

①

$\therefore \pi_i \geq 0 \quad \forall i \in [1, k] \text{ and } p_k(x) \geq 0$

$\therefore p(x) = \sum_k \pi_k p_k(x) \geq 0$ .

$$\int p(x) dx = \int \sum_k \pi_k p_k(x) = \sum_k \sum_k \pi_k p_k(x) = 1$$

$\therefore p(x)$  is a valid distribution.

② -  $p(x) = .25 \cdot N(-4, 1) + .5 N(0, 1) + .25 N(6, 2)$