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Author(s): Yair Mundlak

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EMPIRICAL PRODUCTION FUNCTION FREE OF MANAGEMENT BIAS[1]

YAIR MUNDLAK
The Hebrew University and the Falk Project for Economic Research in Israel

Introduction

IT HAS been felt for a long time that the estimates of the parameters of production functions are subject to bias as a result of excluding the variable which represents management. The reason for omitting management from cross-section analysis is obviously the lack of units for its direct measurement. An attempt to substitute some index of management does not solve the conceptual difficulty. It can be regarded as an ad hoc procedure as long as no criterion for evaluating its performance is available.

Instead of beginning by conceptualizing what we mean by management we shall assume that whatever management is, it does not change considerably over time; and for short periods, say a few years, it can be assumed to remain constant. For the purpose of the following analysis it is sufficient that management remains fixed for a two-year period since at least two observations on each firm are required.[2] In this framework we shall show how covariance analysis is used to obtain unbiased estimates of the coefficients of the linear form of the production function.[3] We shall evaluate the bias which would have been obtained in a regular regression estimate, that is, one subject to a particular specification error. We shall then show how management can be estimated up to a multiplier.

The Model

The procedure deals with the linear form of a production function. This includes the Cobb Douglas function where the variables are written in logarithms. Let it be:

$$(1) \qquad Y_{it} = B_0 + B_1X_{1it} + \cdots + B_kX_{kit} + CM_i + e_{it}$$

where Y is output, $X_1, \cdots, X_k$ are inputs, M is management, e is the disturbance, B_j and C are the true coefficients to be estimated;

$$i = 1, \cdots, I \qquad t = 1, \cdots, T$$

The usual procedure in cross section studies is to fit a regression to data collected for firms at a point of time, ignoring the term CM_i. This term

[1] I have benefited from comments by Y Grunfeld, S. Hoos and J. Putter. I am particularly indebted to N. Liviatan whose valuable suggestions are well reflected in the paper.

[2] It seems desirable to qualify the statement: (a) When there is a reason to believe that management changes from one year to the next, then the approach suggested above cannot be applied to annual data. (b) However, it can be applied for shorter periods, say months.

[3] The same technique was previously used by Hoch: Hoch, Irving, "Estimation of production function parameters and testing for efficiency," *Econometrica* 23:326 (1955).

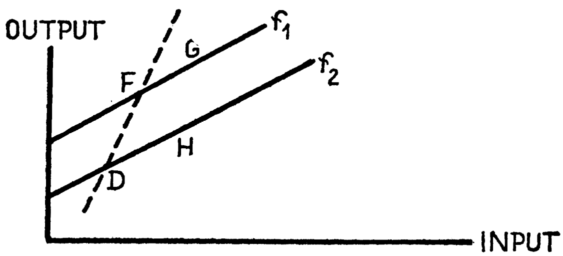

FIG. 1

varies from firm to firm and thus leads to a different production function for each firm. In the linear form of the functions, the difference is in the y-intercept of the lines. The slopes of the linear form are specified to be the same for different firms. In Figure 1 we have two functions, f_1, f_2, which represent output as a function of a single input, both having the same slope but different intercepts. If the level of management were the same, the two lines would coincide. Otherwise, they are not the same and the cross section fit is made over points such as D, F.[4] In the case of a Cobb Douglas function the lines represent the logarithmic form where the actual form can be drawn as in Figure 2. The form of the "interfirm" function DF is not the same as that of the "intrafirm" functions f_i. Furthermore, it is not uniquely determined by the intrafirm function unless the particular competitive form of the market is specified and conditions of equilibrium are imposed.[5] That means, in turn, that the knowledge of DF does not secure the knowledge of f_i.

There are two pertinent questions in cross section analysis:

(1) How to estimate the slope of f_i, the intrafirm function?

(2) Which function is more useful, the "intrafirm" or the "interfirm"?

In this paper we shall deal only with the first question and at this stage will only comment on the second one. The key to the estimation of the slope of the intrafirm function is to have at least two points on each f_i. In this case it is possible to get the slope of each of the lines f_i, average them and get the final estimate. That requires a combination of time series and cross-section data. The two observations can be collected in two successive years on the same sample of firms.

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[4] There is some similarity to problems involved in fitting a production function to time series data. Under conditions of technological change, observations for the various years belong to different functions. The solution is, however, different. This problem is dealt with by: Solow, Robert M., "Technical change and the aggregate production function," *Rev. Econ. and Stat.*, 39:312–21 (1957).

[5] Bronfenbrenner, M., "Production Function: Cobb-Douglas, Interfirm, Intrafirm," *Econometrica*, 12:37–38, (1944).

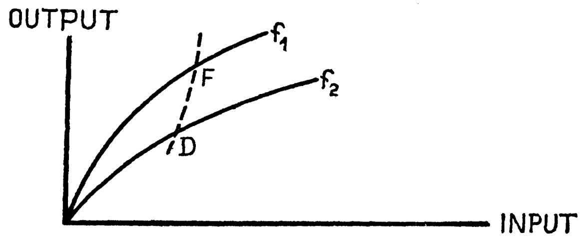

FIG. 2

The statistical treatment is in the framework of the analysis of covariance.[6] The sum of squares

$$\mathcal{S} = \sum_{it} (Y_{it} - B_0 - B_1 X_{1it} - \cdots - B_k X_{kit} - A_i)^2 \tag{2}$$

is minimized with respect to the various parameters.

Since there are no observations on the management variable M, it is impossible to estimate C directly. The minimization is therefore made with respect to the product:

$$A_i = CM_i \tag{3}$$

This does not cause any difficulty. Not only are there no observations on M_i, but also the units by which management is measured are arbitrary and it makes no difference if all the M_i are multiplied by the same constant.

With no loss in generality set $\sum_i A_i = 0$ and obtain the estimates:

$$b_0 = Y_{\cdot\cdot} - b_1 X_{1\cdot\cdot} - \cdots - b_k X_{k\cdot\cdot} \tag{4}$$

$$a_i = Y_{i\cdot} - b_1 X_{1i\cdot} - \cdots - b_k X_{ki\cdot} - b_0 \tag{5}$$

$$(b) = (S_{yx})(S_{xx})^{-1} \tag{6}$$

where

$$X_{1\cdot\cdot} = \frac{1}{IT} \sum_i \sum_t X_{1it}, \qquad X_{1i\cdot} = \frac{1}{T} \sum_t X_{1it}, \text{ etc.}$$

(b) is a row vector of the k regression coefficients, (S_{yx}) is a row vector of k sample moments with typical element:

$$s_{yx_l} = \frac{1}{IT}\left[\sum_i \sum_t Y_{it} X_{it} - T \sum_i Y_{i\cdot} X_{i\cdot} \right] \tag{7a}$$

$$l = 1, \cdots, k$$

$(S_{xx})^{-1}$ is the inverse of the matrix of the sample moments of the independent variables with typical element:

[6] A systematic discussion on covariance analysis can be found in Scheffé, H., *The Analysis of Variance* (Wiley, 1959), ch. 6.

(7b)
$$s_{x_l x_r} = \frac{1}{IT}\left[\sum_i \sum_t X_{lit}X_{rit} - T\sum_i X_{li\cdot}X_{ri\cdot}\right]$$
$$l, r = 1, \cdots, k$$

It is seen that the moments are computed from the average for each firm and represent the variations "within" firms.

If the assumptions of classical regression hold and if the function is completely specified, then the estimates obtained are unbiased and best. This is not the only accomplishment. We also get estimates, a_i of A_i, that is, estimates of the management variable. Such estimates may provide the dependent variable for studies aimed at exploring factors which determine the quality or performance of management.[7]

What is still missing is the estimate of C. This can only be obtained by introducing outside information. If the production function is complete and the factors are divisible we can impose the condition of constant returns to scale, which leads to:[8]

(9)
$$C = 1 - \sum_j^k B_j$$

and the unbiased estimate of C is

(10)
$$c = 1 - \sum_j^k b_j$$

Having obtained c, it is now possible to derive an estimate of the management variable with the original units. That is:

(11)
$$m_i = \frac{a_i}{c}$$

The advantage of (11) over (5) is in yielding a unique estimate of the management variable. Nevertheless it should be clear that for some purposes it is immaterial which of the two estimates is used.

If some other input, besides management, is fixed for all firms over the

[7] Such an approach has been attempted by Yaron in: Yaron, Dan, *Resource allocation for dairy and field crops in the Negev area of Israel*, unpub. Ph .D thesis, Iowa State Univ. Libr., Ames, Iowa, 1960.

[8] There is no unanimous agreement on the role of the divisibility approach to the explanation of economies of scale. The device suggested in the text is only pertinent for the supporters of the approach. For the others it can taken be as an ad hoc procedure Objection to the divisibility approach can be found in: Chamberlin, E. H. "Proportionality, Divisibility and Economies of Scale," *Quar. J Econ.* Feb. 1948, pp. 229–62; or *The Theory of Monopolistic Competition*, Cambridge: Harvard Univ. Press, 6th or 7th ed., Appendix B; also Leibenstein, H., "The proportionality controversy and the theory of production," *Quar. J. Econ.*, Nov. 1955, pp. 619–26.

two year period, its regression coefficient will be zero. The other coefficients remain unbiased but (10) is not directly applicable. In addition, a_i will now be an estimate of management and the "fixed" input combined.

In applying this analysis to agriculture, there is a danger that in what we refer to as management we also include a farm effect; that is, the effect of factors which do not depend on the management but rather on the particular environmental conditions of the farm, such as climate, type of soil, topography, etc. This would not affect the estimates b_j or their properties, but it will change the meaning of a_i. It will now be an estimate of the management and farm effect combined. If there is a possibility of grouping the farms into homogeneous groups, say villages, and if management is randomly distributed over these villages, an introduction of a parameter to represent the village effect may eliminate some of the farm effect.

The Management Bias in the Interfirm Regression

An important feature of the interfirm regression is its dependence on the particular position of the individual firms on their own production function. Referring to Fig. 1, it is noted that a regression connecting the points DF will be different from that connecting DG or FH. Even if we impose the condition that the firms are in equilibrium, the data on which the equilibrium position depends may vary from one year to the next and thus the interfirm regressions will vary over years. Such changes do not affect the intrafirm regression since, if the firms are found on their particular production function, we always obtain estimates of the same slope regardless of the position.

It has been shown that the regression coefficients of an equation not completely specified are subject to a bias.[9] In the case of a cross-section regression for year t we have:

$$E[\,\bar{b}_{jt} - B_j] = Cd_{jt} \tag{12}$$

where $\bar{b}_{jt}$ is the regression coefficient of the biased regression in year t and d_{jt} is the regression coefficient of the jth input in the auxiliary regression of M on all the k inputs included in the analysis. Averaging these coefficients $\bar{b}_{jt}$ over t and subtracting B_j will give the average bias. In order to estimate the bias it is possible to substitute for B_j its unbiased estimate obtained from (6). That will yield the auxiliary regression up to a multiplier C. If we are willing to take c of (10) as an estimate of C, we have the estimated auxiliary regression. It can be written as:

$$M_i = d_1X_{1i} + \cdots + d_kX_{ki} \tag{13}$$

[9] Theil, H., "Specification Errors and the Estimation of Economic Relationships." *Rev. Internat. Stat. Inst.*, 25:41–51 (1957); Griliches, Zvi, "Specification Bias in Estimates of Production Functions," *J. Farm Econ.*, 39:8–20 (1957).

This is an empirical regression which describes the particular relationship between management and the other inputs as they exist in the sample. As such it may be very useful since it suggests a way to compose a management index for firms which belong to the same population from which the sample was drawn. Equation (13) does not explain what determines the performance of the management. It only describes how management is reflected in the level of input utilization.

It is also possible to estimate (13) directly, once the variable a_i is available. This can be done by computing the regression of a_i on all the included variables. This actually amounts to the use of the numerical values ob-

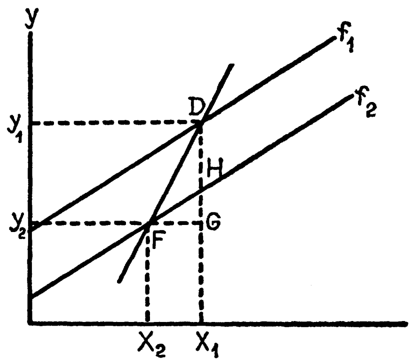

FIG. 3

tained from (5) for a_i instead of substituting the sample statistics as was done here. The results should therefore be the same. It can be shown, however, that statistically either one of these methods is not as efficient as estimating C directly if one could observe M_i; that is, if management could be treated just as another variable.

The relation between the interfirm function, the intrafirm function and the auxiliary equation can be demonstrated graphically for the simple case of $k=1$ (Fig. 3).

The interfirm function passes through points FD, with slope

$$\bar{b} = \frac{Y_1 - Y_2}{X_1 - X_2} \cdot$$

The slope of the intrafirm function is

$$B = \frac{H - G}{X_1 - X_2} \text{ and the difference } \bar{b} - B = \frac{D - H}{X_1 - X_2} = C\frac{(M_1 - M_2)}{X_1 - X_2} = Cd$$

is equal to the slope of the equation expressing CM_i as a linear function of X. It turns out that the assumption of constant slope of the interfirm function over time is actually an assumption about the relationship between the management variable and the level of input utilization. The slope of the interfirm function can be thought to consist of two components, B and Cd. The first is constant over time by assumption. Therefore, assuming that Cd is constant will make $\bar{b}$ constant.

A similar figure can be drawn for a case where $\bar{b}$ is smaller than B but still positive. In that case Cd will be negative but smaller than B. If Cd is negative and larger than B, then $\bar{b}$ becomes negative. That is, if the better managers employ less of the particular input than the worse managers, then the slope of the interfirm function will be smaller than that of the intrafirm and may also turn out to be negative.

Empirical Results

The method is demonstrated by giving results of an analysis of 66 family farms in Israel.[10] The data were collected for the years 1954–58. The variables included in the regression are:

Y = value of product
X_1 = number of labor days
X_2 = variable expense
X_3 = value of livestock at the beginning of the year
X_4 = value of livestock and poultry barns measured in I£. The value was derived by first measuring the capacity of the barns disregarding age differences. The capacity values were then multiplied by the market price.
X_i = amount of land on irrigated area basis (1 irrigated dunam = 4 dunams of dry land; 1 dunam = $\frac{1}{4}$ acre).

All flow variables are annual measures. All value variables are measured in I£ in 1954 prices. In order to give some notion as to the size and structure of farms included in the study, the first line of Table 1 gives the values for geometrical means of the various variables.

Before presenting the results we should consider an additional point which now becomes pertinent to the discussion. Underlying our model is the assumption that neither the production function nor management changes over time within the period under consideration. There is little that we can do about taking variations in management into account. However, with respect to the production function we can allow for a year effect which will shift the intercept of the function in its linear form. Such a procedure will catch changes in the level of productivity which occur in time. The empirical equation will be similar to (1) with additional term

[10] The results are taken from a forthcoming manuscript by the author: "Economic Analysis of Established Family Farms in Israel." The study has been made at the Falk Project for Economic Research in Israel.

A_t which represents the effect of the year. We shall refer to the revised function as the one which represents the model, or better, the unrestricted function. We now present four sets of estimates of this function, obtained under various assumptions: (2a) is obtained under the assumption that there is neither a year nor a firm effect, that is, under the assumption of $A_i = A_t = 0$ for all t and i. (2b) is obtained by allowing for a year effect, that is, under the assumption of $A_i = 0$ for all i. (2c) is obtained by allowing for a firm effect, that is, under the assumption of $A_t = 0$ for all t. (2d) is the unrestricted regression equation which allows for both a firm and a year effect. If we are willing to impose the a priori information that the function has not changed within the period without subjecting it to a statistical test, or if the result of such a test indicates no significant year effect, then the unbiased equation is (2c) and the management bias is evaluated by subtracting the estimates of (2c) from the values obtained in (2a). Otherwise, the unbiased equation is (2d) and the management bias is obtained by subtracting the value in (2d) from those in (2b). In our case the year effect was not significant at the 1 percent level, but passed the 2.5 percent mark. We view it as a borderline case and therefore report the two sets of results. The results appear in Table 1. In both sets of comparison the firm effect turns out to be highly significant. The implication is that the usual regression which is computed by not allowing for the firm effect is likely to be subject to a bias. The rejection of the hypothesis of no firm effect is a necessary condition for a management bias. The sufficient condition is a correlation between management and level of inputs. From (12) we know that the difference between the biased and unbiased coefficients yields estimates of the bias. Those values are estimates and as such are subject to sampling error. We shall not attempt here to test whether the bias of any particular coefficient is significantly different from zero. But we shall comment later on testing the hypothesis that the auxiliary regression is identically zero.

It is seen that both sets of estimates yield positive correlation between management and most of the inputs. In (3a), where no allowance is made for a year effect, negative relations are detected between livestock barns (X_4) and management. In (3b) the bias in X_4 disappears but negative relations are obtained between value of livestock at the beginning of the year (X_3) and management. The interpretation of a negative bias is that the better managers use less of the input with the negative coefficient, other things being equal. In the case of X_3, the regression coefficients were not significantly different from zero in both (2d) and (2b). Hence, it can be argued that the difference is also likely to be zero. In terms of Figure 3, the negative bias would be presented by drawing an interfirm line with a positive slope but smaller in magnitude than that of the intrafirm lines. The estimates of the relative bias appear in section 4 of Table 1. The values reach

TABLE 1

Item	X_1	X_2	X_3	X_4	X_5	Output
1. Geometric means	539	9900	1792	6621	255	18973
2. Estimated elasticities						Σb
a. Data pooled	.130	.692	.0043**	.103	.037	.967
b. Allowing for a year effect	.153	.679	.0042**	.101	.032*	.969
c. Allowing for a firm effect	.083*	.635	.0021**	.156	.002**	.878
d. Allowing for both year & firm effects	.115	.582	.005**	.100*	−.007**	.795
3. Absolute bias						
(a) Assumption of no year effect	.047	.057	.0022	−.053	.035	
(b) Allowing for year effect	.038	.097	−.0008	.001	.039	
4. Relative bias						
(a) Same as 3a	.57	.09	1.00	(−).34	17.5	
(b) Same as 3b	.33	.17	(−).16	.01	5.57	
5. r_{ax}						
(a) Same as 3a	.482	.293	.152	.049	.453	
(b) Same as 3b	.602	.617	.135	.402	.492	
6. Equilibrium estimates	.135	.574	.010	.096	.013	.828

Coefficients which are not marked are significant at the 1 percent level, those marked with * are significant at the 5 percent level and those marked with ** are not significant at the 5 percent level.

(3) a: obtained by subtracting values in 2c from those in 2a.
b: obtained by subtracting values in 2d from those in 2b.
(4) a: obtained by dividing values in 3a by values in 2c.
b: obtained by dividing values in 3b by values in 2d.
(5) Correlation coefficient of a_i and input.
(6) See discussion in text for description.

considerable magnitude for some variables. It is particularly large for land where the unbiased coefficient is nearly zero.

Additional illustration can be performed with the results reported by Hoch. His analysis is "on 63 Minnesota farms over each of the 6 years from 1946 through 1951. Inputs were: X_1, labor; X_2, real estate; X_3, machinery; X_4, feed and fertilizer. Outputs and inputs were in service units, all variables being measured in dollars."[11] His regression coefficients of the Cobb-Douglas function and our computation of the bias are shown in Table 2.

There is a marked difference between the two regressions. In commenting on it Hoch said that a "possible explanation is that entrepreneurial capacity has been winnowed out by . . . (the unbiased regression in our terminology). It may also be that efficiency increases with scale."[12] The first sentence is a conjecture of the bias caused by the correlation between management and input. The second sentence can be interpreted as suggesting auxiliary regression with positive coefficients. Using (10) we can now get an estimate

[11] Hoch, Irving, *op. cit.*
[12] *Ibid.*

for C, the elasticity of management. The sums of the elasticities are .878 and .795 for the estimates without and with a year effect respectively. Consequently, the estimates of C are .122 and .205 respectively. These values are relatively high and rank second in magnitude after the elasticity of current expense. The corresponding value for the Hoch study is .403 which is much higher than our results. It is interesting to note that the sum of the elasticities in the biased regressions are close to one, indicating

TABLE 2

Item	X_1	X_2	X_3	X_4	$\Sigma_j b_j$
1. Biased regression	.2191	.2173	.2690	.3006	1.0060
2. Unbiased regression	.0482	.0550	.2006	.2927	.5965
3. Absolute bias	.1709	.1623	.0684	.0079	
4. Relative bias	.78	.75	.25	.26	

Lines 1 and 2 are taken from Hoch. The estimates in the first line were obtained by covariance analysis, allowing for a year effect. The estimates in the second line were obtained by allowing for both a year effect and a farm effect. The values in the remaining two lines were obtained in the same way as in Table 1.

that the discrepancy in estimating the degree of the function is small, granted that the function is homogeneous of degree one. However, what is important here is the fact that the sum of the elasticities in the biased equation is larger than in the unbiased one. This should be anticipated in most cases unless some resources are used in such excessive amounts that they yield a negative relation with management that overcomes the positive relations with the other factors. That is to say that good management would have decreased the particular resource. When it is felt that this is not the situation, an inclusion of some management index worked out according to common sense should decrease the sum of the elasticities of the k inputs. This is not a perfect indicator; it only gives a criterion for evaluating the direction of expected change but not its magnitude.

Utilizing the estimate of C it is possible to derive the auxiliary regression with the original units:

$$\text{(a)} \qquad \hat{M} = H_a X_1^{.385} X_2^{.467} X_3^{.018} X_4^{-.434} X_5^{.287} \qquad R_m^2 = .263$$

$$\text{(b)} \qquad \hat{M} = H_b X_1^{.185} X_2^{.473} X_3^{-.004} X_4^{.005} X_5^{.190} \qquad R_m^2 = .416$$

$$\text{(c)} \qquad \hat{M} = H_c Z_1^{.421} Z_2^{.400} Z_3^{.168} Z_4^{.195}$$

where $\hat{M}$ is the value for M derived from the auxiliary regression. The Z values of regression (c) are the inputs in the Minnesota study. The H's are the constants of the equations. The coefficient of determination of the auxiliary regression can be computed directly without having to first compute the a's.[13]

[13] Utilizing the general expression for deriving the coefficient of determination, we can

The first two auxiliary regressions explain, respectively, 26 percent and 42 percent of the variations in management. In a regular regression such values lead to the rejection of the hypothesis of $R^2=0$, for a sample of the size used here. Such a rejection is the sufficient condition for having a bias in at least one coefficient of the regression which is subject to specification error. The necessary condition, as was pointed out earlier, is the rejection of the hypothesis of no firm effect.

The gross correlation coefficients between management and the various inputs appear in section 5 of Table 1 and are reported as r_{ax}. They were computed by following a similar procedure to that used in the computation of R_m^2. The values give some indication of the nature of the relationships that exist in the sample.

The Equilibrium Approach

When management is treated symmetrically with the other inputs, the marginal productivity of management is $MP_M=CY/M$. An estimate of it can be obtained by substituting c for C. At the point of the geometric mean of all the inputs the expression becomes $MP_M=c\overline{Y}$ where a bar stands in

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write in our notation:

$$R_m^2 = \frac{\sum_j (cd_j) s_{ax_j}{}^i}{s_a{}^2}$$

where cd_j is the coefficient of x_j, uncorrected for c, in the auxiliary regression,

$$s_{ax_j}{}^i$$

is the cross product between a and x_j, that is:

$$s_{ax_j}{}^i = \sum_i a_i X_{ji}$$

and $s_a{}^2$ is the sample sum of squares of a, that is:

$$s_a{}^2 = \sum_i a_i{}^2.$$

Utilizing (5) we get:

$$s_a{}^2 = [1, \ -b]\left[\begin{array}{c|c} S_{yy}{}^i & S_{yx_j}{}^i \\ \hline S_{yx_j}{}^i & S_{xx}{}^i \end{array}\right]\left[\begin{array}{c} 1 \\ -b \end{array}\right]$$

where b is the $1\times k$ vector of the unbiased regression coefficients and the S^i matrix is symmetrical $(k+1)\times(k+1)$ with typical element

$$s_{uv}{}^i = \sum_i u_{i.}v_{i.} - Iu_{..}v_{..}$$

and $S_{ax}{}^i$ is obtained by

$$S_{ax}{}^i = [1, \ -b]\left[\begin{array}{c} S_{yx_j}{}^i \\ S_{xx}{}^i \end{array}\right]$$

where $S_{ax}{}^i$ is a row vector of the

$$s_{ax_j}{}^i\text{'s}.$$

Thus, the computation of R_m^2 requires very few additional calculations.

this section for the geometric mean of the variable evaluated from the sample. It should be recalled that we previously assumed $\overline{M}=1$ and therefore M does not appear in the expression. The interpretation of this expression is that the additional unit of management at the point of the geometric mean will increase output by $C\overline{Y}$. This is a high value which partially reflects the fact that a unit of M was arbitrarily selected to be large.

Let us now assume perfect competition in which case the condition of equilibrium with respect to management is $CPY=W_M M$ where P is the price of the product and W_M is the wage of management. Under conditions of constant returns to scale we get: $(1-\sum_j B_j)PY=W_M M$. If the firms are also at their equilibrium point we get: $W_M M=PY-\sum_j W_j X_j$. Hence, the value of the marginal product is equal to that part of the total value product which was not imputed to other factors. That suggests that one can get an estimate of that value under the assumptions made above by simply substracting the distributive shares of all the other factors. This is very similar to the procedure suggested by Klein for estimating the production elasticities, except that we cannot do it directly and therefore must employ the assumption of constant returns to scale.[14] Klein's estimate for the elasticity of factor j is

$$\hat{B}_j = \frac{\overline{X}_j\overline{W}_j}{\overline{Y}\overline{P}}$$

which is the proportion of the total product used as payment to factor j, evaluated at the point of geometric means. Our suggestion is to use (10) as before in order to estimate C.

This approach requires two assumptions: (a) Constant returns to scale; (b) That the firms are found at their equilibrium, apart from random variations. The first assumption is only necessary for estimating C and was also used in the first approach introduced in this paper. The second assumption is necessary for estimating the other elasticities. It restricts the analysis and at first sight gives it a sense of being unrealistic. Against this limitation the equilibrium approach is attractive for the following reasons: (a) It is now immaterial whether management includes a farm effect or not. The results are not affected: (b) We do not need more than one observation per firm. Consequently, there is no need to assume that management remains constant over time. These two properties stem from the fact that in equilibrium the firm equates the relative share of the factor with the elasticity. As long as the elasticity remains constant the relative share is invariant to changes in the level of the function which are due to either time or firm effect. Such changes have their influence on the level of inputs and output. We are only concerned however with the relative share.

It seems that the strong properties of the equilibrium approach justify

[14] Klein, Lawrence R., *A Textbook of Econometrics* (Row, Peterson & Co., 1953), pp. 193–94.

some empirical analyses which will make it possible to compare results with those obtained by the use of the first approach. The equilibrium estimates for our study are reported in line (6) of Table 1. It is seen that the results are very close to the values obtained for the unrestricted equation, and thus are somewhat different from the results of the other equations. The sum of the elasticities is .828 which, under the assumption of constant returns to scale, yields an estimate of .172 for the elasticity of management. Hoch also reported estimates obtained by Klein's method. He obtained for the two respective quantities values of .751 and .249. The first estimate for the management elasticity was .406. The second one is not as high and may perhaps be nearer to the true one. But even if the first value is the "true" one, the equilibrium approach is a step in the right direction. The fact is that the two approaches require their own assumptions and it is difficult to judge a priori which set of assumptions is more realistic.

Concluding Remarks

It has been demonstrated how to obtain regression coefficients of production functions that are free of management bias. This only solves one aspect of the problem of handling the management variable in the analysis of factor productivity. It does not solve the problem of estimating the conditional expectation of output for a given bundle of resources. The value to be estimated depends on the value of management and thus varies from firm to firm. The question is for what level of management such an estimate is to be made.

In most cases it is likely that such a question cannot be answered. As long as management is not measurable it is impossible to determine beforehand the level of management for which such an estimate is desirable. This problem also exists in computing the marginal productivities in the case of the Cobb Douglas function where the values are obtained by multiplying the elasticities by the average productivities. The average productivity depends on the level of management. One possibility is to ignore the management variable, in which case the inference will be directed to the average firm since it was assumed that the management variable in the sample is measured from its average.

A better possibility is to utilize the fact that management is correlated with other inputs. Such correlations are reflected in the interfirm regression. It is then preferable to use the interfirm function, because of the fact that it is subject to management bias, for the purpose of inferring about the productivity of a firm whose level of management is not known. This has been shown more formally and will not be reproduced here. It is only mentioned here to call attention to the fact that an unbiased equation is not necessarily better than a biased one for all purposes.