The Strategy for optimal QST and Monte Carlo Simulation Result

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Motivation

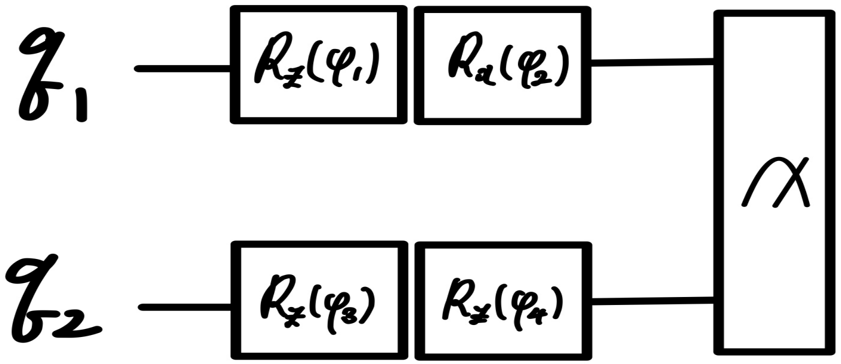
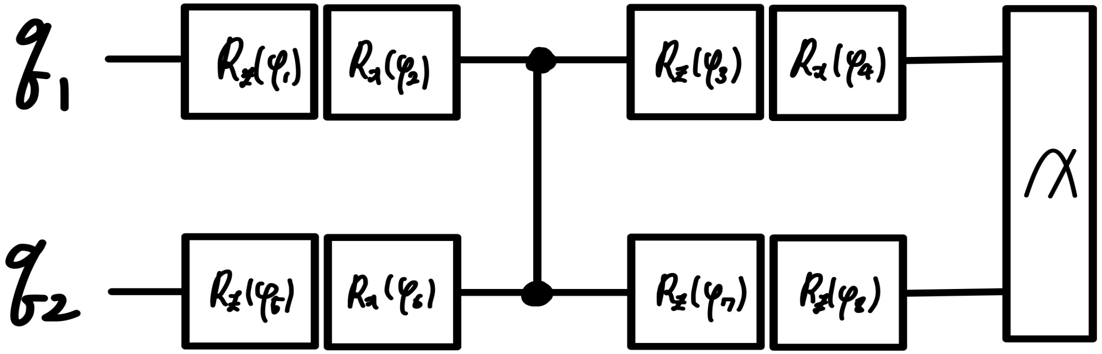
- Individual Readout in NV center system is impossible
- And experimental result is given as the number of detected photon that comes from n qubit

Therefore, QST is performed for more than two qubits with Positive valued measurement operator(PVM). Our PVMs are generated by the following formula.

$$\Omega^i = U_i^\dagger \frac{1}{2}(|0\rangle\langle 0| \otimes I_2 + I_2 \otimes |0\rangle\langle 0|)U_i$$

$$U_i = (R_1 \otimes R_2) \text{ or } (R_1 \otimes R_2)e^{-\frac{i\pi}{4}(\sigma^3 \otimes \sigma^3)}(R_3 \otimes R_4)$$

How the PVM gate looks like...

None CZ gate	One CZ gate
	

$$U_i = (R_1 \otimes R_2)$$

$$U_i = (R_1 \otimes R_2) e^{-\frac{i\pi}{4}(\sigma^3 \otimes \sigma^3)} (R_3 \otimes R_4)$$

Goal

- Conducting QST for more than 2 qubit
  - Done in last presentation : rank 15 was the condition for QST.
- Searching for the PVM set for the optimal QST:

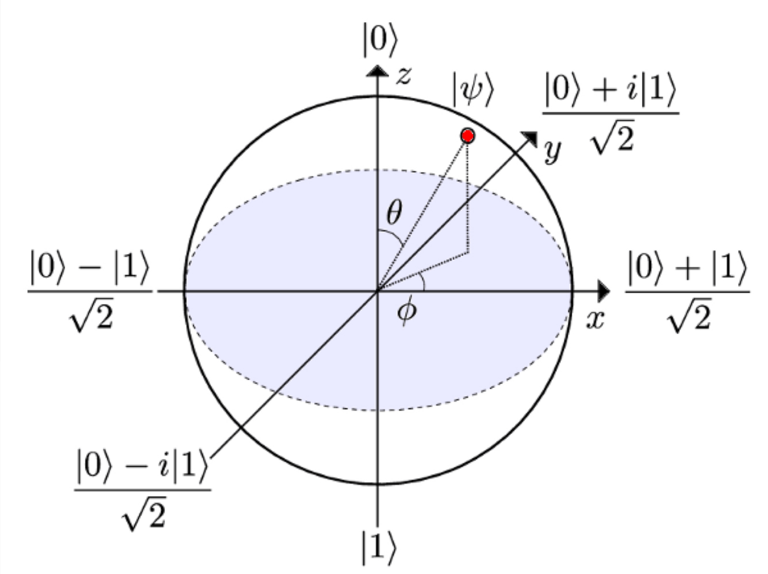
  Using less resources, Achieving Higher Fielity
- Evaluating how well we can do

Goal 2

Search the PVM set for the optimal QST :

Using less resources, Achieving Higher Fielity

Measurement process



The result of measurement process is given by

$$\langle \Omega^k \rangle = \mathrm{Tr}\ \hat{\rho}\Omega^k$$

Here, the measurement process measures the distance between the hyperplane and the state vector.

Ideal QST

The less correlated measurement operators are, the better QST is.

Metric : How to evaluate the correlation between PVM set elements?

von Neumann Entropy

Gram Matrix

The Gram matrix G shows overall correlation between the measurement operators.

The Gram matrix is given by:

$$G = \begin{pmatrix} \mathrm{Tr}\ \Omega_0\Omega_0 & \mathrm{Tr}\ \Omega_0\Omega_1 & \cdots & \mathrm{Tr}\ \Omega_0\Omega_{15} \\ \mathrm{Tr}\ \Omega_1\Omega_0 & \mathrm{Tr}\ \Omega_1\Omega_1 & \cdots & \mathrm{Tr}\ \Omega_1\Omega_{15} \\ \vdots & \vdots & \ddots & \vdots \\ \mathrm{Tr}\ \Omega_{15}\Omega_0 & \cdots & \cdots & \mathrm{Tr}\ \Omega_{15}\Omega_{15} \end{pmatrix}$$

von Neumann Entropy

The von Neumann Entropy of the Gram matrix shows how much and uniformly the information is obtained.

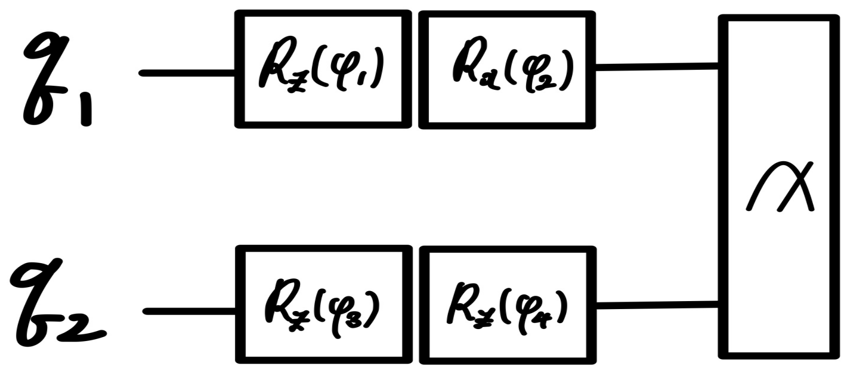
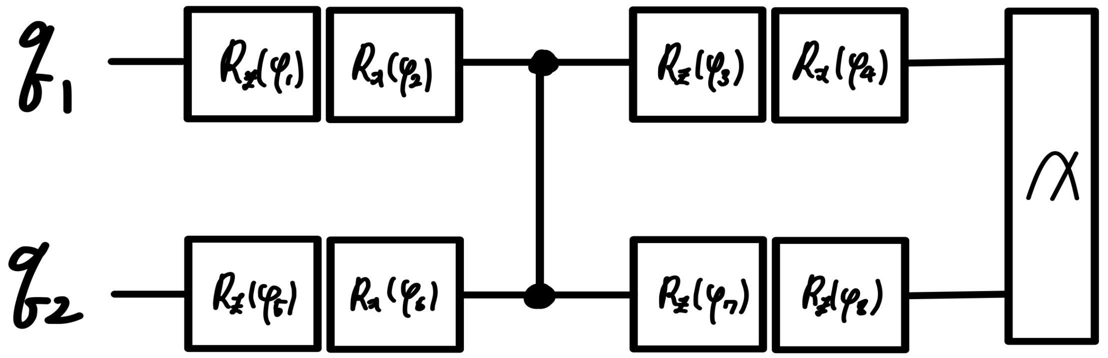
$$S = \sum_i -e_i \ln e_i$$

where e_i is a (normalized) eigenvalue of the Gram matrix.

Note: Any matrix can be represented as a linear combination form.

$$\rho = \sum_i a_i \Gamma^i$$

Strategy for maximzied von Neumann Entropy

None CZ gate	One CZ gate
	

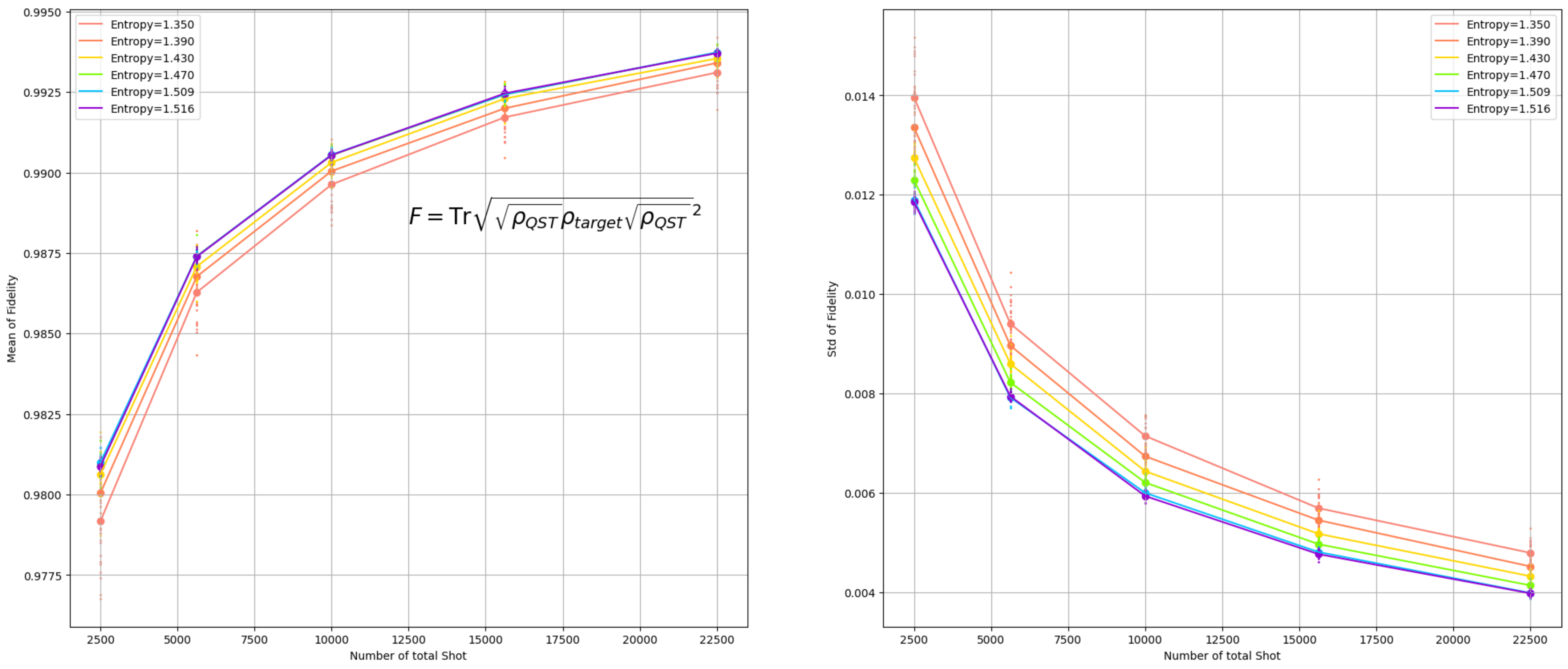
$$U_i = (R_1 \otimes R_2)$$

$$U_i = (R_1 \otimes R_2) e^{-\frac{i\pi}{4}(\sigma^3 \otimes \sigma^3)} (R_3 \otimes R_4)$$

Choose the parameter ϕ_i so that the von Neumann entropy is maximized.

Monte Carlo Simulation Result

The larger the entropy, the more stable the QST is.



Goal 3

Evaluating how well we can do

SIC POVM : The Ideal Case for QST

SIC(Symmetric Informationally Complete) POVM

- Each POVM is rank 1 matrix.
- Existance of the SIC POVM for all dimension remains an open problem.
  The von Neumann entropy of SIC POVM for the two-qubit system is about **2.7**.

GSIC POVM : Ideal Case for QST

GSIC(General Symmetric Informationally Complete) POVM

- Each POVM is not necessarily to be rank 1.
- Existance of GSIC POVM is guaranteed for all dimension.

$$G_i = \lambda A_i + \frac{1-\lambda}{d^2} I_4$$

A_i is a (special) basis such that $\mathbf{Tr}\, A_i A_j = \frac{1}{d}$ and $\sum A_i = I$.

Then, it is possible to construct GSIC POVM set in our set-up using a number of CZ gates. Here the von Neumann Entropy of GSIC POVM for the two-qubit system is about **1.539**.

Conventional PVM set

PVM operators are given by:

$$\Omega^{ij}_{i,j=0,1,2,3} = |i\rangle\langle i| \otimes |j\rangle\langle j|$$

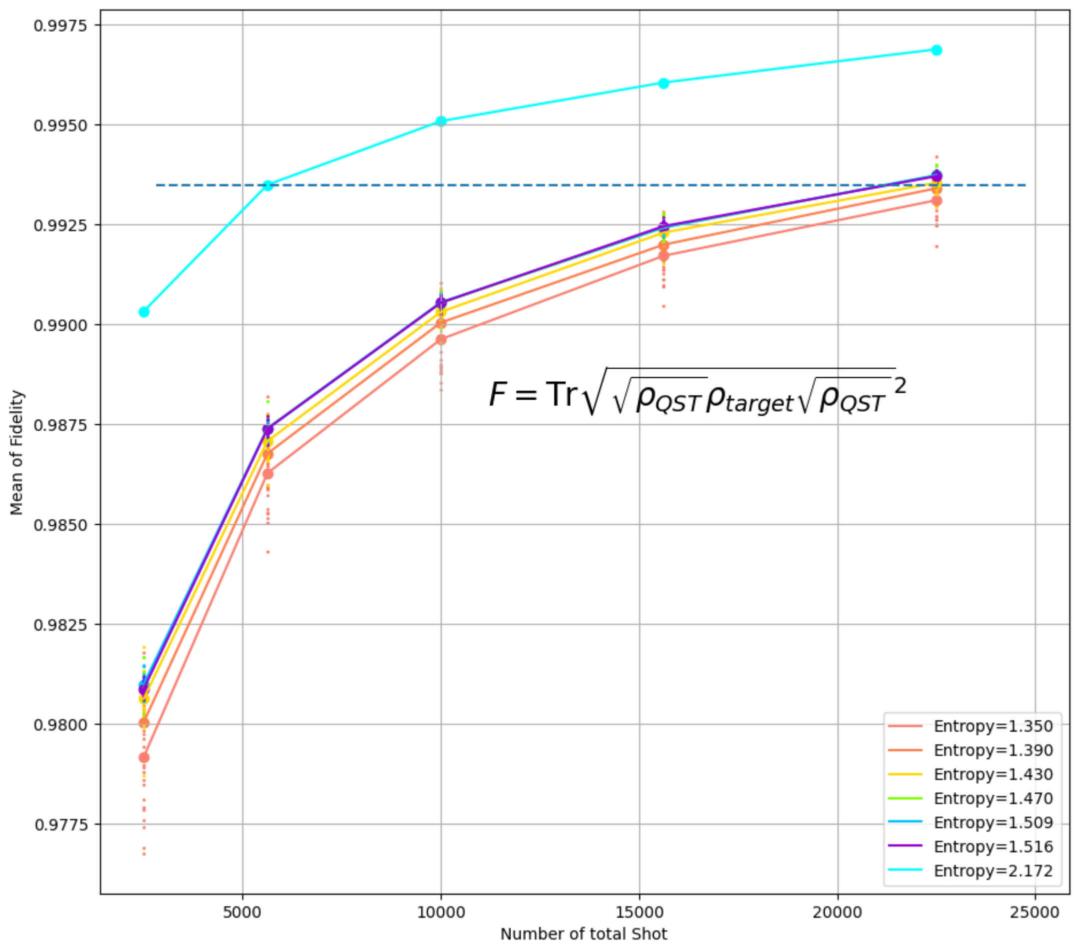
where

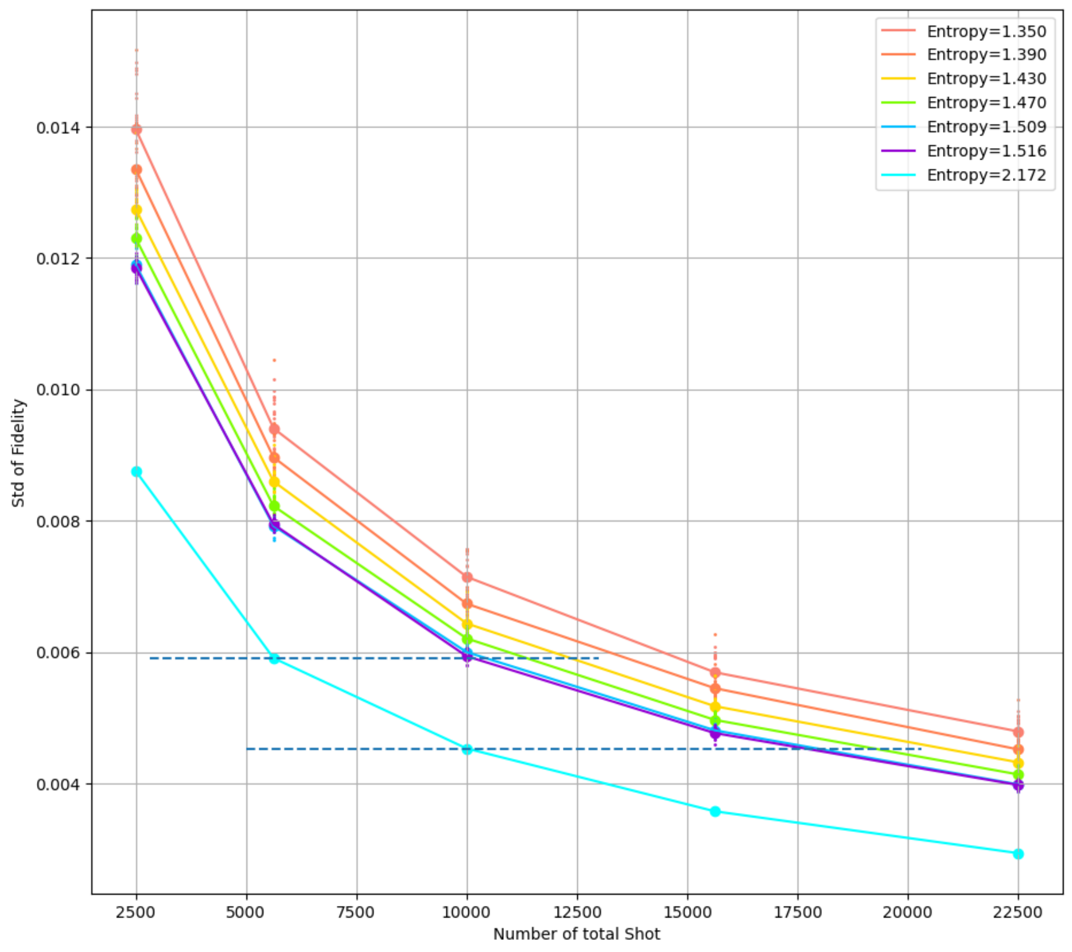
$$\begin{aligned}
|0\rangle &= |0\rangle \\
|1\rangle &= |1\rangle \\
|2\rangle &= |+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \\
|3\rangle &= |i\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)
\end{aligned}$$

This method is mainly used in various settings. In this case, the von Neumann entropy is about 2.17.

Monte Carlo Simulation Result

Our strategy comes up quite good PVM set, and differences in general methodologies can be overcome by getting more photons.





Our strategy is good enough!

Trade-off : The more conditional gate, the lower gate fidelity.

To construct such unitary operator, more than one conditional CZ gates are required for each POVM element.

My Status Report will be uploaded at the end of this week! (23/06/18)