# Causal Inference4

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library(FinMetric)	
library(formatR)	
<pre>knitr::opts_chunk\$set(tidy.opts=list(width.cutoff=60),</pre>	
tidy=TRUE,	
echo = TRUE)	

# Fixed Effects, DID and Panel data

## Fixed effect

## **Individual Fixed Effect**

• Setting

Question: because workers in union, then they wage high; or they earn more because of they are more experienced

 $Y_{it}$ : log earnings of worker i at time t;  $Y_{it} = Y_{0it} + (Y_{1it} - Y_{0it})D_{it}$ 

 $D_{it}$ : union status;

 $A_i$ : unobserved worker ability;  $X_{it}$ : other observed covariate

Suppose:

$$E(Y_{0it}|A_i, X_{it}, t, D_{it}) = E(Y_{0it}|A_i, X_{it}, t)$$

• key fixed-effects estimation assumption:  $A_i$  appears without a time subscrit in a linear model:

$$E(Y_{0it}|A_i, X_{it}, t) = \alpha + \lambda_t + A'_i \gamma + X'_{it} \beta$$

• We assume the causal effect of union membership is additive and constant:

$$E(Y_{1it}|A_i, X_{it}, t) = E(Y_{0it}|A_i, X_{it}, t) + \rho$$

• Thus, we have:

$$E(Y_{1it}|A_i,X_{it},t) = \alpha + \lambda_t + \rho D_{it} + A_i'\gamma + X_{it}'\beta$$
 
$$\Rightarrow Y_{1it} = \alpha_i + \lambda_t + \rho D_{it} + X_{it}'\beta + \varepsilon_{it}$$
 where  $\alpha_i = \alpha + A_i'\gamma$ . Individual effect:  $\alpha_i$  Year effect:  $\lambda_t$ 

By within transformation, we can eliminate the individual effect, we can estimate  $\rho$  consistently.

### **Application**

```
# Balanced panels
data("Grunfeld", package = "plm")
Grunfeld %>%
    select(year, firm) %>%
   table()
##
         firm
## year
          1 2 3 4 5 6 7 8 9 10
     1935 1 1 1 1 1 1 1 1 1
##
##
     1936 1 1 1 1 1 1 1 1 1
     1937 1 1 1 1 1 1 1 1 1
##
##
     1938 1 1 1 1 1 1 1 1 1
##
     1939 1 1 1 1 1 1 1 1 1
     1940 1 1 1 1 1 1 1 1 1
##
     1941 1 1 1 1 1 1 1 1 1
##
##
     1942 1 1 1 1 1 1 1 1 1
     1943 1 1 1 1 1 1 1 1 1
##
##
     1944 1 1 1 1 1 1 1 1 1
##
     1945 1 1 1 1 1 1 1 1 1
##
     1946 1 1 1 1 1 1 1 1 1
##
     1947 1 1 1 1 1 1 1 1 1
     1948 1 1 1 1 1 1 1 1 1
##
##
     1949 1 1 1 1 1 1 1 1 1
##
     1950 1 1 1 1 1 1 1 1 1
##
     1951 1 1 1 1 1 1 1 1 1
##
     1952 1 1 1 1 1 1 1 1 1
##
     1953 1 1 1 1 1 1 1 1 1
##
     1954 1 1 1 1 1 1 1 1 1
# Unbalanced panels
data("EmplUK", package = "plm")
EmplUK %>%
    select(year, firm) %>%
   filter(firm %in% c(1:10)) %>%
   table()
##
         firm
## year
        1 2 3 4 5 6 7 8 9 10
```

```
##
     1976 0 0 0 0 1 1 1 1 1 1
##
     1977 1 1 1 1 1 1 1 1 1
     1978 1 1 1 1 1 1 1 1 1
##
     1979 1 1 1 1 1 1 1 1 1
##
##
     1980 1 1 1 1 1 1 1 1 1
##
     1981 1 1 1 1 1 1 1 1 1 1
##
     1982 1 1 1 1 1 1 1 1 1 1
##
     1983 1 1 1 1 0 0 0 0 0 0
# Balance unbalanced data
# Using 'fill' creates a new row with NAs for each missing
# time point.
EmplUK.balanced1 <- make.pbalanced(EmplUK, balance.type = "fill")</pre>
EmplUK.balanced1[1:8, ]
Balance unbalanced data
     firm year sector
                              wage capital
                        emp
                                              output
## 1
       1 1976
                  NA
                        NA
                                NA
                                         NA
                                                  NA
## 2
        1 1977
                   7 5.041 13.1516 0.5894 95.7072
## 3
       1 1978
                   7 5.600 12.3018 0.6318 97.3569
                   7 5.015 12.8395 0.6771 99.6083
       1 1979
                   7 4.715 13.8039 0.6171 100.5501
## 5
       1 1980
                   7 4.093 14.2897 0.5076 99.5581
## 6
       1 1981
## 7
       1 1982
                   7 3.166 14.8681 0.4229 98.6151
## 8
       1 1983
                   7 2.936 13.7784 0.3920 100.0301
# Using 'shared.times' keeps all available firms in the
# dataset but drops all time periods where at least one
# firm has no data.
EmplUK.balanced2 <- make.pbalanced(EmplUK, balance.type = "shared.times")</pre>
EmplUK.balanced2[1:10, ]
##
      firm year sector
                          emp
                                wage capital
                                                output
## 2
        1 1978
                    7 5.600 12.3018 0.6318 97.3569
## 3
        1 1979
                    7 5.015 12.8395 0.6771 99.6083
## 4
        1 1980
                    7 4.715 13.8039 0.6171 100.5501
## 5
        1 1981
                    7 4.093 14.2897 0.5076 99.5581
## 6
        1 1982
                    7 3.166 14.8681 0.4229 98.6151
## 9
        2 1978
                    7 70.643 14.1036 17.2422 97.3569
                    7 70.918 14.9534 17.5413 99.6083
## 10
      2 1979
## 11
        2 1980
                    7 72.031 15.4910 17.6574 100.5501
## 12
        2 1981
                    7 73.689 16.1969 16.7133 99.5581
## 13
        2 1982
                    7 72.419 16.1314 16.2469 98.6151
# By using 'shared.individuals' all available time periods
# are kept but only for those firms which have information
# for each of them.
EmplUK.balanced3 <- make.pbalanced(EmplUK, balance.type = "shared.individuals")</pre>
EmplUK.balanced3 %>%
   group_by(firm) %>%
```

slice(1)

```
## # A tibble: 14 x 7
## # Groups: firm [14]
##
      firm year sector
                          emp wage capital output
##
     <dbl> <dbl> <dbl> <dbl> <dbl> <
                                     <dbl> <dbl>
##
   1
       127 1976
                     7 1.14 14.7
                                     0.690
                                             94.7
##
  2
       128 1976
                     7 1.70 14.2
                                     0.422
                                             94.7
  3
       129 1976
                     7 0.768 8.83
                                     0.201
                                             94.7
       130 1976
                              29.3
                                    23.6
## 4
                     3 20
                                            102.
##
  5
       131 1976
                     9 4.10 25.6
                                     0.504 104.
##
       132 1976
                     1 0.971 20.1
  6
                                     0.114 127.
##
  7
       133 1976
                     4 2.17 27.6
                                     0.284 118.
       134 1976
## 8
                     8 13.0
                             15.2
                                     1.50
                                            117.
##
  9
       135 1976
                     8 2.37 18.6
                                     0.398 117.
## 10
                     8 0.615 24.1
                                   0.124 117.
       136 1976
## 11
       137 1976
                     2 2.60 38.4
                                     0.664 110.
## 12
       138 1976
                     9 1.92 27.0
                                     0.255 102.
## 13
       139 1976
                     9 1.05 22.2
                                     0.147 103.
## 14
       140 1976
                     3 1.54 29.1
                                     0.651 105.
```

#### **Estimation Methods**

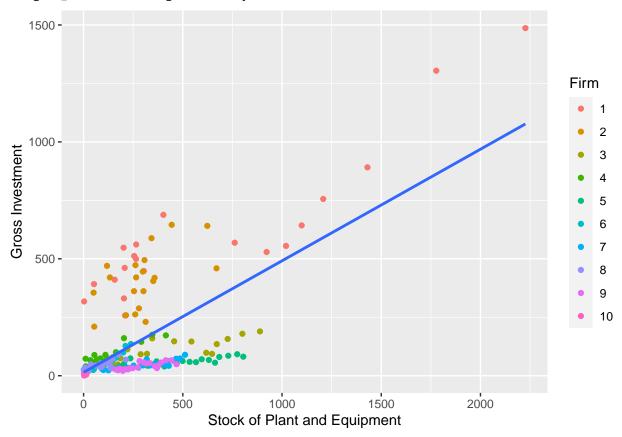
```
# Pooled OLS via lm
pooled_ols_lm <- lm(inv ~ capital, data = Grunfeld)
summary(pooled_ols_lm)</pre>
```

## **Pooled Cross Sections**

## t test of coefficients:

```
##
## Call:
## lm(formula = inv ~ capital, data = Grunfeld)
## Residuals:
##
                                3Q
      Min
               1Q Median
                                       Max
## -316.92 -96.45 -14.43
                            17.07
                                   481.92
##
## Coefficients:
              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 14.23620
                         15.63927
                                      0.91
                                              0.364
                                     12.45
## capital
               0.47722
                          0.03834
                                             <2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 162.9 on 198 degrees of freedom
## Multiple R-squared: 0.439, Adjusted R-squared: 0.4362
## F-statistic: 154.9 on 1 and 198 DF, p-value: < 2.2e-16
# Pooled OLS via plm
pooled_ols_plm <- plm(inv ~ capital, data = Grunfeld, index = c("firm",</pre>
    "year"), effect = "individual", model = "pooling")
coeftest(pooled_ols_plm, vcov = vcovHC, type = "HC1")
```

## `geom\_smooth()` using formula 'y ~ x'



```
se <- list(rob_se(fe_model_lm), rob_se(fe_model_plm), rob_se(fe_model_felm))
stargazer(fe_model_lm, fe_model_plm, fe_model_felm, se = se,
    header = F, keep.stat = c("n", "rsq"), omit = "factor")</pre>
```

Table 1.

	Table 1	•	
	Dependent variable:		
		inv	
	OLS	$egin{aligned} panel \ linear \end{aligned}$	felm
	(1)	(2)	(3)
capital	$0.371^{***}$ $(0.057)$	0.371*** (0.062)	$0.371^{***}$ $(0.056)$
Constant	367.613*** (31.032)		
Observations R <sup>2</sup>	200 0.918	200 0.660	200 0.918
Note:	*p<0.1	; **p<0.05;	***p<0.01

### Fixed Effects Model

### First-difference Estimator

Table 2:

	Dependent variable:		
	inv		
	OLS	$egin{aligned} panel \ linear \end{aligned}$	felm
	(1)	(2)	(3)
capital	$0.414^{***}$ $(0.072)$	$0.414^{***}$ $(0.057)$	$0.414^{***}$ $(0.067)$
Constant	354.917*** (33.413)		
Observations $\mathbb{R}^2$	200 0.931	200 0.599	200 0.931
Note:	*p<0.1	; **p<0.05;	***p<0.01

```
## Oneway (individual) effect First-Difference Model
##
## Call:
## plm(formula = inv ~ capital, data = Grunfeld, effect = "individual",
       model = "fd", index = c("firm", "year"))
##
## Balanced Panel: n = 10, T = 20, N = 200
## Observations used in estimation: 190
## Residuals:
       Min.
             1st Qu.
                        Median 3rd Qu.
## -240.4300 -15.2906 -4.2478 8.4604 339.4974
## Coefficients:
              Estimate Std. Error t-value Pr(>|t|)
##
## (Intercept) 4.456749 4.459346 0.9994 0.318877
## capital
              0.199671
                         0.067274 2.9680 0.003387 **
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Total Sum of Squares:
                           584410
## Residual Sum of Squares: 558250
## R-Squared:
                  0.04476
## Adj. R-Squared: 0.039679
## F-statistic: 8.80927 on 1 and 188 DF, p-value: 0.003387
# only two period Fixed effect model is same as first
# difference estimation Within estimation (two periods)
fe_model_plm_check <- plm(inv ~ capital, data = Grunfeld, subset = year %in%</pre>
    c(1935, 1936), index = c("firm", "year"), effect = "individual",
    model = "within")
coeftest(fe_model_plm_check)
```

```
## t test of coefficients:
##
          Estimate Std. Error t value Pr(>|t|)
##
## capital 0.91353
                      0.85333 1.0705 0.3122
# FD estimation (two periods)
fe_model_fd_check <- plm(inv ~ capital - 1, data = Grunfeld,</pre>
   subset = year %in% c(1935, 1936), index = c("firm", "year"),
   effect = "individual", model = "fd")
coeftest(fe_model_fd_check)
##
## t test of coefficients:
##
          Estimate Std. Error t value Pr(>|t|)
## capital 0.91353 0.85333 1.0705 0.3122
re_model_plm <- plm(inv ~ capital, data = Grunfeld, index = c("firm",</pre>
    "year"), effect = "individual", model = "random")
summary(re_model_plm)
Random Effect Model
## Oneway (individual) effect Random Effect Model
##
      (Swamy-Arora's transformation)
##
## Call:
## plm(formula = inv ~ capital, data = Grunfeld, effect = "individual",
      model = "random", index = c("firm", "year"))
##
## Balanced Panel: n = 10, T = 20, N = 200
##
## Effects:
##
                     var std.dev share
## idiosyncratic 4040.63
                          63.57 0.135
## individual
                25949.52 161.09 0.865
## theta: 0.9121
##
## Residuals:
       Min. 1st Qu.
                        Median 3rd Qu.
## -164.0821 -22.2955 -3.7463 16.9121 319.9564
##
## Coefficients:
               Estimate Std. Error z-value Pr(>|z|)
## (Intercept) 43.246697 51.411319 0.8412
## capital
              0.372120
                         0.019316 19.2652
                                            <2e-16 ***
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Total Sum of Squares:
                           2299300
```

```
## Residual Sum of Squares: 799910
                   0.65211
## R-Squared:
## Adj. R-Squared: 0.65036
## Chisq: 371.149 on 1 DF, p-value: < 2.22e-16
# FE or Pooled OLS
pFtest(fe_model_plm, pooled_ols_plm)
Tests for panel data
##
## F test for individual effects
##
## data: inv ~ capital + factor(firm)
## F = 123.39, df1 = 9, df2 = 189, p-value < 2.2e-16
## alternative hypothesis: significant effects
# The null hypothesis is rejected in favor of the
# alternative that there are significant fixed effects.
# RE or FE Hausman Test
phtest(fe_model_plm, re_model_plm)
##
  Hausman Test
## data: inv ~ capital + factor(firm)
## chisq = 0.93423, df = 1, p-value = 0.3338
## alternative hypothesis: one model is inconsistent
# The null hypothesis cannot be rejected here, hence we
# should use a RE model.
# Pooled OLS or RE
plmtest(pooled_ols_plm, effect = "individual", type = c("bp"))
##
## Lagrange Multiplier Test - (Breusch-Pagan) for balanced panels
## data: inv ~ capital
## chisq = 1285.1, df = 1, p-value < 2.2e-16
## alternative hypothesis: significant effects
# The test shows that there are significant differences
# across firms. Running a pooled OLS regression is thus not
# appropriate and the RE model is the better choice.
# one-way model or two-way model
pFtest(fe_time_plm, fe_model_plm)
## F test for twoways effects
## data: inv ~ capital
## F = 1.594, df1 = 19, df2 = 170, p-value = 0.06242
## alternative hypothesis: significant effects
```

```
# Heteroskedasticity
lmtest::bptest(inv ~ capital + factor(firm), studentize = F,
   data = Grunfeld)
##
## Breusch-Pagan test
##
## data: inv ~ capital + factor(firm)
## BP = 386.81, df = 10, p-value < 2.2e-16
# There is strong evidence for the presense of
# heteroskedasticity. Hence, the use of robust standard
# errors is advised.
# Serial Correlation
pbgtest(fe_model_plm)
##
## Breusch-Godfrey/Wooldridge test for serial correlation in panel models
##
## data: inv ~ capital + factor(firm)
## chisq = 73.785, df = 20, p-value = 4.338e-08
## alternative hypothesis: serial correlation in idiosyncratic errors
# There is strong evidence that the residuals are serially
# correlated.
# Clustered SE(OLS)
coeftest(pooled_ols_plm,
         vcov = vcovHC(pooled_ols_plm,
                       type = "sss",
                       # includes the small sample correction method as applied by Stata
                       cluster = "group"))
Clustered SE
##
## t test of coefficients:
##
##
              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 14.23620 29.63751 0.4803 0.6315130
## capital
               0.47722
                         0.13301 3.5878 0.0004203 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
# Clustered SE(FE)
coeftest(fe_model_plm,
         vcov = vcovHC(fe_model_plm,
                       type = "sss",
                       cluster = "group"))
##
## t test of coefficients:
##
          Estimate Std. Error t value Pr(>|t|)
## capital 0.370750  0.064946  5.7086  4.35e-08 ***
```

```
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
# Clustered SE(RE)
coeftest(re_model_plm,
        vcov = vcovHC(re_model_plm,
                      type = "sss",
                      cluster = "group"))
##
## t test of coefficients:
##
               Estimate Std. Error t value Pr(>|t|)
## (Intercept) 43.246697 37.815768 1.1436
                                            0.2542
## capital
              0.372120
                         0.065803 5.6551 5.389e-08 ***
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

# Differences-in-Differences: pre and post treatment and control

- Goal: Estimate effects of events or policy interventions that take place at an aggregate level.
- Advantages:
  - Policy interventions often take place at an aggregate level;
  - Aggregate /macro data are often available;
- Problems:
  - Selection of Control group is often ambiguous;
  - Standard errors do not reflect uncertainty about hte ability of the control group to reproduce the counterfactual of interest.
- The Synthetic Control Method

\_

#### Application: Card Krueger (1994)

```
# raw data
data_raw <- read_stata("/Users/waynezhang/Library/CloudStorage/OneDrive-UW-Madison/IOS-WIN/Econometric
data_CK1994 <- data_raw %>%
    # chain value label
   mutate(chain = case_when(chain == 1 ~ "bk",
                           chain == 2 ~ "kfc",
                           chain == 3 ~ "roys",
                           chain == 4 ~ "wendys")) %>%
    # state value label
   mutate(state = case_when(state == 1 ~ "New Jersey",
                           state == 0 ~ "Pennsylvania")) %>%
    # Region dummy
   mutate(region = case_when(southj == 1 ~ "southj",
                            centralj == 1 ~ "centralj",
                            northj == 1 ~ "northj",
                            shore == 1 ~ "shorej",
                            pa1 == 1 ~ "phillypa",
                            pa2 == 1 ~ "eastonpa")) %>%
    # meals value label
```

```
# Distribution of restaurants
data_CK1994 %>%
    select(chain, state) %>%
    table() %>%
    prop.table(margin = 2) %>%
    apply(MARGIN = 2, FUN = scales::percent_format(accuracy = 0.01)) %>%
    noquote() %>%
    knitr::kable()
```

#### Descriptive statistics

	New Jersey	Pennsylvania
bk	41.09%	44.30%
kfc	20.54%	15.19%
roys	24.77%	21.52%
wendys	13.60%	18.99%

Table 4: Pre-treatment means 2/15/1992 - 3/4/1992

variable	New Jersey	Pennsylvania
emptot	20.44	23.33
$pct\_fte$	32.85	35.04
$wage\_st$	4.61	4.63
hoursopen	14.42	14.53

```
# Post-treatment means
data_CK1994 %>%
  filter(time == 1) %>%
  group_by(state) %>%
```

Table 5: Post-treatment means 11/5/1992 - 12/31/1992

variable	New Jersey	Pennsylvania
emptot	21.03	21.17
$\operatorname{pct}$ _fte	35.87	30.38
$wage\_st$	5.08	4.62
hoursopen	14.42	14.65

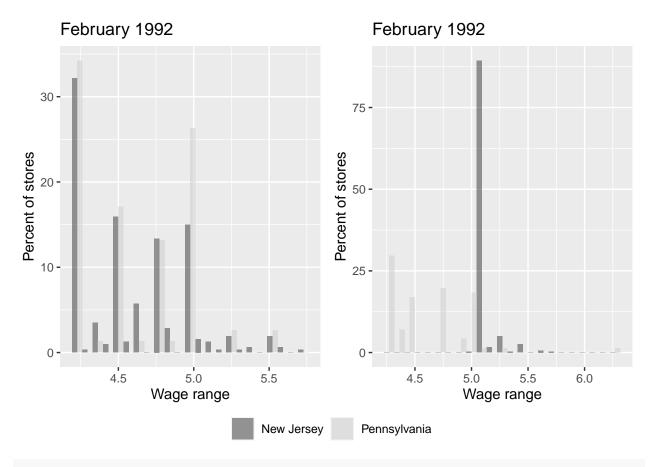
```
# Figure 1
hist_feb <- data_CK1994 %>%
   filter(time == 0) %>%
   ggplot(aes(wage_st, fill = state)) + geom_histogram(aes(y = c(..count..[..group.. ==
   1]/sum(..count..[..group.. == 1]), ..count..[..group.. ==
   2]/sum(..count..[..group.. == 2])) * 100), alpha = 0.5, position = "dodge",
   bins = 23) + labs(title = "February 1992", x = "Wage range",
   y = "Percent of stores", fill = "") + scale_fill_grey()
hist_nov <- data_CK1994 %>%
   filter(time == 1) %>%
   ggplot(aes(wage_st, fill = state)) + geom_histogram(aes(y = c(..count..[..group.. ==
   1]/sum(..count..[..group.. == 1]), ..count..[..group.. ==
   2]/sum(..count..[..group.. == 2])) * 100), alpha = 0.5, position = "dodge",
   bins = 23) + labs(title = "February 1992", x = "Wage range",
   y = "Percent of stores", fill = "") + scale_fill_grey()
library(ggpubr)
ggarrange(hist_feb, hist_nov, ncol = 2, common.legend = TRUE,
   legend = "bottom")
```

### Figure 1

```
## Warning: Removed 20 rows containing non-finite values (stat_bin).
```

<sup>##</sup> Removed 20 rows containing non-finite values (stat\_bin).

<sup>##</sup> Warning: Removed 21 rows containing non-finite values (stat\_bin).



```
# First differences
diff_express <- data_CK1994 %>%
    group_by(time, state) %>%
    summarise(emptot = mean(emptot, na.rm = T)) %>%
    pivot_wider(names_from = state, values_from = emptot) %>%
    mutate(diff = `New Jersey` - Pennsylvania)
```

# Calculating the treatment effect

## `summarise()` has grouped output by 'time'. You can override using the `.groups`
## argument.

```
# The Average Treatment Effect (ATT)
diff_express$diff[2] - diff_express$diff[1]
```

## [1] 2.753606

```
data_CK1994_mod <- data_CK1994 %>%
    mutate(treated = ifelse(state == "New Jersey", ifelse(time ==
          1, 1, 0), 0))

did_mod <- lm(emptot ~ treated + time + factor(state), data = data_CK1994_mod)
coeftest(did_mod, vcov = function(x) vcovHC(x, cluster = "group",
          type = "HC1"))</pre>
```

## Calculating the DID estimator

```
##
## t test of coefficients:
##
##
                            Estimate Std. Error t value Pr(>|t|)
                                       0.50875 40.1756 < 2e-16 ***
## (Intercept)
                            20.43941
## treated
                             2.75361
                                       1.79545 1.5337 0.12551
                            -2.16558
                                       1.64121 -1.3195 0.18738
## factor(state)Pennsylvania 2.89176
                                       1.43870 2.0100 0.04477 *
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
panel_did <- plm(emptot ~ treated + time + factor(state), data = data_CK1994_mod,</pre>
   model = "within", index = "store")
## Warning in pdata.frame(data, index): column 'time' overwritten by time index
coeftest(panel_did, vcov = function(x) vcovHC(x, cluster = "group",
type = "HC1"))
##
## t test of coefficients:
##
          Estimate Std. Error t value Pr(>|t|)
## treated 2.7500
                      1.3359 2.0585 0.04022 *
           -2.2833
                       1.2465 -1.8319 0.06775 .
## time2
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```