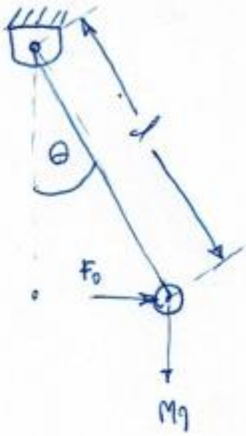
3–8. A simple pendulum of mass m and length l is initially motionless at the bottom equilibrium position, $\theta(0) = 0$. Then a constant horizontal force $F_0 = \frac{1}{2}mg$ is applied to the particle. Find θ_{max} for the motion which follows.



Potential energy term.

$$V = -mgl\cos\theta - \frac{1}{2}mgl\sin\theta$$

$$T_1 = \frac{1}{2}ml^2\dot{\theta}^2$$

$$T_1 + V_1 = T_2 + V_2$$

from conservation of energy

$$\frac{1}{2}ml^2\dot{\theta}^2 - mgl\left(\cos\theta + \frac{1}{2}\sin\theta\right) = -mgl$$

set $\dot{\theta} = 0$ and $\theta = \theta_{max}$, so $\cos\theta + \frac{1}{2}\sin\theta = 1$

But $\cos\theta = \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}$ and $\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

Hence $\sin\frac{\theta}{2}\cos\frac{\theta}{2} = 2\sin^2\frac{\theta}{2}$, so $\tan\frac{\theta_{max}}{2} = \frac{1}{2}$

$\theta_{max} = 2\tan^{-1}\frac{1}{2} = 53.13^{\circ}$ Ans

3–9. A pendulum consists of a particle of mass m and a massless string of length $2R$. As the deflection angle θ increases, the string wraps around one of two fixed cylinders of radius R adjacent to the support point O.

(a) Obtain the differential equation for θ, where θ is assumed to be positive.

(b) Assuming the initial conditions $\theta(0) = 0$, $\dot{\theta}(0) = (g/2R)^{1/2}$, find θ_{max}.

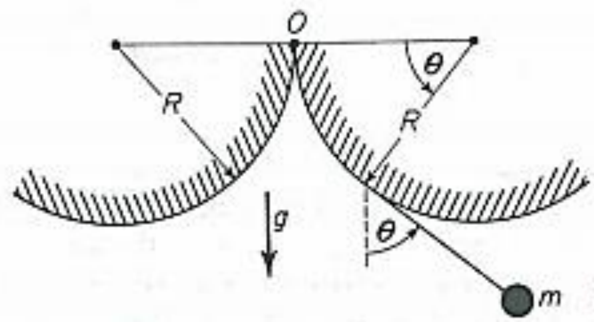
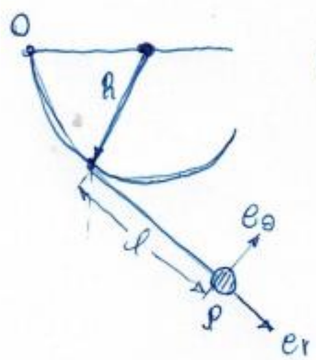


Figure P3-9



a. from $l = 2R - R\theta = (2-\theta)R$

Using tangential and normal components.

$$\vec{a}_p = \ddot{s}\vec{e}_\theta - \frac{\dot{s}^2}{\rho} e_r$$

Where $\dot{s} = l\dot{\theta} = (2-\theta)R\dot{\theta}$ and $\rho = l$

$$\ddot{s} = (2-\theta)R\ddot{\theta} - R\dot{\theta}^2$$

$$\Sigma F_\theta = m a_\theta$$

$-mg\sin\theta = m(R(2-\theta)\ddot{\theta} - R\dot{\theta}^2)$ and then $R(2-\theta)\ddot{\theta} - R\dot{\theta}^2 + g\sin\theta = 0$

b. Use conservation of energy. set $\theta(0) = 0$, $\dot{\theta}(0) = \sqrt{\frac{g}{2R}}$

$$T + V = \frac{1}{2} m (2-\theta)^2 R^2 \dot{\theta}^2 - mgR\left(\sin\theta + (2-\theta)\cos\theta\right) = mgR - 2mgR$$

$$= -mgR$$

θ_{max} at $\dot{\theta} = 0$ so $\sin\theta + (2-\theta)\cos\theta = 1$ or $\theta_{max} = \frac{\pi}{2}$ Ans

3–10. A massless disk of radius R has an embedded particle of mass m at a distance $R/2$ from the center. The disk is released from rest in the position shown and rolls without slipping down the fixed inclined plane. Find: (a) $\dot{\theta}$ as a function of θ; (b) a position θ at which the speed of the particle is momentarily constant.

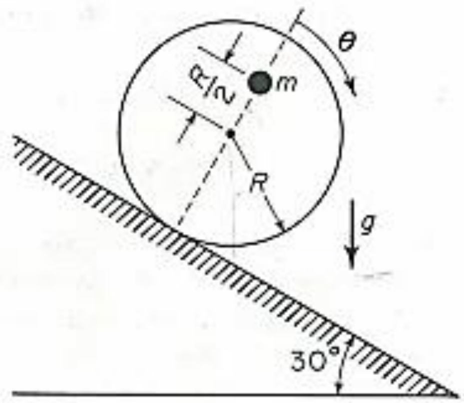


Figure P3-10

Conservative system

a. from $v^2 = \left[R^2 + \left(\frac{R}{2}\right)^2 + 2R\left(\frac{R}{2}\right)\cos\theta\right]\dot{\theta}^2$

$$T = \frac{1}{2}mv^2 = \frac{1}{2}mR^2\dot{\theta}^2\left(\frac{5}{4}+\cos\theta\right)$$

$$V = -\frac{1}{2}mgR\theta + \frac{1}{2}mgR\cos(\theta+30^\circ)$$

Using conservation of energy.

$$T+V = \frac{1}{8}mR^2\dot{\theta}^2(5+4\cos\theta) + \frac{1}{2}mgR\times$$

$$[\cos(\theta+30^\circ)-\theta] = \frac{\sqrt{3}}{4}mg$$

solving for $\dot{\theta}$, $\dot{\theta} = \sqrt{\frac{g}{R}\left[\frac{2\sqrt{3}+4\theta-4\cos(\theta+30^\circ)}{5+4\cos\theta}\right]}$ ANS

b. The speed of the particle is constant when potential energy constant.

$$\frac{dV}{d\theta} = -\frac{1}{2}mgR - \frac{1}{2}mgR\sin(\theta+30^\circ) = 0, \text{ so } \theta = 240^\circ$$

3–13. Given that the force acting on a particle has the following components: $F_x = -x + y$, $F_y = x - y + y^2$, $F_z = 0$. Solve for the potential energy V.

Given $\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$ where $F_x = -x + y$; $F_y = x - y + y^2$

$F_z = 0$

from $\frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x} = 1$, so $(-x+y)\,dx + (x-y+y^2)\,dy = -dV$

from the exact. we have

$-\frac{\partial V}{\partial x} = -x+y$ integrates to $-V = -\frac{1}{2}x^2 + xy + f_1(y)$

$-\frac{\partial V}{\partial x} = x - y + y^2$ integrates to $-V = xy - \frac{1}{2}y^2 + \frac{1}{3}y^3 + f_2(x)$

result $V = \frac{1}{2}x^2 - xy + \frac{1}{2}y^2 - \frac{1}{3}y^3 + C$ Ans

3–15. Initially the spring has its unstressed length l_0 and the particle has a velocity v_0 in

maximum length $4l_0/3$. Assuming no gravity, solve for the spring stiffness k as a function of m, l_0, and v_0.

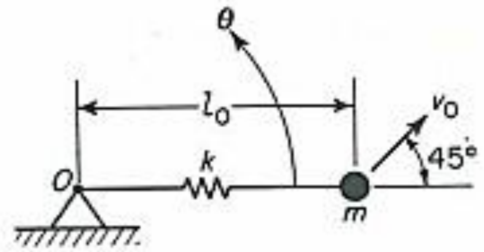


Figure P3-15

Use. polar coordinates.

$$T = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2)$$

$$V = \frac{1}{2}k(r - l_0)^2$$

Conservative system.

$$\frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) + \frac{1}{2}k(r - l_0)^2 = \frac{1}{2}mv_0^2$$

At max stretch $r = \frac{4l_0}{3}$, $\dot{r} = 0$ we obtain

$$\frac{1}{2}m\left(\frac{4l_0}{3}\right)^2\dot{\theta}^2 + \frac{1}{2}k\left(\frac{l_0}{3}\right)^2 = \frac{1}{2}mv_0^2$$

Using conservation of angular momentum.

$mr^2\dot{\theta} = \frac{ml_0}{\sqrt{2}}v_0$ or $\dot{\theta} = \frac{l_0 v_0}{\sqrt{2}r^2}$ at $r = max$

Then $\frac{8}{9}ml_0^2\left(\frac{9v_0}{16\sqrt{2}l_0}\right)^2 + \frac{kl_0^2}{18} = \frac{1}{2}mv_0^2$

or $k = 18\frac{mv_0^2}{l_0^2}\left(\frac{1}{2} - \frac{9}{64}\right) = \frac{207mv_0^2}{32l_0^2} = 6.4668.\frac{mv_0^2}{l_0^2}$ ans