(8)

Basic : Taking first 3 natural nos.

$1^3 + 2^3 + 3^3 = 36$ This is divisible by 9

Inductive : For first consecutive no. being K. Assume it is true.

$\therefore K^3 + (K+1)^3 + (K+2)^3 = 9\lambda$

So, for first consecutive being $K+1$

$(K+1)^3 + (K+2)^3 + (K+3)^3$

$= (K+1)^3 + (K+2)^3 + K^3 + 9K^2 + 27K + 27$

$= K^3 + (K+1)^3 + (K+2)^3 + 9(K^2 + 3K + 3)$

$= 9\lambda + 9(K^2+3K+3)$

$= 9\alpha$

So, Statement holds true for $n = K+1$

Hence proved.

(9)

$$\sum_{i=0}^{n} (2i+1)^2 = \frac{(n+1)(2n+1)(2n+3)}{3}$$

basic : for $n = 0$

$$(2(0)+1)^2 = \frac{(0+1)(2\times0+3)(2\times0+1)}{3}$$

$1 = 3/3 \Rightarrow$ LHS = RHS.

inductive : for $n = K$, Assume true.

$$\sum_{i=0}^{K} (2i+1)^2 = \frac{(K+1)(2K+1)(2K+3)}{3}$$

for $n = K+1$

$$\sum_{i=0}^{K} (2i+1)^2 = \sum_{i=0}^{K} (2i+1)^2 + (2(K+1)+1)^2$$

$$= \frac{(K+1)(2K+1)(2K+3)}{3} + (2K+3)^2$$

$$= \frac{(K+1)(2K+1)(2K+3) + 3(2K+3)^2}{3}$$

$$= \frac{(2k^2+3k+1)(2k+3) + 3(2k+3)^2}{3}$$

$$= \frac{(2k+3)(2k^2+3k+1+6k+9)}{3} = \frac{(2k+3)(2k^2+9k+10)}{3}$$

$$= \frac{(2k+3)(2k^2+5k+4k+10)}{3} = \frac{(2k+3)(2k+5)(k+2)}{3}$$

$$= \frac{[(k+1)+1][2(k+1)+1][2(k+1)+3]}{3} = \underline{RHS}$$

hence proved.