Does Everyone Die?

The answer is of course yes. But it is of intense interest to us when we die. To this end there are several models to estimate the number of diseased and of removed (dead or immune). The first one is as follows.

$$\begin{cases} \dfrac{d\,\mathrm{S}}{dt} = -\beta \mathrm{IS} \\ \dfrac{d\,\mathrm{I}}{dt} = \beta \mathrm{IS} - \gamma \mathrm{I} = \mathrm{I}(\beta \mathrm{S} - \gamma) \\ \dfrac{d\,\mathrm{R}}{dt} = \gamma \mathrm{I} \end{cases}$$

First notice that

$$\frac{d\,(\mathrm{S} + \mathrm{I} + \mathrm{R})}{dt} = \frac{d\,\mathrm{S}}{dt} + \frac{d\,\mathrm{I}}{dt} + \frac{d\,\mathrm{S}}{dt} = 0$$

so $S + I + R = N$ is a constant. This reduces the set of equations to.

$$\frac{d\,\mathrm{S}}{dt} = -\beta \mathrm{IS}$$

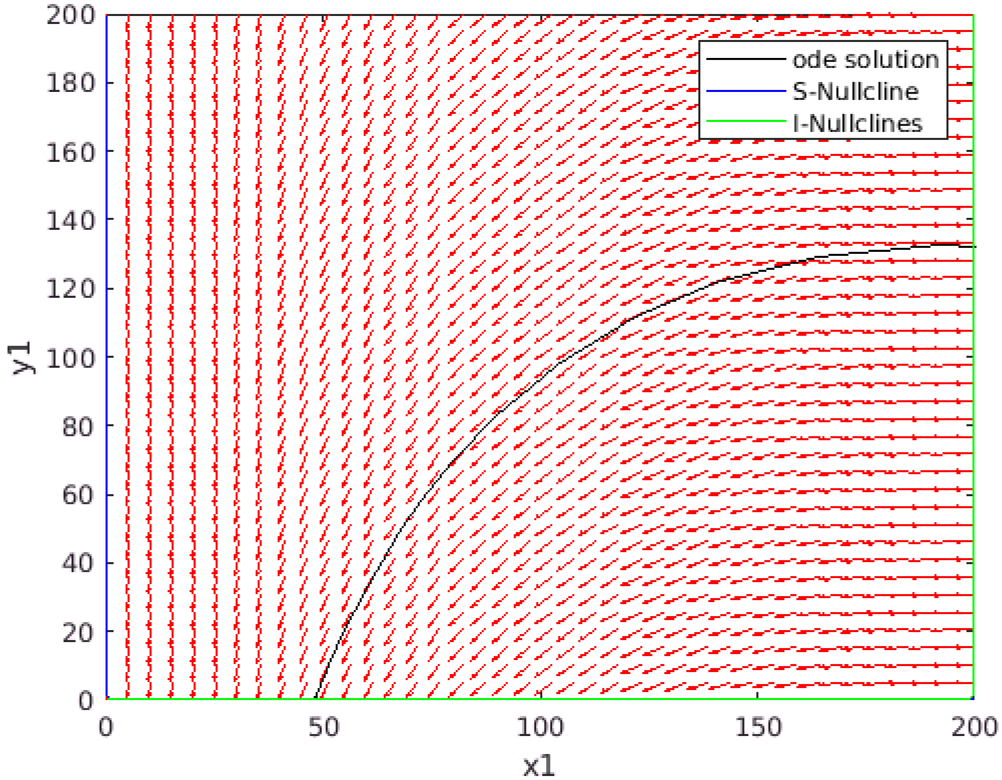
$$\frac{d\,\mathrm{I}}{dt} = \mathrm{I}(\beta \mathrm{S} - \gamma)$$

$$\mathrm{R} = \mathrm{N} - \mathrm{I} - \mathrm{S}$$

Examining the equtions they both have a factor of I so they are both zero on the S axis. There is a S-nullcline on the I axis.The equilbrium line is attractive $S < \frac{\gamma}{\beta}$ and repulsive with $S > \frac{\gamma}{\beta}$. The vector field.

```
beta = .003;
gamma = .6;
N = 400;
percent_sick = .3;
x0 = N*([1,0] + [-1,1]*percent_sick);

f = @(x1,x2) -beta*x1.*x2;
g = @(x1,x2) x2.*(beta*x1 - gamma);
%% A configurable phase plot
phasePlot({f,g},x0,axes)
```



$\frac{dS}{dt}$ is negative where I and S are positive. This means S decreases as long as both S and I are larger then one. But if $0 \leq S \leq \frac{\gamma}{\beta}$ and I is positive then then $\frac{dI}{dt}$ is negative. Since this region this bound on the left by a I axis and on the bottom by a equilibrium line and both $\frac{dI}{dt}$ and $\frac{dS}{dt}$ are negative the solutions go to the equilibriut m line and I goes to zero. But sick people exist, in particularly diseases persist in opulations, they don't all kill the population until they can't persist anymore. to solve this we introduce the followig modifications to $\frac{dS}{dt}$ we add νR to allow susceptible to become survivors. the resulting system

$$\begin{cases} \frac{dS}{dt} = -\beta IS + \eta R \\ \frac{dI}{dt} = I(\beta S - \gamma) \\ \frac{dR}{dt} = \gamma I - \eta S \end{cases}$$

Again $I + S + R = N$ is constant so for a fixed value of N we have