

Consider the integrated variance process $A_t = \int_0^t V_s ds$ which satisfies an eq of the form:

$$A_t = G_0(t) + \int_0^t \kappa(t-s) W_{A_s} ds$$

for some $\kappa \in L^1$ (this allows for $\kappa(t) = \text{const.} \times t^{H-\frac{1}{2}}$ for $H > -\frac{1}{2}$). Consider a discrete-time Monte Carlo scheme for A :

$$A_{j\Delta t} = G_0(t)(j\Delta t) + \sum_{k=1}^j b_{j,k} W_{A_{k\Delta t}} \Delta t \quad (1)$$

for $j = 1, 2, \dots$, where $\tilde{W}_t = W_{A_{j\Delta t}} - W_{A_{(j-1)\Delta t}}$ and

$$b_{j,k} = \int_{t_{k-1}}^{t_k} K(t_j - s) ds$$

and $t_k = k\Delta t$. Now re-write (1) in terms of increments as

$$\Delta A_j := A_{j\Delta t} - A_{(j-1)\Delta t} = Z_j + b_{j,j}(W_{A_{j\Delta t}} - W_{A_{(j-1)\Delta t}})\Delta t$$

for some quantity Z_j which is known at time step $j-1$. Then using the App E argument below, ΔA_j has an Inverse Gaussian distribution, and the ΔA_j 's are independent.

AppE argument recap

$$X_t = G_0(t) + \sigma W_{X_t}. \quad (2)$$

Now let

$$Y_t = -t + \sigma W_t \quad (3)$$

and set $\tilde{X}_t = H_{-G_0(t)}$, where $H_b = \inf\{t : Y_t = b\}$. Then setting $t \mapsto \tilde{X}_t$ in (3), we see that

$$-G_0(t) = -\tilde{X}_t + \sigma W_{\tilde{X}_t}$$

i.e. \tilde{X} satisfies the same equation as X_t in (2), and H_b has an Inverse Gaussian distribution (first hitting time to a barrier for a BM with drift).