

$$R \dot{R}^T = \phi^\wedge$$

$$\dot{R}^T = R^T \phi^\wedge$$

$$\dot{R} = (\phi^\wedge)^T R$$

(ΔR 对应李代数 ϕ)

$$\begin{aligned} \frac{d(R^{-1}p)}{dR} &= \frac{\partial((R\Delta R)^{-1}p)}{\partial(\Delta R)} = \lim_{\phi \rightarrow 0} \frac{(R \exp(\phi^\wedge))^{-1}p - R^{-1}p}{\phi} \\ &= \lim_{\phi \rightarrow 0} \frac{(\exp(\phi^\wedge))^{-1}R^{-1}p - R^{-1}p}{\phi} = \lim_{\phi \rightarrow 0} \frac{\exp(-\phi^\wedge)R^{-1}p - R^{-1}p}{\phi} \\ &= \lim_{\phi \rightarrow 0} \frac{(I - \phi^\wedge)R^{-1}p - R^{-1}p}{\phi} \quad (\phi \text{ 很小, 把 } \exp(-\phi^\wedge) \text{ 泰勒展开, 只保留一阶项}) \\ &= \lim_{\phi \rightarrow 0} \frac{-\phi^\wedge R^{-1}p}{\phi} \\ &= \lim_{\phi \rightarrow 0} \frac{(R^{-1}p)^\wedge \phi}{\phi} \\ &= (R^{-1}p)^\wedge \end{aligned}$$

(指数映射的逆就是旋转矩阵的逆, 对应的旋转向量变号就是旋转回去)

(ϕ 是 ΔR 对应的李代数)

$$\begin{aligned} \frac{d \ln(R_1 R_2^{-1})^v}{dR_2} &= \frac{\partial \ln(R_1 (R_2 \Delta R)^{-1})^v}{\partial \Delta R} \\ &= \lim_{\phi \rightarrow 0} \frac{\ln(R_1 \exp(\phi^\wedge)^{-1} R_2^{-1})^v - \ln(R_1 R_2^{-1})^v}{\phi} \\ &= \lim_{\phi \rightarrow 0} \frac{\ln(R_1 \exp(-\phi^\wedge) R_2^{-1})^v - \ln(R_1 R_2^{-1})^v}{\phi} \\ &= \lim_{\phi \rightarrow 0} \frac{\ln(R_1 R_2^T \boxed{R_2 \exp(-\phi^\wedge) R_2^T})^v - \ln(R_1 R_2^{-1})^v}{\phi} \\ &= \lim_{\phi \rightarrow 0} \frac{\ln(R_1 R_2^T \exp(-R_2 \phi^\wedge R_2^T))^v - \ln(R_1 R_2^{-1})^v}{\phi} \\ &= \lim_{\phi \rightarrow 0} \frac{\ln(R_1 R_2^{-1})^v + J_Y^{-1}(\ln(R_1 R_2^{-1})^v) (-R_2 \phi) - \ln(R_1 R_2^{-1})^v}{\phi} \\ &= -R_2 J_Y^{-1}(\ln(R_1 R_2^{-1})^v) \end{aligned}$$

(SOL3 伴随性质)

(BCH 线性近似)