1

a) mean $= \sum_k \pi_k u_k$

cov $= \sum_k \pi_k \Sigma_k$

3

a) $P(f(x) \neq y) \geq \dfrac{H(Y|x) - 1}{\log(|y|)}$

b) MAP: $\hat{y} = \arg\max_y p(x|y)\,p(y)$

$P(y = \text{fish}) = \frac{3}{8}$ $\quad P(y = \text{cat}) = \frac{1}{4}$ $\quad P(y = \text{dog}) = \frac{3}{8}$

$p(x\|y)\,P(y)$	y= fish	cat	dog
$x=1$	$\frac{3}{32}$	$\frac{1}{32}$	$\frac{3}{8\cdot 16}$
2	$\frac{3}{32\cdot 16}$	0	$\frac{3}{32}$
3	0	$\frac{1}{32}$	$\frac{3}{8\cdot 16}$
4	$\frac{1}{32\cdot 16}$	0	0

$\Rightarrow$

	$\hat{y}$
$x=1$	fish
2	dog
3	cat
4	fish

	error rate
$x=1$	$\frac{1}{8} + \frac{1}{16}$
2	$\frac{1}{16}$
3	$\frac{1}{16}$
4	0

error rate $= \frac{3+2}{16} = \frac{5}{16}$

c) $H(E, Y | \hat{Y}) = H(E | \hat{Y}) + H(Y | E, \hat{Y})$

$$\leq H(E) + H(Y | E, \hat{Y})$$

$$= (1 - P_e) \log_2 \left(\frac{1}{1-P_e}\right) + P_e \log_2 \left(\frac{1}{P_e}\right) + H(Y, E, \hat{Y})$$

$$= (1 - P_e) \log_2 \left(\frac{1}{1-P_e}\right) + P_e \log_2 \left(\frac{1}{P_e}\right) + P_e \log_2 |Y| .$$

While $H(E, Y | \hat{Y}) \geq H(Y|x)$, we have

$$H(Y|x) \leq (1 - P_e) \log_2 \left(\frac{1}{1-P_e}\right) + P_e \left(\log_2 \left(\frac{1}{P_e}\right) + \log_2 |Y|\right) .$$

$$= P_e \left(\log_2 \left(\frac{1}{P_e}\right) + \log_2 |Y| - \log_2 \left(\frac{1}{1-P_e}\right)\right) + \log_2 \left(\frac{1}{1-P_e}\right)$$

$$\therefore P_e \geq \frac{H(Y|x) - \log_2 \left(\frac{1}{1-P_e}\right)}{\log_2 \left(\frac{1}{P_e}\right) + \log_2 |Y| - \log_2 \left(\frac{1}{1-P_e}\right)}$$