The Rough Heston GMC

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We can easily verify that for driftless rHeston model (appropriately normalized) $\mathrm{Cov}(V_s^H, V_t^H) = R_H(s,t)$, so this covariance tends to $R(s,t)$ as $H \to 0$, so V^H tends to a non-negative non-Gaussian random field V with covariance $R(s,t)$ (same as RL field). Then

$$\begin{aligned}\mathbb{E}(e^{\gamma V_s + \gamma V_t}) &= \mathbb{E}(e^{\gamma \int_0^t f(t-u) V_u du}) &= e^{2\gamma V_0 + \frac{1}{2} V_0 \nu^2 \int_0^t \psi(u)^2 du} \\ & \mathbb{E}(e^{\gamma V_s})\mathbb{E}(e^{\gamma V_t}) &= e^{2\gamma V_0 + \frac{1}{2} V_0 \nu^2 \int_0^t \phi(u)^2 du + \frac{1}{2} V_0 \nu^2 \int_0^s \phi(u)^2 du}\end{aligned}$$

where $f(u) = \delta(u) + \delta(u - (t-s))$.

$$\begin{aligned}\psi(u) &= \gamma(ct^{-\frac{1}{2}} 1_{u>t} + cs^{-\frac{1}{2}}) 1_{u>s} + \frac{1}{2}\nu^2 \int_0^t (t-v)^{-\frac{1}{2}} \psi(v)^2 dv \\ \phi(t) &= \gamma t^{-\frac{1}{2}} + \frac{1}{2}\nu^2 \int_0^t (t-u)^{-\frac{1}{2}} \phi(u)^2 du\end{aligned}$$

and $c = \frac{1}{\Gamma(\alpha)}$. Then can define the rHeston GMC associated with

$$Z_t = \frac{e^{\gamma V_t}}{\mathbb{E}(e^{\gamma V_t})}$$

as $H \to 0$