

For Eq 10 in [F24] in the finite option case, set

$$\begin{aligned} u(x) &= \sum_{i=0}^n w_i^X (x - K_i^X)^+ \\ v(y) &= \sum_{i=0}^n w_i^Y (y - K_i^Y)^+ \\ w(z) &= \sum_{i=0}^n w_i^Z (z - K_i^Z)^+ \end{aligned}$$

where  $n = 5$  and  $K_0^X = K_0^Y = K_0^Z = 0$  (which correspond to the forward contracts which pay  $X$ ,  $Y$  and  $Z$  respectively); we then maximize over the  $3*6=18$  weights to get  $e^{u+v+yw}$ . Also need to normalize  $e^{u+v+yw}$  as in Eq 11 in [F24]. Then if done correctly,  $\mu^*(x, y) = \frac{e^{u(x)+v(y)+yw(y/x)}}{\mathbb{E}(e^{u(X)+v(Y)+Yw(X/Y)})} \mu_X(x) \mu_Y(y)$  will correctly price all three smiles. This approach does not require Sinkhorn iterations or the SVI formula or any interpolation or extrapolation, but you may need to run the scipy minimizer for a few thousand iterations to get good results. You can also try doing this in Mosek as in Project 3 (which will likely be quicker since Mosek uses an interior point solver which is specifically designed for convex optimization problems). You can also look at the forward starter problem using this approach (see chap 5 in FM14), Also remember the SVI formula is not a model.