Large deviations and asymptotic methods in finance

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The Large deviation principle (LDP)

- Suppose we have a sequence of random variables (X_n) such that X_n is concentrated around x_0 as $n \to \infty$, and for sets A away from x_0, $\mathbb{P}(X_n \in A)$ tends to zero exponentially rapidly in n:

$$\lim_{n\to\infty} \frac{1}{n} \log \mathbb{P}(X_n \in A) \quad = \quad -I(A) \tag{1}$$

$$\text{i.e.} \quad \forall \delta > 0, \quad e^{-n(I(A)+\delta)} \;\le\; \mathbb{P}(X_n \in A) \;\le\; e^{-n(I(A)-\delta)}$$

  for $n = n(\delta)$ sufficiently large, and some **rate function** $I(.) \ge 0$.

- **Example**: for standard Brownian motion (W_t), we have $\lim_{t\to 0} t \log \mathbb{P}(W_t > x) = -\frac{1}{2}x^2$, $\lim_{t\to\infty} \frac{1}{t} \log \mathbb{P}(\frac{W_t}{t} > x) = -\frac{1}{2}x^2$ for $x > 0$, so here $I(x) = \frac{1}{2}x^2$ in both cases.

- **Definition**. A sequence of random variables (X_n) in a topological space S satisfies the LDP with a LSC rate function $I \ge 0$ if we have the following exponential upper/lower bounds for $A \in \mathcal{B}(S)$:

$$\begin{aligned} -\inf_{x\in A^\circ} I(x) \quad &\le \quad \liminf_{n\to\infty} \frac{1}{n} \log \mathbb{P}(X_n \in A) \\ &\le \quad \limsup_{n\to\infty} \frac{1}{n} \log \mathbb{P}(X_n \in A) \le -\inf_{x\in \bar{A}} I(x)\,. \end{aligned}$$

Large-time asymptotics: the Donsker-Varadhan LDP for the occupation measure of the Ornstein-Uhlenbeck process

- Let $dY_t = -\alpha Y_t dt + dW_t$ be an OU process for $\alpha > 0$, and let

$$\mu_t(A) \;=\; \frac{1}{t}\int_0^t 1_A(Y_s)ds$$

denote the proportion of time that Y spends in A, for $A \in \mathcal{B}(\mathbb{R})$. $\mu_t \in \mathcal{P}(\mathbb{R})$ is a *random* probability measure on $\mathbb{R}$.

- Then from [DV76], μ_t satisfies the LDP as $t \to \infty$ in the topology of weak convergence[1], with a non-negative, convex, LSC rate function

$$I_\alpha(\mu) \;=\; -\inf_{u \in \mathcal{D}^+} \int_{-\infty}^{\infty} \frac{\mathcal{L}u}{u}\, d\mu$$

where $\mathcal{L}$ is the infinitesimal generator for Y and $\mathcal{D}^+$ is the set of u in the domain $\mathcal{D}$ of $\mathcal{L}$ with $u \geq \varepsilon > 0$ for some $\varepsilon > 0$.

- $\mu_t \overset{w}{\to} \mu_\infty$ as $t \to \infty$, where $\mu_\infty(y) = (\frac{\alpha}{\pi})^{\frac{1}{2}}\, e^{-\alpha y^2}$ is the unique stationary distribution for Y, and intuitively for $A \in \mathcal{B}(\mathcal{P}(\mathbb{R}))$ with $\mu_\infty \notin A$, $\mathbb{P}(\mu_t \in A) \approx e^{-t \inf_{\mu \in A} I_\alpha(\mu)}$ as $t \to \infty$.

[1] **Generated by** $U_{\phi,x,\delta} = \{\mu \in \mathcal{P}(\mathbb{R}) : |\int \phi d\mu - x| < \delta, \phi \in C_b(\mathbb{R}), \delta > 0, x \in \mathbb{R}$.

- Let $u \in \mathcal{D}$ and $\psi(y,t) = \mathbb{E}_y(u(Y_t)e^{-\int_0^t \frac{\mathcal{L}u}{u}(Y_t)dt}$. Then from the Feynman-Kac formula $\psi(y,t)$ is the unique solution to

$$\partial_t \psi = \mathcal{L}\psi - \frac{\mathcal{L}u}{u}(y)\psi$$

and $\psi(y,t) = u(y)$ is the solution. Using that $u \geq \varepsilon$, we see that

$$u(y) = \mathbb{E}_y(u(Y_t)e^{-\int_0^t \frac{\mathcal{L}u}{u}(Y_t)dt}) \geq \varepsilon\,\mathbb{E}_y(e^{-\int_0^t \frac{\mathcal{L}u}{u}(Y_t)dt}).$$

- Then for $C \in \mathcal{B}(\mathcal{P}(\mathbb{R}))$, we have

$$\begin{aligned}\frac{u(y)}{\varepsilon} &\geq \mathbb{E}_y(1_{\mu_t \in C}\, e^{-\int_0^t \frac{\mathcal{L}u}{u}(Y_t)dt}) = \mathbb{E}_y(1_{\mu_t \in C}\, e^{-t\int \frac{\mathcal{L}u}{u}(y)\mu_t(dy)}) \\ &\geq \mathbb{E}_y(1_{\mu_t \in C}\, e^{-t\sup_{\mu \in C}\int \frac{\mathcal{L}u}{u}(y)\mu(dy)}).\end{aligned}$$

- Re-arranging, we obtain

$$\begin{aligned}\mathbb{P}_y(\mu_t \in C) &\leq \frac{u(y)}{\varepsilon}\, e^{t\sup_{\mu \in C}\int \frac{\mathcal{L}u}{u}(y)\mu(dy)}) \\ \Rightarrow \limsup_{t \to \infty} \frac{1}{t}\log \mathbb{P}_y(\mu_t \in C) &\leq \inf_{u \in \mathcal{D}^+}\sup_{\mu \in C}\int \frac{\mathcal{L}u}{u}(y)\mu(dy).\end{aligned}$$

- If C is compact, then we can interchange the inf and the sup to obtain

$$
\begin{aligned}
\limsup_{t\to\infty} \frac{1}{t} \log \mathbb{P}_y(\mu_t \in C) &\leq \sup_{\mu\in C} \inf_{u\in\mathcal{D}^+} \int \frac{\mathcal{L}u}{u}(y)\mu(dy) \\
&= -\inf_{\mu\in C} - \inf_{u\in\mathcal{D}^+} \int \frac{\mathcal{L}u}{u}(y)\mu(dy) \\
&= -\inf_{\mu\in C} I_\alpha(\mu)\,.
\end{aligned}
$$

- (see [DV75],[Var84]). For the OU process, it can be shown that the rate function simplifies to

$$I_\alpha(\mu) = \frac{1}{2}\int_{-\infty}^{\infty} \psi'(y)^2\,\mu_\infty(dy)$$

for $\mu \ll \mu_\infty$, and $\frac{d\mu}{d\mu_\infty} = \psi^2$. If μ is not absolutely cts wrt μ_∞, $I_\alpha(\mu) = \infty$.

- If $\mu = \mu_\infty$, then clearly $\psi' = 0$ and $I_\alpha(\mu_\infty) = 0$, and we can show that μ_∞ is the unique minimizer of $I_\alpha(\mu)$ (see [FK13]).

Application to stochastic volatility models

- Now consider a stochastic volatility model

$$\begin{cases} dS_t = S_t\sigma(Y_t)dW_t^1\,, \\ dY_t = -\alpha Y_t dt + dW_t^2 \end{cases}$$

and $dW_t^1 dW_t^2 = \rho dt$. Then under a mild sublinear growth condition on $\sigma(.)$, we can use the D-V LDP to show that $\frac{1}{t}\log S_t$ satisfies the LDP as $t \to \infty$ with rate function:

$$I(x) \quad = \quad \inf_{\mu\in\mathcal{P}(\mathbb{R})}\left[\frac{(x-M(\mu))^2}{2\bar{\rho}^2 F(\mu)} + I_\alpha(\mu)\right]$$

(see Forde&Kumar[FK13]), where $\bar{\rho} = \sqrt{1-\rho^2}$, $b(y) = \alpha y\sigma(y) - \frac{1}{2}\sigma'(y)$, $F(\mu) = \int \sigma^2(y)\mu(dy)$, $G(\mu) = \int b(y)\mu(dy)$, $M(\mu) = -\frac{1}{2}F(\mu) + \rho G(\mu)$.

- Use the **Ritz method** to compute $I(x)$ numerically. Can also deal with a more general process $dY_t = \beta(Y_t)dt + \alpha(Y_t)dW_t^2$ if β has mean reverting behaviour for $|y| \gg 1$, and we can incorporate **stochastic interest rates** with an independent **CIR short rate** process (important for $t \gg 1$).

Small-time asymptotics: the heat kernel expansion

- Consider a general uncorrelated stochastic volatility model for a log stock price X_t:

$$\begin{cases} dX_t &= -\frac{1}{2}Y_t^2 dt \,+\, Y_t dW_t^1 \,, \\ dY_t &= \mu(Y_t)dt + \alpha(Y_t)dW_t^2 \end{cases} \tag{2}$$

- Under suitable regularity conditions on μ, α, the transition density for (X_t, Y_t) satisfies

$$p_t(\mathbf{x},\mathbf{y}) \;\sim\; \frac{1}{2\pi t} u_0(\mathbf{x},\mathbf{y})\, e^{-\frac{1}{2}d(\mathbf{x},\mathbf{y})^2/t \,+\, A(\mathbf{x},\mathbf{y})} \qquad (t \to 0)$$

where $d(\mathbf{x},\mathbf{y}) = \inf_{\gamma\in C[0,1]:\gamma(0)=x,\gamma(1)=y}\int_0^1 \sqrt{(\sum_{i,j} g_{ij}\frac{d\gamma_i}{dt}\frac{d\gamma_j}{dt})}\, dt$ is the shortest distance from $\mathbf{x} = (x_0, y_0)$ to $\mathbf{y} = (x_1, y_1)$ under the metric $ds^2 = \sum_{ij} g_{ij}dx^i dx^j$, where $g_{ij} = a_{ij}^{-1}$ is the inverse of the diffusion matrix for the model (2), and

$$u_0(\mathbf{x},\mathbf{y}) \,=\, |g|(\mathbf{x})^{-\frac{1}{2}}\det\Big(-\frac{\partial^2\phi(\mathbf{x},\mathbf{y})}{\partial x_i \partial y_j}\Big)\,,\;\; A(\mathbf{x},\mathbf{y}) \,=\, \int_0^1 \sum_{ij} g_{ij}\mathcal{A}^i \frac{d\gamma^{*j}}{ds}\, ds$$

and $|g| = |\det g_{ij}|$, $\phi(\mathbf{x},\mathbf{y}) = \frac{1}{2}d(\mathbf{x},\mathbf{y})^2$,
$\mathcal{A}^i = b^i - \frac{1}{2}\sum_j \frac{1}{\sqrt{|g|}}\partial_j(\sqrt{|g|}\,g^{ij})$ where $b = (-\frac{1}{2}y^2, \mu(y))$.

- By integrating in the y variable and again using **saddlepoint methods**, we can integrate the heat kernel expansion to obtain a small-time expansion for call options and **implied volatility**, see Armstrong,Forde&Zhang[AFZ13] for details.

- Our proof also requires use of the **Davies upper bound** for the heat kernel on a Riemmanian manifold with **curvature** bounded from below).

- We can also deal with **local volatility** and **non-zero correlation** using a gauge transformation (for the latter, we have to make restrictions on μ).

Tail asymptotics: the SABR model with $\rho \neq 0$, and perturbations of stoc vol models (NEW)

- Consider the correlated SABR model S_t with $\beta = 1$:

$$\begin{cases} dS_t &= S_t Y_t dW_t^1 \,, \\ dY_t &= \sigma Y_t dW_t^2 \end{cases}$$

with $dW_t^1 dW^2 = \rho dt$, $\rho \leq 0$.

- Using saddlepoint methods, when $y_0 = \sigma = 1$, we have the following right tail behaviour for the transition density of S_t:

$$\begin{aligned} p_t(x) \quad &= \quad \frac{1}{2\sqrt{\pi}} e^{-\frac{\rho}{\bar{\rho}^2} - \frac{t}{8} - \frac{\log^2 2}{2t}} \frac{x^{-1-\frac{1}{\bar{\rho}^2}}}{C(x)^{\frac{3}{2}}} \left[\frac{|\rho| D(x)}{\bar{\rho}^2}\right]^{\frac{1}{2} - \frac{\log 2}{t}} \left[\log\left(\frac{|\rho| D(x)}{\bar{\rho}^2}\right)\right]^{-\frac{1}{2}} \\ &\times \exp\left(-\frac{1}{2t}\left[\log\left(\frac{|\rho| D(x)}{\bar{\rho}^2}\right)\right]^2\right) \left(1 + O([\log(|\rho| C(x)/\bar{\rho}^2)]^{-\frac{1}{2}})\right) \end{aligned}$$

where $u_0(x)$ is the unique positive solution to $\frac{\rho}{\bar{\rho}^2} D(x) = \frac{\log u_x}{u_x}$ and $D(x) = C(x) + \log x + \rho > 0$, $C(x) = \sqrt{(\log x + \rho)^2 + \bar{\rho}^2} > 0$. We can easily adapt this for $y_0, \sigma \neq 0$, which extends the result in [GS10] for the uncorrelated case (see Forde&Zhang[FZ14]).

- Using Girsanov's theorem, we can extend the previous result to characterize the right-tail behaviour of transition density for a **perturbed** SABR model where $dY_t = g(Y_t)dt + \sigma Y_t dW_t^2$, and also for the Heston and Sten-Stein models.

Large deviations and transaction costs

- Consider a market with one safe asset $S_t^0 = 1$, and a risky asset $dS_t = S_t(\mu dt + \sigma dW_t$ with ask (buying) price S_t.
- We assume that the bid (selling) price is $(1-\varepsilon)S_t$, where $\varepsilon \in (0,1)$ is the relative bid-ask spread.
- (φ_t^0, φ_t) is called an **admissible self-financing trading strategy** if both processes are right cts and of a.s. finite variation (or else we incur infinite costs in finite time) which satisfy the **self-financing condition**:

$$d\varphi_t^0 \quad = \quad -S_t d\varphi_t^{\uparrow} + (1-\varepsilon)S_t d\varphi_t^{\downarrow} .$$

- The values of the safe position X_t and of the risky position Y_t then evolve as:

$$\begin{aligned} dX_t &= -S_t d\varphi_t^{\uparrow} + (1-\varepsilon)S_t d\varphi_t^{\downarrow}, \\ dY_t &= \mu Y_t dt + \sigma Y_t dW_t + S_t d\varphi_t^{\uparrow} - S_t d\varphi_t^{\downarrow} . \end{aligned}$$

where $\varphi_t = \varphi_t^{\uparrow} - \varphi_t^{\downarrow}$ is the difference of two increasing processes.

- Let Υ_t denote arithmetic Brownian motion with drift $\mu - \frac{1}{2}\sigma^2$, volatility σ and **reflecting** barriers at 0 and b:

$$d\Upsilon_t = (\mu - \frac{1}{2}\sigma^2)dt + \sigma dW_t + dL_t - dU_t$$

  with $\Upsilon_0 = x$, and $b > 0$, where L_t and U_t are the associated local time processes with $\{dL_t > 0\} \subseteq \{\Upsilon_t = 0\}$ and $\{dU_t > 0\} \subseteq \{\Upsilon_t = b\}$.
- The liquidation value of the wealth associated to an admissible strategy is given by $\Xi_t^{\varphi} = \varphi_t^0 + \varphi_t^+(1-\varepsilon)S_t - \varphi_t^- S_t$.
- It can be shown that under exponential utility $U(x) = -e^{-\theta x}$ for $\theta > 0$, the φ_t process which maximizes

$$\liminf_{T\to\infty} -\frac{1}{\theta T} \log \mathbb{E}(e^{-\theta \Xi_T^{\varphi}})$$

  satisfies

$$d\varphi_t/\varphi_t = d\log\varphi_t = dL_t - dU_t\,.$$

  for some $b > 0$ (see [GM13]). Thus we buy when $\Upsilon_t = 0$ and sell when $\Upsilon_t = b$ and do nothing in between, so the interval $(0, b)$ is called the **no-trade region** (can show that width of the region is $O(\varepsilon^{\frac{1}{3}})$ for $\varepsilon \ll 1$).

- The *relative share turnover* is then defined as

$$\int_0^t \frac{d\|\varphi_s\|}{\varphi_s} \quad = \quad L_t + U_t$$

which is a measure of **trading volume**.

- Using martingale arguments, we compute $\mathbb{E}_x(e^{-\alpha(L_\tau+U_\tau)})$ where τ is an independent $\mathrm{Exp}(\lambda)$ random variable.

- Using this and a Tauberian result, in [FKZ13] we show that $\frac{1}{t}\int_0^t \frac{d\|\varphi_s\|}{\varphi_s} ds$ satisfies the LDP on $[0,\infty)$ as $t \to \infty$ with a convex rate function given by the Fenchel-Legendre transform

$$\Lambda^*(x) \quad = \quad \sup_{\alpha\in\mathbb{R}}[\alpha x - \Lambda(\alpha)]$$

for $x \geq 0$, where $\Lambda(\alpha) = (\alpha^*)^{-1}(\alpha)$ and

$$\alpha^*(\lambda) \quad = \quad \gamma \coth(\gamma b) - \sqrt{\gamma^2/\sinh^2(\gamma b) + \delta^2}$$

and $\delta = \frac{\mu}{\sigma^2} - \frac{1}{2}$, $\gamma = \sqrt{\delta^2 + 2\lambda/\sigma^2}$.

Asymptotics for the Brownian random bridge

- Let $\xi_{tT} = \frac{t}{T}X + \beta_{tT}$ where β_{tT} is a standard 1-d Brownian bridge with $\beta_{0T} = \beta_{TT} = 0$ and $X \sim \nu$ is independent of β.
- Now let $\mathcal{F}_t = \sigma(\{\xi_{sT}\}_{0 \le s \le t})$ be the **filtration generated by** ξ. Then X is $\mathcal{F}_T$-measurable and can be shown that ξ_{tT} is a **Markov** process wrt $\mathcal{F}_t$ but X is not $\mathcal{F}_t$-measurable for $t < T$.
- $X = \xi_{TT} \sim \nu$ at time 0, and X becomes **"known"** at T, but at $t < T$ we only know the value of ξ_{tT} but not X or β_{tT}, so β_{tT} is the **noise**. Model **asset price process** as $X_{tT} = \mathbb{E}(X \,|\, \mathcal{F}_t) = \mathbb{E}(X \,|\, \xi_{tT})$, then $dX_{tT} = \frac{Var(X \,|\, \xi_{tT})}{T-t} dW_t$, for some $\mathcal{F}_t$-Brownian motion.
- From Macrina et al.[HHM11], assuming ν has a smooth density:

$$\mathbb{P}(X \in dy \,|\, \xi_{tT} = x) = \frac{e^{-\frac{1}{2}[\frac{(y-x)^2}{\varepsilon} - \frac{y^2}{T}]}\nu(y)dy}{\int e^{-\frac{1}{2}[\frac{(z-x)^2}{\varepsilon} - \frac{z^2}{T}]}\nu(z)dz} \sim C(x)e^{-\frac{(y-y^*(x))^2}{2\frac{T}{t}\varepsilon}}\nu(y)dy$$

$$\frac{Var(X \,|\, \xi_{tT} = x)}{T-t} = 1 + [\frac{1}{T} + \frac{v(y^*)v''(y^*) - v'(y^*)^2}{v(y^*)^2}]\varepsilon + O(\varepsilon^2)$$

as $t \to T$, where $\varepsilon = T - t$ and $y^*(x) = \frac{T}{t}x \sim X_{tT}$ is the **saddlepoint**. Thus at leading order, X_{tT} behaves like Brownian motion for $T - t \ll 1$ *.

SPDEs

- Consider the **stochastic Burgers equation** with small-noise:

$$\partial_t u^\varepsilon(t,x) = \nu\partial^2_{xx}u^\varepsilon(t,x) + \sqrt{\varepsilon}\dot{W}(t,x) + u^\varepsilon(t,x)\partial_x u^\varepsilon(t,x) \quad (4)$$

  on $[0,T]\times[0,1]$, with Dirichlet boundary condition $u^\varepsilon(t,0) = u^\varepsilon(t,1) = 0$ and $u^\varepsilon(0,x) = u_0(x)$ for $u_0 \in L^2[0,1]$. $\dot{W}$ is **space-time white noise**, i.e. a Gaussian random set function with $W_A \sim N(0,\mathrm{Leb}(A))$ for $A \in \mathcal{B}([0,T]\times[0,1])$ and $\mathbb{E}(W_A W_B) = \mathrm{Leb}(A\cap B)$. ν is the viscosity and we set $\nu = 1$.
- (4) arises in the study of turbulent fluid motion.
- $W(t,x) := W_{[0,t]\times[0,x]}$ is the **Brownian sheet**.
- We can give a rigorous meaning to (4) by writing the solution as

$$\begin{aligned} u^\varepsilon(t,x) &= \int_0^1 G_t(x,y)u_0(y)dy + \sqrt{\varepsilon}\int_0^t\int_0^1 G_{t-s}(x,y)W(ds,dy) \\ &- \sqrt{\varepsilon}\int_0^t\int_0^1 \partial_y G_{t-s}(x,y)g(u^\varepsilon(t,y))dyds \qquad (5) \end{aligned}$$

  where $g(r) = \frac{1}{2}r^2$ and the stochastic integral is defined in similar way to the classical Itō integral, and $G_t(x,y)$ is the Green kernel for $\partial_t - \frac{1}{2}\partial^2$ with the same Dirichlet boundary conditions (test

- The solution has a modification which lies in $C([0,T];L^2[0,1])$, so the modification is a continuous $L^2[0,1]$-valued stochastic process.

- Now let $\mathcal{H}=\{h:\dot{h}=\frac{\partial^2 h}{\partial t\partial x}\in L^2([0,T]\times[0,1])\}$ with norm $\|h\|=(\int_0^t\int_0^x \frac{\partial^2 h}{\partial t\partial x}(u,z)dudz)^{\frac{1}{2}}$ and let Z^h denote the map which takes h to the solution of the associated (non-stochastic) PDE with $\varepsilon=1$:

$$\partial_t u(t,x) \;\;=\;\; \partial^2_{xx}u(t,x)+\dot{h}(t,x)+u(t,x)\partial_x u(t,x)\,.$$

- Under suitable regularity on the initial datum and coefficients, u^ε satisfies the LDP on $C([0,T],L^2[0,1])$ with rate function

$$I(f) \;\;=\;\; \begin{cases} \inf\{\ \frac{1}{2}\int_{[0,T]\times[0,1]}(\frac{\partial^2 h}{\partial s\partial t})^2\,dsdt\ :\ Z^h=f\}\ , & f\in Im(Z^h) \\ +\infty \qquad\qquad (otherwise) \end{cases}$$

($\frac{1}{2}\int_{[0,T]\times[0,1]}(\frac{\partial^2 h}{\partial s\partial t})^2\,dsdt$ is the rate function for the small-noise Brownian sheet).

- Can also derive large-time Donsker-Varadhan-type LDP for the occupation measure of the stochastic Burger eq. The stationary measure and the realized occupation measure are now probability measures on $H = L^2[0,1]$.

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