

①

$$\begin{aligned}
 a) H(X,Y) &= \sum_x \sum_y p(x,y) \log_2 \left(\frac{1}{p(x,y)} \right) \\
 &= \frac{2}{4} \cdot \log_2 4 + \frac{2}{8} \cdot \log_2 8 + \frac{4}{16} \cdot \log_2 16 = \frac{1}{2} \cdot 2 + \frac{1}{4} \cdot 3 + \frac{1}{4} \cdot 4 \\
 &= 1 + \frac{3}{4} + 1 = 2.75 \text{ bits}
 \end{aligned}$$

b)

x	$p(x)$	y	$p(y)$
1	$\frac{7}{16}$	fish	$\frac{6}{16} = \frac{3}{8}$
2	$\frac{5}{16}$	cat	$\frac{1}{4}$
3	$\frac{3}{16}$	dog	$\frac{6}{16} = \frac{3}{8}$
4	$\frac{1}{16}$		

$$\begin{aligned}
 H(X) &= \frac{7}{16} \cdot \log_2 \left(\frac{16}{7} \right) + \frac{5}{16} \cdot \log_2 \left(\frac{16}{5} \right) + \frac{3}{16} \cdot \log_2 \left(\frac{16}{3} \right) + \frac{1}{16} \log_2 16 \\
 &\approx 1.75 \text{ bits}
 \end{aligned}$$

$$c) H(Y) = \frac{6}{8} \cdot \log_2 \left(\frac{8}{6} \right) + \frac{1}{4} \cdot \log_2 4 \approx 1.56 \text{ bits}$$

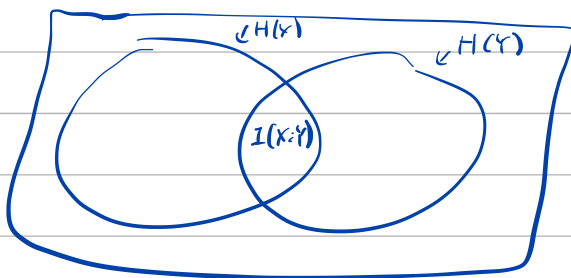
$$d) H(X|Y) = H(X,Y) - H(Y) \approx 1.79 \text{ bits}$$

$$e) H(Y|X) = H(X,Y) - H(X) \approx 1 \text{ bit}$$

$$f) I(X:Y) = H(X) - H(X|Y) = 0.94 \text{ bit}$$

$$g) I(Y:X) = H(Y) - H(Y|X) = 0.94 \text{ bit}$$

h)



$$H(X,Y) = H(X) \cup H(Y)$$

$$H(X,Y) = H(X,Y) + (I(X,Y))$$