Entropy and Mutual Information Basics

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1. Consider the following pmf.

		y		
	$p(x,y)$	fish	cat	dog
x	1	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$
	2	$\frac{1}{16}$	0	$\frac{1}{4}$
	3	0	$\frac{1}{8}$	$\frac{1}{16}$
	4	$\frac{1}{16}$	0	0

Find the following:

a) $H(X,Y)$

b) $H(X)$

c) $H(Y)$

d) $H(X|Y)$

e) $H(Y|X)$

f) $I(X;Y)$

g) $I(Y;X)$

h) Draw a Venn diagram with two partially overlapping circles, and label the diagram with the quantities above.

SOLUTION:

a) $H(X,Y) = 2 \times \frac{1}{4} \times \log_2 4 + 2 \times \frac{1}{8} \times \log_2 8 + 4 \times \frac{1}{16} \times \log_2 16 = 2.75$ bits

b) $H(Y) = 2 \times \frac{3}{8} \times \log_2 \frac{8}{3} + \frac{1}{4} \log_2 4 \approx 1.561$ bits

c) $H(X) = \frac{7}{16} \times \log_2 \frac{16}{7} + \frac{5}{16} \times \log_2 \frac{16}{5} + \frac{3}{16} \times \log_2 \frac{16}{3} + \frac{1}{16} \times \log_2 16 \approx 1.749$bits

d) $H(X|Y) = H(X,Y) - H(Y) \approx 1.189$ bits, since by definition $H(X,Y) = H(Y) + H(X|Y)$.

e) $H(Y|X) = H(X,Y) - H(X) \approx 1.001$ bits

f) $I(X;Y) = H(Y) - H(Y|X) \approx 0.55$ bits

g) since $I(Y;X) = I(X;Y)$ in general, $I(Y;X) \approx 0.55$

h) See the drawing blow: