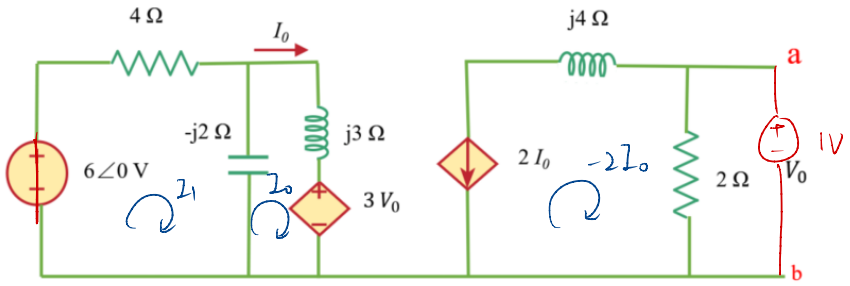
VE215 2022Fall Assignment 6

Due Date: 23:59, December 5th, 2022

In order to get full marks, you shall write all the intermediate steps of calculation or proof unless otherwise indicated.

Exercise 6.1 (40%)
(a) For the given circuit, please calculate the total apparennt power $|S|$, total real power P, total reactive power Q and the total power factor.
(b) Suppose an impedance load Z_L is connected between a and b. Please calculate the value of Z_L that will absorb the maximum power and the value of that maximum power if

- Z_L is an impedance load
- Z_L is a pure resistance load



(a)

$$\begin{cases} -6\angle 0 + 4I_1 - j2\cdot(I_1 - I_0) = 0 \\ -j2(I_0 - I_1) + j3I_0 + 3V_0 = 0 \\ V_0 = -4I_0 \end{cases}$$

(one right equation 5', all right 10')

$$\Rightarrow \begin{cases} I_0 = -0.132 + 0.198j & (2') \\ I_1 = 1.253 + 0.692j & (2') \\ V_0 = 0.527 - 0.791j & (2') \end{cases}$$

$S = I_1^2 \cdot 4 + (I_1 - I_0)^2 \cdot (-j2) + I_0^2 \cdot j3 + (-2I_0)^2 \cdot j0 + (-2I_0)^2 \cdot 2$

$= 8.649 - 3.249j$

$= 9.239\angle -20.587^\circ$

$|S| = 9.239$ (1')

$P = 8.649$ (1')

$Q = -3.249$ (1')

$pf = \cos(-20.587) = 0.934$ (1')

(b) $V_{Th} = V_0 = 0.527 - 0.791j$ (5')

Z_{Th}:

$$\begin{cases} 4I_1 - j2(I_1 - I_0) = 0 & (2') \\ -j2\cdot(I_0 - I_1) + j3\cdot I_0 + 3V_0 = 0 & (2') \\ V_0 = 1 \end{cases}$$

$$\Rightarrow \begin{cases} I_0 = -0.923 + 1.615j & (1') \\ I_1 = 0.462 + 0.692j & (1') \end{cases}$$

$$Z_{Th} = \frac{1}{2I_0 + V_0/2} = 0.110 - 0.264j \quad (2')$$

Impedance load:

$Z_L = Z_{Th}^* = -0.110 + 0.264j$ (2')

$R_{Th} < 0$, $P_m = \infty$ / -2.05 W (1')

Resistive load:

$Z_L = \sqrt{0.110^2 + 0.264^2} = \frac{2}{7}\ \Omega$ (2')

$$P_m = \frac{V_{Th}^2 R_L}{(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2} = 2.57\ \text{W} \quad (1')\ (1')$$

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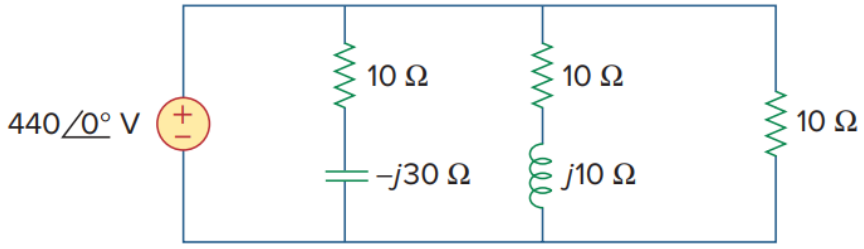
Exercise 6.2 (30%) Consider the power system shown below. Calculate:
(a) the total complex power
(b) the power factor
(c) the parallel capacitance necessary to establish a unity power factor



(a) $S = \frac{440^2}{(10 - j30)^*} + \frac{440^2}{(10 + j10)^*} + \frac{440^2}{10}$ (5')

$= 30976 + 3872j$ (5')

(b) $\theta = \tan^{-1}\left(\frac{3872}{30976}\right) = 7.125$ (5')

$pf = \cos\theta = 0.992$ lagging (5')

(c) $P = 30976$ W.

$C = \frac{P(\tan\theta_1 - \tan\theta_2)}{\omega V_{rms}^2}$ (5')

$= \frac{30976 \cdot \tan(7.125)}{1000 \cdot 440^2}$

$= 2 \times 10^{-5}$ F / $\frac{1}{50\pi}$ F (5')

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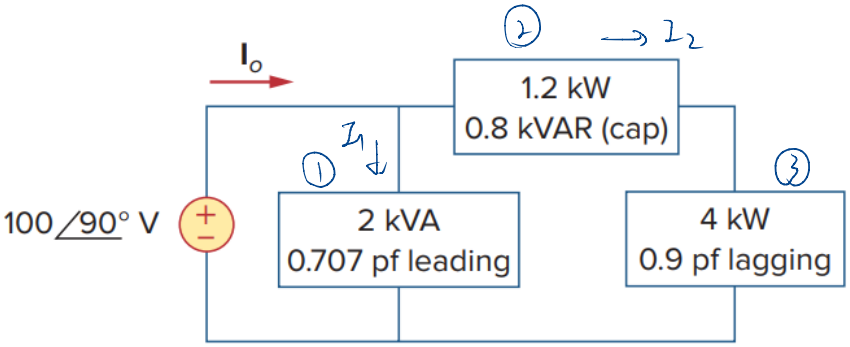
Exercise 6.3 (30%)
Given the circuit below, calculate the equivalent impedance in each of the blocks. Also find I_o and the overall complex power supplied.



$S_1 = 1.414 - 1.414j$ kVA (2')

$S_2 = 1.2 - 0.8j$ kVA (2')

$S_3 = 4 + j4 \cdot \tan(\cos^{-1}(0.9)) = 4 + 1.937j$ kVA (2')

$I_1^* = \frac{S_1}{V} = \frac{1.414 - 1.414j\ k}{j100} = -14.14 - 14.14j$ A $\Rightarrow I_1 = -14.14 + 14.14j$ (2')

$I_2^* = \frac{S_2 + S_3}{V} = 11.37 - 52j$ A $\Rightarrow I_2 = 11.37 + 52j$ (2')

$I_o = I_1 + I_2 = -2.77 + 66.14j = 66.198 \angle 92.398$ A. (4')

$S = S_1 + S_2 + S_3 = 6.614 - 0.277j$ kVA. (4')

$Z_1 = \frac{V}{I_1} = 3.54 - 3.54j = 5 \angle -45°\ \Omega$ (4')

$Z_2 = \frac{S_2}{|I_2|^2} = 0.424 - 0.282j = 0.509 \angle -33.69°\ \Omega$ (4')

$Z_3 = \frac{S_3}{|I_2|^2} = 1.412 + 0.684j = 1.569 \angle 25.839°\ \Omega$ (4')