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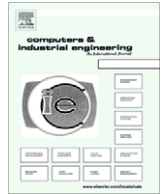
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## Co-op advertising and pricing models in manufacturer–retailer supply chains

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### ABSTRACT

Cooperative (co-op) advertising plays a significant role in marketing programs in conventional supply chains and makes up the majority of promotional budgets in many product lines for both manufacturers and retailers. Nevertheless, most studies to date on co-op advertising have only assumed that the market demand is only influenced by the advertising level but not in any way by the retail price. That is why our work is concerned with co-op advertising and pricing strategies in distribution channels consisting of a manufacturer and a retailer. Four different models are discussed which are based on three non-cooperative games (i.e., Nash, Stackelberg retailer and Stackelberg manufacturer) and one cooperative game. We identify optimal co-op advertising and pricing strategies for both firms mostly analytically but we have to resort to numerical simulations in one case. Comparisons are then made about various outcomes, especially the profits, for all cases. This leads to consider more specifically the cooperation case in which profits are the highest for both the retailer and the manufacturer, and how they should share the extra joint profit achieved by moving to cooperation. We solve this bargain problem using the Nash bargaining model.

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### 1. Introduction

Consider a distribution channel consisting of a manufacturer and a retailer. In the absence of cooperation, channel members determine their decision variables independently and non-cooperatively. It is well known, in the literature and in practice, that such uncoordinated decision making creates “channel inefficiency”. In other words, channel members’ marketing strategies are not at their joint profit maximization levels and their profits are inferior to what could be achieved with coordinated behavior. This creates an incentive for cooperation. If channel members agree to cooperate, they negotiate to make joint decisions that eliminate channel inefficiency. We suppose that although channel members cooperate, they remain independent.

Supply chain models have focused on almost all aspects related to pricing, purchasing, production, and inventories. However, models simultaneously dealing with at least two aspects above are complex and sparse. This paper is to identify optimal pricing and co-op advertising strategies for a manufacturer and a retailer in a distribution channel and concerned with their conflict and coordination.

A fundamental task for supply-chain managers is to determine wholesale prices (and/or retail prices). Such a decision is a core theme in the marketing science literature on distribution channels. For example, [Jeuland and Shugan \(1983, 1988\)](#) and [Moorthy \(1987\)](#) identified two commonly used price mechanisms (quantity-dis-

count schemes and two-part tariffs) that can be used to achieve channel coordination. [Ingene and Parry \(1995a, 1995b, 1998, 2000\)](#) explored wholesale pricing behavior within a two-level vertical channel consisting of a manufacturer selling through multiple independent retailers.

Vertical cooperative (co-op) advertising is a coordinated effort by all members in a distribution channel to increase the customer demand and the overall profits. In a typical distribution channel, the upstream member can be a manufacturer of certain product, who often time promotes its product via the national level advertising to build the long-term image or “brand equity” for the company. Meanwhile, the downstream channel member can be a retailer, who usually advertises the product in its local market to induce short-term purchase. Traditionally, co-op advertising is achieved with the upstream manufacturer sharing a portion of the downstream retailer’s advertising costs, commonly referred to as the manufacturer’s participation rate ([Bergen & John, 1997](#)). It is often used in consumer goods industry and plays a significant role in market strategy for many companies. According to [Nagler \(2006\)](#), the total US expenditure of co-op advertising in 2000 was estimated at \$15 billion, nearly a four-fold increase in real terms in comparison to \$900 million in 1970. The growing importance of co-op advertising motivated us to pay more attention to this subject. [Huang and Li \(2001, 2002\)](#) are two recent papers on co-op advertising, where three one-period co-op advertising models are presented. They explored the role of vertical co-op advertising efficiency with respects to transactions between a manufacturer and a retailer through brand name investments, local advertising expenditures, and sharing rules of advertising expenses. Dynamic

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co-op advertising problems have also discussed, e.g., in [Chin-tagunta and Jain \(1992\)](#); [Jorgensen, Zaccour, and Sigue \(2000\)](#); [Jorgensen, Tiboubi, and Zaccour \(2003\)](#), and among others.

The literature dealing with both pricing and co-op advertising at the same time is sparse, with some exceptions such as [Jorgensen and Zaccour \(1999, 2003\)](#); [Jorgensen, Sigue, and Zaccour \(2001\)](#). In these papers, the authors proposed a dynamic pricing and co-op advertising problem over time, compared coordinated strategies and profits with uncoordinated ones, and then discussed how a coordinated solution could be sustained over time. The difference between this paper and their work is that we consider pricing and co-op advertising models in one period as [Huang and Li \(2001\)](#) did.

The paper proceeds as follows. The next section presents the assumptions and the basic game-theoretic model. In Section 3, four specific models are discussed, which are based on three non-cooperative games (i.e., Nash, Stackelberg retailer and Stackelberg manufacturer), and one cooperative game. In Section 4, the numerical simulation and comparisons between the different scenarios are presented. A bargain problem is identified in Section 5. Section 6 summarizes the findings from the research and proposes future research directions.

## 2. Assumptions and the basic model

Consider a single-manufacturer–single-retailer channel in which the manufacturer sells certain product only through the retailer, and the retailer sells only the manufacturer's brand within the product class. Decision variables for the channel members are their advertising efforts, their prices (manufacturer's wholesale price and retailer's retail price) and the co-op advertising reimbursement policy. Denote by  $a$  and  $q$ , respectively, the retailer's local advertising level and the manufacturer's national advertising investment. The consumer demand function  $V(a, q, p_R)$  depends on the retail price  $p_R$  and the advertising levels  $a$  and  $q$  in a multiplicatively separable way like in [Jorgensen and Zaccour \(1999\)](#), i.e.,  $V(a, q, p_R) = g(p_R)S(a, q)$ ,

where  $g(p_R)$  is linearly decreasing with respect to  $p_R$ , and  $S(a, q)$  is the function that [Huang and Li \(2001\)](#) proposed to model advertising effects on sales in a static way. That is,

$$V(a, q, p_R) = g(p_R)S(a, q) = (\alpha - \beta p_R) \left( A - \frac{B}{a^\gamma q^\delta} \right).$$

The parameters  $\alpha, \beta, B, \gamma, \delta$  are positive constants, and  $A > 0$  is the sales saturate asymptote. Please note that:

$$V(a, q, p_R) > 0 \Rightarrow p_R < \frac{\alpha}{\beta}.$$

Denote by  $t$  the fraction of the local advertising expenditure, which is the percentage the manufacturer agrees to share with the retailer (i.e., the manufacturer's co-op advertising reimbursement policy), and denote by  $p_M$  the manufacturer's transfer price to the retailer. Furthermore, denote by  $c$  (a positive constant) the manufacturer's unit production cost and by  $d$  (a positive constant) the retailer's unit cost incurred in addition to the purchasing cost.

Under these assumptions, the profits of the manufacturer, the retailer and the whole channel can be expressed as follows, respectively:

$$\Pi_M = (p_M - c)(\alpha - \beta p_R) \left( A - \frac{B}{a^\gamma q^\delta} \right) - ta - q, \quad (1)$$

$$\Pi_R = (p_R - p_M - d)(\alpha - \beta p_R) \left( A - \frac{B}{a^\gamma q^\delta} \right) - (1 - t)a, \quad (2)$$

$$\Pi_{M+R} = (p_R - c - d)(\alpha - \beta p_R) \left( A - \frac{B}{a^\gamma q^\delta} \right) - a - q. \quad (3)$$

**Remark 1.** Throughout this paper, the subscript 'M', 'R' and 'M + R' means the parameters corresponding to the manufacturer, the retailer, and the whole system.

Please note that the non-negativity of the retailer's profit implies  $p_R > p_M + d > p_M$ , the non-negativity of the manufacturer's profit implies  $p_M > c$  and the non-negativity of the system profit implies  $p_R > c + d$ . The last inequality and  $p_R < \alpha/\beta$  lead to  $\alpha - \beta(c + d) > 0$ .

In order to handle the problem in an equivalent but more convenient way, we apply an appropriate (and legitimate according the above inequalities) change of variables as shown below:

$$\alpha' = \alpha - \beta(c + d) > 0,$$

$$p'_R = \frac{\beta}{\alpha'}(p_R - (c + d)) > 0,$$

$$p'_M = \frac{\beta}{\alpha'}(p_M - c) > 0,$$

$$B' = \frac{\alpha'^2}{\beta}B,$$

$$A' = \frac{\alpha'^2}{\beta}A.$$

It is easy to see that

$$p_R < \alpha/\beta \iff \beta(p_R - (c + d)) < \alpha - \beta(c + d) \iff \frac{\beta(p_R - (c + d))}{\alpha - \beta(c + d)} < 1 \iff p'_R < 1.$$

Besides, the condition  $p_R > p_M + d$  also implies that  $p'_R > p'_M$ .

Under these redefinitions for variables, the profit functions in (1)–(3) can be rewritten as:

$$\Pi_M = p'_M(1 - p'_R) \left( A' - \frac{B'}{a'^\gamma q'^\delta} \right) - ta - q,$$

$$\Pi_R = (p'_R - p'_M)(1 - p'_R) \left( A' - \frac{B'}{a'^\gamma q'^\delta} \right) - (1 - t)a,$$

$$\Pi_{M+R} = p'_R(1 - p'_R) \left( A' - \frac{B'}{a'^\gamma q'^\delta} \right) - a - q.$$

A final change of variables  $q' = q/B'^{\frac{1}{\gamma+\delta+1}}$ ,  $a' = a/B'^{\frac{1}{\gamma+1}}$ ,  $\Pi' = \Pi/B'^{\frac{1}{\gamma+\delta+1}}$  leads to the following expressions for the manufacturer's, the retailer's and the whole channel's profits:

$$\Pi'_M = p'_M(1 - p'_R) \left( \frac{A'}{B'^{\frac{1}{\gamma+\delta+1}}} - \frac{1}{a'^\gamma q'^\delta} \right) - ta' - q', \quad (4)$$

$$\Pi'_R = (p'_R - p'_M)(1 - p'_R) \left( \frac{A'}{B'^{\frac{1}{\gamma+\delta+1}}} - \frac{1}{a'^\gamma q'^\delta} \right) - (1 - t)a', \quad (5)$$

$$\Pi'_{M+R} = p'_R(1 - p'_R) \left( \frac{A'}{B'^{\frac{1}{\gamma+\delta+1}}} - \frac{1}{a'^\gamma q'^\delta} \right) - a' - q'. \quad (6)$$

For simplicity, hereafter we remove the superscript (') from the above expressions throughout the paper (the only exception is in Section 4.1 where we will discuss how to set these parameters approximately).

## 3. The four game scenarios

### 3.1. Nash game

When the manufacturer and the retailer have the same decision power, they simultaneously and non-cooperatively maximize their own profits. This situation is called a Nash game and the solution provided by this structure is called the Nash equilibrium.

Specifically, the manufacturer's decision problem is

$$\begin{aligned} \text{Max}_{t,q,p_M} \Pi_M &= p_M(1-p_R) \left( \frac{A}{B^{\gamma+\delta+1}} - \frac{1}{a^\gamma q^\delta} \right) - ta - q, \\ \text{s.t. } 0 &\leq t \leq 1, \quad 0 \leq q, \quad 0 \leq p_M \leq 1, \end{aligned} \quad (7)$$

and the retailer's decision problem is

$$\begin{aligned} \text{Max}_{a,p_R} \Pi_R &= (p_R - p_M)(1-p_R) \left( \frac{A}{B^{\gamma+\delta+1}} - \frac{1}{a^\gamma q^\delta} \right) - (1-t)a, \\ \text{s.t. } 0 &\leq a, \quad 0 \leq p_R \leq 1. \end{aligned} \quad (8)$$

It is obvious that the optimal value of  $t$  is zero because of its negative coefficient in the objective (7). It is also obvious that the manufacturer's profit is increasing with  $p_M$ . But  $p_M$  can not be equal to 1, otherwise there is no profit at all for both the manufacturer and the retailer because of the fact that  $p_R \geq p_M = 1 \Rightarrow p_R = 1 \Rightarrow \Pi_R = 0 = \Pi_M$ . In order to solve the problem, we have to add more hypotheses. Here we use the one as in [Jorgensen and Zaccour \(1999\)](#): If the manufacturer and the retailer make their decisions simultaneously, then we could assume that their respective margins are equal. That is:

$$(p_R - p_M) = p_M, \quad \text{or} \quad p_R = p_M/2. \quad (9)$$

The first-order conditions for the manufacturer and the retailer are as following:

$$\frac{\partial \Pi_M}{\partial q} = \delta p_M(1-p_R)a^{-\gamma}q^{-(\delta+1)} - 1 = 0, \quad (10)$$

$$\frac{\partial \Pi_R}{\partial a} = \gamma(p_R - p_M)(1-p_R)a^{-(\gamma+1)}q^{-\delta} - (1-t) = 0, \quad (11)$$

$$\frac{\partial \Pi_R}{\partial p_R} = \left( \frac{A}{B^{\gamma+\delta+1}} - \frac{1}{a^\gamma q^\delta} \right) \{ (1-p_R) - (p_R - p_M) \} = 0. \quad (12)$$

Solving Eqs. (9)–(12) and noticing that  $t$  should be zero under this situation, we can obtain the unique Nash equilibrium as follows (the superscript 'N' means the variables corresponding to the Nash equilibrium).

$$p_M^N = 1/3, \quad p_R^N = 2/3, \quad t^N = 0, \quad (13)$$

$$a^N = \left[ \left( \frac{\gamma}{\delta} \right) \frac{1}{9} \gamma \right]^{\frac{1}{\gamma+\delta+1}}, \quad (14)$$

$$q^N = \frac{\delta}{\gamma} a^N. \quad (15)$$

### 3.2. Stackelberg retailer game

We now model the relationship between the manufacturer and the retailer as a sequential non-cooperative game with the retailer as the leader and the manufacturer as the follower. The solution of this game is called the Stackelberg retailer equilibrium. The retailer, as the leader, first declares the level of its local advertising expenditure that he is willing to invest, and sets the retail price for the product. The manufacturer, as the follower, then sets its own national advertising level and its wholesale price.

In order to determine the Stackelberg retailer equilibrium, we first solve the manufacturer's decision problem (7) to find the best responses of  $t, q, p_M$  to any given values of  $a, p_R$ . Once again we can easily see that  $t = 0$ . Similarly to the Nash game case above, the manufacturer's profit is increasing with  $p_M$ , but, especially in a follower position, it's hard to set the manufacturer's price as big as desired and a natural constraint is that the manufacturer's margin should not be bigger than the retailer's one, and therefore we add the constraint (9) again. Furthermore, because  $\Pi_M$  is a concave function of  $q$ , we can get the optimal value of  $q$  by solving the same first order equation as (10). Therefore, the optimal values for  $t, p_M, q$  are

$$t = 0, \quad p_M = p_R/2, \quad q = [\delta a^{-\gamma} p_R(1-p_R)/2]^{1/(\delta+1)}. \quad (16)$$

Next, the optimal values of  $a, p_R$  are determined by maximizing the retailer's profit subject to the constraints imposed by equations in (16). That is,

$$\begin{aligned} \text{Max}_{a,p_R} \Pi_R &= (p_R - p_M)(1-p_R) \left( \frac{A}{B^{\gamma+\delta+1}} - \frac{1}{a^\gamma q^\delta} \right) - (1-t)a, \\ \text{s.t. } 0 &\leq a, \quad 0 \leq p_R \leq 1 \quad \text{with} \quad t = 0, \quad p_M = p_R/2, \\ q &= [\delta a^{-\gamma} p_R(1-p_R)/2]^{1/(\delta+1)}. \end{aligned}$$

For this optimization problem, we can rewrite the objective function as

$$\Pi_R = \frac{1}{2} p_R(1-p_R) \left\{ \frac{A}{B^{\gamma+\delta+1}} - \delta^{-\frac{\delta}{\delta+1}} a^{-\frac{\gamma}{\delta+1}} \left[ \frac{1}{2} p_R(1-p_R) \right]^{-\frac{\delta}{\delta+1}} \right\} - a.$$

The corresponding first-order condition with respect to  $p_R$  is the following equation:

$$\begin{aligned} \frac{\partial \Pi_R}{\partial p_R} &= \frac{1}{2} (1-2p_R) \left( \frac{A}{B^{\gamma+\delta+1}} - \delta^{-\frac{\delta}{\delta+1}} a^{-\frac{\gamma}{\delta+1}} \left[ \frac{1}{2} p_R(1-p_R) \right]^{-\frac{\delta}{\delta+1}} \right) \\ &+ \frac{1}{2} (1-2p_R) \left( \frac{\delta}{\delta+1} \delta^{-\frac{\delta}{\delta+1}} a^{-\frac{\gamma}{\delta+1}} \left[ \frac{1}{2} p_R(1-p_R) \right]^{-\frac{\delta}{\delta+1}} \right) = 0. \end{aligned} \quad (17)$$

Because the expressions in the second brackets for both of the terms in the left hand side of (17) are always strictly positive (it is the sum of two positive terms: the first term is positive because  $\Pi_R > 0$  and the second term is also positive because  $p_R < 1$ ), Eq. (17) leads to (the superscript 'SR' means the variables corresponding to the Stackelberg retailer equilibrium)

$$p_R^{SR} = 1/2. \quad (18)$$

Now we use the other first-order equation with respect to  $a$ :

$$\frac{\partial \Pi_R}{\partial a} = \delta^{-\frac{\delta}{\delta+1}} \left( \frac{\gamma}{\delta+1} \right) \left[ \frac{1}{2} p_R(1-p_R) \right] a^{-\frac{\gamma+\delta+1}{\delta+1}} \left[ \frac{1}{2} p_R(1-p_R) \right]^{-\frac{\delta}{\delta+1}} - 1 = 0. \quad (19)$$

Eqs. (16), (18) and (19) lead to

$$p_M^{SR} = 1/4, \quad t^{SR} = 0, \quad (20)$$

$$a^{SR} = \left[ \left( \frac{\gamma}{\delta(\delta+1)} \right) \frac{1}{8} \left( \frac{\gamma}{\delta+1} \right) \right]^{\frac{1}{\gamma+\delta+1}}, \quad (21)$$

$$q^{SR} = \frac{\delta(\delta+1)}{\gamma} a^{SR}. \quad (22)$$

### 3.3. Stackelberg manufacturer game

In this part, we model the relationship between the manufacturer and the retailer as a sequential non-cooperative game in the same way as in Section 3.2 but with the manufacturer being the leader and the retailer being the follower. The solution of this game is hence called the Stackelberg manufacturer equilibrium. We proceed exactly as in Section 3.2.

In order to determine the Stackelberg manufacturer equilibrium, we first solve the retailer's decision problem (8) to find the best responses of  $a$  and  $p_R$  to any given values of  $t, q$ . Because  $\Pi_R$  is a concave function of  $a$  and  $p_R$ , we can solve the two first order Eqs. (11) and (12) (like in Section 3.1) to get the following relationship:

$$p_R = \frac{1+p_M}{2}, \quad (23)$$

$$a = \left[ \frac{\gamma(p_R - p_M)(1-p_R)}{(1-t)q^\delta} \right]^{\frac{1}{\gamma+1}} = \left[ \frac{\gamma(1-p_M)^2}{4(1-t)q^\delta} \right]^{\frac{1}{\gamma+1}}. \quad (24)$$

Next, the optimal values of  $q$ ,  $p_M$  and  $t$  are determined by maximizing the manufacturer's profit subject to the constraints imposed by Eqs. (23) and (24). That is,

$$\begin{aligned} \text{Max}_{q, p_M, t} \Pi_M &= p_M(1 - p_R) \left( \frac{A}{B^{\frac{1}{\gamma+\delta+1}}} - \frac{1}{a^\gamma q^\delta} \right) - ta - q, \\ \text{s.t. } 0 &\leq q, \quad 0 \leq p_M, \quad t \leq 1 \quad \text{with (23) and (24)}. \end{aligned}$$

Substituting Eqs. (23) and (24) into the expression of  $\Pi_M$ , we have

$$\begin{aligned} \Pi_M &= \frac{1}{2} p_M(1 - p_M) \left[ \frac{A}{B^{\frac{1}{\gamma+\delta+1}}} - \left( \frac{\gamma}{4} \right)^{\frac{\gamma}{\gamma+1}} (1-t)^{\frac{\gamma}{\gamma+1}} q^{-\frac{\delta}{\gamma+1}} (1-p_M)^{-\frac{2\gamma}{\gamma+1}} \right] \\ &\quad - \left( \frac{\gamma}{4} \right)^{\frac{1}{\gamma+1}} t(1-t)^{-\frac{1}{\gamma+1}} (1-p_M)^{\frac{2}{\gamma+1}} q^{-\frac{\delta}{\gamma+1}} - q. \end{aligned}$$

By taking  $\partial \Pi_M / \partial q = 0$  and  $\partial \Pi_M / \partial t = 0$ , we have

$$\begin{aligned} \frac{1}{2} p_M(1 - p_M) \left[ \frac{\delta}{\gamma+1} \left( \frac{\gamma}{4} \right)^{\frac{\gamma}{\gamma+1}} (1-t)^{\frac{\gamma}{\gamma+1}} q^{-\frac{\delta}{\gamma+1}-1} (1-p_M)^{-\frac{2\gamma}{\gamma+1}} \right] \\ + \frac{\delta}{\gamma+1} \left( \frac{\gamma}{4} \right)^{\frac{1}{\gamma+1}} t(1-t)^{-\frac{1}{\gamma+1}} (1-p_M)^{\frac{2}{\gamma+1}} q^{-\frac{\delta}{\gamma+1}-1} - 1 = 0 \end{aligned} \quad (25)$$

$$\begin{aligned} \frac{\partial \Pi_M}{\partial t} &= \frac{q^{-\frac{\delta}{\gamma+1}}}{\gamma+1} (1-t)^{-\frac{1}{\gamma+1}} \left( \frac{\gamma}{4} \right)^{\frac{1}{\gamma+1}} (1-p_M)^{\frac{2}{\gamma+1}} \\ &\quad \times \left\{ \frac{2p_M}{1-p_M} - \left( \gamma + 1 + \frac{t}{(1-t)} \right) \right\} = 0. \end{aligned} \quad (26)$$

Eq. (26) implies (the superscript  $S_M$  means the variables corresponding to the Stackelberg manufacturer equilibrium)

$$t^{S_M} = \frac{(3+\gamma)p_M - (\gamma+1)}{(2+\gamma)p_M - \gamma}. \quad (27)$$

Recalling that  $t$  should be non-negative in Eq. (27), we should consider two different cases: Either  $p_M^{S_M} > \frac{1+\gamma}{3+\gamma}$  and  $t \neq 0$ , or  $p_M^{S_M} \leq \frac{1+\gamma}{3+\gamma}$  and  $t = 0$ . In the first case, we should solve Eqs. (23)–(25) and (27); while in the second case, we should replace (27) with  $t^{S_M} = 0$ . After tedious algebraic calculations, we can obtain the following solutions in each case expressed as functions of  $p_M$ , respectively:

$$\begin{cases} t^{S_M} = \frac{(3+\gamma)p_M - (\gamma+1)}{(2+\gamma)p_M - \gamma}, \\ a^{S_M} = \left[ \left( \frac{\gamma}{\delta} \right)^{\frac{\gamma}{4}} (1-p_M)((2+\gamma)p_M - \gamma) \right]^{\frac{1}{\gamma+\delta+1}}, \\ q^{S_M} = \frac{\delta}{\gamma} a^{S_M}, \\ p_R^{S_M} = \frac{1+p_M}{2}, \\ t^{S_M} = 0, \\ a^{S_M} = \left[ \left( \frac{\gamma(\gamma+1)(1-p_M)}{2\delta p_M} \right)^{\frac{\delta}{4}} (1-p_M)^2 \right]^{\frac{1}{\gamma+\delta+1}}, \\ q^{S_M} = \left( \frac{\delta}{\gamma} \right)^{\frac{2p_M}{(1-p_M)(\gamma+1)}} a^{S_M}, \\ p_R^{S_M} = \frac{1+p_M}{2}. \end{cases}$$

Now we just have to find out  $p_M^{S_M}$  to obtain all the solutions. This leads to discuss the two cases of  $t \neq 0$ , and  $t = 0$ .

(i) Case  $t \neq 0$

In the case  $p_M^{S_M} > \frac{1+\gamma}{3+\gamma}$  and  $t \neq 0$ ,  $\frac{\partial \Pi_M}{\partial p_M} = 0$  implies

$$\begin{aligned} \frac{1}{2} (1 - 2p_M) \left[ \frac{A}{B^{\frac{1}{\gamma+\delta+1}}} \left( \frac{\gamma}{4} \right)^{\frac{\gamma}{\gamma+1}} q^{-\frac{\delta}{\gamma+1}} (1-t)^{-\frac{\gamma}{\gamma+1}} (1-p_M)^{\frac{2\gamma}{\gamma+1}} - 1 \right] \\ - \frac{\gamma}{\gamma+1} p_M + \frac{2}{\gamma+1} \left( \frac{\gamma}{4} \right)^{\frac{1}{\gamma+1}} t(1-t)^{-1} (1-p_M) = 0. \end{aligned} \quad (28)$$

By substituting the corresponding expressions of  $q^{S_M}$  and  $t^{S_M}$  into (28), we finally obtain

$$\frac{A}{B^{\frac{1}{\gamma+\delta+1}}} \left( \frac{\delta^\delta \gamma^\gamma}{4^{\delta+\gamma}} \right)^{\frac{1}{\delta+\gamma+1}} \{ (1-p_M)((2+\gamma)p_M - \gamma) \}^{\frac{\delta+\gamma}{\delta+\gamma+1}} = 1 + \frac{\gamma}{2} \frac{1-p_M}{\frac{1}{2}-p_M}. \quad (29)$$

(ii) Case  $t = 0$

In the second case  $p_M^{S_M} \leq \frac{1+\gamma}{3+\gamma}$  and  $t = 0$ ,  $\frac{\partial \Pi_M}{\partial p_M} = 0$  implies

$$\frac{1}{2} (1 - 2p_M) \left[ \frac{A}{B^{\frac{1}{\gamma+\delta+1}}} \left( \frac{\gamma}{4} \right)^{\frac{\gamma}{\gamma+1}} q^{-\frac{\delta}{\gamma+1}} (1-p_M)^{\frac{2\gamma}{\gamma+1}} - 1 \right] - \frac{\gamma}{\gamma+1} p_M = 0. \quad (30)$$

By substituting the corresponding expression of  $q^{S_M}$  into (30), we finally obtain

$$\begin{aligned} \frac{A}{B^{\frac{1}{\gamma+\delta+1}}} \left( \frac{\delta^\delta \gamma^\gamma}{4^{\delta+\gamma}} \right)^{\frac{1}{\delta+\gamma+1}} \left( \frac{2}{\gamma+1} \right)^{\frac{\delta}{\delta+\gamma+1}} (1-p_M)^{\frac{\delta}{\delta+\gamma+1}} ((2+\gamma)p_M - \gamma)^{\frac{2\gamma}{\delta+\gamma+1}} \\ = 1 + \left( \frac{\gamma}{\gamma+1} \right) \frac{p_M}{\frac{1}{2}-p_M}. \end{aligned} \quad (31)$$

Eqs. (29) and (31) are difficult to be solved analytical, and their solutions strongly depend on the influence of the parameters  $A$  and  $B$ , whereas in Sections 3.1 and 3.2 all the solutions are independent of them. Hence we will just find numerical solutions for them, which is the main point of Section 4.

### 3.4. Cooperation

In the previous three subsections, we analyzed three non-cooperative game structures (one simultaneous-move game and two sequential-move games). In this part, we focus on a cooperative game structure. That is to say, both the manufacturer and the retailer agree to take decisions to maximize the whole channel's profit.

The channel's profit is described by Eq. (6) and depends only on  $p_R$ ,  $a$  and  $q$ . We hence have the following optimization problem:

$$\text{Max}_{p_R, a, q} \Pi_{M+R} = p_R(1 - p_R) \left( \frac{A}{B^{\frac{1}{\gamma+\delta+1}}} - \frac{B}{a^2 q^\delta} \right) - a - q.$$

This equation can easily be solved by taking the three first order equations equal to zeros. Specifically, we have

$$\frac{\partial \Pi_{M+R}}{\partial p_R} = (1 - 2p_R) \left( \frac{A}{B^{\frac{1}{\gamma+\delta+1}}} - \frac{B}{a^2 q^\delta} \right) = 0, \quad (32)$$

$$\frac{\partial \Pi_{M+R}}{\partial a} = \gamma p_R(1 - p_R) q^{-\delta} a^{-(\gamma+1)} - 1 = 0, \quad (33)$$

$$\frac{\partial \Pi_{M+R}}{\partial q} = \delta p_R(1 - p_R) q^{-(\delta+1)} a^{-\gamma} - 1 = 0. \quad (34)$$

These equations leads to the following solution (the superscript 'co' means the variables corresponding to the equilibrium under cooperation):

$$p_R^{co} = 1/2, \quad q^{co} = \frac{\delta}{\gamma} a^{co}, \quad a^{co} = \left[ \left( \frac{\gamma}{\delta} \right)^{\frac{\delta}{4}} \frac{\gamma}{4} \right]^{\frac{1}{\gamma+\delta+1}}. \quad (35)$$

If  $p_R$ ,  $a$  and  $q$  are, respectively, equal to  $p_R^{co}$ ,  $a^{co}$  and  $q^{co}$ , then the channel's profit is maximized with  $t$  and  $p_M$  being free to take any values between 0 and 1 (provided of course that  $p_M \leq p_R^{co}$ ). But obviously, the manufacturer's profit and the retailer's profit are not independent of  $t$  and  $p_M$ . More precisely, neither the manufacturer nor the retailer would be willing to maximize the system profit and to accept fewer profits with cooperation than those without cooperation. The relevancy of the cooperation game is discussed later in the next section when comparisons with profits gained in other games being made possible.



## 4. Numerical simulations and comparisons

### 4.1. Numerical assumptions

In Section 3.3, we failed to analytically solve for the manufacturer's wholesale price in the Stackelberg manufacturer case. In order to solve Eqs. (29) and (31) numerically, we need an estimation of the following parameters:  $\gamma, \delta$  and  $A'/B^{1/(\delta+\gamma+1)}$ . Once we have fixed these parameters and solved for the manufacturer's price, we can easily numerically compare the differences in profits, advertising expenditures, and the prices for the four scenarios discussed in Section 3.

Let's go back to the profits described by the system of Eqs. (1)–(3) and normalize the cost parameters  $c = d = 0$ . Studies in Barry and Evans (1992) ("Selected Advertising-to-Sales Ratios in 1990 by Type of Retailer", p.493) found out that the average advertising expenditure represents about five percent of the net company sales. Hence we set

$$q + a = \frac{5}{100} p_R (\alpha - \beta p_R) \left( A - \frac{B}{a^\gamma q^\delta} \right). \quad (36)$$

Since  $(A - B/(a^\gamma q^\delta))$  represents the influence of the advertising level on the sales, we reasonably set the estimations for  $A$  and  $(B/a^\gamma q^\delta)$  as

$$A = 2 \text{ and } B/a^\gamma q^\delta = 0.5. \quad (37)$$

In fact, for usual companies in most cases, even a huge advertising campaign cannot double the sales, hence we set  $A = 2$ . We also suppose that the usual national advertising level already increases the sales by 50%, and hence we set  $B/a^\gamma q^\delta = 0.5$  (please note that  $A - B/a^\gamma q^\delta = 1$  means no increase in sales). Indeed, all these parameters are varying within each sector of products, and these general assumptions might be discussible; our point is just to find an approximation for the numerical simulation.

Besides, according to our previous discussions, a good guess for  $p'_R$  is  $p'_R \approx 1/2$ . That is to say,  $p_R \approx \alpha/(2\beta)$ . By substituting this estimation and (36) into (37), we have:

$$\begin{aligned} q + a &= \frac{5}{100} p_R (\alpha - \beta p_R) \left( A - \frac{B}{a^\gamma q^\delta} \right) \\ &\approx \frac{5}{100} \frac{\alpha}{2\beta} \left( \alpha - \beta \frac{\alpha}{2\beta} \right) (2 - 0.5) \approx \frac{0.75}{40} \frac{\alpha^2}{\beta} \approx \frac{1}{53} \frac{\alpha^2}{\beta}. \end{aligned} \quad (38)$$

Once again, according to our previous discussions,

$$q' \approx \delta a' / \gamma \iff q \approx \delta a / \gamma. \quad (39)$$

Combining Eqs. (38) and (39), we have

$$a \approx \frac{\gamma}{\gamma + \delta} \left( \frac{1}{53} \frac{\alpha^2}{\beta} \right) \quad \text{and} \quad q \approx \frac{\delta}{\gamma + \delta} \left( \frac{1}{53} \frac{\alpha^2}{\beta} \right).$$

Substituting this estimation into  $B/a^\gamma q^\delta \approx 0.5$  leads to

$$B \approx 0.5 \frac{\delta^\delta \gamma^\gamma}{(\delta + \gamma)^{\delta+\gamma}} \left( \frac{1}{53} \frac{\alpha^2}{\beta} \right)^{\delta+\gamma}. \quad (40)$$

Finally, our change of variables in Section 1 leads to

$$A'B^{\frac{1}{\delta+\gamma+1}} = AB^{\frac{1}{\delta+\gamma+1}} (\alpha^2/\beta)^{\frac{\delta+\gamma}{\delta+\gamma+1}}. \quad (41)$$

Substituting (40) and  $A = 2$  into (41) leads to

$$A'B^{\frac{1}{\delta+\gamma+1}} \approx 2 \left[ 2(\delta + \gamma)^{\delta+\gamma} 53^{\delta+\gamma} / (\delta^\delta \gamma^\gamma) \right]^{\frac{1}{\delta+\gamma+1}}. \quad (42)$$

With above numerical assumptions,  $\delta$  and  $\gamma$  become the channel's only parameters. For the values of  $\delta, \gamma$ , we will consider  $(\delta, \gamma) \in [0.1, 3]^2$  to see the influence of these two parameters on the solutions (specifically, in the following numerical simulations we discrete the interval with a step of 0.1).

### 4.2. Numerical solutions for Stackelberg manufacturer case

To obtain the manufacturer's price  $p_M^{SM}$  under the condition (42), we use MATLAB to solve Eq. (29) (for the situation of  $t > 0$ ) and Eq. (31) (for the situation of  $t = 0$ ). For each solution of the manufacturer's price, we calculate the associated manufacturer's profit under the condition (42). Of course, the optimal manufacturer's price in the Stackelberg manufacturer case corresponds to the one with the highest profit among the situations of  $t > 0$  and  $t = 0$ . Figs. 1 and 2 present the values of the manufacturer's price  $p_M^{SM}$  and the corresponding participation ratio  $t^{SM}$  at the Stackelberg manufacturer equilibrium for different values of the parameters  $\gamma$  and  $\delta$ .

We notice that the manufacturer's price is quite stable with typical values ranging from 0.45 to 0.5, whereas the manufacturer's participation ratio is positive only if we approximately have  $\gamma \leq 1$ . This finding is consistent with Proposition 1 of Huang and Li (2001). More exactly, as  $\gamma$  increases from 0 to 1, while the man-

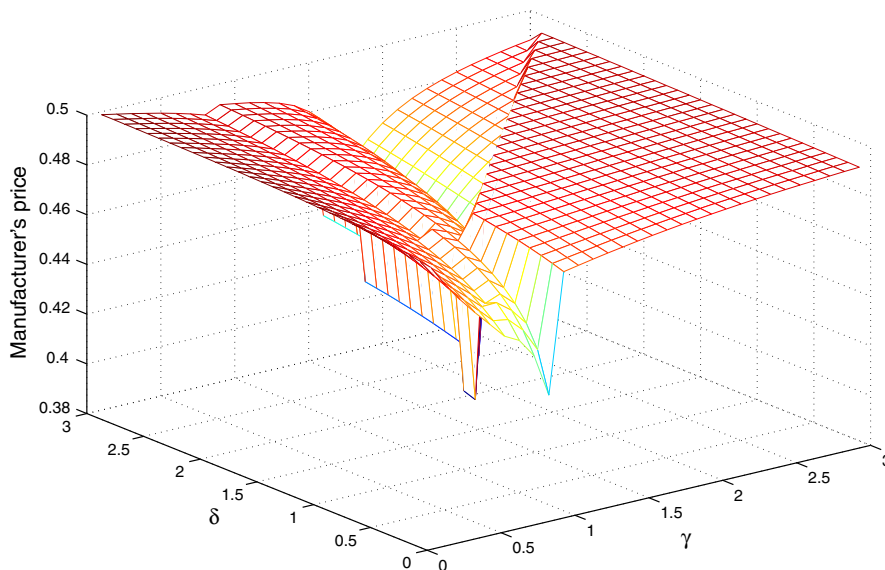


Fig. 1. Manufacturer price  $p_M^{SM}$ .

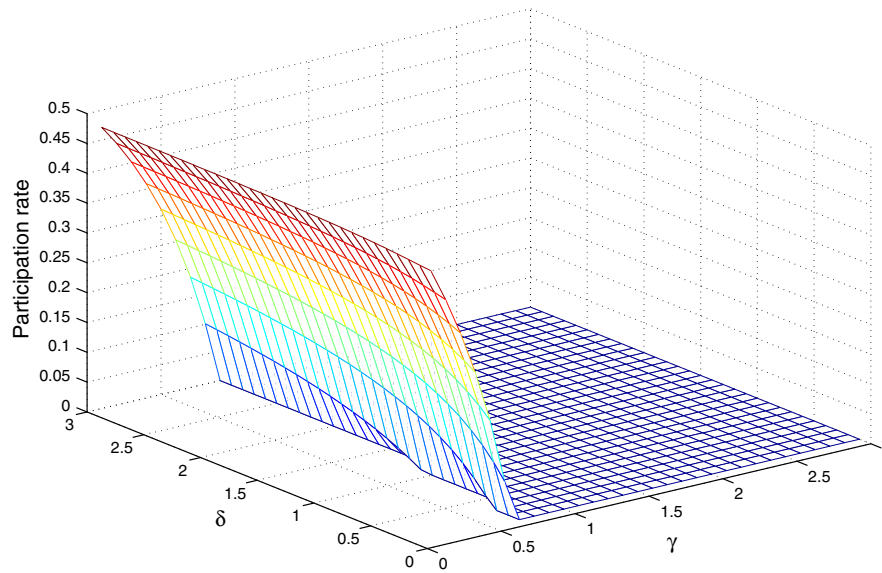
Fig. 2. Participation ratio  $t^{SM}$ .

Table 1

Influence of the four scenarios on prices and margins

	Cooperation	Stackelberg retailer	Nash	Stackelberg manufacturer
Retail price $p_R$	0.5	0.5	0.66	0.73
Retailer "margin" $(p_R - p_M)(1 - p_R)$	To be determined	0.125	0.111	0.07
Manufacturer price $p_M$	To be determined	0.25	0.33	0.47
Manufacturer "margin" $p_M(1 - p_R)$	To be determined	0.125	0.111	0.1245

ufacturer's price  $p_M^{SM}$  is quite stable, the participation ratio  $t^{SM}$  decreases from 0.5 to 0, which is consistent with the formulae obtained for  $t^{SM}$  in the study of the Stackelberg manufacturer case (see Eq. (27) which implies  $\partial t^{SM} / \partial \gamma < 0$ ).

#### 4.3. Comparisons

##### 4.3.1. Comparisons on prices

What's the influence of the scenario on the retailer's and the manufacturer's prices, their advertising expenditures and the profits? When we consider the case where the manufacturer's price in the Stackelberg manufacturer case approximately equals to 0.47, we have, according to our previous analytical results and our numerical simulations, the interesting Table 1.

We notice that the highest retail prices occur at the Stackelberg manufacturer and Nash equilibria whereas the lowest retail prices occur when both the retailer and the manufacturer cooperate or when the retailer is the leader. The high values of the retail price in the conflict cases versus the low value of the retail price in the coordination case are both well known in the literature. Now, if the manufacturer is the leader, he will abuse of its position to impose a very high wholesale price to its retailer who have consequently no choice but to mark a high retail price and yet to obtain a miserable margin. On the other hand, if the retailer is the leader, he marks a lower retail price to get a higher margin and hence induce a lower manufacturer's wholesale price; the higher manufacturer's margin is due to our hypothesis on the equality of the margins for the retailer and the manufacturer in this case.

##### 4.3.2. Comparisons on advertising expenditures

Similarly, we could get a comparison between the different levels of advertising expenditures in the different models. For the lo-

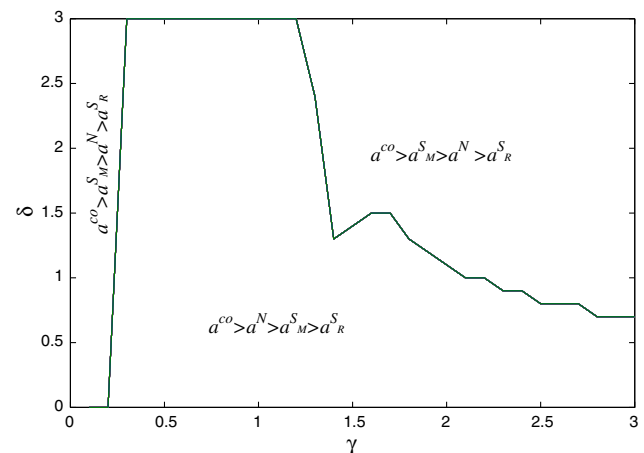


Fig. 3. Local advertising expenditures.

cal advertising expenditures and the national advertising investments, the simulation results are displayed in Figs. 3 and 4, respectively (Using Matlab and the numerical assumptions in Section 4.1, we compute for every couple  $(\delta, \gamma)$  the advertising levels in every scenario).

Concerning the local advertising expenditure of the retailer, it is the highest in the cooperation case, and is the smallest in the Stackelberg retailer case. On the other hand, the manufacturer's national advertising investment is the highest in the Stackelberg retailer case (above the curve) and the cooperation case (below the curve), and is the smallest in the Stackelberg manufacturer case. Indeed, if the retailer is the leader, he will manage to make the manufacturer pay a larger amount of the total advertising ex-

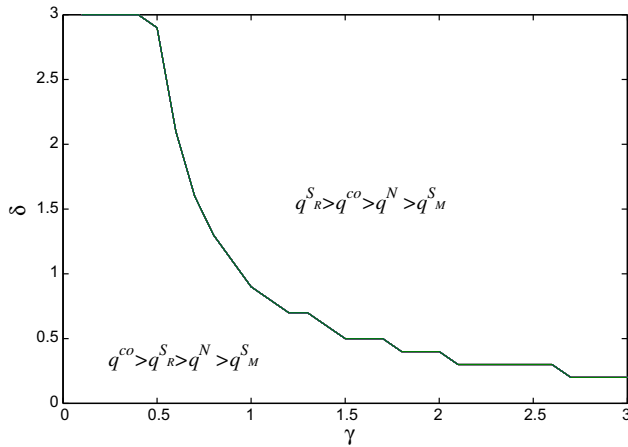


Fig. 4. National advertising expenditures.

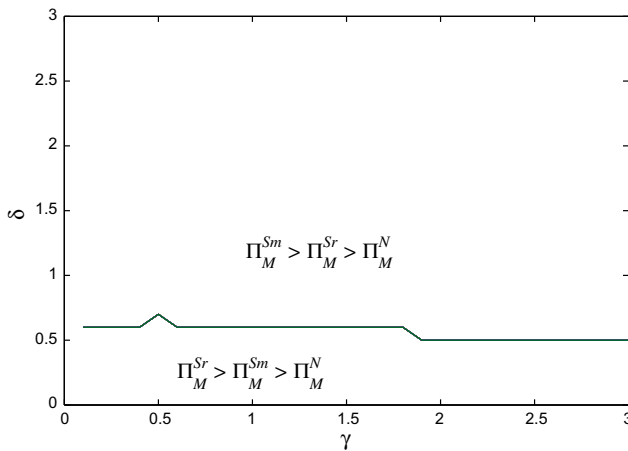


Fig. 5. Manufacturer's profits.

pense with a high national advertising investment and a relatively small level of local advertising to increase its own profit.

The same explanation stands for the case where the manufacturer is the leader. In the same way, in a conflict situation, we have a free-ride problem where both the manufacturer and the retailer has the temptation to invest less in advertising and to benefit from the investment of the other whereas this problem is resolved in the cooperation case.

#### 4.3.3. Comparisons on profits

The most important performance measure of the distribution channel deals with profits. Fig. 5 compares the manufacturer's profit in the Nash, the Stackelberg retailer and the Stackelberg manufacturer cases. There are two striking findings: (1) The manufacturer always prefers to be the follower of the retailer than to be in a conflict situation with the retailer; (2) For small values of  $\delta$  (that is to say, below the curve, where  $\delta$  is approximately smaller 0.5), the manufacturer even prefers to be the retailer's follower rather than to be the leader (though the numerical simulation proves that the difference of profits is very small). There are mainly two explanations to these observations. First, when the retailer is the leader, we have assumed the equality (9) that leads to a quite fair bargain for the manufacturer. Although being the retailer's follower, he gains the same relatively high margin (see also Table 1). The second reason is closely linked to the structure of the Stackelberg manufacturer equilibrium itself. In fact, the manufacturer's "margin" equals

to 0.125 in the Stackelberg retailer case, which drops to 0.1245 in the Stackelberg manufacturer case. Therefore, for small values of  $\delta$ , the manufacturer's profit in the Stackelberg retailer case is higher than in the Stackelberg manufacturer case. But for large values of  $\delta$ , the expensive national advertising investment in the Stackelberg retailer case is much more than the national advertising investment in the Stackelberg manufacturer case, and hence the profit is smaller. Under this situations, the manufacturer always prefers to be the leader.

With regards to the retailer's profits, we have the following very simple result:

$$\forall(\gamma, \delta) : \Pi_R^{S_R} \geq \Pi_R^N \geq \Pi_R^{S_M}$$

Therefore, the retailer always prefers to be the leader and always prefers to be in conflict with the manufacturer rather than to be his follower. Indeed, from Table 1 we can see that the retailer's margin in the Stackelberg manufacturer equilibrium is so small compared to the retailer's margin in the Nash case, that it could never be compensated by the manufacturer's advertising allowance. The only way for the retailer to accept to be the follower would be to have a relatively lower margin compared to the manufacturer's one as showed in Huang and Li (2001). But our model assumes equality between both margins, which is not the situation as Huang and Li (2001).

Fig. 6 compares the whole channel's profit under the three non-cooperative games and the cooperative game. It reveals

$$\forall(\gamma, \delta) : \Pi_{M+R}^{Co} \geq \Pi_{M+R}^{S_R} \geq \Pi_{M+R}^N \geq \Pi_{M+R}^{S_M}$$

In other words, cooperation leads to the best performance for the whole channel. In the next subsection, we will show that cooperation can also guarantee the highest profits for both the retailer and the manufacturer with regards to all other game settings.

#### 4.4. Feasibility of the cooperation game

In Section 3.4, we have found the analytical solution for  $(p_R^{co}, a^{co}, q^{co})$  in the cooperation game with  $t$  and  $p_M$  being free to take any values between 0 and 1. We call a solution  $(p_R^{co}, a^{co}, q^{co}, p_M^{co}, t^{co})$  strongly feasible if and only if both the manufacturer and the retailer cannot get any higher profits in any other games:

$$\Pi_M^{co} = \Pi_M(p_R^{co}, a^{co}, q^{co}, p_M^{co}, t^{co}) \geq \max(\Pi_M^{S_M}, \Pi_M^{S_R}, \Pi_M^N) = \Pi_M^{max}, \quad (43)$$

$$\Pi_R^{co} = \Pi_R(p_R^{co}, a^{co}, q^{co}, p_M^{co}, t^{co}) \geq \max(\Pi_R^{S_M}, \Pi_R^{S_R}, \Pi_R^N) = \Pi_R^{max}. \quad (44)$$

Eqs. (43) and (44) are equivalent to

$$p_M^{co}(1 - p_R^{co}) \left( \frac{A}{B^{\frac{1}{\gamma+\delta+1}}} - \frac{1}{(a^{co})^\gamma (q^{co})^\delta} \right) - t^{co} a^{co} - q^{co} \geq \Pi_M^{max}, \quad (45)$$

$$(p_R^{co} - p_M^{co})(1 - p_R^{co}) \left( \frac{A}{B^{\frac{1}{\gamma+\delta+1}}} - \frac{1}{(a^{co})^\gamma (q^{co})^\delta} \right) - (1 - t^{co}) a^{co} \geq \Pi_R^{max}. \quad (46)$$

Summing up (45) and (46), we have

$$\begin{aligned} \Pi_{M+R}^{co} &= \Pi_M^{co} + \Pi_R^{co} = p_R^{co}(1 - p_R^{co}) \left( \frac{A}{B^{\frac{1}{\gamma+\delta+1}}} - \frac{1}{(a^{co})^\gamma (q^{co})^\delta} \right) - a^{co} - q^{co} \\ &\geq \Pi_M^{max} + \Pi_R^{max} = \Pi_{M+R}^{max}. \end{aligned} \quad (47)$$

Please note that the simulation results in Section 4.3 reveal that  $\Pi_M^{max}$  could be obtained either in the Stackelberg manufacturer game or in the Stackelberg retailer game. That is

$$\Pi_M^{max} = \begin{cases} \Pi_M^{S_R} & \text{if } \delta < 0.5 \\ \Pi_M^{S_M} & \text{otherwise} \end{cases} \quad \text{and} \quad \Pi_R^{max} = \Pi_R^{S_R}.$$



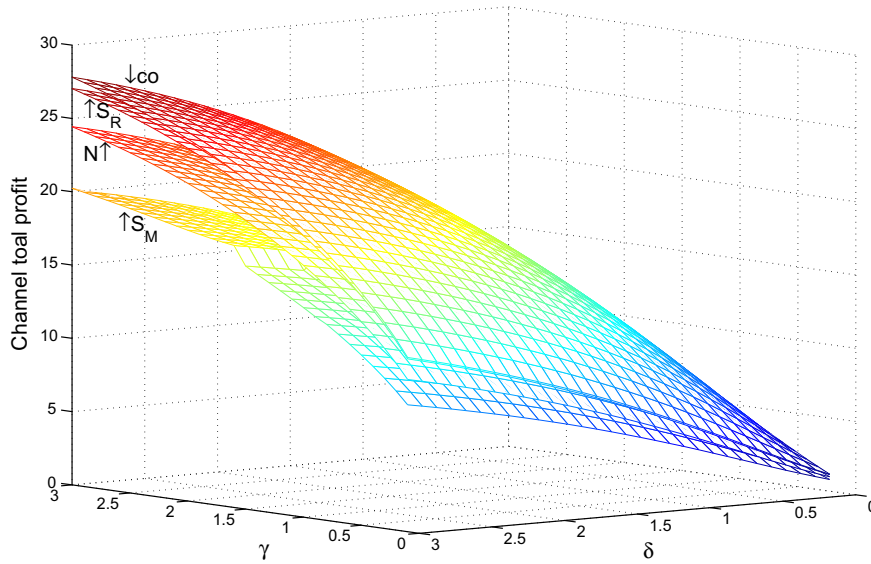


Fig. 6. Whole channel's profits.

For  $\delta < 0.5$ , the inequality (47) is obviously true, because  $\Pi_{M+R}^{\max} = \Pi_{M+R}^{S_R}$  and the cooperation maximizes the system profit. For  $\delta > 0.5$ , we use the numerical simulation with Matlab and Eq. (47) to determine whether or not  $\Pi_{M+R}^{co} > \Pi_M^{\max} + \Pi_R^{\max} = \Pi_M^{S_M} + \Pi_R^{S_R} (= \Pi_{M+R}^{\max})$  (See Fig. 7). In Fig. 7, we show the "relative difference between cooperation and non-cooperation"  $\Delta = \frac{\Pi_{M+R}^{co} - (\Pi_M^{S_M} + \Pi_R^{S_R})}{\Pi_{M+R}^{co}} * 100\%$  as a function of  $\delta$  and  $\gamma$ . We see that (47) is satisfied as long as  $\delta$  and  $\gamma$  are both smaller than 2.5. Even when (47) is not truly satisfied the relative difference ( $\Delta$ ) is less than 0.5%, which is quite negligible.

Therefore, we may regard (47) always satisfied and the inequalities (47) does define a region of feasible solutions in between two parallel non-horizontal lines in the plane  $(p_M, t)$  (see Fig. 10 in the next section). Note that  $0 \leq p_M(t=0) \leq p_R^{co}$  is implied by (46). Moreover, because of the shape of the feasible solutions region, it's easy to see that there are some solutions that satisfy  $0 \leq t \leq 1$ . To conclude, feasible solutions exist and

both the manufacturer and the retailer are willing to cooperate. But we should notice that (47) does also define the constant joint extra-profit that the manufacturer and the retailer achieved by moving to cooperation and that they will have to share the extra-profit:

$$\Delta \Pi = \Pi_{M+R}^{co} - \Pi_{M+R}^{\max} = (\Pi_M^{co} - \Pi_M^{\max}) + (\Pi_R^{co} - \Pi_R^{\max}) = \Delta \Pi_M + \Delta \Pi_R \geq 0.$$

Obviously, the more the manufacturer gets, the less the retailer and vice versa. So they will bargain over  $(p_M, t)$  with boundaries defined by the inequalities (43) and (44),  $0 \leq t \leq 1$  and  $0 \leq p_M \leq p_R^{co}$ . That's the point of the next section.

**Remark 2.** We have showed the existence of strongly feasible solutions: there exists a solution  $(p_R^{co}, a^{co}, q^{co}, p_M, t) = s^*$  such that for any case  $i$  of the four cases ( $i = S_M, S_R, N, co$ ),  $\Pi_M^{co}(s^*) \geq \Pi_M^i$  and  $\Pi_R^{co}(s^*) \geq \Pi_R^i$ . But a weaker definition could be: the cooperation game is said to be feasible if and only if for any case  $i$ , there exists a solution  $(p_R^{co}, a^{co}, q^{co}, p_M, t) = s$  such that

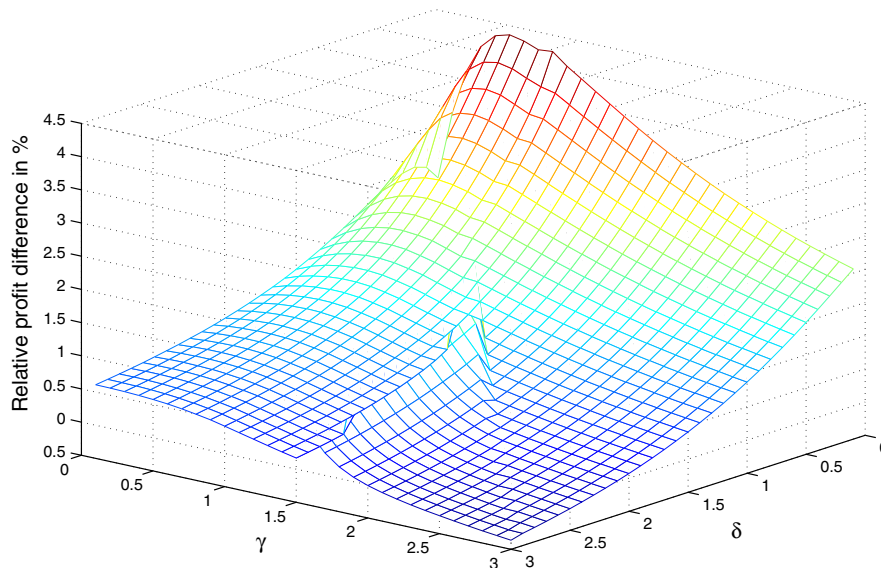


Fig. 7. Relative difference between cooperation and non-cooperation.

$\Pi_M^{co}(s) \geq \Pi_M^i$  and  $\Pi_R^{co}(s) \geq \Pi_R^i$ . Then, using exactly the same explanation as above and the fact that the cooperation game maximizes the total system profit, we can easily know that the cooperation game is always feasible. In other words, whatever the game setting  $i$  is, both the manufacturer and the retailer are willing to move to cooperation where  $t$  and  $p_M$  are bargained with the profits of game  $i$  as boundary references.

We know from previous sections that if the retailer has the possibility to be the leader, he will and the manufacturer will accept to be his follower. On the other hand, if the manufacturer has the possibility to be the leader, he will but the retailer will not accept and we have a conflict situation (the Nash equilibrium). In the first case, numerical simulations reveal a relative joint extra-profit ( $\{\Pi_{R+M}^{co} - (\Pi_{R+M}^S)\} / \Pi_{R+M}^{co} * 100\%$ ) varying from 1% to 3% (it depends on the values of  $\gamma$  and  $\delta$ , (see Fig. 8)). In the second case, we have a relative joint extra-profit ( $\{\Pi_{R+M}^{co} - (\Pi_{R+M}^N)\} / \Pi_{R+M}^{co} * 100\%$ ) varying from 12% to 14% (see Fig. 9). If by any chance the manufacturer manages to make the retailer a follower of him, then moving to cooperation generates a relative joint extra-profit of  $\{\Pi_{R+M}^{co} - (\Pi_{R+M}^S)\} / \Pi_{R+M}^{co} * 100\%$ .

## 5. Bargain problem

We have a bargain problem over  $0 \leq p_M \leq p_R^{co}$  and  $0 \leq t \leq 1$  subject to constraints (43) and (44) (i.e., the feasible solutions) to share the joint extra-profit  $\Delta\Pi = \Pi_{M+R}^{co} - (\Pi_M^{\max} + \Pi_R^{\max}) = \Delta\Pi_M + \Delta\Pi_R \geq 0$  as shown in Fig. 10 where  $A^* = (1 - p_R^{co}) / (A/B^{1/(\delta+\gamma+1)} - 1 / ((a^{co})^\gamma (q^{co})^\delta))$  and the symbol “ $\propto$ ” means “to be proportional to”.

All the couples located in between the two lines  $\Pi_R = \Pi_R^{\max}$  and  $\Pi_M = \Pi_M^{\max}$  are feasible solutions. The closer you get to  $\Pi_R = \Pi_R^{\max}$ , the bigger the manufacturer's share  $\Delta\Pi_M$  and the smaller the retailer's share  $\Delta\Pi_R$ . Couples located on a line parallel to  $\Pi_M = \Pi_M^{\max}$  (or  $\Pi_R = \Pi_R^{\max}$ ) all lead to the same profit  $\Pi_M = \Pi_M^{\max} + \Delta\Pi_M$  for the manufacturer (also the same profit  $\Pi_R = \Pi_R^{\max} + \Delta\Pi_R$  for the retailer).

More precisely, we have

$$\begin{aligned} \Delta\Pi_M &= A^* p_M - a^{co} t - (q^{co} + \Pi_M^{\max}) \\ &= A^* p_M - a^{co} t - C \text{ with } C = (q^{co} + \Pi_M^{\max}) > 0, \\ \Delta\Pi_R &= -A^* p_M + a^{co} t + (A^* p_R^{co} - a^{co} - \Pi_R^{\max}) \\ &= -A^* p_M + a^{co} t + D \text{ with } D = (A^* p_R^{co} - a^{co} - \Pi_R^{\max}) > 0. \end{aligned}$$

The approach we use to solve the bargaining problem is the Nash bargaining model (Nash, 1950). The bargaining outcome  $(p_M^*, t^*)$  is obtained by maximizing the product of individual marginal utilities over the feasible region. We illustrate the bargaining solutions below using three examples.

**Example 1.** Consider a manufacturer and a retailer who are both risk-neutral; i.e., the utility functions for the retailer and the manufacturer are  $u_R(t, p_M) = \Delta\Pi_R(t, p_M)$  and  $u_M(t, p_M) = \Delta\Pi_M(t, p_M)$ , respectively. Nash's model predicts a bargaining solution at a point on the feasible region that maximizes

$$\begin{aligned} u_R(t, p_M) u_M(t, p_M) &= \Delta\Pi_M \Delta\Pi_R \\ &= \{A^* p_M - a^{co} t - C\} \{-A^* p_M + a^{co} t + D\}. \end{aligned} \quad (48)$$

Expression (48) has a maximum at  $A^* p_M^* - a^{co} t^* = (C + D)/2$  (that is, the iso-profit line located in between  $\Pi_R^{\max}$  and  $\Pi_M^{\max}$  on Fig. 10). But we cannot determine more precisely  $(p_M^*, t^*)$  without any further assumptions. Thus, when both the manufacturer and the retailer are risk-neutral, Nash's model predicts that they will equally split the joint extra-profit, which is consistent with the bargaining literature. That is to say,

$$\Delta\Pi_M(t^*, p_M^*) = \Delta\Pi_R(t^*, p_M^*) = \frac{D - C}{2} = \frac{\Delta\Pi}{2}.$$

**Example 2.** Consider a risk-neutral manufacturer and a risk-averse retailer with utility functions  $u_M(t, p_M) = \Delta\Pi_M(t, p_M)$  and  $u_R(t, p_M) = (\Delta\Pi_R(t, p_M))^{1/2}$ . Nash's model predicts that the manufacturer and the retailer will agree at a couple  $(p_M^*, t^*)$  that again maximizes

$$\begin{aligned} u_R(t, p_M) u_M(t, p_M) &= \Delta\Pi_M (\Delta\Pi_R)^{1/2} \\ &= \{A^* p_M - a^{co} t - C\} \{-A^* p_M + a^{co} t + D\}^{1/2} \end{aligned} \quad (49)$$

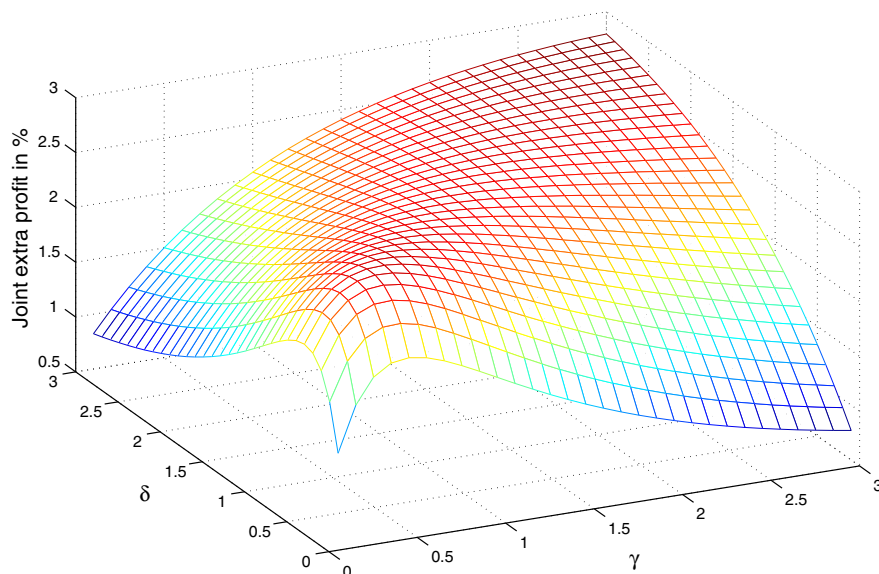


Fig. 8. From  $S_R$  case to cooperation case.

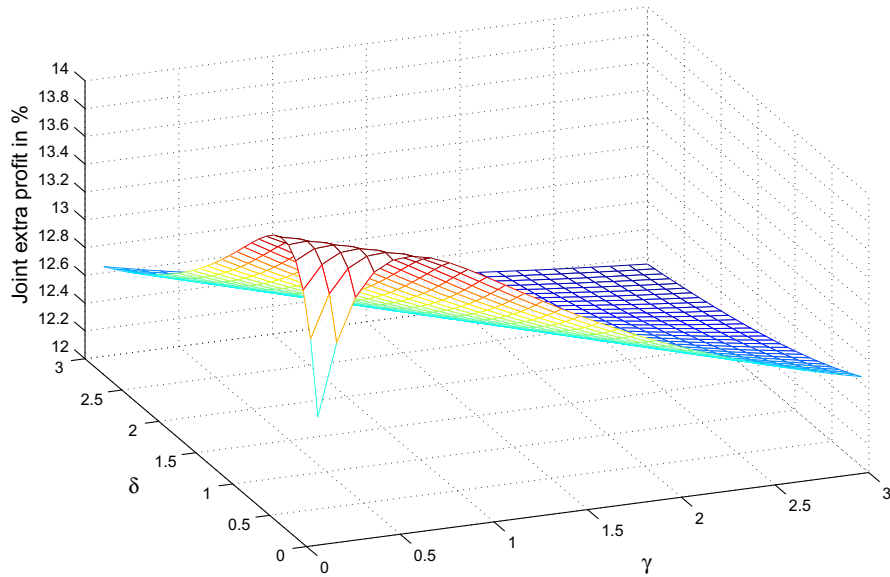


Fig. 9. From Nash case to cooperation case.

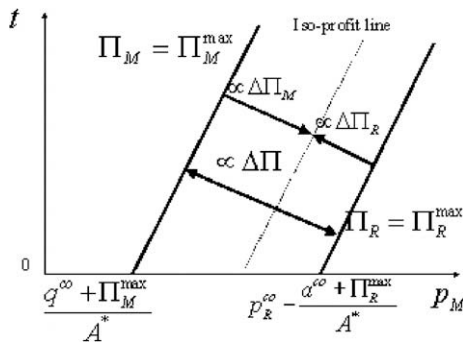


Fig. 10. Feasible region in the bargain problem.

Expression (49) has a maximum at  $A^* p_M^* - a^{co} t^* = (2D + C)/3$ . That is, the iso-profit line now is closer to  $\Pi_R^{\max}$  and the manufacturer will get more of the joint extra-profit, which is intuitively correct because the manufacturer's attitude is more risky than the retailer's one. More specifically,

$$\Delta \Pi_M(t^*, p_M^*) = \frac{2(D - C)}{3} = \frac{2}{3} \Delta \Pi, \Delta \Pi_R(t^*, p_M^*) = \frac{1}{3} \Delta \Pi.$$

**Remark 3.** The situation where the retailer is risk-neutral and the manufacturer is risk-averse leads to exactly the opposite result. That is to say,

$$\Delta \Pi_R(t^*, p_M^*) = \frac{2(D - C)}{3} = \frac{2}{3} \Delta \Pi, \Delta \Pi_M(t^*, p_M^*) = \frac{1}{3} \Delta \Pi.$$

**Example 3.** Consider a manufacturer and a retailer who are both risk-averse, that is  $u_M(t, p_M) = (\Delta \Pi_M(t, p_M))^{1/2}$  and  $u_R(t, p_M) = (\Delta \Pi_R(t, p_M))^{1/2}$ . As the same as Example 1, Nash's model leads to a solution that the manufacturer and the retailer equally split the joint extra-profit.

## 6. Conclusions

This paper attempts to identify the optimal pricing and co-op advertising strategies in four classical types of relationships between a manufacturer and a retailer using the game theory models.

The findings from the research give a clear picture about the competition and cooperation via pricing and co-op advertising strategies between the channel members. First, the advertising expenditures of both members is generally higher in a coordinated situation than in a non-coordinated situation but this does not come at the expense of consumers, since the retail price is the lowest in the coordinated case (and the highest in the case where the manufacturer is the leader). Secondly, the leader will always manage to make the follower invest more in advertising than he usually does in other cases while the leader itself invests a smaller amount of advertising. Thirdly, with regards to profits, the retailer always prefers to be the leader whereas the manufacturer's attitude depends on a certain influence of its brand name investment on sales. He chooses to be the leader only when  $\delta > 0.5$ ; otherwise he always gains by choosing to be the retailer's follower. Furthermore, in our model, the retailer always prefers to be in conflict with the manufacturer rather than to be his follower though he may get some advertising allowance from the manufacturer. This may sound surprising. But our model does not take into account factors other than profit that may lead the retailer to accept to be the manufacturer's follower, e.g., the competition between multiple retailers. Anyway, we have showed that coordination always guarantees higher profits for both the manufacturer and the retailer than in any other cases. But they have to bargain over the manufacturer's price and the advertising allowance to share the system profit gain achieved by moving to cooperation. Using Nash's bargain model on three examples, we obtain that the more risk-averse a member is, the lower his share of the profit gains. In other words, a member has to be more risky in order to get more benefits. Moreover, the manufacturer's price and the advertising allowance can never be fully determined but a linear equation linked both of them at bargain equilibrium.

Our model suffers from some limitations due to the choice of its demand function. Changing the demand function may yield some more interesting results. Besides, we used the traditional Nash bargain model to solve our bargain problem. Other interesting and less 'traditional' models are available (e.g., [Elieshberg, 1986](#)) and might give some new solutions. More interesting issues would be to relax the classical two channel members situation to a three channel members situation (either two manufacturers and one retailer or two retailers and one manufacturer) to move one step towards

the understanding of the role of the competition and the cooperation. This kind of study has only been done in the field of pricing (Choi, 1991; Choi, 1996; Ingene & Parry, 1995a, 1995b, 1998, 2000). Incorporation of more players into the model may yield more interesting results.

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