homework3

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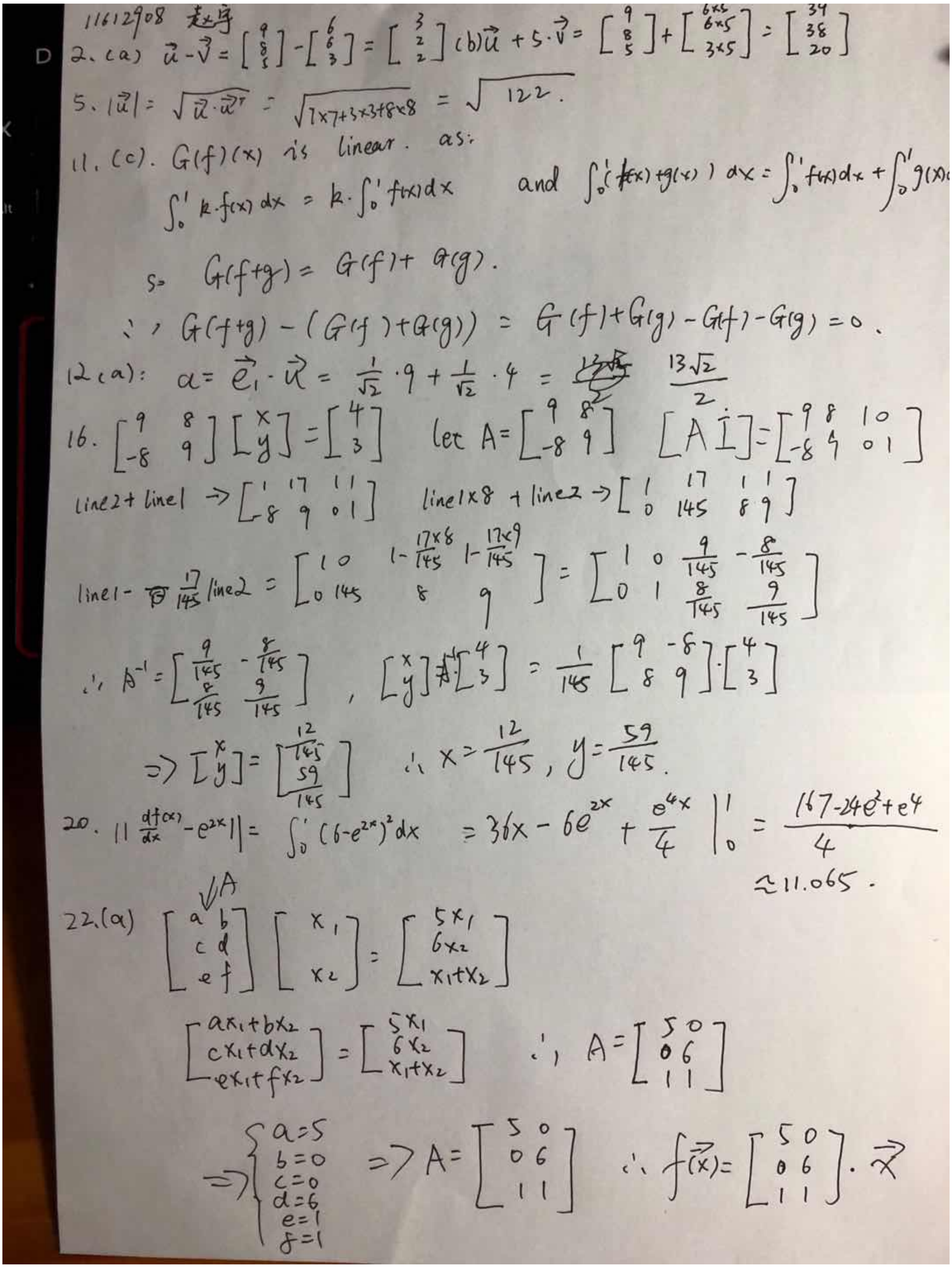
My core mathematical calculations are as follows:



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2.(a) $\vec{u}-\vec{v} = \begin{bmatrix}9\\8\\5\end{bmatrix} - \begin{bmatrix}6\\6\\3\end{bmatrix} = \begin{bmatrix}3\\2\\2\end{bmatrix}$ (b) $\vec{u}+5\cdot\vec{v} = \begin{bmatrix}9\\8\\5\end{bmatrix} + \begin{bmatrix}6\times5\\6\times5\\3\times5\end{bmatrix} = \begin{bmatrix}39\\38\\20\end{bmatrix}$

5. $|\vec{u}| = \sqrt{\vec{u}\cdot\vec{u}^T} = \sqrt{7\times7+3\times3+8\times8} = \sqrt{122}$.

11. (c). $G(f)(x)$ is linear. as:

$\int_0^1 k\cdot f(x)\,dx = k\cdot\int_0^1 f(x)\,dx$ and $\int_0^1 (f(x)+g(x))\,dx = \int_0^1 f(x)\,dx + \int_0^1 g(x)\,d$

So $G(f+g) = G(f) + G(g)$.

$\therefore G(f+g) - (G(f)+G(g)) = G(f)+G(g) - G(f) - G(g) = 0$.

12(a): $a = \vec{e}_1\cdot\vec{u} = \frac{1}{\sqrt2}\cdot 9 + \frac{1}{\sqrt2}\cdot 4 = \frac{13\sqrt2}{2}$

16. $\begin{bmatrix}9 & 8\\-8 & 9\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}4\\3\end{bmatrix}$ let $A = \begin{bmatrix}9 & 8\\-8 & 9\end{bmatrix}$ $[A\ I] = \begin{bmatrix}9 & 8 & 1 & 0\\-8 & 9 & 0 & 1\end{bmatrix}$

line2 + line1 $\to \begin{bmatrix}1 & 17 & 1 & 1\\-8 & 9 & 0 & 1\end{bmatrix}$ line1×8 + line2 $\to \begin{bmatrix}1 & 17 & 1 & 1\\0 & 145 & 8 & 9\end{bmatrix}$

line1 $-\frac{17}{145}$ line2 $= \begin{bmatrix}1 & 0 & 1-\frac{17\times8}{145} & 1-\frac{17\times9}{145}\\0 & 145 & 8 & 9\end{bmatrix} = \begin{bmatrix}1 & 0 & \frac{9}{145} & -\frac{8}{145}\\0 & 1 & \frac{8}{145} & \frac{9}{145}\end{bmatrix}$

$\therefore A^{-1} = \begin{bmatrix}\frac{9}{145} & -\frac{8}{145}\\\frac{8}{145} & \frac{9}{145}\end{bmatrix}$, $\begin{bmatrix}x\\y\end{bmatrix} = A^{-1}\begin{bmatrix}4\\3\end{bmatrix} = \frac{1}{145}\begin{bmatrix}9 & -8\\8 & 9\end{bmatrix}\cdot\begin{bmatrix}4\\3\end{bmatrix}$

$\Rightarrow \begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}\frac{12}{145}\\\frac{59}{145}\end{bmatrix}$ $\therefore x = \frac{12}{145}$, $y = \frac{59}{145}$.

20. $\left\|\frac{df(x)}{dx} - e^{2x}\right\| = \int_0^1 (6-e^{2x})^2\,dx = 36x - 6e^{2x} + \frac{e^{4x}}{4}\Big|_0^1 = \frac{167-24e^2+e^4}{4}$

≈ 11.065.

22.(a) $\overset{A}{\begin{bmatrix}a & b\\c & d\\e & f\end{bmatrix}}\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}5x_1\\6x_2\\x_1+x_2\end{bmatrix}$

$\begin{bmatrix}ax_1+bx_2\\cx_1+dx_2\\ex_1+fx_2\end{bmatrix} = \begin{bmatrix}5x_1\\6x_2\\x_1+x_2\end{bmatrix}$ $\therefore A = \begin{bmatrix}5 & 0\\0 & 6\\1 & 1\end{bmatrix}$

$\Rightarrow \begin{cases}a=5\\b=0\\c=0\\d=6\\e=1\\f=1\end{cases} \Rightarrow A = \begin{bmatrix}5 & 0\\0 & 6\\1 & 1\end{bmatrix}$ $\therefore f(\vec{x}) = \begin{bmatrix}5 & 0\\0 & 6\\1 & 1\end{bmatrix}\cdot\vec{x}$

And my code is easily to understand:

It uses the basic functions in `glm`， and get the same answer as my calculation.

And a method is not included in `glm`, it is called `Adaptive Simpson integral method`.

The code of this algorithm can be found in my github(2 years before), now I provide a python version:

```
EPS = 1e-8


## This is Adaptive Simpson algorithm
class Simpson:

    def __init__(self, fx):
        self.f = fx

    def simpson(self, a, b):
        c = a + (b - a) / 2.0
        return (self.f(a) + 4.0 * self.f(c) + self.f(b)) * (b - a) / 6.0

    def asr(self, a, b, eps, left_a):
        c = a + (b - a) / 2
        l = self.simpson(a, c)
        r = self.simpson(c, b)
        return l + r + (l + r - left_a) / 15. if abs(l + r - left_a) <= 15 * eps
else \
            self.asr(a, c, eps / 2, l) + self.asr(c, b, eps / 2, r)

    def _asr(self, a, b, eps):
        return self.asr(a, b, eps, self.simpson(a, b))

    def solve(self, l, r):
        return self._asr(l, r, EPS)
```

use `solve(left_bound, right_bound)` to get the answer.