SUNGKYUNKWAN UNIVERSITY
DEPARTMENT OF PHYSICS

Status Report

Donghun Jung

Quantum State Tomography and Measurement basis

This report summarizes my studies over the past two weeks until Mar 20 and is submitted on March 21, 2023.

Summary

1. The ‘good’ POVM sets span $SU(2) \otimes SU(2)$ space well.
2. $SU(2) \otimes SU(2)$ is just equal to $SO(1,3)$ which is well known group in special relativity. There is a possibility to reduce the elements in the ‘good’ POVM set, considering that there are only six generators in $SO(1,3)$.
3. The more analytical study for QST is required.

Table of Contents

Chapter 1

Quantum Status Tomography

Chapter 2

Measurement Basis

2.1 $SU(2) \otimes SU(2)$

Our system consists of two qubits, which exhibit $SU(2) \otimes SU(2)$ symmetry. To represent the POVM operators, we choose an orthogonal basis set $\{\Gamma^{ij}\}_{i,j=0}^{3}$ that satisfies $\mathrm{tr}\left(\Gamma^{ij\dagger}\Gamma^{mn}\right) = \delta_{im}\delta_{jn}$. We use the representation basis $\Gamma^{ij} = \sigma^i \otimes \sigma^j$, which is convenient because it is traceless except for Γ^{00}. You can see what the Γ^{ij} look like in Appendix 2.3.

$\{\Gamma^{ij}\}$ possesses some interesting properties. First, the product between Γ^{ij} and Γ^{mn} is given by

$$\begin{aligned}
\Gamma^{ij}\Gamma^{mn} &= (\sigma^i \otimes \sigma^j) \otimes (\sigma^m \otimes \sigma^n) \\
&= (\sigma^i \sigma^m) \otimes (\sigma^j \otimes \sigma^n) \\
&= (\delta^{im} + i\epsilon_{imk}\sigma^k) \otimes (\delta_{jn} + i\epsilon_{jnl}\sigma^l) \\
&= \delta_{im}\delta_{jn} + i\delta_{im}\epsilon_{jnl}\sigma^l + i\delta^{jn}\epsilon_{imk}\sigma^k - \epsilon_{imk}\epsilon_{jnl}\sigma^k\sigma^l \qquad (2.1)
\end{aligned}$$

Note that I supposed that $i, j, m, n \neq 0$ and didn't write $\otimes$ explicitly at the end of equation. And you don't have to pay attention to the upper/lower indecies. In a same way, we have

$$\Gamma^{jn}\Gamma^{ij} = \delta_{im}\delta_{jn} - i\delta_{im}\epsilon_{jnl}\sigma^l - i\delta^{jn}\epsilon_{imk}\sigma^k - \epsilon_{imk}\epsilon_{jnl}\sigma^k\sigma^l \qquad (2.2)$$

in component form. Or more conviniently,

$$\begin{aligned}
\Gamma^{jn}\Gamma^{ij} &= (\sigma^m \otimes \sigma^n) \otimes (\sigma^m \otimes \sigma^n) \\
&= -(\sigma^i \sigma^m - 2\delta_{im}) \otimes (\sigma^n \sigma^j) \\
&= -(\sigma^i \sigma^m) \otimes (\sigma^n \sigma^j) + 2\delta_{im} \otimes \sigma^n \sigma^j \qquad (2.3) \\
&= (\sigma^i \sigma^m - 2i\epsilon_{mik}\sigma^k) \otimes (\sigma^n \sigma^j) \\
&= (\sigma^i \sigma^m) \otimes (\sigma^n \sigma^j) - 2i\epsilon_{mik}\sigma^k \otimes \sigma^n \sigma^j. \qquad (2.4)
\end{aligned}$$

Therefore, the commutator relation and the anticommutator relation are given by

$$\begin{aligned}
\left[\Gamma^{ij}, \Gamma^{mn}\right] &= (\sigma^i \sigma^m) \otimes \left\{\sigma^j, \sigma^n\right\} - 2\delta_{im} \otimes \sigma^n \sigma^j \qquad (2.5) \\
&= (\sigma^i \sigma^m) \otimes \left[\sigma^j, \sigma^n\right] + 2i\epsilon_{mik}\sigma^k \otimes \sigma^n \sigma^j \qquad (2.6) \\
&= 2i\delta_{im}\epsilon_{jnl}\sigma^l + 2i\delta_{jn}\epsilon_{imk}\sigma^k \qquad (2.7)
\end{aligned}$$

$$\begin{aligned}
\left\{\Gamma^{ij}, \Gamma^{mn}\right\} &= (\sigma^i \sigma^m) \otimes \left[\sigma^j, \sigma^n\right] + 2\delta_{im} \otimes \sigma^n \sigma^j \qquad (2.8) \\
&= (\sigma^i \sigma^m) \otimes \left\{\sigma^j, \sigma^n\right\} - 2i\epsilon_{mik}\sigma^k \otimes \sigma^n \sigma^j \qquad (2.9) \\
&= 2\delta_{im}\delta_{jn} - 2\epsilon_{imk}\epsilon_{jnl}\sigma^k \sigma^l \qquad (2.10)
\end{aligned}$$

If one of the elements is zero, i.e. $i = 0$, then,

$$\begin{aligned}
\Gamma^{0j}\Gamma^{mn} &= (\sigma^0 \otimes \sigma^j) \otimes (\sigma^m \otimes \sigma^n) \\
&= (\sigma^0 \sigma^m) \otimes (\sigma^j \otimes \sigma^n) \\
&= \sigma^m \otimes \delta_{jn} + i\sigma^m \otimes \epsilon_{jnl}\sigma^l \qquad (2.11)
\end{aligned}$$

and the commutator relation and the anticommutator relation are given by

$$\left[\Gamma^{0j}, \Gamma^{mn}\right] = -2i\sigma^m \otimes \epsilon_{njk}\sigma^k \qquad (2.12)$$

$$\left\{\Gamma^{0j}, \Gamma^{mn}\right\} = 2\sigma^m \otimes \delta_{jn} \qquad (2.13)$$

If two of the elements are zero, i.e. $i = j = 0$ or $i = m = 0$, then,

$$\begin{aligned}
\Gamma^{00}\Gamma^{mn} &= (\sigma^m) \otimes (\sigma^n) \\
&= \Gamma^{mn} \\
&= \Gamma^{mn}\Gamma^{00}
\end{aligned}$$

And obviously,

$$\left[\Gamma^{00},\Gamma^{mn}\right]=0 \tag{2.14}$$

$$\left\{\Gamma^{00},\Gamma^{mn}\right\}=\Gamma^{mn} \tag{2.15}$$

On the other hand,

$$\Gamma^{0j}\Gamma^{0n}=\sigma^0\otimes\left(\sigma^j\sigma^n\right)$$

and

$$\left[\Gamma^{0j},\Gamma^{0n}\right]=\sigma^0\otimes 2i\epsilon_{njk}\sigma^k \tag{2.16}$$

$$\left\{\Gamma^{0j},\Gamma^{0n}\right\}=\sigma^0\otimes 2\delta_{nj} \tag{2.17}$$

Therefore, $\{\Gamma^{ij}\}$ constructs an orthogonal representation basis. Considering the property of trace, $\operatorname{tr} A\otimes B=\operatorname{tr} A\times\operatorname{tr} B$, we have

$$\operatorname{tr}\left(\Gamma^{ij}\Gamma^{mn}\right)=4\delta_{im}\delta_{jn}. \tag{2.18}$$

Note that here $\dagger$ is removed. The hermitivity of Γ^{ij} is guranteed by the hermitivity of pauli matrices.

It has been recently notice that $SU(2)\otimes SU(2)$ is actually $SO(1,3)$, which is familiar in the special relativity. It seems possible to use generaters in $SO(1,3)$ to represent the system. This study is undergoing.

2.2 Measurement basis

2.2.1 Bell basis measurement

2.2.2 POVM basis Ω^i

The system under consideration is a two-qubit system with $\sigma^3\otimes\sigma^3$ interaction, and projection measurements are performed using a POVM basis consisting of 29 operators. Considering the rank of the set of basis operators is 16, it is expected that the set of POVM operators can be reduced. For more information on the POVM basis, see Appendix B 2.4. There are numerous possible combinations of these operators, but it is assumed that there are approximately 1,600 possible combinations that perform QST well.

Absolutely, the POVM operators can be represented as linear combinations of the representation basis Γ^{ij}.

$$\Omega^k = \theta_{ij}^k \Gamma^{ij} \tag{2.19}$$

Using the property discussed above 2.18, θ_{ij}^k is given by

$$\theta_{ij}^k = \frac{1}{4} \operatorname{tr}\left(\Omega^k \Gamma^{ij}\right) \tag{2.20}$$

In this manner, we find θ_{ij}^k. See the table 2.1. For the case that $(S1, S1)$ is replace to $(S2, S2)$ see the table 2.2 For example, the first POVM basis Ω^1 is given by

$$\begin{aligned}
\Omega^1 &= \frac{1}{4}\Gamma^{00} + \frac{1}{4}\Gamma^{11} - \frac{1}{4}\Gamma^{22} \\
&= \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} + \frac{1}{4}\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} - \frac{1}{4}\begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \\
&= \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}
\end{aligned}$$

The POVM set generated by $(S1, S1)$ has been found to perform QST well, while the set generated by (S_z, S_z) does not, even though it seems that theoretical analysis shows no significant differences between the two except for a constant shift in the measurement process. It was initially believed that the sets consisting of the selected 16 POVM operations that conduct QST well may have almost the same amount of Γ^{ij}, but this was found not to be the case. For example, in a possible POVM basis, such as $\{\Omega_0^1, \Omega_0^2, \Omega_0^3, \Omega_0^7, \Omega_0^9, \Omega_0^{11}, \Omega_0^{13}, \Omega_0^{14}, \Omega_0^{16}, \Omega_0^{18}, \Omega_0^{19}, \Omega_0^{20}, \Omega_0^{21}, \Omega_0^{22}, \Omega_0^{26}, \Omega_0^{29}\}$, one POVM operator Ω_0^{14} consists of Γ^{01}, while Ω_0^1, Ω_0^2, Ω_0^{18}, Ω_0^{19}, Ω_0^{20}, and Ω_0^{26} consist of Γ^{11}. The sum of the square of coefficients for Γ^{11} is $\frac{3}{8}$, while the sum of the square of coefficients for Γ^{01} is $\frac{1}{16}$.

	Γ^{00}	Γ^{01}	Γ^{02}	Γ^{03}	Γ^{10}	Γ^{11}	Γ^{12}	Γ^{13}	Γ^{20}	Γ^{21}	Γ^{22}	Γ^{23}	Γ^{30}	Γ^{31}	Γ^{32}	Γ^{33}
Ω_0^1	$\frac{1}{2}$	0	0	0	0	$\frac{1}{4}$	0	0	0	0	$-\frac{1}{4}$	0	0	0	0	0
Ω_0^2	$\frac{1}{2}$	0	0	0	0	$-\frac{1}{4}$	0	0	0	0	$\frac{1}{4}$	0	0	0	0	0
Ω_0^3	$\frac{1}{2}$	0	0	0	0	0	$-\frac{1}{4}$	0	0	$-\frac{1}{4}$	0	0	0	0	0	0
Ω_0^4	$\frac{1}{2}$	0	0	0	0	0	$\frac{1}{4}$	0	0	$\frac{1}{4}$	0	0	0	0	0	0
Ω_0^5	$\frac{1}{2}$	0	0	0	0	$\frac{1}{4}$	0	0	0	0	$\frac{1}{4}$	0	0	0	0	0
Ω_0^6	$\frac{1}{2}$	0	0	0	0	$-\frac{1}{4}$	0	0	0	0	$-\frac{1}{4}$	0	0	0	0	0
Ω_0^7	$\frac{1}{2}$	0	0	0	0	0	$-\frac{1}{4}$	0	0	$\frac{1}{4}$	0	0	0	0	0	0
Ω_0^8	$\frac{1}{2}$	0	0	0	0	0	$\frac{1}{4}$	0	0	$-\frac{1}{4}$	0	0	0	0	0	0
Ω_0^9	$\frac{1}{2}$	0	0	0	0	0	0	$\frac{1}{4}$	0	$-\frac{1}{4}$	0	0	0	0	0	0
Ω_0^{10}	$\frac{1}{2}$	0	0	0	0	0	0	$-\frac{1}{4}$	0	$-\frac{1}{4}$	0	0	0	0	0	0
Ω_0^{11}	$\frac{1}{2}$	0	0	0	0	0	$\frac{1}{4}$	0	0	0	0	0	0	$\frac{1}{4}$	0	0
Ω_0^{12}	$\frac{1}{2}$	0	0	0	0	0	$\frac{1}{4}$	0	0	0	0	0	0	$-\frac{1}{4}$	0	0
Ω_0^{13}	$\frac{1}{2}$	0	0	$-\frac{1}{4}$	$\frac{1}{4}$	0	0	0	0	0	0	0	0	0	0	0
Ω_0^{14}	$\frac{1}{2}$	$\frac{1}{4}$	0	0	0	0	0	0	$-\frac{1}{4}$	0	0	0	0	0	0	0
Ω_0^{15}	$\frac{1}{2}$	$\frac{1}{4}$	0	0	0	0	0	0	0	0	0	0	$-\frac{1}{4}$	0	0	0
Ω_0^{16}	$\frac{1}{2}$	0	$\frac{1}{4}$	0	$\frac{1}{4}$	0	0	0	0	0	0	0	0	0	0	0
Ω_0^{17}	$\frac{1}{2}$	0	0	0	0	$-\frac{1}{4}$	0	0	0	0	0	$\frac{1}{4}$	0	0	0	0
Ω_0^{18}	$\frac{1}{2}$	0	0	0	0	$-\frac{1}{4}$	0	0	0	0	0	$-\frac{1}{4}$	0	0	0	0
Ω_0^{19}	$\frac{1}{2}$	0	0	0	0	$-\frac{1}{4}$	0	0	0	0	0	0	0	0	$\frac{1}{4}$	0
Ω_0^{20}	$\frac{1}{2}$	0	0	0	0	$-\frac{1}{4}$	0	0	0	0	0	0	0	0	$-\frac{1}{4}$	0
Ω_0^{21}	$\frac{1}{2}$	0	0	$-\frac{1}{4}$	0	0	0	0	$\frac{1}{4}$	0	0	0	0	0	0	0
Ω_0^{22}	$\frac{1}{2}$	0	0	$-\frac{1}{4}$	0	0	0	0	$-\frac{1}{4}$	0	0	0	0	0	0	0
Ω_0^{23}	$\frac{1}{2}$	0	$\frac{1}{4}$	0	0	0	0	0	0	0	0	0	$-\frac{1}{4}$	0	0	0
Ω_0^{24}	$\frac{1}{2}$	0	$-\frac{1}{4}$	0	0	0	0	0	0	0	0	0	$-\frac{1}{4}$	0	0	0
Ω_0^{25}	$\frac{1}{2}$	0	0	0	0	$-\frac{1}{4}$	0	0	0	0	0	0	0	0	0	$\frac{1}{4}$
Ω_0^{26}	$\frac{1}{2}$	0	0	0	0	$-\frac{1}{4}$	0	0	0	0	0	0	0	0	0	$-\frac{1}{4}$
Ω_0^{27}	$\frac{1}{2}$	0	0	$-\frac{1}{4}$	0	0	0	0	0	0	0	0	$\frac{1}{4}$	0	0	0
Ω_0^{28}	$\frac{1}{2}$	0	0	$\frac{1}{4}$	0	0	0	0	0	0	0	0	$-\frac{1}{4}$	0	0	0
Ω_0^{29}	$\frac{1}{2}$	0	0	$-\frac{1}{4}$	0	0	0	0	0	0	0	0	$-\frac{1}{4}$	0	0	0

Table 2.1: The elements of θ_{ij}^k, $(S1, S1) = \frac{1}{2}\left(\left|0\right\rangle\left\langle 0\right| \otimes \sigma^0 + \sigma^0 \otimes \left|0\right\rangle\left\langle 0\right|\right)$

	Γ^{00}	Γ^{01}	Γ^{02}	Γ^{03}	Γ^{10}	Γ^{11}	Γ^{12}	Γ^{13}	Γ^{20}	Γ^{21}	Γ^{22}	Γ^{23}	Γ^{30}	Γ^{31}	Γ^{32}	Γ^{33}
Ω_1^1	$\frac{1}{2}$	0	0	0	0	$-\frac{1}{4}$	0	0	0	0	$\frac{1}{4}$	0	0	0	0	0
Ω_1^2	$\frac{1}{2}$	0	0	0	0	$\frac{1}{4}$	0	0	0	0	$-\frac{1}{4}$	0	0	0	0	0
Ω_1^3	$\frac{1}{2}$	0	0	0	0	0	$\frac{1}{4}$	0	0	$\frac{1}{4}$	0	0	0	0	0	0
Ω_1^4	$\frac{1}{2}$	0	0	0	0	0	$-\frac{1}{4}$	0	0	$-\frac{1}{4}$	0	0	0	0	0	0
Ω_1^5	$\frac{1}{2}$	0	0	0	0	$-\frac{1}{4}$	0	0	0	0	$-\frac{1}{4}$	0	0	0	0	0
Ω_1^6	$\frac{1}{2}$	0	0	0	0	$\frac{1}{4}$	0	0	0	0	$\frac{1}{4}$	0	0	0	0	0
Ω_1^7	$\frac{1}{2}$	0	0	0	0	0	$\frac{1}{4}$	0	0	$-\frac{1}{4}$	0	0	0	0	0	0
Ω_1^8	$\frac{1}{2}$	0	0	0	0	0	$-\frac{1}{4}$	0	0	$\frac{1}{4}$	0	0	0	0	0	0
Ω_1^9	$\frac{1}{2}$	0	0	0	0	0	0	$-\frac{1}{4}$	0	$\frac{1}{4}$	0	0	0	0	0	0
Ω_1^{10}	$\frac{1}{2}$	0	0	0	0	0	0	$\frac{1}{4}$	0	$\frac{1}{4}$	0	0	0	0	0	0
Ω_1^{11}	$\frac{1}{2}$	0	0	0	0	0	$-\frac{1}{4}$	0	0	0	0	0	0	$-\frac{1}{4}$	0	0
Ω_1^{12}	$\frac{1}{2}$	0	0	0	0	0	$-\frac{1}{4}$	0	0	0	0	0	0	$\frac{1}{4}$	0	0
Ω_1^{13}	$\frac{1}{2}$	0	0	$\frac{1}{4}$	$-\frac{1}{4}$	0	0	0	0	0	0	0	0	0	0	0
Ω_1^{14}	$\frac{1}{2}$	$-\frac{1}{4}$	0	0	0	0	0	0	$\frac{1}{4}$	0	0	0	0	0	0	0
Ω_1^{15}	$\frac{1}{2}$	$-\frac{1}{4}$	0	0	0	0	0	0	0	0	0	0	$\frac{1}{4}$	0	0	0
Ω_1^{16}	$\frac{1}{2}$	0	$-\frac{1}{4}$	0	$-\frac{1}{4}$	0	0	0	0	0	0	0	0	0	0	0
Ω_1^{17}	$\frac{1}{2}$	0	0	0	0	$\frac{1}{4}$	0	0	0	0	0	$-\frac{1}{4}$	0	0	0	0
Ω_1^{18}	$\frac{1}{2}$	0	0	0	0	$\frac{1}{4}$	0	0	0	0	0	$\frac{1}{4}$	0	0	0	0
Ω_1^{19}	$\frac{1}{2}$	0	0	0	0	$\frac{1}{4}$	0	0	0	0	0	0	0	0	$-\frac{1}{4}$	0
Ω_1^{20}	$\frac{1}{2}$	0	0	0	0	$\frac{1}{4}$	0	0	0	0	0	0	0	0	$\frac{1}{4}$	0
Ω_1^{21}	$\frac{1}{2}$	0	0	$\frac{1}{4}$	0	0	0	0	$-\frac{1}{4}$	0	0	0	0	0	0	0
Ω_1^{22}	$\frac{1}{2}$	0	0	$\frac{1}{4}$	0	0	0	0	$\frac{1}{4}$	0	0	0	0	0	0	0
Ω_1^{23}	$\frac{1}{2}$	0	$-\frac{1}{4}$	0	0	0	0	0	0	0	0	0	$\frac{1}{4}$	0	0	0
Ω_1^{24}	$\frac{1}{2}$	0	$\frac{1}{4}$	0	0	0	0	0	0	0	0	0	$\frac{1}{4}$	0	0	0
Ω_1^{25}	$\frac{1}{2}$	0	0	0	0	$\frac{1}{4}$	0	0	0	0	0	0	0	0	0	$-\frac{1}{4}$
Ω_1^{26}	$\frac{1}{2}$	0	0	0	0	$\frac{1}{4}$	0	0	0	0	0	0	0	0	0	$\frac{1}{4}$
Ω_1^{27}	$\frac{1}{2}$	0	0	$\frac{1}{4}$	0	0	0	0	0	0	0	0	$-\frac{1}{4}$	0	0	0
Ω_1^{28}	$\frac{1}{2}$	0	0	$-\frac{1}{4}$	0	0	0	0	0	0	0	0	$\frac{1}{4}$	0	0	0
Ω_1^{29}	$\frac{1}{2}$	0	0	$\frac{1}{4}$	0	0	0	0	0	0	0	0	$\frac{1}{4}$	0	0	0

Table 2.2: The elements of θ_{ij}^k, $(S2, S2) = \frac{1}{2}\left(\left|1\right\rangle\left\langle 1\right| \otimes \sigma^0 + \sigma^0 \otimes \left|1\right\rangle\left\langle 1\right|\right)$

Despite of these differences, all the ‘good’ POVM sets include all the elements of $\{\Gamma^{ij}\}$. The POVM operators are generated by using the formula $\Omega_0^k = {U_{\text{ent}}^k}^\dagger \left[\frac{1}{2}\left(|0\rangle\langle 0| \otimes I_2 + I_2 \otimes |0\rangle\langle 0|\right)\right] U_{\text{ent}}^k$, which works for NV center. However, this approach is not applicable to other systems, such as a system using ion-trap. More generally, POVM operations are generated using the formula $\Omega_0^k = {U_{\text{ent}}^k}^\dagger \left[\frac{1}{2}\left(\sigma^3 \otimes I_2 + I_2 \otimes \sigma^3\right)\right] U_{\text{ent}}^k$, but this approach does not work well for QST algorithms because Γ^{00} does not appear in the measurement basis. This is thought to be important because Γ^{00} has a nonzero trace. Further investigation is underway to understand why the QST algorithm does not work in this case.

2.3 Appendix A : Representation basis Γ^{ij}

$$\Gamma^{00} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad \Gamma^{01} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\Gamma^{02} = \begin{pmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{pmatrix} \qquad \Gamma^{03} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\Gamma^{10} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \qquad \Gamma^{11} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$\Gamma^{12} = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix} \qquad \Gamma^{13} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$$

$$\Gamma^{20} = \begin{pmatrix} 0 & 0 & -i & 0 \\ 0 & 0 & 0 & -i \\ i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{pmatrix} \qquad \Gamma^{21} = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}$$

$$\Gamma^{22} = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \qquad \Gamma^{23} = \begin{pmatrix} 0 & 0 & -i & 0 \\ 0 & 0 & 0 & i \\ i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \end{pmatrix}$$

$$\Gamma^{30} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \qquad \Gamma^{31} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

$$\Gamma^{32} = \begin{pmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & i \\ 0 & 0 & -i & 0 \end{pmatrix} \qquad \Gamma^{33} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

2.4 Appendix B : POVM basis Ω_0^{ij}

$$\Omega_0^1 = \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix} \qquad \Omega_0^2 = \begin{pmatrix} \frac{1}{2} & 0 & 0 & -\frac{1}{2} \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ -\frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

$$\Omega_0^3 = \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2}i \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ -\frac{1}{2}i & 0 & 0 & \frac{1}{2} \end{pmatrix} \qquad \Omega_0^4 = \begin{pmatrix} \frac{1}{2} & 0 & 0 & -\frac{1}{2}i \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ \frac{1}{2}i & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

$$\Omega_0^5 = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{pmatrix} \qquad \Omega_0^6 = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

$$\Omega_0^7 = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{1}{2}i & 0 \\ 0 & \frac{1}{2}i & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{pmatrix} \qquad \Omega_0^8 = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2}i & 0 \\ 0 & -\frac{1}{2}i & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

$$\Omega_0^9 = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{4} & \frac{1}{4}i \\ 0 & \frac{1}{2} & \frac{1}{4}i & -\frac{1}{4} \\ \frac{1}{4} & -\frac{1}{4}i & \frac{1}{2} & 0 \\ -\frac{1}{4}i & -\frac{1}{4} & 0 & \frac{1}{2} \end{pmatrix} \qquad \Omega_0^{10} = \begin{pmatrix} \frac{1}{2} & 0 & -\frac{1}{4} & \frac{1}{4}i \\ 0 & \frac{1}{2} & \frac{1}{4}i & \frac{1}{4} \\ -\frac{1}{4} & -\frac{1}{4}i & \frac{1}{2} & 0 \\ -\frac{1}{4}i & \frac{1}{4} & 0 & \frac{1}{2} \end{pmatrix}$$

$$\Omega_0^{11} = \begin{pmatrix} \frac{1}{2} & \frac{1}{4} & 0 & -\frac{1}{4}i \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4}i & 0 \\ 0 & -\frac{1}{4}i & \frac{1}{2} & -\frac{1}{4} \\ \frac{1}{4}i & 0 & -\frac{1}{4} & \frac{1}{2} \end{pmatrix} \qquad \Omega_0^{12} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{4} & 0 & -\frac{1}{4}i \\ -\frac{1}{4} & \frac{1}{2} & \frac{1}{4}i & 0 \\ 0 & -\frac{1}{4}i & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{4}i & 0 & \frac{1}{4} & \frac{1}{2} \end{pmatrix}$$

$$\Omega_0^{13} = \begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ 0 & \frac{3}{4} & 0 & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & 0 & \frac{3}{4} \end{pmatrix} \qquad \Omega_0^{14} = \begin{pmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4}i & 0 \\ \frac{1}{4} & \frac{1}{2} & 0 & \frac{1}{4}i \\ -\frac{1}{4}i & 0 & \frac{1}{2} & \frac{1}{4} \\ 0 & -\frac{1}{4}i & \frac{1}{4} & \frac{1}{2} \end{pmatrix}$$

$$\Omega_0^{15} = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{3}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} & \frac{3}{4} \end{pmatrix} \qquad \Omega_0^{16} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{4}i & \frac{1}{4} & 0 \\ \frac{1}{4}i & \frac{1}{2} & 0 & \frac{1}{4} \\ \frac{1}{4} & 0 & \frac{1}{2} & -\frac{1}{4}i \\ 0 & \frac{1}{4} & \frac{1}{4}i & \frac{1}{2} \end{pmatrix}$$

$$\Omega_0^{17} = \begin{pmatrix} \frac{1}{2} & 0 & -\frac{1}{4}i & -\frac{1}{4} \\ 0 & \frac{1}{2} & -\frac{1}{4} & \frac{1}{4}i \\ \frac{1}{4}i & -\frac{1}{4} & \frac{1}{2} & 0 \\ -\frac{1}{4} & -\frac{1}{4}i & 0 & \frac{1}{2} \end{pmatrix} \qquad \Omega_0^{18} = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{4}i & -\frac{1}{4} \\ 0 & \frac{1}{2} & -\frac{1}{4} & -\frac{1}{4}i \\ -\frac{1}{4}i & -\frac{1}{4} & \frac{1}{2} & 0 \\ -\frac{1}{4} & \frac{1}{4}i & 0 & \frac{1}{2} \end{pmatrix}$$

$$\Omega_0^{19} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{4}i & 0 & -\frac{1}{4} \\ \frac{1}{4}i & \frac{1}{2} & -\frac{1}{4} & 0 \\ 0 & -\frac{1}{4} & \frac{1}{2} & \frac{1}{4}i \\ -\frac{1}{4} & 0 & -\frac{1}{4}i & \frac{1}{2} \end{pmatrix} \qquad \Omega_0^{20} = \begin{pmatrix} \frac{1}{2} & \frac{1}{4}i & 0 & -\frac{1}{4} \\ -\frac{1}{4}i & \frac{1}{2} & -\frac{1}{4} & 0 \\ 0 & -\frac{1}{4} & \frac{1}{2} & -\frac{1}{4}i \\ -\frac{1}{4} & 0 & \frac{1}{4}i & \frac{1}{2} \end{pmatrix}$$

$$\Omega_0^{21} = \begin{pmatrix} \frac{1}{4} & 0 & -\frac{1}{4}i & 0 \\ 0 & \frac{3}{4} & 0 & -\frac{1}{4}i \\ \frac{1}{4}i & 0 & \frac{1}{4} & 0 \\ 0 & \frac{1}{4}i & 0 & \frac{3}{4} \end{pmatrix} \qquad \Omega_0^{22} = \begin{pmatrix} \frac{1}{4} & 0 & \frac{1}{4}i & 0 \\ 0 & \frac{3}{4} & 0 & \frac{1}{4}i \\ -\frac{1}{4}i & 0 & \frac{1}{4} & 0 \\ 0 & -\frac{1}{4}i & 0 & \frac{3}{4} \end{pmatrix}$$

$$\Omega_0^{23} = \begin{pmatrix} \frac{1}{4} & -\frac{1}{4}i & 0 & 0 \\ \frac{1}{4}i & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{3}{4} & -\frac{1}{4}i \\ 0 & 0 & \frac{1}{4}i & \frac{3}{4} \end{pmatrix} \qquad \Omega_0^{24} = \begin{pmatrix} \frac{1}{4} & \frac{1}{4}i & 0 & 0 \\ -\frac{1}{4}i & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{3}{4} & \frac{1}{4}i \\ 0 & 0 & -\frac{1}{4}i & \frac{3}{4} \end{pmatrix}$$

$$\Omega_0^{25} = \begin{pmatrix} \frac{3}{4} & 0 & 0 & -\frac{1}{4} \\ 0 & \frac{1}{4} & -\frac{1}{4} & 0 \\ 0 & -\frac{1}{4} & \frac{1}{4} & 0 \\ -\frac{1}{4} & 0 & 0 & \frac{3}{4} \end{pmatrix} \qquad \Omega_0^{26} = \begin{pmatrix} \frac{1}{4} & 0 & 0 & -\frac{1}{4} \\ 0 & \frac{3}{4} & -\frac{1}{4} & 0 \\ 0 & -\frac{1}{4} & \frac{3}{4} & 0 \\ -\frac{1}{4} & 0 & 0 & \frac{1}{4} \end{pmatrix}$$

$$\Omega_0^{27} = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{pmatrix} \qquad \Omega_0^{28} = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

$$\Omega_0^{29} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\Omega_1^1 = \begin{pmatrix} \frac{1}{2} & 0 & 0 & -\frac{1}{2} \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ -\frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix} \qquad \Omega_1^2 = \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

$$\Omega_1^3 = \begin{pmatrix} \frac{1}{2} & 0 & 0 & -\frac{1}{2}i \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ \frac{1}{2}i & 0 & 0 & \frac{1}{2} \end{pmatrix} \qquad \Omega_1^4 = \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2}i \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ -\frac{1}{2}i & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

$$\Omega_1^5 = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{pmatrix} \qquad \Omega_1^6 = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

$$\Omega_1^7 = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2}i & 0 \\ 0 & -\frac{1}{2}i & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{pmatrix} \qquad \Omega_1^8 = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{1}{2}i & 0 \\ 0 & \frac{1}{2}i & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

$$\Omega_1^9 = \begin{pmatrix} \frac{1}{2} & 0 & -\frac{1}{4} & -\frac{1}{4}i \\ 0 & \frac{1}{2} & -\frac{1}{4}i & \frac{1}{4} \\ -\frac{1}{4} & \frac{1}{4}i & \frac{1}{2} & 0 \\ \frac{1}{4}i & \frac{1}{4} & 0 & \frac{1}{2} \end{pmatrix} \qquad \Omega_1^{10} = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{4} & -\frac{1}{4}i \\ 0 & \frac{1}{2} & -\frac{1}{4}i & -\frac{1}{4} \\ \frac{1}{4} & \frac{1}{4}i & \frac{1}{2} & 0 \\ \frac{1}{4}i & -\frac{1}{4} & 0 & \frac{1}{2} \end{pmatrix}$$

$$\Omega_1^{11} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{4} & 0 & \frac{1}{4}i \\ -\frac{1}{4} & \frac{1}{2} & -\frac{1}{4}i & 0 \\ 0 & \frac{1}{4}i & \frac{1}{2} & \frac{1}{4} \\ -\frac{1}{4}i & 0 & \frac{1}{4} & \frac{1}{2} \end{pmatrix} \qquad \Omega_1^{12} = \begin{pmatrix} \frac{1}{2} & \frac{1}{4} & 0 & \frac{1}{4}i \\ \frac{1}{4} & \frac{1}{2} & -\frac{1}{4}i & 0 \\ 0 & \frac{1}{4}i & \frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{4}i & 0 & -\frac{1}{4} & \frac{1}{2} \end{pmatrix}$$

$$\Omega_1^{13} = \begin{pmatrix} \frac{3}{4} & 0 & -\frac{1}{4} & 0 \\ 0 & \frac{1}{4} & 0 & -\frac{1}{4} \\ -\frac{1}{4} & 0 & \frac{3}{4} & 0 \\ 0 & -\frac{1}{4} & 0 & \frac{1}{4} \end{pmatrix} \qquad \Omega_1^{14} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{4} & -\frac{1}{4}i & 0 \\ -\frac{1}{4} & \frac{1}{2} & 0 & -\frac{1}{4}i \\ \frac{1}{4}i & 0 & \frac{1}{2} & -\frac{1}{4} \\ 0 & \frac{1}{4}i & -\frac{1}{4} & \frac{1}{2} \end{pmatrix}$$

$$\Omega_1^{15} = \begin{pmatrix} \frac{3}{4} & -\frac{1}{4} & 0 & 0 \\ -\frac{1}{4} & \frac{3}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & -\frac{1}{4} \\ 0 & 0 & -\frac{1}{4} & \frac{1}{4} \end{pmatrix} \qquad \Omega_1^{16} = \begin{pmatrix} \frac{1}{2} & \frac{1}{4}i & -\frac{1}{4} & 0 \\ -\frac{1}{4}i & \frac{1}{2} & 0 & -\frac{1}{4} \\ -\frac{1}{4} & 0 & \frac{1}{2} & \frac{1}{4}i \\ 0 & -\frac{1}{4} & -\frac{1}{4}i & \frac{1}{2} \end{pmatrix}$$

$$\Omega_1^{17} = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{4}i & \frac{1}{4} \\ 0 & \frac{1}{2} & \frac{1}{4} & -\frac{1}{4}i \\ -\frac{1}{4}i & \frac{1}{4} & \frac{1}{2} & 0 \\ \frac{1}{4} & \frac{1}{4}i & 0 & \frac{1}{2} \end{pmatrix} \qquad \Omega_1^{18} = \begin{pmatrix} \frac{1}{2} & 0 & -\frac{1}{4}i & \frac{1}{4} \\ 0 & \frac{1}{2} & \frac{1}{4} & \frac{1}{4}i \\ \frac{1}{4}i & \frac{1}{4} & \frac{1}{2} & 0 \\ \frac{1}{4} & -\frac{1}{4}i & 0 & \frac{1}{2} \end{pmatrix}$$

$$\Omega_1^{19} = \begin{pmatrix} \frac{1}{2} & \frac{1}{4}i & 0 & \frac{1}{4} \\ -\frac{1}{4}i & \frac{1}{2} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{2} & -\frac{1}{4}i \\ \frac{1}{4} & 0 & \frac{1}{4}i & \frac{1}{2} \end{pmatrix} \qquad \Omega_1^{20} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{4}i & 0 & \frac{1}{4} \\ \frac{1}{4}i & \frac{1}{2} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{2} & \frac{1}{4}i \\ \frac{1}{4} & 0 & -\frac{1}{4}i & \frac{1}{2} \end{pmatrix}$$

$$\Omega_1^{21} = \begin{pmatrix} \frac{3}{4} & 0 & \frac{1}{4}i & 0 \\ 0 & \frac{1}{4} & 0 & \frac{1}{4}i \\ -\frac{1}{4}i & 0 & \frac{3}{4} & 0 \\ 0 & -\frac{1}{4}i & 0 & \frac{1}{4} \end{pmatrix} \qquad \Omega_1^{22} = \begin{pmatrix} \frac{3}{4} & 0 & -\frac{1}{4}i & 0 \\ 0 & \frac{1}{4} & 0 & -\frac{1}{4}i \\ \frac{1}{4}i & 0 & \frac{3}{4} & 0 \\ 0 & \frac{1}{4}i & 0 & \frac{1}{4} \end{pmatrix}$$

$$\Omega_1^{23} = \begin{pmatrix} \frac{3}{4} & \frac{1}{4}i & 0 & 0 \\ -\frac{1}{4}i & \frac{3}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{4}i \\ 0 & 0 & -\frac{1}{4}i & \frac{1}{4} \end{pmatrix} \qquad \Omega_1^{24} = \begin{pmatrix} \frac{3}{4} & -\frac{1}{4}i & 0 & 0 \\ \frac{1}{4}i & \frac{3}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & -\frac{1}{4}i \\ 0 & 0 & \frac{1}{4}i & \frac{1}{4} \end{pmatrix}$$

$$\Omega_1^{25} = \begin{pmatrix} \frac{1}{4} & 0 & 0 & \frac{1}{4} \\ 0 & \frac{3}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{3}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} \end{pmatrix} \qquad \Omega_1^{26} = \begin{pmatrix} \frac{3}{4} & 0 & 0 & \frac{1}{4} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{3}{4} \end{pmatrix}$$

$$\Omega_1^{27} = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{pmatrix} \qquad \Omega_1^{28} = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$$

$$\Omega_1^{29} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Chapter 3

What to do?

What I've done and haven't

During my one-month study of Quantum State Tomography (QST), I had the opportunity to review QST algorithms based on the Linear Regression (LR) method and Maximum Likelihood Estimation (MLE) method. My focus was on the POVM measurement basis, where I discovered an intriguing relationship between the POVM basis and $SU(2) \otimes SU(2)$, which is equivalent to $SO(1,3)$. This property is crucial in finding a 'good' POVM basis set that can perform QST well. Currently, our 'good' basis sets are known to span $SU(2) \otimes SU(2)$ effectively. However, in the more general case where $(S1, S1)$ is replaced with (S_z, S_z), QST fails. This issue is believed to be caused by the traceless POVM basis set, but it is still not entirely clear.

What I'm going to do

At present, the most urgent issue is to generalize the 'good' POVM set. It is unclear why the POVM set generated by $(S1, S1)$ yields satisfactory QST results, while the one generated by (S_z, S_z) does not. Theoretical analysis reveals no significant differences between the two, except for a constant shift in the measurement process. Over the next few weeks, I will investigate QST algorithms that are unfamiliar to me, seeking potential breakthroughs in this area. Additionally, I will examine the subtle relationship between $SO(1,3)$ and POVM, which may reduce the number of elements in the POVM basis set. In summary, primary objectives for the upcoming weeks are to study

QST algorithms and some physical groups in order to generalize the 'good' POVM set.

1. Study QST algorithms, more analytically.
2. Perform QST with POVM basis $\{\Omega_0^i\} \cup \{\Omega_1^j\}$
3. Study $SO(3,1)$

The followings may be helpful for these study.

1. Zhibo Hou, Han-Sen Zhong, Ye Tian, Daoyi Dong, Bo Qi, Li Li, Yuanlong Wang, Franco Nori, Guo-Yong Xiang, Chuan-Feng Li, et al. Full reconstruction of a 14-qubit state within four hours. *New Journal of Physics*, 18(8):083036, 2016
2. Daniel FV James, Paul G Kwiat, William J Munro, and Andrew G White. Measurement of qubits. *Physical Review A*, 64(5):052312, 2001
3. Schwichtenberg Jakob. Physics from symmetry, 2018

Bibliography

[1] Zhibo Hou, Han-Sen Zhong, Ye Tian, Daoyi Dong, Bo Qi, Li Li, Yuanlong Wang, Franco Nori, Guo-Yong Xiang, Chuan-Feng Li, et al. Full reconstruction of a 14-qubit state within four hours. *New Journal of Physics*, 18(8):083036, 2016.

[2] Schwichtenberg Jakob. Physics from symmetry, 2018.

[3] Daniel FV James, Paul G Kwiat, William J Munro, and Andrew G White. Measurement of qubits. *Physical Review A*, 64(5):052312, 2001.