Consider the integrated variance process $A_t = \int_0^t V_s ds$ which satisfies an eq of the form

$$A_t \;=\; G_0(t) + \nu \int_0^t \kappa(t-s) W_{A_s} ds$$

Now consider a discrete-time MC scheme for A:

$$A_{j\Delta t} \;=\; G_0(t)(j\Delta t) + \sum_{k=1}^{j} b_{j,k} W_{A_{k\Delta t}} \Delta t \;=\; (...) + b_{j,j} W_{A_{j\Delta t}} \Delta t \tag{1}$$

for $j = 1, 2, ...$, where $(...)$ is, $\tilde{W}_t = W_{A_{j\Delta t}} - W_{A_{(j-1)\Delta t}}$ and

$$b_{j,k} \;=\; \int_{t_{k-1}}^{t_k} K(t_j - s) ds$$

and $t_k = k\Delta t$. Now re-write (1) in terms of increments as

$$\Delta A_j \;:=\; A_{j\Delta t} - A_{(j-1)\Delta t} \;=\; Z_j + b_{j,j}(W_{A_{j\Delta t}} - W_{A_{(j-1)\Delta t}})\Delta t$$

for quantity Z_j which is known at time $j-1$. Then using the App E argument below, ΔA_j has an inverse Gaussian distribution, and the ΔA_j's are independent.

AppE argument recap

$$X_t \;=\; G_0(t) + \sigma W_{X_t}. \tag{2}$$

Now let

$$Y_t \;=\; -t + \sigma W_t \tag{3}$$

and set $\tilde{X}_t = H_{-G_0(t)}$, where $H_b = \inf\{t : Y_t = b\}$. Then setting $t \mapsto \tilde{X}_t$ in (3), we see that

$$-G_0(t) \;=\; -\tilde{X}_t + \sigma W_{\tilde{X}_t}$$

i.e. $\tilde{X}$ satisfies the same equation as X_t in (2).