Discrete Random Variables

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1. Recall that a Bernoulli random variable represents a coin flip: if a coin has probability of heads p, and if $X = 1$ represents heads, and $X = 0$ represents tails, then $X \sim \text{Bern}(p)$, which has pmf:

$$p(x) = \begin{cases} p & \text{if } x = 1 \\ 1 - p & \text{if } x = 0 \\ 0 & \text{otherwise} \end{cases}$$

a) Draw a stem plot of the Bernoulli pmf when $p = 3/4$. The horizontal axis should represent the values of the random variable, while the vertical axis should be the probability that the random variable takes on that value.

b) The entropy of a random variable X (measured in bits) is defined as

$$H(X) = -\sum_{x \in \mathcal{X}} p(x) \log_2 p(x)$$

What is the entropy of X when $p = 3/4$? What is the entropy of X when $p = 1/2$?

2. Two random variables are said to be *i.i.d.* (independent, identically distributed) if they are independent and follow the same distribution. Let X_1 and X_2 be i.i.d. Bernoulli random variables. Let $Z = X_1 + X_2$. Find the pmf of Z, and draw a stem plot when $p = 3/4$.

Note: If $Z = X + Y$, and X and Y are independent, then the pmf of Z is the *convolution* of the pmf of X and and the pmf of Y, which is a property we will explore as a homework problem.

3. Consider a sequence of 4 independent fair coin tosses. Define the random variable X to be the number of times that a head is immediately followed by a tail.

a) What is the pmf of X?

b) Write a script to generate random instances of X (that occur with the probability specified by the pmf above).