The Seven Types of Absence

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Abstract

In this paper I reevaluate the statement that 0 is "neither negative nor positive"[0] (i.e. $\neg(+ \vee -)$) and look at how the definition may only be partly correct. Various results include a tentative redefinition of the additive identity element 0 as the conjunction of positive and negative; and more broadly that absence represents a class of concepts that have well-defined relationships to one another that can be expressed through logical connectives as a Hasse diagram. Revealing possible identities allowing for meaningful ways to start to translate between concepts like ordinality, reflexivity, cardinality, and duality.

1. Introduction

To give insight into the motivation behind this foundational redress. Consider, that as an analogy, physicist Paul Dirac once commented that "a vacuum, or nothing, is the combination of matter and antimatter." As an abstraction this seems to get right to the heart of how we think of 0 as a constant. Interestingly Dirac's idea appears to be more than mere conjecture as a team at the University of Michigan has proposed a model to test how to generate matter and antimatter from the vacuum.[1] If we are to consider the vacuum to be a hugely dense source of energy that we can't properly perceive because the observable effects cancel each other out. Then as a conceptualization wouldn't it perhaps be better to think of "nothing" in this sense as a *summation* of $+$ and $-$ rather than neither? Expressed another way on more solid mathematical footing, how can 0 be the identity element for $+$ and $-$ if it's *neither* $+$ nor $-$? This question really comes to the fore as we look at the following expression, solving for the value of *b*:

$$a + b = a - b \tag{1}$$

$$a + 2b = a \tag{2}$$

$$2b = (a - a) \tag{3}$$

$$b = \frac{a - a}{2} = 0 \tag{4}$$

From this we find that $b = 0$ is the additive and subtractive identity element. For brevity I'll collectively refer to both as the additive identity element. Now if we substitute *b* back into (1) we find cardinal zero can be either positive or negative. A strong hint that cardinal zero is a union of $+$ and $-$ as $+ \bigcup - \Rightarrow \{+0, \pm 0, -0\}$ not a logical nor $\neg(+ \vee -) \Rightarrow \{\emptyset\}$, a disjoint set $\{\}$, or an exclusive-or $\{+0, -0\}$.[2] The only thing bothersome about (1), perhaps the simplest statement in all math, is that step (3) can just as easily be transformed to:

$$2 = \frac{a - a}{b} \tag{5}$$

Obviously (5) is invalid because *b* is 0. So for this to be correct it would have to be rewritten as something like $\lim_{b \to 0}\left(\frac{x * b}{b}\right) = x$. However the limit shows there's a direct parallel between canceling the terms and taking the limit. So borrowing a concept from computer science called late evaluation, allows us to consider delaying the evaluation of the contents of *b* in step (5).

$$2 = \frac{a\,(1 - 1)}{b} \tag{6}$$

Since $\frac{c*y}{d*y} = \frac{c}{d}$. Similarly we could just consider canceling the zeros. Thus mimicking the limit:

$$2 = a \tag{7}$$

So where a was free to be anything it wanted previously ($\forall\, a : a \in \mathbb{R}$). Now it's locked to a specific value ($\forall\, a : a = 2$). The question that follows from this is does performing steps (5) through (7) tell us anything meaningful? The answer is an emphatic yes. Here's why.

Imagine if we had two $-b$'s on the right-hand side of the equation in (1) such that the equation was reformulated like so: $a + b = a - 2b$. Going through all the steps all over again would cause step (7) to output: $a = 3$. Thus using induction we can see that what's going on here is we are accumulating all the *multipliers* of b from both sides of the equation and then returning the value. Which is actually useful. It almost seems that dividing by 0 in this one scenario, via the cancellation, allows us to reflect on to the equation itself to count the terms for one of the variables as a sort of metamathematics. Meaning we could now say that a (as a variable, like how it exists in a computer with real physical memory so we don't have to define another) has been redesignated to hold some multiplier x from $x * b$ where the $b = 0$. So rather than a having the meaning of: $a + b = a - b$, where $\forall\, a : a \in \mathbb{R}$. It instead has the meaning of: $\lim_{b \to 0} \left(\frac{x*b}{b}\right) = x = a$. This being a result of, $2 = \frac{a(1-1)}{b}$, in the sense that: $\lim_{b \to 0} \left(\frac{x*b}{b}\right)$ is similar to $\frac{a(1-1)}{b}$. Algorithmically how we change context between these two usages seems to be a result of something like $\{x \in \mathbb{Z} : x = x^2 - x\}$; derived from: $b = 0$; $\frac{a-a}{b} = (a - a) - b$; $\frac{b^2}{b} = b^2 - b$; $b = b^2 - b$, where there's a bijective map between $x = 0$ and the answer from $x = 2$ and vice versa, such that the inputs and outputs are reversed:

$$g[x] = \{-(x - 2) == (x - 2)^2 + (x - 2)\} \tag{8}$$

Where $g(0) = \{T, \{2, 2\}\}$ or $\{T, \{2\}\}$, $g(1) = \{F, \{1, 0\}\}$, and $g(2) = \{T, \{0\}\}$. Or written in its more general form:

$$h[m, x] = \left\{-(x - m) == \frac{2}{m}(x - m)^2 + (x - m)\right\} \tag{9}$$

It is important to note however this does not mean that $a = g(0) = h(2, 0)$, as seen in steps (5) through (7), is correct with regards to the original identity in steps (1) through (4).

Now what is interesting about the property described above is it appears to buttress the notion that the additive identity element 0 is a conjunction of + and −. The reason we are getting the answer of 2 in step (5) is because we are literally quantifying the + and − operators which describe b. Put another way we are self-reflecting on to the equation itself to measure the number of elements of a term. In this case it is telling us exactly what we would expect. Between + and − there are two parts. What is unique however is its telling us this with respect to a very special number, the complete minimal form of the additive identity element.[3] The fact that it gives us 2 seems to indicate that there are an implicit two parts to 0 as + and −. Literally the two units collapse in to one another. To illustrate this take step (2) and look at how we balance the b's: $+b + b = -b + b$. What happens in step (5) is we get the cardinal measure of $\{-, +\}$ on the left-hand side as $+b + b = 2b$. This describes the cancellation that occurred on the right-hand side in (2). Thus steps (5) through (7) can be seen as describing b like so: $2 = \text{Count}(+b_{\text{left}} \text{ and } -b_{\text{right}})$.

Theorem 1.1, then, is 0 as the additive identity element is the conjunction of positive and negative $(+ \wedge -)$.

2. Absence as a Truth-Table

So what of 0 as "neither positive nor negative?" The reason this statement is still somewhat correct is due to the realization that absence, or "zero," encapsulates a class of concepts. To understand this think about how we use absence as an abstraction in numerous, subtle, and multifaceted ways. For instance, we could write: $a - 0 = a$, or, $a + (-0) = a$. Other times we might use: $a + 0 = a$. Thinking about ordinals we can just write $b = \emptyset$ and have no connotation about $+$ or $-$. Then we can show a relationship $\emptyset \subset \{0\}$. Pondering these similar yet dissimilar forms helped me to realize that "zero" represents a collection of concepts. Leading me to conclude that absence, as an axiom, is in essence an ensemble of several mathematical properties (including notions of the empty set, the additive identity element, the idea of nothingness as perhaps a contradiction represented by $\emptyset \neq \emptyset$, et cetera). This is a useful way to think of the types of absence because the properties would appear to relate to one another, over the functional arguments $+$ and $-$, by way of a truth table. Thus as an extension of these interconnections, the results map to the logical connectives as illustrated in a Hasse diagram[4].

Zero as a 7-Fold Truth-Table Mapped to a Spherical Formal System

			A embodies cardinal 0, because z+y=z-y ⇒ 2y=z-z ⇒ 2=(z-z)/y, y=0 assuming ¬¿	B can be expressed as, "positive doesn't imply negative." (i.e. +0).	C can be expressed as, "negative doesn't imply positive." (i.e. -0).	0 is traditionally held to be + nor - . In light of (A) this suggests 0 (as the empty set) ≠ 0 (as summation).	E, as a tautology, resembles the Universal Set because ∀X: (E v X) = E, where X is any characteristic. However in this case E ⇒'s equality because all things are themselves. This can be seen as, ∀X: (E ^ X) = X.	F, as a contradiction, is the closest logical approximation to nothingness (∅ ≠ ∅) as it's devoid of all positive characteristics.	G is the merging point between sets {A, D}, {B, C}, and {E, F}. G requires the extremes to be center.
	α	β	A	B	C	D	E	F	G
	+	-	+ ^ -	~(+ => -)	~(- => +)	~(+ v -)	(A) v (B) v (C) v (D)	~((A) v (B) v (C) v (D))	+ <=> -
0)	T	T	T	F	F	F	T	F	T
1)	T	F	F	T	F	F	T	F	F
2)	F	T	F	F	T	F	T	F	F
3)	F	F	F	F	F	T	T	F	T

There are a total of f[4] = 16 permutations, where $f[x] = 2^x$, with 8 as inverses:

Normal	**Inverses**
1) α = (A v B) (i.e. Q1)	09) ¬(α) = (C v D) (i.e. Q3)
2) β = (A v C) (i.e. Q2)	10) ¬(β) = (B v D) (i.e. Q4)
3) A = (+ ^ -)	11) ¬(A) = (B v C v D)
4) B = (+ ~⇒ -)	12) ¬(B) = (A v C v D)
5) C = (- ~⇒ +)	13) ¬(C) = (A v B v D)
6) D = (+ ~v -)	14) ¬(D) = (A v B v C)
7) E ⇒ ∀X: (X v ¬X)	15) F ⇒ ∀X: (X ^ ¬X)
8) G = (+ ↔ -)	16) ¬(G) = (+ ⊕ -)

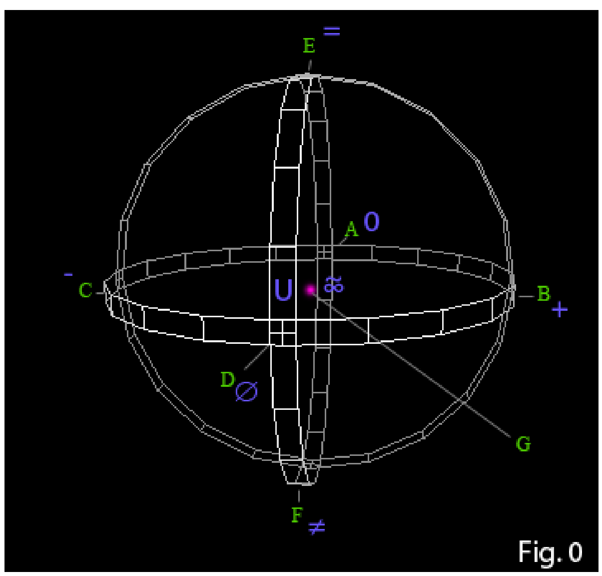


Fig. 0

Notes & Correspondeneces

G defines the extremes of three pairs: {A, D}, {B, C}, {E, F}. By default G expresses (A v D). (B v C) is captured as the inverse and (E v F) is captured through, G ⊕= ¬G ⊕= G ⊕= ¬G, causing G to become ¬G and G to become ¬G; or expressed another way G ↔= ¬G ↔= G ↔= ¬G. (E v F) materializes after step I. G ⊕= ¬G ⇒ G = {T, T, T, T}. Similarly using biconditionals after step I. we see G ↔= ¬G ⇒ G = {F, F, F, F}.

Definition 2.1. Absence as a collection of concepts, hereafter referred to as *zero complete* or $zero_{AG}$, is the set of all permutations of positive and negative modifiers. Representing something akin to an identity matrix (properties A through D in Fig. 0), with the addition of the zero matrix (e.g. $\emptyset \neq \emptyset$ more broadly $\forall\ X : (X \bigwedge \neg\ X)$ as F) and a matrix of ones (i.e. $\{0 = 0,\ \emptyset = \emptyset,\ etc\}$ represented by $\forall\ X : (E \bigvee X)$ as E).

What this intimates is that zero_{AG} is infinite-centric (i.e. $2 = \frac{a-a}{b}$; $b = 0 = \frac{a-a}{2}$; $0 \approx \frac{a-a}{\frac{a-a}{\ddots}} \approx 2$). It would also imply that like infinity, zero_{AG} isn't discrete. So similar to how countable infinity isn't the same as uncountable infinity, this suggests zero_{AG} has numerous properties that can be evaluated and transformed through careful explicit usage of these characteristics. It also seems to express that numerical constructs can in some instances equal themselves and simultaneously not. This follows in line with Dr. Graham Priest's notion, described in his book "*In Contradiction*," that contradictions can occasionally be true (as hard as that is to imagine!).[5]

Here are some of the relationships between the various "types" of absence as I currently see them:

1. Reflexive-Ordinal identity:

$$(A \neq A) \equiv \emptyset \tag{10}$$

Assuming the statement can be true rather than false. Note: $(A = A) \equiv A$, showing that reflexive-equality or -inequality can be simplified in both cases down to a single term as a type of substitution.

2. Cardinal-Dual identity:

$$0 \equiv (+ \wedge -) \tag{11}$$

implying dualness has a cardinality of 2 in the normal case (i.e. $a + b = a - b$; $2\,b = (a - a)$; $2 = (a - a)\,/\,b$, where $b = 0$, assuming not indeterminate due to $\frac{C*y}{D*y} = \frac{C}{D}$ where the 0's cancel thereby mimicking $\lim_{b \to 0} \left(\frac{x*b}{b}\right) = x$; or 4 when accounting for conjunction and negation. This suggests an ...

3. Ordinal-Dual identity:

$$\{\emptyset\} \equiv \neg\ (+ \vee -) \tag{12}$$

4. Cardinal-Ordinal Dual-Reflexive relation:

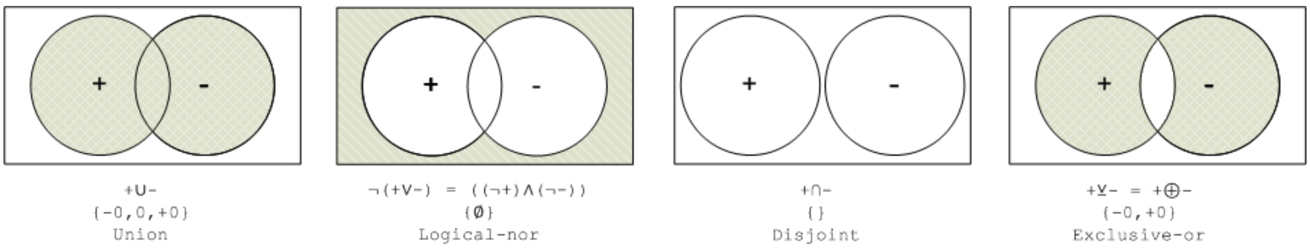
$$(0 \simeq \{+ \bigvee = \ \}) \leftrightarrow (\emptyset \simeq \{- \vee \neq\}) \tag{13}$$

Some fascinating characteristics start to emerge as you think about these adjacencies. For instance, $\forall\ A : \emptyset \subseteq A$ meaning ordinal $\emptyset$ when related to the cardinal-space has a cardinal length of ∞. Suggesting ordinality and cardinality run parallel to one another. The relationship between ordinalness and dualness would seem to be triangular in nature. Probably the easiest way to start to see this is to simply imagine dualness perpendicular to reflexivity; and {cardinal, ordinalness} as parallel and simultaneously orthogonal to {dual, reflexive}. Though there are some strange characteristics when considering $(A \neq A) \equiv \emptyset$. It's hard to imagine $\neq \perp \emptyset$ (where $\perp$ indicates 'perpendicular to') in the instance when $A \neq A$ as being identical to $\emptyset$. Though this may make some sense in that a "not equal to" inequality ($\neq$), as a binary relational operator, doesn't always imply $\emptyset$ in the instances where $A \neq B$ is true and the values are different. Meaning the $\perp$ point of intersection occurs when $\neq$ is a contradiction.

References and Notes

[0]. Weisstein, Eric (1999). "CRC Concise Encyclopedia of Mathematics." Florida: CRC Press. p.1966. ISBN 0849396409. Also see: "Zero." From MathWorld--A Wolfram Web Resource. http://mathworld.wolfram.-com/Zero.html

[1]. "*... following theoretical physicist Paul Dirac, ... a vacuum, or nothing, is the combination of matter and antimatter. Their density is tremendous, but we can't perceive any of them because their observable effects entirely cancel each other out.*" University of Michigan (2010-12-08). "Theoretical breakthrough: Generating matter and antimatter from the vacuum." ScienceDaily. http://www.sciencedaily.com/releas-es/2010/12/101208130038.htm

[2]. 

To give this a semantic meaning as to why the additive identity element is a conjunction of positive and negative. If I was born a Swede (*jus soli*) and had a parent who was a US citizen (*jus sanguinis*). Then I am both a Swede and a US citizen. However at any given moment I can choose to reside in Sweden or the US due to the dual citizenship (i.e. being $a \bigwedge b$ allows for $a \oplus b$). If I am neither Swedish nor a US citizen ($\neg\,(a \bigvee b)$) I wouldn't be allowed to permanently live in either country. So similarly if 0 is neither positive nor negative ($\neg\,(+ \vee -)$) it can never be $+0$ or -0 because it isn't of the same category. Meaning taking 0 as $\neg\,(+ \vee -)$ for granted as true, would indicate $a + 0 \neq a$ because unary $+0$ by its very definition would be invalid. As a consequence of this, since 0 acts a placeholder for positive and negative quantities across all number bases. It naturally follows that 0 is both $(+ \wedge -)$.

[3]. "Complete minimal form" (i.e. $a + 0 = a - 0$; $h(2, 0) = 2$) is meant to differentiate an "incomplete minimal form" (i.e. $a + 0 = a$; $h(1, 0) = 1$) from a "complete nonminimal form" (i.e. $\{x \geq 1,\ y \geq 2 : a + x * 0 = a - y * 0;$ $h(x + y, 0) = x + y\}$). The "incomplete minimal form" is considered incomplete because the traditional additive identity $a + 0 = a = 0 + a$ only accounts for addition. Whereas with subtraction there is additional complexity because the values are anticommutative. To get around this requires either commuting or inverting the signage ($a - b = -b - (-a)$; $a - (-b) = b - (-a)$; $-a - b = -b - a$; $-a - (-b) = b - a$). Nevertheless subtraction is just as applicable for the purpose of returning a values identity because there always exists an isomorphism between addition and subtraction in each of the 2^4 unique $(\pm(a \veebar b)) \pm (\pm(b \veebar a)) \overset{=}{\neq} (\pm(b \veebar a)) \pm (\pm(a \veebar b))$ equal permutations (i.e. [1] $+++ = +--$ [2] $+a+-b = -b--a = +a-+b = -b++a$ [3] $-a++b = +b-+a = -a--b = +b+-a$ [4] $-+- = --+$ where $+++$ and similar variants indicate $(+(a \bigvee b) + (+(a \bigvee b)))$. So just like how $\{+1\}$ is the complete set of multiplicative identity element(s) for multiplication and division ($x * (+1) = x = \frac{x}{(+1)}$). Similarly a complete form of the additive identity needs to account for all variations of the identity elements between the normal and inverse operation ($f(x, b) = x + b$; $f^{-1}(x, b) = x - b$; solving for b in $f(x, b) = f^{-1}(x, b)$) to be complete ($a + 0 = a = a - 0$), and thereby determine the complete set of identity elements $\{-0,\ 0,\ +0\}$.

So where 0 as a conjunction of positive and negative is the identity element for addition and subtraction (or the moment where positive and negative are the same, i.e. $+0 = -0$; cancel the zeros and, $+ = -$, is true in this instance). Meaning $+0$ or -0 are a constituent part, as the sign, of all unary positive ($+a = +a - 0 = -0 - (-(a)) = +a - (-(0)) = 0 - (-(a))$) and negative quantities ($-a = (-a) + 0 = (-0) + (-a) = (-a) + (-0) = 0 + (-a)$) irregardless of whether the expression takes the form of equality, addition, subtraction, or unary double negation. This also reveals a connection between zero and cancellation. As a general rule, whenever something is a tautology we can remove or cancel it (e.g. $\frac{3}{1} + 0 = (3 * 1) - 0$; $3 + 0 = 3 - 0$; $3 = 3$ is the same as $+3$). Implying we can cancel the 0, +, and − because all the properties that zero_{AG} entails (addition, subtraction, positive, negative, equality, and inequality) are reflected as a root of any and all Reals. Similar to how the number 1 imparts quantity to all nonzero numbers. It is these combined properties of zero_{AG} that allows $a + (-b)$ to equal $a - (+b)$ because addition behaves as a type of subtraction and vice versa when the negative quality is preserved in the operand and the combined value decreases.

[4]. cf. Tilman Piesk's diagrammatic representation of the logical connectives as a Hasse diagram http://en.wikipedia.org/wiki/File:Logical_connectives _Hasse _diagram.svg

[5]. Priest, Graham (2006). “In Contradiction: A Study of the Transconsistent.” NY : Oxford University Press. ISBN 978-0199263301. http://books.google.com/books?id=TMztJKtWWSAC