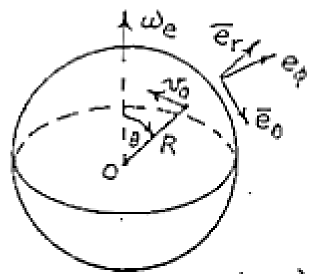
2–5. An airplane flies over a city at 45° north latitude with a relative velocity v_0 towards the northwest. Assuming that it flies in a great circle route of radius R relative to the earth which is rotating at ω_e rad/sec, solve for the airplane's acceleration relative to a nonrotating frame translating with the earth's center. Express the result in terms of the spherical unit vectors $\mathbf{e}_r$, $\mathbf{e}_\theta$, $\mathbf{e}_\phi$.



$$\vec{\omega} = \omega_e\left(\frac{1}{\sqrt{2}}\vec{e}_r - \frac{1}{\sqrt{2}}\vec{e}_\theta\right)$$

$$\dot{\vec{\rho}}_r = v_0\left(-\frac{1}{\sqrt{2}}\vec{e}_\theta - \frac{1}{\sqrt{2}}\vec{e}_\phi\right)$$

$\vec{\omega} = \vec{\omega}_e$, $\theta = 45^\circ$

$$\vec{a} = \ddot{\vec{R}} + \dot{\vec{\omega}} \times \vec{\rho} + \vec{\omega} \times (\vec{\omega} \times \vec{\rho}) + \ddot{\vec{\rho}}_r + 2\vec{\omega} \times \dot{\vec{\rho}}_r$$

C คือ $\ddot{\vec{R}} = 0$

$\dot{\vec{\omega}} = 0$, $\vec{\rho} = R\vec{e}_r$ ดังนั้น $\dot{\vec{\omega}} \times \vec{\rho} = 0$

$$\vec{\omega} \times (\vec{\omega} \times \vec{\rho}) = \frac{R}{\sqrt{2}}\omega_e^2\left(\frac{1}{\sqrt{2}}\vec{e}_r - \frac{1}{\sqrt{2}}\vec{e}_\theta\right)$$

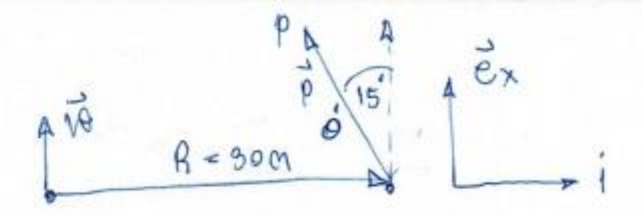
$$\ddot{\vec{\rho}}_r = -\frac{v^2}{R}e_r$$

$$2\vec{\omega} \times \dot{\vec{\rho}}_r = \omega_e v_0\left(-\vec{e}_r \times \vec{e}_\theta - \vec{e}_r \times \vec{e}_\phi + \vec{e}_\theta \times \vec{e}_\phi\right) = \omega_e v_0\left(\vec{e}_r + \vec{e}_\theta - \vec{e}_\phi\right)$$

$$\vec{a} = \left(-\frac{1}{2}R\omega_e^2 - \frac{v_0^2}{R} + \omega_\theta v_0\right)\vec{e}_r + \left(-\frac{1}{2}R\omega_e^2 + \omega_\theta v_0\right)\vec{e}_\theta - \omega_e v_0 \vec{e}_\phi \quad m/s^2$$

Ans

2–8. A cyclist rides around a circular track ($R = 30$ m) such that the point of contact of the wheel on the track moves at a constant speed of 10 m/sec. The bicycle is banked at 15° inward from the vertical. Find the acceleration of a tack in the tire (0.4 m radius) as it passes through the highest point of its path. Use cylindrical unit vectors in expressing the answer.



$\rho = 0.4\,m, \quad \omega = \frac{10}{30}\vec{e}_z$

จาก $a = \ddot{\vec{R}} + \dot{\vec{\omega}} \times \vec{\rho} + \vec{\omega} \times (\vec{\omega} \times \vec{\rho}) + \ddot{\vec{\rho}}_r + 2\,\vec{\omega} \times \dot{\vec{\rho}}_r$

$\vec{R} = (30 - 0.4 \sin 15°)\,\vec{e}_r + 0.4 \cos 15° \cdot \vec{e}_z = 29.896\,\vec{e}_r + 0.3864$

$\vec{\rho} = 0.4 \sin 15°\,\vec{e}_r + 0.4 \cos 15°\,\vec{e}_z = -0.1035\,\vec{e}_r + 0.3864$

$\ddot{\vec{R}} = -29.896 \left(\frac{1}{3}\right)^2 \vec{e}_r = -3.322\,\vec{e}_r$

$\dot{\vec{\omega}} \times \vec{\rho} = 0$

$\vec{\omega} \times (\vec{\omega} \times \vec{\rho}) = 0.1035 \left(\frac{1}{3}\right)^2 \vec{e}_r = 0.0115\,\vec{e}_r$

$\ddot{\vec{\rho}}_r = \frac{10^2}{0.4}\left(\sin 15°\,\vec{e}_r - \cos 15°\,\vec{e}_z\right) = 64.705\,\vec{e}_r - 241.481$

$\dot{\vec{\rho}}_r = 10\,\vec{e}_\phi$

$2\,\vec{\omega} \times \dot{\vec{\rho}}_r = -\frac{20}{3}\vec{e}_r$

$\vec{a} = 54.73\,\vec{e}_r - 241.5\,\vec{e}_z \qquad m/s^2$

2–12. A circular disk of radius r_2 rolls in its plane on the inside of a fixed circular cylinder of radius r_1. Find the acceleration of a point P on the wheel at a distance b from its hub O'. Assume that $\dot{\phi}$ is not constant, where the angle ϕ is measured between $O'P$ and the line of centers $O'O$.

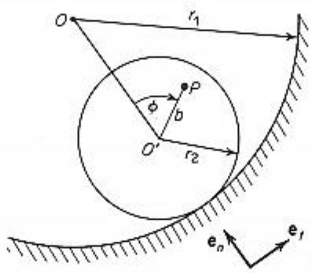


Figure P2-12

$$v_0 = (r_1 - r_2)\omega = r_2(\dot{\phi} - \omega), \quad \vec{\omega} = \frac{r_2}{r_1}\dot{\phi}\,\vec{e}_b$$

จากสูตร

$$\vec{a} = \ddot{\vec{R}} + \dot{\vec{\omega}} \times \vec{p} + \vec{\omega} \times (\vec{\omega} \times \vec{p}) + \ddot{\vec{p}}_r + 2\vec{\omega} \times \dot{\vec{p}}_r$$

$$\vec{p} = b(\sin\phi\,\vec{e}_t + \cos\phi\,\vec{e}_n$$

$$\dot{\vec{p}}_r = b\dot{\phi}(\cos\phi\,\vec{e}_r - \sin\phi\,\vec{e}_n)$$

$$\ddot{\vec{R}} = (r_1 - r_2)\left(\frac{r_2}{r_1}\dot{\phi}\right)^2 \vec{e}_n + (r_1 - r_2)\left(\frac{r_2}{r_1}\ddot{\phi}\right)\dot{e}_t$$

$$\ddot{\vec{\omega}} \times \vec{p} = \frac{b r_2}{r_1}\ddot{\phi}(-\cos\phi\,\vec{e}_t + \sin\phi\,\vec{e}_n)$$

$$\vec{\omega} \times (\vec{\omega} \times \vec{p}) = b\left(\frac{r_2}{r_1}\dot{\phi}\right)^2(-\sin\phi\,\vec{e}_t - \cos\phi\,\vec{e}_n)$$

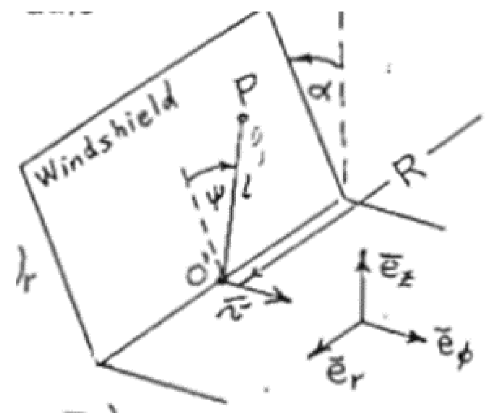
$$\ddot{p}_r = b\ddot{\phi}(\cos\phi\,\vec{e}_t - \sin\phi\,\vec{e}_n) + b\ddot{\phi}^2(-\sin\phi\,\vec{e}_t - \cos\phi\,\vec{e}_n)$$

$$2\dot{\vec{\omega}} \times \dot{\vec{p}}_r = 2\frac{b r_2}{r_1}\ddot{\phi}^2(\sin\phi\,\vec{e}_t + \cos\phi\,\vec{e}_n)$$

$$\vec{a} = \left[\left(1 - \frac{r_2}{r_1}\right)(r_2 + b\cos\phi)\ddot{\phi} - \left(1 - \frac{r_2}{r_1}\right)^2 b\dot{\phi}^2\sin\phi\right]\vec{e}_t$$

$$+ \left[-\left(1 - \frac{r_2}{r_1}\right)b\ddot{\phi}\sin\phi + \left[\left(1 - \frac{r_2}{r_1}\right)\frac{r_2^2}{r_1} - \left(1 - \frac{r_2}{r_1}\right)^2 b\cos\phi\right]\dot{\phi}^2\right]\vec{e}_n \quad m/s^2$$

2–16. The plane of the windshield of a certain auto is inclined at an angle α with the vertical. The windshield wiper blade is of length l and oscillates according to the equation $\psi = \psi_0 \sin \beta t$. Assuming that the auto travels with a constant speed v around a circular path of radius R in a counterclockwise sense, (more exactly, the point O' traces out a circle of this radius), solve for the acceleration of the point P at the tip of the wiper. For what value of $\dot{\psi}$ would you expect the largest force of the blade against the windshield?



$\vec{\omega} = \frac{v}{R}\vec{e}_z$, $\vec{\rho} = l(-\sin\psi\,\vec{e}_r - \cos\psi\sin\alpha\,\vec{e}_\phi + \cos\psi\cos\alpha\,\vec{e}_z$

จาก $\psi = \psi_0 \sin \beta t$

ดังนั้น $a = \ddot{\vec{R}} + \dot{\vec{\omega}} \times \vec{\rho} + \vec{\omega} \times (\vec{\omega} \times \vec{\rho}) + \ddot{\vec{\rho}}_r + 2\vec{\omega} \times \dot{\vec{\rho}}_r$

$\ddot{\vec{R}} = -\frac{v^2}{R}\vec{e}_r$, $\dot{\vec{\omega}} \times \vec{\rho} = 0$

$\vec{\omega} \times (\vec{\omega} \times \vec{\rho}) = \frac{l v^2}{R^2}(\sin\psi\,\vec{e}_r + \cos\psi\sin\alpha\,\vec{e}_\phi)$

$\dot{\vec{\rho}}_r = l\dot{\psi}(-\cos\psi\,\vec{e}_r + \sin\psi\sin\alpha\,\vec{e}_\phi - \sin\psi\cos\alpha\,\vec{e}_z)$

$2\vec{\omega} \times \dot{\vec{\rho}}_r = \frac{2vl\dot{\psi}}{R}(-\sin\psi\sin\alpha\,\vec{e}_r - \cos\psi\,\vec{e}_\phi)$

$\ddot{\vec{\rho}}_r = l\ddot{\psi}(-\cos\psi\,\vec{e}_r + \sin\psi\sin\alpha\,\vec{e}_\phi - \sin\psi\cos\alpha\,\vec{e}_z)$
$\quad + l\dot{\psi}^2(\sin\psi\,\vec{e}_r + \cos\psi\sin\alpha\,\vec{e}_\phi - \cos\psi\cos\alpha\,\vec{e}_z)$

$\vec{a} = \left[-\frac{v^2}{R} + \frac{v^2}{R^2}l\sin\psi + l\dot{\psi}^2\sin\psi - l\ddot{\psi}\cos\psi - \frac{2vl\dot{\psi}}{R}\sin\psi\sin\alpha\right]\vec{e}_r$

$\quad + \left[\frac{v^2}{R^2}l\cos\psi\sin\alpha + l\dot{\psi}^2\cos\psi\sin\alpha + l\ddot{\psi}\sin\psi\sin\alpha - \frac{2vl\dot{\psi}}{R}\cos\psi\right]\vec{e}_\phi$

$\quad + \left[-l\ddot{\psi}\sin\psi\cos\alpha - l\dot{\psi}^2\cos\psi\cos\alpha\right]\vec{e}_z$ m/s²

Ans

2–20. A cone of base radius r and semivertex angle β rolls without slipping inside a conical depression of semi-vertex angle 2β having a vertical axis of symmetry. The center of the base at O' moves about the vertical axis at a constant angular rate Ω. The unit vector system **i, j, k** also rotates about the vertical axis at the angular rate Ω.

(a) Find the acceleration of a point P on the edge of the base in terms of the angle ϕ measured clockwise from $O'Q$ to $O'P$ as viewed from above.

(b) Find the radius of curvature of the path of P at the instant when $\phi = 0$.

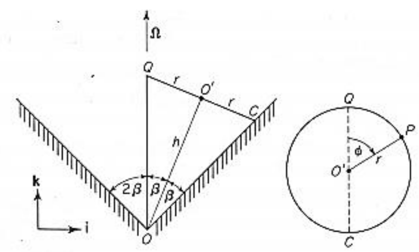


Figure P2-20

จาก $\vec{\omega} = \Omega \vec{k}$, $v_Q = r\dot{\phi}$

$2v_0 = 2(r\Omega\cos\beta) = r\dot{\phi}$ หรือ $\dot{\phi} = 2\Omega\cos\beta$

จาก $\vec{a} = \ddot{\vec{R}} + \dot{\vec{\omega}}\times\vec{\rho} + \vec{\omega}\times(\vec{\omega}\times\vec{\rho}) + \ddot{\vec{\rho}}_r + 2\vec{\omega}\times\dot{\vec{\rho}}_r$

$\ddot{\vec{R}} = -r\Omega^2\cos\beta\hat{i}$, $\dot{\vec{\omega}}\times\vec{\rho} = 0$, $\vec{\omega}\times(\vec{\omega}\times\vec{\rho}) = r\Omega^2(\cos\phi\cos\beta\hat{i} - \sin\phi\hat{j})$

$\dot{\vec{\rho}}_r = r\dot{\phi}(\sin\phi\cos\beta\hat{i} + \cos\phi\hat{j} - \sin\phi\sin\beta\hat{k})$

$= 2r\Omega(\sin\phi\cos^2\beta\hat{i} + \cos\phi\cos\beta\hat{j} - \sin\phi\sin\beta\cos\beta\hat{k})$

$\ddot{\vec{\rho}}_r = -\dot{\phi}^2\vec{\rho} = r\dot{\phi}^2(\cos\phi\cos\beta\hat{i} - \sin\phi\hat{j} - \cos\phi\sin\beta\hat{k})$

$= 4r\Omega^2(\cos\phi\cos^3\beta\hat{i} - \sin\phi\cos^2\beta\hat{j} - \cos\phi\sin\beta\cos^2\beta\hat{k})$

$2\vec{\omega}\times\dot{\vec{\rho}} = 4r\Omega^2(-\cos\phi\cos\beta\hat{i} + \sin\phi\cos^2\beta\hat{j})$

$\vec{a} = r\Omega^2\cos\beta(-1r\cos\phi - 4\cos\phi\sin^2\beta)\hat{i} - r\Omega^2\sin\phi\hat{j}$

$= -4r\Omega^2\cos\phi\sin\beta\cos^2\beta\vec{k}$

(b) สำหรับ $\phi = 0$ $\vec{a} = -4r\Omega^2\sin\beta\cos\beta(\sin\beta\hat{i} + \cos\beta\vec{k})$

$a = \frac{v^2}{\rho} = 4r\Omega^2\sin\beta\cos\beta$

$v = v_Q = 2r\Omega\cos\beta$.

$\rho = \frac{v^2}{a} = r\cot\beta = h$ Ans