# VE215 2022Fall Assignment 1

**Due Date: 23:59, , 2022**

**Exercise 1.1** (20%)
The voltage $v$ (unit:V) across a device and the current $i$ (unit:A) through it are

$$v(t) = 4e^{-t/2} \qquad\qquad i(t) = \begin{cases} 0 & t<0 \\ 5\sin 3t & 0 \le t < \dfrac{\pi}{6} \\ 5 & t \ge \dfrac{\pi}{6} \end{cases}$$

(a) (10%) Calculate the total charge in the device at $t = 2$.
(b) (10%) Calculate the energy consumed by the device in the time period $3 \le t \le 5$.

(a) $$q = \int_{-\infty}^{2} i\,dt = \int_{0}^{\frac{\pi}{6}} 5\sin(3t)\,dt + \int_{\frac{\pi}{6}}^{2} 5\,dt = -\frac{5}{3}\cos 3t \Big|_{0}^{\frac{\pi}{6}} + 5t \Big|_{\frac{\pi}{6}}^{2}$$

$$= \left(\frac{35}{3} - \frac{5\pi}{6}\right) C = 9.05\,C$$

(b) $$E = \int_{3}^{5} vi\,dt = \int_{3}^{5} 4e^{-\frac{t}{2}} \cdot 5\,dt = -40e^{-\frac{t}{2}} \Big|_{3}^{5}$$

$$= 40\left(e^{-\frac{3}{2}} - e^{-\frac{5}{2}}\right) J = 5.64\,J$$

for (a), considering the use of "through", $q=0$ is also acceptable.
if the answer is right, can give full points

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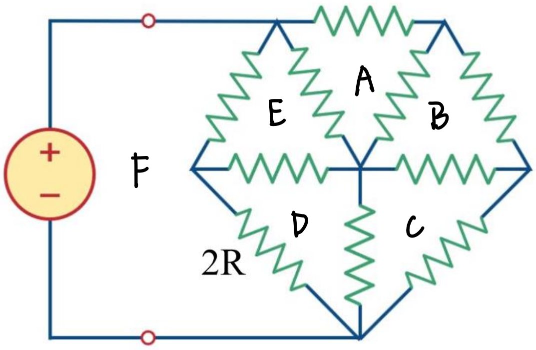
**Exercise 1.2** (35%)
In the circuit below, all the resistors have a resistance of R except the labeled one on the left bottom.
(a) (10%) Determine the number of branches, nodes, loops and meshes. Write your answers directly.
(b) (25%) Calculate the equivalent resistance between the terminals.




(a) branch: 11, node: 6, mesh: 6 (each 2')

6 loops covering 1 area: A, B, C, D, E, F

7 loops covering 2 areas: AB, BC, CD, DE, EA, FE, FD

8 loops covering 3 areas: ABC, BCD, CDE, DEA, EAB, FEA, FED, FDC

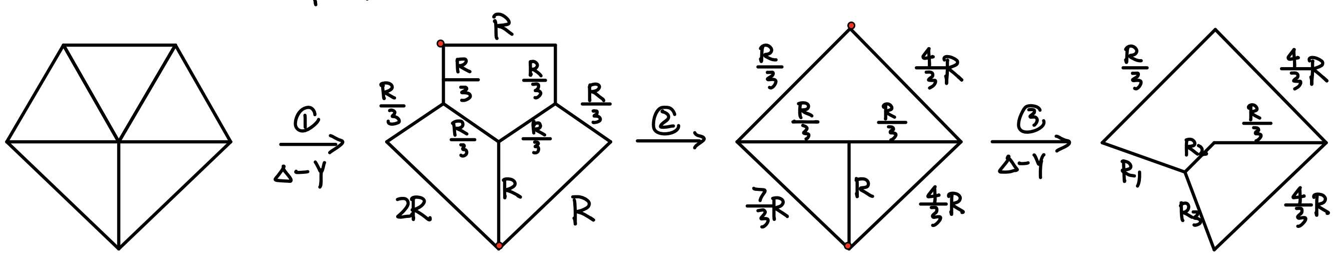
9 loops covering 4 areas: ABCD, BCDE, CDEA, DEAB, EABC, FEAB, FDEA, FDEC, FDCB

4 loops covering 5 areas: ABCDE, FDEAB, FDEAC, FDECB

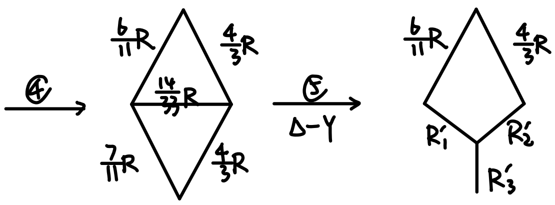
1 loop covering 6 areas: ABCDEF

Therefore, the number of loops is 35. (4')

(b)




$R_1 = \dfrac{\frac{R}{3}R\cdot\frac{7}{3}R}{\frac{11}{3}R} = \dfrac{7}{33}R$, $R_2 = \dfrac{\frac{R}{3}\cdot R}{\frac{11}{3}R} = \dfrac{1}{11}R$, $R_3 = \dfrac{\frac{7}{3}R\cdot R}{\frac{11}{3}R} = \dfrac{7}{11}R$




$R_1' = \dfrac{\frac{7}{11}R\cdot\frac{14}{33}R}{\frac{79}{33}R} = \dfrac{98}{869}R$, $R_2' = \dfrac{\frac{4}{3}R\cdot\frac{14}{33}R}{\frac{79}{33}R} = \dfrac{56}{237}R$, $R_3' = \dfrac{\frac{7}{11}R\cdot\frac{4}{3}R}{\frac{79}{33}R} = \dfrac{28}{79}R$

$R_{eq} = (\frac{6}{11}R + R_1') \,||\, (\frac{4}{3}R + R_2') + R_3' = \frac{9}{11}R$ (answer: 5')

①~⑤: each step 4'

Use other kind of transformation is also okay.

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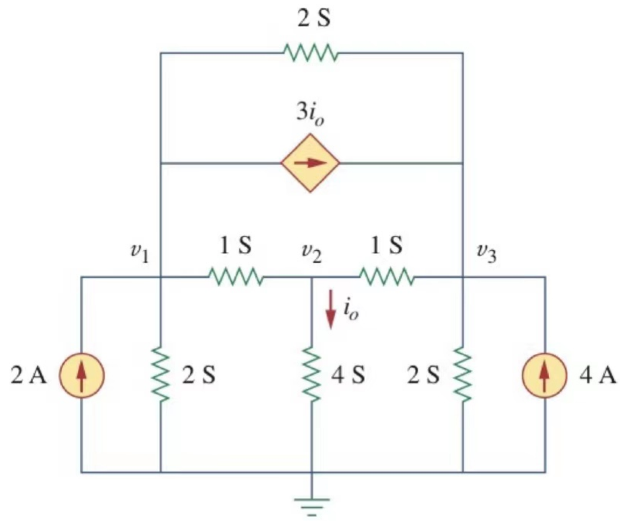
**Exercise 1.3** (30%)
Use nodal analysis to determine voltages $v_1$, $v_2$ and $v_3$ in the circuit.




$4V_2 = i_o$

by KCL:
$$\begin{cases} 2 = 2V_1 + (V_1 - V_2) + 3i_o + 2(V_1 - V_3) \\ V_1 - V_2 = 4V_2 + (V_2 - V_3) \\ 2(V_1 - V_3) + 3i_o + (V_2 - V_3) + 4 = 2V_3 \end{cases}$$

right equation of KCL / inspection: 25'
one right equation: 10'
two right equation: 20'

we can solve that
$$\begin{cases} V_1 = \frac{19}{56}V \approx 0.339V \\ V_2 = \frac{3}{8}V \approx 0.375V \\ V_3 = \frac{107}{56}V \approx 1.911V \end{cases}$$
(5' for answer)

or by inspection: $G_{11} = 2+1+2 = 5S$, $G_{22} = 1+4+1 = 6S$, $G_{33} = 2+1+2 = 5S$

$$\begin{bmatrix} 5 & -1 & -2 \\ -1 & 6 & -1 \\ -2 & -1 & 5 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 2-3i_o \\ 0 \\ 3i_o+4 \end{bmatrix}$$

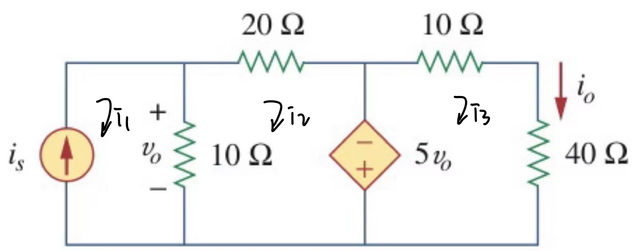
$4V_2 = i_o$

$$\begin{bmatrix} 5 & -1 & -2 \\ -1 & 6 & -1 \\ -2 & -1 & 5 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 2-12V_2 \\ 0 \\ 12V_2+4 \end{bmatrix}$$

if mistake S as an unknown number and get answer of:
$$\begin{cases} V_1 = \frac{16}{5}S \\ V_2 = \frac{48}{11}S \\ V_3 = \frac{364}{55}S \end{cases}$$
5 points should be deducted.

**Exercise 1.4** (15%)

Calculate the current gain $i_o/i_s$ in the circuit.




Apply mesh analysis.

Obviously, $i_s = i_1$, $i_o = i_3$, $V_o = 10(i_1 - i_2) \Rightarrow i_1 = \frac{1}{10}V_o + i_2$

By KVL: $\begin{cases} 20i_2 - 5V_o - V_o = 0 \Rightarrow i_2 = \frac{3}{10}V_o, \quad i_1 = \frac{2}{5}V_o \\ 10i_3 + 40i_3 + 5V_o = 0 \Rightarrow i_3 = -\frac{1}{10}V_o \end{cases}$ (10')

$i_o/i_s = i_3/i_1 = -\frac{1}{10}\Big/\frac{2}{5} = -\frac{1}{4}$ (5')