MPM Research Community

Anura3D MPM Software

Verification Manual

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Anura3D MPM Software, Verification Manual

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Introduction

Anura 3D is a software product that uses the material point method (MPM) for numerical modelling and simulation of large deformations and soil-water-structure interaction. The software is implemented by the MPM Research Community, which is a collaboration of six partners:

- ◇ **Geotechnical and Environmental Research Group**
  Engineering Department, University of Cambridge, United Kingdom
- ◇ **Soil and Rock Mechanics Research Group**
  Civil Engineering School, Universitat Politècnica de Catalunya, Barcelona, Spain
- ◇ **Institute of Geotechnical Engineering and Construction Management**
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The MPM Research Community has committed to quality control and quality assurance. Therefore, benchmark tests have been introduced to check the correct functioning of the main code as well as the extra features. These benchmark tests are run on a regular basis to control the functioning of the software during the development phase and to verify the results after its release. In this verification manual the benchmark tests are introduced and the performance of Anura 3D is expressed by means of relative errors with reference solutions.

The collection of benchmark tests is subdivided into five separate groups. Each chapter contains one group

- ◇ **Chapter 1 – Benchmark tests from literature (exact solution)**:
  Simple benchmark tests for which an exact analytical result is available from literature.
- ◇ **Chapter 2 – Benchmark tests from literature (approximate solution)**:
  More complex benchmark tests described in literature for which an approximate solution is known.
- ◇ **Chapter 3 – Benchmark tests from spread sheets**:
  Benchmark tests which test program features specific to Anura 3D.
- ◇ **Chapter 4 – Benchmark tests generated by Anura 3D**:
  Benchmark tests for which the reference results are generated using Anura 3D.
- ◇ **Chapter 5 – Benchmark tests compared with other programs**:
  Benchmark tests for which the results of Anura 3D are compared with the results of other programs.

It should be noted that it it is impossible for software developers to prove the correctness of any non-trivial calculation. This verification manual shows to some degree that the software works as it should work. Nevertheless, there will always be combinations of input values that will cause the software to crash or produce wrong results. However, the use of benchmark tests limits the number of times this occurs.

1 Benchmark tests from literature (exact solution)

The benchmark tests in this chapter are compared to the corresponding analytical solutions available from literature.

1.1 Quasi-static oedometer test with dry soil

1.1.1 Description

A column of dry soil is compressed under a constant load of 20 kPa, while lateral movement is prohibited. The column of soil has an initial height of 1.0 m. The solid phase is assumed to be linear-elastic. The material properties are listed in Table 1.1.

Table 1.1: *Material properties of dry soil*

material property	value
initial porosity [-]	0.3
density solid [kg/m^3]	2600
Young's modulus [kPa]	100
Poisson ratio [-]	0.2

The performance of the quasi-static calculation is determined by the tolerated error (0.001), the damping factor (0.75) used with local damping and the Courant number (0.80). The corresponding computational mesh is shown in Figure 1.1, as well as the material point configuration with 4 material points per element.

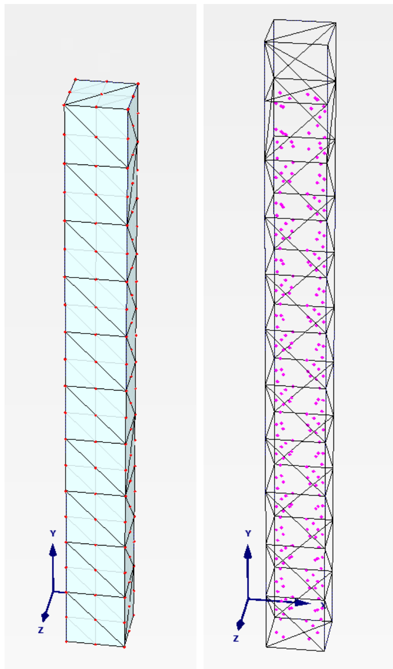

Figure 1.1: *Computational mesh for quasi-static calculations*

1.1.2 Benchmark result

From Malvern [1], page 150-151, it follows that the engineering strain ($e = \Delta L/L_0$) should be considered with FEM calculations, since they are based on the initial configuration. The logarithmic strain ($\epsilon = \log(1 + e)$) should be considered with UL-FEM and MPM, since these calculations use the updated configuration. In addition, Van Langen [2], page 13, gives the linear-elastic stress-strain relations using normal versus objective stress. The equations are summarized in Table 1.2. It should be noted that it is necessary to work with the constrained modulus, given by

$$E^c = \frac{(1-\nu)E}{(1+\nu)(1-2\nu)}.$$

Table 1.2: *Linear-elastic stress-strain relations for dry soil*

	normal stress	objective stress
FEM	$\sigma = E^c e$	$\sigma = E^c(1 - \exp(-e))$
UL-FEM / MPM	$\sigma = E^c \epsilon$	$\sigma = E^c(1 - \exp(-\epsilon))$

1.1.3 Anura3D result

The numerical and analytical value for the total displacement in equilibrium state are compared. The values and the corresponding relative error are presented in Table 1.3.

Table 1.3: *Total displacement* (m) *of node 1 / material point 207*

benchmark number	calculation type	stress type	numerical value	analytical value	relative error
1001	FEM	normal	-0.17992	-0.18000	0.00044
1002	FEM	objective	-0.16548	-0.16551	0.00021
1003	UL-FEM	normal	-0.16458	-0.16473	0.00091
1004	UL-FEM	objective	-0.15245	-0.15254	0.00061
1005	MPM-MP	normal	-0.23108	-0.15790	0.46343
1006	MPM-MP	objective	-0.20830	-0.14622	0.42455
1007	MPM-MIXED	normal	-0.16087	-0.15790	0.01884
1008	MPM-MIXED	objective	-0.15120	-0.14622	0.03410

It should be noted that the large relative error of benchmark number 1005 and 1006 is the result of grid crossing. The cause of the grid crossing error is explained in Appendix A, as well as the fact that the grid crossing error is significantly reduced by switching from MPM-MP to MPM-MIXED.

The influence of the tolerated error and the damping factor is taken into account by deviating from the default values. The results are presented in Table 1.4.

Table 1.4: *Total displacement* (m) *of material point 207; MPM-MIXED with normal stress*

benchmark number	tolerated error	damping factor	numerical value	analytical value	relative error
1009	0.0001	0.75	-0.16325	-0.15790	0.03390
1010	0.0100	0.75	-0.15925	-0.15790	0.00853
1011	0.0010	0.55	-0.16262	-0.15790	0.02986
1012	0.0010	0.95	-0.16302	-0.15790	0.03242

1.2 Quasi-static oedometer test with saturated soil

1.2.1 Description

A column of saturated soil is compressed under a constant load of –20 kPa acting on both phases, while lateral movement is prohibited. The column of soil has an initial height of 1.0 m. The solid phase is assumed to be linear-elastic. The material properties of the solid and liquid phase are given in Table 1.5.

Table 1.5: *Material properties of saturated soil*

material property	value	material property	value
initial porosity [-]	0.35	intrinsic permeability [m^2]	$1.0 \cdot 10^{-9}$
density solid [kg/m^3]	2650	density liquid [kg/m^3]	1000
Young modulus [kPa]	50	bulk modulus liquid [kPa]	50
Poisson ratio [-]	0.3	dynamic viscosity liquid [kPa/s]	$1.0 \cdot 10^{-6}$

The performance of the quasi-static calculation is determined by the tolerated error (0.001), the damping factor (0.75) used with local damping and the Courant number (0.80). The computational mesh is equal to the computational mesh of the quasi-static oedometer test with dry soil in Section 1.1.

1.2.2 Benchmark result

In case of an undrained situation, as is the case in this benchmark, the load is proportionally carried by the soil and the water. Verruijt [3], page 96, gives the analytical value for the effective stress in equilibrium state, being

$$\sigma' = \frac{E^c}{E^c + K_w/n}\sigma.$$

It should be noted that with objective stress the stiffness increase must be taken into account. The value for effective stress can be obtained from

$$\sigma' = \frac{E^c - \sigma'}{E^c - \sigma' + K_w/n}\sigma,$$

being the negative root

$$\sigma' = \frac{1}{2}(E^c + K_w/n + \sigma) - \frac{1}{2}\sqrt{(E^c + K_w/n + \sigma)^2 - 4E^c\sigma}.$$

The stress-strain relations in Table 1.2, following from Malvern [1] and Van Langen [2], also hold for saturated soil when the total stress is replaced by the effective stress. The new stress-strain relations are presented in Table 1.6.

Table 1.6: *Linear-elastic stress-strain relations for saturated soil*

	normal stress	objective stress
FEM	$\sigma' = E^c e$	$\sigma' = E^c(1 - \exp(-e))$
UL-FEM / MPM	$\sigma' = E^c \epsilon$	$\sigma' = E^c(1 - \exp(-\epsilon))$

1.2.3 Anura3D result

The numerical and analytical value for the total displacement in equilibrium state are compared. The values and the corresponding relative error are presented in Table 1.7.

Table 1.7: *Total displacement* (m) *of node 1 / material point 207*

benchmark number	calculation type	stress type	numerical value	analytical value	relative error
1051	FEM	normal	-0.09512	-0.09516	0.00043
1052	FEM	objective	-0.09368	-0.09090	0.03050
1053	UL-FEM	normal	-0.09072	-0.09078	0.00064
1054	UL-FEM	objective	-0.08940	-0.08689	0.02886
1055	MPM-MP	normal	-0.10304	-0.08701	0.18425
1056	MPM-MP	objective	-0.10300	-0.08329	0.23665
1057	MPM-MIXED	normal	-0.08536	-0.08701	0.01904
1058	MPM-MIXED	objective	-0.08436	-0.08329	0.01278

It should be noted that the large relative error of benchmark number 1055 and 1056 is the result of grid crossing. The cause of the grid crossing error is explained in Appendix A, as well as the fact that the grid crossing error is significantly reduced by switching from MPM-MP to MPM-MIXED.

The influence of the Courant number is taken into account by deviating from the default value. The results are presented in Table 1.8.

Table 1.8: *Total displacement* (m) *of material point 207; MPM-MIXED with normal stress*

benchmark number	Courant number	numerical value	analytical value	relative error
1059	0.65	-0.08532	-0.08701	0.01940
1060	0.95	-0.08523	-0.08701	0.02047

1.3 K0-initialisation with dry soil

1.3.1 Description

A cubic box with ribs of 5.0 m is filled with 75 m^3 of dry soil, such that the top surface is at 3.0 m. The dry soil, with material properties in table 1.9, is at rest under a compressive load of p = –2.0 kPa and/or a gravitational force with g = 9.81 m/s^2. The lateral stresses are determined by the value K0 = 0.2.

Table 1.9: *Material properties of dry soil*

material property	value
initial porosity [-]	0.5
density solid [kg/m^3]	1000
Young's modulus [kPa]	10
Poisson ratio [-]	0.1

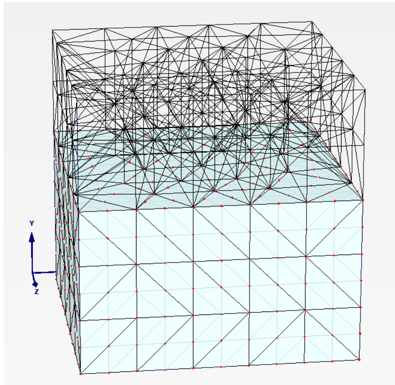

Figure 1.2: *Computational mesh for K0-initialisation*

A coarse semi-structured mesh with tetrahedral elements of 1.0 m is used, see figure 1.2. MPM-MIXED is used as computational method with 4 material points per element.

1.3.2 Benchmark result

Verruijt [4], page 29, gives the analytical value for the total stress profile. With load q and gravitational acceleration g, the stress at height y is given by

$$\sigma_{yy}(y) = q - (1 - n)\rho g(y_{top} - y).$$

By definition it follows that

$$\sigma_{xx}(y) = \sigma_{zz}(y) = K_0\sigma_{yy}(y)$$

1.3.3 Anura3D result

The numerical and analytical value for the initial stress are compared for element 40. The material points 157 to 160 all have the same value due to the low-order basis functions. This value is based on the position of the single Gauss point at y = 1.75 m. The results and the corresponding relative error are presented in Table 1.10.

Table 1.10: *Stress values (*kPa*) of element 40*

benchmark number	load q	gravitational acceleration g	numerical value	analytical value	relative error
3051	-2.0	0.0	σ_{xx} = -0.40000	-0.40000	0.000
			σ_{yy} = -2.00000	-2.00000	0.000
			σ_{zz} = -0.40000	-0.40000	0.000
3052	0.0	9.81	σ_{xx} = -1.22625	-1.22625	0.000
			σ_{yy} = -6.13125	-6.13125	0.000
			σ_{zz} = -1.22625	-1.22625	0.000
3053	-2.0	9.81	σ_{xx} = -1.62625	-1.62625	0.000
			σ_{yy} = -8.13125	-8.13125	0.000
			σ_{zz} = -1.62625	-1.62625	0.000

1.4 K0-initialisation with saturated soil

1.4.1 Description

A cubic box with ribs of 5.0 m is filled with 75 m^3 of saturated soil, such that the top surface is at 3.0 m. The saturated soil, with material properties in table 1.11, is at rest under a compressive load of q = –2.0 kPa and/or a gravitational force with g = 9.81 m/s^2. It should be noted that the load is fully carried by the solid phase, while the gravity affects both phases. The lateral stresses are determined by the value K0 = 0.1.

Table 1.11: *Material properties of dry soil*

material property	value	material property	value
initial porosity [-]	0.5	intrinsic permeability [m^2]	$1.0 \cdot 10^{-9}$
density solid [kg/m^3]	1000	density liquid [kg/m^3]	250
Young modulus [kPa]	10	bulk modulus liquid [kPa]	100
Poisson ratio [-]	0.1	dynamic viscosity liquid [kPa/s]	$1.0 \cdot 10^{-6}$

The mesh from figure 1.2 is reused with this benchmark. MPM-MIXED is used as computational method with 4 material points per element.

1.4.2 Benchmark result

Verruijt [3], page 96, and Verruijt [4], page 29, give the analytical value for the effective stress and pore water pressure profile. With load q and gravitational acceleration g, the effective stress at height y is given by:

$$\sigma'_{yy}(y) = q - (1 - n)(\rho_s - \rho_w)g(y_{top} - y).$$

By definition it follows that

$$\sigma'_{xx}(y) = \sigma'_{zz}(y) = K_0\sigma'_{yy}(y).$$

The pore water pressure at height y is given by:

$$p(y) = -\rho_w g(y_{top} - y).$$

1.4.3 Anura3D result

The numerical and analytical value for the initial stress are compared for element 40. The material points 157 to 160 all have the same value due to the lower order basis functions. This value is based on the position of the single Gauss point at y = 1.75 m. The results and the corresponding relative error are presented in Table 1.12.

Table 1.12: Stress values (kPa) of element 40

benchmark number	load p	gravity g	numerical value	analytical value	relative error
3054	-2.0	0.0	σ_{xx} = -0.20000	-0.20000	0.000
			σ_{yy} = -2.00000	-2.00000	0.000
			σ_{zz} = -0.20000	-0.20000	0.000
			p = -0.00000	0.00000	0.000
3055	0.0	9.81	σ_{xx} = -0.45984	-0.45984	0.000
			σ_{yy} = -4.59844	-4.59844	0.000
			σ_{zz} = -0.45984	-0.45984	0.000
			p = -3.06563	-3.06563	0.000
3056	-2.0	9.81	σ_{xx} = -0.65984	-0.65984	0.000
			σ_{yy} = -6.59844	-6.59844	0.000
			σ_{zz} = -0.65984	-0.65984	0.000
			p = -3.06563	-3.06563	0.000

1.5 Quasistatic gravity test with dry soil

1.5.1 Description

A column of dry soil is compressed under influence of gravity with a gravitational acceleration of g = 9.81 m/s, while lateral movement is prohibited. The column will reach equilibrium in 1.0 s due to local damping. The column of soil has an initial height of H = 1.0 m. The solid phase is assumed to be linear-elastic. The material properties of the solid phase are given in Table 1.13.

Table 1.13: Material properties of dry soil

material property	value
initial porosity [-]	0.0
density solid [kg/m^3]	2000
Young modulus [kPa]	10000
Poisson ratio [-]	0.33

The computational mesh consists of 240 elements filled with 4 material points and 6 empty elements, as shown in Figure 1.3. The Courant number (0.98) is kept constant and the damping factor is set to 0.75. In two follow-up calculations the displacements are reset to zero. In one case the gravitational force is maintained such that the column stays in equilibrium. In the other case the gravitational force is removed such that the column expands to its original height.

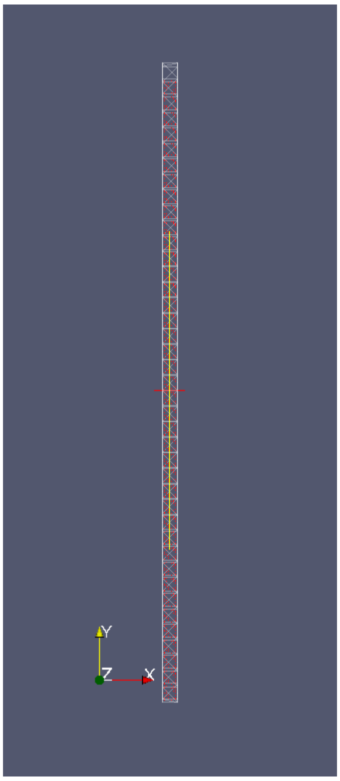

Figure 1.3: *Computational mesh for gravity tests*

1.5.2 Benchmark result

Mieremet [5] analytically solved the equilibrium equation based on the original configuration. In this benchmark we are interested in the analytical solution of the equilibrium equation based on the updated configuration, which correspondingly becomes

$$\bar{u}_{eq}(y) = \frac{1}{2}\frac{\rho g}{E^c}y^2 - \frac{\rho g H}{E^c}y.$$

For this benchmark, the equilibrium solution is rewritten in terms of the original configuration

$$u_{eq}(y_0) = \bar{u}_{eq}(y_0 + u_{eq}(y_0)) = \frac{1}{2}\frac{\rho g}{E^c}(y_0 + u_{eq}(y_0))^2 - \frac{\rho g H}{E^c}(y_0 + u_{eq}(y_0)).$$

The negative root of this quadratic equation gives the equilibrium solution in terms of the original configuration.

1.5.3 Anura3D result

The calculations are performed with the computational method MPM-MIXED with the use of normal stress. The numerical and approximate value for the displacement are compared at material point 33 (y_0 = 0.94309 m) at t = 1.0 s, and at t = 2.0 s in case of a follow-up calculation. The values and the corresponding relative error are presented in Table 1.14.

Table 1.14: *Displacement* (mm) *of material point 33*

benchmark number	time	numerical value	analytical value	relative error
3062	1.0	-0.65939	-0.65991	0.00079
3063	2.0	0.00005	0.00000	
3064	2.0	0.65947	0.65991	0.00067

1.6 Dynamic gravity test with saturated soil

1.6.1 Description

A column of saturated soil is compressed under influence of gravity with a gravitational acceleration of g = 9.81 m/s, while lateral movement is prohibited. The column will reach equilibrium in 2.0 s due to damping caused by the relative movement of the solid and water phase. The column of soil has an initial height of H = 1.0 m. The solid phase is assumed to be linear-elastic. The material properties of both phases are given in Table 1.15.

Table 1.15: *Material properties of saturated soil*

material property	value	material property	value
initial porosity [-]	0.25	intrinsic permeability [m^2]	$1.0 \cdot 10^{-9}$
density solid [kg/m^3]	2650	density liquid [kg/m^3]	1000
Young modulus [kPa]	10000	bulk modulus liquid [kPa]	10000
Poisson ratio [-]	0.33	dynamic viscosity liquid [kPa/s]	$1.0 \cdot 10^{-6}$

The computational mesh consists of 240 elements filled with 4 material points and 6 empty elements, which is equal to the mesh for the quasistatic gravity test with dry soil as shown in Figure 1.3. The Courant number (0.98) is kept constant.

1.6.2 Benchmark result

The equilibrium displacement of this benchmark test depends on the submerged density $\rho' = \rho_{sat} - \rho_w$ since the water level is equal to the soil level or higher [**?**]. Therefore, it is possible to use the analytical equilibrium solution from Section 1.5 with replacement of the solid density by the submerged density. This solution, in terms of the original configuration, is the negative root of the following quadratic equation:

$$u_{eq}(y_0) = \bar{u}_{eq}(y_0 + u_{eq}(y_0)) = \frac{1}{2}\frac{\rho' g}{E^c}(y_0 + u_{eq}(y_0))^2 - \frac{\rho' g H}{E^c}(y_0 + u_{eq}(y_0)).$$

1.6.3 Anura3D result

The calculations are performed with the computational method MPM-MIXED with the use of normal stress. The numerical and approximate value for the displacement are compared at material point 33 (y_0 = 0.94309 m) at t = 2.0 s. The values and the corresponding relative error are presented in Table 1.16.

Table 1.16: *Displacement (*mm*) of material point 33*

benchmark number	time	numerical value	analytical value	relative error
3065	2.0	-0.40787	-0.40833	0.00113

1.7 Quasistatic gravity test with saturated soil

1.7.1 Description

A column of saturated soil is compressed under influence of gravity with a gravitational acceleration of g = 9.81 m/s, while lateral movement is prohibited. The column will reach equilibrium in 1.0 s due to local damping. The column of soil has an initial height of H = 1.0 m. The solid phase is assumed to be linear-elastic. The material properties of both phases are given in Table 1.17.

Table 1.17: *Material properties of saturated soil*

material property	value	material property	value
initial porosity [-]	0.25	intrinsic permeability [m^2]	$1.0 \cdot 10^{-9}$
density solid [kg/m^3]	2650	density liquid [kg/m^3]	1000
Young modulus [kPa]	10000	bulk modulus liquid [kPa]	10000
Poisson ratio [-]	0.33	dynamic viscosity liquid [kPa/s]	$1.0 \cdot 10^{-6}$

The computational mesh consists of 240 elements filled with 4 material points and 6 empty elements, which is equal to the mesh for the quasistatic gravity test with dry soil as shown in Figure 1.3. The Courant number (0.98) is kept constant and the damping factor is set to 0.75. In two follow-up calculations the displacements are reset to zero. In one case the gravitational force is maintained such that the column stays in equilibrium. In the other case the gravitational force is removed such that the column expands to its original height.

1.7.2 Benchmark result

The equilibrium displacement of this benchmark test depends on the submerged density $\rho' = \rho_{sat} - \rho_w$ since the water level is equal to the soil level or higher [**?**]. Therefore, it is possible to use the analytical equilibrium solution from Section 1.5 with replacement of the solid density by the submerged density. This solution, in terms of the original configuration, is the negative root of the following quadratic equation:

$$u_{eq}(y_0) = \bar{u}_{eq}(y_0 + u_{eq}(y_0)) = \frac{1}{2}\frac{\rho' g}{E^c}(y_0 + u_{eq}(y_0))^2 - \frac{\rho' g H}{E^c}(y_0 + u_{eq}(y_0)).$$

1.7.3 Anura3D result

The calculations are performed with the computational method MPM-MIXED with the use of normal stress. The numerical and approximate value for the displacement are compared at material point 33 (y_0 = 0.94309 m) at t = 1.0 s, and at t = 2.0 s in case of a follow-up calculation. The values and the corresponding relative error are presented in Table 1.18.

Table 1.18: *Displacement* (mm) *of material point 33*

benchmark number	time	numerical value	analytical value	relative error
3066	1.0	-0.40787	-0.40833	0.00113
3067	2.0	0.00023	0.00000	
3068	2.0	0.40765	0.40833	0.00167

1.8 Quasistatic implicit consolidation test

1.8.1 Description

A column of saturated soil is compressed under a constant surface traction of 5 kPa (total stress). The vertical boundaries are fixed for horizontal displacement and closed for flow. The bottom horizontal boundary is fixed for vertical displacement. The prescribed pore pressure at the top and bottom horizontal boundary equals zero. The column of soil has an initial height of 1 m and the solid phase is assumed to be linear elastic. For this problem an unstructured mesh of 0.025 m was used with 1 element in the out-of plane direction. Figures 1.4 and 1.5 show the problem geometry, boundary conditions and mesh. In Figure 1.4 the blue lines refer to liquid fixities and the black lines refer to solid fixities. Table 1.19 lists the material properties.

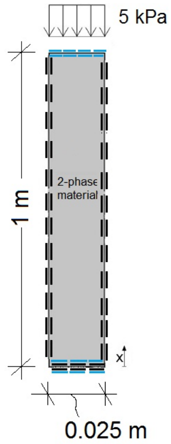

Figure 1.4: *Problem Sketch*

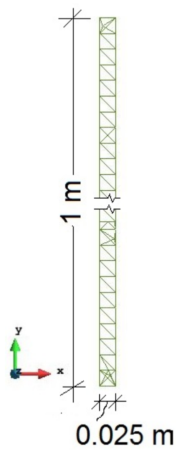

Figure 1.5: *Problem Mesh*

Table 1.19: *Material properties of saturated soil*

material property	value	material property	value
initial porosity [-]	0.33	intrinsic permeability [m^2]	$1.157 * 10^{-17}$
density solid [kg/m^3]	2650	density liquid [kg/m^3]	1000
Young modulus [kPa]	10000	bulk modulus liquid [kPa]	$2 * 10^6$
Poisson ratio [-]	0.0	dynamic viscosity liquid [kPa*s]	$1 * 10^{-6}$

The calculation time is $2.59 * 10^6$ seconds. The loading is immediately applied in 1 load step for a total of 250 time steps. The implicit quasistatic integration scheme [**?**] was used for the calculation. The computation method is MPM-MIXED.

1.8.2 Benchmark Result

Terzaghi's one dimensional consolidation theory provides the following analytical solution for double-draining soil layers:

$$p(y,t) = \sum_{n=0}^{\infty} \frac{2p_0}{\pi n}(1-\cos n\pi)\sin(\frac{n\pi y}{2h})\exp(\frac{-n^2\pi^2}{4h^2}c_v t)$$

Where t is the time, y the position along the height of the column, p_0 the applied load, h the total height and c_v is a function of κ, the darcy permeability, $gamma_l$ or the unit weight of the liquid phase, and coefficients α and γ. These coefficients are represented by:

$$\alpha = \frac{1}{K + 4G/3}, \beta = 1/K_l$$

Where G, K, K_l are the shear modulus of soil skeleton, bulk modulus of the soil skeleton amd bulk modulus of the liquid phase, respectively. The darcy permeability is represented by $\kappa = k\gamma_l/\mu$, where k is the intrinsic permeability, and μ is the viscosity of the liquid.

The equation for c_v following [**?**] is given by:

$$c_v = \frac{k}{\gamma_l(\alpha + n * \beta)}$$

Where n is the porosity. For double drainage the drainage length h must equal half the height of the column. Therefore the boundary conditions here are $p(0, t) = 0, p(2h, t) = 0$ and the initial condition is $p(y, 0) = p_0$.

The analytical solution above originates from:

$$\frac{\partial p}{\partial t} = c_v \frac{\partial^2 p}{\partial y^2}$$

Which is a diffusion equation. For the boundary conditions provided it starts as a step function at $t = 0$ equalling the total stress (pore water takes the entire stress at beginning of calculation). As time approaches infinity the liquid gradually drains and transfers the stresses to the soil skeleton. At very large times most of the pore pressure has dissipated and the soil skeleton carries the entire load.

1.8.3 Anura3D result

Results of the calculation are shown in figure 1.6 with an overlay of Anura3D and the analytical results for several time steps and height values. The oscillations that appear in the Anura3D-MPM result are inherent to the MPM formulation. Table 1.20 shows an error breakdown for the benchmark and Figure 1.7 . High error values at the top and bottom occur due to the oscillations associated with MPM-mixed computation method.

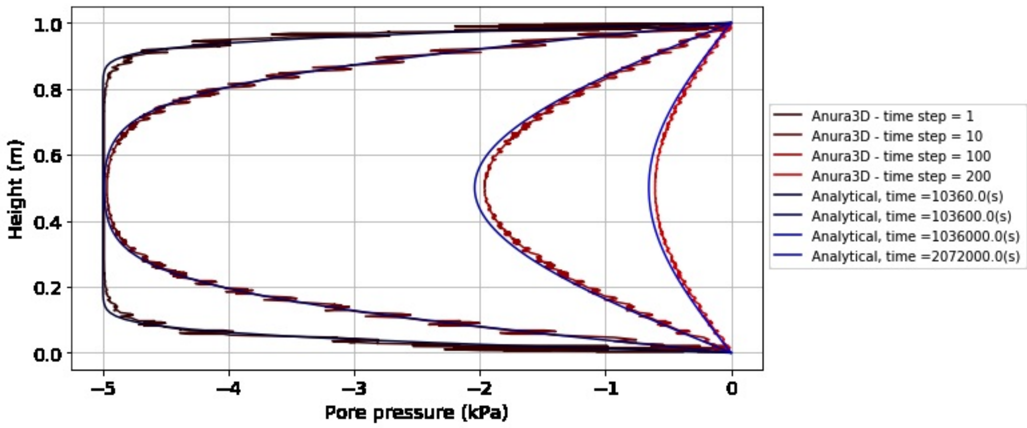

Figure 1.6: *Problem Results*

Table 1.20: *Error (in percent) for selected material points (Benchmark number 1002)*

MPID	Height(m)	time step 1	time step 10	time step 100	time step 200
909	0.9965	24.4	7.48	46.9	144
71	0.8534	1.28	0.022	4.22	8.68
1226	0.7069	0.18	1.92	6.5	10.28
427	0.6443	0.16	0.8	4.11	7.85
745	0.5305	0.16	0.45	3.93	7.57
898	0.4319	0.16	0.38	3.22	6.86
258	0.3309	0.16	0.31	2.53	6.17
1260	0.2333	0.22	1.48	0.23	3.92
1202	0.0965	1.32	1.35	3.92	7.30
367	0.00355	27.3	1.26	3.51	5.71

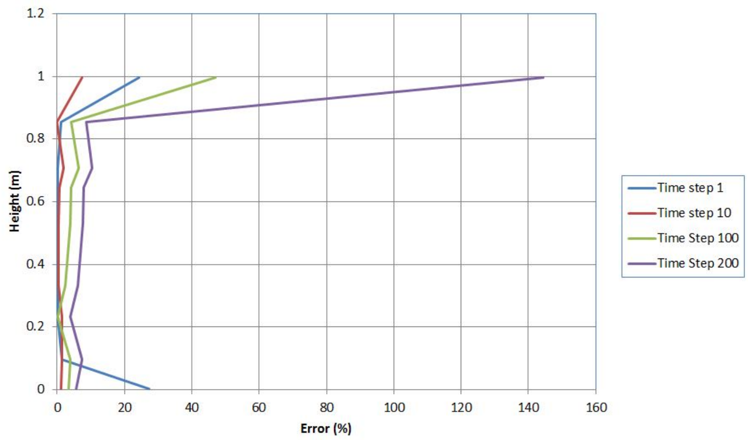

Figure 1.7: *Error plot*

1.9 Free-falling mass

1.9.1 Description

A mass is dropped from a height of 100 meters down to a soil layer below. The mass is assumed to be rigid. No external loading is applied and gravity is present throughout the analysis. Figure 1.8 shows an exaggerated view of the problem geometry and boundary conditions. Figure 1.9 shows a portion of the computational mesh.

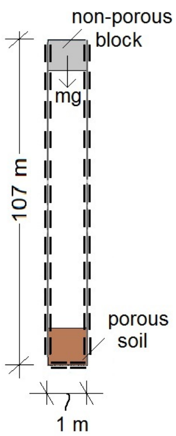

Figure 1.8: *Problem Sketch*

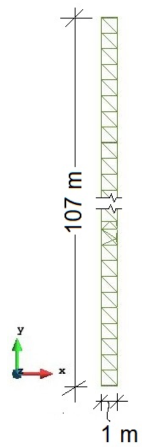

Figure 1.9: *Problem Mesh*

The model consists of a fully structured mesh with tetrahedral elements of 1m in length. Furthermore empty elements are provided between the block and soil materials to allow the block to fall down the column. The active elements contain 4 material points per element

Both materials are linear elastic, their parameters are provided in table 1.21

Table 1.21: *Material properties*

material property	value (soil)	value (block)
Material Type	1-phase solid	1-phase solid
Initial Porosity	0.3	0.0
Density (kg/m^2)	2650	3000
k0-value	0.5	0.5
Material Model	Linear Elasticity	Linear Elasticity
Young Modulus	10000	100000
Poisson Ratio	0.3	0.3

1.9.2 Results

The analytical solution for the displacement of the block arises from the energy balance

$$E_{potential} = E_{kinetic} \rightarrow gh = \frac{1}{2}mv^2$$

Therefore, $v = gt$ and since $v = \frac{du}{dt}$ then the displacement $u = \frac{1}{2}gt^2$

The Anura3D analysis comprised of 100 loadsteps of 0.04 seconds. Key calculation parameters in Anura3D include:

- Setting gravity multiplier in vertical direction equal to 1
- Applying no damping factor
- Ensuring that the gravity vector in vertical direction is active

Figures 1.10 and 1.11 compare Anura3D results to the analytical result. The maximum root mean square value for this benchmark equals 0.037.

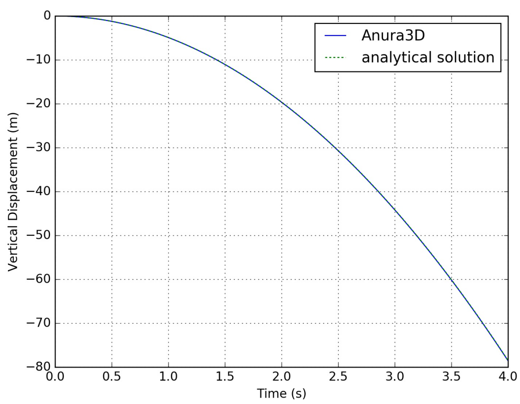

Figure 1.10: *Problem Sketch*

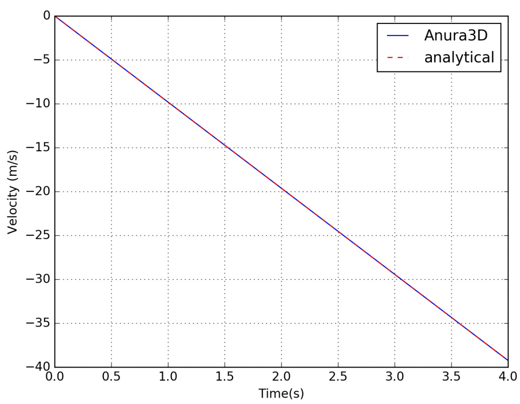

Figure 1.11: *Problem Mesh*

2 Benchmark tests from literature (approximate solution)

The benchmark tests in this chapter are compared to the corresponding approximate solutions available from literature.

2.1 Dynamic oedometer test with dry soil

2.1.1 Description

A column of dry soil is compressed under a constant load of 1 kPa, while lateral movement is prohibited. The column of soil has an initial height of 1.0 m. The solid phase is assumed to be linear-elastic. The material properties are listed in Table 2.1.

Table 2.1: *Material properties of dry soil*

material property	value
initial porosity [-]	0.0
density solid [kg/m^3]	2000
Young's modulus [kPa]	180
Poisson ratio [-]	0.2

The performance of the dynamic calculation is determined by the number of material points per element (4) and the Courant number (0.75). The corresponding computational mesh is shown in Figure 2.1, as well as the material point configuration with 4 material points per element.

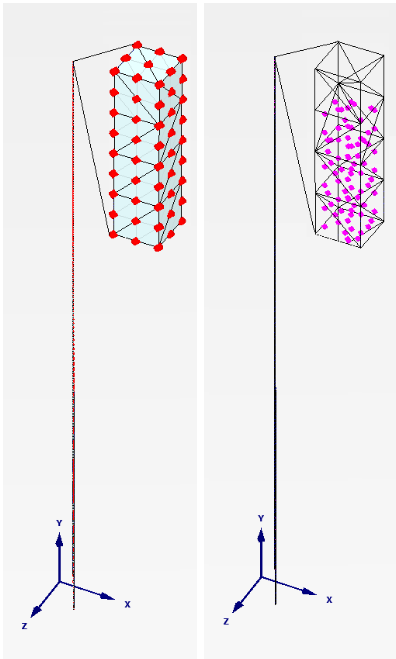

Figure 2.1: *Computational mesh for dynamic calculations*

2.1.2 Benchmark result

Mieremet [5] derived the analytical solution of this benchmark for FEM calculations using normal stress. Since only small deformations are considered, it also approximates the solution for UL-FEM and MPM calculations using objective stress. The analytical solution shows a shock wave traveling with speed $\sqrt{E^c/\rho}$ through the column of dry soil, such that the displacement of the top of the column equals

$$u(H, t_1) = \frac{1}{2}\frac{\sigma}{E^c}H \qquad \text{and} \qquad u(H, t_2) = \frac{\sigma}{E^c}H$$

at time

$$t_1 = \frac{1}{2}\frac{H}{\sqrt{E^c/\rho}} \qquad \text{and} \qquad t_2 = \frac{H}{\sqrt{E^c/\rho}}.$$

2.1.3 Anura3D result

The numerical and approximate value for the total displacement are compared at t = 0.1 s. The values and the corresponding relative error are presented in Table 2.2.

Table 2.2: *Total displacement (*mm*) of node 21 / material point 1 at* t = 0.1 s

benchmark number	calculation type	stress type	numerical value	approximate value	relative error
2001	FEM	normal	-5.00076	-5.00000	0.00015
2002	FEM	objective	-4.99595	-5.00000	0.00081
2003	UL-FEM	normal	-4.99145	-5.00000	0.00171
2004	UL-FEM	objective	-4.98679	-5.00000	0.00264
2005	MPM-MP	normal	-6.87724	-4.99931	0.37564
2006	MPM-MP	objective	-6.68822	-4.99931	0.33783
2007	MPM-MIXED	normal	-4.98581	-4.99931	0.00270
2008	MPM-MIXED	objective	-4.98207	-4.99931	0.00345

It should be noted that the large relative error of benchmark number 2005 and 2006 is the result of grid crossing. The cause of the grid crossing error is explained in Appendix A, as well as the fact that the grid crossing error is significantly reduced by switching from MPM-MP to MPM-MIXED.

The influence of the number of material points per element is taken into account by deviating from the default value. The results are presented in Table 2.3. Note that the total displacement is compared at t = 0.05 s.

Table 2.3: *Total displacement (*mm*) of material point 1 at* t = 0.05 s*; MPM-MIXED with normal stress*

benchmark number	number of material points	numerical value	analytical value	relative error
2009	1	-2.48974	-2.49875	0.00361
2007	4	-2.50045	-2.49931	0.00046
2010	7	-2.49480	-2.49931	0.00180
2011	8	-2.49399	-2.49938	0.00215
2012	10	-2.50187	-2.49931	0.00102
2013	13	-2.50331	-2.49931	0.00160
2014	20	-4.24731	-2.49963	0.69918

2.2 Dynamic oedometer test with saturated soil

2.2.1 Description

A column of saturated soil is compressed under a constant load of –2 kPa acting on both phases, while lateral movement is prohibited. The column of soil has an initial height of 0.5 m. The solid phase is assumed to be linear-elastic. The material properties of the solid and liquid phase are given in Table 2.4.

Table 2.4: *Material properties of saturated soil*

material property	value	material property	value
initial porosity [-]	0.1	intrinsic permeability [m^2]	$1.0 \cdot 10^{-9}$
density solid [kg/m^3]	2222	density liquid [kg/m^3]	1000
Young modulus [kPa]	180	bulk modulus liquid [kPa]	20
Poisson ratio [-]	0.2	dynamic viscosity liquid [kPa/s]	$1.0 \cdot 10^{-6}$

The performance of the dynamic calculation is determined by the number of material points per element (4) and the Courant number (0.75). The computational mesh and the material point representation are shown in Figure 2.2.

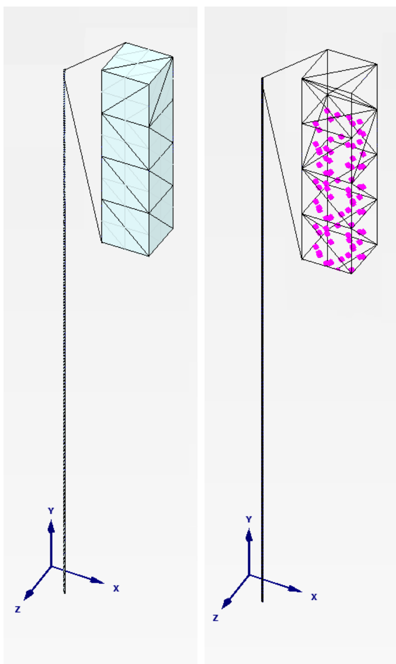

Figure 2.2: *Computational mesh for dynamic calculations*

2.2.2 Benchmark result

Verruijt [3], page 108, gives the analytical solution for the response of a long column to a block wave. The numerical solution of this benchmark for FEM calculations using normal stress can be compared with the analytical solution. Since only small deformations are considered, it also approximates the solution for UL-FEM and MPM calculations using objective stress. The analytical solution shows an undrained wave traveling with speed

$$c_1 = \sqrt{\frac{E^c + K_w/n}{\rho_{sat}}}$$

and a damped wave traveling with speed

$$c_2 = \sqrt{\frac{nE^c}{nE^c + (1-n)K_w}}\sqrt{\frac{K_w}{\rho_w}}.$$

2.2.3 Anura3D result

The numerical and approximate value for the effective stress are compared at the Gauss point of element 301 (x = 0.40025 m) or at material point 1197 (x = 0.40014) after 0.04 s. At this moment, both the undrained wave and the damped wave have past the considered position. The values and the corresponding relative error are presented in Table 2.5.

Table 2.5: *Effective stress (kPa) of element 301/material point 1197 at* t = 0.04 s

benchmark number	calculation type	stress type	numerical value	approximate value	relative error
2051	FEM	normal	-0.72497	-0.73352	0.01164
2052	FEM	objective	-0.72736	-0.73352	0.00838
2053	UL-FEM	normal	-0.72971	-0.73352	0.00519
2054	UL-FEM	objective	-0.72728	-0.73352	0.00849
2055	MPM-MP	normal	-0.75503	-0.73396	0.02165
2056	MPM-MP	objective	-0.27006	-0.73396	0.63205
2057	MPM-MIXED	normal	-0.70260	-0.73396	0.04273
2058	MPM-MIXED	objective	-0.69446	-0.73396	0.05382

It should be noted that the large relative error of benchmark number 2057 is the result of grid crossing. The cause of the grid crossing error is explained in Appendix A, as well as the fact that the grid crossing error is significantly reduced by switching from MPM-MP to MPM-MIXED.

2.3 Consolidation test

2.3.1 Description

A column of saturated soil is compressed under a constant load of –1 kPa, while lateral movement is prohibited. Initially, the compressive load is fully carried by the water phase, such that the initial pore pressure equals –1 kPa. The water can flow out at the top of the column from the start of the calculation. The column of soil has an initial height of 1.0 m. The solid phase is assumed to be linear-elastic. The material properties of the solid and liquid phase are given in Table 4.3.

Table 2.6: *Material properties of saturated soil*

material property	value
initial porosity [-]	0.3
density solid [kg/m^3]	2650
Young modulus [kPa]	10000
Poisson ratio [-]	0.0
intrinsic permeability [m^2]	$1.0 \cdot 10^{-10}$ (high)
	$1.0 \cdot 10^{-12}$ (low)
density liquid [kg/m^3]	1000
bulk modulus liquid [kPa]	75000
dynamic viscosity liquid [kPa/s]	$1.0 \cdot 10^{-6}$

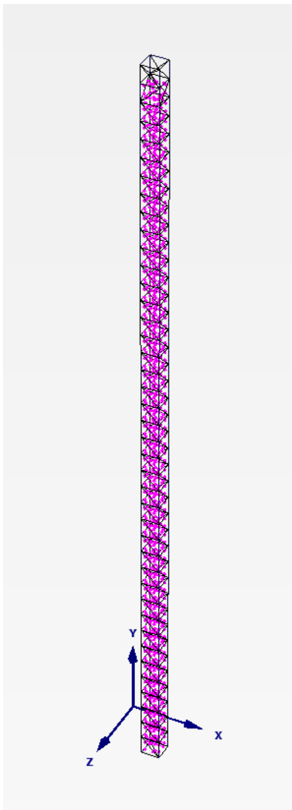

Figure 2.3: *Computational mesh for consolidation calculations*

The computational mesh, with 4 material points per element, is shown in Figure 2.3. Strain smoothing, local damping (α = 0.05) and/or mass scaling (β = 100) is considered in some of the benchmarks. The Courant number (0.98) is kept constant.

2.3.2 Benchmark result

During the consolidation process, the water flows out of the column and the load is transferred to the solid phase. The dissipation of the pore pressure, and consequently the increase of the effective stress, is captured in Terzaghi's one-dimensional consolidation theory [**?**]. The analytical solution to the corresponding equation is exact for FEM calculations, but can be used as an approximate solution for MPM calculations as well, when small deformations are considered. The analytical solution is given in terms of the excess pore pressure and is plotted in Figure 4.9.

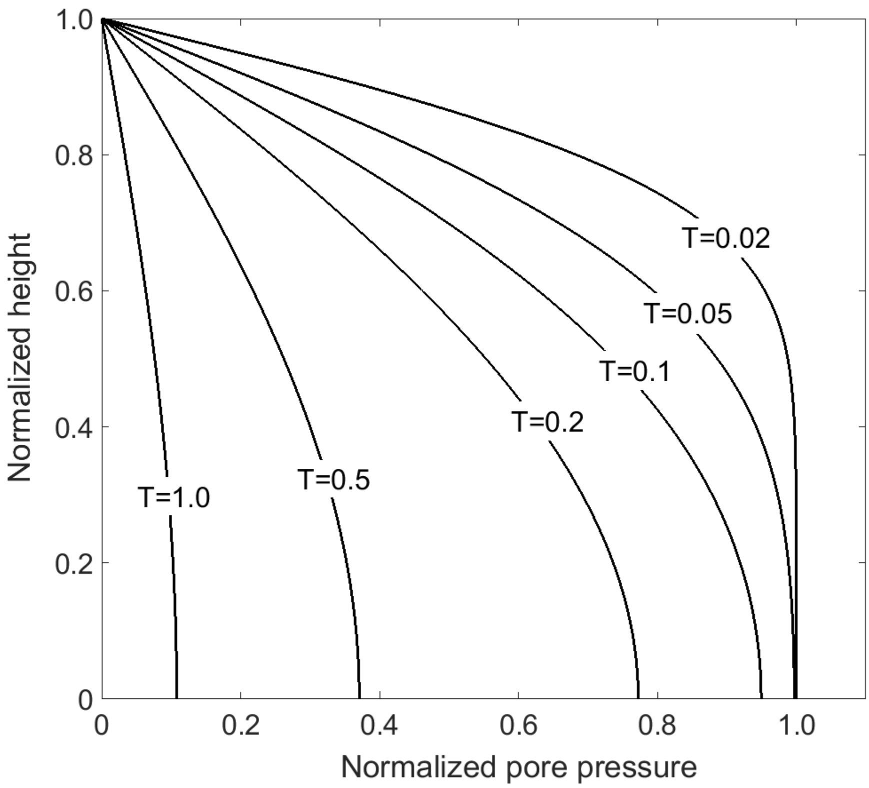

Figure 2.4: *Terzaghi's analytical solution for consolidation*

It should be noted that the time is normalized. The non-dimensional time factor is defined as

$$T = \frac{c_v t}{h^2}.$$

Here, h is the drainage length and c_v is the consolidation coefficient. The latter is given by

$$c_v = \frac{k}{\rho_w g(1/E^c)}.$$

2.3.3 Anura3D result

All calculations are performed with the computational method MPM-MP with the use of normal stress. The numerical and approximate value for the pore pressure are compared at material point 350 (y = 0.25345) at multiple times. Strain smoothing and local damping are considered in case of high permeability, while strain smoothing and mass scaling are used to obtain the results in case of low permeability. The values and the corresponding relative error are presented in Table 2.7.

Table 2.7: *Pore pressure (kPa) of material point 350*

benchmark number	calculation specifications	normalized time	numerical value	approximate value	relative error
3031	high permeability	T = 0.02	-1.11444	-0.99981	0.11465
		T = 0.05	-0.95501	-0.98169	0.02718
		T = 0.10	-0.95459	-0.89988	0.06080
		T = 0.20	-0.72753	-0.71468	0.01798
		T = 0.50	-0.35927	-0.34178	0.05117
		T = 1.00	-0.11002	-0.09953	0.10539
3032	high permeability strain smoothing	T = 0.02	-1.06159	-0.99981	0.06179
		T = 0.05	-0.95111	-0.98169	0.03115
		T = 0.10	-0.94065	-0.89988	0.04531
		T = 0.20	-0.72508	-0.71468	0.01455
		T = 0.50	-0.36053	-0.34178	0.05486
		T = 1.00	-0.11032	-0.09953	0.10841
3033	high permeability strain smoothing local damping	T = 0.02	-1.01383	-0.99981	0.01402
		T = 0.05	-0.99187	-0.98169	0.01037
		T = 0.10	-0.92459	-0.89988	0.02746
		T = 0.20	-0.75283	-0.71468	0.05338
		T = 0.50	-0.38307	-0.34178	0.12081
		T = 1.00	-0.12423	-0.09953	0.24817
3034	low permeability strain smoothing mass scaling	T = 0.02	-1.00081	-0.99981	0.00100
		T = 0.05	-0.99121	-0.98169	0.00970
		T = 0.10	-0.91921	-0.89988	0.02148
		T = 0.20	-0.73468	-0.71468	0.02798
		T = 0.50	-0.36107	-0.34178	0.05644
		T = 1.00	-0.11038	-0.09953	0.10901

2.4 Dynamic gravity test with dry soil

2.4.1 Description

A column of dry soil is compressed under influence of gravity with a gravitational acceleration of 9.81 m/s, while lateral movement is prohibited. The column of soil has an initial height of 1.0 m. The solid phase is assumed to be linear-elastic. The material properties of the solid phase are given in Table 2.8.

Table 2.8: *Material properties of dry soil*

material property	value
initial porosity [-]	0.0
density solid [kg/m^3]	2000
Young modulus [kPa]	10000
Poisson ratio [-]	0.33

The computational mesh consists of 240 elements with 4 material points and 6 empty elements on top, as shown in Figure 2.5. The Courant number (0.98) is kept constant.

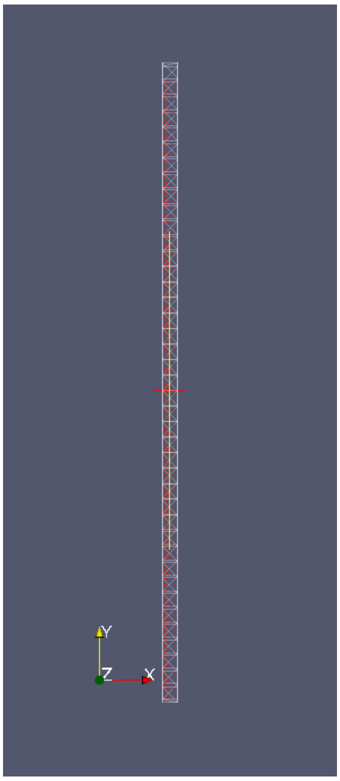

Figure 2.5: *Computational mesh for gravity test*

2.4.2 Benchmark result

Mieremet [5] derived the analytical solution of this benchmark for FEM calculations using normal stress. Since only small deformations are considered, it also approximates the solution for MPM calculations. The analytical solution shows oscillating behaviour arount an equilibrium displacement of

$$u_{eq}(y_0) = \frac{1}{2}\frac{\rho g}{E^c}y_0^2 - \frac{\rho g H}{E^c}y_0.$$

Here, H is the initial height of the column and y_0 is the initial vertical coordinate.

2.4.3 Anura3D result

The calculations are performed with the computational method MPM-MIXED with the use of normal stress. The numerical and approximate value for the displacement are compared at material point 33 (y_0 = 0.94309 m) every 0.2 s. The values and the corresponding relative error are presented in Table 2.9.

Table 2.9: *Displacement (*mm*) of material point 165*

benchmark number	time	numerical value	approximate value	relative error
3061	0.2	-0.92028	-0.91109	0.01009
	0.4	-1.17730	-1.19839	0.01760
	0.6	-0.05055	-0.08456	0.40220
	0.8	-0.52934	-0.48610	0.08895
	1.0	-1.30146	-1.31657	0.01148

3 Benchmark tests compared with spreadsheets

Currently, no benchmark tests exist that are compared with spreadsheets.

4 Benchmark tests generated by Anura 3D

Currently, no benchmark tests exist that are compared with reference results generated using Anura3D.

4.1 Application of pore pressure traction

4.1.1 Description

A pressure traction of -10kPa is applied at the top surface. Load is applied at the NODES. Material type is assumed to be 2-phase linear elastic. 4 material points per element are used and the mesh counts 30 tetrahedral element.

Table 4.1: *Material properties of saturated soil*

material property	value
initial porosity [-]	0.4
density solid [kg/m^3]	2650
Young modulus [kPa]	1000
Poisson ratio [-]	0.3
intrinsic permeability [m^2]	$1.0 \cdot 10^{-10}$
density liquid [kg/m^3]	1000
bulk modulus liquid [kPa]	20000
dynamic viscosity liquid [kPa/s]	$1.0 \cdot 10^{-6}$

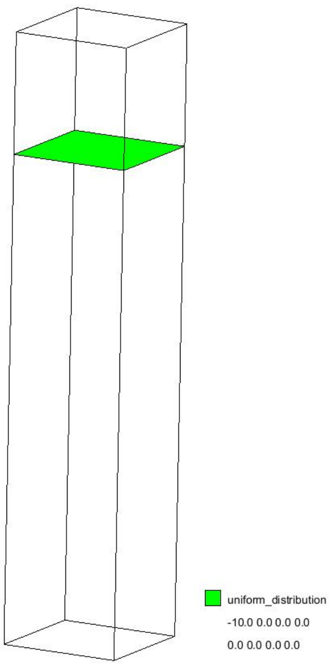

Figure 4.1: *Pressure traction*

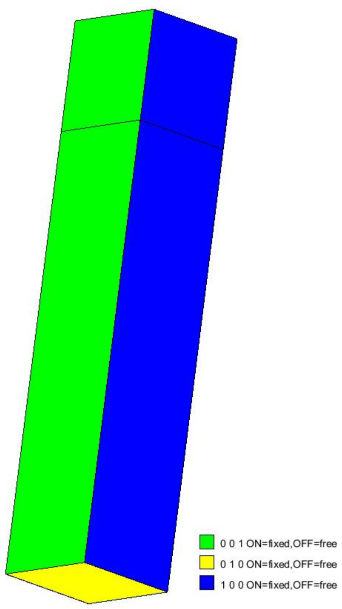

Figure 4.2: *Solid Fixities*

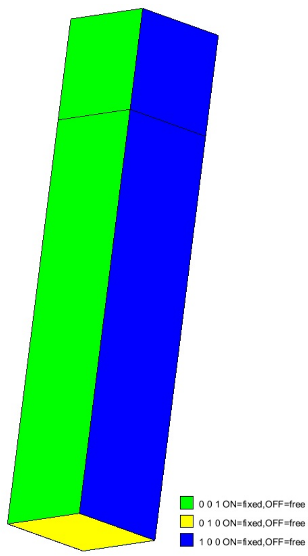

Figure 4.3: *Liquid Fixities*

4.1.2 Anura3D results

Gravity loading is initialized using quasi-static convergence and a damping factor of 0.75. The loading phase then follows with4 additional load steps. Table 4.4 summarizes the key parameter changes between the gravity initialization phase and the subsequent dynamic loading phases.

Table 4.2: *Calculation Parameters for Pore Pressure Traction Benchmarks*

Calculation Parameter	Initialization	Dynamic Phase
Apply Quasistatic Convergence	True	False
Liquid Load Multiplier	0.0	1.0
Damping Factor	0.75	0.0
Total Time	-	2.0 s

The results of the analysis are shown in figures 4.5 and **??** The pore pressure increases with time as expected.

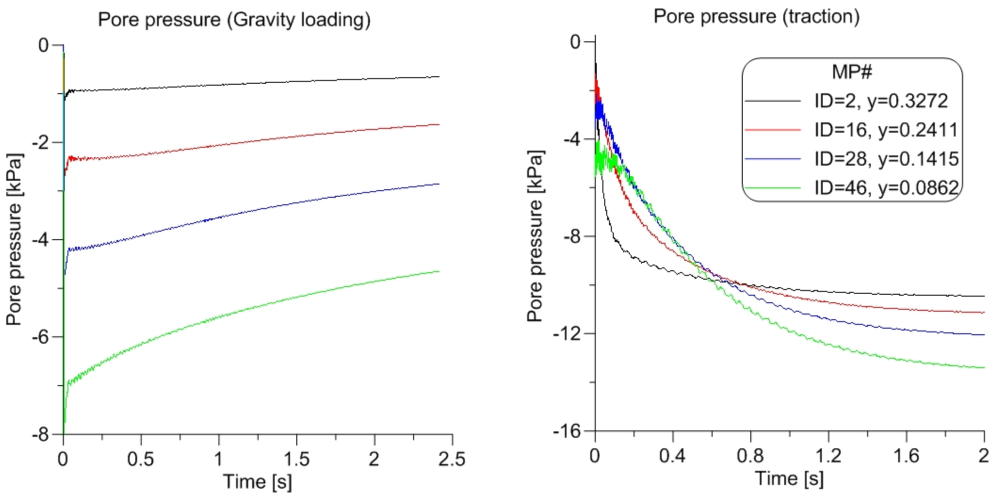

Figure 4.4: *Pore pressure results: Gravity loading (left) and constant loading (right)*

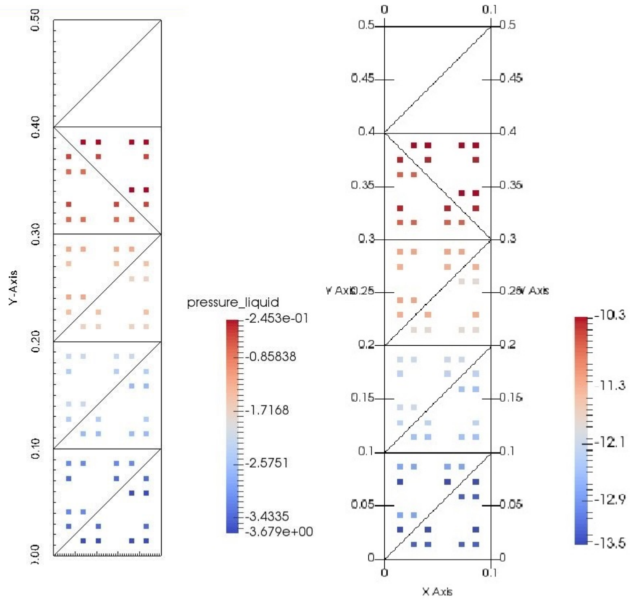

Figure 4.5: *Pore pressure results: Gravity loading hydrostatic distribution (left) and constant loading (right)*

4.2 Collapse of a dry soil column by removal of fixities

4.2.1 Description

In the first loadstep the initial stresses are generated using gravity loading. At the right side of the column are horizontal fixities. At the second loadstep horizontal fixities at the right side of the column are removed. The column collapses.

The soil is dry and modeled as Mohr-Coulomb material, 1-phase-1-point formulation. MPM-MIXED with strain smoothing is used. First loadstep damping of 75

Column is 1x1x0.1m, whole domain is 3x1.1.0.1m. Mesh size is 0.1m structured. 1980 elements of which 600 are initially active. 4 material points per element. The material properties are given below:

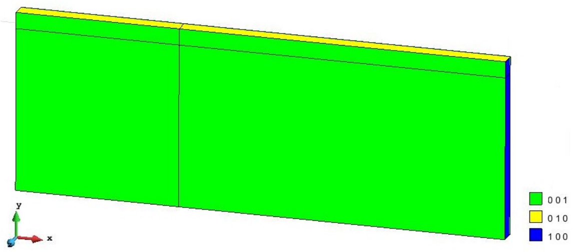

Figure 4.6: *General boundary conditions*

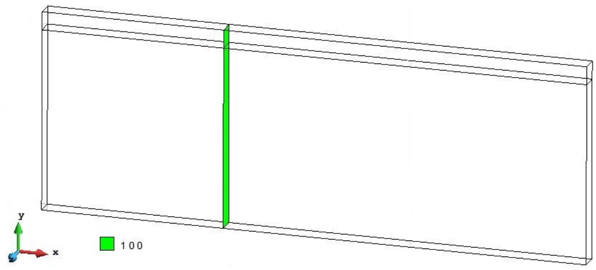

Figure 4.7: *Removal fixity boundary conditions*

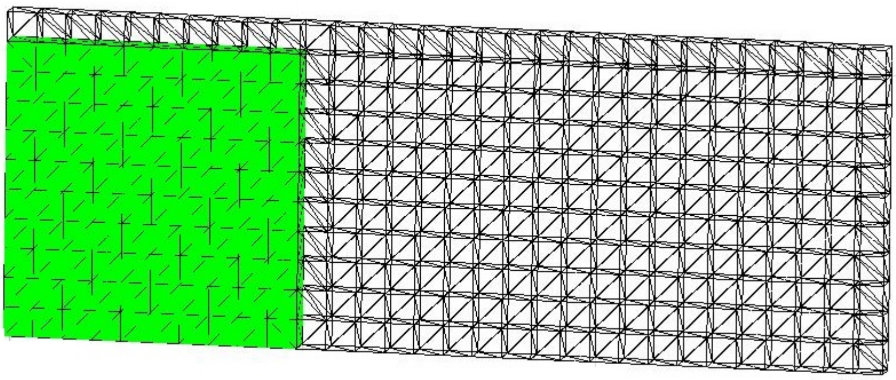

Figure 4.8: *Mesh and Material assignment*

Table 4.3: *Material properties of saturated soil*

material property	value
initial porosity [-]	0.4
density solid [kg/m^3]	2650
Young modulus [kPa]	1000
Poisson ratio [-]	0.3
Effective Cohesion [kPa]	0.0
Effective Friction Angle	30.0
Effective Dilatancy Angle	0.0
Tensile Strength	0.0

4.2.2 Anura3D results

Gravity loading is initialized using quasi-static convergence and a damping factor of 0.75. The loading phase then follows with4 additional load steps. Table 4.4 summarizes the key parameter changes between the gravity initialization phase and the subsequent dynamic loading phases.

Table 4.4: *Calculation Parameters for Pore Pressure Traction Benchmarks*

Calculation Parameter	Initialization	Dynamic Phase
Number of Load Steps	1	10
Apply Quasistatic Convergence	True	False
Apply Strain Smoothing	True	True
Damping Factor	0.75	0.05
Total Time	Convergence Criterium	0.05 s/step

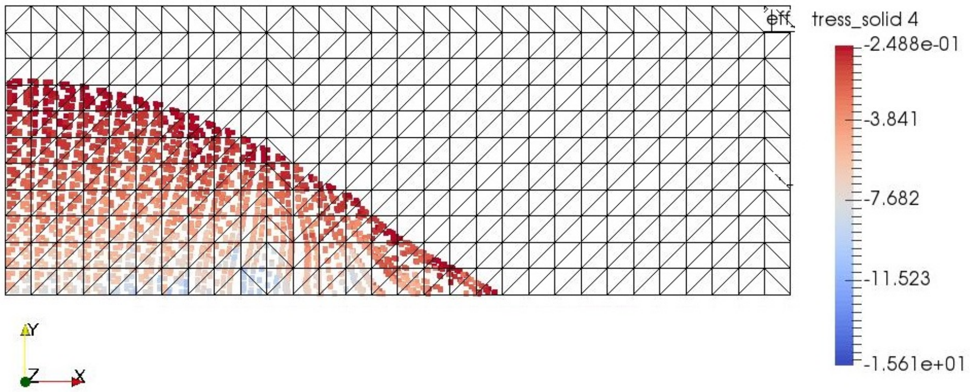

Figure 4.9: *Effective stress results: Remove Fixities benchmark*

5 Benchmark tests compared with other numerical programs

The benchmark tests in this chapter are compared with the results of other numerical programs.

5.1 Triaxial tests using linear-elastic model

5.1.1 Description

A soil cube is compressed with a constant velocity of 0.01 m/s. Gravity is neglected, such that the stress is independent of depth. Boundary conditions are typical for a triaxial test. The bottom surface is constrained in normal direction. On the lateral surfaces a compressive load of 50 kPa is applied, normal to the surface. The soil is a linear-elastic 1-phase material using the parameters listed in Table 5.1. It should be noted that two situations are considered, one with zero Poisson ratio (no lateral expansion) and one with nonzero Poisson ratio (lateral expansion).

Table 5.1: *Material properties of the soil cube*

material property	value
initial porosity [-]	0.0
density solid [kg/m^3]	2600
Young's modulus [kPa]	10000
Poisson ratio [-]	0.0 / 0.25

The soil cube is discretized by 6 tetrahedral elements, see Figure 5.1. The calculation consists of 50 load steps of 0.2 s, such that the total time equals 10.0 s.

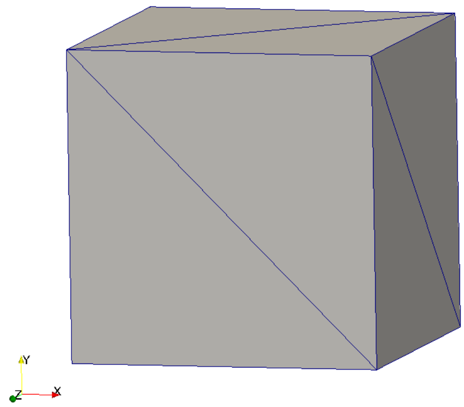

Figure 5.1: *Computational mesh for triaxial tests*

The calculation is performed with FEM in order to focus on the material model. With the K0 procedure the initial stresses are set to –50 kPa. The compression is realized by a prescribed velocity of 0.01 m/s on the top nodes. Finally, the Courant number is set to 0.50 and no additional smoothing is added.

5.1.2 Benchmark result

It is possible to perform a triaxial test with Plaxis 3D (Version 2013.1) in the soil test environment. The material properties are set to the values in Table 5.1 and a drained compressive triaxial test with $|\sigma_3|$ = 50 kPa and $|\varepsilon_{1,\max}|$ = 10% is run.

The result is a vertical stress that is linearly decreasing from –50 kPa to –1050 kPa. The volumetric strain equals 10% when the zero Poisson ratio is considered and 5% when the nonzero Poisson ratio is considered. Graphs are plot in the next subsection.

5.1.3 Anura3D result

In this section, the Plaxis 3D and Anura3D output are compared. The effective stress and the volumetric strain are plot as a function of the vertical strain in Figure 5.2 and 5.3.

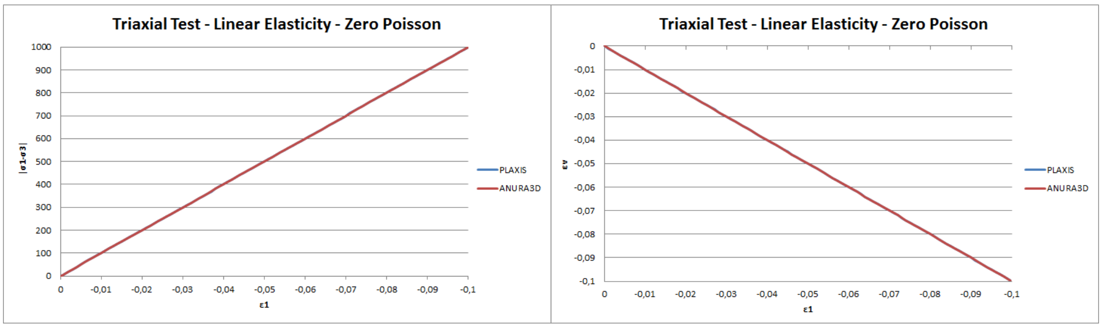

Figure 5.2: *Triaxial test of linear-elastic 1-phase material with zero Poisson ratio (benchmark number 3011)*

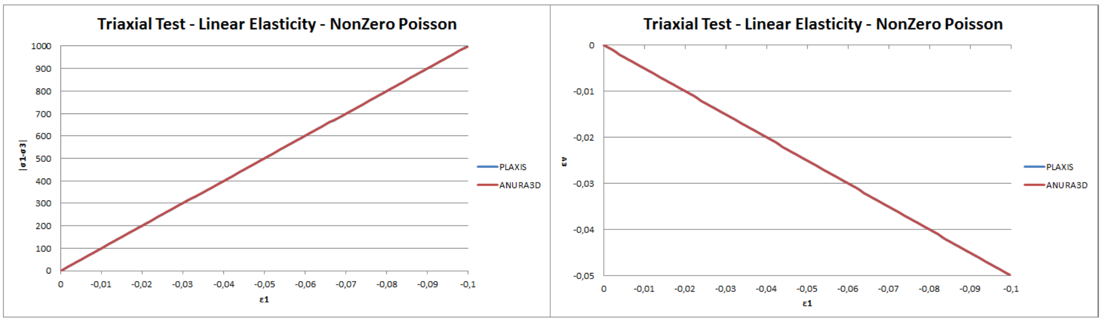

Figure 5.3: *Triaxial test of linear-elastic 1-phase material with nonzero Poisson ratio (benchmark number 3012)*

The final values of the effective stress are presented in Table 5.2, as well as the corresponding relative error. It should be noted that σ_1 represents the vertical stress and σ_2 and σ_3 represent the horizontal stresses.

Table 5.2: *Effective stress (*kPa*) of element 1*

benchmark number	calculation type	Poisson ratio	numerical value	analytical value	relative error
3011	FEM	0.0	$\sigma_1 = -1050.00$	$\sigma_1 = -1050.00$	0.00000
			$\sigma_2 = -50.00$	$\sigma_2 = -50.00$	0.00000
			$\sigma_3 = -50.00$	$\sigma_3 = -50.00$	0.00000
3012	FEM	0.25	$\sigma_1 = -1050.00$	$\sigma_1 = -1050.00$	0.00000
			$\sigma_2 = -50.08$	$\sigma_2 = -50.00$	0.00160
			$\sigma_3 = -49.92$	$\sigma_3 = -50.00$	0.00160

5.2 Triaxial tests using Modified Cam-Clay model

5.2.1 Description

A soil cube is compressed with a constant velocity of 0.01 m/s. Gravity is neglected, such that the stress is independent of depth. Boundary conditions are typical for a triaxial test. The bottom surface is constrained in normal direction. On the lateral surfaces a compressive load of 100 kPa is applied, normal to the surface. The soil is a 2-phase material following the Modified Cam-Clay model. Its parameters are listed in Table 5.3. Three situations are considered: a drained triaxial test of normally consolidated soil, an undrained triaxial test of normally consolidated soil and an undrained triaxial test of overconsolidated soil.

Table 5.3: *Material properties of the soil cube*

material property	value
initial porosity [-]	0.655
density solid [kg/m^3]	2600
Poisson ratio [-]	0.2
slope of the critical state line on p-q-plane [-]	1.2
slope of the recompression line on e-log(p')-plane [-]	0.06
slope of the critical state line on e-log(p')-plane [-]	0.3
initial void ratio [-]	1.9
overconsolidation ratio [-]	1.0 / 3.0
density liquid [kg/m^3]	1000
bulk modulus [kPa]	65000
intrinsic permeability [m^2]	$1.0 \cdot 10^{-6}$
dynamic viscosity [kPa/s]	$1.0 \cdot 10^{-4}$

The soil cube is discretized by 6 tetrahedral elements, see Figure 5.1 in Section 5.1. The calculation consists of 50 load steps of 0.2 s, such that the total time equals 10.0 s.

The calculation is performed with FEM in order to focus on the material model. With the K0 procedure the initial stresses are set to –100 kPa. The compression is realized by a prescribed velocity of 0.01 m/s on the top nodes. The Courant number is set to 0.50 and no additional smoothing is added. It should be noted that the drained situation is captured by a 1-phase calculation, such that the water phase is neglected, and the undrained situation is captured by a 1-phase calculation with effective stress analysis.

5.2.2 Benchmark result

It is possible to perform a triaxial test with Plaxis 3D (Version 2013.1) in the soil test environment. After choosing the Modified Cam-Clay model, the material properties are set to the values in Table 5.3. In addition, the interface material properties are set to c_{ref} = 0.001, ϕ = 89 and ψ = 89. It should be noted that these values only influence the calculation of the equivalent isotropic preconsolidation stress in case of overconsolidated soil and are chosen in such a way that $c_{ref} \cot \psi \approx 0$.

First, the drained triaxial test with normally consolidated soil is run with $|\sigma_3|$ = 100 kPa, $|\varepsilon_{1,max}|$ = 10% and |vertical precons. stress| = 0 kPa. Second, the undrained triaxial test with normally consolidated soil is run with the same input parameters.

The third test is the undrained triaxial test with overconsolidated soil. Here, the input parameters are $|\sigma_3|$ = 100 kPa, $|\varepsilon_{1,max}|$ = 10% and |vertical precons. stress| = 316.6 kPa. The reason for the latter value is the fact that Plaxis 3D calculates K_0^{nc} and the equivalent isotropic preconsolidation stress with a different equation than Anura3D does. Anura3D uses an equivalent isotropic preconsolidation stress of 278.1kPa for OCR = 3.0, while Plaxis 3D uses an equivalent isotropic preconsolidation stress of 278.1 kPa when |vertical precons. stress| = 316.6 kPa.

5.2.3 Anura3D result

In this section, the Plaxis 3D and Anura3D output are compared. The effective stress and the volumetric strain are plot as a function of the vertical strain in Figure 5.4, 5.5 and 5.6.

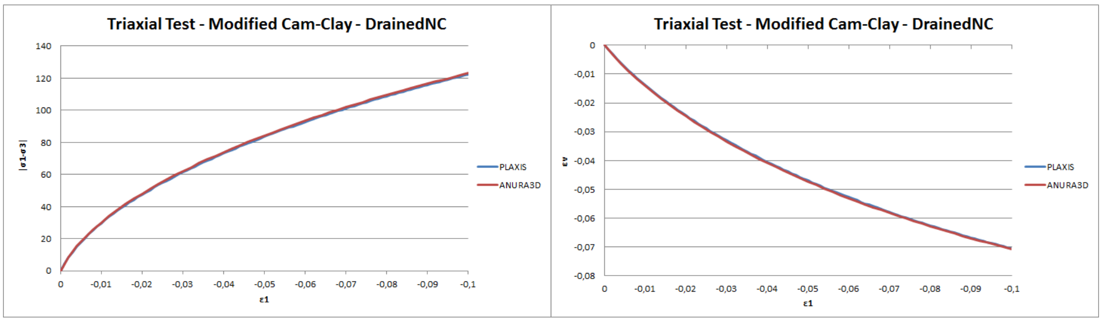

Figure 5.4: *Drained triaxial test of normally consolidated soil using Modified Cam-Clay (benchmark number 3013)*

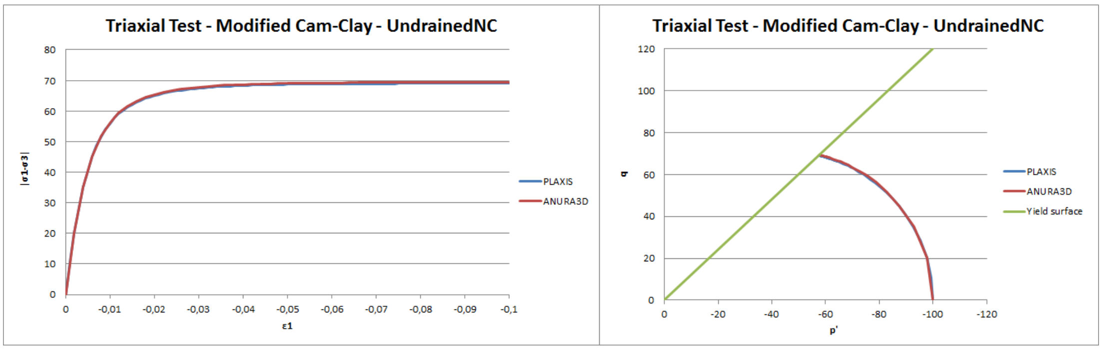

Figure 5.5: *Undrained triaxial test of normally consolidated soil using Modified Cam-Clay (benchmark number 3014)*

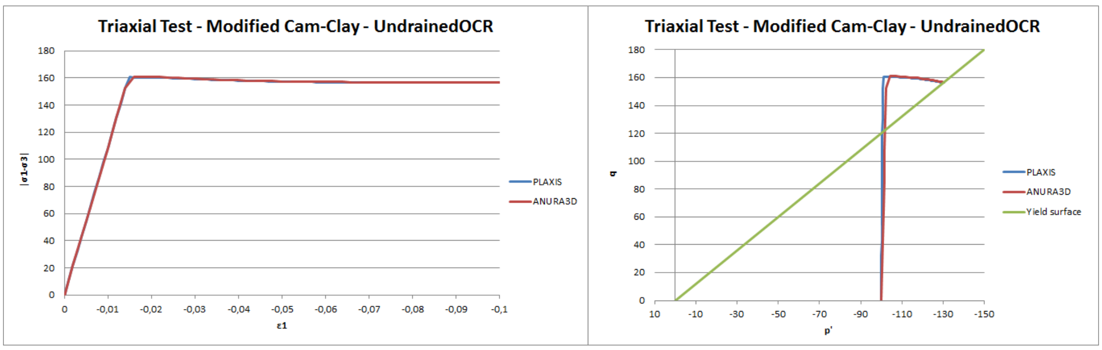

Figure 5.6: *Undrained triaxial test of overconsolidated soil using Modified Cam-Clay (benchmark number 3015)*

The final values of the effective stress are presented in Table 5.4, as well as the corresponding relative error. It should be noted that σ_1 represents the vertical stress and σ_2 and σ_3 represent the horizontal stresses.

Table 5.4: *Effective stress (*kPa*) of element 1*

benchmark number	calculation type	overconsolidation ratio	numerical value	analytical value	relative error
3013	FEM	1.0	$\sigma_1 = -223.08$	$\sigma_1 = -222.30$	0.00350
			$\sigma_2 = -100.00$	$\sigma_2 = -100.00$	0.00000
			$\sigma_3 = -100.00$	$\sigma_3 = -100.00$	0.00000
3014	FEM	1.0	$\sigma_1 = -104.06$	$\sigma_1 = -103.50$	0.00541
			$\sigma_2 = -34.72$	$\sigma_2 = -34.53$	0.00550
			$\sigma_3 = -34.71$	$\sigma_3 = -34.53$	0.00521
3015	FEM	3.0	$\sigma_1 = -234.76$	$\sigma_1 = -234.40$	0.00154
			$\sigma_2 = -78.17$	$\sigma_2 = -78.03$	0.00179
			$\sigma_3 = -78.17$	$\sigma_3 = -78.03$	0.00179

5.3 Triaxial tests using Mohr-Coulomb model

5.3.1 Description

A dry soil cube is compressed with a constant velocity of 0.01 m/s. Gravity is neglected, such that the stress is independent of depth. Boundary conditions are typical for a triaxial test. The bottom surface is constrained in normal direction. On the lateral surfaces a compressive load of 50 kPa is applied, normal to the surface. The soil is a 1-phase material following the Mohr-Coulomb model. Its parameters are listed in Table 5.5. Two situations are considered: loose soil and dense soil.

Table 5.5: *Material properties of the soil cube*

material property	value (loose)	value (dense)
initial porosity [-]	0.45	0.15
density solid [kg/m^3]	2600	2600
Young's modulus [kPa]	10000	10000
Poisson ratio [-]	0.15	0.15
cohesion [kPa]	5	3
friction angle [°]	35	47
dilatancy angle [°]	0	14

The soil cube is discretized by 6 tetrahedral elements, see Figure 5.1 in Section 5.1. The calculation consists of 50 load steps of 0.2 s, such that the total time equals 10.0 s.

The calculation is performed with FEM in order to focus on the material model. With the K0 procedure the initial stresses are set to –50 kPa. The compression is realized by a prescribed velocity of 0.01 m/s on the top nodes. The Courant number is set to 0.50 and no additional smoothing is added.

5.3.2 Benchmark result

It is possible to perform a triaxial test with Plaxis 3D (Version 2013.1) in the soil test environment. The material properties are set to the values in Table 5.5 and a drained compressive triaxial test with $|\sigma_3|$ = 50 kPa and $|\varepsilon_{1,\max}|$ = 10% is run.

The result is a vertical stress that is linearly decreasing until the yield surface is reached. From that moment, the vertical stress stays constant, as only plastic deformation occurs. The evolution of the volumetric strain is determined by the Poisson ratio until the yield surface is reached. In case of loose soil, the volumetric strain does not change anymore from that moment (zero dilatancy angle), while the volumetric strain starts increasing in case of dense soil (positive dilatancy angle). Graphs are plot in the next subsection.

5.3.3 Anura3D result

In this section, the Plaxis 3D and Anura3D output are compared. The effective stress and the volumetric strain are plot as a function of the vertical strain in Figure 5.7 and 5.8.

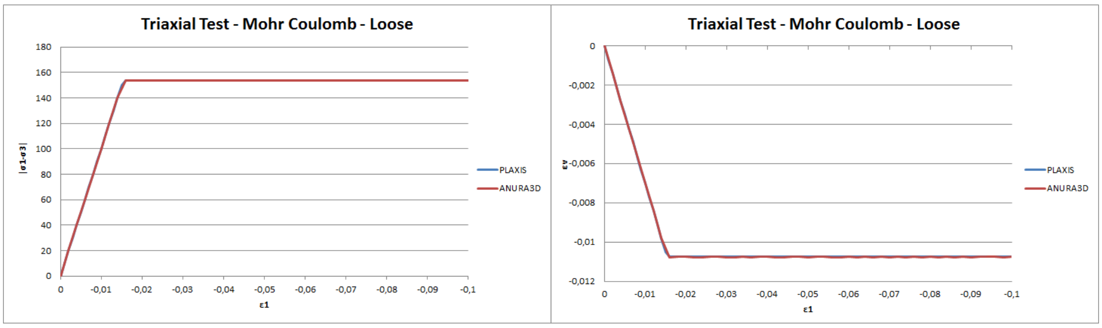

Figure 5.7: *Triaxial test of loose 1-phase material using Mohr-Coulomb (benchmark number 3016)*

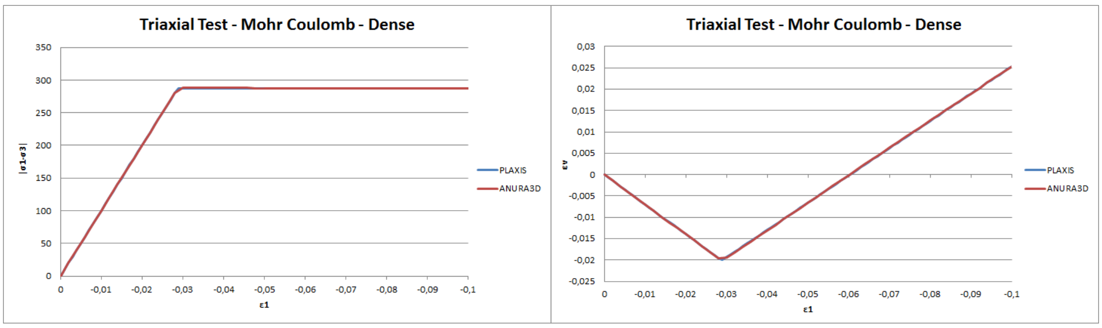

Figure 5.8: *Triaxial test of dense 1-phase material using Mohr-Coulomb (benchmark number 3017)*

The final values of the effective stress are presented in Table 5.8, as well as the corresponding relative error. It should be noted that σ_1 represents the vertical stress and σ_2 and σ_3 represent the horizontal stresses.

Table 5.6: Effective stress (kPa) of element 1

benchmark number	calculation type	situation ratio	numerical value	analytical value	relative error
3016	FEM	loose	σ_1 = –203.67	σ_1 = –203.72	0.00025
			σ_2 = –49.99	σ_2 = –50.00	0.00020
			σ_3 = –49.99	σ_3 = –50.00	0.00020
3017	FEM	dense	σ_1 = –337.75	σ_1 = –337.47	0.00083
			σ_2 = –50.04	σ_2 = –50.00	0.00080
			σ_3 = –50.04	σ_3 = –50.00	0.00080

5.4 Triaxial tests using Mohr-Coulomb model with strain smoothing

5.4.1 Description

A dry soil cube is compressed with a constant velocity of 0.01 m/s. Gravity is neglected, such that the stress is independent of depth. Boundary conditions are typical for a triaxial test. The bottom surface is constrained in normal direction. On the lateral surfaces a compressive load of 50 kPa is applied, normal to the surface. The soil is a 1-phase material following the Mohr-Coulomb model with strian softening. Its parameters are listed in Table 5.7. Two situations are considered: loose soil and dense soil.

Table 5.7: Material properties of the soil cube

material property	value (loose)	value (dense)
initial porosity [-]	0.45	0.15
density solid [kg/m^3]	2600	2600
Young's modulus [kPa]	10000	10000
Poisson ratio [-]	0.15	0.15
peak cohesion [kPa]	5	3
residual cohesion [kPa]	3	1
peak friction angle [$^\circ$]	35	47
residual friction angle [$^\circ$]	30	40
peak dilatancy angle [$^\circ$]	0	14
residual dilatancy angle [$^\circ$]	0	10

The soil cube is discretized by 6 tetrahedral elements, see Figure 5.1 in Section 5.1. The calculation consists of 50 load steps of 0.2 s, such that the total time equals 10.0 s.

The calculation is performed with FEM in order to focus on the material model. With the K0 procedure the initial stresses are set to –50 kPa. The compression is realized by a prescribed velocity of 0.01 m/s on the top nodes. The Courant number is set to 0.50 and no additional smoothing is added.

5.4.2 Benchmark result

Plaxis 3D (Version 2013.1) does not include a triaxial test for the Mohr Coulomb model with strain softening in the soil test environment, therefore the results are compared to the triaxial test for the Mohr Coulomb model with peak/residual values. The material properties are set to the peak/residual values in Table 5.7 and a drained compressive triaxial test with $|\sigma_3|$ = 50 kPa and $|\varepsilon_{1,max}|$ = 10% is run.

The result is a vertical stress that is linearly decreasing until the yield surface is reached. From that moment, the vertical stress stays constant, as only plastic deformation occurs. The evolution of the volumetric strain is determined by the Poisson ratio until the yield surface is reached. In case of loose soil, the volumetric strain does not change anymore from that moment (zero dilatancy angle), while the volumetric strain starts increasing in case of dense soil (positive dilatancy angle). Graphs are plot in the next subsection.

By definition of strain softening, the effective stress first follows the path for the peak values and smoothly softens to the path for the residual values after reaching the yield surface. The graph of the volumetric strain first follows the path for the peak values and must end parallel to the path for the residual values.

5.4.3 Anura3D result

In this section, the Plaxis 3D and Anura3D output are compared. The effective stress and the volumetric strain are plot as a function of the vertical strain in Figure 5.9 and 5.10.

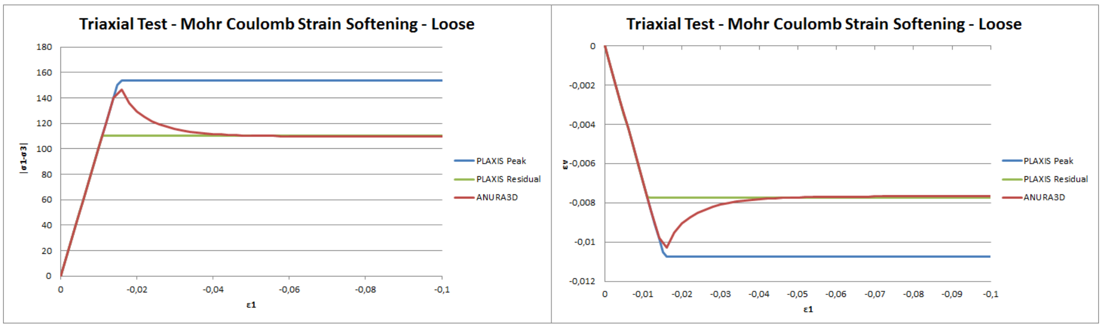

Figure 5.9: *Triaxial test of loose 1-phase material using Mohr-Coulomb with strain smoothing (benchmark number 3018)*

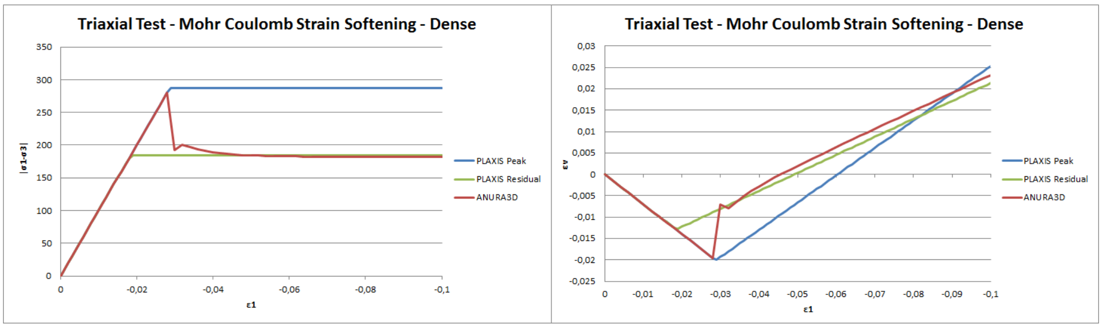

Figure 5.10: *Triaxial test of dense 1-phase material using Mohr-Coulomb with strain smoothing (benchmark number 3019)*

The oscillation in Figure 5.10 seems to be a numerical issue and only happens with certain combinations of peak and residual values.

The final values of the effective stress are presented in Table **??**, as well as the corresponding relative error. Here, the Anura3D output is compared to the Plaxis 3D output with residual values. It should be noted that σ_1 represents the vertical stress and σ_2 and σ_3 represent the horizontal stresses.

Table 5.8: *Effective stress (*kPa*) of element 1*

benchmark number	calculation type	situation ratio	numerical value	analytical value	relative error
3018	FEM	loose	σ_1 = –159.57	σ_1 = –160.39	0.00511
			σ_2 = –50.00	σ_2 = –50.00	0.00000
			σ_3 = –50.00	σ_3 = –50.00	0.00000
3019	FEM	dense	σ_1 = –232.18	σ_1 = –234.23	0.00875
			σ_2 = –50.01	σ_2 = –50.00	0.00020
			σ_3 = –50.01	σ_3 = –50.00	0.00020

References

[1] L.E. Malvern. *Introduction to the mechanics of a continuous medium*. Prentice-Hall, 1969.

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[3] Arnold Verruijt. *An introduction to soil dynamics*, volume 24. Springer, 2010.

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[5] M.M.J. Mieremet. Numerical stability for velocity-based 2-phase formulation for geotechnical dynamic analysis. Report 15-03, ISSN 1389-6520, Reports of the Delft Institute of Applied Mathematics, Delft University of Technology, Delft, The Netherlands, 2015.

[6] R.P.W.M. Tielen. *High-order Material Point Method*. MSc Thesis, Applied Mathematics, Delft University of Technology, Delft, The Netherlands, 2016.

A Grid crossing error

The material point method is based on a collection of material points that move through a finite element mesh. The method is well-suited for large deformation calculations since mesh deformation does not occur. Instead, material points move from one element to another, which is called grid crossing.

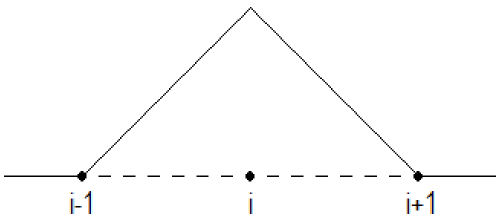

Figure A.1: *Low-order basis function (1D)*

The first release of Anura3D is based on low-order basis functions. Such basis function, see Figure A.1 for its one-dimensional equivalent, is piece-wise linear and has a discontinuity in the gradient. This discontinuity is the reason for unphysical oscillations in the computed stresses when grid crossing occurs. The oscillations significantly affect the numerical solution as can be seen in Figure A.2. Here, the displacement of material point 1 of benchmark test 2005 (MPM-MP) is compared with the analytical solution of the dynamic 1-phase oedometer test [5].

Tielen [6] shows that the problem can be solved by implementing high-order basis functions. This is not yet available with Anura3D, but can be an improvement for future releases. With the first release of Anura3D the problem can be reduced by switching from MPM-MP to MPM-MIXED. Figure A.3 shows that the numerical and analytical solution now coincide.

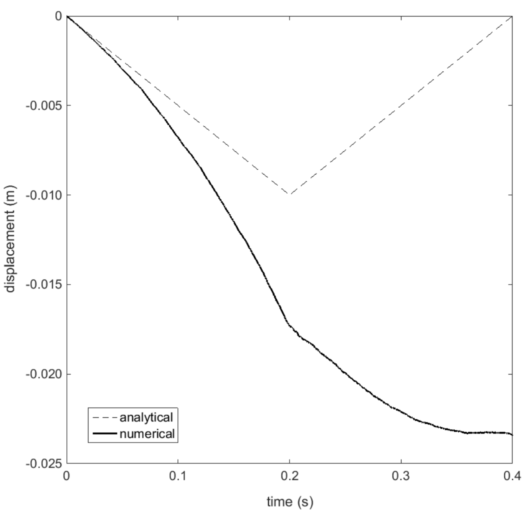

Figure A.2: *Numerical solution of benchmark test 2005 (MPM-MP)*

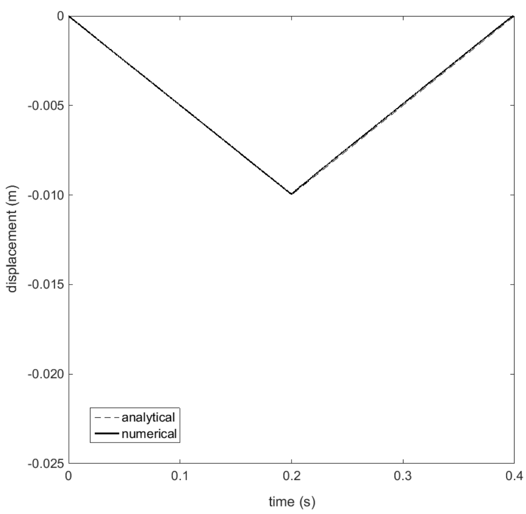

Figure A.3: *Numerical solution of benchmark test 2007 (MPM-MIXED)*