

Need to numerically solve the Volterra integral eq:

$$\phi(t) = c_1 \int_0^t (t-s)^{H-\frac{1}{2}} f(\phi(s)) \quad (1)$$

(with $c_1 = \frac{1}{\Gamma(\alpha)}$, where $\alpha = H + \frac{1}{2}$) and initial condition $\phi(0) = 0$, with a simple recursive Adams scheme (essentially just Euler scheme for the fractional ODE above) of the form:

$$\phi(t_i) \approx \frac{c_1}{H + \frac{1}{2}} \sum_{j=1}^i [(t_i - s_j)^{H+\frac{1}{2}} - (t_i - s_{j-1})^{H+\frac{1}{2}}] f(\phi(s_{j-1}))$$

where $s_i = \frac{i}{n} dt$, and for our rough Heston application $f(w) = \frac{1}{2}(p^2 - p) + (\rho p \nu - \lambda)w + \frac{1}{2}\nu^2 w^2$. $H = \frac{1}{2}$ corresponds to usual ODE for the standard Heston model. Note there's no randomness here

The characteristic function of the log stock price X_T for rHeston is then given in terms of ϕ as

$$\phi(u) = \mathbb{E}(e^{iuX_T}) = e^{V_0(I^{1-\alpha}\phi)(T) + \lambda\theta(I^1\phi)(T)}$$

(assuming $S_0 = 1$ WLOG), where I^α is the α th order fractional integral operator (see end of pg 16 in EuchRosenbaum article for definition), and the Lewis Fourier inversion formula for the price of a call option with strike $K = e^x$ is then

$$\mathbb{E}((e^{X_T} - e^x)^+) = 1 - e^{\frac{1}{2}x} \int_{-\infty}^{\infty} \frac{e^{-iux}}{u^2 + \frac{1}{4}} \phi(u - \frac{1}{2}i) du$$

So in short, we have to solve Eq 1, then compute $1 - \alpha$ th order fractional integral of it, then do the Fourier inversion