

CHAPTER 22

Sediment Oxygen Demand (SOD) in Rivers, Lakes, and Estuaries

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22.1 INTRODUCTION

Sediment oxygen demand (SOD) is defined as the rate of oxygen consumption exerted by the bottom sediment on the overlying water (Lee et al. 2000b). The uptake of dissolved oxygen (DO) by sediment is usually attributed to the aerobic decomposition of organic material by microorganisms at the sediment surface and the reaction of oxygen with anaerobic respiration by-products (Sweerts et al. 1991). Because the sediments are a repository for decaying organic material, SOD is often a major contributor to DO depletion in rivers and lakes (Ellis and Stefan 1990; Sweerts et al. 1991; Nakamura and Stefan 1994; Seiki et al. 1994).

The amount of DO in a water body is an indication of the level of microbiological activity and the amount of decaying organic matter present, and often limits the amount of waste that a water body can safely assimilate from municipal and industrial discharges (Hatcher 1986; Lung and Soback 1999). In addition, DO is critical for the sustainability of fish habitat in temperate rivers and lakes. Low DO levels influence the composition of fish fauna, favoring species with tolerance for low oxygen levels, such as common carp (*Cyprinus carpio*), and can cause winterkill in ice-covered lakes (Moyle and Cech 2000). Because SOD is often a major component of the DO budget, accurate estimation of the flux of DO across the sediment-water interface is of paramount importance.

Davis and Lathrop-Davis (1986) have provided a brief history of the early investigations in SOD. Early reports relating water quality to sediments described the effects of wastewater sanitation practices and settled sludge deposits on surface-water quality in heavily populated areas (Hering et al. 1887; Stearns and Drown 1890; Forbes and Richardson 1913; Metcalf and Eddy 1916; Richardson 1928; Purdy 1930). Studies of oxygen dynamics in lakes considered the influence of bottom sediments on DO depletion (Birge 1906; Birge and Juday 1911; Alsterberg 1922; Welch 1935).

Early models of DO dynamics include the work of Streeter and Phelps (1925), Wisely and Klassen (1938), Jansa and Akerlindh (1941), and Bouldin (1967).

Experimental investigations of SOD have considered chemical, biological, and hydrodynamic influences on the rate of DO consumption by sediments (Baity 1938; Fair et al. 1941; Lardieri 1954; Odum 1956; Hayes and MacAulay 1959; Teal and Kanwisher 1961; Isaac 1962; Edwards and Rolley 1965; O'Connell and Thomas 1965; Knowles et al. 1962; Lenard et al. 1962). More recently, the effect of velocity and boundary-layer interactions has become a major focus for SOD research (NCASI 1978; Jorgensen and Revsbech 1985; Whittemore 1986; Hall et al. 1989; Rahm and Svensson 1989; Sweerts et al. 1991; Dade 1993; Nakamura and Stefan 1994; Maran et al. 1995).

Current work related to DO dynamics and SOD is extensive and is being performed in many disciplines. Ecologists and aquatic biologists have recently published several studies on the effect of oxygen stress on the growth and survival of aquatic organisms (Matthews and Berg 1997; Sparks and Strayer 1998; Hale 1999; Harris et al. 1999; Lowell and Culp 1999; Rosas et al. 1999; Buentello et al. 2000; Ruggerone 2000), the effect of DO on habitat utilization and the distribution of organisms (McKinsey and Chapman 1998; Sellers et al. 1998; Elliott 2000), and the relationships between DO in nesting areas and mate selection and reproduction (Jones and Reynolds 1999a; 1999b; Takegaki and Nakazono 1999). In addition, biologists have been interested in the diurnal variation in DO as it relates to metabolism (Guasch et al. 1998; Marzolf et al. 1998; Young and Huryn 1998), the influence of benthic activity on DO consumption (Brekhovskikh et al. 1998; Moodley et al. 1998; Schallenberg and Burns 1999; Schol et al. 1999; Caraco et al. 2000), and relationships between ecosystem functioning and DO (Moore and Townsend 1998; Nishri et al. 1998).

Other recent work of biological importance includes studies of DO and SOD in estuaries, tidal rivers, and oceans (Bertuzzi et al. 1997; Borodkin and Makkaveev 1997; Najjar and Keeling 1997; Summers et al. 1997; Tishchenko et al. 1998; Boyer et al. 1999; Chen et al. 1999; Engle et al. 1999). Additional studies have recently been published on DO and SOD in large regulated rivers and ancient lakes (Martin et al. 1998; Bachmann and Usseglio-Polatera 1999).

Modeling DO dynamics and SOD for management and regulation of water quality continues to be an active area of research in environmental engineering (Chambers et al. 1997; Houck et al. 1997; Chaudhury et al. 1998; Leu et al. 1998; Neal et al. 1998; Sun and Wakeham 1998; Chapra and Runkel 1999; Park and Jaffe 1999; Lung and Sobeck 1999; Ansa-Asare et al. 2000; Kayombo et al. 2000). Thermal stratification in DO-limited systems also continues to be an important area of research, especially in relation to climate change (Fang and Stefan 1997; Jonas 1997; Chapman et al. 1998; Fang et al. 1999; Kelly and Doering 1999; Fang and Stefan 2000). Engineers are also concerned with the effect of velocity and turbulence on SOD as it relates to the physical mechanisms of DO mass transfer (Parkhill and Gulliver 1997; Guss 1998; Hondzo 1998; Mackenthun and Stefan 1998; Josiam and Stefan 1999; Steinberger and Hondzo 1999; Lee et al. 2000a; 2000b).

22.2 DIFFUSIVE SUBLAYER THICKNESS

The mechanisms of mass transfer at the sediment-water interfaces in rivers and lakes involve a combination of molecular diffusion and turbulent transport of DO from the overlying water to the sediment bed. Consider the physical effects occurring in the region adjacent to an interface (Fig. 22-1). In the near-bed region lies the diffusive sublayer, where molecular diffusional transport dominates over

turbulent transport (Levich 1962; Dade 1993). The diffusive sublayer thickness has been reported as $\delta_c = 10 \frac{\nu}{u_*} \text{Sc}^{-1/3}$, where ν is the kinematic viscosity, u_* is the shear stress velocity, $\text{Sc} = \nu/D$ is the Schmidt number, and D is the molecular diffusion coefficient. The dependence of the Schmidt number on water temperature is displayed in Fig. 22-2. According to this relationship, the Schmidt number decreases with increasing water temperature. A typical magnitude of the Schmidt number for DO in water is about 500 at 20°C (Fig. 22-2).

Therefore, the diffusive sublayer is on the order of one tenth the thickness of the viscous sublayer. The presence of this diffusive sublayer can act as a region of resistance to mass transport, and thus may limit chemical reactions occurring at the sediment-water interface (Jorgensen and Revsbech 1985; Hall et al. 1989; Rahm and Svensson 1989; Dade 1993; Steinberger and Hondzo 1999).

Consider a turbulent flow between two parallel plates as an example of shear dispersion (Fig. 22-3). The lower plate is a DO sink, whereas the upper plate constitutes a no-flux boundary condition. The boundary layer between the plates is considered to be either developed, or so slowly varying that the absence of variation in the streamwise direction can be assumed. For developed flow the mean velocity in the vertical direction, y , is zero (note that the turbulent velocity fluctuation v' is not zero), and streamwise velocity is a function only of y . As the fluid moves along the channel, DO diffuses from the fluid to the lower plate, causing the growth of the diffusive sublayer (Fig. 22-3). The bulk DO concentration at any distance x can be expressed as

$$C_B - C_w = \frac{\int_0^{2H} u (C - C_w) dy}{\int_0^{2H} u dy} \quad (22-1)$$

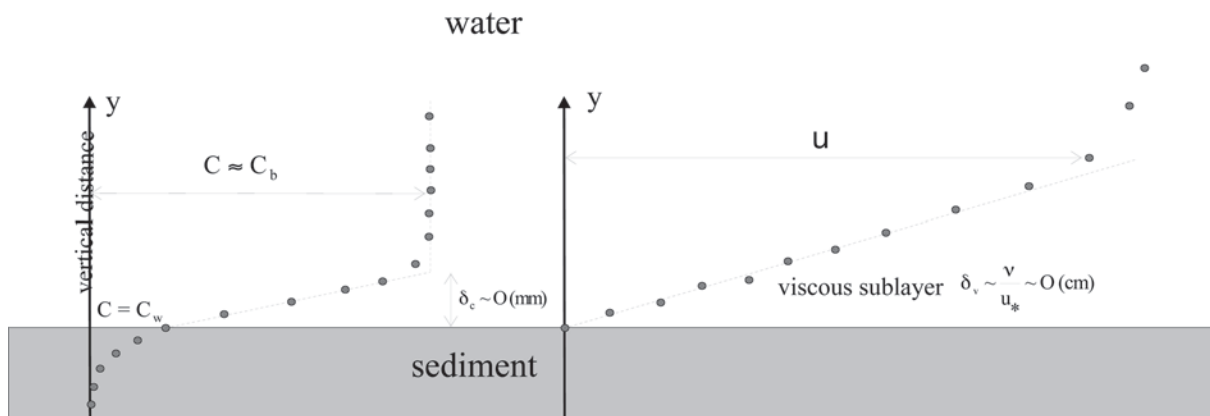


Fig. 22-1. Conceptual sketch for near-bed concentration and velocity distributions (δ_c is the diffusive sublayer; δ_v is the viscous sublayer).

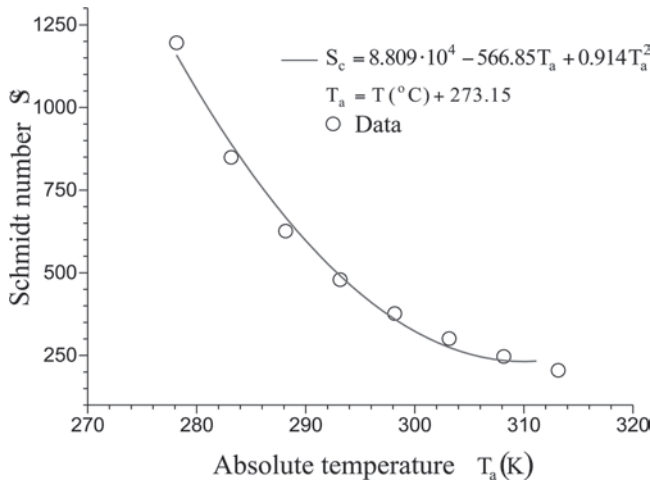


Fig. 22-2. Relationship of Schmidt number (Sc) for dissolved oxygen in water to temperature.

where

- C = mean (time-averaged) DO concentration;
- C_w = DO concentration at the sediment-water interface;
- u = mean velocity in the x direction; and
- H = the half-distance between the plates.

For developed flow the diffusive sublayer experiences no further growth in the streamwise direction. This suggests that the local mass-transfer coefficient is constant in the streamwise direction. The constant mass-transfer coefficient condition is equivalent to the condition that the concentration profiles at all x stations are mappable onto a single curve. This mapping is accomplished by the use of a concentration similarity group $\frac{C-C_w}{C_B-C_w}$. Note that C , C_w , C_B may vary with x , but not the group $\frac{C-C_w}{C_B-C_w}$. From the invariance condition

$$\frac{\partial}{\partial x} \left(\frac{C-C_w}{C_B-C_w} \right) = 0, \quad (22-2)$$

it follows that

$$\frac{C-C_w}{C_B-C_w} = f(y) \quad (22-3)$$

Therefore, the concentration gradient follows from Eq. (22-3) as

$$\frac{\partial C}{\partial y} = (C_B - C_w) \frac{df}{dy} \quad (22-4)$$

The local mass flux per unit area from the fluid to the sediment-water interface can be evaluated as

$$\begin{aligned} M &= -D \frac{\partial C}{\partial y} \Big|_{y=0} = -D(C_B - C_w) \left(\frac{df}{dy} \right) \Big|_{y=0} \\ &= \frac{D}{\delta_c} (C_B - C_w) \end{aligned} \quad (22-5)$$

where M is the mass flux across the sediment-water interface, and δ_c is the diffusive-sublayer thickness or the “unstirred-layer” thickness. The diffusive-sublayer thickness is given by

$$\delta_c = \frac{1}{\left(\frac{df}{dy} \right) \Big|_{y=0}} \quad (22-6)$$

The first derivative of the DO concentration similarity variable profile $\frac{C-C_w}{C_B-C_w}$ is evaluated at the sediment-water interface. The derivative represents the slope of the similarity group in the diffusive sublayer region. The local mass-transfer coefficient is related to the diffusive sublayer thickness by

$$k = \frac{D}{\delta_c} \quad (22-7)$$

The diffusive-sublayer thickness is constant in a developed flow. Therefore, for a given fluid temperature, the mass-

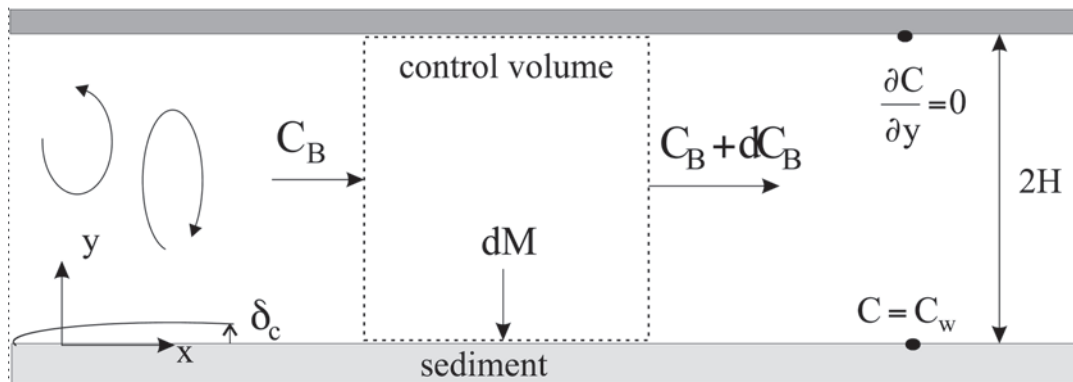


Fig. 22-3. Definition sketch for flow between two parallel plates.

transfer coefficient is constant as well. Once the diffusive-sublayer thickness is available the mass-transfer coefficient can be estimated from the above expression. The constant mass-transfer coefficient and concentration-profile similarity are important characteristics of the developed flow regime. Note that the DO flux across the sediment-water interface should always occur across the diffusive sublayer thickness. The thickness of this layer may be determined by turbulent activity, i.e., by mean frequency of large or small eddies in the fluid above the unstirred layer. The time-averaged micro-DO concentration profiles of natural sediments are shown for four experimental runs in Fig. 22-4 (Steinberger and Hondzo 1999). The data acquisition system, operating at 25 Hz, obtained a minimum of 200 readings at each vertical step. These readings were then averaged as represented by a single measurement point denoted by a circle in Fig. 22-4. The consistent shape of the microprofiles is an indication of the high level of repeatability of the experiments. As shown on the DO microprofiles, all DO arriving at the sediment bed was quickly utilized within the first few millimeters of the surface, indicating that the experiments were water-side-controlled as opposed to reaction-limited. The concentration-sublayer

thickness near the sediment-water interface is the thin diffusive region within which the DO concentration changes rapidly. The diffusive-sublayer thickness ranged from 0.12 to 1.23 mm. In Fig. 22-4 it can be seen that δ_c decreased with higher Reynolds number. This smaller thickness corresponds to an increase in the flux of DO to the sediment bed.

Scaling arguments (Steinberger and Hondzo 1999) yield an expression for δ_c (Fig. 22-5) of the form

$$\delta_c = (19.4 \pm 5.5) \frac{V}{u_*} S_c^{-1/3} \quad (22-8)$$

where

δ_c = diffusive-sublayer thickness in mm

and ± 5.5 are the 90% confidence intervals for the mean slope. Prasad and Russell (2000) derived a similar expression, $\delta_c = 14.5 \nu / u_* S_c^{-1/3}$, that falls within the 90% confidence limits of Eq. (22-8). This result is not in close agreement to

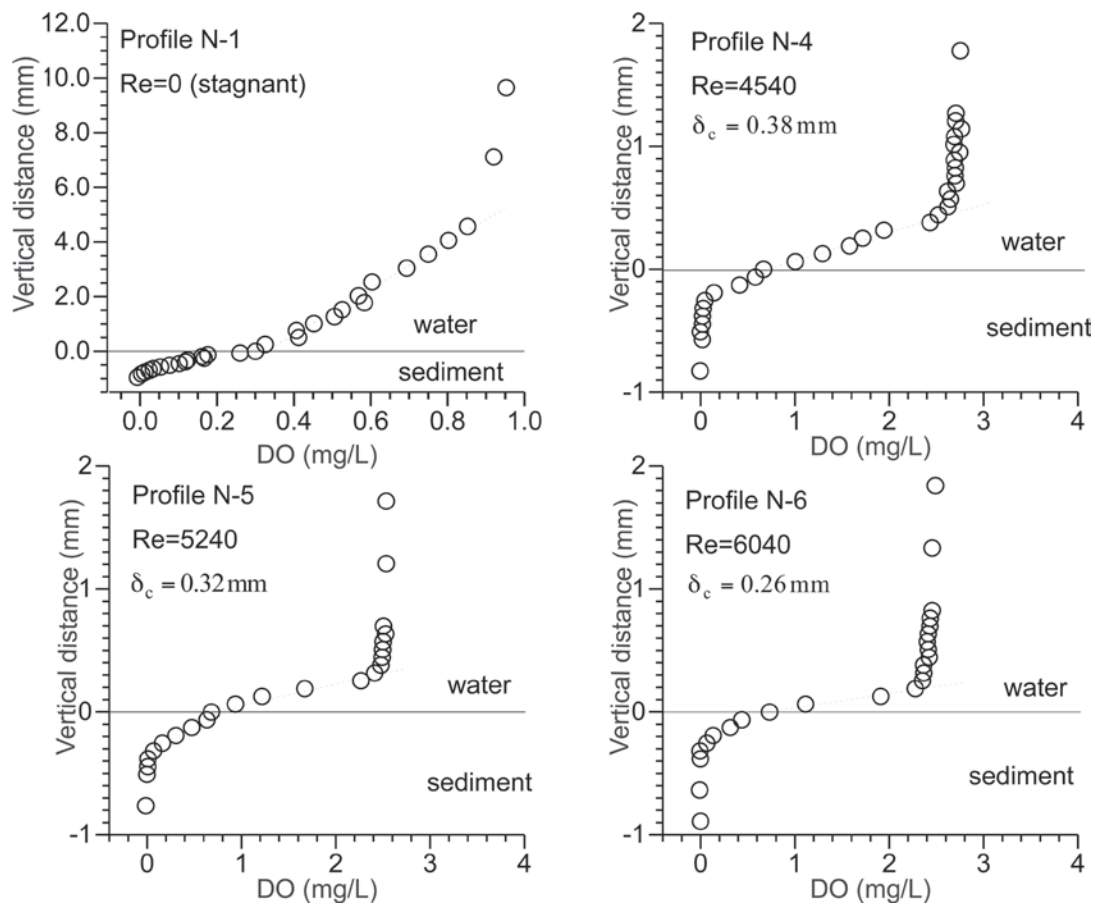


Fig. 22-4. Dissolved oxygen concentration profiles in near-bed region.

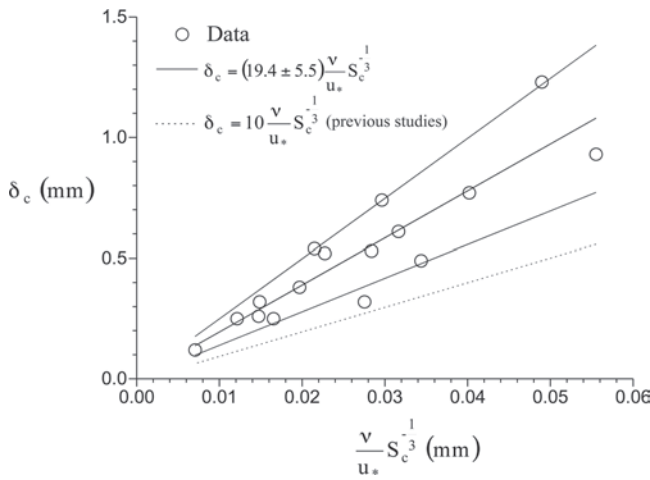


Fig. 22-5. Experimental data for diffusive sublayer thickness (δ_c) as function of viscous length scale (ν/u_*) and Schmidt number (Sc).

the theoretical formulation of previous studies (e.g., Levich 1962; Dade 1993), as presented by the dashed line in Figure 22-5. The cited studies provided useful scaling parameters and an order-of-magnitude estimate for the diffusive sublayer thickness; however, the nature of the function δ_c was not verified by experiment in the previous work. In Fig. 22-5 it can be seen that δ_c decreased with shear stress velocity. An increase in mean flow velocity produces a corresponding increase in the shear stress velocity, and therefore a decrease in the diffusive sublayer thickness.

22.3 MASS-TRANSFER COEFFICIENT

22.3.1 Dimensional Analysis: Bulk Flow

To relate the mass flux of DO to relevant system parameters, dimensional analysis was invoked. Let us consider the situation shown in Fig. 22-1, where a reaction at the sediment surface is causing a reduction in DO concentration of the turbulent water above it. It is assumed that the concentration at the sediment-water interface and the average bulk concentration are known. The remaining independent variables that influence the mass-transfer coefficient appear in the expression

$$k = f(D, H, U, \rho, \mu) \quad (22-9)$$

where

- H = plate half-spacing (boundary layer thickness);
- ρ = density; and
- μ = dynamic viscosity.

The quantity of interest that is dependent upon these parameters is the mass-transfer coefficient. Using k as the dependent variable and D , H , and ρ as the repeating independent variables, π -theorem analysis can be used to obtain the following dimensionless variables, which are important to this system:

$$Sh = \frac{kH}{D}; \quad R = \frac{UH}{\nu}; \quad Sc = \frac{\nu}{D} \quad (22-10)$$

where Sh is the Sherwood number, R is the Reynolds number based on H and U , and Sc is the Schmidt number. It therefore follows that the mass-transfer coefficient of oxygen to the sediment bed can be represented by an expression of the form

$$Sh = f(R, Sc) \quad (22-11)$$

Investigators have reported mass-transfer data in terms of Sherwood-Reynolds-Schmidt number correlations of the form $Sh = aR^bSc^{1/3}$. The values for the constants a and b are typically determined from experimental measurements. In the next section, we will derive a Sherwood-Reynolds-Schmidt number correlation for DO transport in a turbulent flow between two parallel plates.

22.3.2 Mass-Transport Analysis

The mean DO concentration between two parallel plates (Fig. 22-3) is governed by the equation (Fischer et al. 1979)

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = \frac{\partial}{\partial x} \left[(D_{cx} + D) \frac{\partial C}{\partial x} \right] + \frac{\partial}{\partial y} \left[(D_{cy} + D) \frac{\partial C}{\partial y} \right] \quad (22-12)$$

where t is time and D_{cx} and D_{cy} are the turbulent diffusion coefficients. Neither biological oxygen demand nor primary production-respiration is considered in the fluid between the plates. From the invariance condition $\frac{\partial}{\partial x} \left(\frac{C - C_w}{C_b - C_w} \right) = 0$, it follows that $\frac{\partial C}{\partial x} = \frac{\partial C_b}{\partial x} = \text{constant}$ (Appendix). Therefore, for steady developed flow Eq. (22-12) reduces to the form

$$u \frac{dC_B}{dx} = \frac{\partial}{\partial y} \left[(D_{cy} + D) \frac{\partial C}{\partial y} \right] \quad (22-13)$$

This two-process model implies a first-order balance between the advection and vertical diffusion in a developed flow. Equation (22-13) is subject to the boundary conditions

$$C = C_w \text{ at } y = 0 \quad \text{and} \quad \frac{\partial C}{\partial y} = 0 \text{ at } y = 2H \quad (22-14)$$

Equation (22-13) may be integrated twice subject to the boundary conditions (22-14) so that

$$C - C_w = \frac{dC_B}{dx} U \int_0^{y'} \frac{y - 2H}{D_{cy} + D} dy \quad (22-15)$$

where

U = discharge velocity.

To facilitate closed-form solution, we assumed that $u = U$.

The slope of the variation of the streamwise bulk concentration, dC_B/dx , is obtainable from the conservation-of-mass equation for the control volume shown in Fig. 22-3. The net mass flux into the control volume yields

$$\frac{dC_B}{dx} = - \frac{M}{2UH} \quad (22-16)$$

Substituting Eq. (22-16) into Eq. (22-15) yields

$$C - C_w = -M \int_0^{y'} \frac{\frac{y}{D} - 1}{D + D_{cy}} dy \quad (22-17)$$

Introducing $\tilde{y} = yu_*/\nu$ into Eq. (22-17) as the independent variable yields

$$\tilde{C} = \frac{(C - C_w)u_*}{M} = - \int_0^{\tilde{y}'} \frac{\frac{\tilde{y}\nu}{u_*} \frac{1}{2H} - 1}{\frac{1}{S_c} + \frac{D_{cy}}{\nu}} d\tilde{y} \quad (22-18)$$

The integral in Eq. (22-18) can be evaluated using different degrees of approximation. We will neglect the terms D_{cy}/ν and $\tilde{y}\nu/u_*$ in the diffusive sublayer and we will neglect $1/S_c$ in the region outside the sublayer. The diffusive sublayer extends to $\tilde{\delta}_c = 19.4S_c^{-1/3}$ (Eq. 22-8). When these approximations are taken, the following equation for the concentration profile results:

$$\tilde{C} = \frac{(C - C_w)u_*}{M} = - \left(\int_0^{\tilde{\delta}_c} S_c d\tilde{y} + \int_{\tilde{\delta}_c}^{\tilde{y}'} \frac{\tilde{y}\nu}{u_*} \frac{1}{2H} - 1 d\tilde{y} \right) \quad (22-19)$$

The turbulent diffusion of DO for the entire region outside the diffusive sublayer is expressed as (Hondzo 1998)

$$\frac{D_{cy}}{\nu} = \frac{\kappa \tilde{y}}{4Sc_t} \left(2 - \frac{y}{H} \right) \left[1 + 2 \left(1 - \frac{y}{H} \right)^2 \right] \quad (22-20)$$

where κ is the von Karman constant, taken as equal to 0.4, and Sc_t is the turbulent Schmidt number, taken as equal to 1.0. Substituting Eq. (22-20) into Eq. (22-19), integrating, evaluating the value of \tilde{C} at the half distance between the plates ($y = H$), and defining $R_* = u_*H/\nu = R\sqrt{C_f}$ where, C_f is the friction coefficient, yields

$$\begin{aligned} \tilde{C}_c = 19.4S_c^{2/3} + 10S_{ct} \left\{ \frac{1}{6} \ln \left(\frac{19.4S_c^{-1/3}}{R\sqrt{C_f}} \right) \right. \\ \left. + \frac{1}{2} \ln \left(\frac{4.5}{3 + 752S_c^{-2/3} \frac{1}{R^2 C_f} - 77S_c^{-1/3} \frac{1}{R\sqrt{C_f}}} \right) \right. \\ \left. + \frac{\sqrt{2}}{6} \operatorname{atan} \left[\left(\frac{19.4S_c^{-1/3}}{R\sqrt{C_f}} - 1 \right) \sqrt{2} \right] \right\} \end{aligned} \quad (22-21)$$

The local mass-transfer coefficient can be represented by an expression of the form

$$Sh = \frac{R\sqrt{C_f} Sc}{\tilde{C}_c} \frac{(C_c - C_w)}{(C_B - C_w)} \quad (22-22)$$

The ratio $C - C_w / C_B - C_w \approx 1$; thus the nondimensional mass-transfer coefficient follows from Eq. (22-22) as

$$Sh = \frac{R\sqrt{C_f} Sc}{\tilde{C}_c} \quad (22-23)$$

where \tilde{C}_c is given by Eq. (22-21) and C_f is the friction coefficient (e.g., $C_f = 0.0791 \times R^{-1/4}$; Dawson and Trass 1972).

A comparison of Eq. (22-23) with experimental data for DO transfer at the sediment-water interface (Steinberger and Hondzo 1999) is given in Fig. 22-6. Linear regression of the experimental data yields a mean exponent of 0.89 on R in the equation

$$Sh = (0.012 \pm 0.001) R^{0.89 \pm 0.05} Sc^{0.33} \quad (22-24)$$

where

± 0.001 and ± 0.05 = the 90% confidence intervals for the mean coefficient and the mean exponent, respectively.

The exponents of 0.89 closely match the reported values of: 0.87 (Probstein et al. 1972), 0.91 (Harriott and Hamilton 1965), and 0.80 (Colburn 1933; Incropera and DeWitt 1990).

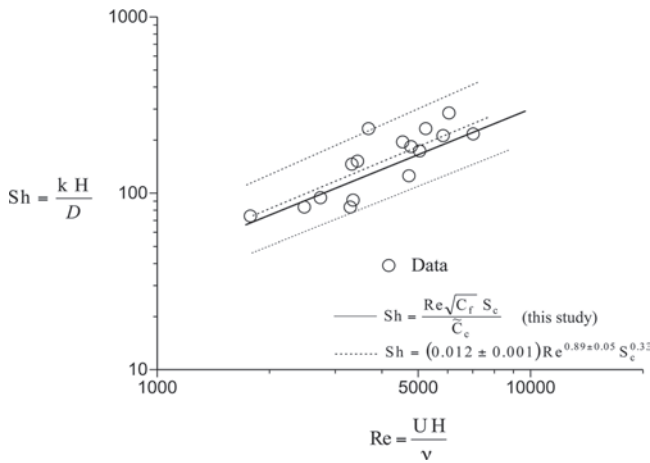


Fig. 22-6. Experimental data for Sherwood number (Sh) as function of Reynolds number (Re) with $Sc = 500$.

The results of the cited studies were not based specifically on diffusional DO transfer at the sediment-water interface. They were based on a semianalytical integral theory for electro dialysis (Probstein et al. 1972), benzoic acid dissolution in glycerine-water solutions (Harriott and Hamilton 1965), and heat transfer data (Colburn 1933; Incropera and DeWitt 1990). The model developed in this study (Eq. (22-23)) compares very well with the data. The predicted values of the Sherwood number are within the 90% confidence intervals of the empirical relationships. Using Eq. (22-24) or Eq. (22-23), it is possible to estimate the mass-transfer coefficient, k , from the bulk measured quantities such as the mean flow velocity, the flow depth, and the mean water temperature.

Example: Consider a wide irrigation channel with a smooth channel bed. The channel has the following characteristics: mean depth $H = 0.5$ m, cross-section average velocity $U = 0.5$ m/s, and mean water temperature $T = 20^\circ\text{C}$. Using Eqs. (22-23) and (22-24) estimate the mass-transfer coefficient and DO flux at the sediment-water interface.

Solution: For water temperature at 20°C the kinematic viscosity is $1.005 \times 10^{-6} \text{ m}^2/\text{s}$. The Schmidt number (Fig. 22-2) is $S_c = 8.809 \times 10^4 - 566.85 \times (20 + 273.15) + 0.914 \times (20 + 273.15)^2 = 464$. The DO diffusion coefficient in water is

$$D = \frac{\nu}{S_c} = \frac{1.005 \times 10^{-6}}{464} \approx 2.16 \times 10^{-9} \text{ m}^2/\text{s}.$$

The Reynolds number for the wide channel is

$$R = \frac{UH}{\nu} = \frac{0.5 \times 0.5}{1.005 \times 10^{-6}} \approx 2.5 \times 10^5.$$

The mass-transfer coefficient can be estimated from Eq. (22-24) as

$$\begin{aligned} k_1 &= 0.012 \frac{D}{H} R^{0.89} S_c^{0.33} \\ &= 0.012 \frac{2.16 \times 10^{-9}}{0.5} (2.5 \times 10^5)^{0.89} 464^{0.33} \\ &= 2.5 \times 10^{-5} \text{ m/s}. \end{aligned}$$

With the mass-transfer coefficient, it is possible to estimate the DO flux using values for DO concentration at the bed and in the bulk flow. Assuming that $C_w = 0$ and $C_b = 6 \text{ mg/L}$, and using Eq. (22-5), the DO flux is

$$\begin{aligned} M_1 &= k_1 (C_w - C_b) = 2.5 \times 10^{-5} (0 - 6) \times 10^3 \\ &= -0.15 \text{ mg/m}^2\text{s}. \end{aligned}$$

The minus sign designates DO flux from water toward the sediment (downward flux).

The nondimensional mass-transfer coefficient from Eq. (22-23) is

$$\begin{aligned} Sh &= \frac{R \sqrt{C_f} S_c}{C_c} \\ &= \frac{2.5 \times 10^5 \times \sqrt{(0.0791 \times (2.5 \times 10^5 \times 4)^{-1/4} \times 464)}}{1151.03} \\ &= 5043.22. \end{aligned}$$

The mass-transfer coefficient is

$$k_2 = \frac{Sh \times D}{H} = \frac{5043.22 \times 2.16 \times 10^{-9}}{0.5} = 2.18 \times 10^{-5} \text{ m/s}.$$

The DO flux is

$$\begin{aligned} M_2 &= k_2 (C_w - C_b) = 2.18 \times 10^{-5} (0 - 6) \times 10^3 \\ &= -0.13 \text{ mg/m}^2\text{s}. \end{aligned}$$

The relative difference between the DO flux and the corresponding mass-transfer coefficient is

$$\left(\frac{M_1 - M_2}{M_1} \right) = \left(\frac{-0.15 + 0.13}{-0.15} \right) \times 100 = 13\%.$$

APPENDIX: DEVELOPED FLOW CONCEPTS

The statement that the DO concentration similarity variable is invariant with x can be expressed as

$$\frac{\partial}{\partial x} \left(\frac{C - C_w}{C_B - C_w} \right) = 0. \quad (22-25)$$

Differentiating and solving for $\frac{\partial C}{\partial x}$

$$\frac{\partial C}{\partial x} = \frac{dC_w}{dx} + \frac{C - C_w}{C_B - C_w} \frac{dC_B}{dx} - \frac{C - C_w}{C_B - C_w} \frac{dC_w}{dx} \quad (22-26)$$

The constant DO flux and mass-transfer coefficient are characteristics of the developed regime.

$$M = k(C_B - C_w) = \text{const} \quad (22-27)$$

where M is the DO flux at the sediment water interface, and k is the mass-transfer coefficient. Therefore, if k is a constant, then $C_B - C_w = \text{const}$. From this condition

$$\frac{dC_B}{dx} = \frac{dC_w}{dx} \quad (22-28)$$

Thus, substituting Eq. (22-28) into Eq. (22-26),

$$\frac{\partial C}{\partial x} = \frac{\partial C_B}{\partial x} = \frac{\partial C_w}{\partial x} \quad (22-29)$$

From Eq. (22-29), it follows that $\partial^2 C / \partial x^2 = 0$ in the developed regime.

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