

ECE/Comp Sci 561
Practice Exam 1, Fall 2020, Malloy

Name: _____

ID #: _____

1: / 7

2: / 4

3: / 4

TOTAL: / 15

Duration: 1.25 hours

**Show your work to receive credit unless noted
otherwise**

- I will not/did not communicate with anyone, other than the TAs or instructors, regarding the content of this examination. I will not discuss the content of the exam with anyone until after the close of the canvas submission time. I will adhere to the University of Wisconsin Honor Code.

Your signature in agreement with the above statement:

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- This exam is open notes and you are allowed to use numerical software such as Python.

1. Label each statement with $=, \leq, \geq$, or NONE. Label with \leq or \geq only when strict equality does not hold. Justify your answers.

a) $\mathbb{P}(A|B)$ vs. $\frac{\mathbb{P}(A,B)}{\mathbb{P}(A_1)\mathbb{P}(B|A_1)+\mathbb{P}(A_2)\mathbb{P}(B|A_2)}$ if $A_1 \cup A_2 = \Omega$

b) $H(X)$ vs. 1 bit when $p(x) = \begin{cases} 0.4 & \text{if } x = 0 \\ 0.6 & \text{if } x = 1 \end{cases}$

c) $\mathbb{P}(A)$ vs. $E[g(X)]$ where $g(x) = \mathbb{I}_{\{x \in A\}}$

d) $E[X^4Y^4]$ vs. $E[X^4]E[Y^4]$ if X, Y independent.

e) $\mathbb{P}((X+Y)^2 \geq 16)$ vs. $1/2$ if $E[X^2] = E[Y^2] = 4$, $E[X] = 0$, and X, Y independent.

f) $[\Sigma_{\mathbf{x}}]_{i,j}$ (i.e, the i, j element of the covariance matrix) vs. $E[x_i x_j]$ if $E[x_i] = 0$

2. Let X and Y be random variables with joint pdf

$$f(x, y) = cx\mathbb{I}_{\{0 \leq x \leq 1, 0 \leq y \leq x\}}$$

where \mathbb{I} is the indicator function.

a) Find the constant c .

b) Find $E[X|Y = y]$. Your answer should be in terms of y only.

c) Are X, Y independent? Why or why not?

3. You are tasked with building a binary *apple/pear* classifier. You collect two features: $X_1 \in \{0.5, 1, 2\}$ which represents the top-to-bottom ratio of the fruit, and $X_2 \in \{0, 1\}$, which is an indicator if the fruit is green ($X_2 = 1$ if the fruit is green, and 0 otherwise). The joint pmf is specified below:

$p(\mathbf{x}, y = 0)$			
		$x_2 = 0$	$x_2 = 1$
$x_1 = 0.5$	0.1	0.15	
	0	0.15	
	0	0	

$p(\mathbf{x}, y = 1)$	$x_2 = 0$	$x_2 = 1$
$x_1 = 0.5$	0.1	0
$x_1 = 1$	0.1	0.1
$x_1 = 2$	0.2	0.1

- a) Are X_1 and X_2 independent?
- b) Specify the MAP classifier. Be explicit. Your answer should specify \hat{y} for each value in $\mathbf{x} \in \{0.5, 1, 2\} \times \{0, 1\}$.
- c) What is the true risk of your classifier (assuming 0/1 loss)?