same though out the process and the events are independent.

Binomial distribution

Consider a Bernoulli trail which results in success with probability p and a failure with probability $q = 1 - p$.

Let X: the number of success in n trails.

Then the probability distribution of the binomial random variable X is

$$P(X = x) = f(x) = b(x; n, p) = \binom{n}{x} p^x q^{n-x}, \qquad x = 0, 1, 2, ...n$$

Example-2:

If Probability of hitting the target is $\frac{3}{4}$ and three shots are fired, then

(i)Find the probability of hitting the target 2 times.

(ii)Formulate the binomial Probability distribution function

Ans:

Total no. of trails n=3.

Let X = no. of times hitting the target (no. of success), $x = 0, 1, 2, 3$

P (success) = P (hitting the target) =$\frac{3}{4}$

P (failure) = $\frac{1}{4}$

Therefore

(i) $P(X = 2) = f(2) = b\left(2; 3, \frac{3}{4}\right) = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \left(\frac{3}{4}\right)^2 \left(\frac{1}{4}\right)^{3-2}$

(ii) $P(X = x) = f(x) = b\left(x; 3, \frac{3}{4}\right) = \begin{pmatrix} 3 \\ x \end{pmatrix} \left(\frac{3}{4}\right)^x \left(\frac{1}{4}\right)^{3-x}, x = 0, 1, 2, .3$

Note: The mean and variance of binomial distribution$\boldsymbol{f(x) = b\,(x; n, p)}$ are $\boldsymbol{\mu = np}$ and $\boldsymbol{\sigma^2 = npq}$

(Q.11) The probability that a patient recovers from a delicate heart operation is 0.9. What is the probability that exactly 5 of the next 7 patients having this operation survive?

Ans:

Let X = no. of patients recovered from the heart operation i.e. $x = 0, 1, 2, ...7$

Here, $n = 7$, $p = 0.9$, $q = 0.1$

hence,

$$P(X = 5) = f(5) = b\,(5; 7, 0.9) = \begin{pmatrix} 7 \\ 5 \end{pmatrix} (0.9)^5 (0.1)^{7-5}$$

$$= 0.1240$$

Binomial distribution Table:

The cumulative distribution for the binomial distribution is pre-calculated and given in the form of a table.

Examples-3:(Use of binomial distribution Table)

We know

$$\begin{aligned} P(X \leq 4) &= F(4) \\ &= P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) \end{aligned}$$

Suppose, $n = 5$, $p = 0.6$

Hence,

$$\begin{aligned} P(X \leq 4) &= F(4) \\ &= \binom{5}{0} (0.6)^0 (0.4)^{5-0} + \binom{5}{1} (0.6)^1 (0.4)^{5-1} + ... + \binom{5}{4} (0.6)^4 (0.4)^{5-4} \\ &= \sum_{x=0}^{4} b(x; 5, 0.6) = 0.9222 \end{aligned}$$

$P(X \leq 4) = B(4; 5, 0.6) = 0.9222$ (from binomial distribution table)

In general $P(X \leq x) = B(x; n, p)$ and $b(x; n, p) = B(x; n, p) - B(x-1; n, p)$

(Q.15) It is known that 60% of mice inoculated with a serum are protected from a certain disease. If 5 mice are inoculated, find the probability that (a) none contracts the disease; (b) fewer than 2 contract the disease; (c) more than 3 contract the disease.

Ans:

Let X = no. of mice from the disease after inoculated, $x = 0, 1, 2, 3, 4, 5$

Here, $n = 5$, $p = 0.4$, $q = 0.6$

(i) $P(X = 0) = f(0) = \binom{5}{0} (0.4)^0 (0.6)^5 = 0.0778$

(ii) $P(X < 2) = P(X \leq 1)$

$$= P(X = 0) + P(X = 1)$$
$$= \begin{pmatrix} 5 \\ 0 \end{pmatrix} (0.4)^0 (0.6)^5 + \begin{pmatrix} 5 \\ 1 \end{pmatrix} (0.4)^1 (0.6)^{5-1} = 0.3370$$

(iii) $P(X > 3) = P(X = 4) + P(X = 5)$

$$= \begin{pmatrix} 5 \\ 4 \end{pmatrix} (0.4)^4 (0.6)^{5-4} + \begin{pmatrix} 5 \\ 5 \end{pmatrix} (0.4)^5 (0.6)^{5-5} = 0.087$$

(Q.16) Suppose that airplane engines operate independently and fail with probability equal to 0.4. Assuming that a plane makes a safe fight if at least one-half of its engines run, determine whether a 4-engine plane or a 2 engine plane has the higher probability for a successful fight.

Ans:

Case-1

The plane has 4 engines; $n = 4$

The plane will make a safe flight if 2 or more engines are working.

$$P(X \geq 2) = 1 - P(X \leq 1)$$
$$= 1 - [P(X = 0) + P(X = 1)]$$
$$= 1 - \left[\begin{pmatrix} 4 \\ 0 \end{pmatrix} (0.6)^0 (0.4)^{4-0} + \begin{pmatrix} 4 \\ 1 \end{pmatrix} (0.6)^1 (0.4)^{4-1} \right] = 0.8208$$

Case-2

The plane has 2 engines; $n = 2$

The plane will make a safe flight if 1 or more engines are working.

$$P(X \geq 1) = 1 - P(X = 0)$$
$$= 1 - \left[\begin{pmatrix} 2 \\ 0 \end{pmatrix} (0.6)^0 (0.4)^{2-0} \right] = 0.84$$

Conclusion: comparing the above two cases, a 2 engine flight has higher probability for a successful flight.