

**ECE/Comp Sci 561**  
**Practice Exam 1, Fall 2020, Malloy**

Name: \_\_\_\_\_

ID #: \_\_\_\_\_

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TOTAL:        / 15

**Duration: 1.25 hours**

**Show your work to receive credit unless noted  
otherwise**

- I will not/did not communicate with anyone, other than the TAs or instructors, regarding the content of this examination. I will not discuss the content of the exam with anyone until after the close of the canvas submission time. I will adhere to the University of Wisconsin Honor Code.

Your signature in agreement with the above statement:

\_\_\_\_\_

- This exam is open notes and you are allowed to use numerical software such as Python.

1. Label each statement with  $=, \leq, \geq$ , or NONE. Label with  $\leq$  or  $\geq$  only when strict equality does not hold. Justify your answers.

a)  $\mathbb{P}(A|B)$  vs.  $\frac{\mathbb{P}(A,B)}{\mathbb{P}(A_1)\mathbb{P}(B|A_1)+\mathbb{P}(A_2)\mathbb{P}(B|A_2)}$  if  $A_1 \cup A_2 = \Omega$

b)  $H(X)$  vs. 1 bit when  $p(x) = \begin{cases} 0.4 & \text{if } x = 0 \\ 0.6 & \text{if } x = 1 \end{cases}$

c)  $\mathbb{P}(A)$  vs.  $E[g(X)]$  where  $g(x) = \mathbb{I}_{\{x \in A\}}$

d)  $E[X^4Y^4]$  vs.  $E[X^4]E[Y^4]$  if  $X, Y$  independent.

e)  $\mathbb{P}((X+Y)^2 \geq 16)$  vs.  $1/2$  if  $E[X^2] = E[Y^2] = 4$ ,  $E[X] = 0$ , and  $X, Y$  independent.

f)  $[\Sigma_{\mathbf{x}}]_{i,j}$  (i.e, the  $i, j$  element of the covariance matrix) vs.  $E[x_i x_j]$  if  $E[x_i] = 0$

2. Let  $X$  and  $Y$  be random variables with joint pdf

$$f(x, y) = cx \mathbb{I}_{\{0 \leq x \leq 1, 0 \leq y \leq x\}}$$

where  $\mathbb{I}$  is the indicator function.

- a) Find the constant  $c$ .
- b) Find  $E[X|Y = y]$ . Your answer should be in terms of  $y$  only.
- c) Are  $X, Y$  independent? Why or why not?

3. You are tasked with building a binary *apple/pear* classifier. You collect two features:  $X_1 \in \{0.5, 1, 2\}$  which represents the top-to-bottom ratio of the fruit, and  $X_2 \in \{0, 1\}$ , which is an indicator if the fruit is green ( $X_2 = 1$  if the fruit is green, and 0 otherwise). The joint pmf is specified below:

$p(\mathbf{x}, y = 0)$	$x_2 = 0$	$x_2 = 1$
$x_1 = 0.5$	0.1	0.15
$x_1 = 1$	0	0.15
$x_1 = 2$	0	0

$p(\mathbf{x}, y = 1)$	$x_2 = 0$	$x_2 = 1$
$x_1 = 0.5$	0.1	0
$x_1 = 1$	0.1	0.1
$x_1 = 2$	0.2	0.1

- a) Are  $X_1$  and  $X_2$  independent?
- b) Specify the MAP classifier. Be explicit. Your answer should specify  $\hat{y}$  for each value in  $\mathbf{x} \in \{0.5, 1, 2\} \times \{0, 1\}$ .
- c) What is the true risk of your classifier (assuming 0/1 loss)?