SDM366 Project 1
Due April 13, 2025

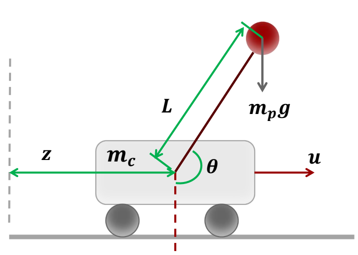
- Please attach all of your codes and relevant figures. To receive credits, please write down all the necessary steps leading to your final solution. Make sure your discussions about the results are clear, brief, and to the point.
- Please form a team with 2 people. Each team needs to turn in one report. Please make sure everyone in the team is involved and understand the solutions.
- This course has a zero-tolerance policy on plagiarism.

Consider a cart of mass M slides on a horizontal frictionless track, and is pulled by a horizontal force $u(t)$. On the cart an inverted pendulum of mass m_p is attached via a frictionless hinge, as shown in the figure below. The pole's center of mass is located at a distance L from the revolute joint located at the center of mass of the cart. The variable $\theta(t)$ is the counter-clockwise angle of the pendulum (zero is hanging straight down). The dynamics of the system is given by:

$$\begin{cases} \ddot{z} = \frac{1}{m_c + m_p \sin^2\theta}\left[u + m_p \sin\theta\left(L\dot{\theta}^2 + g\cos\theta\right)\right] \\ \ddot{\theta} = \frac{1}{L(m_c + m_p \sin^2(\theta))}\left[-u\cos(\theta) - m_p L\dot{\theta}^2\cos\theta\sin\theta - (m_c + m_p)g\sin\theta\right] \end{cases} \tag{1}$$

where $m_p = 1$, $m_c = 10$, $g = 9.81$, and $L = 0.5$.

1. **State space model**: Define the state vector: $x = \left[z, \pi - \theta, \dot{z}, \dot{\theta}\right]^T$ (note with this definition x_2 becomes the clockwise angle the pendulum makes with respect to the upright direction). Derive the continuous time state space model $\dot{x} = f(x, u)$ based on the dynamic relation given in equation (1).
2. **Linearized model**: Consider the state space model obtained in Part 1. Derive the linearized continuous time model around $\hat{x} = [0, 0, 0, 0]^T$ and $\hat{u} = 0$, i.e., find $\hat{A}$, $\hat{B}$, such that $\dot{x} \approx \hat{A}x + \hat{B}u$. (note that $f(\hat{x}, \hat{u}) = 0$, $\Delta x \triangleq x - \hat{x} = x$, $\Delta u \triangleq u - \hat{u} = u$).
3. **Discrete Time Dynamics Model**: Based on the continuous-time linearized model in Part 3 , find the matrices A, B for the discrete time model $x(k+1) = Ax(k) + Bu(k)$.
4. **Simulation Setup and Testing**: Download the "CartPoleControlMujoco.ipynb" file from github. Simulate the unconstrolled cart-pole system with at least two sets of initial conditions. Attach your codes and a few snapshots of your simulation.
5. **Simulation studies for state-feedback control**
   (a) **(LQR)**: Use the model (A, B) derived in part 3 to design a LQR feedback controller $u = -Kx$. You should compute K using LQR theory introduced in class with properly selected Q, R matrices .
   (b) **(Closed-loop Simulation)**: Implement your controller in Mujoco. Simulate & visualize the response for a few different initial conditions. Attach your codes and a few snapshots/plots.
6. Repeat the design and corresponding closed-loop simulation with two different sets of Q, R weighting matrices. Compare the responses with the previous case and discuss your observations.



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