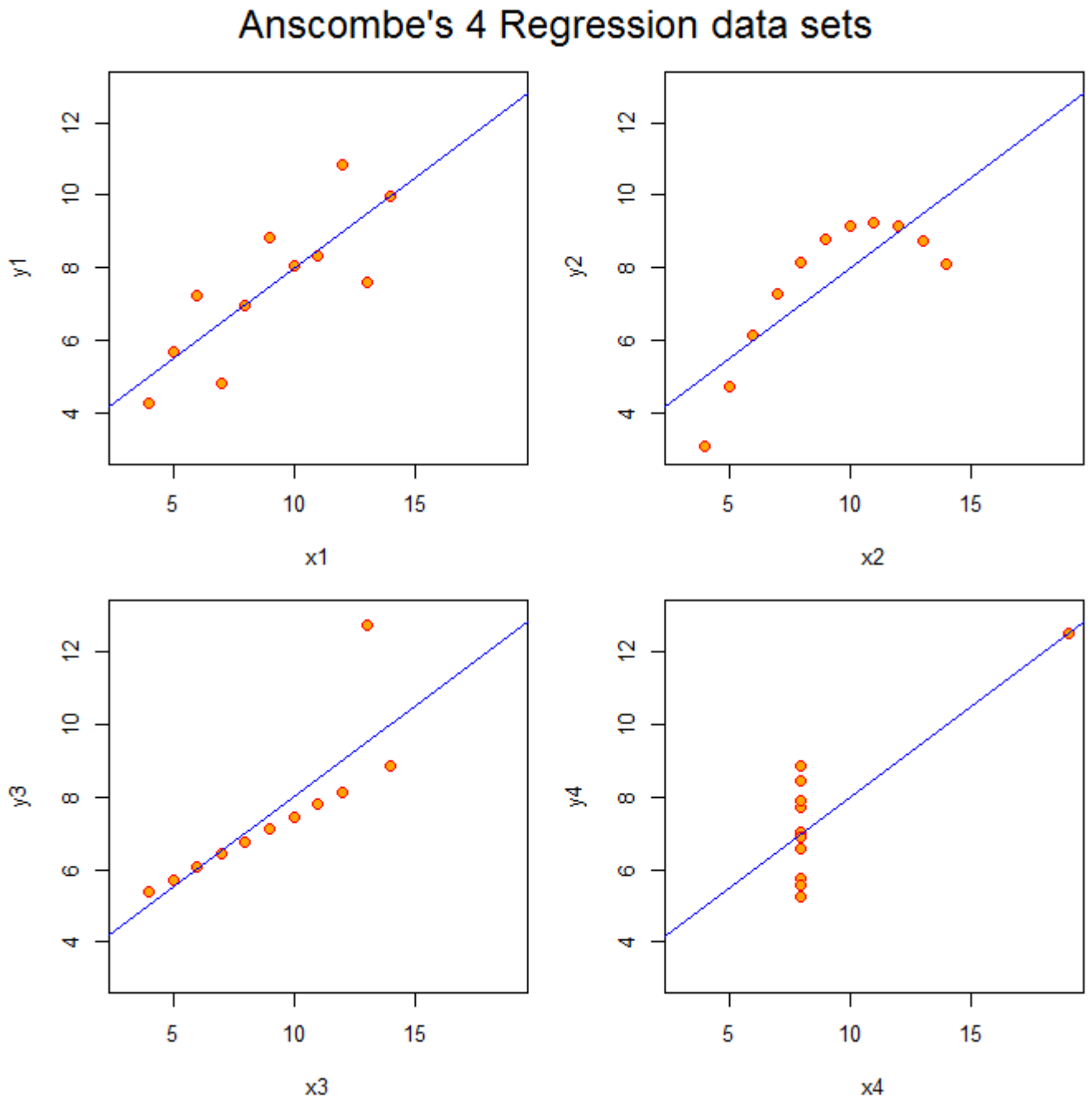
# BIAS FROM SPECIFICATION OR MEASUREMENT

# ANSCOMBE'S QUARTET
## A LESSON IN MODEL FIT

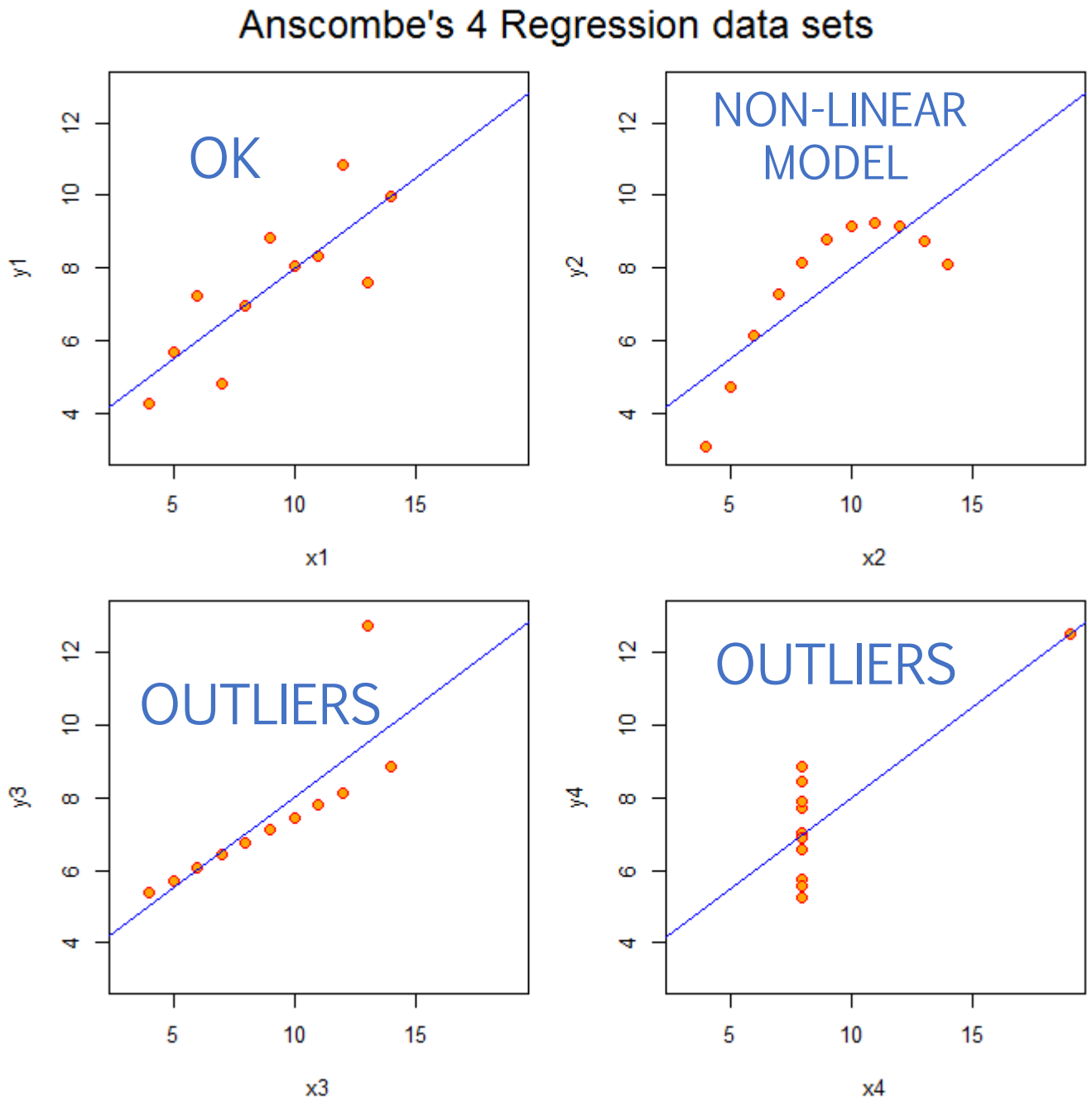
# Anscombe's Quartet

| | x1 | y1 | x2 | y2 | x3 | y3 | x4 | y4 |
|---|---|---|---|---|---|---|---|---|
| 1 | 10 | 8.04 | 10 | 9.14 | 10 | 7.46 | 8 | 6.58 |
| 2 | 8 | 6.95 | 8 | 8.14 | 8 | 6.77 | 8 | 5.76 |
| 3 | 13 | 7.58 | 13 | 8.74 | 13 | 12.74 | 8 | 7.71 |
| 4 | 9 | 8.81 | 9 | 8.77 | 9 | 7.11 | 8 | 8.84 |
| 5 | 11 | 8.33 | 11 | 9.26 | 11 | 7.81 | 8 | 8.47 |
| 6 | 14 | 9.96 | 14 | 8.1 | 14 | 8.84 | 8 | 7.04 |
| 7 | 6 | 7.24 | 6 | 6.13 | 6 | 6.08 | 8 | 5.25 |
| 8 | 4 | 4.26 | 4 | 3.1 | 4 | 5.39 | 19 | 12.5 |
| 9 | 12 | 10.84 | 12 | 9.13 | 12 | 8.15 | 8 | 5.56 |
| 10 | 7 | 4.82 | 7 | 7.26 | 7 | 6.42 | 8 | 7.91 |
| 11 | 5 | 5.68 | 5 | 4.74 | 5 | 5.73 | 8 | 6.89 |
| Mean | 9 | 7.5 | 9 | 7.5 | 9 | 7.5 | 9 | 7.5 |
| Variance | 11 | 4.12 | 11 | 4.12 | 11 | 4.12 | 11 | 4.12 |
| Correlation | 0.816 | | 0.816 | | 0.816 | | 0.816 | |
| Regression | y = 3 + 0.5x | | y = 3 + 0.5x | | y = 3 + 0.5x | | y = 3 + 0.5x | |

Four datasets that produce IDENTICAL descriptive stats, correlations, and regression models




BUT THEY ARE VERY DIFFERENT RELATIONSHIPS!

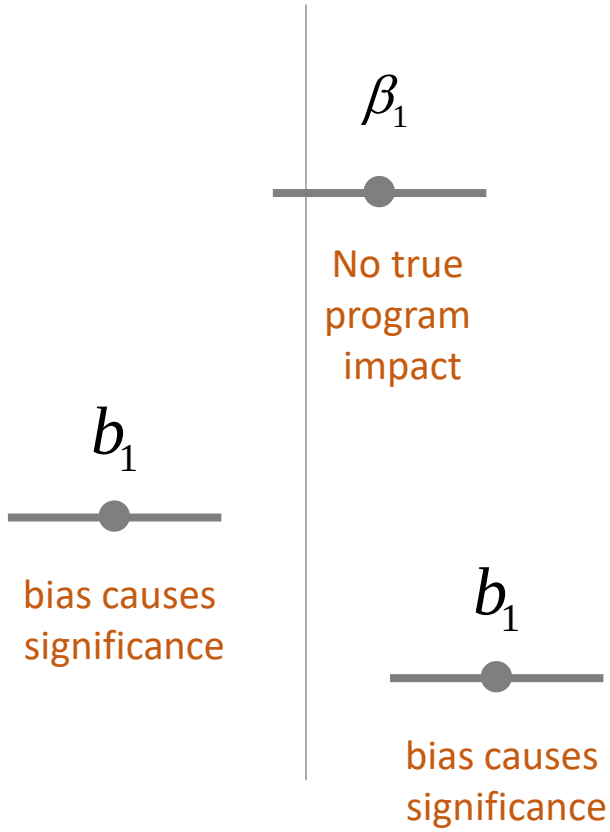



Anscombe's Quartet is often cited because (a) whoever created this example is a genius, and (b) it is a vivid demonstration of causes and consequences of **SPECIFICATION BIAS.**

We will consider what happens to slopes when outliers are present, or we use a linear specification when the relationship is non-linear.

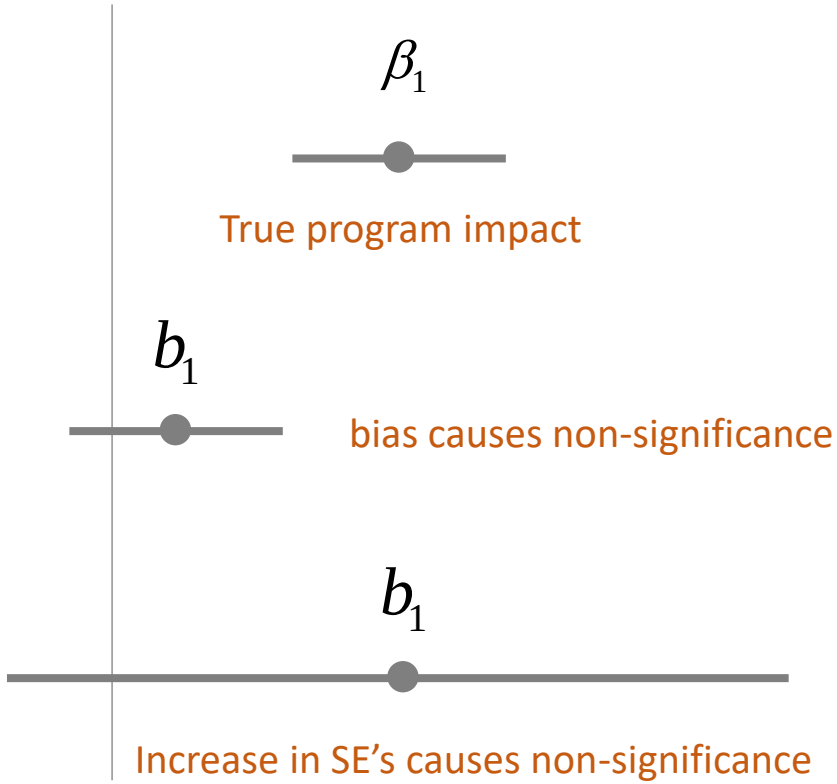
# CLASSES OF INFERENTIAL FAILURE
## TYPE I AND TYPE II ERRORS

## TYPE I ERROR
## FALSE POSITIVE
## CLAIMING PROGRAM HAS IMPACT WHEN IT DOESN'T



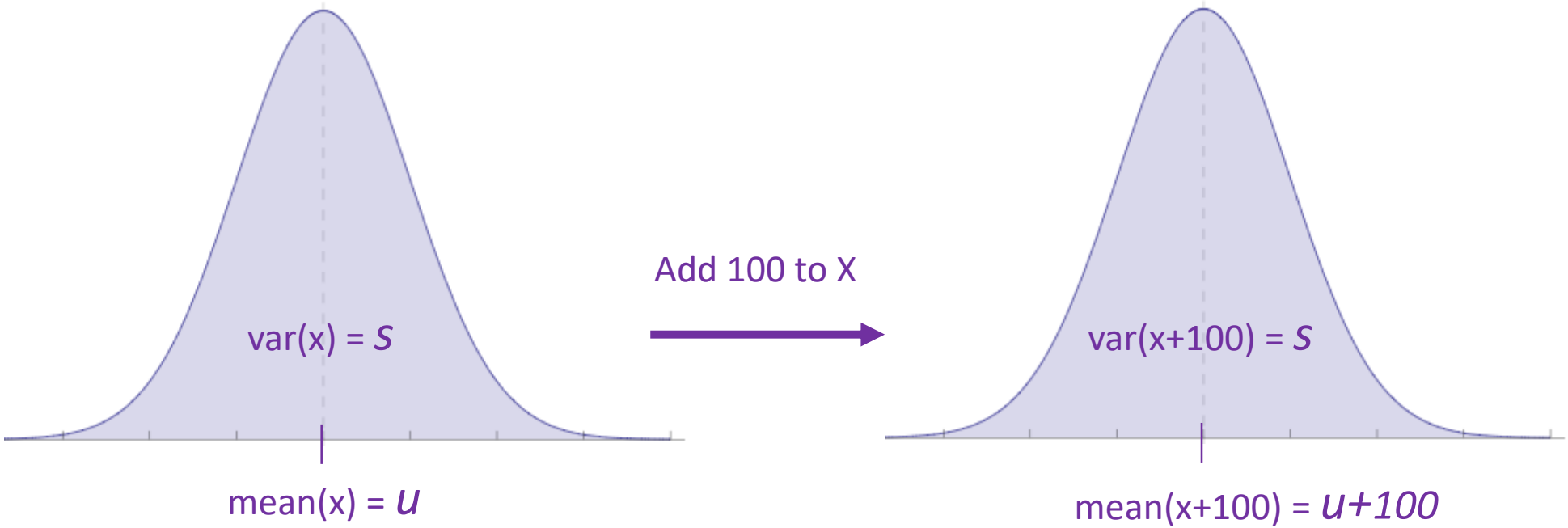

Type I errors are typically caused by OVB

## TYPE II ERROR
## FALSE NEGATIVE
## FAILING TO IDENTIFY TRUE PROGRAM IMPACT




Type II Errors can be caused by bias or inflated standard errors

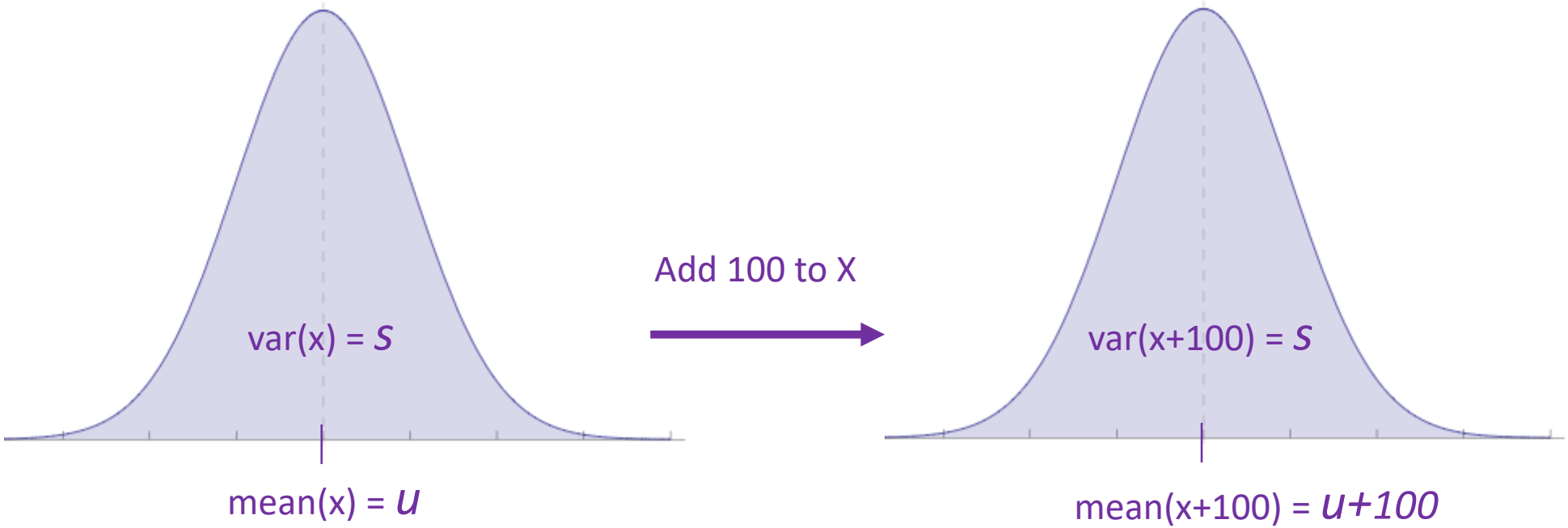
# IMPLICATIONS OF MEASUREMENT ERROR

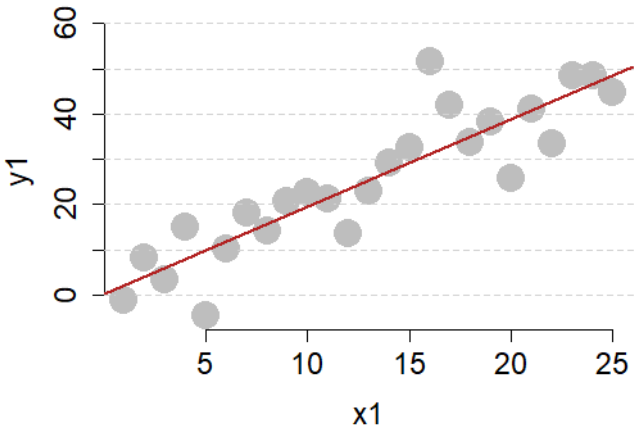


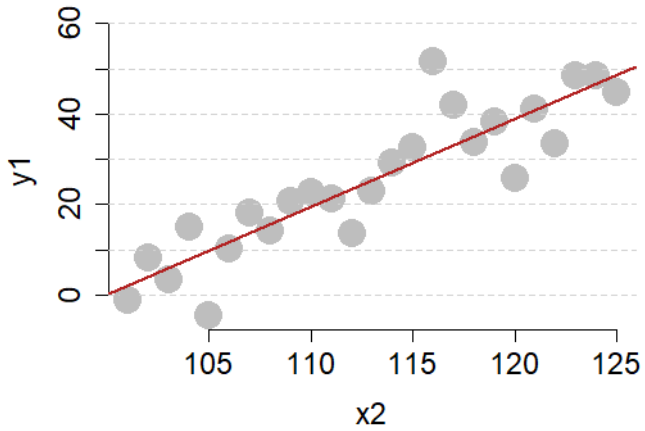

"Linear Transformations"

X2 = X1 + 100

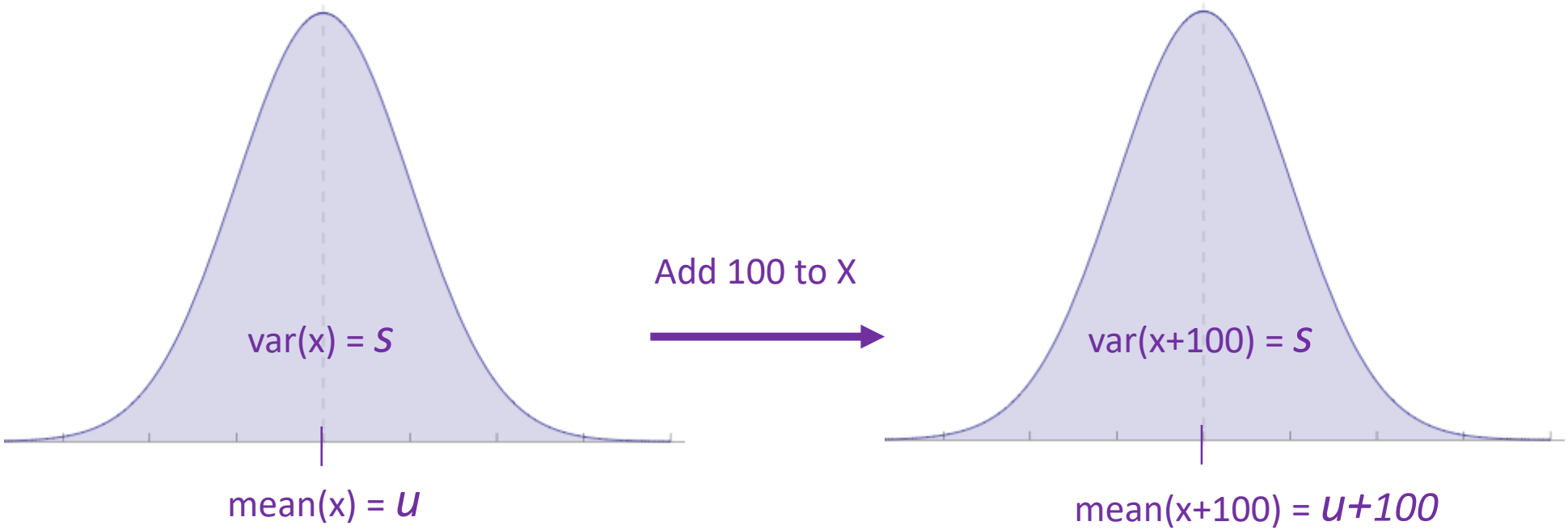
Variance of X is unchanged




After linear transformations:
Regressions are unaffected

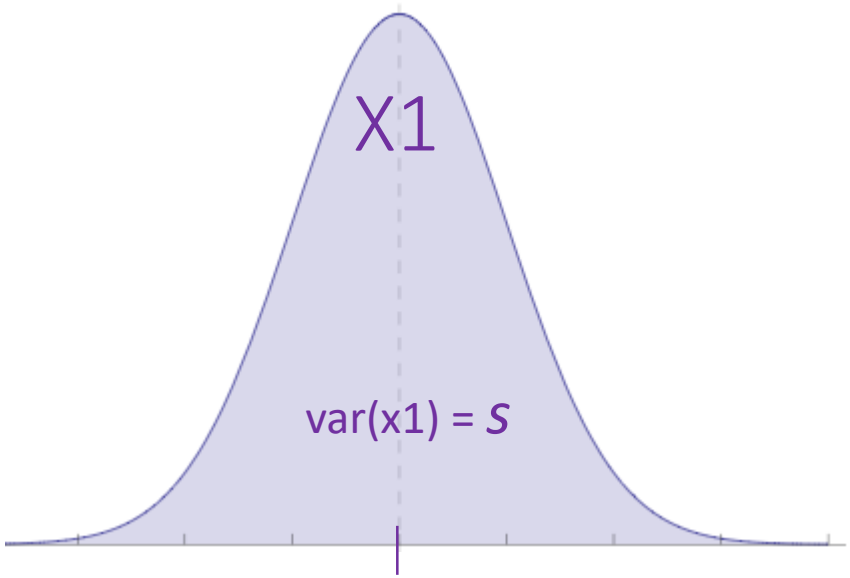


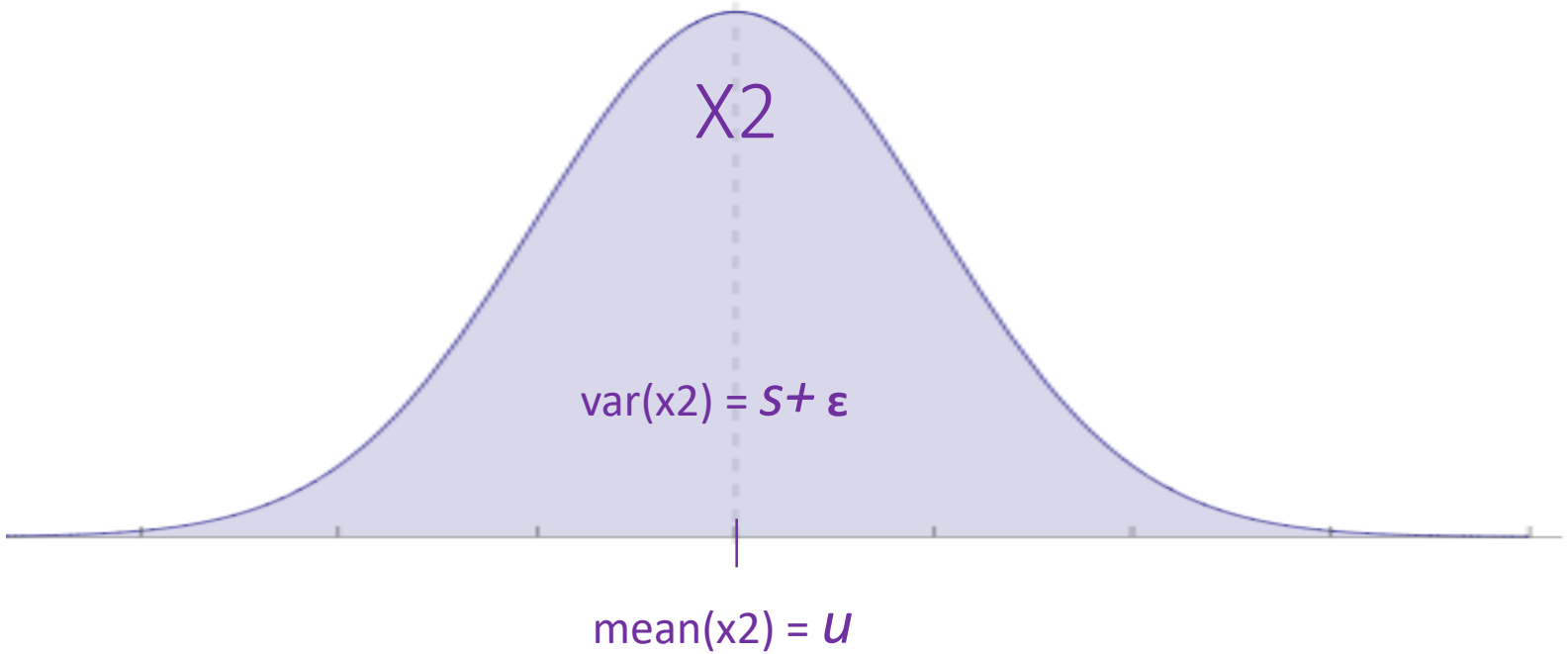

"Linear Transformations"

X2 = X1 + **100**

Must add the **same constant** to every value of X

Just moves the distribution to right or left




# Measurement Error

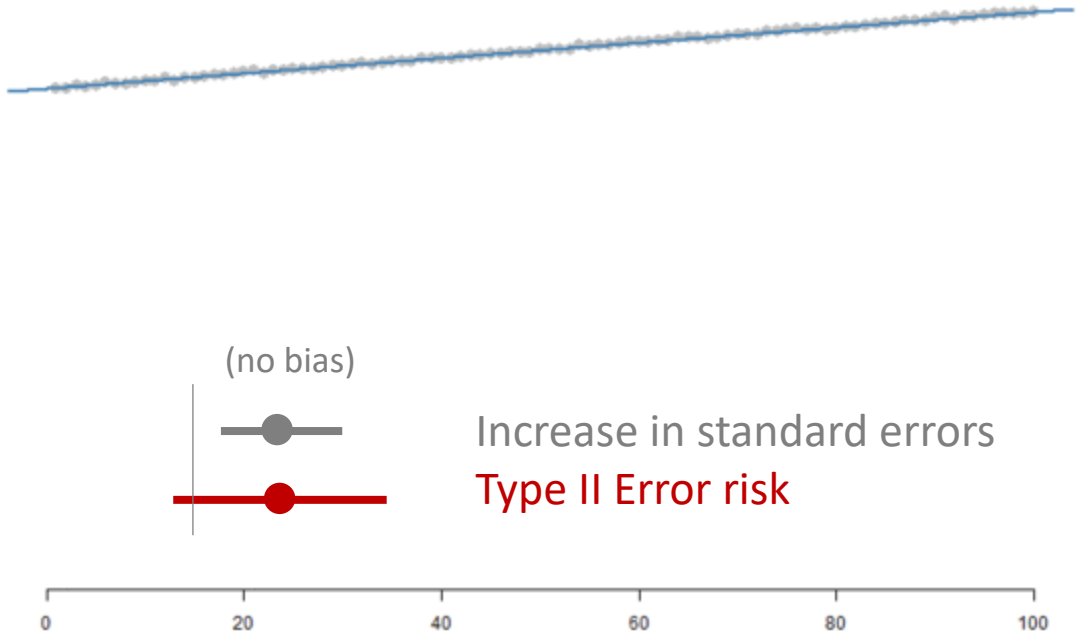
$$X2 = X1 + \varepsilon$$

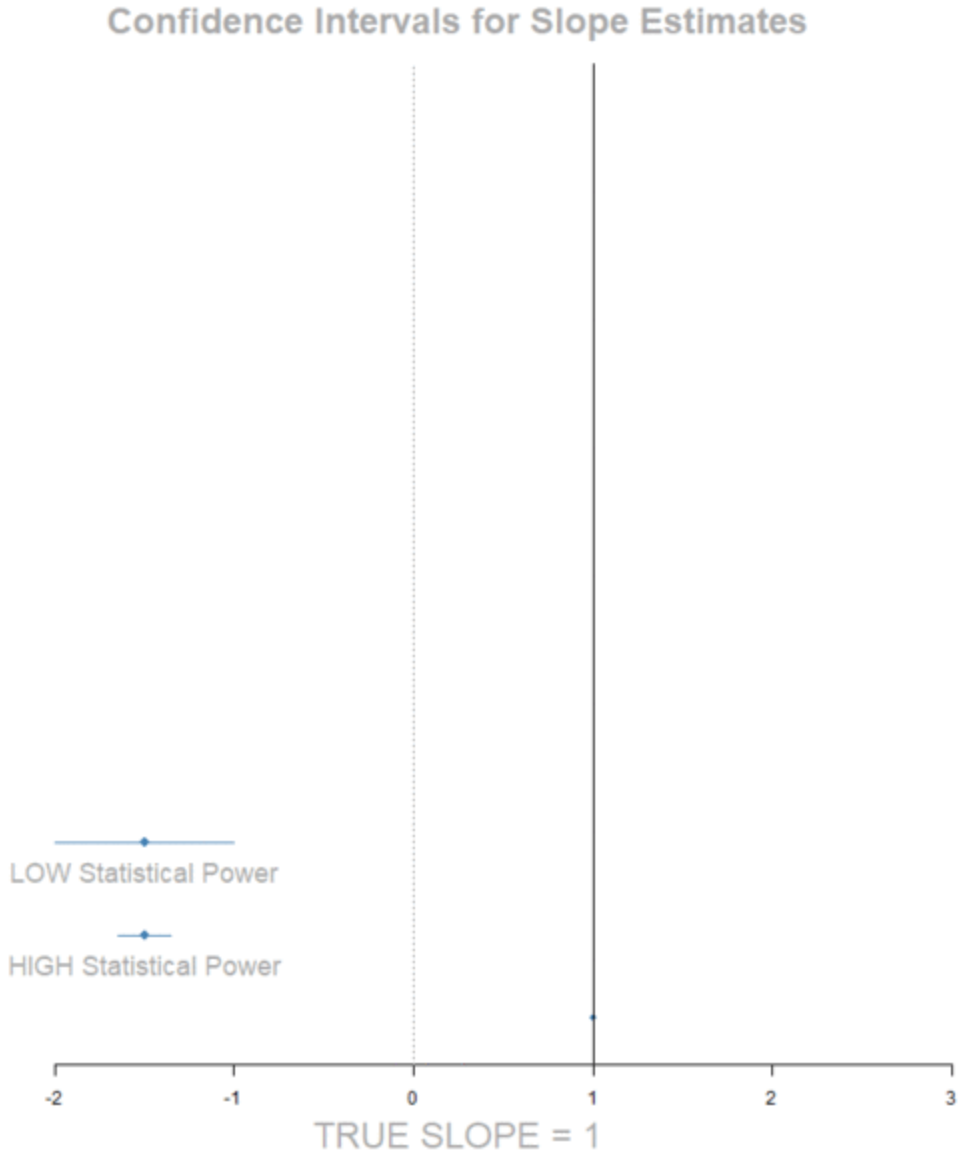
Add random error to every X.

*Random means each X is equally likely to be over-measured as under-measured.*

X2 has the same mean as X1, but more variance

# ADDING MEASUREMENT ERROR TO THE DV



# ADDING MEASUREMENT ERROR TO THE INDEPENDENT VARIABLE: "ATTENUATION BIAS"

slope with measurement error

$\beta_1$

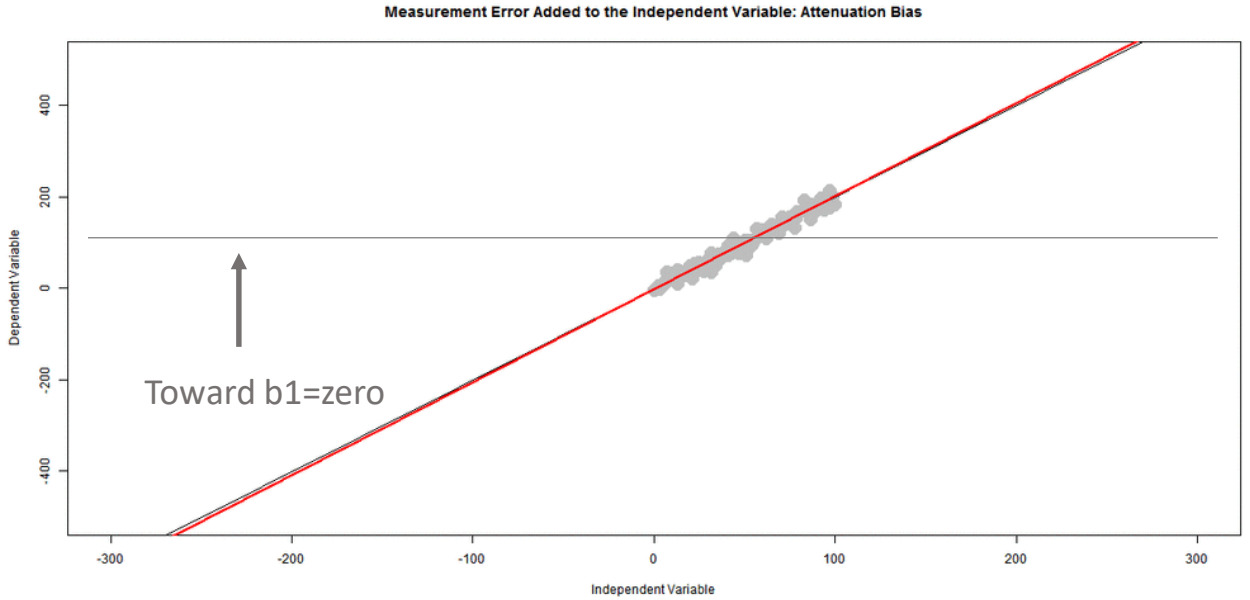
True program impact

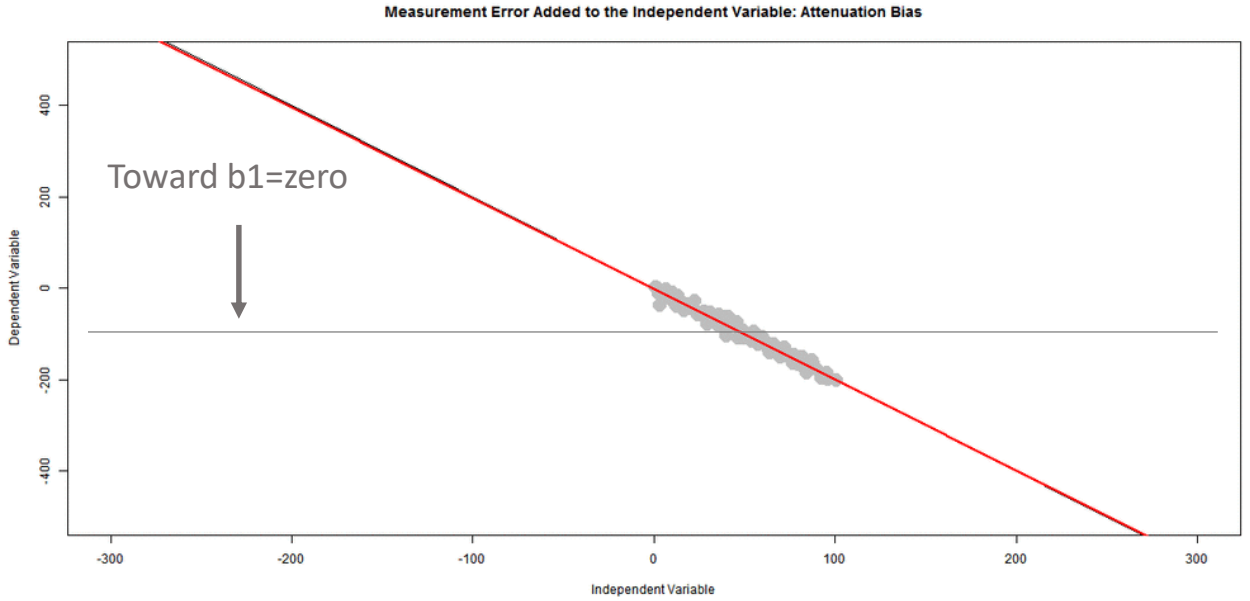
$b_1$

Smaller slope
Smaller SE

$$b_1 \downarrow = \frac{\text{cov}(x_1, y)}{\text{var}(x_1) \uparrow}$$

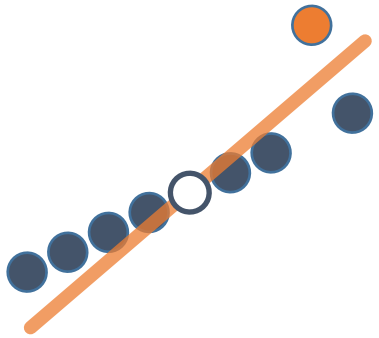
$$SE_{b1} \downarrow = \frac{residual}{samplesize \cdot \text{var}(x_1) \uparrow}$$

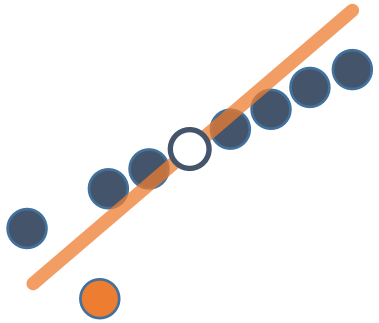


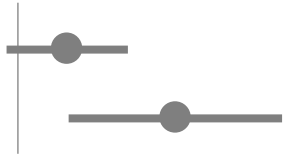
# OUTLIERS

## SLOPES TOO LARGE
## SE LARGER

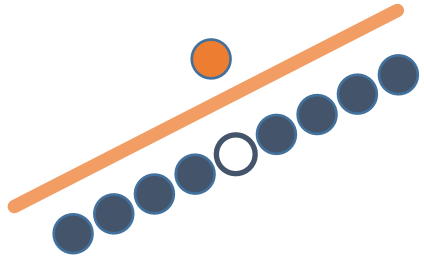


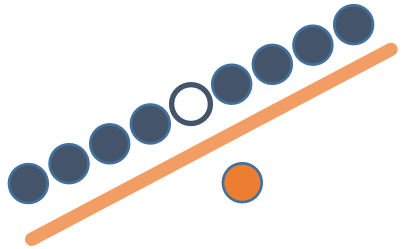


Extreme of X:
Risk of bias in slope ↑
Risk of false positive

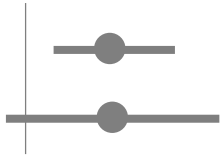


## SLOPES OK
## SE LARGER

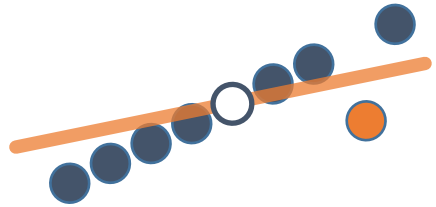


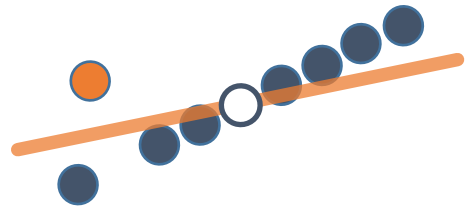


Middle of X:
Don't bias slope
Increased risk of false negative

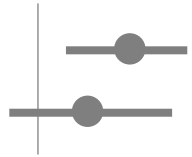


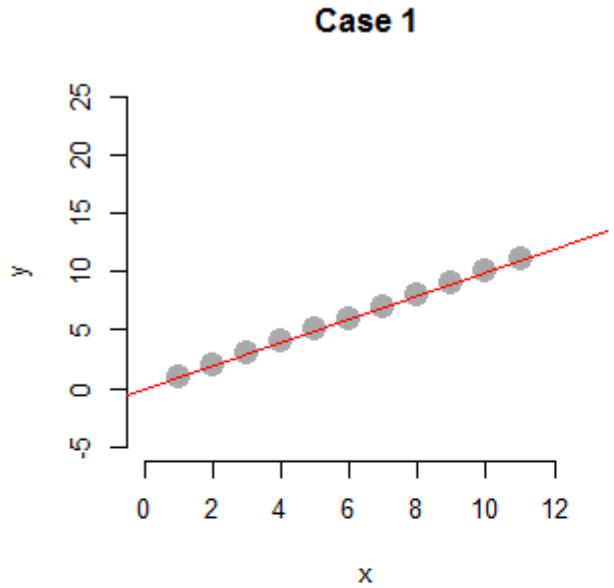
## SLOPES TOO SMALL
## SE LARGER

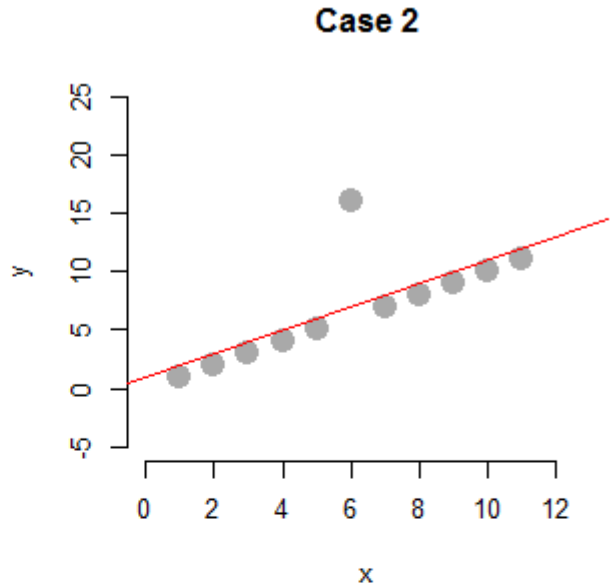


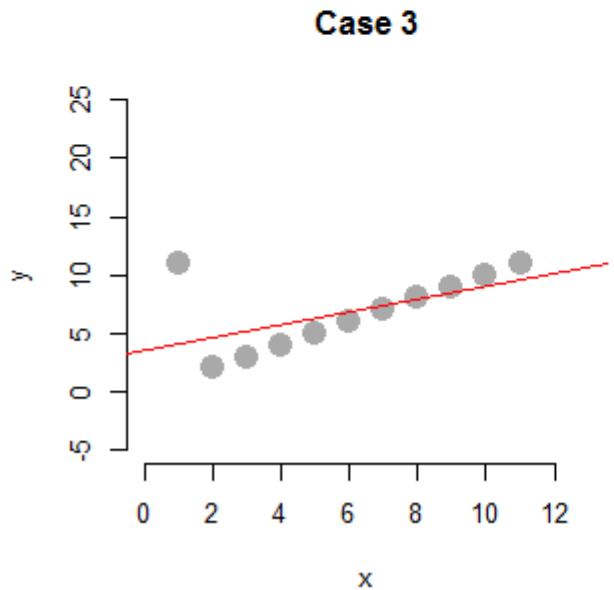


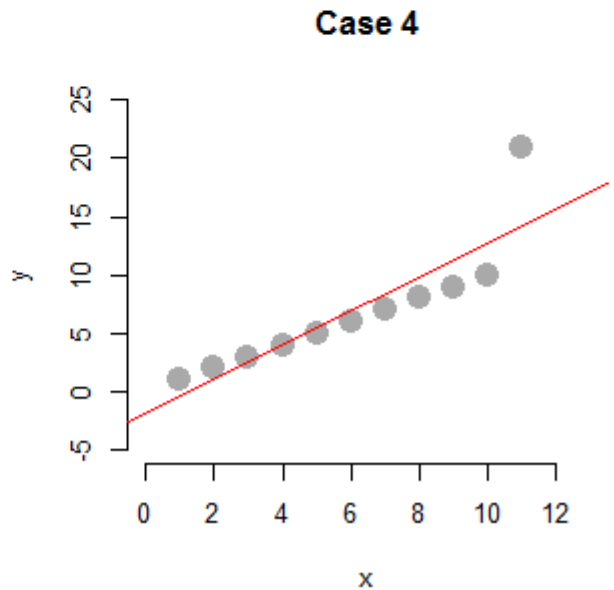
Extreme of X:
Risk of bias in slope ↓
Increased risk of false negative





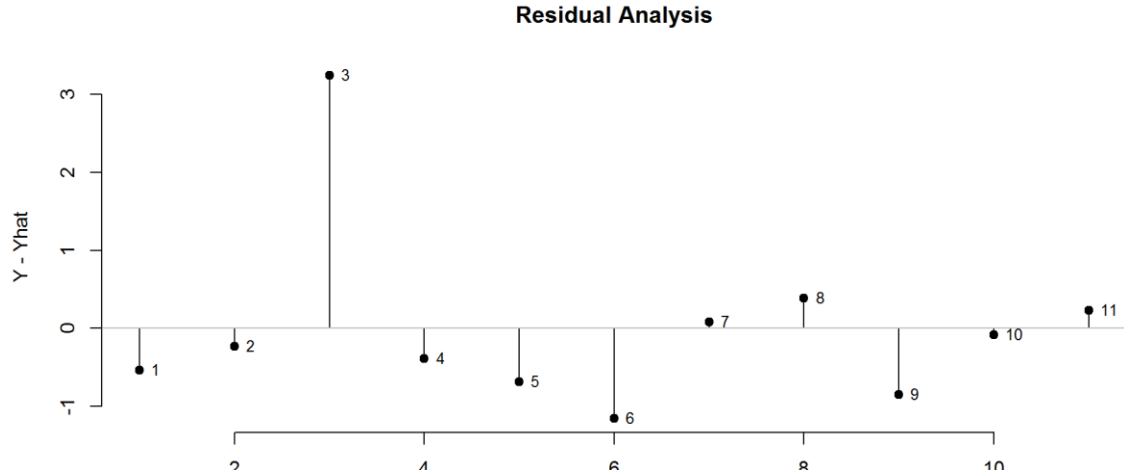



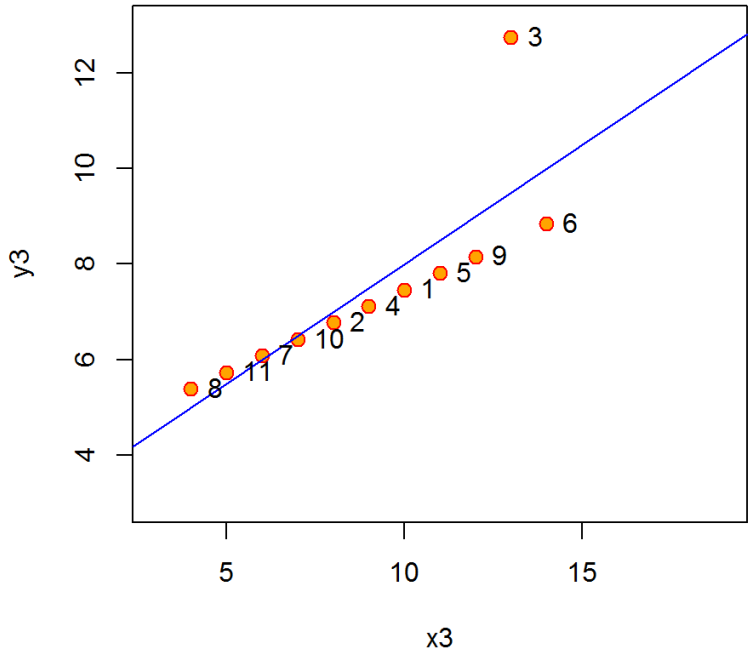



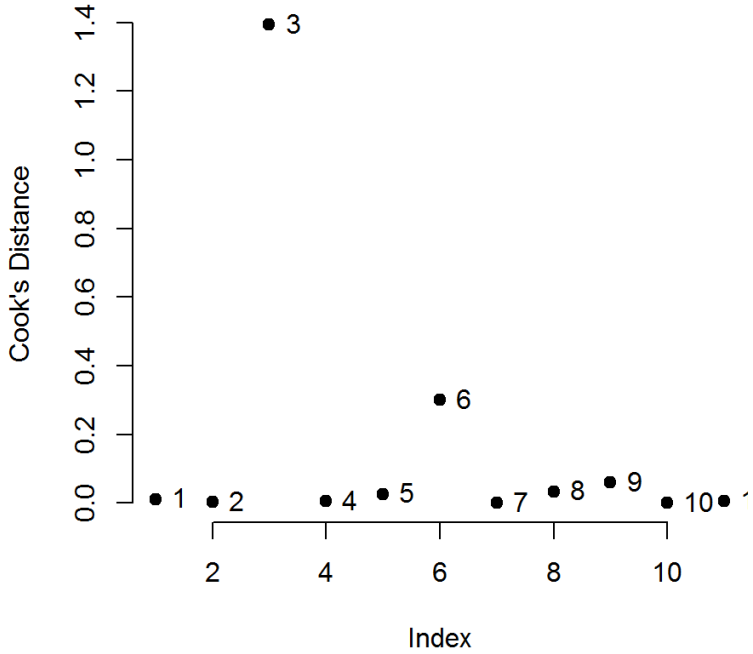




| | *Dependent variable:* | | | |
|---|---|---|---|---|
| | y | | | |
| | (1) | (2) | (3) | (4) |
| x | 1.00*** | 1.00*** | 0.55* | 1.45*** |
| | (0.00) | (0.30) | (0.26) | (0.26) |
| Constant | 0.00*** | 0.91 | 3.64* | -1.82 |
| | (0.00) | (2.06) | (1.78) | (1.78) |

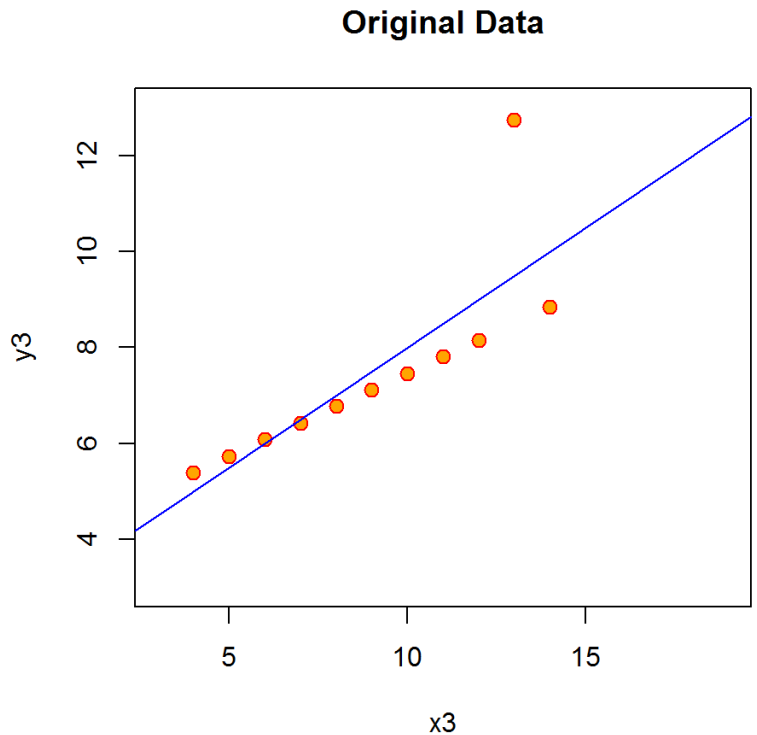
# IDENTIFYING OUTLIERS USING RESIDUALS AND COOK'S DISTANCE

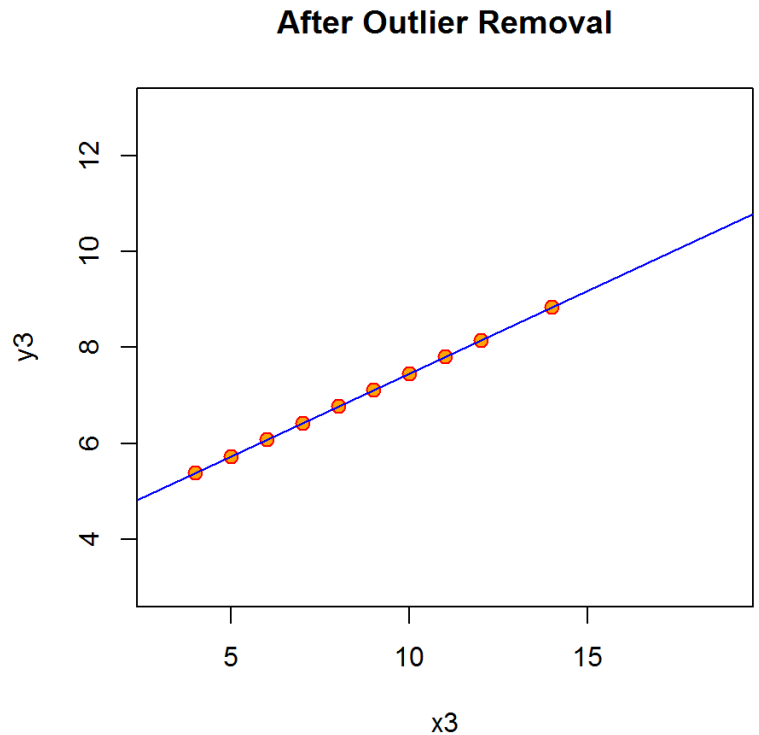


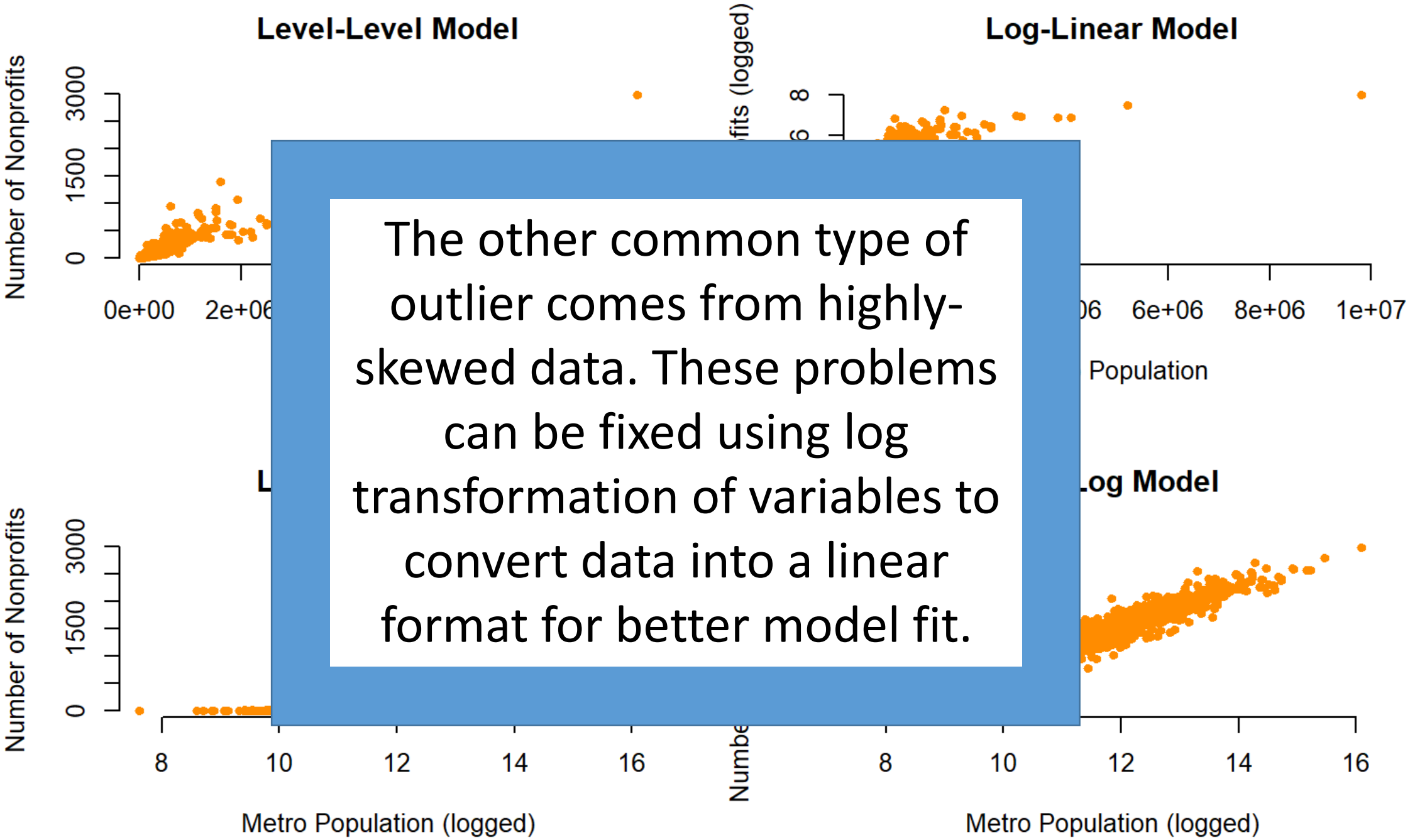



| | *Dependent variable:* | |
|---|---|---|
| | y3 | |
| | (1) | (2) |
| x3 | 0.50*** | 0.35*** |
| | (0.12) | (0.0003) |
| Constant | 3.00** | 4.01*** |
| | (1.12) | (0.003) |
| Observations | 11 | 10 |
| $R^2$ | 0.67 | 1.00 |
| Adjusted $R^2$ | 0.63 | 1.00 |
| *Note:* | *p<0.1;* ***p<0.05;*** p<0.01 | |

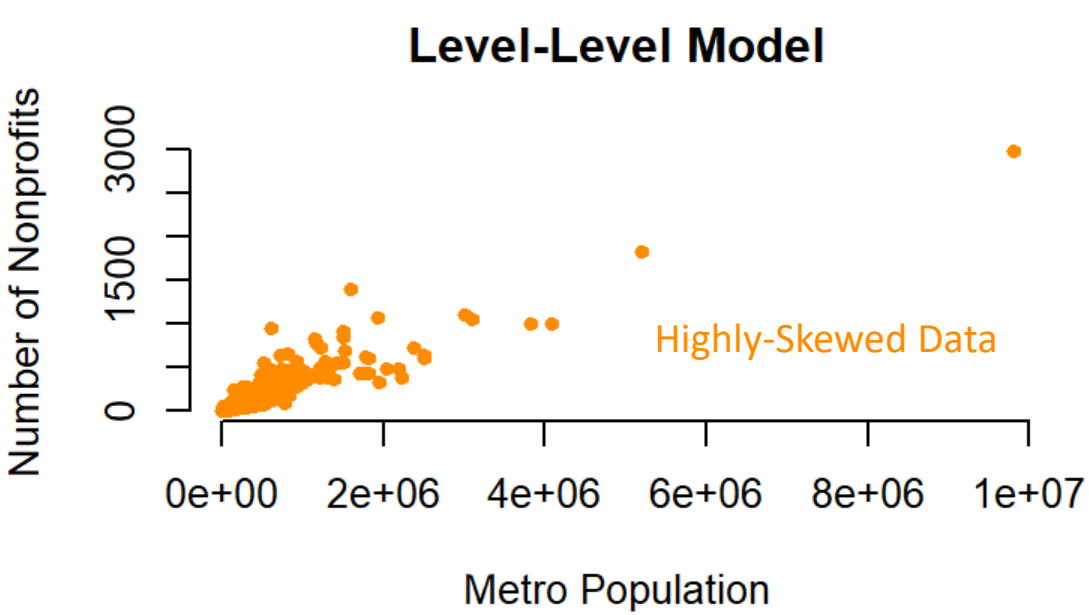


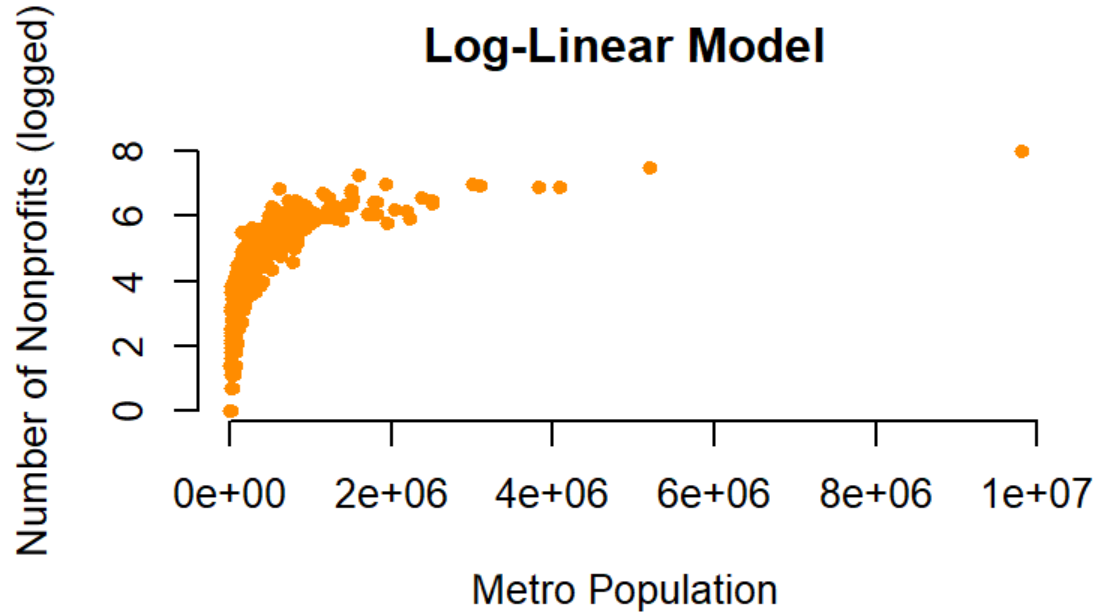



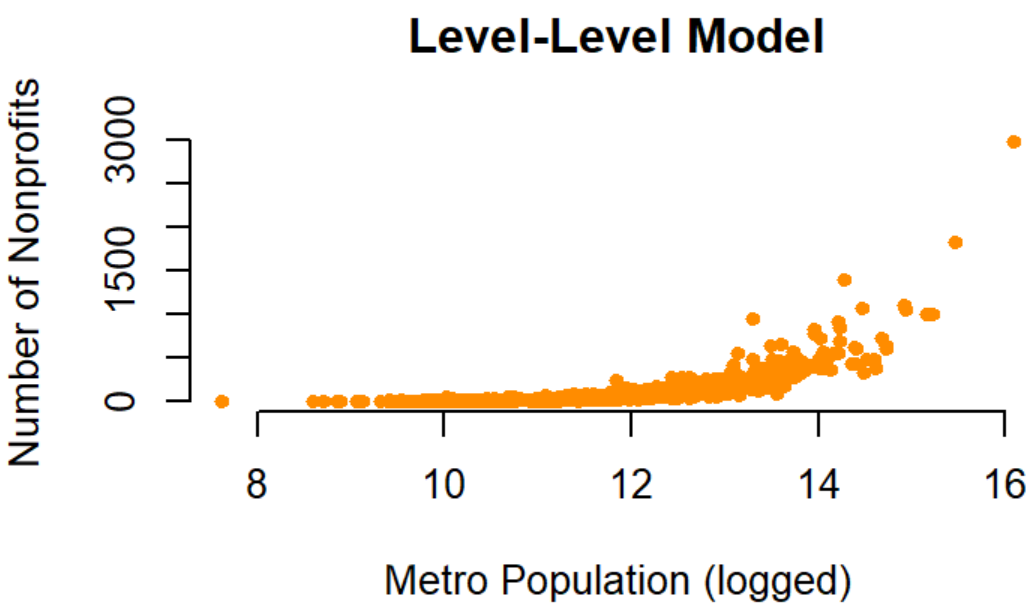
# LOGGED
# REGRESSION MODELS

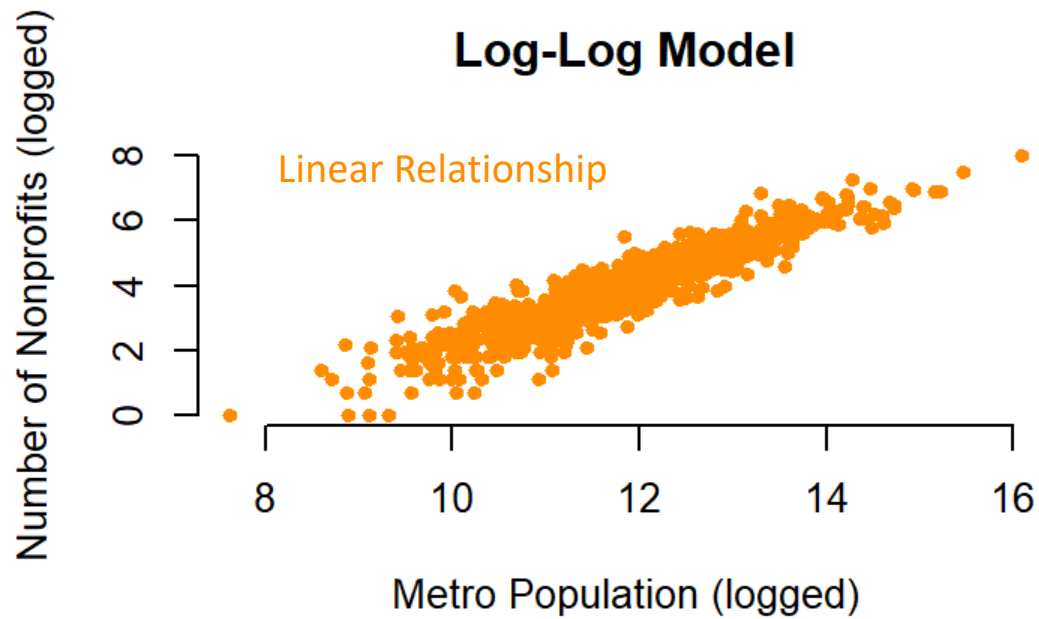



The other common type of outlier comes from highly-skewed data. These problems can be fixed using log transformation of variables to convert data into a linear format for better model fit.


Level-Level Model
Number of Nonprofits
0
1500
3000
Highly-Skewed Data
0e+00
2e+06
4e+06
6e+06
8e+06
1e+07
Metro Population


Log-Linear Model
Number of Nonprofits (logged)
0
2
4
6
8
0e+00
2e+06
4e+06
6e+06
8e+06
1e+07
Metro Population


Level-Level Model
Number of Nonprofits
0
1500
3000
8
10
12
14
16
Metro Population (logged)


Log-Log Model
Number of Nonprofits (logged)
Linear Relationship
0
2
4
6
8
8
10
12
14
16
Metro Population (logged)

# NON-LINEAR RELATIONSHIPS
# QUADRATIC MODELS

Linear: $Y = b0 + b1(X_1) + e$

Quadratic: $Y = b0 + b1(X_1) + b2(X_1)^2 + e$