2

$\frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$

(a) $w = V\Sigma^{-1}U^T y$

$$\min_{\underline{w}} \|y - Xw\|_2^2 = \frac{1}{2\sqrt{2}}\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \frac{1}{\gamma} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right)$$

$$= \frac{1}{2\sqrt{2}} \left(\begin{bmatrix} 2 \\ 2 \end{bmatrix} + \begin{bmatrix} \frac{2}{\gamma} \\ -\frac{2}{\gamma} \end{bmatrix} \right) = \frac{1}{\sqrt{2}\gamma} \begin{bmatrix} \gamma+1 \\ \gamma-1 \end{bmatrix}$$

$$\|\underline{w}\|_2^2 = \frac{1}{\gamma}\sqrt{\gamma^2+1}$$

$\gamma \to 0$, $\|\underline{w}\|_2^2 \to \infty$.

(c) $\min_{\underline{w}} \|y - X\underline{w}\|_2^2 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\|w\|_2^2 = 1$