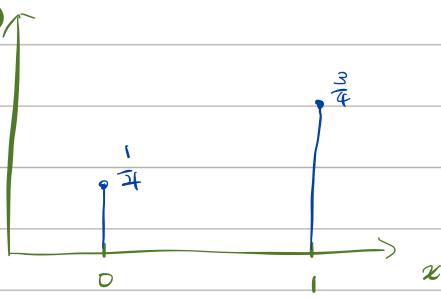


①

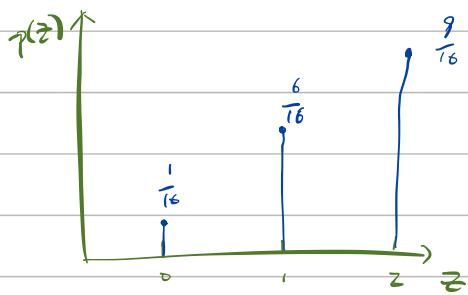
a)



b) when $p = \frac{3}{4}$ $H(x) = -\left[\frac{1}{4} \cdot \log_2\left(\frac{1}{4}\right) + \frac{3}{4} \cdot \log_2\left(\frac{3}{4}\right)\right] \approx 0.811$

when $p = \frac{1}{2}$ $H(x) = -2 \cdot \frac{1}{2} \cdot \log_2\left(\frac{1}{2}\right) = 1$

②

a) pmf of Z : $p(z) = \begin{cases} \binom{2}{0} p^0 (1-p)^2 = \frac{1}{16}, & z=0 \\ \binom{2}{1} p^1 (1-p)^1 = 2 \cdot \frac{3}{4} \cdot \frac{1}{4} = \frac{3}{8}, & z=1 \\ \binom{2}{2} p^2 (1-p)^0 = \frac{9}{16}, & z=2 \end{cases}$ ③ a) $X \sim \text{Geom}\left(\frac{1}{2}\right)$

$$p(x) = \frac{1}{2} \cdot \left(\frac{1}{2}\right)^{x-1} \quad \text{for } x=1, 2, 3, \dots$$

see attached pdf for script.