

## CHAPTER 8

# *River Meandering and Channel Stability*

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### 8.1 INTRODUCTION

River meandering is a planform process that generates a series of bends of alternate curvature connected by straight reaches. The outer banks of the bends tend to erode, causing the channel planform to gradually shift or migrate (Fig. 1). The rate of shift depends on the properties of the surrounding material, which can vary from alluvium to rock. Depending on the surrounding material, the channel is termed either *freely meandering* or *rock incised*. Most rock-incised channels are formed by down-cutting over long periods of time (Leopold et al. 1964; Dury 1966). This chapter deals with freely meandering channels.

The planforms of freely meandering channels migrate by down-valley translation, lateral expansion, or a combination of both. Down-valley translation is essentially a longitudinal shift of the meander pattern, whereas lateral expansion is a widening of the meander pattern. This chapter presents a summary of analytical approaches to the description of this process.

The chapter starts with a brief review of historical relationships, which is then followed by a summary of recent approaches to the calculation of flow and bed topography in rivers, migration rates, and dominant meander wavelength. The chapter concludes with a brief review of technologies for channel stabilization.

The review is not inclusive. It is focused on concepts and findings that have direct bearing on river engineering practice more than on scientific discourse. As such, important contributions may have been omitted. For a more inclusive review, readers are referred to Callander (1978); Richards (1982); and Howard (1992; 1996).

### 8.2 MEANDERING PROCESS

It is generally assumed that meandering is the result of channel instability. This assumption implies that a straight

channel is unstable and that a slight perturbation in any of its flow or boundary characteristics causes an increase in shear stress along one bank, resulting in erosion, and a decrease in shear stress along the opposite bank, promoting deposition. The result is gradual increase in channel sinuosity (ratio of channel length to valley length) with time. As the length of the channel increases, the channel slope decreases and becomes less than the valley slope.

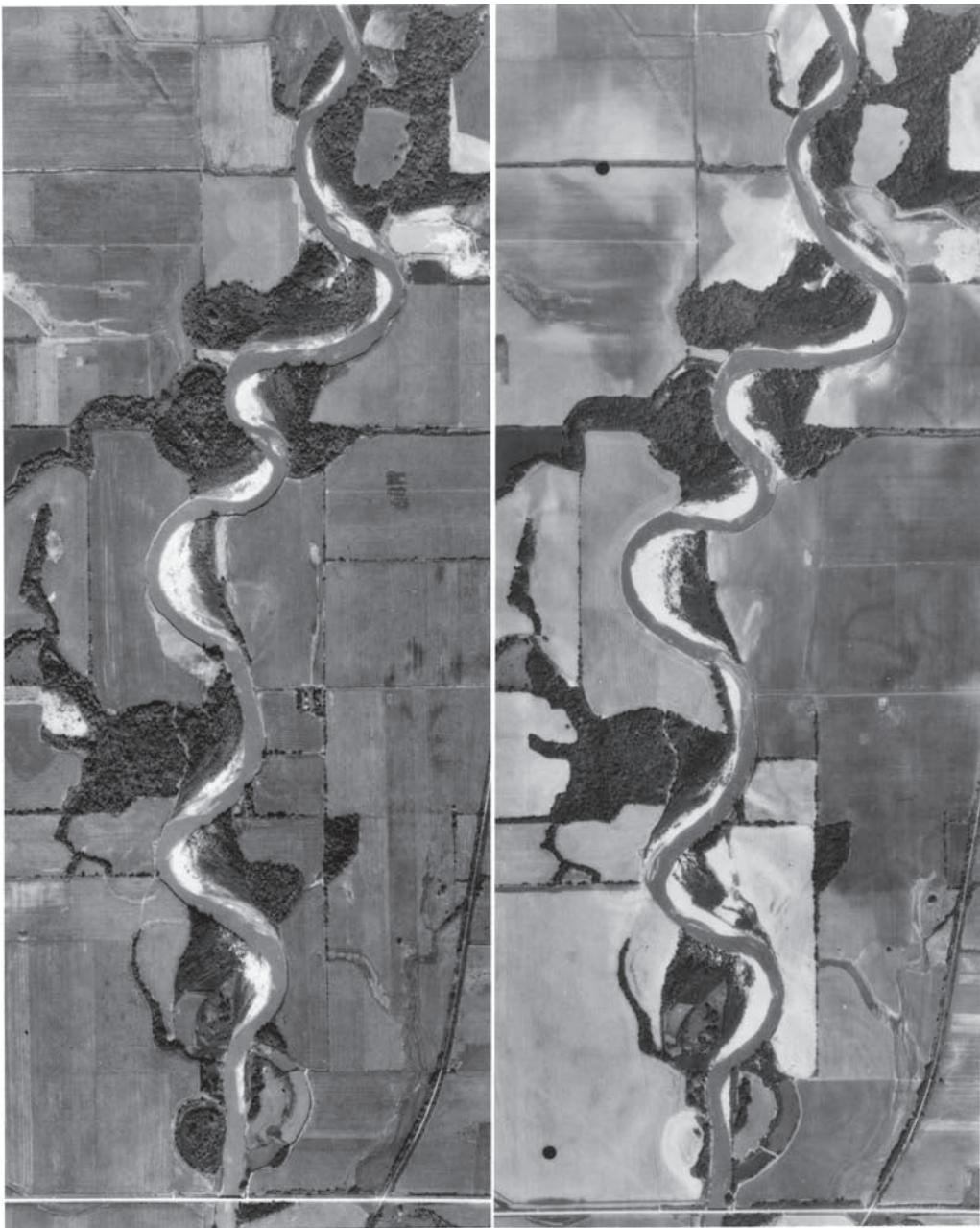
The process of erosion and deposition results in sideways migration of the channel, which, as indicated previously, may or may not be combined with down-valley translation. Sideways migration may eventually result in a cut-off, after which the process starts over again.

Field observations have been described by, among others, Leopold and Wolman (1957); Wolman and Leopold (1957); Schumm (1963); Kondrat'yev (1968); Konditerova and Ivanov (1969); Schumm and Khan (1971, 1972); Brice (1973); Kulemina (1973); Brice (1974); Hickin (1974); Hickin and Nanson (1975); Lewin (1976); Allen (1977); Hooke (1977); Lewin and Brindle (1977); Dort (1978); Lewin (1978); Nanson (1980); Allen (1982); Beck et al. (1983b); Lewis and Lewin (1983); Nanson and Hickin (1983); Schumm (1983); Hooke (1984); Schumm (1985); Carson and Lapointe (1986); Lapointe and Carson (1986); Nanson and Hickin (1986).

The meandering process has also been demonstrated in numerous laboratory studies. An early demonstration was by Friedkin (1945). By studying the evolution of a laboratory channel from straight to meandering (Fig. 8-2), Friedkin defined some of the key variables of the process. He also made a first attempt to establish qualitative relationships between the variables.

#### 8.2.1 Meandering Criteria

A significant amount of literature suggests that this process takes place only when certain combinations of variables are in place. Data suggest that as the channel slope becomes



**Fig. 8-1.** Aerial photos of the East Nishnabotna River just south of Red Oak, Iowa: (left) October 5, 1973; (right) May 25, 1979.

steeper, there is a tendency for the river to become braided, that is, split into several channels. Leopold and Wolman (1957) analyzed data from a large number of rivers in the United States and in India and found that the threshold between the two classifications (meandering and braided) for these rivers is a function of channel slope  $S$  and bank-full discharge  $Q(\text{m}^3/\text{s})$ ,

$$S = 0.012 Q^{-0.44} \quad (8-1)$$

For a braided river,  $S$  is greater, and for a meandering river, it is less than the value given by Eq. (8-1). As indicated in Fig. 8-3, the data suggest that one and the same river can have both braided and meandering reaches.

Henderson (1963) attempted to refine Eq. (8-1) by accounting for the effect of size of bed material and proposed the relation

$$S = 0.0002 D^{1.14} Q^{-0.44} \quad (8-2)$$

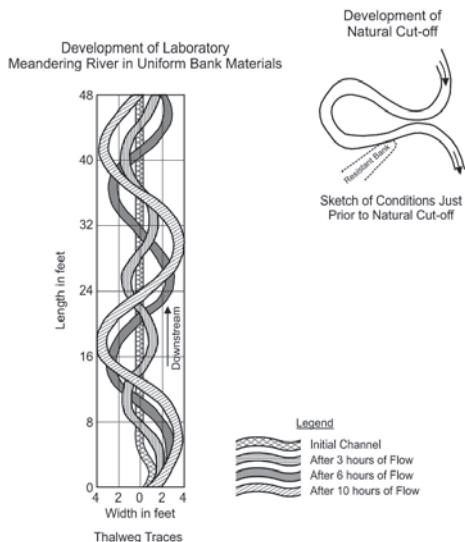


Fig. 8-2. Meandering process demonstration by Friedkin (1945).

in which  $D$  = median-particle diameter in millimeters and  $Q$  = discharge in  $\text{m}^3/\text{s}$ .

Based on observations of sand-bed rivers in the United States, Lane (1957) proposed a slightly different criterion for braided and meandering rivers,

$$S = KQ^{-0.25} \quad (8-3)$$

in which  $K$  = constant. Figure 8-4 summarizes Lane's plots and shows that when  $K \leq 0.0017$  English units ( $K \leq 0.0007$  metric units), a sand-bed river will tend toward a meandering pattern, and when  $K \geq 0.01$  (0.004 metric units), it will tend toward a braided pattern. It is noted that channel slopes for these two extremes differ by a factor of nearly 6. It is also noted that many U.S. rivers fall in between these extremes.

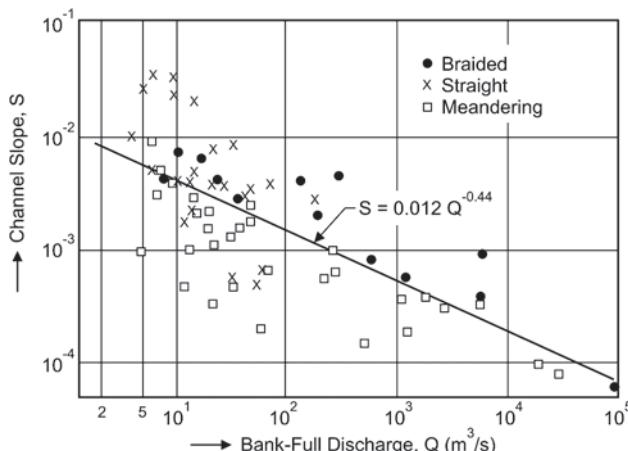


Fig. 8-3. Threshold between meandering and braided channels (Leopold and Wolman 1957).

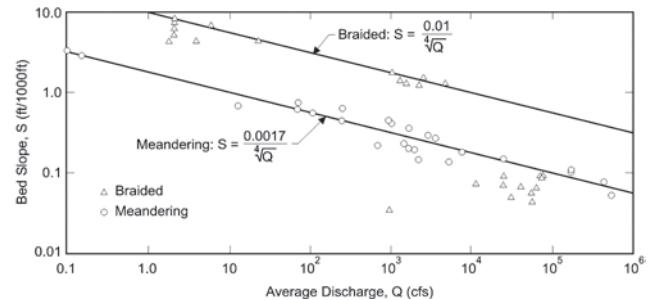


Fig. 8-4. Slope-discharge relationships in meandering and braided sand-bed streams (after Lane 1957).

Recently Millar (2000) showed that bank vegetation influences channel patterns and that the meandering-braiding transition slope increases with the erosional resistance of the banks. Millar's relation, which is based on theoretical analysis and curve fitting, using data from 137 rivers, reads as

$$S = 0.0002 D^{0.61} \phi^{1.75} Q^{-0.25} \quad (8-4)$$

where

$D$  = median sediment diameter for the banks and bed surface (meters);

$\phi$  = bank sediment friction angle (degrees); and

$Q$  = bank-full discharge ( $\text{m}^3\text{s}^{-1}$ ).

With no vegetation on the bank,  $\phi$  is the angle of repose for noncohesive bank sediment, which for coarse gravel is up to  $40^\circ$ . As vegetation increases,  $\phi$  increases. Millar states that  $\phi$  represents a lumped calibration parameter that probably accounts for several different processes, including reduction of near-bank velocity and shear stress, binding of the bank sediment by root networks, packing and imbrication, and cementing of the gravel clasts by interstitial fines.

These relations are just a few of the many criteria for meandering that have been proposed over the years (for a more complete summary, see Bridge 1993). Although they have been, and are still being, challenged by engineers and scientists, the relations offer some guidance to river engineers.

## 8.2.2 Meander Planform

Measurements of the dimensions of meander patterns suggest that there are relations between certain planform characteristics that are relatively consistent for a wide range of stream sizes. The planform characteristics, defined in Fig. 8-5, are wavelength  $\lambda$ , amplitude  $A$ , bank-full channel width  $b$ , and minimum radius of curvature  $r_c$ . Leopold and Wolman (1960) have suggested the following relationships between these variables:

$$L = 11.0 b^{1.01} \quad (8-5)$$

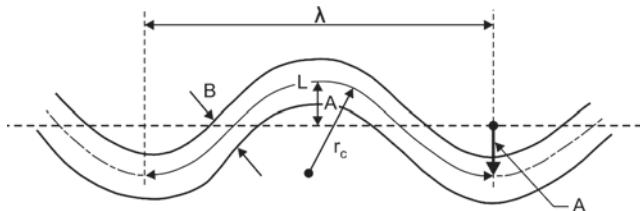


Fig. 8-5. Meander planform characteristics.

$$A = 3.0b^{1.1} \quad (8-6)$$

$$\lambda = 4.6r_c^{0.98} \quad (8-7)$$

All dimensions are in meters. These relationships imply that  $r_c \approx 2.4 b$ . Leopold et al. (1964) and Zeller (1967) later confirmed these relationships in a comprehensive data analysis that included furrow meanders, meanders in glacier ice, and meanders of the Gulf Stream.

A number of theories have been proposed to explain the observed regularity of plan form. They include theories based on optimization concepts such as minimum energy dissipation and minimum variance (Langbein and Leopold 1966; Chang 1979; 1984; 1988b). For a review of these theories, readers are referred to Chang (1988a).

The consistency of these relations may suggest that the meandering process, as described in Section 8.2, is a transitional (transient) process that, as sinuosity increases, tends toward some form of planform equilibrium or order, which is disrupted only during extreme events when cutoffs occur.

The notion of a time-limited equilibrium or order has been promoted in recent studies that attempt to simulate meandering through chaotic dynamics and self-organization. These simulations show the meandering process as oscillating in space and time between a state in which the river planform is ordered and one in which it is chaotic (Stølum 1996; 1997; 1998).

### 8.2.3 Meander Migration—Bank Erosion

As indicated in the Introduction, the planforms of meanders tend to migrate. The aforementioned relations do not predict rate and direction of migration. Specific migration relations have been developed by, among others, Hickin (1974); Hickin and Nanson (1975); Hooke (1980); Brice (1982); Nanson and Hickin (1983). Their relations are in the form of measured correlations between rates of bank retreat and width or width-radius ratio.

Brice demonstrated that the rate of bank retreat increases with increasing channel width. His data consist of 43 data points from four different stream types (equiwidth, wide bend, braided point bar, braided) with rates ranging from 0.1 m/year on a 10-m-wide channel to about 9 m/year on a 600-m-wide channel. The approximate relationship is mean erosion rate in meters per year = 0.01 times channel width in meters.

An increase in erosion rate with channel width is also indicated indirectly in Hooke's (1980) data. Her plot of erosion rates versus drainage area for 11 streams in Devon, England, and 43 streams compiled from the literature covers rates from 0.05 m/year for a drainage area of 3 km to 800 m/year for a drainage area of 1 million km<sup>2</sup>. The approximate relationship is mean erosion rate in meters per year = 0.05 times square root of drainage area in square kilometers.

Hickin and Nanson (1975) and Nanson and Hickin (1983) demonstrated that channel curvature plays an important role in determining the rate of bank retreat. They used the technique of dendrochronology to determine the relative ages of scroll bars on the floodplain of the Beatton River, Canada, and they correlated local migration rate with local radius-width ratio. Their data (Nanson and Hickin 1983) conform, approximately, to the relation

$$\begin{aligned} v_e \text{ (m/year)} &= 2.0 b/r_c \quad b/r_c \leq 0.3 \\ v_e \text{ (m/year)} &= 0.2 r_c/b \quad b/r_c \geq 0.3 \end{aligned} \quad (8-8)$$

in which  $v_e$  is the erosion rate,  $b$  is the channel width, and  $r_c$  is the radius of curvature of the channel.

Ikeda et al. (1981), in their theory of river meanders, assume that the rate of bank retreat  $v_e$  is proportional to the difference between near-bank depth-averaged mean velocity  $u_b$  and the reach-averaged mean velocity  $u$  at bank-full discharge,

$$v = E(u_b - u) = Eu_b' \quad (8-9)$$

in which  $E$  = parameter describing the erodibility of the bank material and  $u_b'$  = near-bank velocity increment. This relationship is based on the assumption that soil particles on the bank are eroded and removed by the flow whenever the near-bank velocity exceeds the reach-averaged velocity. Field data support the assumption of a linear relationship between erosion rate and near-bank velocity increment (Odgaard 1987; Hasagawa 1989; Pizzuto and Meckelnburg 1989).

By determining  $u_b$  using Engelund's (1974) second approximation, Parker (1983) and Parker and Andrews (1986) developed a convolutional relation between migration rate and curvature. For developed bend flow in a constant-radius curve, their relation reduces to

$$\frac{v_e}{r} = EA \frac{u}{b} \quad (8-10)$$

in which  $A$  is "an order-one scour factor parameterizing the role of secondary currents" (Parker 1983, p. 727). Equation (8-10) supports Hickin and Nanson's (1975) notion that the rate of channel migration is a function of width-radius ratio. Odgaard (1987) used Eq. (8-10) for analysis of stream bank erosion along rivers in Iowa.

A convolutional relationship between migration rate and curvature has also been suggested by Howard and Knutson (1984) and Furbish (1988; 1991) and has been used in several simulation models. A convolutional relationship is often

appropriate because it allows the migration rate at a given point to depend not only on the local channel curvature but also on the upstream curvatures. The merits of a convolutional model have been discussed and demonstrated by Furbish (1991), among others.

Odgaard (1989a) has suggested that erosion rate may also be related to increase in scour depth at the bank,

$$\frac{v_e}{u} = E' \frac{d'_b}{d} \quad (8-11)$$

in which  $d'_b = d_b - d$  = near-bank depth increment. The rationale behind this relation is that as the height of the outer bank increases, the stability of the bank decreases, which is indicated by the analyses of Osman and Thorne (1988) and Thorne and Osman (1988). Hasegawa also includes near-bank depth increment as a factor in determining the rate of bank migration; however, in his equation, the effect is negative under certain conditions. Howard (1992) has suggested that the migration rate may be related to both  $u'_b$  and  $d'_b$ . Such a relationship may read

$$\frac{v_e}{u} = E'' \left( C_1 \frac{u'_b}{u} + C_2 \frac{d'_b}{d} \right) \quad (8-12)$$

in which  $C_1$  and  $C_2$  are weighting factors. The value of  $C_1$  is positive, whereas  $C_2$  may be positive, negative, or zero.

The mechanism of bank failure varies from river to river and there are cases where none of the aforementioned relationships come even close to a description of it. For example, bank failure by piping and sapping (Hagerty 1991a; 1991b), which occurs along many rivers in the midwestern states of the United States, may have little or no relationship to stream-flow variables. The same applies to bank failure triggered by vegetative growth or climate-influenced deterioration (weathering) of the bank material. A more extended description of these mechanisms can be found in ASCE (1998); Langendoen (2000); and in chapter 7 of this volume. Lawler et al. also describe bank erosion measurement techniques that are under development, including a photo-electronic erosion pin (PEEP) automatic erosion and deposition monitoring system.

### 8.3 FLOW AND BED TOPOGRAPHY IN MEANDERS

It is the dynamics of the flow in the river, in particular in bends, that determines whether the bends migrate sideways or down-valley or both.

As the flow enters a bend, the centrifugal acceleration drives the faster-moving surface current toward the outer bank and the flow near the bed toward the inner bank (secondary current). The result is a spiraling flow that produces greater depths and higher velocities near the outer bank. The channel-deepening undermines the bank, and the higher

velocity and shear stress attack it, setting the stage for bank erosion. Near the inner bank a point bar tends to form. Thomson (1876) may have been the first to suggest that this is the process that causes rivers to meander. His qualitative description of the bend flow has remained unchallenged.

There are other features of flow and bed topography in meander curves that also must be recognized. As a result of the secondary current, pressure builds along the outer bank, causing the water surface to rise or superelevate (Thomson 1876; Ippen and Drinker 1962; Yen 1965; Yen and Yen 1971). In sharp curves of the river there is a tendency for flow to separate at the point bar (Dietrich and Whiting 1989; Kawai and Julien 1996; Hodskinson and Ferguson 1998; Ferguson et al. 2003). In fact, the sharper the curve the more complex are the secondary currents and boundary shear stresses (Hey and Thorne 1975; Bathurst et al. 1979; Cheng and Shen 1983; Allen 1985; Thorne et al. 1985; Hey and Rainbird 1996; Blanckaert and Graf 2001; Blanckaert 2003). The complexity of flow is also reflected in the sediment transport. A sorting of bed sediment often occurs (Parker and Andrews 1985; Ikeda 1989; Bridge 1992; Yen and Lee 1995; Julien and Anthony 2002). Readers are referred to Chapter 3 for more details about sediment transport of mixtures.

Because, as indicated in the previous section, the rate of bank erosion may be closely related to near-bank depth and velocity, many attempts have been made over the years to relate these variables to mean-flow properties (Thomson 1879; van Bendegom 1947; Rozovskii 1957; Yen 1965; Yen 1970; Yen and Yen 1971; Apmann 1972; Yen 1972; Engelund 1974; Ikeda 1974; Engelund 1975; Hooke 1975; Ikeda 1975; Yen 1975; Bridge 1976; Bridge and Jarvis 1976; Gottlieb 1976; Kikkawa et al. 1976; Bridge 1977; Bridge and Jarvis 1977; DeVriend 1977; Allen 1978; Zimmermann and Kennedy 1978; Dietrichet al. 1979; Begin 1981; Odgaard 1981; Bridge and Jarvis 1982; DeVriend and Geldof 1983; Dietrich and Smith 1983; Falcon and Kennedy 1983; Geldof and DeVriend 1983; Parker et al. 1983; Thorne et al. 1983; Bridge 1984; Chang 1984; Dietrich and Smith 1984; Kitanidis and Kennedy 1984; Odgaard 1984; Smith and McLean 1984; Ikeda and Nishimura 1985; Parker and Andrews 1985; Struiksma et al. 1985; Odgaard 1986a; 1986b; Parker and Andrews 1986; Dietrich 1987; Odgaard 1987; Furbish 1988; Odgaard and Bergs 1988; Dietrich and Whiting 1989; Ikeda 1989; Nelson and Smith 1989; Odgaard 1989a; 1989b; Parker and Johannesson 1989; Shimizu and Itakuru 1989; Bridge 1992; Mosselman 1995; 1998; Seminara et al. 2001; Zolezzi and Seminara 2001). Summaries have been given by, among others, Odgaard (1984) and Chang (1988a).

#### 8.3.1 Governing Equations and Sample Solution

Several attempts to relate near-bank depth and velocity to mean-flow properties are based on solving the equations for conservation of mass (water and sediment) and momentum and using a stability criterion for sediment particles on the

streambed. The attempts differ in the way the equations are reduced. As an example, Odgaard (1989a; 1989b) employs an order-of-magnitude consideration and linearization and reduces the equations to those of a damped oscillating system. He utilizes the observation, from both laboratory and field, that both  $u$  and  $d$  are essentially constant along the river channel's centerline, and that their variation in transverse direction is nearly linear over the central portion of the cross section (Fig. 8-6). Consequently, the following description is deemed appropriate:

$$\frac{\bar{u}}{\bar{u}_c} = 1 + \frac{n}{d_c} U_{tc} \quad (8-13)$$

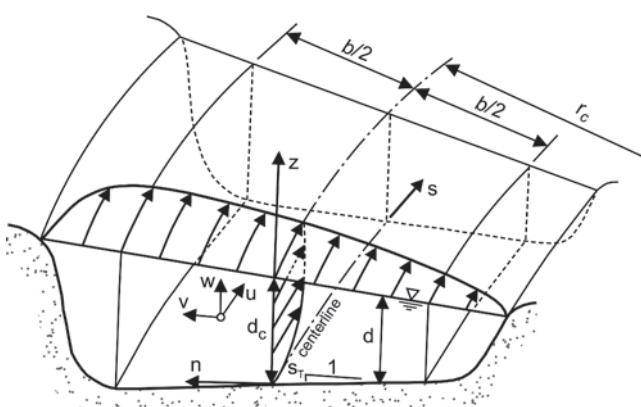
$$\frac{d}{d_c} = 1 + \frac{n}{d_c} S_{tc} \quad (8-14)$$

in which  $S_c$  = transverse bed slope at the centerline =  $(\partial d / \partial n)_c$  and  $U_c$  = normalized transverse velocity gradient at the centerline =  $d_c [\partial / \partial n (\bar{u} / \bar{u}_c)]_c$ , in which subscript  $c$  = the centerline values, and overbars denote the depth-averaged values. In Fig. 8-6 the  $s$ -axis is along the channel centerline, and positive in the streamwise direction, the  $n$ -axis is perpendicular to the  $s$ -axis and positive toward the concave bank, and the  $z$ -axis is vertically upward. The velocity components (time-averaged) in the  $s$ -,  $n$ -, and  $z$ -directions are denoted  $u$ ,  $v$ , and  $w$ , respectively.

Odgaard assumes a transverse distribution of sediment transport  $q_s$  described by a power law,

$$q_s = q_{sc} \left( \frac{\bar{u}}{\bar{u}_c} \right)^M \quad (8-15)$$

in which exponent  $M$  is a function of sediment characteristics and  $q_s$  is the volumetric rate per unit width. Such a power law is often used for description of bed load transport in straight alluvial channels, where the value of  $M$  is generally between 2 and 4 (Simons and Sentürk 1977).



**Fig. 8-6.** Definition sketch for sinusoidal channel flow.

Odgaard is then able to reduce the equations to two ordinary differential equations,

$$\frac{dU_{tc}}{d\sigma} + a_1 U_{tc} = \frac{1}{2} a_1 S_{tc} \quad (8-16)$$

$$\frac{d^2S_{tc}}{d\sigma^2} + a_2 \frac{d^2U_{tc}}{d\sigma^2} + a_3 \frac{dS_{tc}}{d\sigma} + a_4 \frac{dU_{tc}}{d\sigma} + a_5 S_{tc} = a_6 \quad (8-17)$$

in which  $\sigma = s/b$  and

$$a_1 = \frac{2k^2}{m^2} \frac{b}{d_c} \quad (8-18)$$

$$a_2 = 1 - \frac{m+1}{m+2} M \quad (8-19)$$

$$a_3 = \frac{8 B \sqrt{\theta}}{\alpha \kappa F_{D_c}} \frac{m(m+1)}{m+2} \frac{d_c}{b} + \frac{2 \kappa^2 m}{(m+1)(m+2)} \frac{b}{d_c} \quad (8-20)$$

$$a_4 = \frac{2\kappa^2 m}{(m+1)(m+2)} \left[ 1 - M \left( 1 + \frac{1}{2m} + \frac{1}{2m^2} \right) \right] \frac{b}{d_c} \quad (8-21)$$

$$a_5 = \frac{8B\kappa\sqrt{\theta}}{\alpha(m+2)F_{D_0}} \left(1 + \frac{2m^2}{m+1}\right) \quad (8-22)$$

$$a_6 = \frac{8}{\alpha} \frac{2m+1}{m(m+2)} \frac{d_c}{r} \quad (8-23)$$

in which factor  $F_{Dc}$  = particle densimetric Froude number, defined as  $F_{Dc} = \bar{u}_c / \sqrt{\Delta g D}$ .  $m$  = friction parameter, whose relationship to shear velocity  $u_*$ , Darcy-Weisbach's friction factor  $f$ , and Chezy's coefficient  $C$  is  $m = \kappa \bar{u} / u_* = \kappa \sqrt{8/f} = \kappa C / \sqrt{g}$  is in which  $\kappa$  = von Karman's constant ( $\sim 0.4$ ), and  $u_* = \sqrt{\tau_{bs}/\rho}$ ,  $\tau_{bs}$  = bed shear stress in the s-direction,  $\rho$  = density of water,  $\Delta$  = specific weight of submerged sediment =  $(\rho_s - \rho)/\rho_s$ ;  $\rho_s$  = density of sediment (for quartz sand,  $\Delta = 1.65$ ),  $B$  = transverse bed-slope factor (see Odgaard 1989a),  $\alpha$  = transverse-mass flux factor (Odgaard 1989a),  $\theta$  = dimensionless critical bed shear stress (Shield's parameter), and  $g$  = acceleration due to gravity. Typical values of  $\alpha$ ,  $B$ , and  $\theta$  are 0.4, 6, and 0.03, respectively.

By eliminating  $S_{tc}$ , using Eq. (8-16), Eq. (8-17) is reduced to

$$\frac{d^3 U_{tc}}{d\sigma^3} + h_1 \frac{d^2 U_{tc}}{d\sigma^2} + h_2 \frac{d U_{tc}}{d\sigma} + h_3 U_{tc} = h_4 \quad (8-24)$$

in which  $h_1 = a_1 + (\frac{1}{2})a_1 a_2 + a_3$ ;  $h_2 = a_1 a_3 + (\frac{1}{2})a_1 a_4 + a_5$ ;  $h_3 = a_1 a_5$ ; and  $h_4 = (\frac{1}{2})a_1 a_6$ . The system ( $U_{tc}$  and  $S_{tc}$ ) described by these equations is a damped oscillation forced by curvature ( $h_4$ ). With given boundary conditions, the solution is readily obtained.

In fully developed bend flow, where  $d/d\sigma = 0$ , Eqs. (8-16) and (8-17) yield

$$\frac{\bar{u}}{u_c} = \sqrt{\frac{d}{d_c}} \quad (8-25)$$

and

$$S_{tc} = H F_{Dc} \frac{d_c}{r_c} \quad (8-26)$$

in which  $H = (2m + 1)(m + 1)/[B\kappa\sqrt{\theta} m(m + 1 + 2m^2)]$ . These equations are well supported by both laboratory and field data (Kikkawa et al. 1976; Falcon and Kennedy 1983; Ikeda and Nishimura 1985; Odgaard and Bergs 1988). The composition of factor  $H$ , however, varies somewhat from author to author.

### 8.3.2 Sample Simulation of Bed Topography in Laboratory Channel

The oscillatory behavior of the flow system is illustrated by a simulation of  $S_{tc}$  in a recirculating 180° constant-radius alluvial-bend model at IIHR Hydroscience and Engineering. This model has width 2.44 m and centerline radius  $r_c = 13.11$  m. At a discharge of 0.153 m<sup>3</sup>/s, centerline values of depth, velocity, particle Froude number, and water surface slope are  $d_c = 0.15$  m,  $u_c = 0.45$  m/s,  $F_{Dc} = 6.5$ , and  $S_c = 0.00116$ , respectively; resistance parameters are  $m = 5.3$  and  $\kappa = 0.52$ ; and bed load transport  $q_s \approx 4$  g/cm/min. Flow and sediment conditions were described earlier by Odgaard and Bergs (1988) and Bergs (1989). The bend is preceded by a 20-m-long straight reach. Under such conditions,  $d^2 U_{tc}/d\sigma^2$  is negligibly small, and Eqs. (8-16) and (8-17) yield

$$\frac{d^2 S_{tc}}{d\sigma^2} + \left( a_3 + \frac{1}{2} a_4 \right) \frac{d S_{tc}}{d\sigma} + a_5 S_{tc} = a_6 \quad (8-27)$$

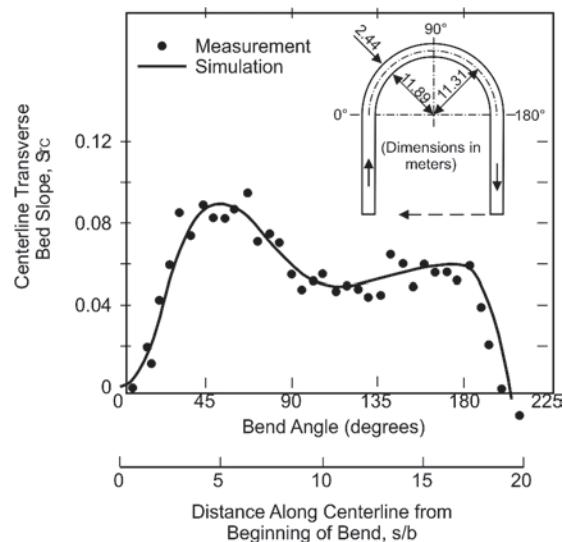
With  $S_{tc}$  and  $dS_{tc}/d\sigma$  being zero at the beginning of the bend at  $\sigma = 0$ , the solution is

$$S_{tc} = S_{tco} - S_{tco} \sqrt{1 + \left( \frac{a'}{2\omega} \right)^2} \cos(\omega\sigma - \psi) \exp\left(-\frac{1}{2}a'\sigma\right) \quad (8-28)$$

in which  $\omega = (1/2)\sqrt{4a_5 - a'^2}$ ;  $a' = a_3 + (\frac{1}{2})a_4$ ;  $\psi = \arctan[a'/(2\omega)]$ ; and  $S_{tco}$  = fully developed value of  $S_{tc}$  (Eq. (8-26)). The simulation (using values  $B = 3$ ,  $\alpha = 1$ ,  $\theta = 0.03$ , and  $M = 2.7$ ) is shown in Fig. 8-7 with measured data. The oscillation of  $S_{tc}$  is very distinct in this flow situation. The transverse bed slope is seen to overshoot its equilibrium (fully developed) value by a factor of about 1.5. A similar overshoot was measured in Struiksma et al.'s (1985) experiments and also predicted by their model. The overshoot is associated with redistribution of flow and sediment transport in the beginning of the bend, where both  $q_n$  and  $\partial u/\partial n$  are greater than zero ( $q_n$  = volumetric bed load transport per unit width in the  $n$ -direction). A positive value of  $q_n$  is necessary there to provide the increase of sediment transport associated with the increase in velocity along the outer bank. It is apparent that the system generates such a transverse transport of sediment by locally increasing transverse bed slope beyond that of fully developed bend flow (to make the downslope gravity force component larger than the upslope drag force component). The magnitude of the overshoot is dependent on the value of  $M$ . Overshoot of "overdeepening" is also discussed by Zolezzi and Seminara (2001). Using a linear, depth-averaged flow model coupled with the Exner equation, they simulate, and obtain good agreement with, the overshoot measured by Struiksma et al. (1985).

## 8.4 CHANNEL STABILITY

One of the critical questions from a river-engineering point of view is the extent to which a given channel alignment is prone to future changes. There is an obvious need for answers to this question. The problem is one of stability of river channel alignment.



**Fig. 8-7.** Measured and computed transverse bed slopes in IIHR bend model experiment.

#### 8.4.1 Regime Theories

Many attempts have been made in the past to establish guidelines for assessment of channel stability. Among the guidelines are the so-called regime theories (Kennedy 1895; Lacey 1930; Blench 1952; Kellerhals 1967; Charlton et al. 1978), which are empirical techniques, used primarily for the design of stable, straight channels. These theories generally predict that channel width must be less than 6 to 10 times depth for the channel to remain stable.

Most natural channels have a width-depth ratio larger than 6 to 10, and their planforms are unstable. They consist of meanders that, as mentioned above, usually migrate by both downstream translation and lateral expansion.

#### 8.4.2 Perturbation Stability Analyses

As indicated earlier, it is generally believed that meandering is the result of channel instability. Building on this assumption, many researchers have attempted to simulate the initiation of the meandering process by a perturbation stability analysis. Early attempts were made by, among others, Callander 1969; Hansen 1967; Engelund and Skovgaard 1973; Fredsoe 1978; and Parker 1976. As they progressed in sophistication, the perturbation stability analyses were also used to evaluate the stability of given channel alignments. That is, if a given channel alignment had characteristics similar to those calculated by the stability analysis, the alignment was considered "relatively stable" or "minimally destructive" in terms of bank erosion. Early alternate approaches are reviewed by Yang (1971), Chitale (1973), and Callander (1978) among others. The theory of minimum variance has also been offered as a possible cause of meandering (von Schelling 1951; Langbein and Leopold 1966; and Chang 1988a).

In a perturbation stability analysis, small traveling perturbations are introduced into the system of equations governing river flow, and their effect on channel planform is determined by calculating the rate of growth of the perturbations. The primary advantage of the perturbation stability analysis is that it allows channel planform stability to be described as a feature of the basic flow equations. Whereas the regime formulas and empirically based meander relations correlate flow and meander variables using data and simple, one-dimensional, straight-channel resistance formulas (Manning, Chezy, etc.), the perturbation analyses generally employ models that are based on the complete set of governing equations, including those of sediment transport, and describe flow and depth distributions in the channel in at least two dimensions.

Two categories of stability theories exist: (1) bar theories, which examine conditions for formation of alternating bars in straight channels, and (2) bend theories, which examine migration features of weakly meandering flows. It is the latter category that is addressed in this section. It should be noted that the two categories may in fact be related. Theoretical analyses by Blondeaux and Seminara (1983; 1985), Seminara and Tubino (1989), Parker and Johannesson

(1989), and Tubino and Seminara (1990) suggest that alternating bars may under certain circumstances trigger bend instability and lead to meandering.

Representative bend theories are those of Ikeda et al. (1981); Kitanidis and Kennedy (1984); Blondeaux and Seminara (1985); and Odgaard (1989a). These theories differ in their treatment of bank erosion and of centrifugally induced secondary flow and its effect on bed topography and primary flow. Kitanidis and Kennedy assume that the rate of bank retreat is proportional to and in phase with the secondary current, whereas the other authors assume that the rate is proportional to and in phase with the difference between near-bank and section-average velocity. Kitanidis and Kennedy account for effects of secondary current and assume that transverse bed slope has negligible effect on stability. Ikeda et al., on the other hand, consider effects of transverse bed slope and neglect effects of secondary current. Blondeaux and Seminara (1983; 1985) and Odgaard (1989a) include effects of both secondary current and transverse bed slope. In Blondeaux and Seminara's model, the secondary current is controlled by an external stress relation, whereas in Odgaard's model it is controlled by the basic flow equations. Odgaard's approach allows for phase lag between channel curvature and secondary current and thus calculates the direction of migration. None of these analyses account for convective transport of primary flow momentum by the secondary current. One reason for this is that depth averaging of the governing flow equations eliminates it; in addition, in mildly curved channels, the effect is minor.

In Odgaard's stability analysis, the bed topography is calculated based on a coupling between flow field and sediment transport. This was done in response to the findings of Struiksma et al. (1985) and Johannesson (1988) that redistribution of sediment transport can have a significant effect on bed topography. In most other stability analyses, sediment redistribution is not considered.

#### 8.4.3 Example of a Perturbation Stability Analysis

The following example illustrates the principles of stability analysis (Odgaard 1989a) based on the equations of a damped oscillating system. The stability of the system is tested by subjecting it to a channel alignment perturbation in the form of a traveling sinusoid,

$$\eta(x, t) = A(t) \sin[k(x - ct)] \quad (8-29)$$

in which  $x$  = coordinate distance along the unperturbed channel axis;  $k = 2\pi/\lambda$  is the wave number;  $A$  = amplitude;  $\lambda$  = meander wavelength;  $t$  = time; and  $c$  = celerity of sinusoid. The channel-centerline displacement  $\eta(t)$  is limited to values much smaller than the meander wavelength. The centerline curvature is then

$$\frac{1}{t_c} = -\frac{d^2\eta}{dx^2} = k^2 A(t) \sin[k(x - ct)] \quad (8-30)$$

The differential equation for  $U_{tc}$  is obtained by substituting Eq. (8-30) into Eq. (8-24). The solution, which is periodic and independent of the initial condition, is

$$U_{tc} = \frac{Nb k^2 A}{\sqrt{e_1^2 + e_2^2}} \sin [k(x - ct) - \gamma] \quad (8-31)$$

in which  $N = 8\kappa^2(2m + 1)/[\alpha m^3(m + 2)]$ ;  $e_1 = h_3 - 2h_1 k^2 b^2$ ; and  $e_2 = h_2 k b - k^3 b^3$ . The phase shift between  $U_{tc}$  and the channel-axis displacement is  $\gamma = \arctan(e_2/e_1)$  ( $0 \leq \gamma \leq \pi/2$ ). The corresponding transverse bed slope is obtained by substituting Eq. (8-31) into Eq. (8-16),

$$S_{tc} = \frac{2Nb k^2 A}{\sqrt{e_1^2 + e_2^2}} \sqrt{1 + \left(\frac{bk}{a_1}\right)^2} \sin [k(x - ct) - \phi] \quad (8-32)$$

in which  $\phi = \gamma - \arctan(bk/a_1)$ .

To determine  $A(t)$ , an equation is introduced that describes the rate of lateral shifting of the channel axis due to erosion of the concave bank and deposition on the convex bank. In this example, two alternatives are used. One is the relation proposed by Ikeda et al. (1981), which assumes that the rate of bank retreat is proportional to the difference between near-bank depth-averaged velocity  $u_{bank}$  and the section-averaged velocity  $\bar{u}_o$ . By using the depth-averaged centerline velocity  $\bar{u}_c$  for  $\bar{u}_o$ , the relation reads

$$v_e = E \bar{u}_c \left( \frac{\bar{u}_{bank}}{\bar{u}_c} - 1 \right) \quad (8-33)$$

in which  $v_e$  = rate of bank retreat and  $E$  = parameter describing the erodibility of the bank material. This model is labeled IKD. The other relation, which is proposed by Odgaard (1989) and labeled ODG, assumes that rate of bank retreat is linearly related to increase in scour depth at the bank,

$$v_e = E' \bar{u}_c \left( \frac{d_{bank}}{d_c} - 1 \right) \quad (8-34)$$

in which  $E'$  = erosion parameter. It follows that  $E' \cong (1/2)E$ . Because of the assumed mild curvature of the channel,  $v_e$  may be equal to the rate of change of channel alignment,  $\partial\eta/\partial t$ . The parenthetical expressions in Eqs. (8-33) and (8-34) equal  $bU_{tc}/2d_c$  and  $bS_{tc}/2d_c$ , respectively. The closing of the problem is achieved by substituting Eq. (8-29) into the left-hand sides of Eqs. (8-33) and (8-34), and Eqs. (8-31) and (8-32) into the right-hand sides of Eqs. (8-33) and (8-34), respectively. After some reduction, relationships for amplitude growth rate,  $\partial A/\partial t$ , and celerity,  $c$ , are obtained. Using the IKD bank erosion model (Eq. (8-33)), the relations are

$$\frac{1}{A} \frac{\partial A}{\partial t} = \frac{E \bar{u}_c}{b} K b k \cos \gamma \quad (8-35)$$

in which

$$K = \frac{1}{2} N \frac{b}{d_c} \frac{kb}{\sqrt{e_1^2 + e_2^2}} \quad (8-37)$$

The ODG bank erosion model [Eq. (8-34)] yields

$$\frac{1}{a} \frac{\partial A}{\partial t} = 2 \frac{E' \bar{u}_c}{b} K b k \sqrt{1 + \left(\frac{bk}{a_1}\right)^2} \cos \phi \quad (8-38)$$

$$c = 2E' \bar{u}_c K \sqrt{1 + \left(\frac{bk}{a_1}\right)^2} \sin \phi \quad (8-39)$$

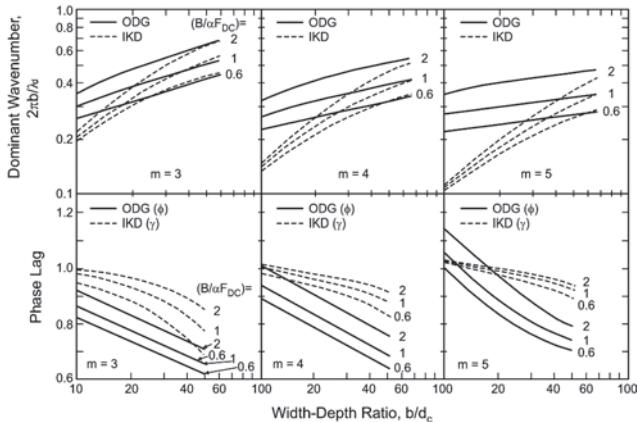
#### 8.4.4 Dominant Wavelength

It is generally assumed that the dominant wavelength is the wavelength that is associated with the conditions that yield maximum growth rate of alignment amplitude. The wave number at which the maximum amplitude growth rate occurs is termed the dominant wave number, and it is determined from the equation

$$\partial^2 A / \partial t \partial k = 0. \quad (8-40)$$

Sample calculations are shown in Fig. 8-8, which shows dominant wavelengths  $\lambda_d$  and corresponding phase shifts  $\phi_d$  and  $\gamma_d$  as a function of width-depth ratio for different friction factors, densimetric Froude numbers, and transverse bed slope factor. A value of  $M = 3$  is used. It is seen that the calculations based on the IKD bank erosion model yield a stronger dependence of wavelength on width-depth ratio than do calculations based on the ODG bank erosion model. The typical bank-full range for  $F_{Dc}$  and  $m$  are  $5 \leq F_{Dc} \leq 15$  and  $3 \leq m \leq 5$ . For width-depth ratios between 10 and 60 (the typical range), the ODG model then yields dominant wavelengths between 9 and 24 times the width, which is in agreement with data presented by Zeller (1967) and Leopold and Wolman (1957; 1960). For the same width-depth ratios, the IKD model yields wavelengths between 9 and 57 times the width, somewhat larger than those indicated by data.

The calculations based on the IKD bank erosion model yield dominant-wavelength relationships ranging from  $\lambda_d \sim d_c$  at small width-depth ratios to  $\lambda_d \sim \sqrt{bd_c}$  at large width-depth ratios. This range covers that represented by the theories of Ikeda et al. and Kitanidis and Kennedy. The IKD model yields a nearly linear dependence of  $\lambda_d$  on  $m$ , or the inverse of  $\sqrt{f}$ , which is also predicted by Ikeda et al. and Kitanidis and Kennedy. The ODG model yields a roughly linear dependence on  $m$  only at large width-depth ratios; the dependence on  $m$  is weaker at smaller width-depth ratios. For large width-depth ratios, the two analyses yield essentially the same results. The two analyses also differ in their prediction



**Fig. 8-8.** Results of sample stability analysis: dominant wavelengths and phase shifts as functions of width-depth ratio.

of dominant phase lag, and thus of distance from crossover to first outer-bank erosion occurrence. The phase lags predicted based on the ODG model are generally smaller than those predicted when the IKD bank erosion model is used. Computed data points, obtained using the ODG model and  $M = 3$ , conform roughly to the following curve-fitted relations:

$$\frac{2\pi b}{\lambda_d} = 0.11m^{1/4} \left( \frac{B}{\alpha F_{Dc}} \right)^{0.41} \left( \frac{b}{d_c} \right)^{1/m} \quad (8-41)$$

and

$$\phi_d = 0.16(m+5) \left( \frac{B}{\alpha F_{Dc}} \right)^{0.1} \left( \frac{d}{b} \right)^{0.125m^{0.27}} \quad (8-42)$$

Within the range  $2 \leq M \leq 4$ , dominant meander wavelength and phase shift are relatively insensitive to  $M$ . For width-depth ratios between 10 and 40, a 50% increase (decrease) of  $M$  causes  $\lambda_d$  to decrease (increase) by less than 10%.

#### 8.4.5 Finite-Amplitude Meanders

In the analysis presented, it is assumed that curvature is small and that the meander wavelength is the same whether it is measured along the down-valley axis or along the channel centerline. As the process of meandering progresses, the wavelength measured along the centerline,  $L$ , becomes larger than that measured along the down-valley axis,  $\lambda$ . The ratio  $L/\lambda$  is often termed the sinuosity of the channel. Stochastic analysis, as well as field data (Langbein and Leopold 1966), indicates that a sine-generated alignment persists during the migration of many meanders. The curvature may then be written

$$\frac{1}{r_c} = \frac{1}{R_c} \sin \left( \frac{2\pi s}{L} \right) \quad (8-43)$$

in which  $R_c$  = minimum value of  $r_c$  at apex; and transverse bed slope and velocity may be obtained from the aforementioned equations with  $x$  replaced by  $s$ ,  $k$  by  $2\pi/L$ , and  $k^2 A(t)$  by  $1/R_c$ . It easily can be shown (Langbein and Leopold 1966) that  $L$  and  $R_c$  are related as

$$L = 4.4\pi R_c \sqrt{1 - \frac{\lambda}{L}} \quad (8-44)$$

#### 8.4.6 Prediction Uncertainties

This sample stability analysis shows that the description of meander migration is very sensitive to the manner in which bank erosion is related to primary flow variables. The rates of bank retreat are, of course, particularly sensitive to the values of  $E$  and  $E'$ . The direction of channel migration (lateral expansion versus downstream translation) is different depending on the bank erosion model used. There are not enough data available to determine which of the models, ODG or IKD, performs better. In fact, it is still an open question whether any of them comes even close to complete description of the relationship between flow variables and bank erosion.

In the sample analysis, the transverse bed slope factor  $B$  and the transverse-mass flux factor  $\alpha$  play significant roles. Factor  $B$  represents the bed sediment's motion-resistive properties. Its value has been reported to range from 3 to 6, possibly depending on sediment gradation. For the field cases analyzed by Odgaard (1989b), its value is about 6, which is in agreement with findings of Kikkawa et al. (1976). The transverse-mass flux factor  $\alpha$  corrects the cross-channel flows of water and sediment when these are calculated based on linear distributions of  $u$  and  $d$  in the cross-channel direction. Its value is defined by comparing the calculation (with the continuity equation) of transverse flow of water using linear  $u$  and  $d$  distributions with that computed with measured  $u$  and  $d$  distributions. A value of  $\alpha = 0.4$  is found to be reasonable for field cases.

The sediment transport relation is another uncertain element in the analysis. By using a simple power law (Eq. (8-15)), as is done in the preceding example, all sediment properties are embodied in the exponent  $M$ . Consequently,  $M$  varies from river to river. The value of  $M$  has a significant influence on transverse bed slope in accelerating bend flow, although not as dominant as that of  $B$ . In Odgaard's analysis of field data, a value of  $M = 3$  is used.

It must be kept in mind that the formulas presented in the previous example are based on linear analysis; they cannot be expected to apply to river channels with large curvature. The studies by Nelson (1988), Blanckaert and Graf (2001) and Blanckaert (2003) show that in channels with large curvature,

nonlinear terms in the flow equations can have a significant effect on the description of flow. Blanckaert (2003) demonstrates that in a large-curvature channel bend, an additional secondary flow cell develops near the outer bank. There is even a tendency for stacking of cells. Moreover, multiple point bars may develop as has been demonstrated by Whiting and Dietrich (1993a; 1993b; 1993c).

## 8.5 APPLICATIONS OF FLOW AND STABILITY RELATIONS

The flow and stability analysis in the preceding sections provides formulas and graphs for calculation of (1) rate and direction of channel migration; (2) dominant meander wavelength and phase shift; and (3) velocity and depth distributions in meandering channels.

Input consists of primary channel characteristics: slope  $S$ , width  $b$ , centerline depth  $d_c$ , median grain size  $D$ , friction factor  $f$ , and bank-erosion constants  $E$  and  $E'$ . Lateral and down-valley migration rates are then calculated by Eqs. (8-35) to (8-39), velocity and depth distributions by Eqs. (8-13), (8-14), (8-31), and (8-32), and dominant wavelength and phase lag by Eqs. (8-40) and (8-41) or Fig. 8-8.

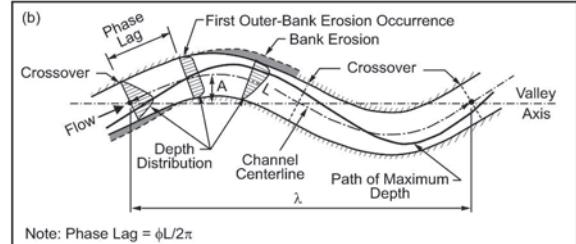
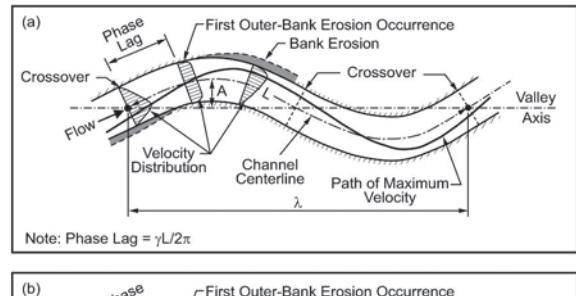
Velocity and depth distribution in channels with arbitrary curvature are obtained by solving Eqs. (8-16) and (8-17) (or (8-24)) with appropriate boundary conditions, and by using Eqs. (8-13) and (8-14). In a constant-radius channel with a long straight approach reach, velocity and depth distributions may be calculated by Eqs. (8-28) and (8-16) together with Eqs. (8-13) and (8-14).

Two alternative bank-erosion models have been tested, the Ikeda et al. model (1981), denoted by IKD, which assumes that the rate of bank retreat is proportional to and in phase with the difference between near-bank and section-averaged velocity (Eq. (8-33)), and a model proposed by Odgaard (1989), denoted by ODG, which relates the rate of bank retreat to increase in near-bank scour depth (Eq. (8-34)).

The principal quantities and concepts are shown in Figs. 8-9(a) and 8-9(b). The figures show the paths of maximum velocity and flow depth through two consecutive meander bends. As indicated, the velocity and depth distributions respond to the change in curvature with a certain lag, which equals  $\gamma L/2\pi$  for velocity and  $\varphi L/2\pi$  for depth. In the IKD bank-erosion model, it is assumed that bank erosion occurs with the same lag as velocity, whereas the ODG model assumes that bank erosion occurs with the same lag as depth. A basic assumption is that  $\lambda$  is nearly equal to  $L$ .

### 8.5.1 Numerical Example

The application of the previously given formulas is best illustrated by an example with data from a hypothetical river (Odgaard 1989b). The bank-full characteristics of the river channel are taken to be  $S = 0.0005$ ;  $b = 150$  m;  $d_c = 6$  m;



**Fig. 8-9.** Applications of flow and stability relations. Definition sketch for principal quantities and concepts: (a) utilizing IKD bank erosion model; (b) utilizing ODG bank erosion model.

$f = 0.08$  (i.e.,  $m = \kappa\sqrt{8/f} = 4$ );  $D = 1$  mm;  $\bar{u}_c = 1.72$  m/s ( $\approx \sqrt{8gSd_c/f}$ );  $M = 3$ ;  $\theta = 0.06$ ; and  $F_{dc} = \bar{u}/\Delta g D = 13.5$ . Transverse bed slope and mass flux factors are  $B = 6$  and  $\alpha = 0.4$ , and erosion constants are  $E = 3 \times 10^{-7}$  and  $E' = 1.5 \times 10^{-7}$ , values typical of rivers in the Midwest (Odgaard 1987).

To estimate dominant wavelength and phase lag, the graphs in Fig. 8-8 (or Eqs. (8-40) and (8-41)) are used. With  $B/\alpha F_{dc} = 1.1$ ,  $m = 4$ , and  $b/d_c = 25$ , the ODG curve yields  $\lambda_d = 2,700$  m and  $\varphi = 0.8$ , and the IKD curve  $\lambda_d = 3,500$  m and  $\gamma = 0.95$ . The phase shifts indicate that the first outer-bank erosion occurrence may occur at a distance from crossover of 0.12 to 0.15 times meander length, or slightly more. Lateral and down-valley migration rates are estimated by Eqs. (8-35), (8-36), or (8-38) and (8-39). The values of pertinent variables are listed in Table 8-1. If  $A = 200$  m, then  $\partial A/\partial t$  (ODG) = 5 m/year; and  $\partial A/\partial t$  (IKD) = 2 m/year. The variation of transverse bed slope through the meander is obtained from Eq. (8-32) with  $A = 200$  m (or, if  $L$  is given instead of  $A$ , with  $k^2 A = 1/R_c$ , and  $R_c$  obtained from Eq. (8-43))

$$S_{tc}(\text{ODG}) = 0.071 \sin\left(\frac{2\pi s}{L} - 0.8\right) \quad (8-45)$$

$$S_{tc}(\text{IKD}) = 0.039 \sin\left(\frac{2\pi s}{L} - 0.8\right) \quad (8-46)$$

and near-bank depth by Eq. (8-14) with  $n = 75$  m and  $d_c = 6$  m (or by Eq. 49 in Odgaard (1986a)). Maximum depth of scour is estimated to be 11.3 m based on ODG and 8.9 m based on IKD, and to occur at  $s/L = 0.38$  (downstream from bend apex). Migration rates of and flow and bed topography

**Table 8-1 Computation of Lateral and Down-Valley Migration Rates for Hypothetical River**

Variable	Bank erosion model	
	ODG	IKD
$kb$	0.35	0.27
$a_1$	0.50	0.50
$a_2$	-1.50	-1.50
$a_3$	1.793	1.793
$a_4$	-2.633	-2.633
$a_5$	1.074	1.074
$h_1$	1.918	1.918
$h_2$	1.312	1.312
$h_3$	0.537	0.537
$e_1$	0.067	0.257
$e_2$	0.416	0.335
$N$	0.075	0.075
$K$	0.778	0.600
$(1/A)\partial A/\partial t$	$8.0 \times 10^{-10} \text{ s}^{-1}$	$3.2 \times 10^{-10} \text{ s}^{-1}$
$c$	11 m/year	8 m/year

in channels with planform different from that of the dominant wave are computed in the same manner with  $k$  = actual wave number. Note that if  $L$  is significantly larger than  $\lambda$ , the calculations should be performed with  $k = 2\pi/L$  instead of  $2\pi/\lambda$ .

## 8.6 SIMULATION OF MEANDER EVOLUTION

Many attempts have been made over the years to develop models that can simulate the evolution or long-term behavior of a meandering river. They range from purely stochastic models to more rigorous process models.

The stochastic models include models based on the “most probable path” assumption with various degrees of simulated randomness (von Schelling 1951; Langbein and Leopold 1966; Thakur and Scheidegger 1968; Surkan and van Kan 1969; Thakur and Scheidegger 1970; Ferguson 1973; 1976; 1977; Stølum 1996; 1997; 1998). These models generally attempt to reproduce the evolution of meander patterns on a large scale with no or little consideration of local floodplain characteristics and local sedimentary processes.

The process models attempt to reproduce the relationship between rates of migration and flow and channel variables quantitatively. The relationship is typically one of the equations listed in Section 8.2.3 or a convolutional relation between migration and curvature. The process models attempt to predict the long-term evolution of rivers, taking

into consideration flood-plain characteristics that modulate, in both time and space, the channel parameters and erosion coefficient (Parker 1982; Beck et al. 1983a; 1983b; Howard 1983; Beck 1984; Beck et al. 1984; Howard and Knutson 1984; Johannesson and Parker 1985; Parker and Andrews 1986; Parker et al. 1988; Crosato 1989; Furbish 1991; Howard 1992; Garcia et al. 1994; Mosselman 1995; Meakin et al. 1996; Sun et al. 1996; Mosselman 1998; Sun et al. 2001a; 2001b; 2001c; Lancaster and Bras 2002). A few process models are developed in which bank erosion is calculated by a separate process model that accounts for near-bank scour, bank collapse, and deposition and removal of bank material (Nagata et al. 2000; Duan et al. 2001; Darby 2002; Darby and Delbono 2002).

The simulations by Stølum (1996; 1997; and 1998) are examples of a combination of process and stochastic modeling. Stølum assumes that meander evolution is the result of two opposing processes: lateral migration, which acts to increase sinuosity, and cutoffs, which act to decrease it. Lateral migration results form bend erosion and deposition, whereas cutoffs result from local geometry. According to Stølum, these opposing processes self-organize the sinuosity into a steady state around a mean value of 3.14, the sinuosity of a circle  $\pi$ .

Recently, several attempts have been made to overcome the limitations of using a calibrated bank erosion coefficient. They include two-dimensional flow-field (mass and momentum), sediment-transport and bank erosion models (Nagata et al. 2000; Duan et al. 2001; Darby 2002; Darby and Delbono 2002).

### 8.6.1 Sample Simulations

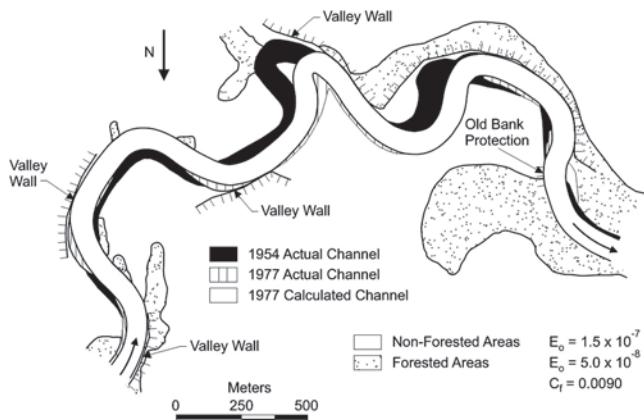
Figure 8-10 shows a simulation by Johannesson and Parker (1985) of the evolution of Red Lake River, Minnesota. The evolution of the channel is obtained by tracking the channel migration over time. Channel width is assumed constant. It is also assumed that the channel centerline is displaced at the same rate as the bank. The migration is described by a Hickin mapping (so called in recognition of the original work of Hickin (1974)), according to which the centerline displacement is described as

$$\frac{dx_p}{dt} = v_e \sin \theta \quad (8-47)$$

$$\frac{dy_p}{dt} = -v_e \cos \theta \quad (8-48)$$

in which  $x_p$  and  $y_p$  are the coordinates of point P of the channel centerline, and  $\theta$  = local angle of centerline with x-axis. See Fig. 8-11.

A slightly modified Johannesson and Parker model was used by Garcia et al. (1994) to simulate the evolution of rivers in Illinois. In order to better determine the magnitude



**Fig. 8-10.** Simulation of the evolution of the Red Lake River, Minnesota, from 1954 to 1977 (from Johannesson and Parker 1985).

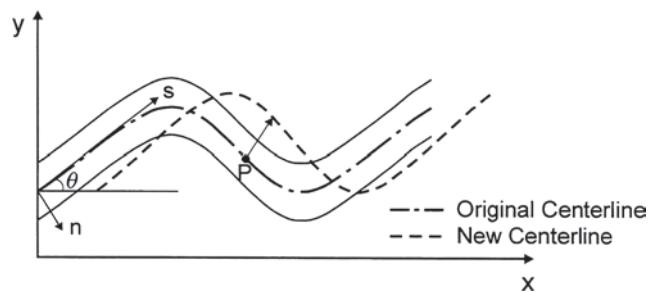
and characteristics of channel shifts, Garcia et al. used the computer program MEANDER, developed by MacDonald et al. (1992). MEANDER measures various components of channel shift, the most important of which is the average normal shift. It also measures sinuosity, time rate of change of sinuosity, and average rate of curvature. Figure 8-12a shows the simulation of the evolution of the Big Muddy River in Illinois (Garcia et al. 1994). Abad and Garcia (2004) have also presented a methodology for simulating the evolution of meandering streams in restoration and naturalization processes. The remeandering of Poplar Creek, Illinois, was analyzed. In this application, Kinoshita curves (Kinoshita 1961; Kinoshita and Miwa 1974; Parker et al. 1982; Parker et al. 1983; Parker and Andrews 1986; Seminara et al. 2001) were used to delineate the new channel. Figure 8-12b shows the planform migration of Poplar Creek at bank-full flow over a period of 100 years. Recently, Abad and Garcia (2006) developed a Windows-based and geographical information system-based interface for the analysis and modeling of planform migration (this program contains the models of Garcia et al. (1994) and MacDonald et al. (1992)).

## 8.7 CHANNEL STABILIZATION

Channel stabilization is an important part of floodplain management. Channels are stabilized to enhance the utility of floodplains, whether for business or recreation. Specific objectives are to (1) prevent bank erosion and loss of property, including bridges and other infrastructure; (2) enhance conveyance, in particular for floods; (3) facilitate traffic (commercial navigation and recreation); and (4) facilitate water usage (utilities, irrigation, diversion, etc.).

### 8.7.1 Strategy

The basic strategy is to stabilize the channel alignment and the channel cross section. The river should maintain a natu-



**Fig. 8-11.** Schematic showing migration of Point P on channel centerline.

ral alignment (a path of easy bends of reverse curvature) and have a cross section that can accommodate the river's water and sediment regime. A good practice is to find a relatively stable reach of the river, determine channel and alignment characteristics for that reach and then apply those characteristics to the reach to be stabilized.

A complementary or supplementary approach is to calculate alignment characteristics using stability theory. This approach is described in detail in the previous sections of this chapter. The approach is based on (1) equations for conservation of mass (water and sediment) and momentum and (2) a stability criterion for sediment particles on the bed. The equations are reduced to those of a damped oscillating system, which is then subjected to a traveling small-amplitude channel alignment wave. It is the growth characteristics of this wave that defines the natural alignment. This approach results in (1) planform development in terms of lateral and downstream migration rates; (2) flow and bed topography in terms of transverse gradients of depth and depth-averaged velocity; and (3) formulas for estimates of dominant meander wavelength and phase shift.

### 8.7.2 Technologies

Several technologies are available for stabilizing a channel. Reviews are given by Biedenharn et al. (1997) and Petersen (1986). The techniques range from the construction of revetments and dikes, vanes or weirs, to dredging. They function by adjusting bank resistance and/or bank erodibility and/or flow and bed topography.

**8.7.2.1 Revetments** Revetments are structures that are aligned parallel to the current. They are used most often to protect eroding banks and to form a smooth bank line. Petersen 1986 classifies revetments into the following types: (1) standard revetment with mattresses (e.g., gabions); (2) woven wooden mattresses; (3) articulated concrete mattresses; (4) standard trench-fill revetments; (5) pile revetments; and (6) stone-fill revetments. Biomattresses are also used to promote vegetation on banks.

**8.7.2.2 Dikes, Submerged Vanes, Bendway Weirs** Dikes, submerged vanes, and bendway weirs are structures placed at

an angle to a current. They are typically used for (1) fairing out sharp bends to a larger radius of curvature to provide a more desirable channel alignment (and thus stabilize concave banks); (2) closing off secondary channels and old bend ways; (3) redistributing flow within a channel cross section (for example, to constrict a channel to increase depth in certain areas or to concentrate a braided river into a single channel); and (4) protecting bridges, utility crossings, and structures along the bank.

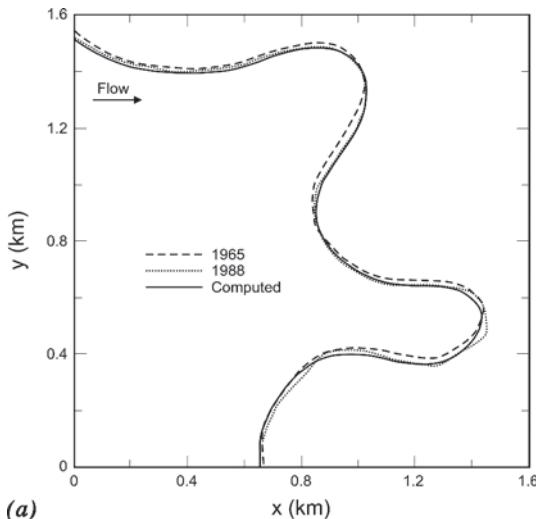
Most dikes are made with stone fill, but other materials are used. Petersen (1986) provides a comprehensive review of standard techniques. The submerged vane technique has received less coverage in the literature and will be described in more detail in a subsequent section. The technique for bendway weirs also has received little coverage so far. Made of rocks, they function like dikes. They are oriented upstream, at an angle with the bank of, typically, 60° to 80°. Reference is made to Pokrefke (1993) and U.S. Army Corps of Engineers (2002).

**8.7.2.3 Dredging** Dredging is the process of moving material from one part of a channel to another or to a disposal site on land. It is used most often for deepening or widening navigation channels or for land reclamation. This technique is also described in detail in Petersen (1986).

### 8.7.3 Submerged Vanes

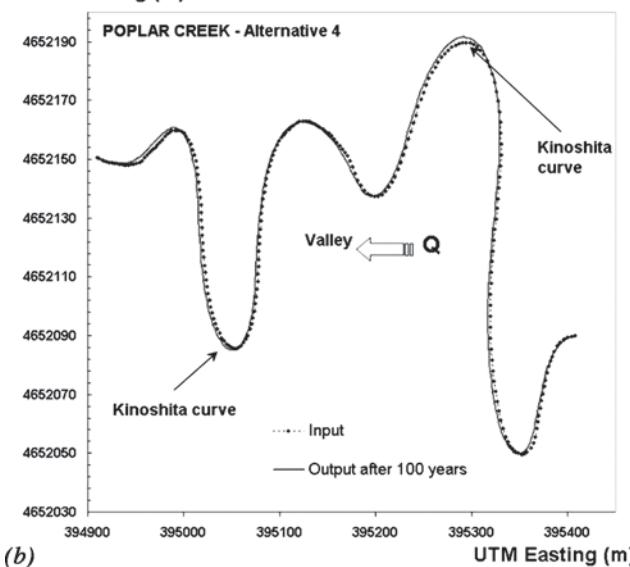
Submerged vanes are small flow-training structures (foils) designed to modify the near-bed flow pattern and redistribute flow and sediment transport within the channel cross section. The structures are installed at an angle of attack 15° to 25° with the flow, and their initial height is 0.2 to 0.4 times local water depth at the design stage. The vanes function by generating secondary circulation in the flow (Fig. 8-14). The circulation alters the magnitude and direction of the bed shear stresses and causes a change in the distribution of velocity, depth, and sediment transport in the area affected by the vanes. As a result, the riverbed aggrades in one portion of the channel cross section and degrades in another (Fig. 8-15).

Vanes or panels for flow training have been discussed previously by Potapov and Pyshkin (1947); Potapov (1950, 1951); Chabert et al. (1961); and Jansen et al. (1979). However, it is only recently that efforts have been made to optimize vane design and document performance. The first known attempts to develop a theoretical design basis were those of Odgaard and Kennedy (1983) and Odgaard and Spoljaric (1986). Odgaard and Kennedy's efforts are aimed at designing a system of vanes to stop or reduce bank erosion in river curves. In such an application, the vanes are laid out so that the vane-generated secondary current eliminates the centrifugally induced secondary current, which is the root cause of bank undermining. Centrifugally induced secondary current in river bends results from the difference in centrifugal acceleration along a vertical line in the flow because of the nonuniform vertical profile of the velocity. The sec-



(a)

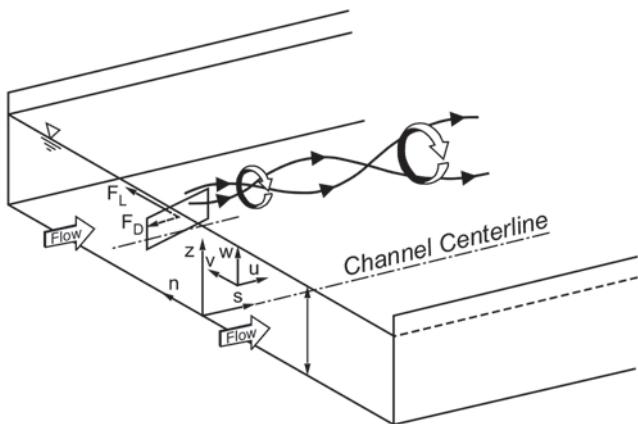
UTM Northing (m)



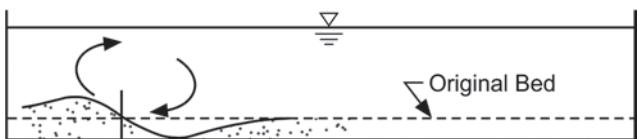
(b)

Fig. 8-12. (a) Simulation of the evolution of the Big Muddy River, Illinois (from Garcia et al. (1994)); (b) prediction of planform migration for Poplar Creek, Illinois (from Abad and Garcia (2006)).

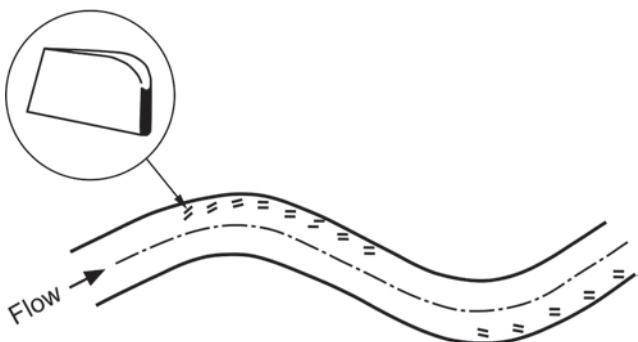
ondary current forces high-velocity surface current outward and low-velocity near-bed current inward. The increase in velocity at the outer bank increases the erosive attack on the bank, causing it to fail. By directing the near-bed current toward the outer bank, the submerged vanes counter the centrifugally induced secondary current and thereby inhibit bank erosion. The vanes can be laid out to make the water and sediment move through a river curve as if it were straight. Figure 8-15 shows a typical layout, and Fig. 8-16 indicates the primary design variables. Field tests with this application have been conducted by Odgaard and Mosconi (1987); Fukuoka and Watanabe (1989); and others. Figure 8-17(a) shows vanes being installed in a bend of the Wapsipicon River, Iowa, in the summer of 1988 during low flow. Figure 8-17(b) shows the same bend 2 years later.



**Fig. 8-13.** Schematic of flow situation showing vane-induced circulation.



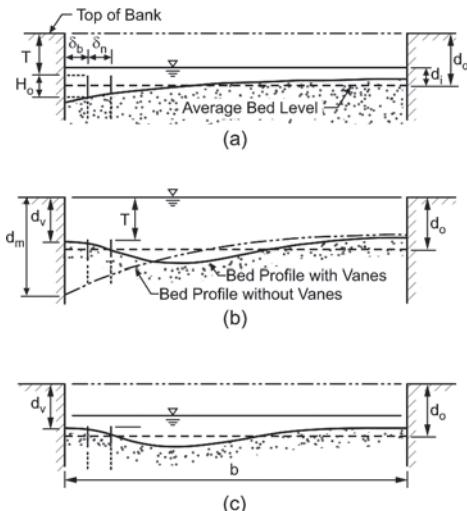
**Fig. 8-14.** Schematic showing vane-induced circulation.



**Fig. 8-15.** Layout of vane systems in a curved channel.

The technique has been further developed to ameliorate shoaling problems in rivers. This application is suggested by laboratory tests by Odgaard and Spoljaric (1986), in which vanes were laid out to change the cross-sectional profile of the bed in a straight channel. The tests showed that significant changes in depth could be achieved without causing significant changes in cross-sectional area, energy slope, or downstream sediment transport. The changes in cross-sectional average parameters are small because the vane-induced secondary current changes the direction of the bed shear stresses by only a small amount.

Further field and laboratory studies (Odgaard and Wang 1991a; 1991b; Pokrefke 1993; Wang et al. 1996; Sinha and Marelius 2000; Zijlstra 2003; Van Zwol 2004) and three-dimensional numerical modeling of the flow around vanes



**Fig. 8-16.** Schematic showing primary design variables and flow sections at (a) installation, (b) subsequent bank-full (design) flow, and (c) subsequent low flow.



(a)



(b)

**Fig. 8-17.** (a) Installation of Iowa vanes in the Wapsipinicon River bend, 1988; (b) Iowa vanes 2 years after installation, 1990.

(Marelius and Sinha 1998; Marelius 2001; Flokstra et al. 2003; Abad et al. 2004) have resulted in an improved understanding of the functioning of vanes and an improved design basis.

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