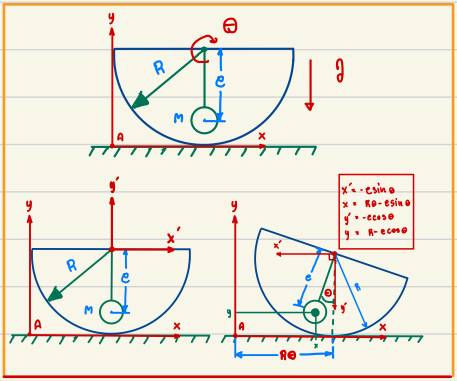
Analytical dynamics 254721: Quiz-1 Wathanyu Chaiya ID 630631081

1.



Fundamental form of Lagrange's equation.

$$\boxed{\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{q}}\right] - \frac{\partial L}{\partial q} = Q}$$

We have $L[q,\dot{q}] = T - V$

$$L[q,\dot{q}] = \frac{1}{2}mv^2 - mgy \qquad (1)$$

From: $v^2 = \left(\sqrt{\dot{x}^2 + \dot{y}^2}\right)^2 = \dot{x}^2 + \dot{y}^2$

Can be written in cartesian coordinates.

$$x = R\theta - e\sin\theta \quad \text{and} \quad y = R - e\cos\theta$$

And then. $\dot{x} = R\dot{\theta} - e\dot{\theta}\cos\theta$ and $\dot{y} = \dot{\theta}e\sin\theta$

We get: $v^2 = (R\dot{\theta} - e\dot{\theta}\cos\theta)^2 + (\dot{\theta}e\sin\theta)^2 = \dot{\theta}^2\left[(R^2 + 2Re\cos\theta + e^2\cos^2\theta) + e^2\sin^2\theta\right]$

$$v^2 = \dot{\theta}^2(R^2 + 2Re\cos\theta + e^2)$$

From (1) get: $L(q,\dot{q}) = \left[\frac{1}{2}m\dot{\theta}^2(R^2 - 2Re\cos\theta + e^2)\right] - \left[mg(R - e\cos\theta)\right]$

From lagrange equation.

Find $\frac{\partial L}{\partial \theta} = \left[\cancel{\frac{1}{2}}m\dot{\theta}^2(\cancel{2}Re\sin\theta)\right] - \left[mge\sin\theta\right] = m\dot{\theta}^2Re\sin\theta - mg\sin\theta$

$$\frac{\partial L}{\partial \dot{\theta}} = m\dot{\theta}(R^2 - 2Re\cos\theta + e^2)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = 2m\dot{\theta}^2(Re\sin\theta) + m\ddot{\theta}(R^2 - 2Re\cos\theta + e^2)$$

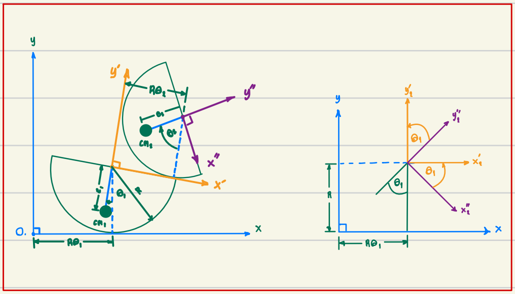
From $\boxed{\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{q}}\right] - \frac{\partial L}{\partial q} = Q}$ $\qquad Q = 0$

$$\cancel{2}\cancel{m}\dot{\theta}^2(Re\sin\theta) + \cancel{m}\ddot{\theta}(R^2 - 2Re\cos\theta + e^2) - \cancel{m}\dot{\theta}^2\cancel{Re}\sin\theta + \cancel{m}g\sin\theta = 0$$

$$(R^2 - 2Re\cos\theta + e^2)\ddot{\theta} + (Re\sin\theta)\dot{\theta}^2 + g\sin\theta = 0$$

$$\ddot{\theta} = \frac{-\left[(Re\sin\theta)\dot{\theta}^2 + g\sin\theta\right]}{(R^2 - 2Re\cos\theta + e^2)} \qquad \underline{\text{Ans}}$$

2.



Find Position of mass at fixed frame

From No. 1 We have kinetic and Potential energy of mass 1

$v_1^2 = \dot{\theta}_1^2 (R_1^2 - 2R_1e_1\cos\theta_1 + e_1^2)$ and $y_1 = (R_1 - e_1\cos\theta_1)$

And then consider of mass (2)

$x_2' = R\theta_2 - e_2\sin\theta_2$, $y_2' = R - e_2\cos\theta_2$

We can find x_2 from. $x_2 = R\theta_1 + x_2''\cos\theta_1 + y_2''\sin\theta_1 = R\theta_1 + (R\theta_2 - e_2\sin\theta_2)\cos\theta_1 + (R - e_2\cos\theta_2)\sin\theta_1 \Rightarrow R\theta_1 + R\theta_2\cos\theta_1 - e_2\sin\theta_2\cos\theta_1 + R\sin\theta_1 - e_2\cos\theta_2\sin\theta_1$

$x_2 = R\theta_1 + R\theta_2\cos\theta_1 + R\sin\theta_1 - e_2(\sin\theta_2\cos\theta_1 + \cos\theta_2\sin\theta_1)$ From $\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$ We get $x_2 = R\theta_1 + R\theta_2\cos\theta_1 + R\sin\theta_1 - e_2\sin(\theta_2 + \theta_1)$

And then find y_2 from. $y_2 = R - x_2''\sin\theta_1 + y_2''\cos\theta_1 = R - (R\theta_2 - e_2\sin\theta_2)\sin\theta_1 + (R - e_2\cos\theta_2)\cos\theta_1 = R - R\theta_2\sin\theta_1 + e_2\sin\theta_2\sin\theta_1 + R\cos\theta_1 - e_2\cos\theta_2\cos\theta_1$

$y_2 = R - R\theta_2\sin\theta_1 + R\cos\theta_1 - e(\cos\theta_2\cos\theta_1 - \sin\theta_2\sin\theta_1)$ From $\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$ We get $y_2 = R - R\theta_2\sin\theta_1 + R\cos\theta_1 - e\cos(\theta_2+\theta_1)$

We get $x_2 = R\theta_1 + R\theta_2\cos\theta_1 + R\sin\theta_1 - e_2\sin(\theta_2+\theta_1)$, $y_2 = R - R\theta_2\sin\theta_1 + R\cos\theta_1 - e\cos(\theta_2+\theta_1)$

Fundamental form of lagrange's equation. $\boxed{\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{q}}\right] - \frac{\partial L}{\partial q} = Q}$

Defind $L(q,\dot{q}) = T - V$

$L(q,\dot{q}) = \frac{mv^2}{2} - mgy$ From $v^2 = \left(\sqrt{\dot{x}^2 + \dot{y}^2}\right)^2 = \dot{x}^2 + \dot{y}^2$ and $S_2 = x_2 i + y_2 j$

Find $\dot{s}_2$ from $\dot{s}_2 = \dot{x}_2\hat{i} + \dot{y}_2\hat{j} = (R\dot{\theta}_1 - R\theta_2\dot{\theta}_1\sin\theta_1 + R\dot{\theta}_2\cos\theta_1 + R\dot{\theta}_1\cos\theta_1 - e_2(\dot{\theta}_2+\dot{\theta}_1)\cos\theta_2+\theta_1)\hat{i} + (-R\theta_2\dot{\theta}_1\cos\theta_1 - R\dot{\theta}_2\sin\theta_1 - R\dot{\theta}_1\sin\theta_1 - e(\dot{\theta}_2+\dot{\theta}_1)\cos(\theta_2+\theta_1))\hat{j}$

$v_2^2 = \dot{s}_2^2 = (R\dot{\theta}_1 - R\theta_2\dot{\theta}_1\sin\theta_1 + R\dot{\theta}_2\cos\theta_1 + R\dot{\theta}_1\cos\theta_1 - e_2(\dot{\theta}_2+\dot{\theta}_1)\cos\theta_2+\theta_1)^2 + (-R\theta_2\dot{\theta}_1\cos\theta_1 - R\dot{\theta}_2\sin\theta_1 - R\dot{\theta}_1\sin\theta_1 - e(\dot{\theta}_2+\dot{\theta}_1)\cos(\theta_2+\theta_1))^2$

And then find kinetic energy of system. $T = \frac{mv_1^2}{2} + \frac{mv_2^2}{2}$

$T = \frac{m\dot{\theta}_1^2}{2}(R_1^2 - 2R_1e_1\cos\theta_1 + e_1^2) + \frac{m}{2}((R\dot{\theta}_1 - R\theta_2\dot{\theta}_1\sin\theta_1 + R\dot{\theta}_2\cos\theta_1 + R\dot{\theta}_1\cos\theta_1 - e_2(\dot{\theta}_2+\dot{\theta}_1)\cos\theta_2+\theta_1)^2 + (-R\theta_2\dot{\theta}_1\cos\theta_1 - R\dot{\theta}_2\sin\theta_1 - R\dot{\theta}_1\sin\theta_1$

Find Potential energy of system. $V = mgy_1 + mgy_2 = mg[(R - e_1\cos\theta_1) + (R - R\theta_2\sin\theta_1 + R\cos\theta_1 - e\cos(\theta_2+\theta_1))]$

$V = mg(\cancel{R} - e_1\cos\theta_1 + \cancel{R} - R\theta_2\sin\theta_1 + R\cos\theta_1 - e\cos(\theta_2+\theta_1)) \Rightarrow -mg((e_1 - R)\cos\theta_1 + R\theta_2\sin\theta_1 + e\cos(\theta_2+\theta_1))$

And then we have $L = T - V = \frac{m}{2}\left[(R^2\dot{\theta}_1^2 - 2\dot{\theta}^2Re_1\cos\theta_1 + e^2) + (R\dot{\theta}_1 - R\theta_2\dot{\theta}_1\sin\theta_1 + R\dot{\theta}_2\cos\theta_1 + R\dot{\theta}_1\cos\theta_1 - e_2(\dot{\theta}_2+\dot{\theta}_1)\cos\theta_2+\theta_1)^2\right.$

$\left.+ (-R\theta_2\dot{\theta}_1\cos\theta_1 - R\dot{\theta}_2\sin\theta_1 - R\dot{\theta}_1\sin\theta_1 - e(\dot{\theta}_2+\dot{\theta}_1)\cos(\theta_2+\theta_1))^2\right] + mg((e_1 - R)\cos\theta_1 + R\theta_2\sin\theta_1 + e\cos(\theta_2+\theta_1))$ #

And then solve equation from matlab ($\sin A = sA$, $\cos A = cA$). we get

$$\frac{\partial L}{\partial \theta_1} = Rgs\theta_1 - egs\theta_1 - R^2\dot{\theta}_1^2 s\theta_1 - egs\theta_2{+}\theta_1 - R^2\theta_2\dot{\theta}_1^2 c\theta_1 + Rg\theta_2 c\theta_1 + Re\dot{\theta}_1^2 s\theta_2{+}\theta_1 + Re\dot{\theta}_1^2 s\theta_1 - R^2\dot{\theta}_1\dot{\theta}_2 s\theta_1 + Re\dot{\theta}_1\dot{\theta}_2 s\theta_2{+}\theta_1 \quad \#$$

$$\frac{\partial L}{\partial \theta_2} = Rgs\theta_1 + R^2\theta_2\dot{\theta}_1^2 - R^2\dot{\theta}_1^2 s\theta_1 - egs\theta_2{+}\theta_1 + Re\dot{\theta}_1^2 s\theta_2{+}\theta_1 + Re\dot{\theta}_1^2 s\theta_2 - Re\theta_2\dot{\theta}_1^2 c\theta_2 + Re\dot{\theta}_1\dot{\theta}_2 s\theta_2{+}\theta_1 + Re\dot{\theta}_1\dot{\theta}_2 s\theta_2 - Re\theta_2\dot{\theta}_1\dot{\theta}_2 c\theta_2 \quad \#$$

$$\frac{\partial L}{\partial \dot{\theta}_1} = \frac{m}{2}\Big[2(Rg\theta_1 - es\theta_2{+}\theta_1 + R\theta_2 c\theta_1)(Rs\theta_1)(\dot{\theta}_2+\dot{\theta}_1) - es\theta_2{+}\theta_1(\dot{\theta}_2+\dot{\theta}_1) + R\theta_2\dot{\theta}_1 c\theta_1 + 2(R\dot{\theta}_1 - ec\theta_2{+}\theta_1(\dot{\theta}_2+\dot{\theta}_1) + Rc\theta_1(\dot{\theta}_2+\dot{\theta}_1) - R\theta_2\dot{\theta}_1 s\theta_1)(R - ec\theta_2{+}\theta_1 + Rc\theta_1 - R\theta_2 s\theta_1)\Big]$$

$$+ m\dot{\theta}_1(R^2 - 2c\theta_1) \quad \#$$

$$\frac{\partial L}{\partial \dot{\theta}_2} = m(R^2\dot{\theta}_1 + R^2\dot{\theta}_2 + e^2\dot{\theta}_1 + e^2\dot{\theta}_2 + R^2\dot{\theta}_1 c\theta_1 - Re\dot{\theta}_1 c\theta_2{+}\theta_1 - 2Re\dot{\theta}_1 c\theta_2 - 2Re\dot{\theta}_2 c\theta_2 - Re\dot{\theta}_1 s\theta_2 \quad \#$$

$$\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{\theta}_1}\right] = 3R^2\ddot{\theta}_1 + R^2\ddot{\theta}_2 + 2e^2\ddot{\theta}_1 + e^2\ddot{\theta}_2 + R^2\theta_2^2\ddot{\theta}_1 + 2R^2\ddot{\theta}_1 c\theta_1 + R^2\ddot{\theta}_2 c\theta_1 - 2R^2\dot{\theta}_1^2 s\theta_1 - 2R^2\theta_2\dot{\theta}_1^2 - 2Re\ddot{\theta}_1 c\theta_2{+}\theta_1 - Re\ddot{\theta}_2 c\theta_2{+}\theta_1 - 2Re\ddot{\theta}_1 c\theta_1 - 2Re\ddot{\theta}_1 c\theta_2 - 2Re\ddot{\theta}_2 c\theta_2 + 2R^2\theta_2\dot{\theta}_1\dot{\theta}_2 + 2Re\dot{\theta}_1^2 s\theta_2{+}\theta_1 \quad \#$$

$$\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{\theta}_2}\right] = R^2\ddot{\theta}_1 + R^2\ddot{\theta}_2 + e^2\ddot{\theta}_1 + e^2\ddot{\theta}_2 + R^2\ddot{\theta}_1 c\theta_1 - R^2\dot{\theta}_1^2 s\theta_1 - Re\ddot{\theta}_1 c\theta_2{+}\theta_1 - 2Re\ddot{\theta}_1 c\theta_1 - 2Re\ddot{\theta}_2 c\theta_2 + Re\dot{\theta}_2^2 s\theta_2 + Re\dot{\theta}_1 s\theta_2{+}\theta_1(\dot{\theta}_2+\dot{\theta}_1) - Re\theta_2\ddot{\theta}_1 s\theta_2 + Re\dot{\theta}_1\dot{\theta}_2 s\theta_2 - Re\theta_2\dot{\theta}_1\dot{\theta}_2 c\theta_2 \quad \#$$

From Lagrange equation. We have $\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{\theta}_i}\right] - \frac{\partial L}{\partial \theta_i} = Q$

Thus We can obtian. equation motion of $\ddot{\theta}_1$ and $\ddot{\theta}_2$

$$\ddot{\theta}_1 = \Big[R^2\ddot{\theta}_2 + e^2\ddot{\theta}_2 - Rgs\theta_1 + egs\theta_1 + R^2\ddot{\theta}_2 c\theta_1 - R^2\dot{\theta}_1^2 s\theta_1 + eg\sin\theta_2{+}\theta_1 - R^2\theta_2\dot{\theta}_1^2 c\theta_1 - Re\ddot{\theta}_2 c\theta_2{+}\theta_1 - 2Re\ddot{\theta}_2 c\theta_2 - Rg\theta_2 c\theta_1 + 2R^2\theta_2\dot{\theta}_1\dot{\theta}_2 + Re\dot{\theta}_1^2 s\theta_2{+}\theta_1 + Re\dot{\theta}_2^2 s\theta_2{+}\theta_1 + Re\dot{\theta}_1^2 s\theta_1 + Re\dot{\theta}_2^2 s\theta_2 - 2R^2\dot{\theta}_1\dot{\theta}_2 s\theta_1 - Re\theta_2\dot{\theta}$$

$$+ 2Re\dot{\theta}_1\dot{\theta}_2 s\theta_2{+}\theta_1 - Re\theta_2\ddot{\theta}_2 s\theta_2 - 2Re\theta_2\dot{\theta}_1\dot{\theta}_2 c\theta_2\Big] \Big/ \Big[2R^2\theta_2 s\theta_1 - 2e^2 - R^2\theta_2^2 - 2R^2 c\theta_1 - 3R^2 + 2Rec\theta_2{+}\theta_1 + 2Re\cos\theta_1 + 2Re\cos\theta_2 + 2Re\theta_2 s\theta_2\Big] \quad \text{Ans}$$

$$\ddot{\theta}_2 = -\frac{R^2\ddot{\theta}_1 + e^2\ddot{\theta}_1 - Rg\sin\theta_1 - R^2\theta_2\dot{\theta}_1^2 + R^2\ddot{\theta}_1\cos\theta_1 + eg\sin\theta_2{+}\theta_1 - Re\ddot{\theta}_1\cos\theta_2{+}\theta_1 - 2Re\ddot{\theta}_1\cos\theta_2 + Re\dot{\theta}_2^2\sin\theta_2 + Re\theta_2\dot{\theta}_1^2\cos\theta_2 - Re\theta_2\ddot{\theta}_1\sin\theta_2}{R^2 - 2\cos\theta_2 Re + e^2} \quad \text{Ans}$$

Solution of all show at "Lagrange.mlx"