# INTERPRETING PROGRAM IMPACT

Fundamentals of

**PROGRAM EVALUATION** 

**JESSE LECY** 

# WHICH MODEL IS THE "RIGHT" ONE?

	Dependent Variable: Test Scores				
	Model 1 (1)	Model 2 (2)	Model 3 (3)	Model 4 (4)	Model 5 (5)
tqual		57.771*** (0.263)			57.755*** (0.259)
csize	$-4.224^{***}$ $(0.174)$	-4.129*** $(0.025)$		-2.327 (1.634)	$-2.781^{***} (0.229)$
ses			44.079*** (1.818)	19.920 (17.057)	14.160*** (2.389)
Constant	702.385*** (4.928)	409.739*** (1.507)	504.439*** (3.986)	613.247*** (76.487)	346.459*** (10.781)
Observations Adjusted R <sup>2</sup>	1,000 0.370	1,000 0.987	1,000 0.370	1,000 0.371	1,000 0.988

*Note:* 

\*p<0.1; \*\*p<0.05; \*\*\*p<0.01

# INTERPRETING PROGRAM IMPACT

#### BET#1

The bet costs \$1,000 to place There is a 75% chance you win \$1,500 There is a 25% chance you win \$1,100

### **BET #2**

The bet costs \$1,000 to place There is a 75% chance you win \$4,000 There is a 25% chance you lose \$2,000

#### BET#1

The bet costs \$1,000 to place There is a 75% chance you win \$1,500 There is a 25% chance you win \$1,100

Expected value = (0.75)(1500) + (0.25)(1100) = **\$1,400** 

#### **BET #2**

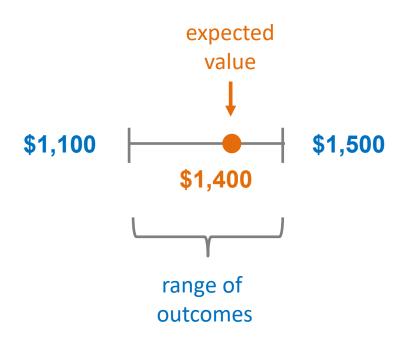
The bet costs \$1,000 to place There is a 75% chance you win \$4,000 There is a 25% chance you **lose \$2,000** 

Expected value = (0.75)(4000) - (0.25)(2000) = **\$2,500** 

#### BET#1

The bet costs \$1,000 to place There is a 75% chance you win \$1,500 There is a 25% chance you win \$1,100

Expected value = 
$$(0.75)(\$1,500) + (0.25)(\$1,100) = \$1,400$$



#### BET#1



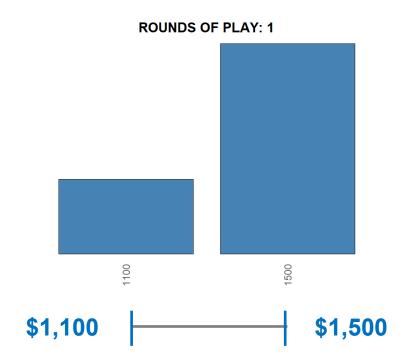
#### A BET AS METAPHOR FOR REGRESSION RESULTS:

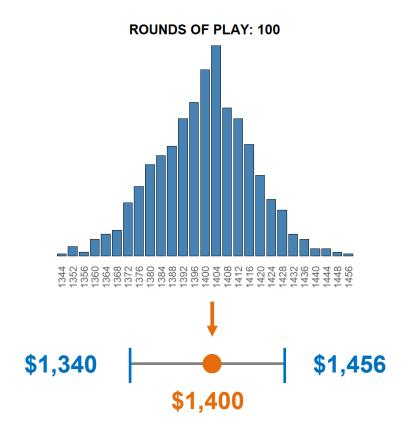
In regression terms, the coefficient in the model is like the expected value. It is our best guess of the most likely outcome given all information that we have available.

The standard error allows us to build a confidence interval around that guess, which provides a range of plausible outcomes.

These values allow us to do scenario planning by thinking about the coefficient as the most likely outcome while also considering the full range of possible outcomes.

Note that the range of outcomes corresponds with our selected model tolerance, a 95% confidence range, 99% confidence range, etc.





### A NOTE ON THE EXPECTED VALUE METAPHOR OF A BET:

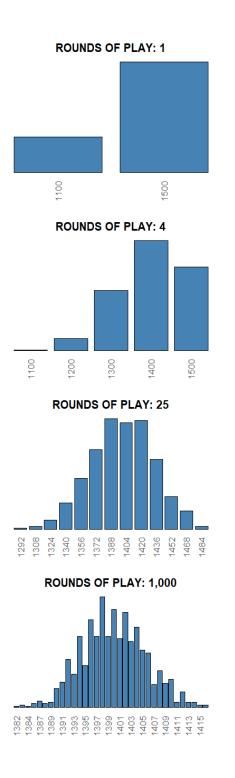
If you a making a single bet with only two possible outcomes, then the distribution is bimodal and it's impossible to achieve the exact expected value. The range out outcomes give you best case and worst case. The expected value is theoretical.

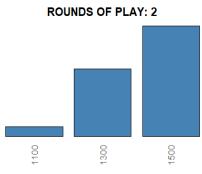
The idea of an expected value is easier to understand in a game that can be played many times. The outcome is then the average payoff across all plays.

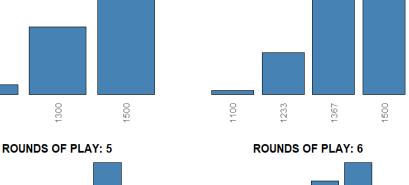
As you increase the number of plays you move toward the central limit theorem with the expected value becoming the most likely outcome.

As you increase your rounds of play the range of *likely* outcomes also narrows. Theoretically you can still win \$1,100, but you would have to roll that outcome 100 times in a row, which has the following probability:

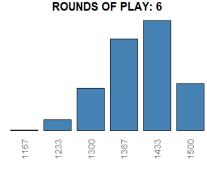
These outcomes generated by simulations approximate a confidence intervals with the number of rounds of play acting like the sample size in a study. As you increase the sample size (number of rounds), the interval of likely outcomes (average across all games) narrows.



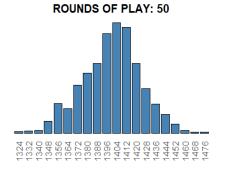


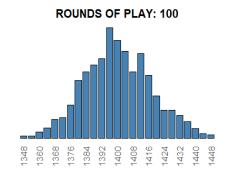


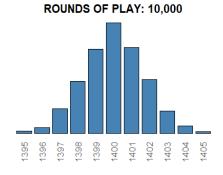


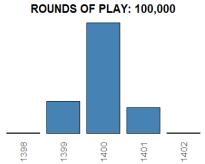


**ROUNDS OF PLAY: 3** 









```
rep_samp <- function( N=3 ){</pre>
  res <- NULL
  for( i in 1:10000 ){
    res[i] <-
      sample( c(1500,1500,1100,1500), size=N, replace=T ) |>
     mean() |> round(0)
  return( res )
}
par(mfrow=c(4,3))
table( rep_samp( 1 ) ) |> prop.table() |>
barplot(col="steelblue",yaxt="n",main="ROUNDS OF PLAY:
1",cex.main=2,cex.names=1.5,las=2,col.axis="gray40")
table( rep_samp( 2 ) ) |> prop.table() |>
barplot(col="steelblue",yaxt="n",main="ROUNDS OF PLAY:
2",cex.main=2,cex.names=1.5,las=2,,col.axis="gray40")
table( rep_samp( 3 ) ) |> prop.table() |>
barplot(col="steelblue",yaxt="n",main="ROUNDS OF PLAY:
3",cex.main=2,cex.names=1.5,las=2,,col.axis="gray40")
table( rep_samp( 4 ) ) |> prop.table() |>
barplot(col="steelblue",yaxt="n",main="ROUNDS OF PLAY:
4",cex.main=2,cex.names=1.5,las=2,,col.axis="gray40")
table( rep_samp( 5 ) ) |> prop.table() |>
barplot(col="steelblue",yaxt="n",main="ROUNDS OF PLAY:
5",cex.main=2,cex.names=1.5,las=2,,col.axis="gray40")
table( rep_samp( 6 ) ) |> prop.table() |>
barplot(col="steelblue",yaxt="n",main="ROUNDS OF PLAY:
6",cex.main=2,cex.names=1.5,las=2,,col.axis="gray40")
table( rep_samp( 25 ) ) |> prop.table() |>
barplot(col="steelblue",yaxt="n",main="ROUNDS OF PLAY:
25",cex.main=2,cex.names=1.5,las=2,,col.axis="gray40")
table( rep_samp( 50 ) ) |> prop.table() |>
barplot(col="steelblue",yaxt="n",main="ROUNDS OF PLAY:
50",cex.main=2,cex.names=1.5,las=2,,col.axis="gray40")
table( rep_samp( 100) ) |> prop.table() |>
barplot(col="steelblue",yaxt="n",main="ROUNDS OF PLAY:
100",cex.main=2,cex.names=1.5,las=2,,col.axis="gray40")
table( rep_samp( 1000) ) |> prop.table() |>
barplot(col="steelblue",yaxt="n",main="ROUNDS OF PLAY:
1,000",cex.main=2,cex.names=1.5,las=2,,col.axis="gray40")
table( rep_samp( 10000) ) |> prop.table() |>
barplot(col="steelblue",yaxt="n",main="ROUNDS OF PLAY:
10,000",cex.main=2,cex.names=1.5,las=2,,col.axis="gray40")
table( rep_samp( 100000) ) |> prop.table() |>
barplot(col="steelblue",yaxt="n",main="ROUNDS OF PLAY:
100,000",cex.main=2,cex.names=1.5,las=2,,col.axis="gray40")
```

#### **BET #2**

The bet costs \$1,000 to place There is a 75% chance you win \$4,000 There is a 25% chance you **lose \$2,000** 

Expected value = (0.75)(4000) - (0.25)(2000) =

\$2,500

**—** \$2,000

† expected

value

\$4,000

1 in 4 chance of losing money

3 in 4 chances of winning money

### BET#1

100% chance of positive return

#### **BET #2**

75% chance of a positive return

Expected value = (0.75)(4000) - (0.25)(2000) = **\$2,500** 

### LOW RISK, LOW RETURN

100% chance of positive return

Expected value = (0.75)(1500) + (0.25)(1100) = **\$1,400** 

### HIGH RISK, HIGH RETURN

75% chance of a positive return

Expected value = (0.75)(4000) - (0.25)(2000) = **\$2,500** 

RISK AND RETURN IN DECISION-MAKING:

There is no "right" answer about which bet is best. It really depends on context.

Theory would suggest that a "rational" actor would focus on the expected value. However, in the real world we also must consider consequences.

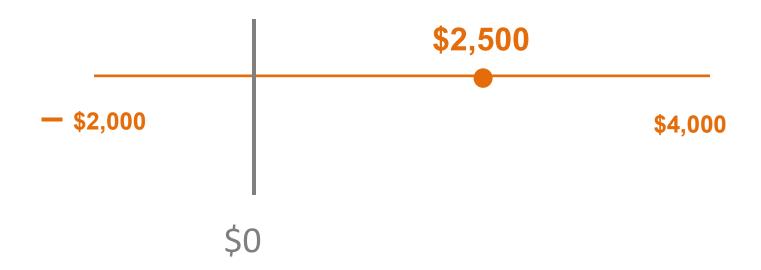
If you are betting your birthday money, then go big! The more extreme the outcomes the more adrenaline you will get from the bet, so it is more exciting either way. And since it was not money you had in your budget you will not experience the loss in the same way. You have the same likelihood of winning as BET #1, and the payoff is \$2,500 better.

If you are betting your rent, then the implications of losing the \$1,000 are tangible and significant. The low risk, low return option probably maximizes your utility, even if it doesn't maximize the expected value or total returns.

#### STATISTICAL SIGNIFICANCE:

People often use statistical significance as a decision criteria, which is an odd convention that has evolved in statistics because it only tells us one thing: do outcomes only include positive results?

If the range of likely outcomes spans zero, then it would NOT be statistically significant. Even if the expected value is positive.



#### **BET** #3

The bet costs \$1,000 to place There is a 75% chance you win \$3,300 There is a 25% chance you win \$100

Expected value: (0.75)(3300) + (0.25)(100) = \$2,500



#### **BET** #4

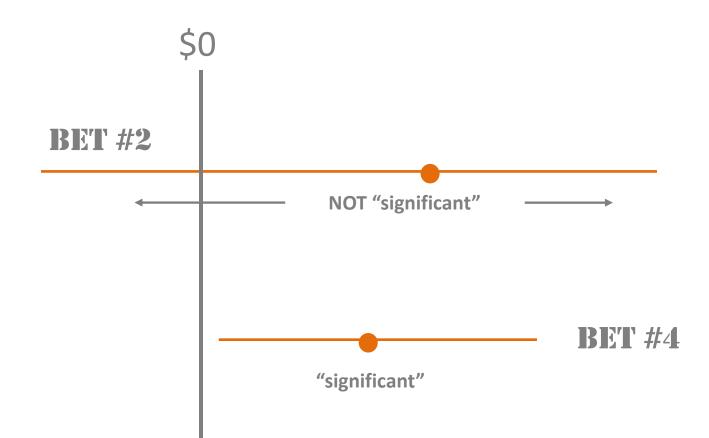
The bet costs \$1,000 to place There is a 50% chance you win \$2,000 There is a 50% chance you win \$100

Expected value: (0.50)(2000) + (0.50)(100) = \$1,050



### COMPLICATED MEANING OF SIGNIFICANCE:

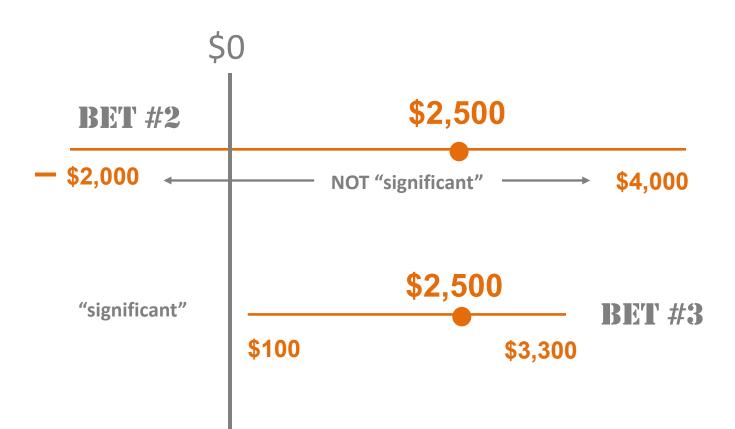
Statistical significance tells us if we can be certain about the **DIRECTION** of our program effects (the confidence interval does NOT contain zero).



#### **COMPLICATED MEANING OF SIGNIFICANCE:**

In some cases, we can gather more information (a larger sample size, or in this metaphor play a game repeatedly) and narrow the range of plausible outcomes (make the confidence interval smaller).

When comparing bets with similar expected values a narrower range of outcomes might help hedge the risk of "making the bet" (or investing in a program).



#### **COMPLICATED MEANING OF SIGNIFICANCE:**

It is only one piece of information, though. And a somewhat arbitrary criteria in the world of expected values.

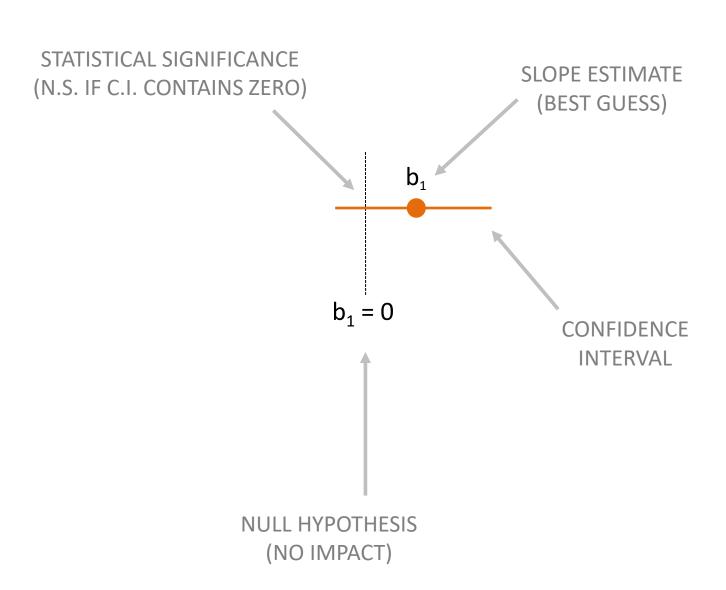
Which bet has the higher payoff (both in max payout and in expected value terms)?

Which is "significant"?

Statistical significance should NOT be the first or only piece of information to consider.

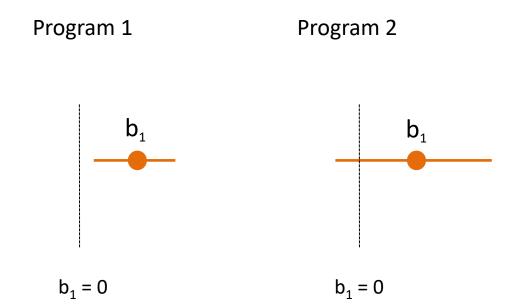


### REGRESSION COEFFICIENT PLOTS



### WHICH PROGRAM IS BETTER?

Reading Speed =  $b_0 + b_1$  Hours of Tutoring + e



(assume these are all 95% confidence intervals)

The cost of the program is the bet we are making.

The expected value of the program is represented by the point estimate of the slope (b1).

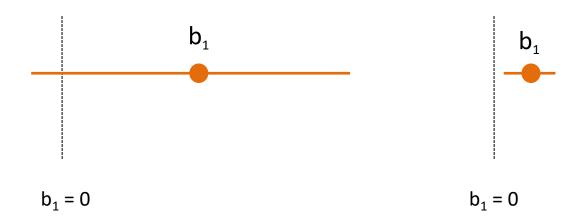
The risk (certainty) of the bet is symbolized by the confidence interval.

Preferences for bets is always a balance between expected pay-off and risk (uncertainty).

Reading Speed =  $b_0 + b_1$ Hours of Tutoring + e

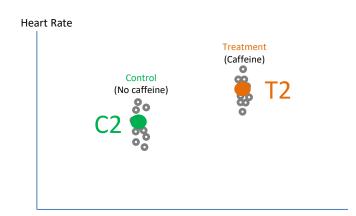
Program 1

Program 2

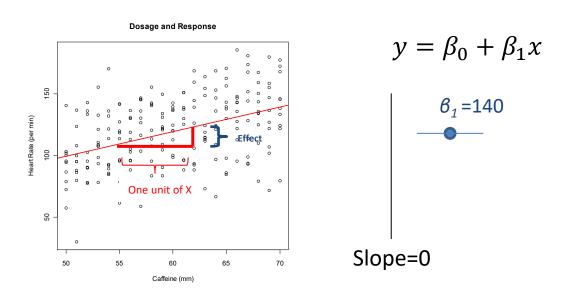


# EXPERIMENT WITH GROUPS (TREATED: YES OR NO)

 $Heartrate = b_0 + b_1 \cdot Caffeine + \varepsilon$ 



### EFFECTS SIZES FOR LEVELS OF A TREATMENT



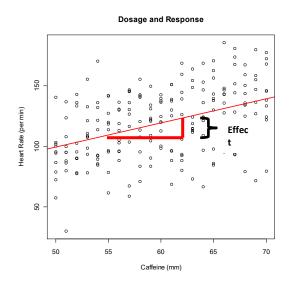
For a one-unit change in X, we expect a  $\theta_1$  change in Y.

How big is the effect?

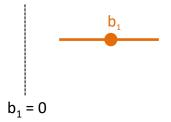
Is it significant?

### **EFFECT SIZE**

 $Heartrate = b_0 + b_1 \cdot Caffeine + \varepsilon$ 

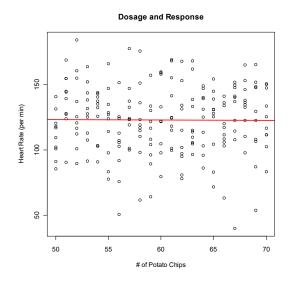


### Positive & Significant Impact on Outcome



### EFFECT SIZE

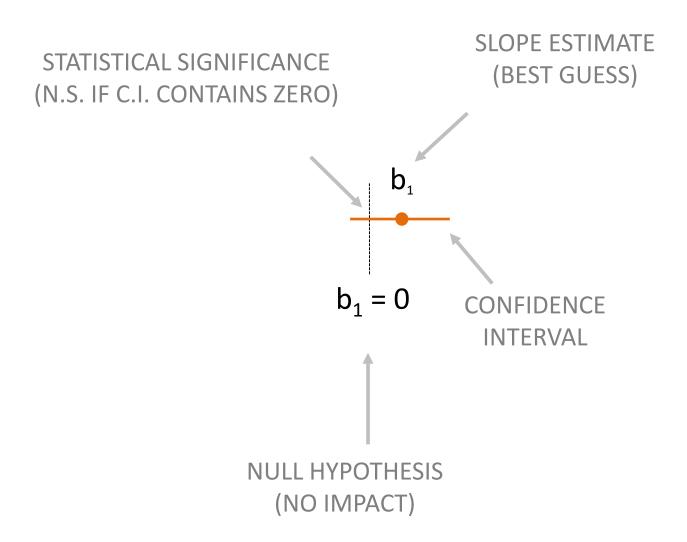
 $Heartrate = b_0 + b_1 \cdot Potato\ Chips + \varepsilon$ 





Not statistically significant – i.e. we can't tell whether the program has a positive or negative impact since the confidence interval is on both sides of zero.

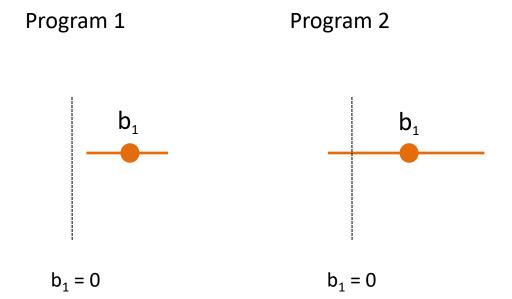
### **HYPOTHESIS TESTING**



### WHICH PROGRAM IS BETTER?

Consider two programs that are meant to improve reading comprehension. The dependent variable is a score on a reading comprehension exam (higher being better). Which program do you prefer and why?

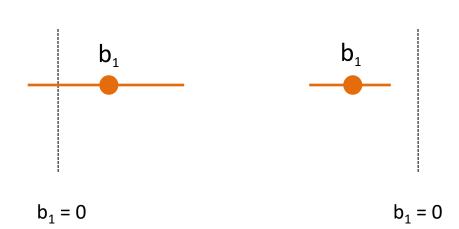
Reading Speed =  $b_0 + b_1$ Hours of Tutoring + e



Reading Speed =  $b_0 + b_1$ Hours of Tutoring + e

Program 1

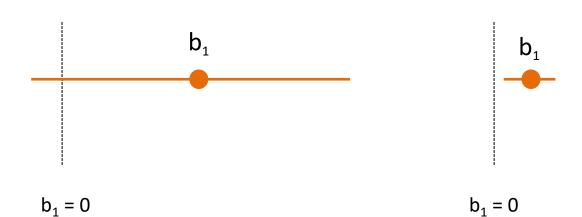
Program 2



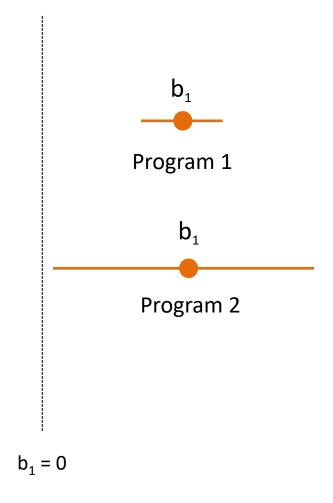
Reading Speed =  $b_0 + b_1$ Hours of Tutoring + e

Program 1

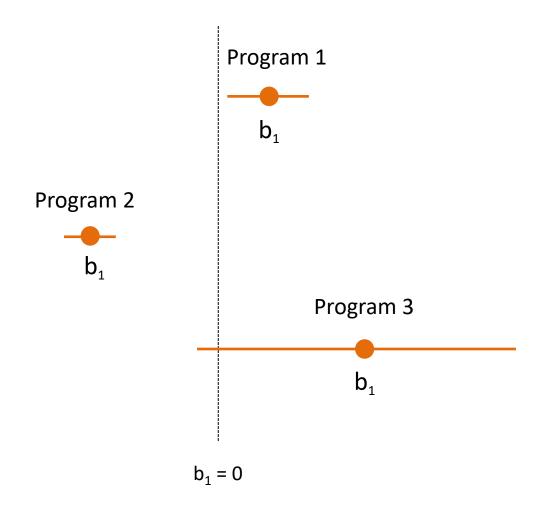
Program 2



Reading Speed =  $b_0 + b_1$ Hours of Tutoring + e

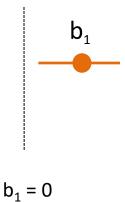


Reading Speed =  $b_0 + b_1$ Hours of Tutoring + e

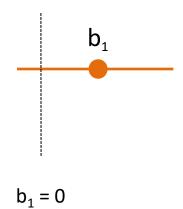


### Reading Speed = $b_0 + b_1$ Hours of Tutoring + e

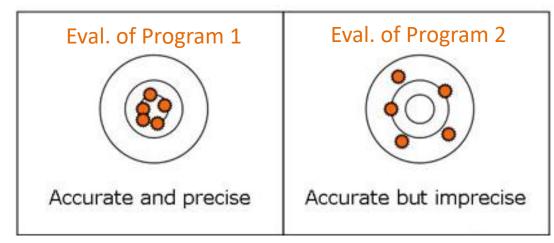
### Program 1



### Program 2



### Model precision

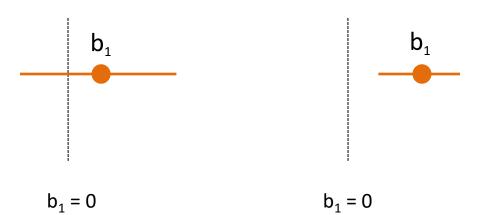


### **LOOKING AHEAD**

For now we are focusing on the interpretation of coefficient plots. But next week we will look at how adding control variables change models. They can shift coefficients, and change standard errors, changing the interpretations of program effectiveness.

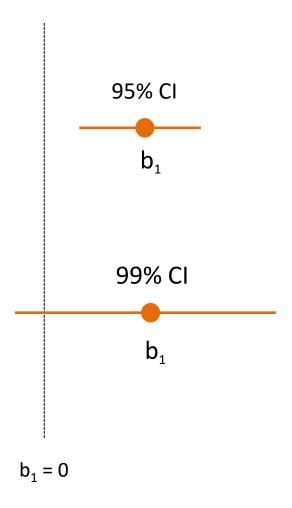
#### Model 1

#### Model 2 w Controls

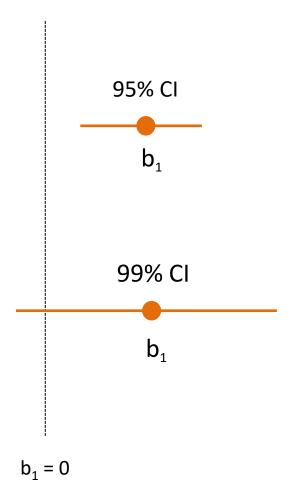


### WHAT IS A **P-VALUE?**

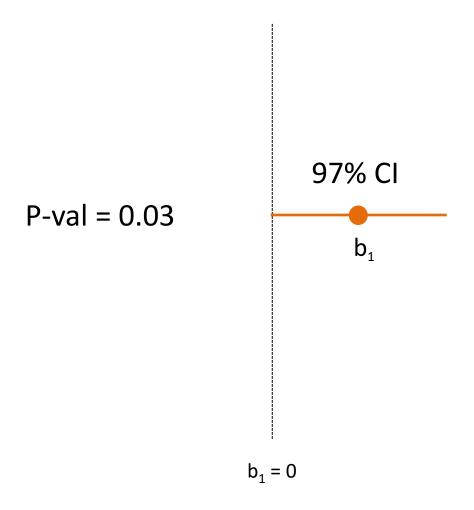
### WHICH OF THESE IS STATISTICALLY SIGNIFICANT?



## WHAT IS THE P-VALUE IN THIS CASE?



## WHAT IS THE P-VALUE IN THIS CASE?



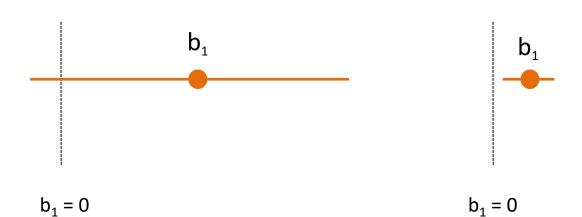
The p-value tells you how large you can draw your confidence interval before it contains the null.

### WHICH PROGRAM IS BETTER?

Reading Speed =  $b_0 + b_1$ Hours of Tutoring + e

#### Program 1

Program 2



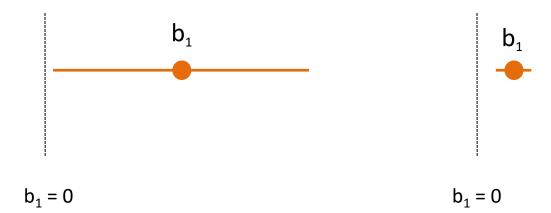
### 95% CONFIDENCE INTERVALS

### WHAT ABOUT NOW?

Reading Speed =  $b_0 + b_1$ Hours of Tutoring + e

Program 1

Program 2



### 90% CONFIDENCE INTERVALS

# WHICH BET WOULD YOU PREFER?

#### BET#1

The bet costs \$1,000 to place There is a 75% chance you win \$1,500 There is a 25% chance you win \$1,100

#### **BET #2**

The bet costs \$1,000 to place
There is a 75% chance you win \$4,000
There is a 25% chance you win \$0

## MECHANICS OF CONFIDENCE INTERVALS

### THE ROAD MAP (AGAIN)

#### Of the Mean:

#### Of the Slope:

Sampling Variance:



$$\sigma_x^2 = \frac{\sum (x_i - \bar{x})^2}{n - 1}$$

(for x)

$$\sigma_{\varepsilon}^2 = \frac{SSE}{n-2} = \frac{\sum e_i^2}{n-2}$$

(using the residual)

Standard **Deviation:** 





$$\sigma_{\varepsilon} = \sqrt{\sigma_{\varepsilon}^2}$$

Standard **Error**:

$$SE_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}}$$

$$SE_{b_1} = \sqrt{\frac{\sigma_{\varepsilon}^2}{\sum (x_i - \bar{x})^2}}$$

Confidence Interval

$$\mu = \bar{x} \pm t \cdot SE_{\bar{x}}$$

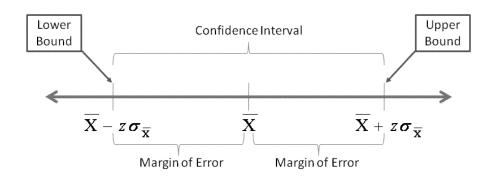
(of the mean)

$$\beta_1 = b_1 \pm t \cdot SE_{b_1}$$

(of the slope)

### THE FORMULA

If we were sure of ourselves we wouldn't need a margin of error! We only have a sample, though, so we can't be certain.



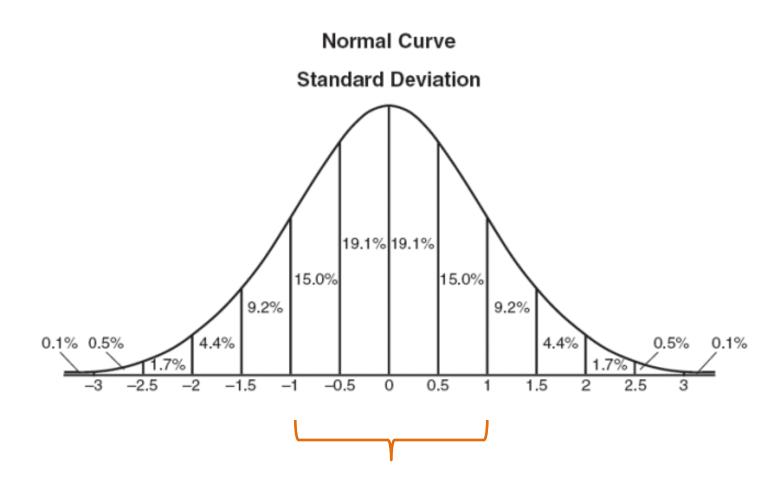
CI for 
$$\mu = \overline{x} \pm t \frac{s}{\sqrt{n}}$$

 $\frac{1}{\sqrt{n}}$  are never known, so we use t-stats and the formula for the sample standard error.

The population parameters

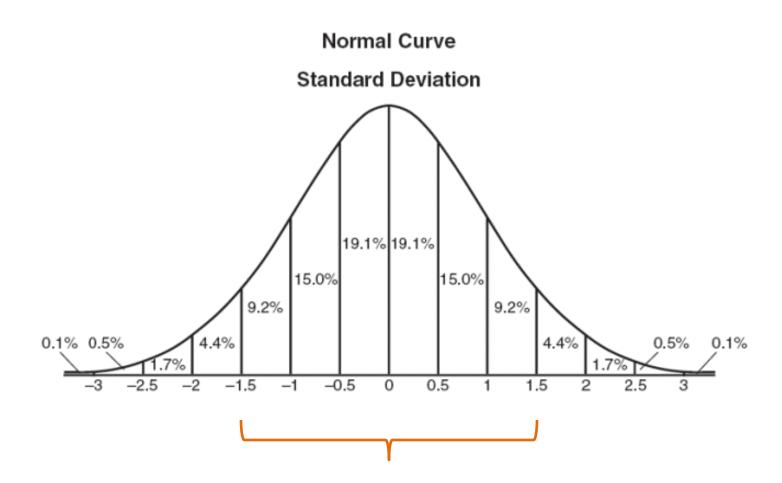
(CI of the mean)

### WHAT IS A T-VALUE?



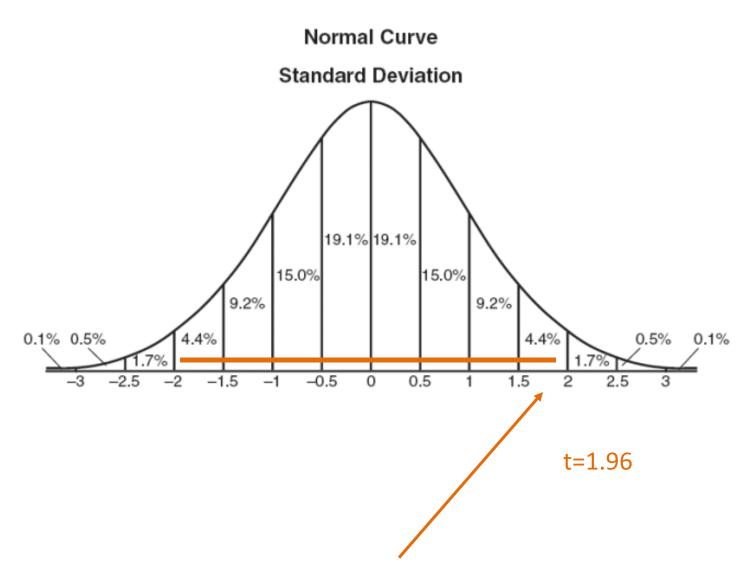
If we examine an interval that is 1 standard deviation from the mean in both directions, we know that this will include 68.2% of the cases.

### WHAT IS A T-VALUE?



If we examine an interval that is 1.5 standard deviations from the mean in both directions, we know that this will include 86.6% of the cases.

### WHAT IS A T-VALUE?



I want a 95% confidence interval, so I find the t-value where 95% of the data falls within the interval (in a 2-sided test).

### DETERMINE T-VALUE:

CI for 
$$\beta_1 = b_1 \pm t \cdot SE_{b_1}$$

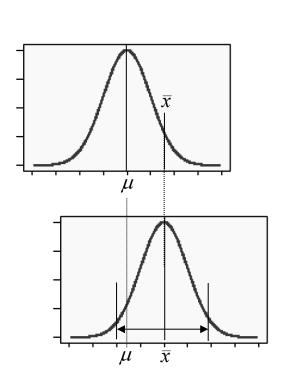
CI for 
$$\mu = \bar{x} \pm t \cdot SE_{\bar{x}}$$

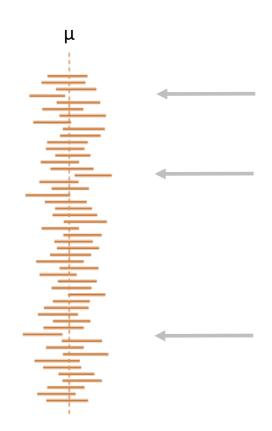
- 1) Select a level of confidence
- 2) Figure out your sample size
- 3) Find a t-table
- 4) Match level of confidence to sample size

Or just use software like a normal person

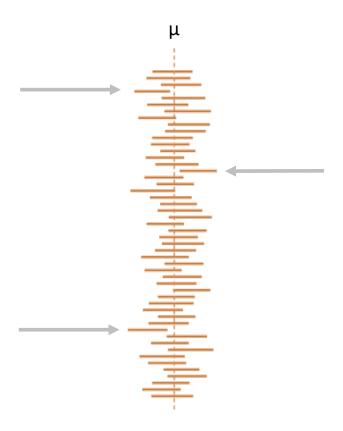
```
# create some fake data
                                                  head( data.frame( y, t ) )
# y = heartrate
                                                            y t
                                                  1 67.67494 0
# t = treatment (e.g. caffeine)
                                                  2 71.19501 1
t < -rep(c(0,1), 50)
                                                  3 69.81960 0
                                                  4 73.58000 1
y < -70 + rnorm(100,0,3) + 5*t
                                                  5 65.47680 0
                                                  6 74.04290 1
plot( factor(t), y,
        main="Comparison of Group Means",
        xlab="Treatment Group",
        ylab="Heart Rate" )
             Comparison of Group Means
               Treatment Group
mean.t \leftarrow mean(y[t == 1])
mean.c \leftarrow mean(y[t == 0])
mean.c
[1] 70.29385
mean.t
[1] 75.25103
# effect size
mean.t - mean.c
[1] 4.957187
t.test(y \sim t)
        Welch Two Sample t-test
data: y by t
t = -8.1231, df = 97.927, p-value = 1.391e-12
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -6.168235 -3.746139
sample estimates:
mean in group 0 mean in group 1
       70.29385
                        75.25103
```

If alpha=0.05, what is our level of confidence?



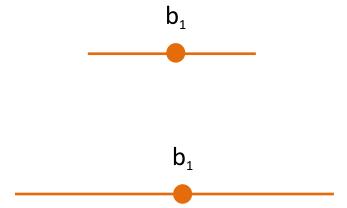


Chose an **alpha-level**, which determines the size of the confidence interval. This example uses alpha=0.05. We would expect five samples in one-hundred to result in confidence intervals that do not contain the true mean. We see 3 in 50 draws here, which is consistent with expectations.

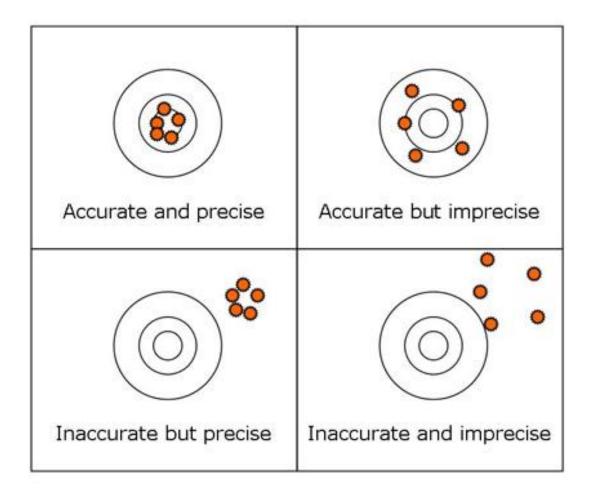


What if we change our alpha from 0.05 to 0.10, how many of these confidence intervals would not contain the true mean?

Is a **90% confidence interval** bigger or small than a **95% confidence interval**?



## WHERE WE ARE GOING:



Regression estimates should be:

- 1. UNBIASED (accurate)
- 2. **EFFICIENT** (precise)

# COEFFICIENT PLOTS AS AN ALTERNATIVE TO DENSE REGRESSION TABLES

Variable	Co	efficient (Standard Error)
Constant		.41 (.93)
Countries		
Argentina		1.31 (.33)### B,M
Chile		.93 (.32)### B,M
Colombia		1.46 (.32) ### B,M
Mexico		.07 (.32) <sup>A,CH,CO,V</sup>
Venezuela		.96 (.37)## B,M
Threat		
Retrospective egocentric economic perceptions		.20 (.13)
Prospective egocentric economic perceptions		.22 (.12)#
Retrospective sociotropic economic perceptions		21 (.12)#
Prospective sociotropic economic perceptions		32 (.12)##
Ideological Distance from president		
Ideology		
Ideology		.23 (.07) ###
Individual Differen	ces	
Age		.00 (.01)
Female		03 (.21)
Education		.13 (.14)
Academic Sector		.15 (.29)
Business Sector		.31 (.25)
Government Sector		10 (.27)
R <sup>2</sup>		.15
Adjusted R <sup>2</sup>		.12
n		500
###p < .01, ##p < .05, #p < .10	(two-tailed)	

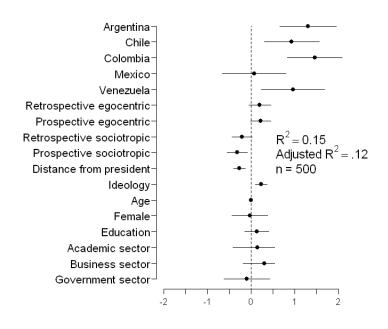


Table 2 from Stevens (2006):

Determinants of Authoritarian Aggression

This figure has everything we need to interpret the regression. It shows the magnitude of the relationship between each variable and Y, the standard error of each coefficient (encoded in the confidence interval), and statistical significance (does it cross zero?).

Which do you prefer?

# INTERPRETING PROGRAM IMPACT

### What should be clear in my mind?

- 1. Our interpretation of program impact involves an understanding of the "effect size" (regression slope) and the precision with which it is estimated (the confidence interval).
- 2. The level of confidence we select determines the t-value, which determines the size of the confidence interval.
- 3. For a program to be statistically significant, the confidence interval around the slope should not contain the null hypothesis (slope=0).
- 4. We can choose an arbitrary level of confidence such that our confidence interval will not contain the null.
- 5. The p-value tells us the largest confidence interval that we can draw that does not contain the null.
- 6. Program investments are bets that balance effect size plus confidence.

## ASIDE: STATISTICAL POWER

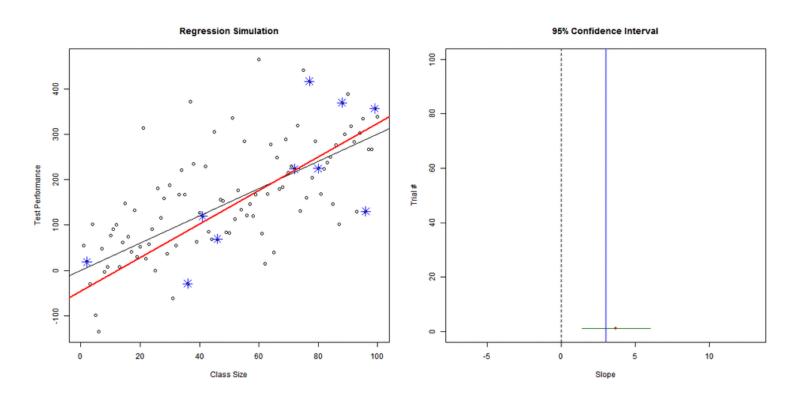
## STATISTICAL POWER

Power: the ability to detect a program impact when it exists.

Type I Error: Claiming a program has impact when it doesn't (false positive). This type of error is usually caused by bias in our estimate of impact.

Type II Error: Failure to detect program impact when it exists (false negative). This type of error is usually caused by a lack of adequate statistical power (large standard errors).

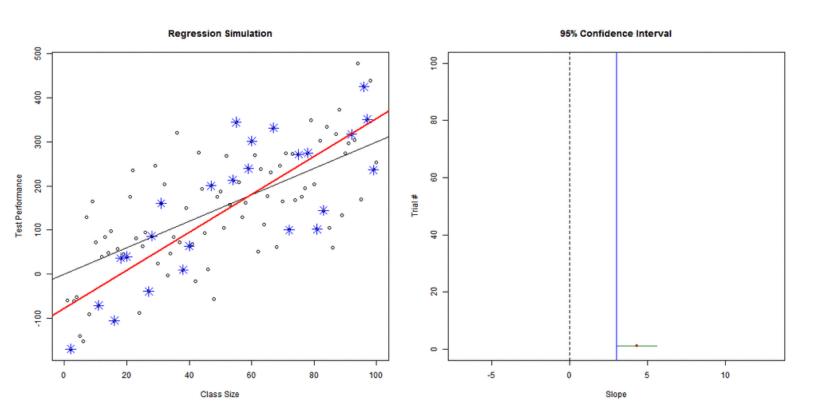
## LOW POWER



In many cases we fail to reject the null, even though our true program impact is a slope of 3.

Note that our model is unbiased – our estimates all cluster around the true slope.

## HIGH POWER

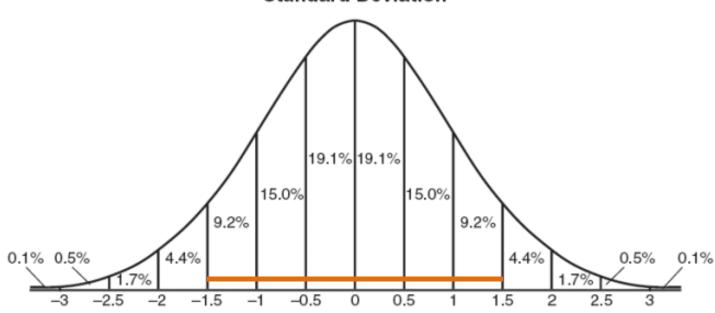


We are sure here that we have enough power to say something concrete about the program impact. We do not worry about Type II Errors in this evaluation.

What has changed?



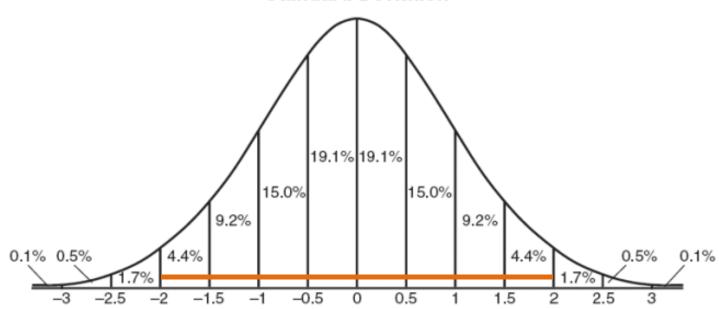




An interval with a width of of 1.5 stan. dev.'s ensures that we capture 86.6% of the data



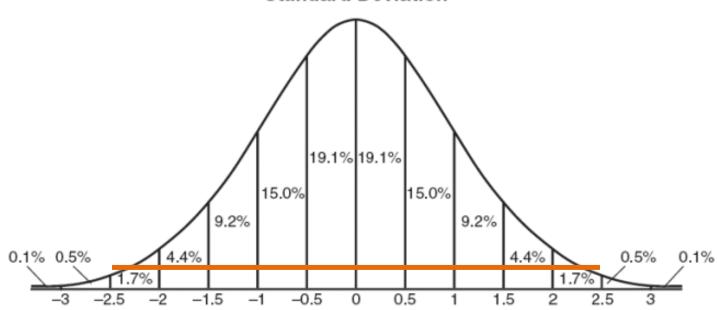




If we increase the interval to 2 standard deviations we now capture 95.4% of the data for a gain of 8.8 points of confidence.





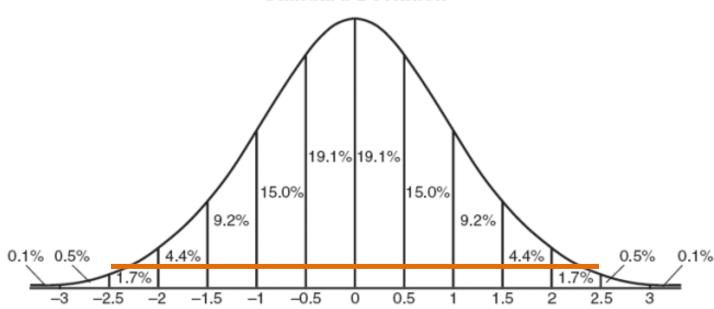


If we take another half-unit step to 2.5 standard deviations from the mean we now capture 98.8% of the data, but we gain only 3.4 points from the same increase in interval size, less than half the confidence gain as before.

Increasing the interval from 2.5 to 3 standard deviations results in only 1 more point of confidence gained.

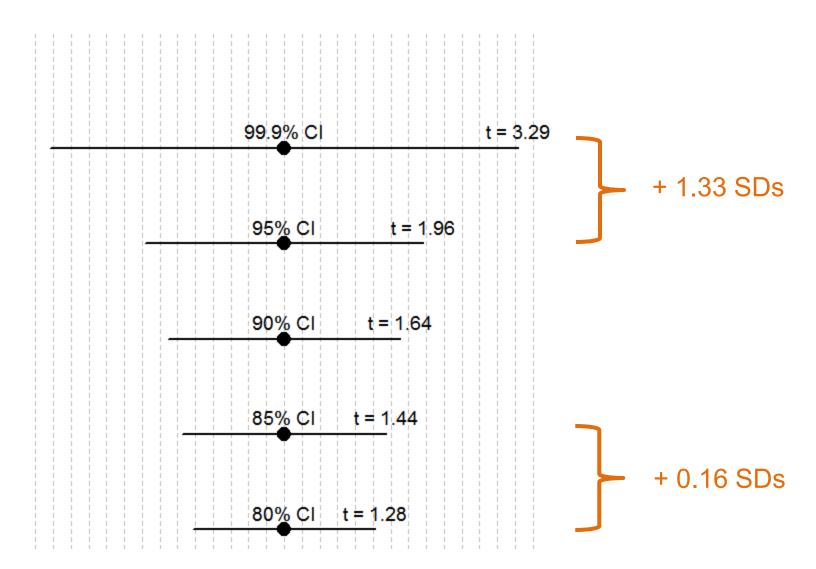






Each additional unit of confidence become more and more expensive as you approach 100%.

What is the relationship between a "unit of confidence" and a confidence interval?



There is an increasing marginal cost of gaining confidence. The

```
x.85 < - round(qnorm(0.075, mean = 0, sd = 1), 2)
x.90 \leftarrow round(qnorm(0.05, mean = 0, sd = 1), 2)
x.95 < - \text{ round}(\text{ gnorm}(0.025, \text{ mean} = 0, \text{ sd} = 1), 2)
x.999 \leftarrow round(qnorm(0.0005, mean = 0, sd = 1), 2)
ci.lower < c(x.80, x.85, x.90, x.95, x.999)
par( mar=c(0,0,0,0) )
plot.new()
plot.window(xlim=c(-3.5,3.5), ylim=c(1,6))
abline( v=seq(-3.5,3.5,by=0.25), lty=2, col="gray")
points ( rep (0,5), 1:5, pch=19, cex=2 )
segments (x0=ci.lower, x1=abs (ci.lower), y0=1:5,
          lwd=2 )
text( rep(0,5), 1:5, c("80% CI", "85% CI", "90% CI", "95% CI", "99.9% CI"),
      cex=1.2, pos=3)
text(abs(ci.lower), 1:5,
      paste("t = ",abs(ci.lower),sep=""), cex=1.2, pos=3)
```

