Gradient

$$L = -\sum_j \sum_i y_{ji} \log(P_j = i) + \frac{\lambda}{2} \sum_i w_i^2$$

$$y = \begin{array}{cc} & D \\ N & \begin{bmatrix} \quad \end{bmatrix} \end{array}$$

W_1 $\quad$ $n = \#\text{input}$ $\quad$ $m = \#\text{neuron}$

$\frac{\partial L}{\partial W_i}$

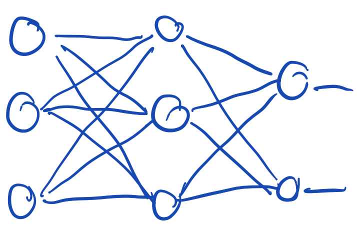
x . $N * D$

$W_1 = D * H$

$X_1 = XW_1 + b$ $\quad$ $N * H$

$X_2 = X_1 W_2 + b$ $\quad$ $N * C$.

$$\begin{array}{cc} & m \\ n & \begin{bmatrix} \frac{\partial L}{\partial W_i} \end{bmatrix} \end{array}$$



Sigmoid

$$P(X_j = i) = \frac{e^{W_2 X_2 + b_2}}{\sum e^{W_2 X_2 + b_2}}$$

$h = w_2 x_2 + b_2$

$$\frac{\partial L}{\partial h} \cdot \frac{\partial h}{\partial W_2}$$

$P(X_j = 1) =$ $\quad$ (H).

$$\frac{dL_1}{dh} = \begin{bmatrix} \frac{\partial L}{\partial h_{i1}} & \frac{\partial L}{\partial h_{i2}} & \frac{\partial L}{\partial h_{i3}} \\ \vdots & \vdots & \vdots \end{bmatrix}_{N*C} = \begin{bmatrix} P(y_{i1}=1)-y_{i1} & P(y_{i2}=2)-y_{i2} & P(y_{i3}=1)-y_{i3} \\ & & \end{bmatrix}$$

$$\frac{dL_2}{dw_1} = \begin{bmatrix} \frac{\lambda}{\cancel{2}} \cdot \cancel{2} w_1 \\ \\ \end{bmatrix}_{D*H} \qquad \frac{dL_2}{dw_2} = \begin{bmatrix} \\ \\ \end{bmatrix}_{H*C.}$$

$$h = x_1 w_2 + b_2$$

$$\frac{dh}{dw_2} = \begin{bmatrix} x_1 & \cdots \\ \vdots & \ddots \end{bmatrix}_{N*H.} \qquad \frac{dh}{db_2} = \begin{bmatrix} 1 & \cdots \\ \vdots & \ddots \end{bmatrix}_{N*1.}$$

$$x_1 = Relu(xw_1 + b_1) \;.\; x' = Relu(xw_1 + b_1)$$

$$\frac{dx_1}{dw_1} = \begin{bmatrix} \text{ismorethan}(x_1)w_1 & \cdots \\ \vdots & \end{bmatrix}_{D*H} \qquad \frac{dh}{dx_1} = \begin{bmatrix} w_2 & \cdots \\ \vdots & \ddots \end{bmatrix}_{H*C.}$$

$$\frac{dx_1}{db_1} = \begin{bmatrix} \text{ismorethan0}(x_1) & \cdots \\ \vdots & \end{bmatrix}_{N*H.}$$

$$L = -\sum_j \sum_i y_{ji} \log(P_j = i) + \frac{\lambda}{2} \sum_i w_i^2$$

$$= -\sum_j \sum_i y_{ji} (\log \exp(h_{ji}) - \log \sum_i \exp(h_{ji})) + \frac{\lambda}{2} \sum_i w_i^2$$

$$h = x_1 w_2 + b_2$$

$$x_1 = x w_1 + b_1$$

$$\frac{dL}{dw_2} = \frac{\partial L}{\partial h} \frac{\partial h}{\partial w_2}$$

$(H, C)^2$

$$\frac{\partial L}{\partial w_{2_{ij}}} = \frac{\partial L}{\partial h} \frac{\partial h}{\partial w_{2_{ij}}}$$

$$\underset{(1,D)}{\vec{x}} \xrightarrow[(D,H)]{W_1} \oplus \xleftarrow{} \underset{(1,H)}{\vec{b}_1} \rightarrow \underset{(1,H)}{\vec{x}_1} \rightarrow \text{Relu} \xrightarrow[(H,C)]{W_2} \oplus \leftarrow \underset{(1,C)}{b_2} \rightarrow \underset{(1,C)}{\vec{x}_2} \xrightarrow{\text{Softmax}} \underset{(1,C)}{\vec{h}} \rightarrow L.$$

$$\frac{\partial L}{\partial b_{2i}} = \frac{\partial L}{\partial \vec{h}} \frac{\partial \vec{h}}{\partial \vec{x}_2} \frac{\partial \vec{x}_2}{\partial b_{2i}}$$

$$\frac{\partial L}{\partial h}$$

$$L = -\sum_j y_j \log(P_j = i) + \frac{\lambda}{2} \sum_i w_i^2 \qquad h = \frac{e^{x_i}}{\sum_j e^{x_j}}$$

$$= -\sum_i y_j \log h + \frac{\lambda}{2} \sum w_i^2$$

$$\frac{\partial L}{\partial h} = \left[-\frac{y_1}{h_j} \quad -\frac{y_2}{h_2} \quad -\frac{y_3}{h_3} \right].$$

$$\frac{\partial L}{\partial w_{2i}} = \frac{\partial L}{\partial \vec{h}} \frac{\partial \vec{h}}{\partial \vec{x}_2} \frac{\partial \vec{x}_2}{\partial w_{2i}} \qquad \vec{x}_2 = \vec{x}_1 W_1 + \vec{b}$$

$1H \; HC \; . \; 1C$

$$\underbrace{[\quad\quad]}_{H} \quad \begin{bmatrix} w_{11} & \\ w_1 & \\ \vdots & \\ w_{1H} & \end{bmatrix}$$

$\vec{x}\vec{w}_{1:}, \ \vec{x}\vec{w}_{2:}, \ \vec{x}\vec{w}_{3:}, \ \ldots$

อันที่ m

$$\text{อันที่ } m\left(\frac{\partial \vec{x}_2}{\partial w_{ij}}\right) = [\underbrace{0 \;\; 0 \;\; 0 \;\; x}_{i} \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0].$$

$i \to 1H$, $j \to 1C$

$\frac{\partial L_j}{\partial h} = \begin{bmatrix} P(y=1)-y_1 & P(y=2)-y_2 & P(y=3)-y_3 \end{bmatrix}$.

$\quad\quad\quad\quad \frac{\partial L}{\partial h_1}$

$C \times 1$.

$\frac{\partial \vec{h}}{\partial \vec{x}_2} = \begin{bmatrix} \frac{\partial h_1}{\partial x_1} & \cdots & \frac{\partial h_1}{\partial x_c} \\ \vdots & & \vdots \\ \frac{\partial h_c}{\partial x_1} & \cdots & \frac{\partial h_c}{\partial x_c} \end{bmatrix}$.

$\frac{\partial \frac{e^{x_1}}{\sum e^{x_i}}}{\partial x_1} = \frac{\sum e^{x_i} e^{x_1} - e^{x_1} e^{x_1}}{(\sum e^{x_i})^2}$ $\quad \frac{\partial \frac{e^{x_1}}{\sum e^{x_1}}}{\partial x_2} = \frac{\cancel{x} \; - e^{x_1} e^{x_2}}{(\sum e^{x_i})^2}$

$\frac{\partial \vec{x}_2}{\partial b_{2i}} = \begin{bmatrix} 1 & 0 & 0. \end{bmatrix}$ $\quad x_2 = \vec{x}_1 W_2 + \vec{b}_2$

$b_{21} \quad b_{22} \quad b_{23}$

$\frac{\partial L}{\partial h} = \frac{1}{N} \sum_j \frac{\partial L_j}{\partial b_{2i}}$

$\cancel{h} \; \cdots \; \cancel{h}$

$x_1 \quad\quad x_c$

$x_1 \quad x_2$

$b_1 \quad b_c$

$\frac{\partial L_j}{\partial x_{2i}} = \frac{\partial \vec{x}_2}{\partial x_{2i}} \cdot \frac{\partial \vec{h}}{\partial \vec{x}_2} \cdot \frac{\partial L_j}{\partial h}$

$(1 \times C)\,(C \times C)\,(C \times 1)$.

$\cancel{x}_2$

b_1

$\frac{\partial x_{11}}{\partial x_{21}}$

$\frac{\partial x_{11}}{\partial x_{22}}$

$\vdots$

$\frac{\partial x_{11}}{\partial x_{2c}}$

$\cancel{x}_c$

b_1

$\frac{\partial L}{\partial x_{1i}} = \frac{\partial L}{\partial h} \cdot \frac{\partial h}{\partial \vec{x}_2} \frac{\partial \vec{x}_2}{\partial x_{1i}}$

(จำนวน$\times$dim) $(1 \times C)\,(C \times C)\,(C, 1)$

$\frac{\partial L}{\partial b_{1i}} = \frac{\partial L}{\partial \vec{x}_1} \cdot \frac{\partial \vec{x}_1}{\partial b_{1i}}$

$1 \times H \times H \times 1$.

$$L_1 = -\Sigma_j y_j log P(y=j) \quad (2)$$

where y_j is 1 if y is class j, and 0 otherwise.

Pc

$\frac{\partial L}{\partial b_2} = \frac{\partial \vec{h}}{\partial b_2} \frac{\partial L}{\partial \vec{h}}$ $\quad \frac{\partial L}{\partial \vec{h}}$

$h = [\ -\ -\ -\ -\ -\ ].$

$h \quad N \times C$ $\quad L \quad 1$

$b \quad 1 \times C$ $\quad h \quad N \times C$

$L = -\sum_i \sum_j^c y_i \log P(x_i = j) + \frac{\lambda}{2} \sum_k w_k^2.$

$L = L_1 + L_2$

$\frac{\partial L_j}{\partial \vec{b}} = \frac{\partial L}{\partial h} \cdot \frac{\partial h}{\partial b_{2i}}$ $\quad 1 \times C$

$= \frac{\partial \vec{x}_2}{\partial \vec{b}_{2i}} \frac{\partial \vec{h}}{\partial \vec{x}_2} \cdot \frac{\partial L}{\partial \vec{h}}$

$N \times [C, 1] \quad C \times C. \quad C \times C \quad C \times 1.$

$\vec{x}_2 = Relu(\vec{y}_1).$

$\frac{\partial L}{\partial b} = \frac{1}{N} \cdot \sum_j \frac{\partial L_j}{\partial b_{2i}}$

$\vec{x}_2 = Rulu(x_1 w_2 + b_2)$

$(1, H)(H, C) \quad (1, C)$

$[x_1 \ \ ]$ $\begin{bmatrix} w_1 & \end{bmatrix}.$

$1 \times H$

$\frac{\partial \vec{x}_2}{\partial \vec{x}_1} = \vec{x}_1 \left\{ \overbrace{\begin{bmatrix} \partial x_{21} & \\ & \end{bmatrix}}^{\vec{x}_2} \right.$

$\frac{\partial \vec{x}_2}{\partial x_{1i}} = 0 \quad 0$

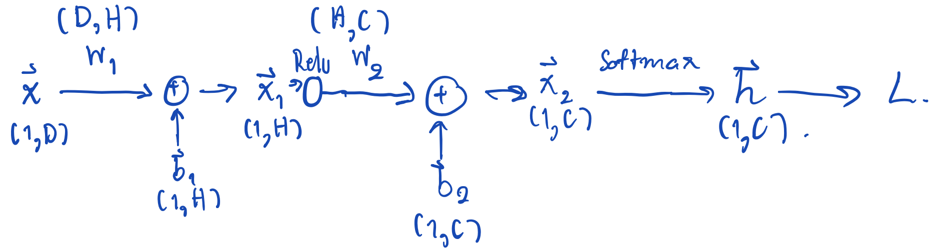
$\frac{\partial x_{2i}}{\partial x_{1j}} = w_{ji} \cdot sign(x_{2,i})$

$\frac{\partial L}{\partial \vec{x}_2} \quad \frac{\partial \vec{X}_2}{\partial \vec{X}_1}$

$1 \times C \times \quad C \times H$

$\frac{\partial \vec{x}_2}{\partial Rule} \quad \frac{\partial Relu}{\partial \vec{x}_1} = Relu'(x_2)$

$\frac{\partial \vec{x}_2}{\partial \vec{x}_1} = \frac{\partial \vec{x}_2}{\partial \vec{y}_1} \frac{\partial \vec{y}_1}{\partial \vec{x}_1}$



$\frac{\partial L}{\partial \vec{x}_2}$: N, 1, C

$\frac{\partial l}{d b_2}$ C, N, 1, 1 $\quad\quad \vec{b}$ (1, C.

$\frac{d l}{d W_2}$

$$\frac{\partial l}{\partial b_1} = \frac{\partial l}{\partial \vec{x}_1} \cdot \frac{\partial \vec{x}_1}{\partial b_1}$$

$$\frac{\partial l}{\partial W_1} = \frac{\partial l}{\partial x_1} \cdot \frac{\partial x_1}{\partial W_1}$$