Q.1 If X is a discrete random variable with probability distribution

$$f(x) = \begin{cases} \frac{1}{4}, & x = 1,2,3,4 \\ 0, & otherwise \end{cases}$$

Then $P(X<4 \mid X>1)$ is:

(A) $\frac{1}{2}$

(B) $\frac{2}{3}$

(C) $\frac{1}{3}$

(D) 1

Q.2 Let A and B be two events. Suppose the probability that neither A nor B occurs is 2/3. The probability that one or both occur is :

A. $\frac{1}{6}$

B. $\frac{1}{3}$

C. $\frac{2}{3}$

D. None

Q.3 For any constant k , Var(k) is :

(A) k
(B) 1
(C) k^2
(D) 0

Q.4 Consider the density function

$$f(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & elsewhere \end{cases}.$$

The cumulative distribution function at x = 0.5, i.e., F(0.5) is :

(A) 0

(B) 0.25

(C) 0.75

(D) 1

Q.5 Suppose that P(A) = 0.4, P(B) = 0.3, then P(A ∩ B) *is necessarily* :

(A) ≤ 0.3

(A) $\leq$ 0.3

(B) ≤ 0.7

(C) $\geq$ 0.3

(D) ≥ 0.7

Q.6 A random variable X has a mean $\mu = 10$ and a variance $\sigma^2 = 4$. Using Chebyshev's theorem, the value of constant c such that $P(|X - 10| \geq c) \leq 0.01$ is:

(A) 10

(B) 100

(C) 20

(D) None

Q.7 If X and Y are independent random variables with variances $\sigma_X^2 = 5$ and $\sigma_Y^2 = 3$, the variance of the random variable $Z = -2X + 4Y - 3$ is:

(A) 8

(B) 2

(C) 68

(D) None

Q.8 The probability of getting a total of 5 or 10 when a pair of fair dice is tossed is:

(A) $\frac{1}{2}$

(B) $\frac{1}{18}$

(C) $\frac{7}{36}$

(D) None

Q.9 If 2 books are picked at random from a shelf containing 5 novels, 3 books of poems, and a dictionary, then the probability that the dictionary is not selected is:

(a) $\frac{2}{9}$

(b) $\frac{7}{9}$

(c) $\frac{1}{3}$

(d) None

Q.10 One bag contains 3 white balls and 2 black balls, and a second bag contains 2 white balls and 3 black balls. One ball is drawn from the first bag and placed unseen in the second bag. The probability that a ball now drawn from the

ball is drawn from the first bag and placed unseen in the second bag. The probability that a ball now drawn from the second bag is black is:

(A) 0.6

(B) 0.57

(C) 0.1

(D) None

Q.11 For any constant k, E(k) is:

(A) 0

(B) k

(C) k^2

(D) 1

Q.12 A manufacturing firm employs three analytical plans for the design and development of a particular product. For cost reasons, all three are used at varying times. In fact, plans 1, 2, and 3 are used for 40%, 10%, and 50% of the products, respectively. The defect rate is different for the three procedures as follows: P(D|P1)=0.01, P(D|P2)=0.03, P(D|P3)=0.02, where P(D|Pj) is the probability of a defective product, given plan j. If a random product was observed and found to be defective, what is the probability that plan 2 was used and thus responsible?

(A) 0.127

(B) 0.116

(C) 0.176

(D) None

Q.13 The probability that a patient recovers from a delicate heart operation is 0.8. What is the probability that none of the next 3 patients who have this operation survive ?

(a) 0.8

(b) 0.2

(c) 0.512

(d) 0.008

Q.14 Let X be a continuous random variable with pdf f(x) and cdf F(x). Which of the following is NOT necessarily true?

(A) $F(-\infty) = 0$

(B) $F(\infty) = 1$

(C) $f(0) = 0$

(D) P(X=0) = 0

Q.15 A certain form of cancer is known to be found in women over 60 with probability 0.07. A blood test exists for the detection of the disease, but the test is not infallible. In fact, it is known that 10% of the time the test gives a false negative (i.e., the test incorrectly gives a negative result) and 5% of the time the test gives a false positive (i.e., incorrectly gives a positive result). If a woman over 60 is known to have taken the test and received a favorable (i.e., negative) result, the probability that she has the disease is

(A) 0.001

(B) 0.005

(C) 0.008

(D) None

Q.16 If cumulative distribution function of a random variable is given by $F(0) = \frac{1}{16}$, $F(1) = \frac{5}{16}$, $F(3) = \frac{11}{16}, F(4) = \frac{15}{16}, F(5) = 1$, *then* $P(X \geq 2)$ *is*

(A) $\frac{15}{16}$

(B) $\frac{1}{16}$

(C) $\frac{7}{16}$

(D) $\frac{11}{15}$

Q.17 The value c so that the function

$$f(x) = c\binom{100}{x}\binom{400}{400-x}, x = 0,1,2,\dots,100$$

can serve as a probability distribution of the discrete random variable X is

(a) $\binom{100}{2}^{-1}$

(b) $\binom{500}{100}^{-1}$

(c) $\binom{400}{100}^{-1}$

(d) None

Q.18 On a laboratory assignment, if equipment is working, the density function of the observed outcome, X , is

$$f(x) = \begin{cases} 2(1-x), 0 < x < 1 \\ 0, \quad elsewhere \end{cases}$$

Given that $X \geq 0.5$, the probability that X will be less than 0.75 is

(A) 0.25

(B) 0.50

$$P(X < 0.75 \mid X \geq 0.5) = \frac{P(X < 0.75 \wedge X \geq 0.5)}{P(X \geq 0.5)}$$

(B) 0.50

(C) 0.75

(D) 1

$$P(X < 0.75 \mid X \geq 0.5) = \frac{P(X < 0.75 \cap X \geq 0.5)}{P(X \geq 0.5)}$$

$$= \frac{P(0.5 \leq X < 0.75)}{P(X \geq 0.5)} = \frac{\int_{0.5}^{0.7} 2(1-x)\,dx}{\int_{0.5}^{1} 2(1-x)\,dx}$$

Q.19 Let A, B, and C be three events with P(A) = 0.1, P(B) = 0.7, and P(C) = 0.5. If A and B are disjoint, then $P(A \cap B \cap C)$ *is*:

(a) 1

(b) 0.8

(c) 0.3

(d) 0

Q.20 To find out the prevalence of a virus in a city's population of size 1,00,000, a blood test was carried out on 200 randomly selected citizens. If the test returned 8 positive results, the distribution of number of affected persons in a random sample of size 500 from the population can be approximately be taken as

(a) Poisson(40)

(b) Poisson(20)

(c) Poisson(8)

(d) Poisson(16/5)

Q.21 A traffic control engineer reports that 75% of the vehicles passing through a checkpoint are from within the state. The probability that fewer than 4 of the next 9 vehicles are from out of state is

(a) 0.0571

(b) 0.0142

(c) 0.0101

(d) None

p = prob that a vehicle passing through the checkpoint is from out of the state

$= 1 - 0.75 = 0.25$

$X \sim B(n = 9, p = 0.25)$

$P(X < 4) = P(X \leq 3) = \sum_{x=0}^{3} b(x; n = 9, p = 0.25)$ [Use binomial probsums table]

Q.22 The probability that a person will die when he or she contracts a virus infection is 0.002. Of the next 10,000 people infected, the mean number who will die is

(A) 100

(B) 20

(C) 50

(D) None

Q.23 Suppose the data set is the following: 1.7, 2.2, 3.9, 3.11, and 14.7. 20% trimmed mean of the data set is

(A) 5.122

(B) 4.6

(C) 4

(D) 3.07

Q.24 Sample variance of the data set D = {1,2,...,n} is

(A) $\frac{n(n+1)}{12}$

(B) $\frac{n^2-1}{12}$

(C) $\frac{n(n+1)(2n+1)}{12}$

(D) n^2

Q.25 Three cards are drawn in succession, without replacement, from an ordinary deck of playing cards. The probability that all the three cards are BLACK is

(A) 0.125

(B) 0.5

(C) 0.333

(D) 0.118

Q.26 : The probability that a patient recovers from a rare blood disease is 0.4. If 15 people are known to have contracted this disease, the probability that from 6 to 8 survive is

(A) 0.5018

(B) 0.4032

(C) 0.9050

(D) None

Q.27 The probability that a person flipping a coin gets 4^{rth} head on the 10^{th} flip is

(A) $(0.5)^{10}$

(B) $84\times (0.5)^{10}$

(C) $210\times (0.5)^{10}$

(D) None

$X \sim NB\ (b = 0.5)$

$P(X=4) = \binom{9}{3} (0.5)^3 (0.5)^{7-3} \times (0.5)$ (last one is success)

Q.28 3 different objects 1,2, and 3 are distributed at random in 3 places marked 1,2,3. The probability that none of the objects occupies the place corresponding to its number is

(A) 0.677

(B) 0.5

(C) 0.333

(D) None

A_i : object 1 occupies the place corresponding to the number i, $i = 1, 2, 3$

$P(\bar{A}_1 \cap \bar{A}_2 \cap \bar{A}_3) = P(\overline{A_1 \cup A_2 \cup A_3}) = 1 - P(A_1 \cup A_2 \cup A_3) \quad -(i)$

$P(A_1 \cup A_2 \cup A_3) = P(A_1) + P(A_2) + P(A_3) - P(A_1 \cap A_2) - P(A_1 \cap A_3) - P(A_2 \cap A_3) + P(A_1 \cap A_2 \cap A_3)$

$= \frac{2!}{3!} + \frac{2!}{3!} + \frac{2!}{3!} - \frac{1!}{3!} - \frac{1!}{3!} - \frac{1!}{3!} + \frac{1}{3!} = \frac{2}{3}$

From (i), $P(\bar{A}_1 \cap \bar{A}_2 \cap \bar{A}_3) = 1 - \frac{2}{3} = \frac{1}{3} = 0.333$

Q.29 Which of the following is not true?

Q.29 Which of the following is not true?

(A) The sample space S = $\{(x,y)|x^2+y^2<9\}$ is continuous.

(B) The sample space S = { H, TH, TTH, ...} is discrete and finite.

(C) The sample space S = { HT, TH, TT, HH} is discrete.

(D) All the outcomes cannot have the same positive probability if the sample space is not finite.

Q.30 Suppose X is a random variable with cdf

$$F(x) = \begin{cases} 0, & for\ x < 0 \\ x(2-x), & for\ 0 \le x \le 1 \\ 1, & for\ x > 1 \end{cases}$$

E(X) is

(A) 2

(B) 1

(C) 0.5

(D) 0.333

$$f(x) = \begin{cases} \frac{d}{dx}(0), & x<0 \\ \frac{d}{dx}(x(2-x)), & 0 \le x \le 1 \\ \frac{d}{dx}(1), & x>1 \end{cases} = \begin{cases} 2(1-x), & 0<x<1 \\ 0, & e.w. \end{cases}$$

$$E(X) = \int_0^1 x f(x)\,dx$$