JOINT LAB ASSIGNMENT #1

GROUP "ARMS"

1 Mathematical Formulation

It is desired to describe the equations of motion of the two-wheeled skid-steer robots in the following nonlinear state-space form:

$$\begin{aligned} \mathbf{x}_{k+1} &= A\mathbf{x}_k + B(\mathbf{x}_k)\mathbf{u}_k + \mathbf{w}_k, \\ \mathbf{y}_k &= \mathbf{c}(\mathbf{x}_k) + D\mathbf{u}_k + \mathbf{v}_k. \end{aligned} \tag{1}$$

Here $\mathbf{w}_k$ and $\mathbf{v}_k$ encapsulate the disturbances and noises of the system. The state at time k is defined to be

$$\mathbf{x}_k = \begin{bmatrix} x_k \\ y_k \\ \theta_k \end{bmatrix}, \tag{2}$$

where x and y are the absolute coordinates of the robot with respect to the bottom-left corner of the rectangular "arena", and θ is the orientation of the robot, defined as the angle between the front of the robot and the positive y-axis in a clockwise manner.

The input to the robots are the two angular velocities of the wheels ω_R and ω_L. These angular velocities are defined with the right-hand rule, where the normal vector faces away from the robot.

There are five sensor outputs which are measured as the robot moves. l_1 and l_2 represent the distance to the nearest wall detected by the front and right facing lasers respectively. Ω is the angular velocity of the car through the axis normal to the floor (equal to $\dot{\theta}$), and b_1 and b_2 are the components of the measured magnetic field along each of the coordinate axes in the car's rotating frame. Thus the state space formulation from (1) is shown below in (3):

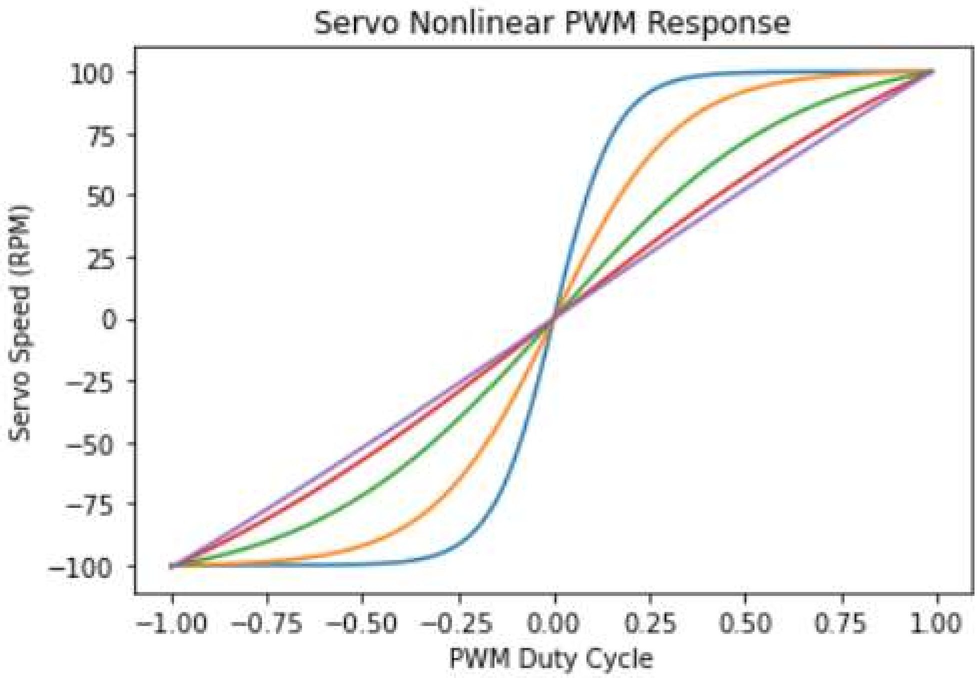
$$\begin{aligned} \begin{bmatrix} x_{k+1} \\ y_{k+1} \\ \theta_{k+1} \end{bmatrix} &= \mathbf{I}_3 \begin{bmatrix} x_k \\ y_k \\ \theta_k \end{bmatrix} + \begin{bmatrix} -\frac{d\Delta t}{4}\sin\theta_k & \frac{d\Delta t}{4}\sin\theta_k \\ -\frac{d\Delta t}{4}\cos\theta_k & \frac{d\Delta t}{4}\cos\theta_k \\ \frac{d\Delta t}{2w} & \frac{d\Delta t}{2w} \end{bmatrix} \begin{bmatrix} (\omega_R)_k \\ (\omega_L)_k \end{bmatrix} + \mathbf{w}_k, \\ \begin{bmatrix} (l_1)_k \\ (l_2)_k \\ \Omega_k \\ (b_1)_k \\ (b_2)_k \end{bmatrix} &= \begin{bmatrix} H - y_k \\ L - x_k \\ 0 \\ B_x\cos(\theta_k) - B_y\sin(\theta_k) \\ B_x\sin(\theta_k) + B_y\cos(\theta_k) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{d}{2w} & \frac{d}{2w} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} (\omega_R)_k \\ (\omega_L)_k \end{bmatrix} + \mathbf{v}_k. \end{aligned} \tag{3}$$

Here d and w are given wheel diameter and width of the robot, Δt is the system timestep, and B_x and B_y are the components of the earth's magnetic field aligned with the x and y-axes respectively. H and L are the height and length of the arena the robot exists in.

2 Simulation

3.1.1 Actuation Non-Linearity

Depending on the severity of the nonlinearity, the curve goes from purely linear to a logistic curve that asymptotically approaches the maximum servo speed.



3.1.2 Sensor Noises

For the 3DM-GX5-10 IMU, the datasheet has the following entry for the gyroscope's noise density:

(0.005°/sec)/sqrt(Hz) (300°/sec)

We interpret this as meaning that at an angular velocity of 300 degrees per second, the "noise density" is 0.005 degrees per second per square root Hertz. This is a relatively unfamiliar unit, but it seems reasonable that this too is an additive noise with no bias, so the expected value is still the actual value. The noise may have a wider Gaussian distribution as the angular velocity increases and a narrower distribution as the angular velocity decreases, given that the noise is provided at a specific angular velocity.

The datasheet also lists a "bias instability" of 8 degrees per hour, which may mean that the measured values can drift from actual values at that rate. As such, it is best not to rely on the data to remain the same when operating this IMU for an extended time.

For the GYVL53L0X laser sensor, we have the following table about accuracy (searching for noise in the datasheet does not produce any results):

Table 12. Ranging accuracy

	Indoor (no infrared)			Outdoor		
Target reflectance level (full FOV)	Distance	33 ms	66 ms	Distance	33 ms	66 ms
White Target (88%)	At 120 cm	4 %	3 %	At 60 cm	7 %	6 %
Grey Target (17%)	At 70 cm	7 %	6%	at 40 cm	12 %	9 %

For this laser sensor, then, there is a percent error from the exact value, which we interpret as meaning that for e.g. the 4% accuracy, the measured value can be anywhere between 96% and 104% of the real value. This is therefore an additive noise centered on the real value (no bias, and the expected value is still the actual value), although we are not sure whether the noise is uniformly distributed or whether it is more of a Gaussian.

Note that we looked at the IMU for the Segway and the laser sensor for the paperbot because the other datasheets were more unclear in their information. The Segway laser sensor has no actual values for its noise, but the MPU9250 IMU for the paperbot has a “rate noise spectral density” of 0.01 degrees per second over sqrt(Hz), similar to the Segway’s IMU.

3.1.3 Simulation

An example path and input data is produced below. The source code of the simulation is available at: https://github.com/WaveWarrior3/Lab1

```
Input Data: (Left Wheel (+ is forward), Right Wheel (- is forward))
(1.0, -0.2) for 100 time steps
(1.0, -1.0) for 100 time steps
(0.2, -1.0) for 100 time steps
(1.0, -1.0) for 500 time steps
```