①

a) $w(a_i) \approx b_i \iff w_i a^i \approx b_i$

b) $A = \begin{bmatrix} a_1 \\ \vdots \\ a_m \end{bmatrix}$

②

(a) $\min_{\underline{w}} \|x - Tw\|_2^2 \Rightarrow \underline{w} = (T^T T)^{-1} T^T \underline{x} = *$

since T is an orthonormal basis, $(T^T \cdot T)^{-1} \cdot T^T \cdot x = I^{-1} T^T \cdot x = T^T x$.

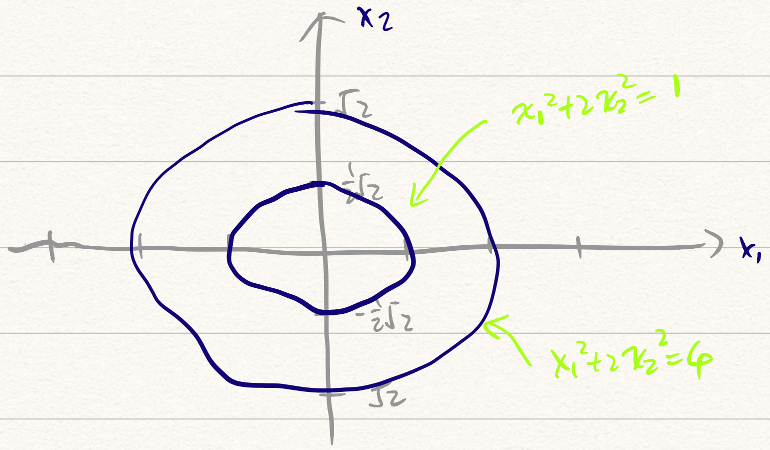
(b) $W_i = (T^T \cdot T)^{-1} \cdot T^T \cdot x_i$

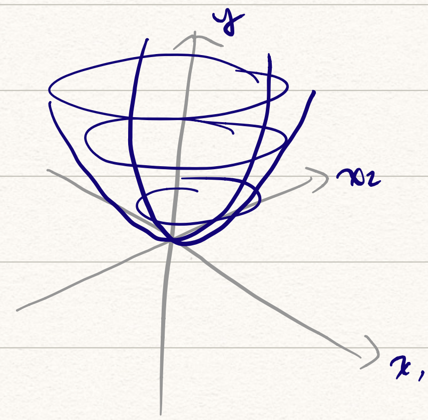
$\therefore W = (T^T \cdot T) \cdot T^T \cdot [x_1 \ x_2 \ \cdots \ x_p]$

④

a) Yes.

b) $y = [x_1 \ x_2] \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = [x_1 \ 2x_2] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = x_1^2 + 2x_2^2$





③

$\because Q$ symmetric, $Q = Q^T$

Let $v \neq 0$.

$v^T \cdot Q \cdot P \cdot Q \cdot v = (v^T Q^T) \cdot P(Q \cdot v) = (Qv)^T \cdot P(Q \cdot v)$

$\because \mathrm{rank}(Q) = n$, $Q \cdot v \neq 0$.

$\because P \succ 0$, $(Q \cdot v)^T \cdot P(Q \cdot v) > 0$

$\therefore v^T \cdot Q \cdot PQ \cdot v > 0 \ \forall v \neq 0$ $\therefore QPQ \succ 0$.