4–1. What is the minimum spring compression δ necessary to cause m_2 to leave the floor after m_1 is suddenly released with zero velocity? Measure δ from the unstressed position of the spring and assume that all motion is in the vertical direction.

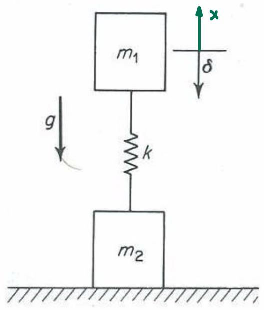


Figure P4-1

Consider in cause M_2 to leave the floor

kX_{max}

M_2

$M_2 g$

Mass (M_2) $\uparrow +$

$\Sigma F_y = 0$

$kX_{max} = M_2 g$; M_2 leaves the floor if $kx_{max} > M_2 g$

$X_{max} = \dfrac{M_2 g}{K}$ —— ①

And then consider M_1

M_1

$M_1 g, kX_{st}$

$\Sigma F_y = 0 \quad \uparrow +$

$M_1 g + kX = 0$

$X_{st} = -\dfrac{M_1 g}{K}$; st ⇒ static —— ②

Thus we get. $\quad X_{st} = \dfrac{1}{2}(X_{max} + X_{min})$

From: $\delta = X_{min}$ $\quad -\dfrac{1}{K} m_1 g = \dfrac{1}{2}\left(\dfrac{m_2 g}{K} - \delta\right)$

And then. $\quad -\delta = -\dfrac{2mg}{K} - \dfrac{M_2 g}{K}$

Thus we obtain $\quad \delta = \dfrac{2m_1 g + M_2 g}{K} = \dfrac{(2m_1 + m_2) g}{K}$ $\quad$ Ans.

4–2. A chain of length L and mass m rests on a horizontal table. If there is a coefficient of friction μ between the chain and the table top, find the velocity of the chain as it leaves the table. Assume that it is released from the position $x = a$, where $a > \mu L/(1+\mu)$, and the chain is guided without friction around the corner.

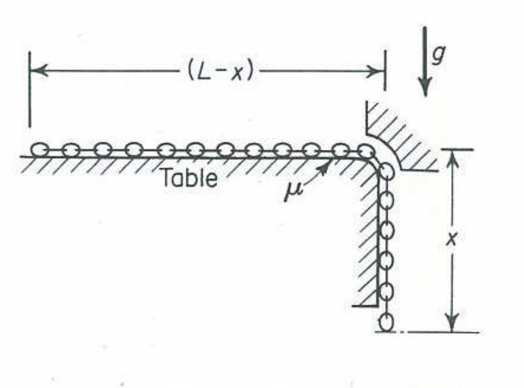


Figure P4-2

From the work and energy principle

$$T_i + V_i = T_f + V_f + W_f \quad \text{——— (1)}$$

The initial kinetic energy (T_i)

$u_0 = 0$ Thus : $T_i = 0$

Initial of potential energy (V_i)

From linear density $\rho = \frac{m}{L}$

Thus $V_i = \frac{1}{2}\rho g a^2 = -\frac{1}{2L} m g a^2$ at. $x(0) = a$

And then the final of kinetic energy (T_f)

$$T_f = \frac{1}{2} m v^2$$

The final potential energy. (V_f)

$$V_f = -\frac{1}{2} m g L$$

We have energy lost in friction

$$= \int_a^L \frac{\mu m g}{L} \cdot (L-x)\, dx = \frac{\mu m g}{2L} \cdot x^2 \Big|_a^L = \frac{\mu m g \cdot (L-a)^2}{2L}$$

From equation (1)

$$0 - \frac{m g a^2}{2L} = \frac{m v^2}{2} - \frac{m g L}{2} + \frac{\mu m g}{2L} \cdot (L-a)^2$$

$$-\frac{\cancel{m}}{\cancel{2}} \cdot \frac{g a^2}{L} = \frac{\cancel{m}}{\cancel{2}}\left[v^2 - gL + \frac{\mu g}{L} \cdot (L-a)^2\right]$$

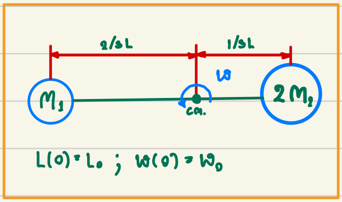
$$-g a^2 = L v^2 - g L^2 + \mu g \cdot (L-a)^2$$

$$L v^2 = g L^2 - g a^2 - \mu g (L-a)^2$$

$$v^2 = \frac{g}{L}\left[L^2 - a^2 - \mu (L-a)^2\right.$$

$$v = \sqrt{\frac{g}{L}\left[L^2 - a^2 - \mu (L-a)^2\right.} \quad \text{Ans}$$

4–4. Particles $m_1 = m$ and $m_2 = 2m$ are connected by a massless string and undergo free planar motion. Initially they rotate about each other with a constant separation l_0 and an angular velocity ω_0. Then a device in m_1 reels in the connecting string until its length is $\frac{1}{2}l_0$ and is again constant. At this later time, find: (a) the angular velocity ω; (b) the tension in the string.



Using conservation of angular momentum about the center of mass

Thus we get $\left[m\left(\frac{2}{3}L\right)^2 + 2m\left(\frac{1}{3}L\right)^2\right]\omega = \frac{2}{3} m L_0^2 \omega_0$; $L = \frac{L_0}{2}$

$$\left[m\left(\frac{2}{3}\cdot\frac{1}{2}\right)^2 L_0^2 + 2m\left(\frac{1}{3}\cdot\frac{1}{2}\right)^2 L_0^2\right]\omega = \frac{2}{3} m L_0^2 \omega_0$$

$$\left(\frac{1}{9} + \frac{2}{36}\right) m L_0^2 \omega = \frac{2}{3} m L_0^2 \omega_0$$

$$\left(\frac{\cancel{6}^{1}}{\cancel{36}_{6}}\right) m L_0^2 \omega = \frac{2}{3} m L_0^2 \omega_0$$

$$\omega = \frac{\cancel{6}^{2}}{\cancel{mL_0^2}} \cdot \frac{2\cancel{mL_0^2}\omega_0}{\cancel{3}_{1}} = 4\omega_0$$ Ans

b. The string tension is $F = m_1 \frac{L_0}{3} \omega^2$

when $\omega = 4\omega_0$

Thus we get $F = m_1 \frac{L_0}{3} (4\omega_0)^2$

$$F = \frac{16 m_1 L_0 \omega_0^2}{3}$$ Ans