VE215 2022Fall Assignment 4

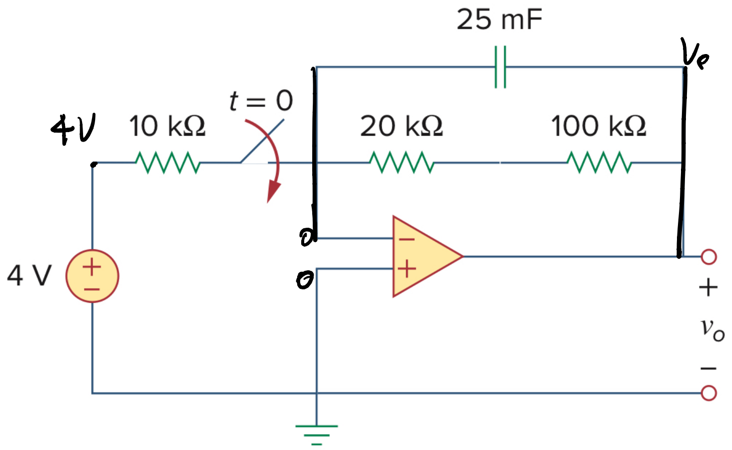
Due Date: 23:59, November 6th, 2022

Exercise 4.1 (25%)

The following figure shows a op-amp circuit. The switch is closed at t=0.

(a) (10%) Derive the differential equation that relates to the output voltage v_o.

(b) (15%) Derive $v_o(t)$ of $t > 0$.



(a) At $t \le 0$ $\quad V_o(t) = 0$ $\quad$ (Not a must)

At $t > 0$ $\quad \frac{0-4}{10k\Omega} + \frac{0-V_o}{120k\Omega} + \left(-25\times10^{-3}F \times \frac{dV_o}{dt}\right) = 0$ $\quad$ 5'

Hence $\quad \boxed{\frac{dv_o}{dt} + \frac{V_o}{3000} + \frac{2}{125} = 0}$ $\quad$ 5'

(b) At $t=0$. $\quad V_o(0^-) = V_o(0^+) = 0$ $\quad$ 2'

At $t=\infty$. $\quad \frac{dV_o}{dt}(\infty) = 0$, According to the differential equation ($t>0$)

$\Rightarrow V_o(\infty) = -48V$ $\quad$ 3'

$V_o(t) = \left(Ae^{-\frac{t}{3000}} + B\right)V$ $\quad$ 5'

$V_o(\infty) = -48V \Rightarrow B = -48$

$V_o(0^-) = V_o(0^+) = 0$, $A + B = 0 \Rightarrow A = 48$

Hence $\quad \boxed{V_o(t) = 48\left(e^{-\frac{t}{3000}} - 1\right)V}$ $\quad$ for $t > 0$. 5'

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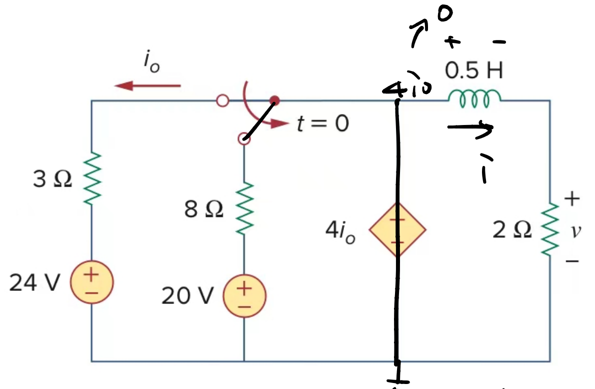
Exercise 4.2 (25%)
For the op-amp circuit shown below, the switch is connected to the branch connected with a 3Ω resister and a $24V$ independent voltage source at $t < 0$, and it is switched to the branch connected with a 8Ω resister and a $20V$ independent voltage source at $t \geq 0$.
(a) (10%) Find $v(t)$ for $t < 0$.
(b) (15%) Find $v(t)$ for $t \geq 0$.



(a) At $t<0$. steady state. → Indutor as a wire.

$i_o = \frac{4i_o - 24V}{3\Omega} \Rightarrow i_o = 24A$ 5'

$\boxed{v(t) = 4i_o = 96V}$ for $t<0$ 5'

(b) At $t \geqslant 0$

At $t=0$ $v(0^-) = v(0^+) = 96(V)$

At $t>0$ $i_o = 0A$.

Hence the branch with $4i_o$ dependent source is treated as a wire.

$L\frac{di(t)}{dt} + 2i(t) = 0$, $L = 0.5H$ 5'

Hence $\frac{di(t)}{dt} + 4i(t) = 0 \Rightarrow i(t) = Ae^{-4t}$ 5'

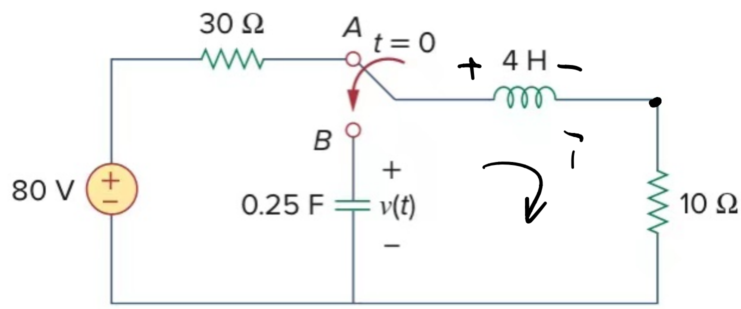
with $v(0^-) = v(0^+) = 96V \Rightarrow i(0^-) = i(0^+) = \frac{96v}{2\Omega} = 48A$

Hence $A = 48$

$\Rightarrow i(t) = 48e^{-4t} A$

$\boxed{v(t) = 2i(t) = 96e^{-4t} V}$ for $t \geqslant 0$ 5'

Exercise 4.3 (20%)
The switch in the following figure moves form position A to position B at $t = 0$ (please note that the switch must connect to point B before it breaks the connection at A, a make-before-break switch). Let $v(0) = 0V$, find $v(t)$ for $t > 0$.



At $t<0$ steady state → inductor as a wire

$$i(t) = \frac{80V}{(30+10)\Omega} = 2A \quad 3'$$

At $t \geq 0$ $i(0^-) = i(0^+) = 2A$, $1'$ $V(0^-) = V(0^+) = 0V$. $1'$

$$i(t) = -C\frac{dv(t)}{dt} \quad 3'$$

$$10i - v(t) + L\frac{di(t)}{dt} = 0 \quad 3'$$

$$\Rightarrow \quad -LC\frac{d^2v(t)}{dt^2} - 10C\frac{dv(t)}{dt} - v(t) = 0 \qquad L = 4H. \quad C = 0.25F$$

$$\Rightarrow \quad \frac{d^2v(t)}{dt^2} + 2.5\frac{dv(t)}{dt} + v(t) = 0 \quad 5'$$

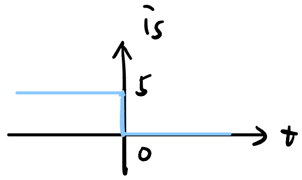
with $s^2 + 2.5s + 1 = 0 \Rightarrow s_1 = -2, \ s_2 = -\frac{1}{2}$

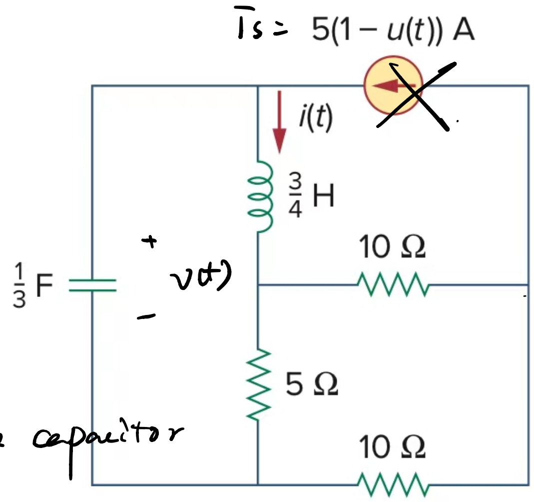
Hence $v(t) = Ae^{-2t} + Be^{-\frac{1}{2}t}$

$$\left.\begin{array}{l} v(0^-) = v(0^+) = 0V \\ \frac{dv(0)}{dt} = -\frac{i(0)}{C} = -8 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} A + B = 0 \\ -2A - \frac{1}{2}B = -8 \end{array}\right. \Rightarrow \left\{\begin{array}{l} A = \frac{16}{3} \\ B = -\frac{16}{3} \end{array}\right.$$

Hence $\boxed{v(t) = \frac{16}{3}\left(e^{-2t} - e^{-\frac{1}{2}t}\right)V}$ for $t \geq 0$ $4'$

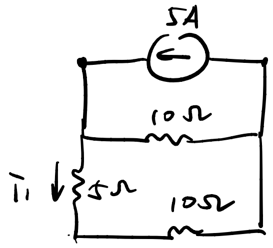
Exercise 4.4 (30%)
The input current source of the following circuit is $5(1 - u(t))$A. Please find $i(t)$ for $t > 0$.





Assume $v(t)$ across the capacitor

At $t \leq 0$ $\quad i_s = 5A \quad$ steady state → capacitor cut-off. inductor as wire

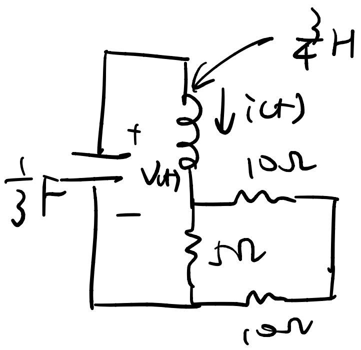


$15 i_1 = 10(5 - i_1) \Rightarrow i_1 = 2A$

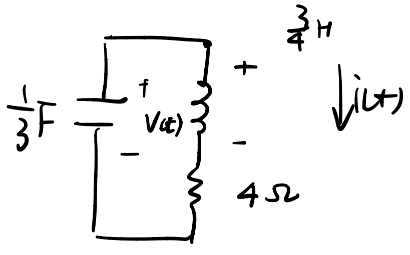
Hence $v(t) = 2A \times 5\Omega = 10V$. 5'

At $t > 0$ $\quad v(0^-) = v(0^+) = 10V$ 1', $\quad i(0^-) = i(0^+) = 5A$. 1'

$i_s = 0A$ ⇔ the current source cut-off



⇔



3'

$$L\frac{di(t)}{dt} + i(t)\cdot 4\Omega - v(t) = 0$$ 5'

$$\left.\begin{aligned} & L\frac{di(t)}{dt} + i(t)\cdot 4\Omega - v(t) = 0 \\ & i(t) = -C\frac{dv(t)}{dt} \Rightarrow v(t) = -\frac{1}{C}\int_0^{\infty} i(t)\,dt \end{aligned}\right\} \Rightarrow L\frac{di(t)}{dt} + 4i(t) + \frac{1}{C}\int_0^{\infty} i(t)\,dt = 0$$

↓ differentiate by t

$$L\frac{d^2 i(t)}{dt^2} + 4\frac{di(t)}{dt} + \frac{1}{C} i(t) = 0$$

$$\Rightarrow \frac{d^2 i(t)}{dt^2} + \frac{16}{3}\frac{di(t)}{dt} + 4i(t) = 0$$ 5'

$$\Rightarrow s^2 + \frac{16}{3}s + 4 = 0 \Rightarrow s_1 = \frac{-8+2\sqrt{7}}{3},\ s_2 = \frac{-8-2\sqrt{7}}{3}$$

Hence $i(t) = Ae^{\frac{-8+2\sqrt{7}}{3}t} + Be^{\frac{-8-2\sqrt{7}}{3}t}$ A 3'

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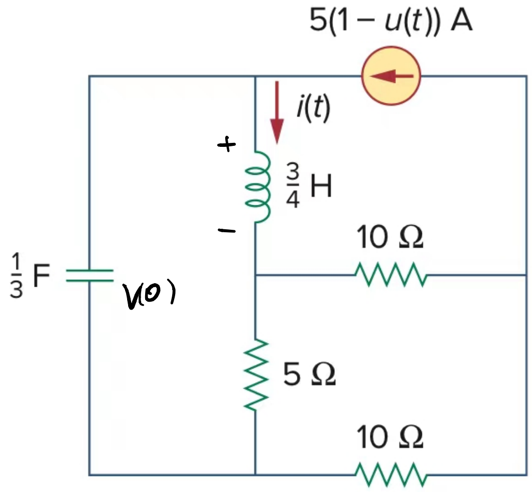
Exercise 4.4 (30%)

The input current source of the following circuit is $5(1-u(t))$A. Please find $i(t)$ for $t>0$.



(continue)

With initial condition: $V(0) = 10V, \quad i(0) = 5A$ ✓

$$V(0) = L\frac{di(0)}{dt} + 4i(0)$$

$$\Rightarrow \frac{di(0)}{dt} = -\frac{40}{3}$$ ✓ 2'

$$\Rightarrow \left.\begin{array}{l} A+B=5 \\ \frac{-8+2\sqrt{7}}{3}A + \frac{-8-2\sqrt{7}}{3}B = -\frac{40}{3} \end{array}\right\} \Rightarrow \begin{cases} A = \frac{5}{2} \\ B = \frac{5}{2} \end{cases}$$

Hence $\boxed{i(t) = \frac{5}{2}\left[e^{\frac{-8+2\sqrt{7}}{3}} + e^{\frac{-8-2\sqrt{7}}{3}}\right] A}$ for $t>0$. 5'

△注：若答案正确，给满分，若有错，给0分（全卷）