

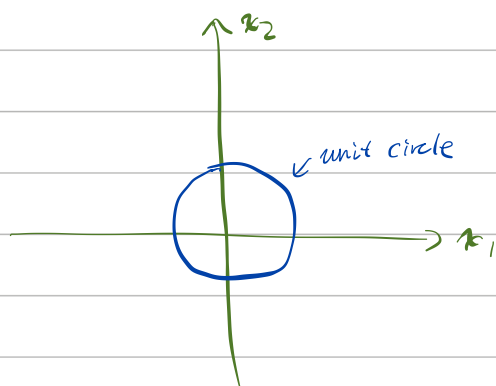
1)

$$a) p(x_2) = \sum_{x_1 \in X_1} p(x_2 | x_1) =$$

$\therefore x_1$  distribution centered around 0

$$\therefore p(x_2) = p(x_1).$$

b)



c) Yes.

$$2) \log \left( \frac{\frac{1}{(2\pi)^{\frac{n}{2}} |\Sigma_0|^{\frac{1}{2}}} \exp\left(-\frac{1}{2} (x-u_0)^T \Sigma_0^{-1} (x-u_0)\right)}{\frac{1}{(2\pi)^{\frac{n}{2}} |\Sigma_1|^{\frac{1}{2}}} \exp\left(-\frac{1}{2} (x-u_1)^T \Sigma_1^{-1} (x-u_1)\right)} \right)$$

$$= -\frac{1}{2} (x-u_0)^T \Sigma_0^{-1} (x-u_0) + \frac{1}{2} (x-u_1)^T \Sigma_1^{-1} (x-u_1)$$

$$= -\frac{1}{2} \Sigma_0^{-1} \left( (x-u_0)^T (x-u_0) - (x-u_1)^T (x-u_1) \right)$$

$$= -\frac{1}{2} \Sigma_0^{-1} \left( x^T x - 2u_0^T x + (u_0)^2 - (x^T x - 2u_1^T x + (u_1)^2) \right)$$

$$= -\frac{1}{2} \Sigma_0^{-1} \left( 2(u_1 - u_0)^T x + (u_0)^2 - (u_1)^2 \right) \geq 0$$

$$\Rightarrow (u_1 - u_0)^T x \geq 2 \Sigma_0 + u_1^2 - u_0^2$$