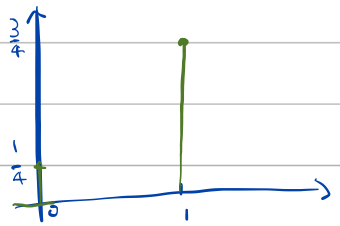
① (a)



(b) when $p = \frac{3}{4}$:

$$H(x) = -\frac{3}{4}\log_2\left(\frac{3}{4}\right) - \frac{1}{4}\log_2\left(\frac{1}{4}\right) \approx 0.811$$

when $p = \frac{1}{2}$

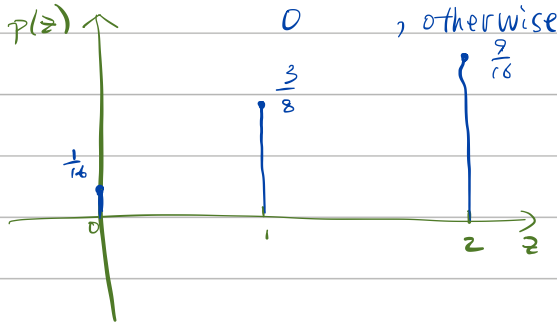
$$H(x) = -\frac{1}{2}\log_2\left(\frac{1}{2}\right)\cdot 2 = 1$$

$\frac{3}{2} \cdot \frac{1}{4}$

② pmf:

$$p(z) = \begin{cases} (1-p)^2, & z=0 \\ 2p(1-p), & z=1 \\ p^2, & z=2 \\ 0, & \text{otherwise} \end{cases}$$

$$p=\frac{3}{4} = \begin{cases} \frac{1}{16}, & z=0 \\ \frac{3}{8}, & z=1 \\ \frac{9}{16}, & z=2 \\ 0, & \text{otherwise} \end{cases}$$



③ $z=1 = \{$ HTHH, HTTT, HHTH, HHTT, THTH, THTT, HHHT, TTHT, THHT $\}$

$z=2 = \{$ HTHT $\}$

$z \geq 3 = \emptyset$

$|\Omega| = 2^4 = 16$.

$\therefore$ pmf:

$$p(z) = \begin{cases} \frac{1}{2}, & z=0 \\ \frac{9}{16}, & z=1 \\ \frac{1}{16}, & z=2 \\ 0, & \text{otherwise} \end{cases}$$

④ n = np.random.rand()

if $0 < n \leq \frac{1}{2}$:

$z = 0$

else if $\frac{1}{2} < 0 \leq \frac{15}{16}$;

```
    z=1
else:
    z=2
```