# Binary arithmetic (Addition and Subtraction)
# Complements of Numbers (r and r-1)

**Lecture-3**

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# Binary Addition

Basic mathematical operations with binary numbers works similar to the decimal system. However there are a few rules specific to the binary system.

| Case | A + B | Sum | Carry |
|---|---|---|---|
| 1 | 0 + 0 | 0 | 0 |
| 2 | 0 + 1 | 1 | 0 |
| 3 | 1 + 0 | 1 | 0 |
| 4 | 1 + 1 | 0 | 1 |

$$
\begin{array}{rcr}
{\scriptstyle 0\,1\,1\,1\,\phantom{0}} & & \\
00111 & & 7 \\
\underline{10101} & & \underline{21} \\
11100 & = & 28
\end{array}
$$

# Binary Addition

**Addition Rules:**

0 + 0 = 0

0 + 1 = 1

1 + 0 = 1

1 + 1 = 0 carry 1

1 + 1 + 1 = 1 carry 1

## Example 1:

$$\begin{array}{r} {\scriptstyle 1}\phantom{000} \\ 0101 \\ +\,0110 \\ \hline 1011 \end{array}$$

## Example 2:

$$\begin{array}{r} {\scriptstyle 1\ 1}\phantom{00} \\ 10111 \\ +\,10010 \\ \hline \boxed{1}\ 01001 \end{array}$$

OVERFLOW → 1

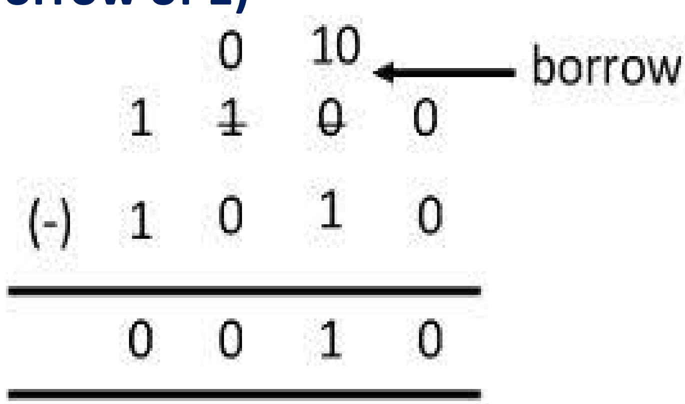
# Binary Subtraction

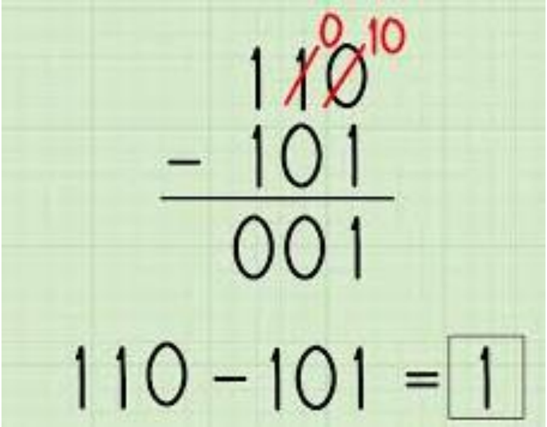
Binary subtraction is also similar to that of decimal subtraction with the difference that when 1 is subtracted from 0, it is necessary to borrow 1 from the next higher order bit and that bit is reduced by 1 (or 1 is added to the next bit of subtrahend) and the remainder is 1.

**Binary Subtraction Rule Chart**

**Rules and tricks:** Binary subtraction is much easier than the decimal subtraction when you remember the following rules:

- **0 – 0 = 0**
- **0 – 1 = 1 ( with a borrow of 1)**
- **1 – 0 = 1**
- **1 – 1 = 0**



# COMPLEMENTS OF NUMBERS

- Complements are used in digital computers to **simplify the subtraction operation** and for **logical manipulation**.
- Simplifying operations leads to simpler, less expensive circuits to implement the operations.
- There are two types of complements for each base-*r* *system:* the radix complement and the diminished radix complement.
- The first is referred to as the *r's complement and the second as the (r - 1)'s complement.*
- *When the value of the* base *r is substituted in the name, the two types are referred to as the 2's complement and* 1's complement for binary numbers and the 10's complement and 9's complement for decimal numbers.

# Diminished Radix Complement

- Given a number *N in base r having n digits, the (r - 1)'s complement of N , i.e., its* diminished radix complement, is defined as $(r^n - 1) - N$.

- *For decimal numbers, r = 10* and *r - 1 = 9, so the 9's complement of N is* $(10^n - 1) - N$.

- *In this case,* $10^n$ *represents* a number that consists of a single 1 followed by *n 0's.*

# Diminished Radix Complement

- $10^n - 1$ *is a number represented* by *n 9's.*

- *For example, if n = 4, we have* $10^4$ *= 10,000 and* $10^4$ *- 1 = 9999.*

- *It follows* that the 9's complement of a decimal number is obtained by subtracting each digit from 9.

## Calculation of the 9's complement of 546700

999999

-546700

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453299

# Calculation of the 9's complement of 012398

999999

-012398

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**987601**

# Diminished Radix Complement

- For binary numbers, *$r = 2$ and $r - 1 = 1$, so the 1's complement of $N$ is $(2^n - 1) - N$.*

- Again, *$2^n$ is represented by a binary number that consists of a 1 followed by n 0's.*

- *$2^n - 1$* is a binary number represented by *n 1's*.

# Diminished Radix Complement

- *For example, if n = 4, we have $2^4$ =* $(10000)_2$ and $2^4 - 1 = (1111)_2$ .

- Thus, the 1's complement of a binary number is obtained by subtracting each digit from 1. However, when subtracting binary digits from 1, we can have either 1 - 0 = 1 or 1 - 1 = 0, which causes the bit to change from 0 to 1 or from 1 to 0, respectively.

- Therefore, **the 1's complement of a binary number is formed by changing 1's to 0's and 0's to 1's.**

# 1's complement of 1011000

$$
\begin{array}{r}
1\,1\,1\,1\,1\,1\,1 \\
-1\,0\,1\,1\,0\,0\,0 \\
\hline
0\,1\,0\,0\,1\,1\,1
\end{array}
$$

# The 1's complement of 0101101

$$\begin{array}{r} 1111111 \\ -\ 0101101 \\ \hline 1010010 \end{array}$$

- The *(r - 1)'s complement of octal or hexadecimal numbers is obtained by subtracting* each digit from 7 or F (decimal 15), respectively.

# Radix Complement

- The *r's complement of an n-digit number N in base r is defined as* $r^n - N$ *for* $N \neq 0$ *and* as 0 for $N = 0$.
- *Comparing with the (r - 1)'s complement, we note that the r's complement* is obtained by adding 1 to the (*r - 1)'s complement, since*

  $r^n - N = [(r^n - 1) - N] + 1.$

# 10’s complement of decimal 2389

- Thus, the 10’s complement of decimal 2389 is 7610 + 1 = 7611 and is obtained by adding 1 to the 9’s complement value.

$$
\begin{array}{r}
9999 \\
-\ 2389 \\
\hline
7610 \\
+\quad 1 \\
\hline
7611
\end{array}
$$

# 2’s complement of binary 101100

- The 2’s complement of binary 101100 is 010011 + 1 = 010100 and is obtained by adding 1 to the 1’s-complement value.

1’s-complement of 101100= 010011

$$
\begin{array}{lr}
 & 010011 \\
+ & 1 \\
\hline
 & 010100
\end{array}
$$

# Radix Complement

Since *$10^n$* is a number represented by a 1 followed by *n 0's*, *$10^n$ - N, which is the 10's* complement of *N, can be formed also by leaving all least significant 0's unchanged,* subtracting the first nonzero least significant digit from 10, and subtracting all higher significant digits from 9.

# 10’s complement of 012398 is 987602

$$
\begin{array}{r}
1000000 \\
-\quad 012398 \\
\hline
987602
\end{array}
$$

# 10’s complement of 246700 is 753300

$$
\begin{array}{r}
1000000 \\
-\quad 246700 \\
\hline
753300
\end{array}
$$

- The 10’s complement of the first number is obtained by subtracting 8 from 10 in the least significant position and subtracting all other digits from 9.

- The 10’s complement of the second number is obtained by leaving the two least significant 0’s unchanged, subtracting 7 from 10, and subtracting the other three digits from 9.

- Similarly, the 2’s complement can be formed by leaving all least significant 0’s and the first 1 unchanged and replacing 1’s with 0’s and 0’s with 1’s in all other higher significant digits.

## Obtain 2's complement of the binary number 1101100

1's complement of 1101100=0010011

2's complement of 1101100= 1's complement of 1101100+1

$$
\begin{array}{rr}
= & 0010011 \\
+ & 1 \\
\hline
= & 0010100
\end{array}
$$

## Obtain 2's complement of the binary number 0110111

1's complement of 0110111 =1001000

2's complement of 0110111 = 1's complement of 0110111 +1

$$
\begin{array}{rr}
= & 1001000 \\
+ & 1 \\
\hline
= & 1001001
\end{array}
$$

- The 2’s complement of the first number is obtained by leaving the two least significant 0’s and the first 1 unchanged and then replacing 1’s with 0’s and 0’s with 1’s in the other four most significant digits.

- The 2’s complement of the second number is obtained by leaving the least significant 1 unchanged and complementing all other digits.

- It is also worth mentioning that **the complement of the complement restores the number to its original value .**

- **To see this** relationship, note that the *r's complement of N is* $r^n$ - *N, so that the complement of the* complement is

  $r^n - (r^n - N) = N$ *and is equal to the original number.*

# Assignment

1. **Add and multiply the following numbers without converting them to decimal.**
   (a) Binary numbers 1011 and 101.
   (b) Hexadecimal numbers 2E and 34.

2. **Obtain the 1's and 2's complements of the following binary numbers:**
   (a) 00010000
   (b) 00000000
   (c) 11011010
   (d) 10101010
   (e) 10000101
   (f) 11111111.

3. **Find the 9’s and the 10’s complement of the following decimal numbers:**

(a) 25,478,03 (b) 63, 325, 600

(c) 25,000,000 (d) 00,000,000.

**4. (a) Find the 16’s complement of C3DF.**

(b) Convert C3DF to binary.

(c) Find the 2’s complement of the result in (b).

(d) Convert the answer in (c) to hexadecimal and compare with the answer in (a).