① 

a) $Var(Y) = E[(Y - E[Y])^2] = n\sigma^2$

b) $var(\frac{1}{n}\sum_{i=1}^{n} X_i) = \frac{1}{n}\sigma^2$

② a) $E[Y] = E[\alpha^T X] = \alpha^T E[X] = \alpha^T u_x$

b) $Var(Y) = E[(Y - E[Y])^2] = E[(\alpha^T x - \alpha^T u_x)^2] = \alpha^T \Sigma_x$

c) Yes.

$\alpha = \begin{bmatrix} \frac{1}{n} \\ \vdots \\ \frac{1}{n} \end{bmatrix} \Rightarrow \alpha^T x = \frac{1}{n}\sum x_i$

If $x_i$ is idd

$Var(Y) = \alpha^T \Sigma_x = \frac{1}{n}\sum var(x_i) = \frac{\sigma^2}{n}.$