3 Random Variables and Probability Distribution

by-91

3.3)

HHH →	3	TTT →	−3
HHT →	~~3~~ 1	TTH →	−1
HTH →	~~2~~ 1	THT →	−1
THH →	~~2~~ 1	HTT →	−1

$\therefore$ S = {−3, −1, 1, 3}

3.4) S = { HHH,
THHH,
HTHHH,
TTHHH,
HHTHHH,
HTTHHH,
THTHHH,
TTTHHH }

3.7

a) $P\left(x < \frac{120}{100}\right) = P(x < 1.2)$

$$= \int_0^{1.2} f(x)\,dx$$

$$= \int_0^1 x\,dx + \int_1^{1.2} 2 - x\,dx$$

$$= \left[\frac{x^2}{2}\right]_0^1 + \left[2x - \frac{x^2}{2}\right]_1^{1.2}$$

$$= \frac{1}{2} + \left[2.4 - \frac{1.44}{2} - 2 + \frac{1}{2}\right]$$

$$= 0.68$$

b) $P\left(\frac{80}{100} \leq X < \frac{100}{100}\right) = P(0.8 \leq x < 1.00)$

$$\cancel{= \int_{0.8}^{1} x\,dx + \int_{1}^{1.2} 2-x\,dx}$$

$$= \int_{0.5}^{1} x\,dx = 0.375$$

3.10

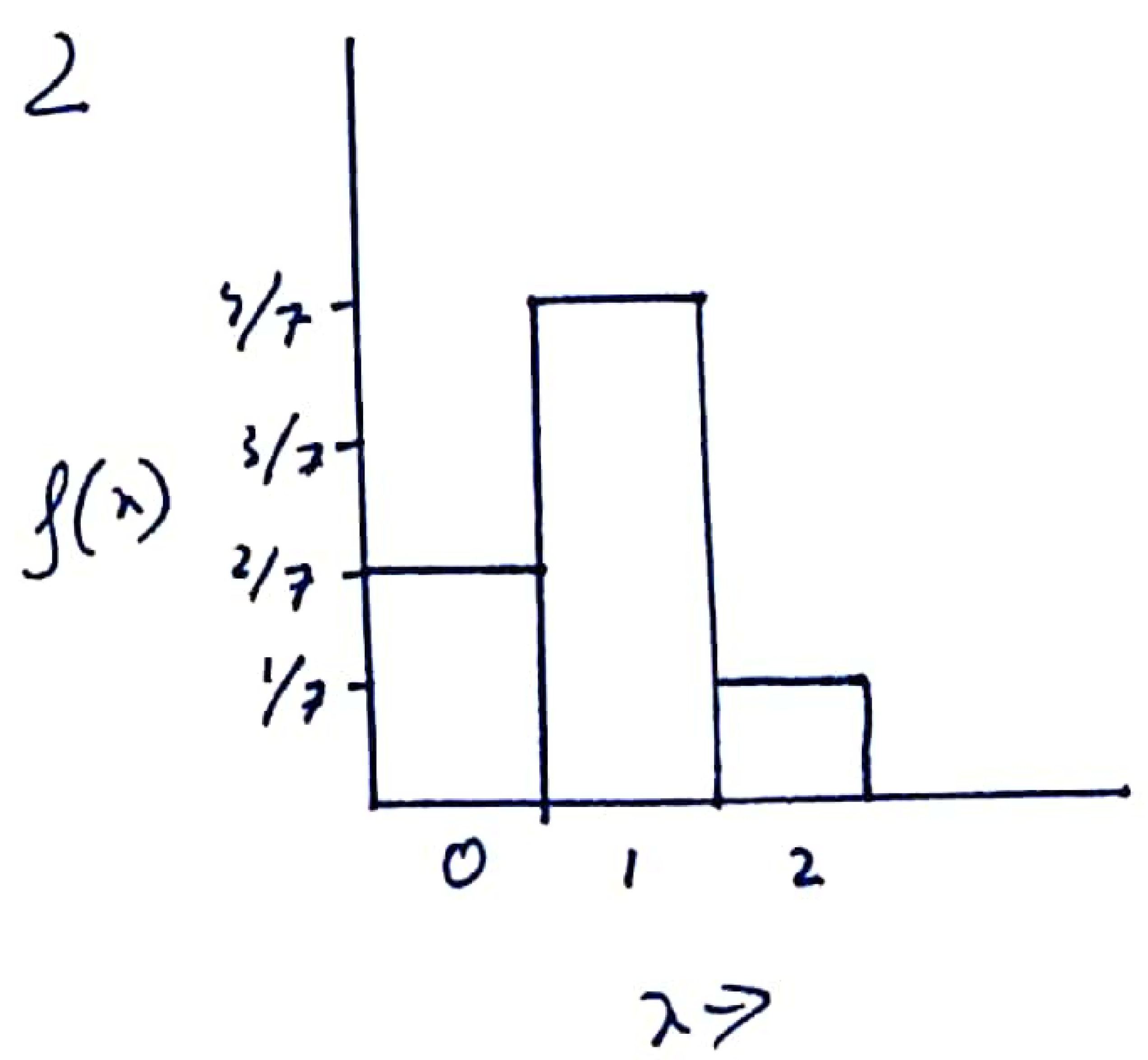
When a single die is rolled once, there are 6 equally likely outcomes {1, 2, 3, 4, 5, 6}. Hence, the probability distribution function is given by

$$f(x) = \frac{1}{6}, \quad x = 1, 2, \ldots, 6$$

3.11

Possible values of $x = 0, 1, 2$

x	0	1	2
$f(x)$	$\frac{{}^5C_3}{{}^7C_3}$	$\frac{{}^5C_2\,{}^2C_1}{{}^7C_3}$	$\frac{{}^5C_1\,{}^2C_2}{{}^7C_3}$
	$= \frac{2}{7}$	$= \frac{4}{7}$	$= \frac{1}{7}$



3.12

a) $P(T = 5) = P(T \leq 5) - P(T < 5)$

$$= F(5) - \lim_{t \to 5^-} P(T \leq t)$$

$$= \frac{3}{4} - \lim_{t \to 5^-} F(t)$$

$$= \frac{3}{4} - \frac{1}{2} = \frac{1}{4}$$

b) $P(T>3) = 1 - P(T \le 3)$

$= 1 - F(3)$

$= 1 - \frac{1}{2} = \frac{1}{2}$

c) $P(1.4 < T < 6) =$

$\lim_{t \to 6^-} F(t) - F(1.4)$

$= \frac{3}{4} - \frac{1}{4} = \frac{1}{2}$

d) $P(T \le 5 \mid T \ge 2)$

$$= \frac{P(T \le 5 \cap T \ge 2)}{P(T \ge 2)}$$

$$= \frac{P(2 \le T \le 5)}{P(T \ge 2)}$$

$$= \frac{\frac{3}{4} - \frac{1}{4}}{1 - P(T \le 2)} = \frac{\frac{1}{2}}{1 - \frac{1}{4}} = \frac{\frac{1}{2}}{\frac{3}{4}} = \frac{2}{3}$$

3-14

12 mins $= \frac{12}{60}$

$= 0.2$ hrs

a) $P(x < 0.2)$

$= \lim_{x \to 0.2^-} P(X \le x)$

$= \lim_{x \to 0.2} (1 - e^{-8x})$

$= 1 - e^{-8(0.2)} = 0.7981$

b) Probability density function is the derivative of cumulative distribution function.

$$f(x) = \frac{dF}{dx} = (1-e^{-8x})'$$

$$= 0 - (e^{-8x} \times -8)$$

$$= 8e^{-8x}$$

Now

$$P(X \le 0.2) = \int_0^{0.2} f(x)\,dx$$

$$= 8\int_0^{0.2} e^{-8x}\,dx$$

$$= -\frac{8}{8}\int_0^{-1.6} e^{z}\,dz$$

$$= -\left[e^{z}\right]_0^{-1.6}$$

$$= -(0.202 - 1)$$

$$= 0.7981$$

substitution

$-8x = z$

$-8\,dx = dz$

$dx = -\frac{1}{8}dz$

3.29

a)

$$\int_{-\infty}^{\infty} f(x)\,dx$$

$$= \int_{-\infty}^{1} f(x)\,dx + \int_{1}^{\infty} 3x^{-4}\,dx$$

$$= 0 + \left[\cancel{3}\frac{x^{-3}}{-\cancel{3}}\right]_1^{\infty} = 0 + \frac{1}{1^3} = 1$$

hence, its possible

b) $F(x) = \int_{x} f(x)\,dx$

$= \int_{1}^{x} 3x^{-4}\,dx = \left[\frac{3x^{-3}}{-3}\right]_{1}^{x} = \frac{-1}{x^3} + 1 = \boxed{1 - x^{-3}}$

c) ~~P(E) = 1 - F(4)~~

$P(x > 4)$

$= 1 - F(4)$

$= 1 - (1 - 4^{-3})$

$= 1 - 1 + 4^{-3} = 0.0156$

3.30

a) $\int_{-1}^{1} K(3 - x^2)\,dx = 1$

$K\left[3x - \frac{x^3}{3}\right]_{-1}^{1} = 1$

$\Rightarrow K\left[3 - \frac{1}{3} - \left(-3 + \frac{1}{3}\right)\right] = 1$

$\Rightarrow K\left[3 - \frac{1}{3} + 3 - \frac{1}{3}\right] = 1$

$\Rightarrow K\left[6 - \frac{2}{3}\right] = 1$

$\Rightarrow K\,\frac{16}{3} = 1$

$\Rightarrow K = \frac{3}{16}$

b) $\int_{-1}^{0.5} \frac{3}{16}(3 - x^2)\,dx$

$= \frac{3}{16}\left[3x - \frac{x^3}{3}\right]_{-1}^{0.5}$

$= \frac{3}{16}\left[\frac{3}{2} - \frac{1}{24} - \left(-3 + \frac{1}{3}\right)\right]$

$= \frac{99}{128} = 0.7734$

c) $P(|X| \leq 0.8)$

$= 1 - P(-0.8 \leq x \leq 0.8)$

$= 1 - \int_{-0.8}^{0.8} \frac{3}{16}(3 - x^2)\,dx$

$= 1 - 0.836$

$= 0.164 = \frac{41}{250}$

3.35

ⓐ $P(x > 8) = 1 - P(x \leq 8)$

$= 1 - \sum_{x \leq 8} f(x)$

$= 1 - \sum_{i=0}^{8}\left(e^{-6}\frac{6^x}{x!}\right) = 1 - e^{-6}\left[\frac{6^0}{0!} + \frac{6^1}{1!} + \dots + \frac{6^8}{8!}\right]$

$= 1 - 0.8472$

$= 0.1528$

ⓑ $f(2) = e^{-6}\frac{6^2}{2!} = 0.0446$