VE215 2022Fall Assignment 7

JOINT INSTITUTE 交大密西根学院

Due Date: 23:59, Dec.7th, 2022

In order to get full marks, you shall write all the intermediate steps of calculation or proof unless otherwise indicated.

Exercise 7.1 (35%)
(a) (15%) A 480/2400-V rms step-up ideal transformer delivers 60 kW to a resistive load. Calculate: (1) the turns ratio (2) the primary current (3) the secondary current

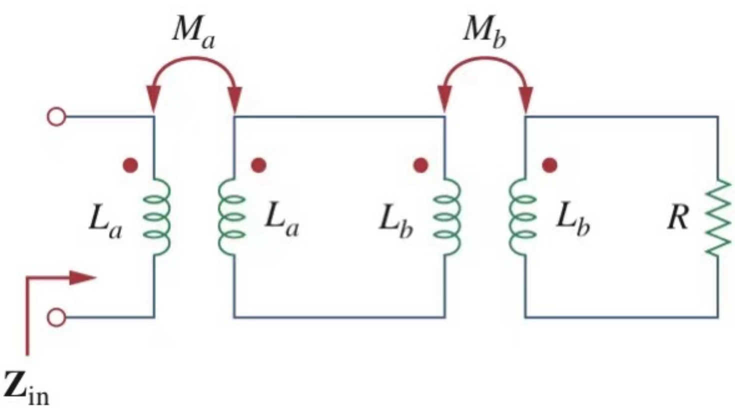
(a) $V_1 = 480\text{V}$, $V_2 = 2400\text{V}$

$n = \frac{V_2}{V_1} = 5$ (5')

(b) $I_1 = \frac{60\times10^3}{480} = 125\text{A}$ (5')

(c) $I_2 = \frac{1}{n}I_1 = \frac{1}{5}I_1 = 25\text{A}$ / $I_2 = \frac{60\times10^3}{2400} = 25\text{A}$ (5')

(b) (20%) Two linear transformers are cascaded as shown below. Calculate Z_{in}.



method 1: $Z_a = j\omega L_a$, $Z_b = j\omega L_b$

$Z_{in} = \frac{V_{in}}{I_1}$

$$\begin{cases} (R+Z_b)I_3 - j\omega M_b I_2 = 0 \Rightarrow I_3 = \frac{j\omega M_b}{R+Z_b} I_2 \\ (Z_a+Z_b)I_2 - j\omega M_b I_3 - j\omega M_a I_1 = 0 \Rightarrow I_2 = \frac{j\omega M_a}{Z_a+Z_b - j\omega M_b \cdot \frac{j\omega M_b}{R+Z_b}} I_1 = \frac{(R+Z_b)j\omega M_a}{(R+Z_b)(Z_a+Z_b)+\omega^2 M_b^2} I_1 \\ Z_a I_1 - j\omega M_a I_2 - V_{in} = 0 \Rightarrow V_{in} = Z_a I_1 - j\omega M_a I_2 = \left(Z_a + \frac{\omega^2 M_a^2 (R+Z_b)}{(R+Z_b)(Z_a+Z_b)+\omega^2 M_b^2}\right) I_1 \end{cases}$$

(5' for each KVL equation)

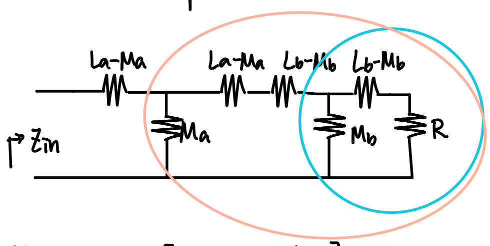
$$Z_{in} = Z_a + \frac{\omega^2 M_a^2 (R+Z_b)}{(R+Z_b)(Z_a+Z_b)+\omega^2 M_b^2}$$

$$= j\omega L_a + \frac{\omega^2 M_a^2 (R+j\omega L_b)}{j\omega R(L_a+L_b) - \omega^2(L_aL_b+L_b^2-M_b^2)}$$

$$= j\omega L_a + \frac{\omega^2 M_a^2}{j\omega(L_a+L_b) + \frac{\omega^2 M_b^2}{j\omega L_b + R}} \Big/ j\omega L_a - \frac{j\omega M_a^2}{j\omega(L_a+L_b) - \frac{j\omega M_b^2}{j\omega L_b + R}}$$ (most students may have this answer)

$$= \frac{\omega^2 R(L_a^2+L_aL_b-M_a^2) + j\omega^3(L_a^2L_b+L_aL_b^2-L_aM_b^2-L_bM_a^2)}{\omega^2(L_aL_b+L_b^2-M_b^2) - j\omega R(L_a+L_b)}$$ (5' for the answer)

method 2: use T-transformation



(equivalent circuit: 5')

$$Z_1 = j\omega M_b \,\|\, [R + j\omega(L_b - M_b)]$$

$$= \frac{j\omega M_b (R + j\omega L_b - j\omega M_b)}{j\omega L_b + R} = j\omega M_b - \frac{j\omega M_b}{j\omega L_b + R} \quad (5')$$

$$Z_2 = [Z_1 + j\omega(L_a - M_a) + j\omega(L_b - M_b)] \,\|\, (j\omega M_a)$$

$$= \frac{j\omega M_a [Z_1 + j\omega(L_a - M_a) + j\omega(L_b - M_b)]}{Z_1 + j\omega L_a + j\omega(L_b - M_b)}$$

$$= \frac{j\omega M_a \left[j\omega(L_a + L_b - M_a - M_b) + \frac{j\omega M_b (R + j\omega L_b - j\omega M_b)}{j\omega L_b + R}\right]}{j\omega(L_a + L_b - M_b) + \frac{j\omega M_b (R + j\omega L_b - j\omega M_b)}{j\omega L_b + R}}$$

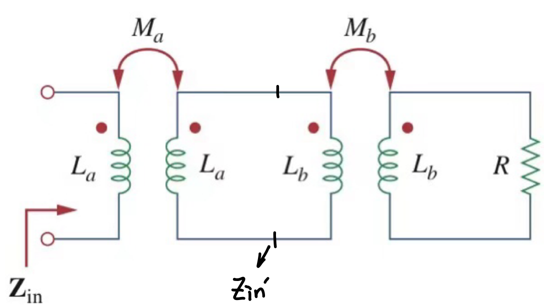
$$= \frac{j\omega M_a \left[j\omega(L_a + L_b) - j\omega M_a - \frac{j\omega M_b}{j\omega L_b + R}\right]}{j\omega(L_a + L_b) - \frac{j\omega M_b}{j\omega L_b + R}} \quad (5')$$

$$Z_{in} = Z_2 + j\omega(L_a - M_a)$$

$$= j\omega(L_a - M_a) + \frac{j\omega M_a \left[j\omega(L_a + L_b - M_a - M_b) + \frac{j\omega M_b (R + j\omega L_b - j\omega M_b)}{j\omega L_b + R}\right]}{j\omega(L_a + L_b - M_b) + \frac{j\omega M_b (R + j\omega L_b - j\omega M_b)}{j\omega L_b + R}}$$

$$= \frac{\omega^2 R (L_a^2 + L_a L_b - M_a^2) + j\omega^3 (L_a^2 L_b + L_a L_b^2 - L_a M_b^2 - L_b M_a^2)}{\omega^2 (L_a L_b + L_b^2 - M_b^2) - j\omega R (L_a + L_b)} \quad (5')$$

method 3:



$$Z_{in}' = j\omega L_b + \frac{\omega^2 M_b^2}{j\omega L_b + R} \quad (10')$$

$$Z_{in} = j\omega L_a + \frac{\omega^2 M_a^2}{j\omega L_a + j\omega L_b + \frac{\omega^2 M_b^2}{j\omega L_b + R}}$$

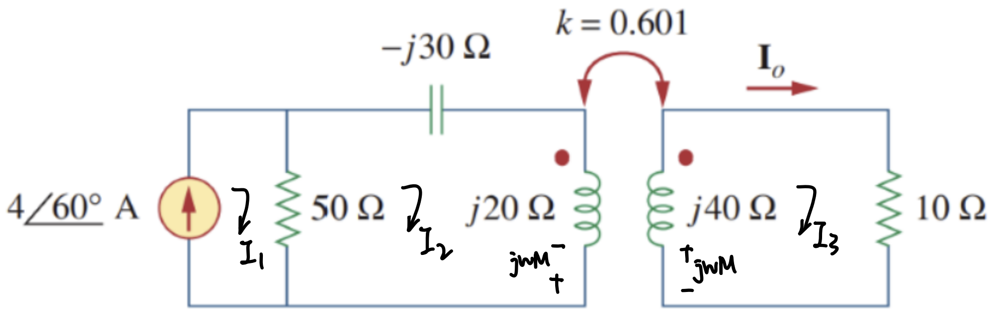
(10')

right answer: full points

Since there're many methods, each right step worths 5'

VE215 2022Fall Assignment 7

Exercise 7.2 (25%)
Find current I_0 in the circuit.



$$k = \frac{M}{\sqrt{L_1 L_2}} = 0.601$$

$$M = 0.601\sqrt{L_1 L_2}$$

$$j\omega M = 0.601\, j\omega\sqrt{L_1 L_2} = 0.601\sqrt{20\times 40} = 17.00j\ V$$ get right $j\omega M$: 10'

$I_1 = 4\angle 60° A$, $I_0 = I_3$

by KVL: $(-j30 + j20)I_2 - j\omega M I_3 + 50(I_2 - I_1) = 0$ (5')

$10I_3 - j\omega M I_2 + 40jI_3 = 0$ (5')

$$(50-10j)I_2 - 17.00jI_3 = 50I_1 = 200\angle 60°$$

$$-17.00jI_2 + (10+40j)I_3 = 0$$

$$\Delta = \begin{vmatrix} 50-10j & -17j \\ -17j & 10+40j \end{vmatrix} = (50-10j)(10+40j) + 289 = 1189 + j1900$$

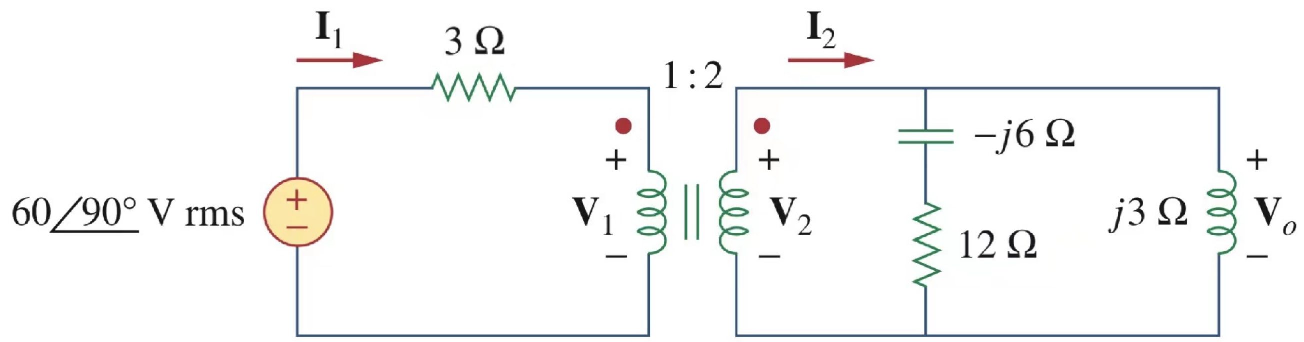
$$\Delta_2 = \begin{vmatrix} 50-10j & 200\angle 60° \\ -17j & 0 \end{vmatrix} = 3400\angle 150°$$

$$I_0 = I_3 = \frac{\Delta_2}{\Delta} = -0.0539 + j1.516$$

$$= 1.52\angle 92.04° A$$ (5')

VE215 2022Fall Assignment 7

Exercise 7.3 (40%)
For the ideal transformer circuit below, find:
(a) I_1 and I_2
(b) V_1 and V_2
(c) the complex power supplied by the source



(a) $\frac{V_2}{V_1} = n = \frac{I_1}{I_2} = 2$ (5')

$Z_R = \frac{(-j6+12)\|j3}{2^2} = \frac{3}{17} + j\frac{27}{34} = 0.176 + j0.794\ \Omega$ (5')

$I_1 = \frac{60\angle 90^\circ}{3+Z_R} = \frac{40}{9} + j\frac{160}{9} = 4.44 + j17.78\ \Omega = 18.32\angle 75.97^\circ$ A (5')

$I_2 = \frac{I_1}{2} = \frac{20}{9} + j\frac{80}{9} = 2.22 + j8.89\ \Omega = 9.16\angle 75.97^\circ$ A (5')

(b) $3I_1 + V_1 - 60\angle 90^\circ = 0 \Rightarrow V_1 = -\frac{40}{3} + j\frac{20}{3} = -13.33 + j6.67\ \text{V} = 14.91\angle 153.43^\circ$ V (5')

$V_2 = 2V_1 = -\frac{80}{3} + j\frac{40}{3} = -26.67 + j13.33\ \text{V} = 29.82\angle 153.43^\circ$ V (5')

(5') (rms value!)

(c) $S = V_s I_1^* = 60\angle 90^\circ \cdot \left(\frac{40}{9} - j\frac{160}{9}\right)$

$= \frac{3200}{3} + j\frac{800}{3} = 1066.67 + j266.67$ VA $= 1099.49\angle 14.04^\circ$ VA (5')