① (a) RHS $= P(A^c) - P(B) = 1 - P(A) - P(B) \ngeq P(A^c, B^c)$ ∴ false

(b) from (a), we know (b) is false

(c) $P(A) - P(B) \le P(B^c) - P(A^c) \Rightarrow P(A) + P(A^c) \le P(B) + P(B^c) \Rightarrow 1 \le 1$ ∴ false

a)

② define P = door player selected has gold, H = door host opened has gold

L = the door that played didn't select and host didn't open has gold

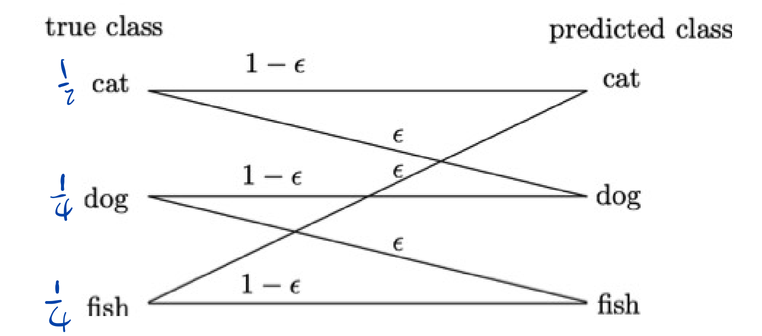
$\Omega = \{P, H, L\}$

b) given $P(P) = P(H) = P(L) = \frac{1}{3}$

prob of winning if stayed $= P(P \mid H^c) = \dfrac{P(P, H^c)}{P(H^c)} = \dfrac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$

prob of winning if moved $= P(L \mid H^c) = \dfrac{P(L, H^c)}{P(H^c)} = \frac{1}{2}$

③



(a)

$P(p = cat) = P(p = cat \mid t = cat)\, P(t = cat) + P(p = cat \mid t = fish) \cdot P(t = fish)$

$= \frac{1}{2}(1-\varepsilon) + \frac{1}{4}\varepsilon = \frac{1}{2} - \frac{1}{4}\varepsilon$

$P(p = dog) = \frac{1}{2}\varepsilon + \frac{1}{4}(1-\varepsilon) = \frac{1}{4} - \frac{1}{4}\varepsilon$

$P(p = fish) = \frac{1}{4}\varepsilon + \frac{1}{4}(1-\varepsilon) = \frac{1}{4}$

(b) $P(p = dog, t = cat) = \frac{1}{2}\varepsilon$

$P(p = dog, t = dog) = \frac{1}{4}(1-\varepsilon)$

$P(p = dog, t = fish) = 0$.

④ (a) $\Omega = \{(x_1, x_2, x_3, x_4, x_5, x_6) \mid x_1 + \dots + x_6 = 7$ where $x_i \ge 0\ \forall i\}$

see attached pdf for the remaining qestions.