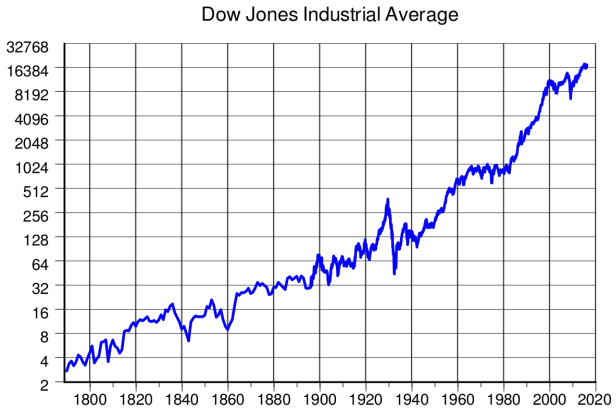
CS/ECE/ME 532
Practice Exam 1 Activity

The following are problems from the 2017 Midterm Exam 1. Write out and submit a solution to at least two problems.

1. You see the below plot of the Dow Jones Industrial Average stock market index and hypothesize that at time t, the index value is $v_t \approx e^{at+b}$. For simplicity, assume we measure the index value at times $t = 1, 2, \ldots, T$, where $t = 1$ corresponds to 1795 and $t = T$ corresponds to 2017.



a) Explain how you would estimate a and b. Specify any matrices and vectors you would need to use.

b) Partially solve the least squares problem. That is, if you had a least squares problem of the form $(X^\top X)^{-1} X^\top y$, then give expressions for $(X^\top X)^{-1}$ and $X^\top y$.

HINT: recall that the inverse of a 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

HINT 2: $\sum_{t=1}^{T} t = \frac{T(T+1)}{2}$ and $\sum_{t=1}^{T} t^2 = \frac{T(T+1)(2T+1)}{6}$.

2. Congratulations! You have been hired by Google's machine learning group. Your first task is to develop a classifier than will indicate whether a web page is political in nature or not. You first write a script that, for any web page, counts the number of times different $p = 6$ keywords (Trump, dog, refugee, clearance, Xbox, and football) appear in the page. You also have an intern who has looked at n different web pages and labeled them as "political" or "apolitical".

a) Explain in detail how you would design your classifier.

b) What pitfalls do you need to be wary of?

c) How can you predict how well your classifier will work?

d) Write a few lines of matlab or python code illustrating how you would compute your classifier given the training data and how you would classify a new web page.

3. AllRecipes.com has a large collection of different receipes and ratings their customers have given to each recipe. They hire you to help them analyze this data so they can better help customers find recipes they are likely to enjoy. They start with $p = 5$ recipes – Apple pie, Beef stew, Carrot salad, Devil's food cake, and Eggs benedict. Here are some customer ratings (each column corresponds to a different customer):

$$\begin{bmatrix} 4 \\ 4 \\ 2 \\ 4 \\ 2 \end{bmatrix}, \quad \begin{bmatrix} 8 \\ 8 \\ 4 \\ 8 \\ 4 \end{bmatrix}, \quad \begin{bmatrix} 10 \\ 4 \\ 8 \\ 4 \\ 8 \end{bmatrix}, \quad \begin{bmatrix} 10 \\ 8 \\ 6 \\ 8 \\ 6 \end{bmatrix}, \quad \begin{bmatrix} 9 \\ 6 \\ 6 \\ 6 \\ 6 \end{bmatrix}$$

You are asked to identify some representative customers (in terms of their tastes in recipes), so that each customer's tastes can be modeled in terms of these representatives.

a) Find a basis for the customer tastes.

b) A new customer joins who rates Beef stew 5 and Carrot salad 7. (i) What do you predict will be their rating for Egg's benedict? (ii) Could you have figured this out if we only knew their rating for Carrot Salad? (iii) Could you have figured this out if we only knew rating for Beef stew?

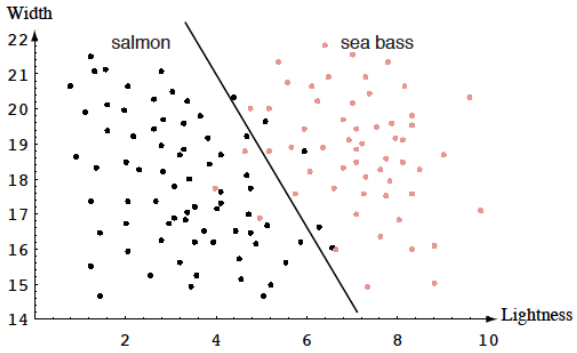
4. Suppose that a fish packing plant wants to automate the process of sorting incoming fish on a conveyor belt according to species. You decide to try to separate sea bass from salmon using optical sensing. You set up a camera, take some sample images and begin to note some physical differences between the two types of fish, such as length and lightness.

a) You first try to build a linear classifier using least squares, with a decision rule of

$$\tilde{y} = \begin{cases} +1 & \text{if } w_1 \cdot \text{length} + w_2 \cdot \text{lightness} > t \\ -1 & \text{otherwise} \end{cases}.$$

Describe (1) how you would formulate this as a least-squares problem to find w_1, w_2, and t that best fit the training data you've gathered and (2) how you would solve this least squares problem.

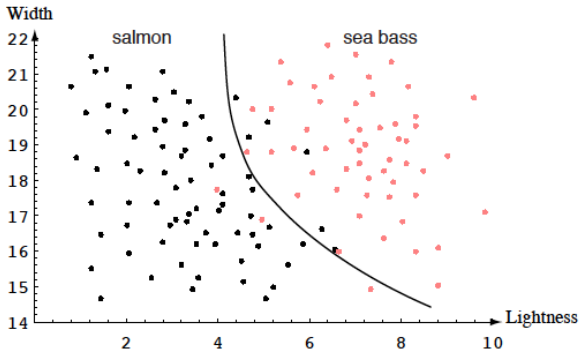
b) After learning your linear least squares classifier, you plot the decision boundary and see this:



You then decide to see whether you can build a classifier with a *quadratic* decision boundary. That is, the decision boundary is the set of (length,lightness) pairs that satisfy

$$w_1 + w_2 \cdot \text{length} + w_3 \cdot \text{lightness} + w_4 \cdot \text{length}^2 + w_5 \cdot \text{lightness}^2 + w_6 \cdot \text{length} \cdot \text{lightness} = 0$$

for some set of coefficients $w_1, \ldots, w_6$, as in this image:



Describe a method for learning this classifier from the same data as you had with the linear classifier above.