CS/ECE/ME532 Assignment 7

1. **Recovering a blurred signal.** This continues the deblurring problem that you started in Period 15 Activity. The original problem statement is repeated here for convenience.

   Many sensing and imaging systems produce signals that may be blurred or distorted, such as an out-of-focus camera, and have noise in them. In such situations, algorithms are needed to deblur the data to obtain a more accurate estimate of the true signal. A famous example is the Hubble telescope, which was launched with imperfections in the mirror. These imperfections blurred the image. The blur could be identified by measuring how much a star (a point) was distorted. Eventually a shuttle mission installed corrective optics, but prior to that algorithms were used to deblur the images.

   Suppose our true signal is a vector $\boldsymbol{x} \in \mathbb{R}^n$ with ith element x_i and the blurring produces a new signal $b \in \mathbb{R}^n$ according to the model:

$$b_i = \frac{1}{k}\left(x_i + x_{i-1} + \cdots + x_{i-k+1}\right) + w_i \qquad \text{for } i = 1, \ldots, n$$

   In other words, each b_i is the average of the previous k values of x_i, plus some extra noise w_i. Here we treat x_j as zero when $j < 1$.

   We rewrite these n scalar equations in matrix-vector form

$$\boldsymbol{b} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{w}$$

   where the n-by-1 vector $\boldsymbol{b}$ is the noisy, blurred, measured signal, the n-by-1 vector $\boldsymbol{x}$ is the true signal, and the n-by-1 vector $\boldsymbol{w}$ contains the noise samples. In this formulation the n-by-n matrix $\boldsymbol{A}$ is a model for the blurring operation.

   a) Write code that generates $\boldsymbol{A} \in \mathbb{R}^{n\times n}$ matrix as a function of n and k.

   b) Suppose the true $\boldsymbol{x}$ is given in the file `xsignal.csv`. Generate $\boldsymbol{b}$ by using $k = 30$. To generate $\boldsymbol{w}$, make each w_i normally distributed with standard deviation σ. For example, you can do this in Pyhton via: `w=sigma*np.random.randn(n,1)`. Plot $\boldsymbol{x}$ and $\boldsymbol{b}$ using $\sigma = 0.01$ and $\sigma = 0.1$.

   c) Reconstruct $\boldsymbol{x}$ in the three following ways, and for each one plot the true $\boldsymbol{x}$ and its reconstruction.

   (i) Ordinary least squares

   (ii) Truncated SVD; only keep the largest m singular values of $\boldsymbol{A}$ and try different values of m.

   (iii) Regularized (Tikhonov) least squares; try different values of the parameter λ.

d) Experiment with different degrees of averaging (i.e., different values of k). and with different noise levels (σ in the code). How do the blurring and noise level affect the value of the regularization parameters that produce the best estimates?