Expectation of the sample mean, Joint pdfs

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1. *Sample mean.* Let $X_i \overset{i.i.d.}{\sim} \mathcal{N}(\mu, \sigma^2)$, and define $Y = \sum_{i=1}^{n} X_i$.

a) Find $E[Y]$ using the properties of expectation.

b) The sample mean is defined as $\frac{1}{n}\sum_{i=1}^{n} X_i$. What is the expected value of the sample mean?

Note: The sum of Gaussian random variables is also Gaussian (a property will show later in the course). Next time we will find $\text{var}(Y)$ using the properties of expectation and independence.

2. *Joint pdf.* Let $(X, Y) \sim f(x, y)$ where

$$f(x,y) = \begin{cases} c \text{ if } x, y \geq 0 \text{ and } x + y \leq 1 \\ 0 \text{ otherwise} \end{cases}$$

a) Sketch the region of the x, y plane where $f(x, y)$ is non-zero.

b) Find c.

c) Find $f(y)$.

d) Are X, Y independent?

e) Find $\mathbb{P}(2X \geq Y)$.