①

a) (i) A experiment gives you a set of particular outcomes each of a particular probability. The question states that you want "amount" of new information, not the content, thus I is determined by probability.

(ii) $\because x_1 \perp x_2$

$\therefore P(X_1 = x_1, X_2 = x_2) = P(X_1 = x_1) + P(X_2 = x_2)$

$\therefore I(P(X_1 = x_1, X_2 = x_2)) = I(P(X_1 = x_1) \cdot P(X_2 = x_2))$

$= I(P(X_1 = x_1) + I(P(X_2 = x_2))$

(iii) You either learn nothing or something new from an experiment.

(iv) When an outcome of a experiment is more definitive, you expect to learn less new info.

(v) A definite outcome leads to no new info.

b) (i) $I(P(X=x)) = \log_2 \left(\frac{1}{P(X=x)}\right)$

$\therefore I$ is a function of probability

(ii) $I(P(X_1 = x_1, X_2 = x_2)) = \log_2 \left(\frac{1}{P(X_1 = x_1, X_2 = x_2)}\right) = -\log_2 (P(X_1 = x_1, X_2 = x_2))$

$\overset{\perp}{=} -\log_2 (P(X_1 = x_1) \cdot P(X_2 = x_2)) = -\log_2 (P(X_1 = x_1)) - \log_2 (P(X_2 = x_2))$

$= \log_2 \left(\frac{1}{P(X_1 = x_1)}\right) + \log_2 \left(\frac{1}{P(X_2 = x_2)}\right) = I(P(X_1 = x_1)) + I(P(X_2 = x_2))$

(iii) $P(X=x) \geq 0 \Rightarrow \frac{1}{P(X=x)} \geq 0 \Rightarrow \log_2 \left(\frac{1}{P(X=x)}\right) \geq 0 \Rightarrow I(P(X=x)) \geq 0$

(iv) $P(X=x) \uparrow \Rightarrow \frac{1}{P(X=x)} \downarrow \Rightarrow \log_2 \left(\frac{1}{P(X=x)}\right) \downarrow \Rightarrow I(P(X=x)) \downarrow$.

(v) $I(P(1)) = \log_2 (1) = 0$.

② want to show $H(\hat{y}) \leq H(x)$ or $H(f(x)) \leq H(x)$.

$H(x, \hat{y}) = H(\hat{y}) + H(x \mid \hat{y}) \geq H(\hat{y})$ because $H(x \mid \hat{y}) \geq 0$.

$H(x, \hat{y}) = H(x) + H(\hat{y} \mid x) \Rightarrow H(\hat{y}) \leq H(x) + H(\hat{y} \mid x)$.

$\because H(\hat{y} \mid x) \geq 0 \quad \therefore H(\hat{y}) \leq H(x)$.

③ a) see other pdf

b) $x = \left[\frac{x_1}{x_2}\right] \quad x \sim N\left(\left[\frac{\mu_1}{\mu_2}\right], \left[\begin{array}{c|c} \Sigma_{11} & \Sigma_{12} \\ \hline \Sigma_{21} & \Sigma_{22} \end{array}\right]\right)$

$$x_1 | x_2 = 0 \sim N\left(3 \cdot \frac{1}{2}(0+1) + 3,\ 7 - 3 \cdot \frac{1}{2} \cdot 3\right)$$
$$\sim N\left(\frac{9}{2}, \frac{5}{2}\right)$$

c) see other pdf