①

$$f(\vec{x}) = \lim_{\Delta \to 0} \frac{P(x_1 < X_1 < x_1+\Delta, \dots)}{\Delta^n}$$

$$\therefore f(\vec{x}, y) = \lim_{\Delta \to 0} \frac{P(x_1 < X_1 < x_1+\Delta, \dots, Y = y_1)}{\Delta^n}$$

$$= P(y) \lim_{\Delta \to 0} \frac{P(x_1 < X < x_1+\Delta, \dots | Y = y_1)}{\Delta^n} = P(y) f(\vec{x}|y)$$

$$\therefore P(y|\vec{x}) = \frac{f(\vec{x}, y)}{f(\vec{x})} = \frac{f(\vec{x}|y) P(y)}{f(\vec{x})}$$

②

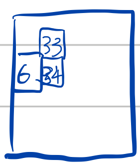
a) $E[\ell(f(x) - y)] = E[(y - f(x))^2]$ $\quad \therefore \hat{y} = f(x) = E[y|x]$.

We have $y|x \sim U[0, 1-x_1-x_2]$

$\therefore E[y|x] = \dfrac{1-x_1-x_2}{2}$

b) $E[\ell(f(x), y)] = E[(f(x) - y)^2] = \int_0^1 \int_0^{1-x_1} \int_0^{1-x_1-x_2} \left(\frac{1-x_1-x_2}{2} - y\right)^2 dy\, dx_1\, dx_2$

③

a) 

$p(x_{34}|x_1, \dots, x_{33}) = p(x_{34}|x_6, x_{33})$

b) $p(x|y) = p(x_1) \prod_{i=2}^{n} p(x_i | x_{i-28}, x_{i-1}, y)$

c) need 80×28^2 params

Less than general case, more than naive Bayes