Description

Have a random amount of money from range [A,B], build a thing cost N steps, and each step cost C_i money. When the remain money can not afford a step, stop. Calculate the expected amount of final remaining money. Note: If there is still enough money left after N steps, then continue building another thing.

Take a example:
Initial money: [40,140]
N = 5 steps:
i = [0] [1] [2] [3] [4]
C_i = 4 9 1 12 7
S_i = 4 13 14 26 33

1. Continue building, until [40%33, 140-40/33*33] = [7,107]
2. Afford step 0, remain [3,103]
3. Let A becomes 0, consider step 1:

$$E_0 = \frac{min(C_i, B) - 3}{103 - 3} \times \frac{3+9}{2} = \frac{9-3}{103-3} \times \frac{3+9}{2} = 0.36$$

4. Consider step 2:

$$E_1 = \frac{103-3-6}{103-3} \times \frac{1}{103-3-6} \times \frac{1}{2} + \frac{94}{100} \times \frac{93}{94} \times \frac{1}{93} \times \frac{1}{2} + ... = \frac{1}{100} \times \frac{1}{2} \times (2+1)$$

(Note: 94/33 = 2.xxx, 94%33 = 28, 28 > 1, so +1. And then 28-1=27)
5. Consider step 3:

$$E_2 = \frac{12}{100} \times \frac{12}{2} \times (2+1)$$

(Note: 27>12, 27-12=15)
6. Consider step 4:

$$E_3 = \frac{7}{100} \times \frac{7}{2} \times (2+1)$$

(Note: 15>7, 15-7=8)
7. Consider step 0:

$$E_4 = \frac{4}{100} \times \frac{4}{2} \times (2+1)$$

(Note: 8>4, 8-4=4)
8. Consider step 1:

$$E_5 = \frac{9}{100} \times \frac{9}{2} \times 2 + \frac{4}{100} \times \frac{4}{2}$$

(Note: 4<9)
9. Finally, Ans = $E_0+E_1+E_2+E_3+E_4+E_5$