VE215 2022Fall Assignment 8

Due Date: 23:59, Dec.11 ,2022

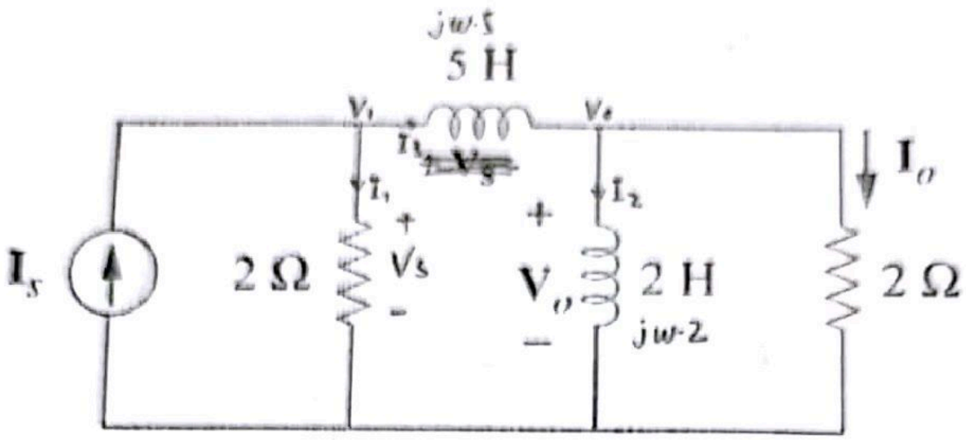
Exercise 8.1 (30%)

For the circuit shown below, find the transfer function

(a) (15%) $I_o(\omega)/I_s(\omega)$, and its zeros and poles.

(b) (15%) $\cancel{V_o}(\omega)/\cancel{I_s}(\omega)$, and its zeros and poles.

V_s $\quad$ I_s



(a)

$$\begin{cases} \cancel{\frac{V_1}{2}+\frac{V_1-V_0}{5wj}=I_s} \\ \cancel{\frac{V_1-V_0}{2wj}+\frac{V_0}{2}=I_s} \end{cases}$$

$\therefore I_2=\frac{V_0}{2wj}=\frac{2I_0}{2wj}=\frac{I_0}{wj}$

$I_3=I_2+I_0=I_0(1+\frac{1}{jw})$

$\therefore V_1=jw5I_3+V_0=5wj\cdot I_0(1+\frac{1}{jw})+2I_0=(5wj+7)I_0$

$I_1=\frac{V_1}{2}=\frac{7+5wj}{2}I_0$

$\therefore I_s=I_1+I_3=\frac{-5w^2+jw\cdot 9+2}{jw\cdot 2}I_0$

$\therefore \frac{I_0(w)}{I_s(w)}=\frac{jw\cdot 2}{5(jw)^2+jw\cdot 9+2}=\frac{2s}{5s^2+9s+2}$ $(s=jw)$ (5')

$\therefore$ Zeros: $2s=0$ $\quad\therefore s=0$ $\quad\therefore z=0$ (5')

Poles: $5s^2+9s+2=0$ $\quad\therefore s=\frac{-9\pm\sqrt{41}}{10}$ $\quad\therefore P_1=-0.2597,\ P_2=-1.5403$ (5')

(b) $V_s=2I_1=(7+5jw)I_0$

$\therefore \frac{V_s(w)}{I_s(w)}=\frac{(7+5jw)I_0}{\frac{-5w^2+9\cdot jw+2}{jw\cdot 2}I_0}=\frac{2jw(7+5jw)}{-5w^2+9jw+2}$

$=\frac{2s(7+5s)}{5s^2+9s+2}$ $(s=jw)$ (5')

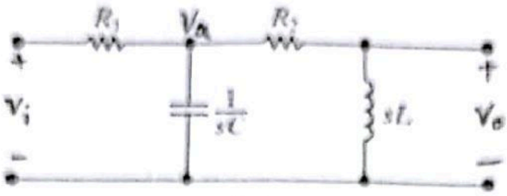
$\therefore$ Zeros: $s(7+5s)=0$ $\quad\therefore z_1=0,\ z_2=-\frac{7}{5}$ (5')

Poles: $5s^2+9s+2=0$

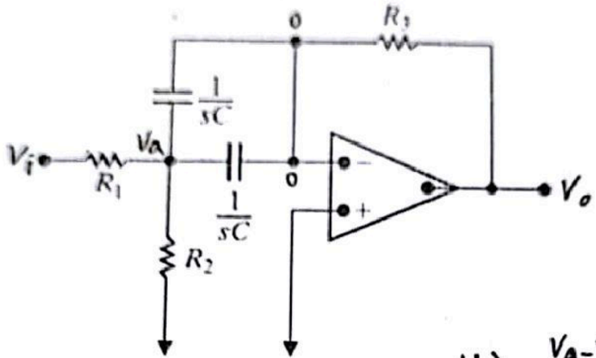
$\therefore P_1=-0.2597,\ P_2=-1.5403$ (5')

Exercise 8.2 (30%)

Find $H(s) = V_0(s)/V_i(s)$, where $s = j\omega$ in both circuits. Assume that $R_1 = R_2 = R_3 = 100\Omega$, $L = 1H$ and $C = 1mF$ for (a) and (b).



(a)



(b)

(a)

$$\therefore \frac{V_a - V_i}{R_1} + sCV_a + \frac{V_a}{R_2 + sL} = 0$$

$$\therefore V_a = \frac{(R_2 + sL)V_i}{R_1LCs^2 + (R_1R_2C + L)s + (R_1 + R_2)}$$

$$\because V_o = \frac{sL}{R_2 + sL} V_a,$$

$$\therefore V_o = \frac{sLV_i}{R_1LCs^2 + (L + R_1R_2C)s + (R_1 + R_2)}$$

$$\therefore H(s) = \frac{V_o}{V_i} = \frac{sL}{R_1LCs^2 + (R_1 + R_2) + (L + R_1R_2C)s}$$

$$\therefore H(j\omega) = \frac{j\omega L}{[(R_1 + R_2) - R_1LC\omega^2] + j\omega(L + R_1R_2C)} = \frac{j\omega}{200 - 0.1\omega^2 + j\omega \cdot 11}$$

(15')

(b)

$$\therefore \frac{V_a - V_i}{R_1} + \frac{V_a}{R_2} + V_a sC + V_a sC = 0$$

$$(0 - V_a)sC + (0 - V_a)sC + \frac{0 - V_o}{R_3} = 0$$

$$\therefore V_a = \frac{R_2V_i}{2R_1R_2Cs + R_1 + R_2}$$

$$V_a = -\frac{V_o}{2R_3Cs}$$

$$\therefore H(s) = \frac{V_o}{V_i} = \frac{-2R_2R_3Cs}{2R_1R_2Cs + R_1 + R_2}$$

$$H(j\omega) = \frac{-j\omega \cdot 20}{200 + j\omega \cdot 20} = \frac{-j\omega}{10 + j\omega}$$

(15')

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Exercise 8.3 (40%)

Obtain the Bode plots ($H-\omega$ and $\psi-\omega$ relationship) for

$$(a) H(j\omega) = \frac{0.1(20+j\omega)}{j\omega(5+j\omega)}$$

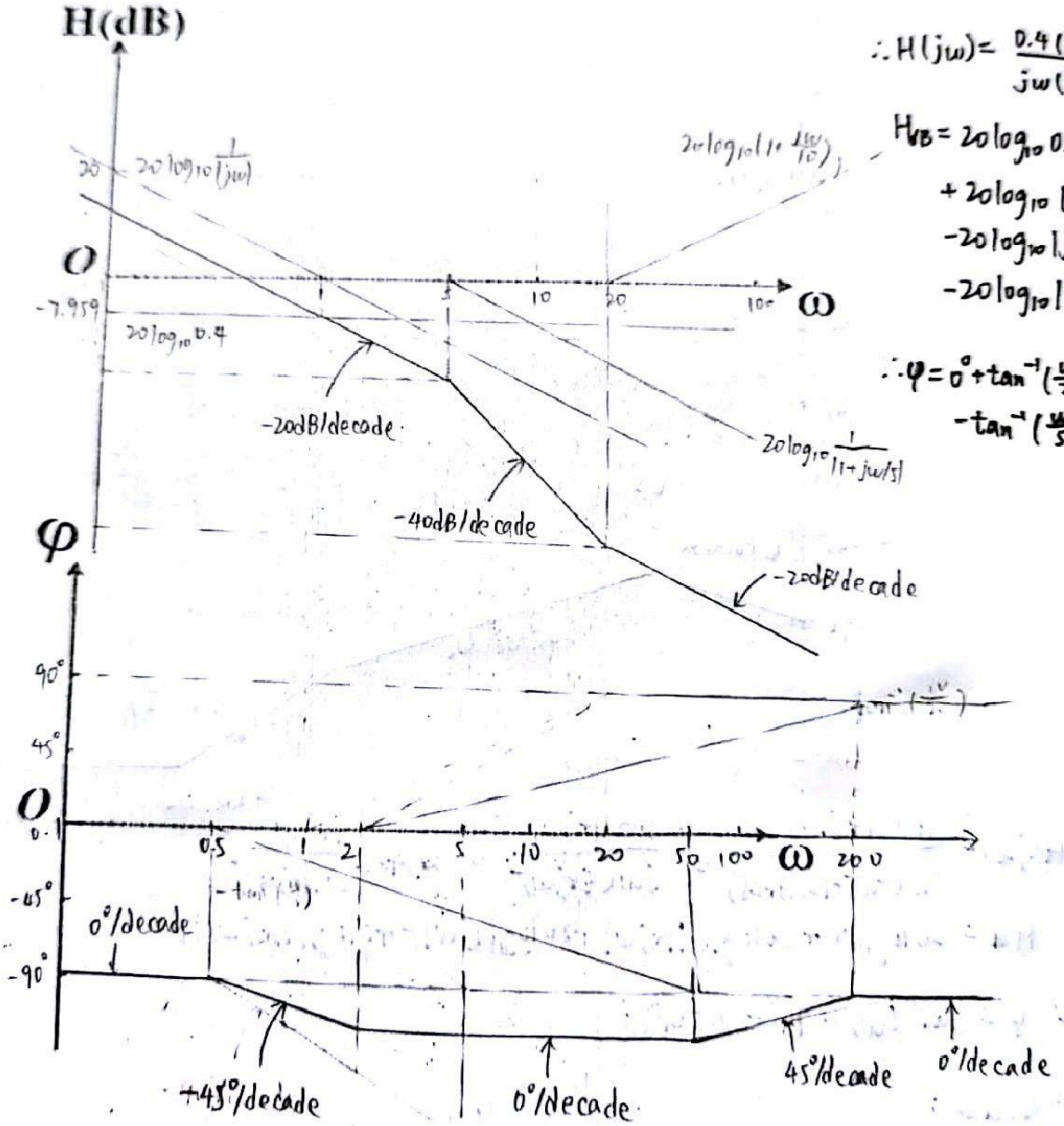
$$(b) H(j\omega) = \frac{100(1+j\omega)}{j\omega(-\omega^2+10j\omega+25)}$$

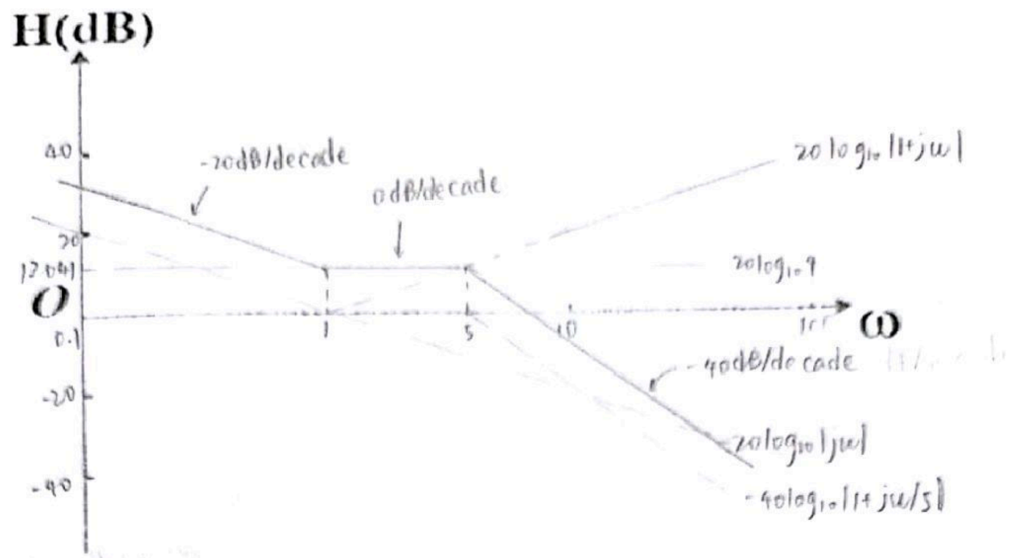
(a)

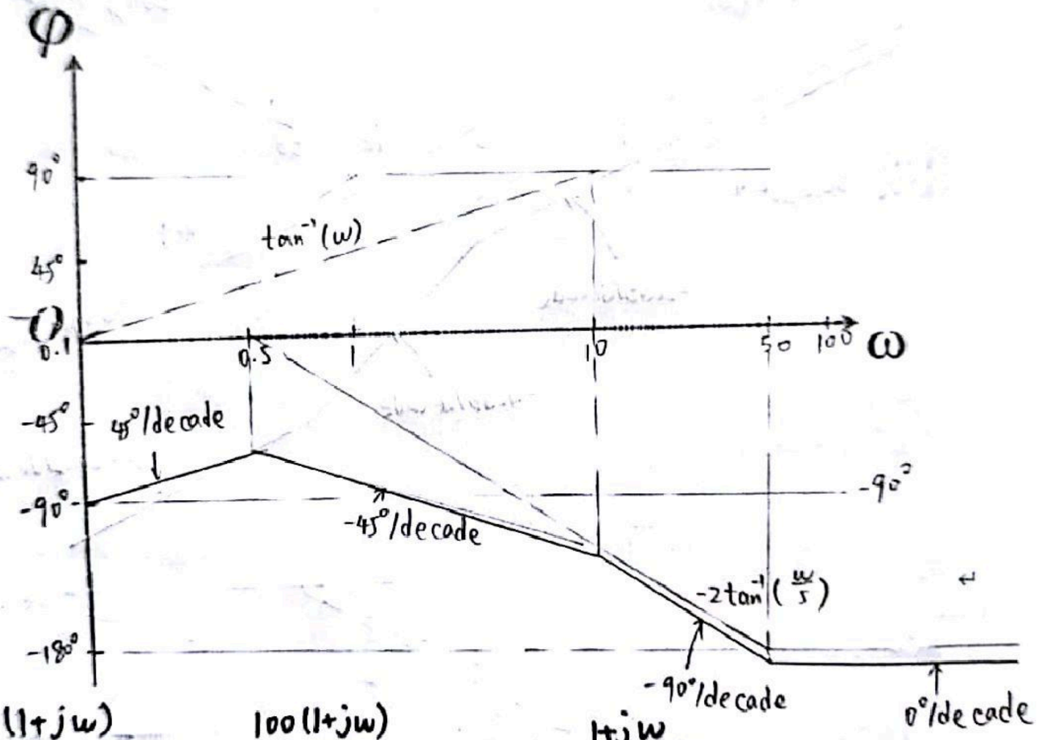
$$\therefore H(j\omega) = \frac{0.4(1+j\omega/20)}{j\omega(1+j\omega/5)}$$

$$H_{dB} = 20\log_{10}0.4 + 20\log_{10}|1+j\omega/20| - 20\log_{10}|j\omega| - 20\log_{10}|1+j\omega/5|$$

$$\therefore \varphi = 0^\circ + \tan^{-1}\left(\frac{\omega}{20}\right) - 90^\circ - \tan^{-1}\left(\frac{\omega}{5}\right)$$







$$\therefore H(jw) = \frac{100(1+jw)}{jw(-w^2+10jw+25)} = \frac{100(1+jw)}{jw(5+jw)^2} = \cancel{30}\, 4 \cdot \frac{1+jw}{jw(1+jw/5)^2}$$

$$H_{dB} = 20\log_{10}4 + 20\log_{10}|1+jw| - 20\log_{10}|jw| - 40\log_{10}|1+jw/5|$$

$$\therefore \varphi = \tan^{-1}(w) - 90^\circ - 2\tan^{-1}\left(\frac{w}{5}\right)$$

- Notice:
  - Lines (1 wrong -3')
  - Slopes / φ-coordinate & H-coordinate (miss 1 -2')
  - w-coordinate (miss 1 -2')