Analytical dynamics 254721: HW4-2 Wathanyu Chaiya ID 630631081

4–6. A smooth sphere A of mass m, traveling with velocity $\mathbf{v} = v_0\mathbf{i}$, hits a similar motionless sphere B such that the angle between the line of centers and the negative x axis is 45°. Assuming that the coefficient of restitution is $e = 0.90$, find the velocities of A and B after impact.

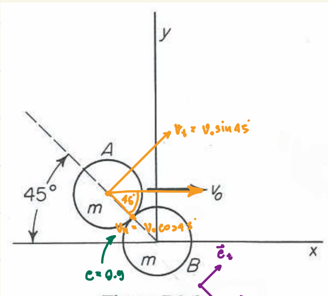


Figure P4-6

(v is velocities of before impact. and v' is after impact.)

In case before impact. from figure P4-6

$v_{nA} = v_{tA} = v_0 \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\cancel{2} v_0}{\cancel{2}\sqrt{2}}$; because $\sin 45° = \cos 45°$

$v_{nB} = v_{tB} = 0$; because motionless spheres B

And then, consider after impact in the normal velocities

Thus we can obtain

$$v'_{nA} = \left(\frac{m_1 - e m_2}{m_1 + m_2}\right) \cdot v_{nA} + \left(\frac{(1+e) m_2}{m_1 + m_2}\right) \cdot v_{nB}$$

$$v'_{nA} = \frac{m_1 - 0.9 m_2}{m_1 + m_2} \cdot \frac{v_0}{\sqrt{2}} + \frac{(1 + 0.9) m_2}{m_1 + m_2} \cdot 0 \quad ; \langle m_1 = m_2 = 1 \rangle$$

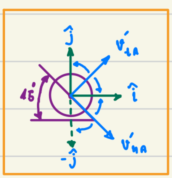
$$v'_{nA} = \frac{0.1 v_0}{2\sqrt{2}} + 0 = \frac{v_0}{20\sqrt{2}}$$

$$v'_{nB} = \left(\frac{(1+e) m_1}{m_1 + m_2}\right) \cdot v_{nA} + \left(\frac{m_2 - e m_1}{m_1 + m_2}\right) \cdot v_{nB}$$

$$v'_{nB} = \frac{(1 + 0.9)}{2} \cdot \frac{v_0}{\sqrt{2}} + 0 = \frac{1.9 v_0}{2\sqrt{2}} \times \frac{10}{10} = \frac{19 v_0}{20\sqrt{2}}$$

And then taking $\vec{i}$ and $\vec{j}$ in components.

We get. $v = v_0(\hat{i} + \hat{j})$; $v'_{nA} = \frac{v_0}{20\sqrt{2}}$, from $m_1 v_{tA} + m_2 v_{tB} = m_1 v'_{tA} + m_2 v'_{t2}$



Consider of mass A

We get $v'_{tA} = v_{tA} - v'_{t2}$ in case $m_1 = m_2$

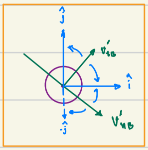
$$v'_{tA} = \frac{v_0}{\sqrt{2}} - 0 = \frac{v_0}{\sqrt{2}}$$

Thus $\vec{v}'_A = v_0 \left[(v'_{tA} \cos 45° + v'_{nA} \cos 45°) i + (v'_{tA} \sin 45° - v'_{nA} \sin 45°) j \right]$

$$v'_A = \left[\left(\frac{v_0}{\cancel{\sqrt{2}}}\cdot\frac{\cancel{\sqrt{2}}}{2} + \frac{v_0}{20\cancel{\sqrt{2}}}\cdot\frac{\cancel{\sqrt{2}}}{2}\right)i + \left(\frac{v_0}{\cancel{\sqrt{2}}}\cdot\frac{\cancel{\sqrt{2}}}{2} - \frac{v_0}{20\cancel{\sqrt{2}}}\cdot\frac{\cancel{\sqrt{2}}}{2}\right)j\right] = \left[\left(\frac{1}{2}+\frac{1}{40}\right)i + \left(\frac{1}{2}-\frac{1}{4}\right)i\right]v_0$$

$$v'_A = \left[\left(\left(\frac{v_0}{\sqrt{2}}\cdot\frac{\sqrt{2}}{2}\right)+\left(\frac{v_0}{20\sqrt{2}}\cdot\frac{\sqrt{2}}{2}\right)\right)i + \left(\left(\frac{v_0}{\sqrt{2}}\cdot\frac{\sqrt{2}}{2}\right)-\left(\frac{v_0}{20\sqrt{2}}\cdot\frac{\sqrt{2}}{2}\right)\right)j\right] = \left[\left(\frac{1}{2}+\frac{1}{40}\right)i + \left(\frac{1}{2}-\frac{1}{4}\right)j\right]v_0$$

$$v'_A = (0.525i + 0.475j)v_0 \quad \text{Ans}$$



Consider spheres B

from $v'_{tB} = 0$

Thus $\vec{v}'_B = (v'_{nB}\cos 45^\circ)i - (v'_{nB}\sin 45^\circ)j$

$$\vec{v}'_B = \left[\frac{19v_0}{20\cancel{\sqrt{2}}}\cdot\frac{\cancel{\sqrt{2}}}{2}\right]i - \left[\frac{19v_0}{20\cancel{\sqrt{2}}}\cdot\frac{\cancel{\sqrt{2}}}{2}\right]j = v_0\left[\frac{19}{40}i - \frac{19}{40}j\right]$$

$$\vec{v}'_B = 0.475i - 0.475j \quad \text{Ans}$$

4–7. Bodies B_1 and B_2 each consist of two small spheres of equal mass m connected by a rigid massless rod of length l. Initially, B_2 is motionless and B_1 is moving such that $\omega_1 = 0$ and the velocity of its mass center is $\mathbf{v}_1 = v_0\mathbf{i}$. Two of the smooth spheres hit, as shown in the figure, with their line of centers at 45° to each rod during impact. Assuming that all motion takes place in a plane, and the coefficient of restitution is $e = 0.5$, solve for the linear and angular velocities of B_1 and B_2 immediately after impact. Consider the spheres as particles.

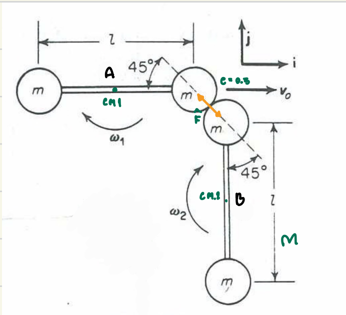
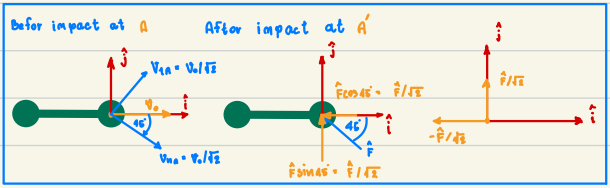


Figure P4-7



Consider the bodies A.

We have. $v_{nA} = v_{nB} = v_0/\sqrt{2}$, $v_{iA} = v_0$, $v_{jA} = 0$, $v'_{iA} = -\hat{F}_i/\sqrt{2}$, $v'_{jA} = \hat{F}_j/\sqrt{2}$

In $\hat{i}$ coordinate. we have $m = 2M$ (massless rod of length l)

We get $-\frac{\hat{F}_i}{\sqrt{2}} = 2M(v'_{iA} - v_0)$

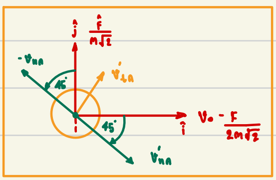
And then. find v'_{iA} using $\hat{F} = m(v'_A - v_A)$

$$v'_{iA} = \left(-\frac{\hat{F}_i}{\sqrt{2}}\cdot\frac{1}{2M}\right) + v_0 = v_0 - \frac{\hat{F}_i}{2M\sqrt{2}}$$

And then. in $\hat{j}$ coordinate. we have $m = M$

Using $\hat{F} = m(v'_A - v_A)$ we get. $\frac{\hat{F}_j}{\sqrt{2}} = M(v'_{jA} - 0)$ $\quad v'_{jA} = \frac{F_j}{M\sqrt{2}}$

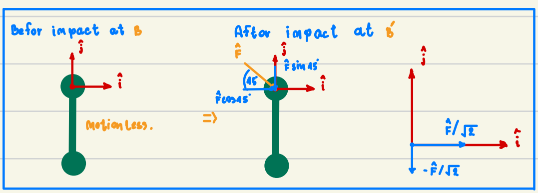
Thus we can obtain $v'_A = \left(v_0 - \frac{\hat{F}}{2M\sqrt{2}}\right)\hat{i} + \left(\frac{\hat{F}}{M\sqrt{2}}\right)\hat{j}$ ——— (1)



Then, set to normal frame. we get.

$$v'_{nA} = \left(v_0 - \frac{\hat{F}}{2M\sqrt{2}}\right)\cdot\cos 45^\circ - \left(\frac{\hat{F}}{M\sqrt{2}}\cdot\cos 45^\circ\right)$$

$$v'_{nA} = \frac{v_0}{\sqrt{2}} - \frac{\hat{F}}{4M} - \frac{\hat{F}}{2M} = \frac{v_0}{\sqrt{2}} - \frac{\hat{F}}{M}\cancelto{3/4}{\left(\frac{1}{4} + \frac{1}{2}\right)} = \frac{v_0}{\sqrt{2}} - \frac{3\hat{F}}{4M}$$



Consider the bodies B

We have. $v_{iB} = v_{jB} = 0$, $v'_{iB} = \hat{F}/\sqrt{2}$, $v'_{jB} = -\hat{F}/\sqrt{2}$

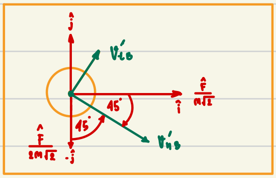
In $\hat{i}$ coordinate. we have $m = 2M$ (massless rod of length l)

Using $\hat{F} = m(v'_0 - v_0)$ we get $\frac{\hat{F}_i}{\sqrt{2}} = M(v'_{iB} - 0)$

$$v'_{iB} = \frac{\hat{F}_i}{M\sqrt{2}}$$

And then in $\hat{j}$ coordinate, we have $m = M$ (massless rod of length L), using $\hat F = m(v'_B - v_B)$ we get $-\frac{\hat F_j}{\sqrt2} = 2M(v'_{jB} - 0) \Rightarrow v'_{jB} = -\frac{\hat F_j}{2M\sqrt2}$

Thus we can obtain $v'_B = \left(\frac{\hat F}{M\sqrt2}\right) i - \left(\frac{\hat F}{2M\sqrt2}\right) j$ ——— (2)



Then, set to normal frame, we get.

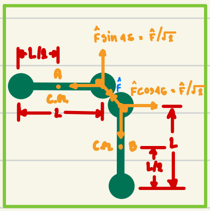
$$v'_{nB} = \left(\frac{\hat F}{M\sqrt2}\cdot\cos45^\circ\right) + \left(\frac{F}{2M\sqrt2}\cdot\cos45^\circ\right)$$

$$v'_{nB} = \left(\frac{\hat F}{M\sqrt2}\cdot\frac{1}{\sqrt2}\right) + \left(\frac{\hat F}{2M\sqrt2}\cdot\frac{1}{\sqrt2}\right) = \frac{\hat F}{2M} + \frac{\hat F}{4M} = \frac{\hat F}{M}\left(\cancel{\frac12 + \frac14}^{\,3/4}\right) = \frac{3\hat F}{4M}$$

Find $\hat F$ from equation is $v'_{nB} - v'_{nA} = e(v_{nA} + v_{nB})$

Thus we can obtain $\frac{3\hat F}{4M} - \frac{v_0}{\sqrt2} + \frac{3\hat F}{4M} = \frac12\left(\frac{v_0}{\sqrt2} + 0\right) \Rightarrow \frac{\hat F}{M}\left(\cancel{\frac34 + \frac34}^{\,3/2}\right) = \frac{v_0}{2\sqrt2} + \frac{v_0}{\sqrt2}$

$$\frac{3\hat F}{2M} = \frac{v_0(1+2)}{2\sqrt2} \Rightarrow \hat F = \frac{\cancel{3}v_0}{\cancel{2}\sqrt2}\cdot\frac{\cancel{2}M}{\cancel{3}} = \frac{v_0 M}{\sqrt2}$$



Using angular impulse and momentum of dumbbell about its center

$$\circlearrowleft M'_A = -\frac{\hat F}{\sqrt2}\cdot\frac L2 = \overbrace{2M\left(\frac L2\right)^2}^{I_A} w'_A \Rightarrow w'_A = -\frac{\hat F \cancel{L}}{\cancel{2}\sqrt2}\cdot\frac{1}{\cancel{2}M}\cdot\frac{\cancel{4}}{L^{\cancel{2}}} = -\frac{\hat F}{\sqrt2 ML} \text{ ——— (3)}$$

$$\circlearrowleft M'_B \quad \frac{\hat F}{\sqrt2}\cdot\frac L2 = \underbrace{2M\left(\frac L2\right)^2}_{I_B} w'_A \Rightarrow w'_A = \frac{\hat F}{\sqrt2 ML} \text{ ——— (4)}$$

From eq 1 Thus we can obtain

$$v'_A = \left(v_0 - \frac{v_0 M}{\sqrt2}\cdot\frac{1}{2\sqrt2 M}\right)\hat i + \left(\frac{v_0 M}{\sqrt2}\cdot\frac{1}{2\sqrt2 M}\right)\hat j \Rightarrow v'_A = \left(\frac34 v_0\right)\hat i + \left(\frac{v_0}{4}\right)\hat j = \frac{v_0}{4}(3\hat i + \hat j) \quad \text{Ans}$$

From eq 2 Thus we can obtain

$$v'_B = \frac{\hat F}{2\sqrt2 M}\hat i - \frac{\hat F}{2\sqrt2}\hat j \qquad v'_B = \left(\frac{v_0\cancel{M}}{\sqrt2}\cdot\frac{1}{2\sqrt2\cancel{M}}\right)\hat i - \left(\frac{v_0\cancel{M}}{\sqrt2}\cdot\frac{1}{2\sqrt2\cancel{M}}\right)\hat j \Rightarrow v'_B = \left(\frac{v_0}{4}\right)\hat i - \left(\frac{v_0}{4}\right)\hat j = \frac{v_0}{4}(\hat i - \hat j) \quad \text{Ans}$$

From eq 3

$$w'_A = -\frac{\hat F}{\sqrt2 ML} \quad \text{We have } \hat F = \frac{v_0 M}{\sqrt2} \text{ so we get } w'_A = -\left(\frac{v_0\cancel{M}}{\sqrt2}\cdot\frac{1}{\sqrt2\cancel{M}L}\right) = -\frac{v_0}{2L} \quad \text{Ans}$$

From eq 4

$$w'_B = \frac{\hat F}{\sqrt2\cdot ML} \quad \text{We have. } \hat F = \frac{v_0 M}{\sqrt2} \text{ so we get } w'_B = \frac{v_0\cancel{M}}{\sqrt2}\cdot\frac{1}{\sqrt2\cancel{M}L} = \frac{v_0}{2L} \quad \text{Ans}$$

4–13. Masses m_2 and m_3 are initially at rest and the spring of length l_0 is unstressed. Then mass m_1, traveling with velocity v_0 in a direction perpendicular to the spring, hits m_2 inelastically and sticks to it. In the ensuing motion, the spring stretches to a maximum length $3l_0$. Solve for v_0, assuming that the masses are equal and can be considered as particles.

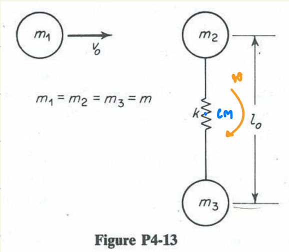


Figure P4-13

In the situation just after impact is as shown.

$$\omega_i = \frac{v_2 - v_3}{l_0} = \frac{v_0}{2l_0}$$

From conservation $m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$ M_2 stichs to the motion. so $e = 0$

$$\cancel{e}^{\,0} = \frac{v_2' - v_1'}{v_1 - v_2} \Rightarrow v_2' = v_1'$$

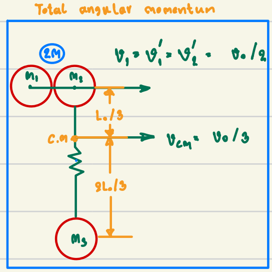
From figure P4-13 we get $\cancel{m_1}v_1 + 0 = \cancel{m_1}v_1' + \cancel{m_2}v_2'$; $m_1 = m_2$

$$\cancel{v_1}^{\,v_0} = v_1' + v_2' \Rightarrow 2v_2' = v_0 \Rightarrow v_2' = v_1' = \frac{v_0}{2}$$

Find linear velocity at c.m.

$$v_{cm} = \omega_i r_c = \frac{v_0}{\cancel{2}l_0} \cdot \frac{\cancel{2}l_0}{3} = \frac{v_0}{3}$$

Total angular momentum



The total angular momentum about the c.m. is

$$H_c = 2M\left(\frac{L_0}{3}\right)\left(\frac{v_0}{2} - \frac{v_0}{3}\right) + M\left(\frac{2L_0}{3}\right)\left(\frac{v_0}{3}\right) = \frac{M}{3}\left(L_0v_0 - \cancel{\frac{2L_0v_0}{3}} + \cancel{\frac{2L_0v_0}{3}}\right) = \frac{L_0v_0M}{3}$$

At the maximum stretch using conservation of H_c. $(3L_0)$

$$(2ML_0^2 + M(2L_0)^2)\omega = \frac{L_0v_0M}{3} \Rightarrow (2ML_0^2 + 4ML_0^2)\omega = \frac{L_0v_0M}{3}$$

$$6\cancel{M}L_0^{\cancel{2}}\omega = \frac{\cancel{L_0}v_0\cancel{M}}{3} \Rightarrow \omega = \frac{v_0}{18L_0}$$

And then using conservation of energy $(T_1 + V_1 = T_2 + V_2)$

We have $s = 3L_0 - L_0 = 2L_0$ and then we have $I = 2Mr^2 = (2M)L_0^2 + M(2L_0)^2 = 2ML_0^2 + 4ML^2 = 6ML_0^2$

$T_1 + \cancel{V_1}^{\,0} = T_2 + V_2$ we get $T_1 = \frac{1}{2}Mv^2 = \frac{1}{2}(2M)\left(\frac{v_0}{2}\right)^2 = \frac{Mv_0^2}{4}$ and $T_2 = T(\text{Linear}) + T(\text{angular})$ $(T_L + T_a)$

$T_L = \frac{1}{2}Mv_{cm}^2 = \frac{1}{2}(3M)\left(\frac{v_0}{3}\right)^2 = \frac{\cancel{3}^{\,1}Mv_0^2}{2\cdot\cancel{9}_{\,3}} = \frac{Mv_0^2}{6}$ and then $T_a = \frac{1}{2}Mv^2 = \frac{1}{2}I\omega^2 = \frac{\cancel{6}^{\,3}ML^2}{\cancel{2}_{\,1}}\left(\frac{v_0}{18L_0}\right)^2$

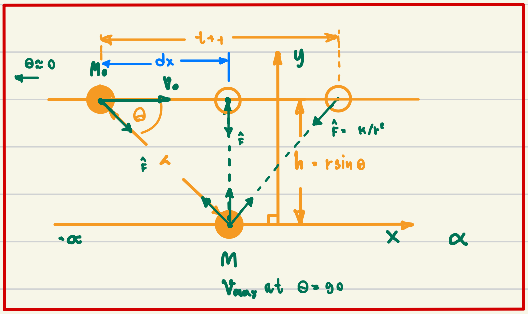
and $V_2 = \frac{1}{2}ks^2 = \frac{1}{2}k(2L_0)^2$

$$\frac{Mv_0^2}{4} = \frac{Mv_0^2}{6} + 3ML^2\left(\frac{v_0}{18L_0}\right)^2 + \frac{\cancel{4}^{\,2}kL_0^2}{\cancel{2}_{\,1}} \Rightarrow \frac{Mv_0^2}{6} - \frac{Mv_0^2}{4} + \frac{3Mv_0^2}{324} + 2kL_0^2 = 0 \Rightarrow Mv_0^2\left(\frac{1}{6} - \frac{1}{4} + \frac{3}{324}\right) + 2kL_0^2 = 0$$

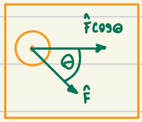
$$-\frac{24Mv_0^2}{324} = -2kL_0^2 \Rightarrow v_0^2 = +\cancel{2}^{\,1}kL_0^2 \cdot \frac{\cancel{324}^{\,27}}{\cancel{24}_{\,\cancel{12}\,1}M} = \frac{27kL_0^2}{M} \Rightarrow v_0 = \sqrt{\frac{27kL_0^2}{M}} = L_0\sqrt{\frac{(9\times3)k}{M}} \Rightarrow v_0 = 3L_0\sqrt{\frac{3k}{M}}$$

Ans

4–15. A particle of mass m can slide without friction along a fixed horizontal wire coinciding with the x axis. Another particle of mass m_0 moves with a constant speed v_0 along the line $y = h$ from $x = -\infty$ to $x = \infty$. If the particle m is initially at the origin and if an attractive force of magnitude K/r^2 exists between the two particles, where r is their separation, solve for the maximum speed of m.



We have $\hat F = \frac{K}{r^2} \Rightarrow \hat F = \frac{K}{\frac{h^2}{\sin^2\theta}} = \frac{K\sin^2\theta}{h^2}$, $v_A = v_0$, $v_B = v'$

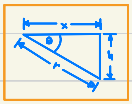


$$\hat F\cos\theta = \frac{K\sin^2\theta\cdot\cos\theta}{h^2}$$

Using work and kinetic energy

$$W = \int_A^B \frac{K}{h^2}\sin^2\theta\cdot\cos\theta\,dx = \frac{mv_B^2}{2} - \frac{mv_A^2}{2} \quad ①$$

Convers dx to dθ



$\cos\theta = \frac{x}{r} \Rightarrow x = r\cos\theta$ $(r = h/\sin\theta)$

$$x = \frac{h\cos\theta}{\sin\theta} = h\tan^{-1}\theta \Rightarrow dx = h\tan^{-1}\theta\,d\theta$$

$$dx = \frac{h\sec^2\theta}{\tan^2\theta}\,d\theta = h\left(\frac{\frac{1}{\cancel{\cos^2\theta}}}{\frac{\sin^2\theta}{\cancel{\cos^2\theta}}}\right)d\theta$$

$$dx = h\cdot\frac{1}{\sin^2\theta}\,d\theta$$

From equation 1. Thus we can obtain

$$W = \int_A^B \frac{K}{h^{\cancel{2}}}\cancel{\sin^2\theta}\cdot\cos\theta\cdot\frac{\cancel{h}}{\cancel{\sin^2\theta}}\,d\theta = \frac{mv'^2}{2} - \frac{mv_0^2}{2} = \int_0^{90}\frac{K}{h}\cdot\cos\theta\,d\theta = \frac{mv'^2}{2} - \frac{mv_0^2}{2} = -\frac{K}{h}\sin\theta\Big|_0^{90} = \frac{mv'^2}{2} - \frac{mv_0^2}{2}$$

$$-\frac{K}{h}\cancel{\sin 90}^{\,1} + \frac{K}{h}\cancel{\sin 0}^{\,0} = \frac{mv'^2}{2} - \frac{mv_0^2}{2} \Rightarrow -\frac{K}{h} = \frac{mv'^2}{2} - \frac{mv_0^2}{2} \quad ②$$

Find v' from equation 2 (v' is speed of the particle m)

$$\frac{mv'^2}{2} = -\frac{K}{h} + \frac{mv_0^2}{2} \Rightarrow v'^2 = -\left(\frac{K}{h} - \frac{mv_0^2}{2}\right)\cdot\frac{2}{m} = -\frac{2K}{hm} + v_0^2 \Rightarrow v' = \sqrt{\frac{v_0^{\cancel{2}}}{\cancel{v_0}} - \frac{2K}{hmv_0}} = \sqrt{v_0 - \frac{2K}{hmv_0}}$$

But rerative to the fixed xy frames, and moving by v_0

$$\dot x = v_0 + v' = v_0 + \sqrt{v_0 - \frac{2K}{hmv_0}\cdot\frac{v_0}{v_0}} = v_0\left(1 + \sqrt{1 - \frac{2K}{hmv_0^2}}\right)$$

Giving a maximum speed

$$v'_{max} = |\dot x| = v_0\left(1 + \sqrt{1 - \frac{2K}{hmv_0^2}}\right)$$

4–17. Particles $m_1 = 2m$ and $m_2 = m$ can slide without friction on parallel fixed horizontal wires separated by a distance h. A spring of stiffness k and unstressed length h connects the two particles. If m_1 has an initial velocity v_0, m_2 is initially motionless, and the spring is initially unstressed, find: (a) the maximum velocity v_2 of m_2; (b) the maximum stretch δ in the spring.

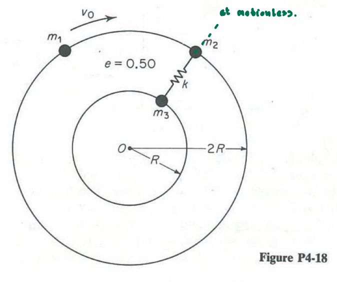


Figure P4-18

Using conservation of linear momentum.

$$m_1v_1 + \cancel{m_2v_2}^{\,0} = m_1v_1' + m_2v_2'$$

$2mv_0 = 2mv_1' + mv_2'$ if $v_1' = v_2'$ we get $3mv' = 2mv_0 \Rightarrow v_1' = v_2' = \frac{2}{3}v_0$

Find v_1' $\quad v_1' = \frac{2mv_0 - mv_2'}{2m} = \frac{\cancel{m}(2v_0 - v_2')}{2\cancel{m}}$

$$v_1' = \frac{2v_0 - v_2'}{2} = v_0 - \frac{v_2}{2}$$

And then using conservation of energy

$\frac{1}{\cancel{2}}(\cancel{2}m)v_0^2 = \frac{1}{\cancel{2}}(\cancel{2}m)v_1'^2 + \frac{1}{2}mv_2'^2 + \frac{1}{2}k\delta^2$ We get $mv_0^2 = mv_1'^2 + \frac{mv_2^2}{2} + \frac{k\delta^2}{2}$

Thus we can obtion $m\left(v_0 - \frac{v_2}{2}\right)^2 + \frac{mv_2^2}{2} + \frac{k\delta^2}{2} = mv_0^2 \Rightarrow m\left(v_0^2 - \frac{\cancel{2}v_0v_2}{\cancel{2}} + \frac{v_2^2}{4}\right) + \frac{mv_2^2}{2} + \frac{k\delta^2}{2} = mv_0^2$

$\cancel{mv_0^2} - mv_0v_2 + \frac{mv_2^2}{4} + \frac{mv_2^2}{2} + \frac{k\delta^2}{2} = \cancel{mv_0^2} \Rightarrow -mv_0v_2 + mv_2^2\left(\cancel{\frac{1}{4} + \frac{1}{2}}^{\,3/4}\right) + \frac{k\delta^2}{2} = 0$

$\frac{1}{m}\cdot\left(\frac{3}{4}mv_2^2 - mv_0v_2 + \frac{k\delta^2}{2} = 0\right) \Rightarrow \frac{3}{4}v_2^2 - v_0v_2 + \frac{k\delta^2}{2m} = 0$

From $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ we get $v_2 = \frac{v_0 \pm \sqrt{(-v_0)^2 - \cancel{4}\cdot(3/\cancel{4})\cdot k\delta^2/2m}}{\cancel{2}\cdot 3/\cancel{4}_2} = \frac{v_0 \pm \sqrt{v_0^2 - 3k\delta^2/2m}}{3/2}$

$v_{2\text{-}max}$ occurs for $\delta = 0$ $\quad v_{2\text{-}max} = \frac{v_0 + v_0}{3/2} = \frac{4}{3}v_0$ Ans

δ_{max} occurs when $\dot{\delta} = 0$ and $v_1 = v_2 = \frac{2}{3}v_0$, Then from the momentum equation.

$$\frac{3}{4}\cdot\left[\frac{2}{3}v_0\right]^2 - \frac{2}{3}v_0^2 + \frac{k\delta^2}{2m} = 0$$

$\frac{\cancel{3}}{\cancel{4}}\cdot\frac{\cancel{4}}{\cancel{9}_3}v_0^2 - \frac{2}{3}v_0^2 + \frac{k\delta^2}{2m} = 0 \Rightarrow v_0^2\left(\cancel{\frac{1}{3} - \frac{2}{3}}^{\,-1/3}\right) + \frac{k\delta^2}{2m} = 0 \Rightarrow \delta^2 = \frac{v_0^2}{3}\cdot\frac{2m}{k} = \frac{2mv_0^2}{3k}$

$\delta_{max} = v_0\sqrt{\frac{2m}{3k}}$ Ans