LECTURE - 11

CHEPTER-4

4.2 Variance and Covariance of random variables

The most important measure of variability of a random variable X is the variance of the random variable X or the variance of the probability distribution of X and is denoted by Var(X) or the symbol σ_X^2, or simply by σ^2

Definition 4.3

Let X be a random variable with probability distribution f(x) and mean μ.
The variance of X is
$\sigma^2 = E[(X-\mu)^2] = \sum_x (x-\mu)^2 f(x)$, if X is discrete, and
$\sigma^2 = E[(X-\mu)^2] = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$, if X is continuous.
The positive square root of the variance σ is called the standard deviation of X.

Theorem-4.2:

The variance of a random variable X is $\sigma^2 = E(X^2) - \mu^2$.

Theorem-4.3:

Let X be a random variable with probability distribution f(x). The variance of the random variable g(X) is

$\sigma_{g(X)}^2 = E[g(X) - \mu_{g(X)}]^2 = \sum_x [g(x) - \mu_{g(X)}]^2 f(x)$
if X is discrete, and

$\sigma_{g(X)}^2 = E\{[g(X) - \mu_{g(X}]^2\} = \int_{-\infty}^{\infty} [g(x) - \mu_{g(X)}]^2 f(x)dx$
if X is continuous.

Definition 4.4

Let X and Y be random variables with joint probability distribution f(x, y).
The covariance of X and Y is
$\sigma_{XY} = E[(X-\mu_X)(Y-\mu_Y)] = \sum_x \sum_y (x-\mu_x)(y-\mu_y)f(x,y)$
if X and Y are discrete, and

$\sigma_{XY} = E[(X-\mu_X)(Y-\mu_Y)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x-\mu_x)(y-\mu_y)f(x,y)dxdy$
if X and Y are continuous

Theorem-4.4:

The covariance of two random variables X and Y with means μ_X and μ_Y , respectively, is given by

$\sigma_{XY} = E(XY) - \mu_X\mu_Y$.

Example-4.9
Let the random variable X represents the number of defective parts for a machine when 3 parts are sampled from a production line and tested. The following is the probability distribution of X.

x	0	1	2	3
f(x)	0.51	0.38	0.10	0.01

Calculate σ^2.
Solution :
$\mu = (0)(0.51) + (1)(0.38) + (2)(0.10) + (3)(0.01) = 0.61.$

$E(X^2) = (0)(0.51) + (1)(0.38) + (4)(0.10) + (9)(0.01) = 0.87.$
Therefore,
$\sigma^2 = 0.87 - (0.61)^2 = 0.4979.$

Example-4.10
The weekly demand for a drinking-water product, in thousands of liters, from a local chain of efficiency stores is a continuous random variable X having the probability density

$$\text{f(x)}=\begin{cases} 2(x-1) & 1 < x < 2 \\ 0 & elsewhere \end{cases}$$

Find the mean and variance of X.
Solution :
Calculating E(X) and$E(X^2)$,
we have $\mu = E(X) = 2\int_1^2 x(x-1)dx = \frac{5}{3}$
and
$E(X^2) = 2\int_1^2 x^2(x-1)dx = \frac{17}{6}$
$Therefore,$
$\sigma^2 = \frac{17}{6} - (\frac{5}{3})^2 = \frac{1}{18}.$

Exercise-4.34
Let X be a random variable with the following probability distribution:

x	-2	3	5
f(x)	0.3	0.2	0.5

Find the standard deviation of X.
Solution :

Calculating E(X) and$E(X^2)$,
we have
$\mu = (-2)(0.3) + (3)(0.2) + (5)(0.5) = 2.5$

$E(X^2) = (4)(0.3) + (9)(0.2) + (25)(0.5) = 15.5$
$Therefore,$
$\sigma^2 = 15.5 - 2.5^2 = 9.25.$
The standard deviation $\sigma = \sqrt{9.25} = 3.04138.$

Exercise-4.35
The random variable X, representing the number of errors per 100 lines of software code, has the following probability distribution:

x	2	3	4	5	6
f(x)	0.01	0.25	0.4	0.3	0.04

Solution :
E(X)=$\sum_x xf(x) = 2(0.01) + 3(0.25) + 4(0.4) + 5(0.3) + 6(0.04) = 4.11$

$E(X^2) = \sum_x x^2 f(x) = 4(0.01)+9(0.25)+16(0.4)+25(0.3)+36(0.04) = 17.63$

$\sigma^2 = E(X^2) - (E(X))^2 = 17.63 - (4.11)^2 = 0.7379$

Exercise-4.50
For a laboratory assignment, if the equipment is working, the density function of the observed outcome X is

$$f(x)=\begin{cases} 2(1-x) & 0 < x < 1 \\ 0 & elsewhere \end{cases}$$

Find the variance and standard deviation of X.
Solution :
Calculating E(X) and$E(X^2)$,
we have $\mu = E(X) = 2\int_0^1 x(1-x)dx = \frac{1}{3}$
and
$E(X^2) = 2\int_0^1 x^2(1-x)dx = \frac{1}{6}$
$Therefore,$
$\sigma^2 = \frac{1}{6} - (\frac{1}{3})^2 = \frac{1}{18}$ and $\sigma = \sqrt{\frac{1}{18}} = 0.2357$

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