# CS/ECE/ME532 Period 2 Activity

1) Let $\boldsymbol{X} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & -2 & -2 \\ 3 & -3 & 3 & -3 \end{bmatrix}$ and $\boldsymbol{w} = \begin{bmatrix} 1 \\ b \\ 1 \\ c \end{bmatrix}$.

   a) Write out and evaluate the vector $\boldsymbol{y} = \boldsymbol{X}\boldsymbol{w}$.

   b) Find $b$ and $c$ so that $\boldsymbol{y} = \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix}$.

   c) Find $b$ and $c$ so that $\boldsymbol{y} = \begin{bmatrix} 0 \\ 0 \\ 12 \end{bmatrix}$.

2) Consider the fourth-order polynomial $y = x^4 - 2x^3 + 3x^2 - 4x + 5$. Suppose you have seven (arbitrary) values of this polynomial, $(x_i, y_i), i = 1, 2, \ldots, 7$. Write the vector

$$\boldsymbol{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_7 \end{bmatrix} = \boldsymbol{X}\boldsymbol{w}.$$ Define the matrix $\boldsymbol{X}$ in terms of the $x_i$ and the vector $\boldsymbol{w}$.

3) Recall from the last activity that food involves fats, proteins and carbohydrates. There are 9 calories for every gram of fat, 4 calories for every gram of protein, and 4 calories for every gram of carbohydrates. If we define a vector $\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ where $x_1$ is the number of grams of fat, $x_2$ is the number of grams of protein, and $x_3$ is the number of grams of carbohydrate, then the number of calories is $y = \boldsymbol{x}^T \boldsymbol{w}$ where $\boldsymbol{w} = \begin{bmatrix} 9 \\ 4 \\ 4 \end{bmatrix}$.

   Your nutrition expert has a way of defining a food as "low carb" based on the ratio of carbohydrate calories to total calories. Let

$$z = \frac{\text{carbohydrate calories}}{\text{total calories}}$$

A food is classified as low carb if $z < 1/4$.

a) Express the rule for classifying foods given by your nutritionist as the sign of an inner product between $\boldsymbol{x}$ and a vector of weights $\tilde{\boldsymbol{w}}$. In other words, specify $\tilde{w}_1, \tilde{w}_2$ and $\tilde{w}_3$ so that when $\text{sign}(\boldsymbol{x}^T\tilde{\boldsymbol{w}}) = 1$ then the food is low carb, and when $\text{sign}(\boldsymbol{x}^T\tilde{\boldsymbol{w}}) = -1$ then the food is not low carb.

b) Nutritionists like to look at the ratios of the types of calories. Consider the features $r_f = x_1/x_3$, the ratio of the number of grams of fat to carbohydrate, and $r_p = x_2/x_3$, the ratio of the number of grams of protein to carbohydrate. Express low carb criterion as a function of the features $r_f$ and $r_p$.

c) Define the decision boundary as the line where $z = 1/4$, since a food with $z < 1/4$ is classified as low carb while a food with $z \geq 1/4$ is not low carb. Graph the decision boundary assuming feature $r_p$ is on the vertical axis and $r_f$ is on the horizontal axis. Shade the portion of the $r_f$-$r_p$ plane that corresponds to low-carb foods. Note that $r_p$ and $r_f$ cannot be negative. *Hint:* Recall that the equation $y = mx + b$ describes a line with slope $m$ and $y$-intercept $b$.

d) Consider the four cereals:

Cereal 1: 1 gram fat, 8 grams protein, 44 grams carbohydrate
Cereal 2: 0.5 grams fat, 2 grams protein, 25 grams carbohydrate
Cereal 3: 1.3 grams fat, 2.7 grams protein, 29.3 grams carbohydrate
Cereal 4: 9 grams fat, 4 grams protein, 16 grams carbohydrate

Plot the features $r_f, r_p$ for each cereal in the $r_f$-$r_p$ plane and label each pair of features with the corresponding cereal number. Are any of these classified as low carb?

e) Almond butter has 9 grams fat, 3.4 grams protein, and 3 grams carbohydrate per serving. Plot the features $r_f, r_p$ for almond butter in the $r_f$-$r_p$ plane. Is almond butter classified as a low-carb food?

f) A serving of marinated grilled salmon has 19 grams fat, 23 grams protein, and 1 gram carbohydrate per serving. Plot the features $r_f, r_p$ in the $r_f$-$r_p$ plane. Is this salmon classified as a low-carb food?

g) Suppose you were designing a classifier for low fat foods based on the ratio of total calories to calories from fat. What features would you choose and why?