

For CGMY process  $X$

$$\mathbb{E}(e^{pX_t}) = e^{tV(p)}$$

for some function  $V(p)$  (including drift term  $bp$ ) for  $p \in (-G, M)$ . Choose  $b$  so that  $V(1) = 0$  (martingale condition).

Let  $dV_t = \kappa(\theta - V_t)dt + \sigma\sqrt{V_t}dW_t$  denote usual CIR process and let  $\tau_t = \int_0^t V_s ds$ .  $\mathbb{E}(X_t) = V'(0)t \leq \log \mathbb{E}(e^{X_t}) = 0$ , so under the CGMY-SV model, using iterated expectations

$$\begin{aligned} \text{VIX}_T^2 &= -\frac{2}{\Delta} \mathbb{E}(X_{\tau_{T+\Delta}} - X_{\tau_T}) = -\frac{2}{\Delta} \mathbb{E}(\mathbb{E}(X_{\tau_{T+\Delta}} - X_{\tau_T} | \tau_{T+\Delta} - \tau_T)) = -2V'(0) \frac{1}{\Delta} \mathbb{E}(\tau_{T+\Delta} - \tau_T) \\ &= -2V'(0) \frac{1}{\Delta} \mathbb{E}\left(\int_T^{T+\Delta} V_s ds\right) \\ &= -2V'(0)(aV_T + b) \end{aligned}$$

from e.g. Eq 11 in [FS23], for two constants  $a, b$  which depend only on  $\kappa, \theta, \Delta$  (and  $a = 1, b = 0$  if  $\kappa = 0$  since  $V$  is a martingale in this case). As a sanity check note if  $dX_t = -\frac{1}{2}dt + dW_t$ , then  $V(p) = \frac{1}{2}(p^2 - p)$ , and  $-2V'(0) = 1$ , and we recover Eq 11 in [FS23] for the standard Heston model.