CS51501 Spring 2016
Homework set # 3

Part a: due at 11:59 pm on Thursday Match. 3, 2016
Part b: due at 11:59 pm on Thursday Match. 10, 2016

Block LU Factorization with boosting

Solve the dense linear system $\boldsymbol{A}\mathbf{x} = \mathbf{b}$ using the approximate block LU Factorization algorithm with the procedure of "diagonal boosting"[1]. Your code should be implemented using the MPI/OpenMP programming paradigm, and tested on eight nodes of the MC cluster.

Let α be a multiple of the unit roundoff, e.g. 10^{-6}, and $\mathbf{a}_j$ be the jth column of the updated matrix $\boldsymbol{A}$ after step $j-1$. In step j, if the diagonal pivot does not satisfy

$$|pivot| > \alpha||\mathbf{a}_j||_1,$$

its value is "boosted" as,

$$pivot = pivot + \beta||\mathbf{a}_j||_1, \text{ if } pivot > 0,$$
$$pivot = pivot - \beta||\mathbf{a}_j||_1, \text{ if } pivot < 0,$$

where β is often taken as the square root of α.

Part a:$\boldsymbol{A}$ is diagonally dominant

If $\boldsymbol{A}$ is a diagonally dominant matrix, no "boosting" will occur, and $\boldsymbol{A} = \boldsymbol{LU}$. Solve $\boldsymbol{A}\mathbf{x} = \mathbf{b}$ by solving

$$\boldsymbol{L}\mathbf{y} = \mathbf{b}, \text{ and}$$
$$\boldsymbol{U}\mathbf{x} = \mathbf{y}.$$

Compute the relative residual $||\boldsymbol{A}\mathbf{x} - \mathbf{b}||_2/||\mathbf{b}||_2$, and print it out. You also need to print out the time of the block LU factorization, the time of the block backward sweep using pipeline, and the time of the block forward sweep using pipeline.

[1] Prof. Sameh's note on Feb 2, or Chapter 4.3.1 on page 85-87 in the textbook

Part b:$\boldsymbol{A}$ is not diagonally dominant

If $\boldsymbol{A}$ is not diagonally dominant, "boosting" will occur, and $\boldsymbol{LU} = \boldsymbol{A} + \boldsymbol{E}$, where $\boldsymbol{E}$ is a perturbation matrix with $||E|| \ll ||A||$. Please print out how many boosting steps are used.

(1) Try to solve $\boldsymbol{A}\mathbf{x} = \mathbf{b}$ as in Part a, and print out the relative residual.

(2) Solve $\boldsymbol{A}\mathbf{x} = \mathbf{b}$ using GMRES with $\boldsymbol{M} = \boldsymbol{L} \times \boldsymbol{U}$ as a preconditioner. Please print out the iteration number and the relative residual.

FGMRES is available in Intel's MKL. TA will provide you with an example to show how to call GMRES and use the preconditioner.

The Intel MKL library

The Intel Math Kernel Library accelerates math processing routines that increase application performance and reduce development time. In HW3, you will learn how to use Intel's MKL library to develop your own application.

The Reference Manual is available at https://software.intel.com/en-us/intel-mkl/documentation . BLAS 1, BLAS 2, BLAS 3 operations, and the sparse BLAS operations are in the manual's Chapter 2 BLAS and Sparse BLAS Routines . The common decomposition methods and direct solves are in the manual's Chapter 3 LAPACK Routines. Examples could be found at /p/intel/mkl/examples/. Please copy the tgz file to your own directory and extract it.

Please download the example of Makefile.

Submission

turnin –c cs51501 –p HW3 your_folder_name

Your submission should include the following files:

1. The source code.
2. A Readme file or a Makefile, which includes all the compiling commands.

Please **do not** include the test cases in your submission.