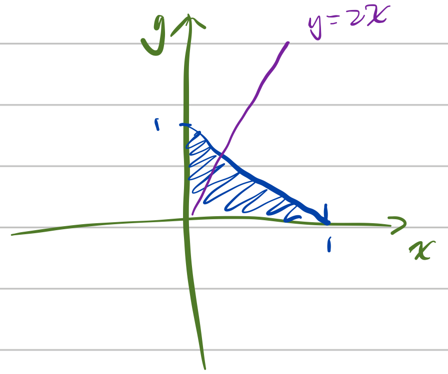
①

a) $E[Y] = E[X_1 + X_2 + \cdots + X_n] = E[X_1] + \cdots + E[X_n] = n\mu$

b) $E\left[\frac{1}{n}\sum_{i=1}^{n} X_i\right] = \frac{1}{n}E[Y] = \mu$

②

a)



$y = 2x$

b) $\int_0^1 \int_0^{1-y} c \, dx \, dy = c\int_0^1 1-y \, dy = c\left[y - \frac{1}{2}y^2\right]_0^1 = c\left[(1-\frac{1}{2}) - \cancel{(1-0)}\right]$

$= \frac{1}{2}c$

$\frac{1}{2}c = 1 \qquad c = 2$

c) $f(y) = \int_0^{1-y} 2 \, dx = 2x \Big|_0^{1-y} = 2-2y$

d) No. with $x, y \geq 0$ and $x+y \leq 1$, the greater the x, the more likely we will see a small y.

e) $P(2X \geq Y) = \int_0^{1/3}\int_0^{2x} c \, dy \, dx + \int_{\frac{1}{3}}^1 \int_0^{1-x} c \, dy \, dx$

$= \int_0^{1/3} 4x \, dx + \int_{\frac{1}{3}}^1 2-2x \, dx = 2x^2\Big|_0^{\frac{1}{3}} + (2x - x^2)\Big|_{1/3}^1 = \frac{2}{3}$