ECE3110J/VE311 Electronic Circuits

Differential Amplifiers

Design of Analog CMOS Integrated Circuits, Chapter 4
Fundamentals of Microelectronics, Chapter 10

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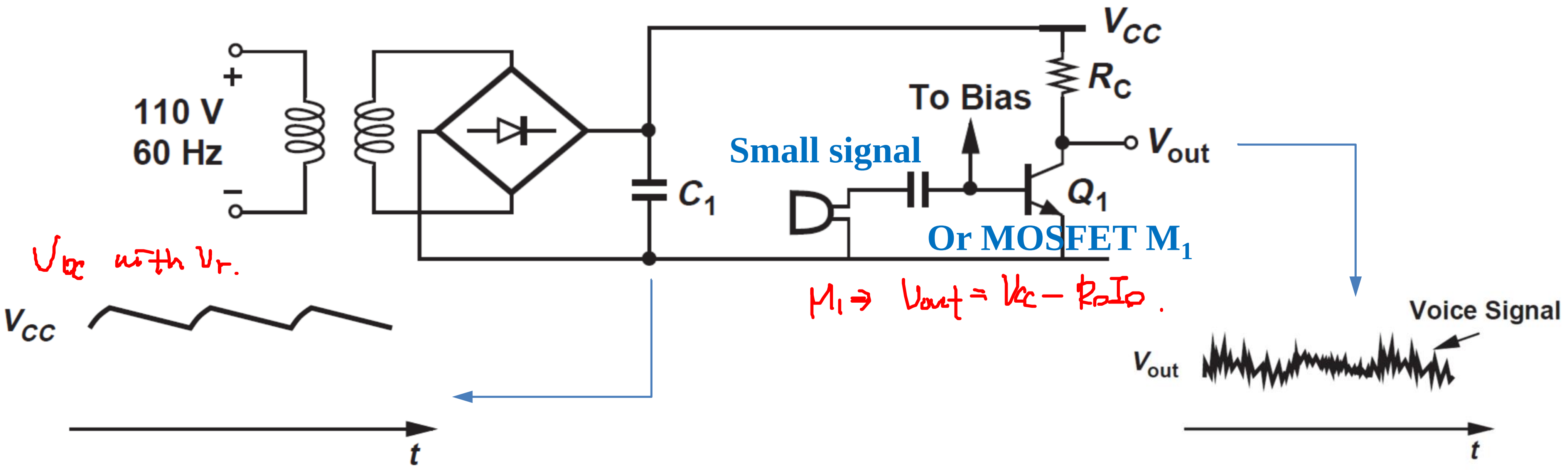
Need for Differential Circuits

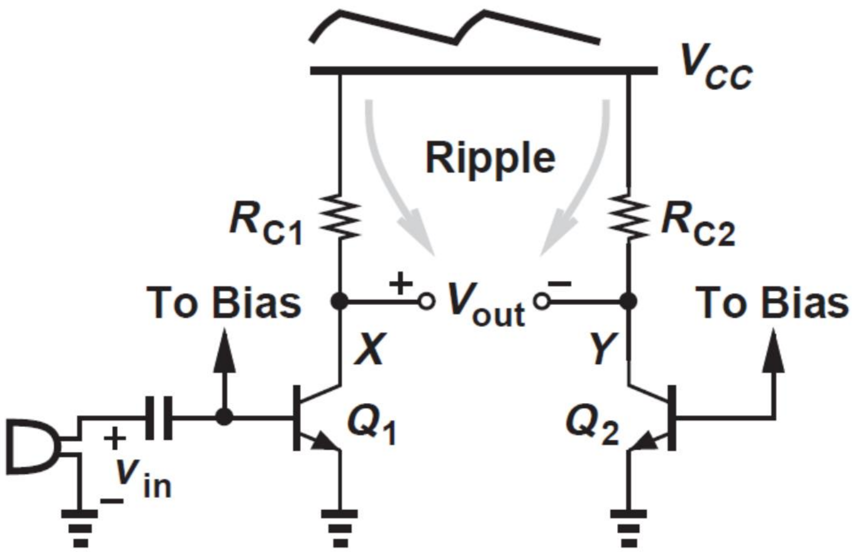
Having learned the design of rectifiers and basic amplifier stages, a student A constructed the circuit shown below to amplify the signal of a microphone. Unfortunately, the student observed that the amplifier output contained a strong humming noise, i.e. a steady low-frequency component.



Examining the output $V_{out} = V_{CC} - I_C R_C$, we can identify two components: (1) the **amplified microphone signal** and (2) the **ripple waveform** present on V_{CC}.

To suppress the hum, we can increase C_1, thus lowering the ripple amplitude. A potential issue is that the required capacitor value may become prohibitively large if many circuits draw current from the rectifier.

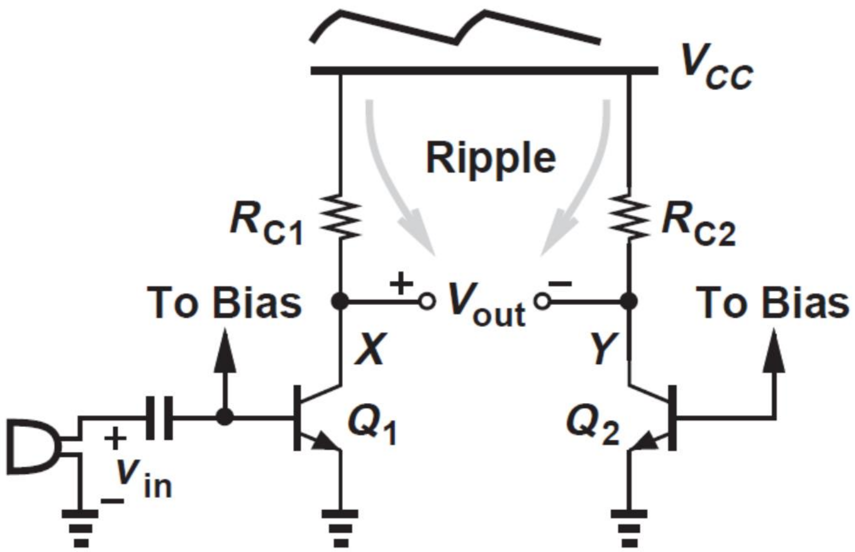
$$*\boldsymbol{V_r} \cong (\mathbf{V}_{\boldsymbol{P}} - 2\mathbf{V}_{\mathbf{on}})\left(\frac{\mathbf{T}}{\mathbf{2RC}}\right)$$



Or MOSFET M_1 and M_2 CS MOSFET stages

Alternatively, we can modify the amplifier topology such that the output is insensitive to V_{CC}. $V_{out} = V_{CC} - I_C R_C$ implies that a change in V_{CC} directly appears in V_{out}, fundamentally because both are measured **with respect to ground** and differ by $R_C I_C$.

What if V_{out} is measured with respect to another point that itself experiences the supply ripple to the same extent? It is thus possible to eliminate the ripple from the "net" output.



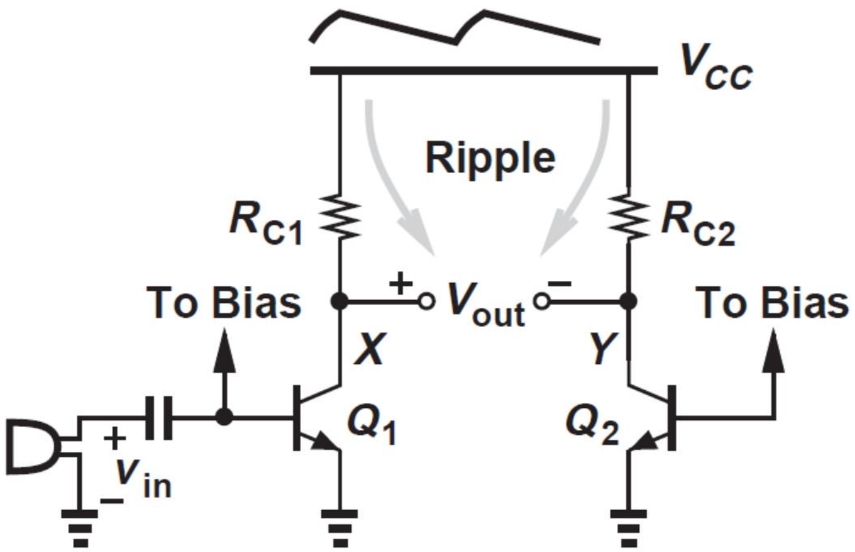
$$v_X = A_v v_{in} + v_r$$

$$v_Y = v_r .$$

$$v_X - v_Y = A_v v_{in} .$$

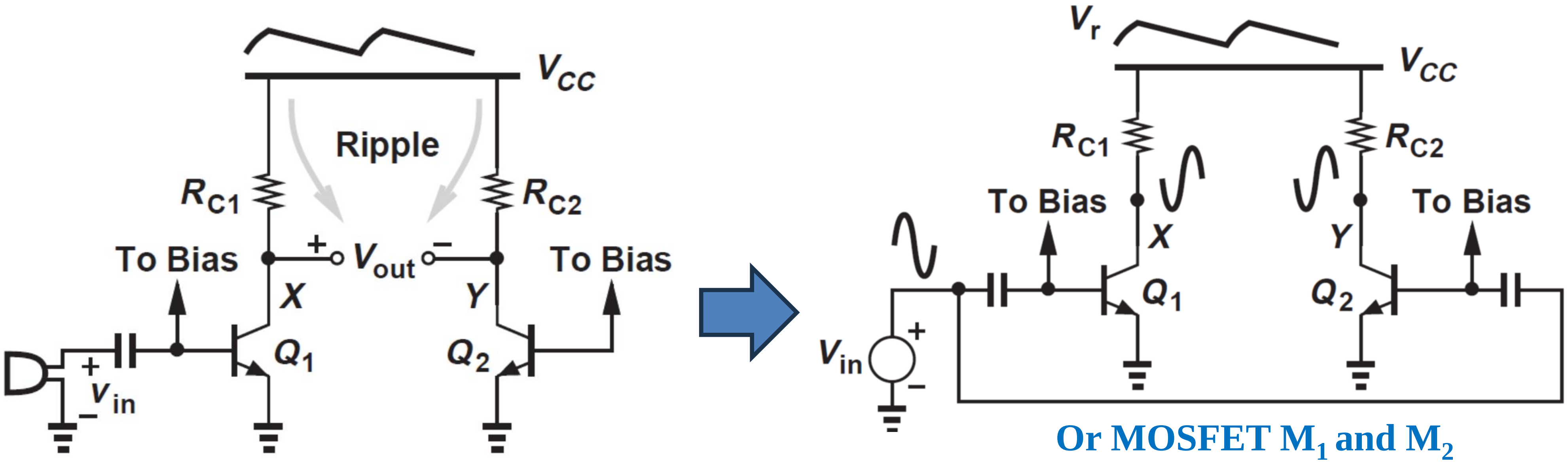
Or MOSFET M_1 and M_2

In the circuit above, the output is measured **between nodes *X* and *Y*** rather than from *X* to ground. If V_{CC} contains ripple (v_r), both V_X and V_Y rise and fall by the same amount and hence the ***difference* between V_X and V_Y remains free from the ripple (v_r)**.



Or MOSFET M_1 and M_2

However, Q_2 carries no signal, simply serving as a constant current source—improvement needed. Overall, the above development serves as the **foundation for differential amplifiers**: the **symmetric CE stages provide *two* output nodes** whose voltage difference remains free from the supply ripple.

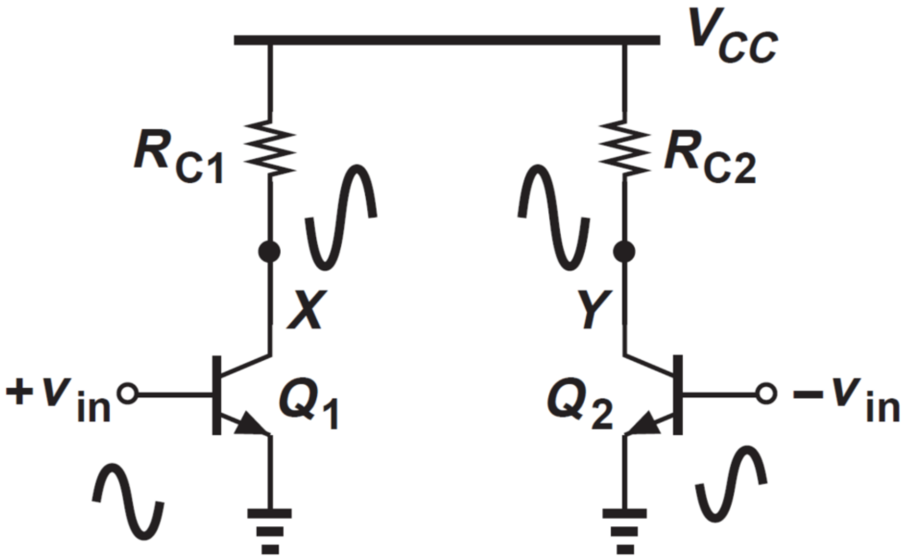


Or MOSFET M_1 and M_2

In the circuit left, Q_2 and R_{C2} remains idle, thereby wasting current. To improve the circuit, we apply the input signal to the base of Q_2. Unfortunately, the **signal components at *X* and *Y* are in phase**, canceling each other as they appear in $v_X - v_Y$.

$$v_X = A_v v_{in} + v_r; \quad v_Y = A_v v_{in} + v_r$$

Thus, $\mathbf{v_X - v_Y = 0.}$



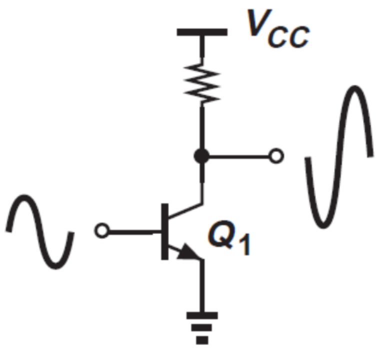
For the signal components to enhance each other at the output, we invert one of the input phases, obtaining

$$v_X = A_v v_{in} + v_r; \quad v_Y = -A_v v_{in} + v_r$$

$$\text{Thus, } \mathbf{v_X - v_Y = 2A_v v_{in}}$$

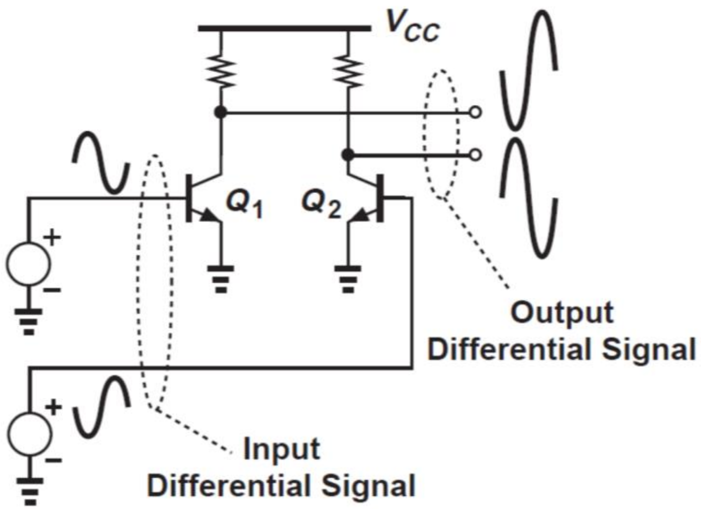
This topology provides twice the output swing by exploiting the amplification capability of the duplicate stage. The circuit senses two inputs that vary by equal and opposite amounts and generates two outputs that behave in a similar fashion. These waveforms are examples of **differential signals** and stand in contrast to single-ended signals.

Single-Ended *vs* Differential Signals



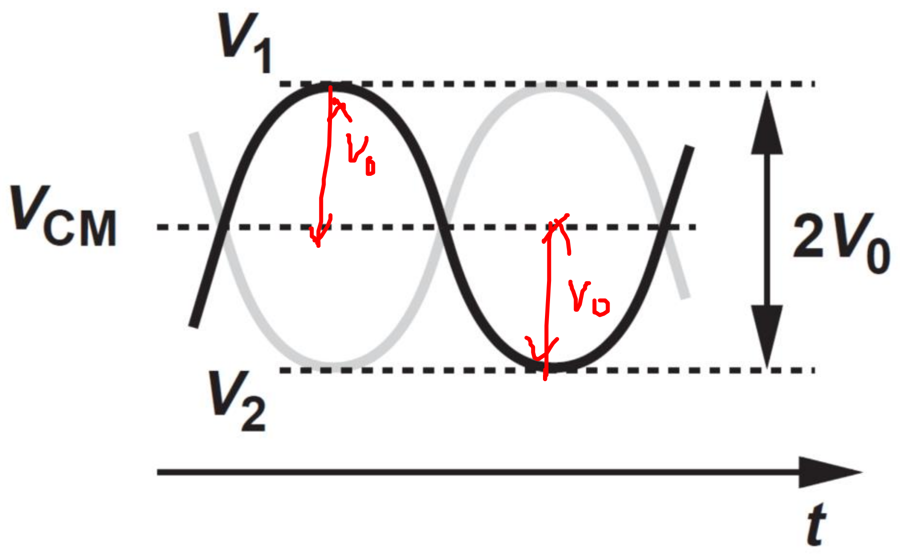
Or MOSFET M_1

A single-ended signal: Signal measured with respect to the common ground and carried by one line.



Or MOSFET M_1 and M_2

A differential signal: Signal measured between two nodes that have equal and opposite swings and is thus carried by two lines.

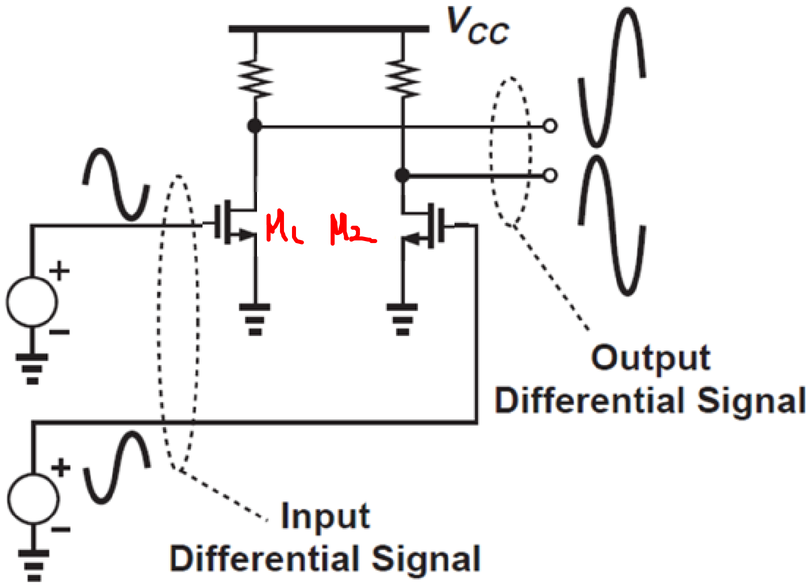


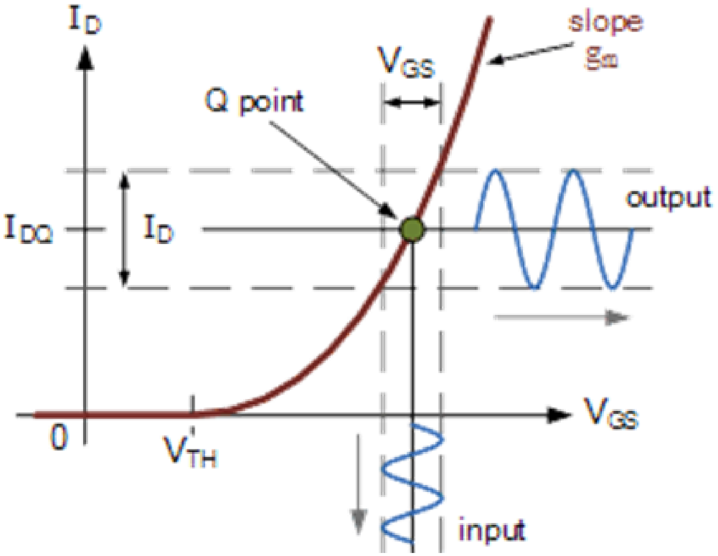
The differential signals V_1 and V_2 vary by equal and opposite amounts around a fixed level and have the same average (dc) level with respect to ground:

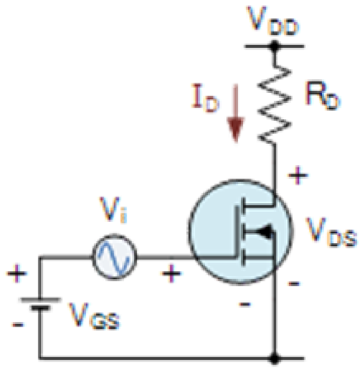
$$V_1 = V_0 \sin \omega t + V_{CM}; \qquad V_2 = -V_0 \sin\omega t + V_{CM}.$$

V_1 and V_2 has a peak-to-peak swing of $2V_0$ and thus the differential swing is $4V_0$.

V_{CM}, the dc voltage that is common to both V_1 and V_2, is called the **common-mode (CM) level**. That is, in the absence of differential signals, the two nodes remain at a potential equal to V_{CM} with respect to the global ground.

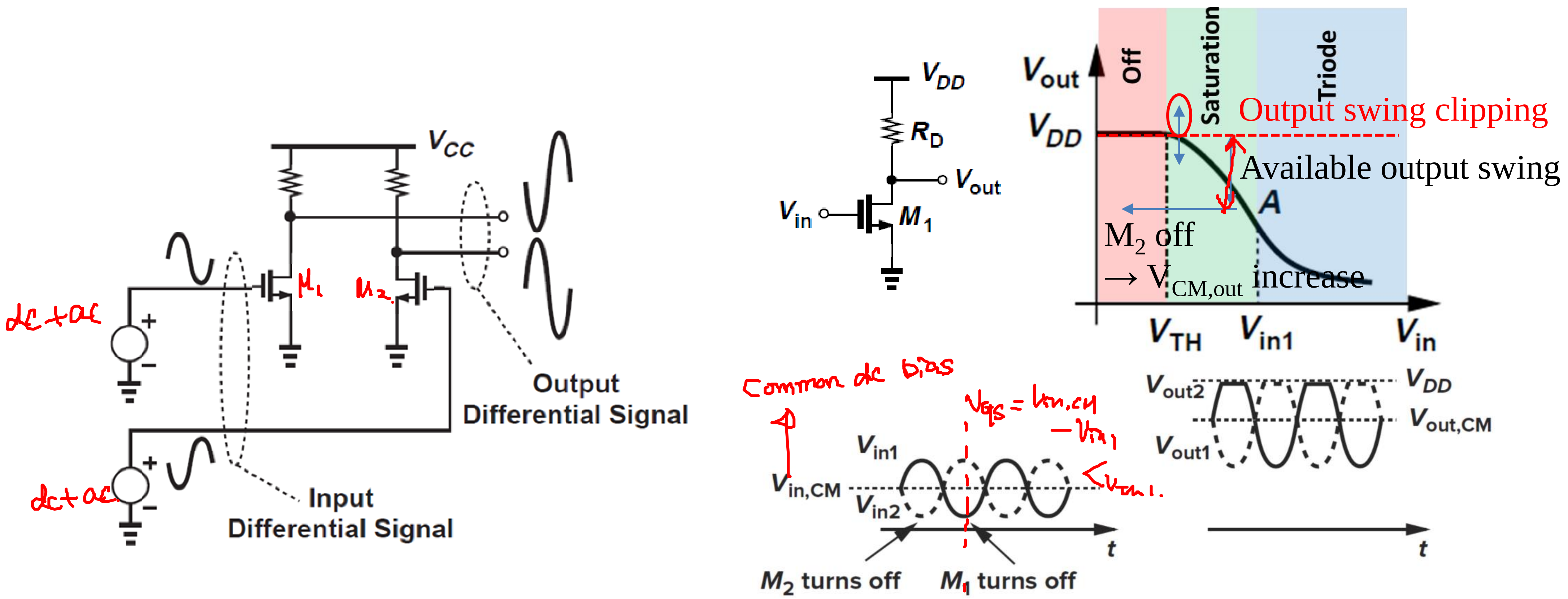






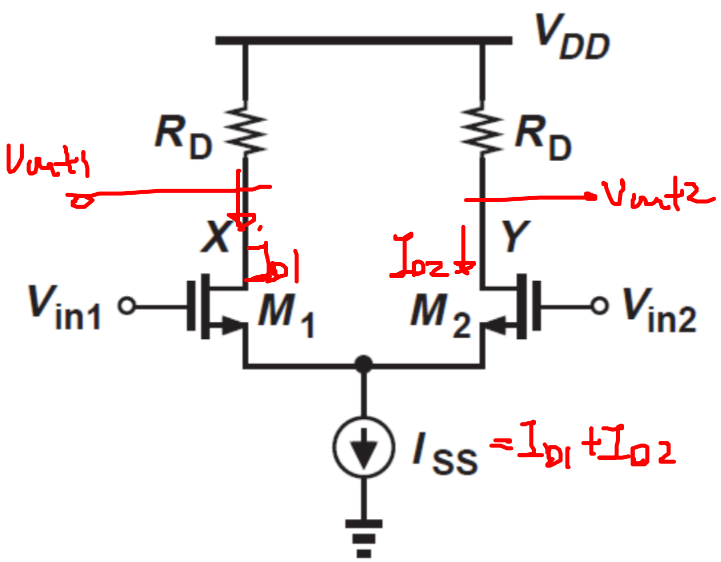
common dc bias.

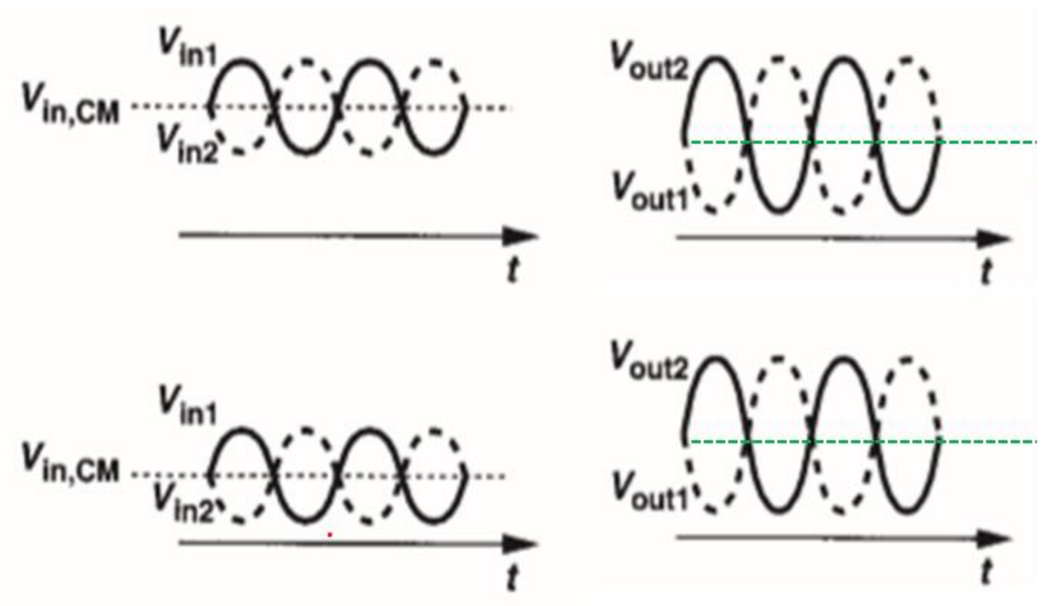
While sensing and producing differential signals, the circuit above suffers from some drawbacks: As the $V_{in,CM}$ changes, so do the bias currents of M_1 and M_2, thus varying both the transconductance of the devices and the $V_{out,CM}$ level. The variation of the transconductance, in turn, leads to a **change in the small-signal gain.**



Also, the departure of the $V_{out,CM}$ level from its ideal value lowers the maximum allowable output swings. For example, if the $V_{in,CM}$ is excessively low, the minimum values of V_{in1} and V_{in2} may in fact turn off M_1 and M_2, leading to severe **clipping at the output**. Thus, it is important that the bias currents of the devices have **minimal dependence on the $V_{in,CM}$.**

Differential Pair



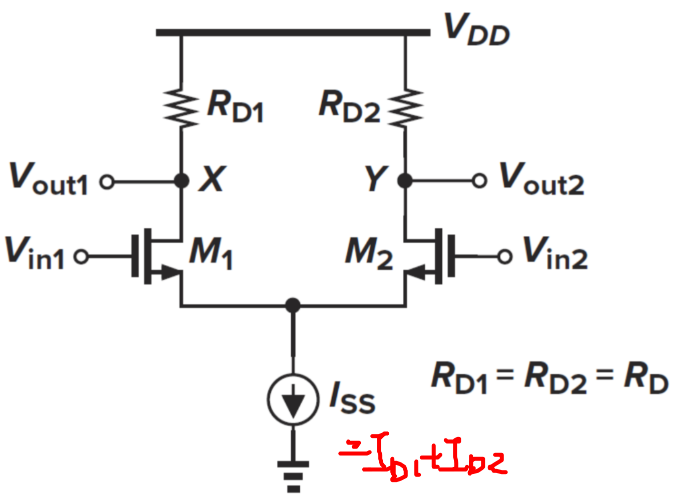


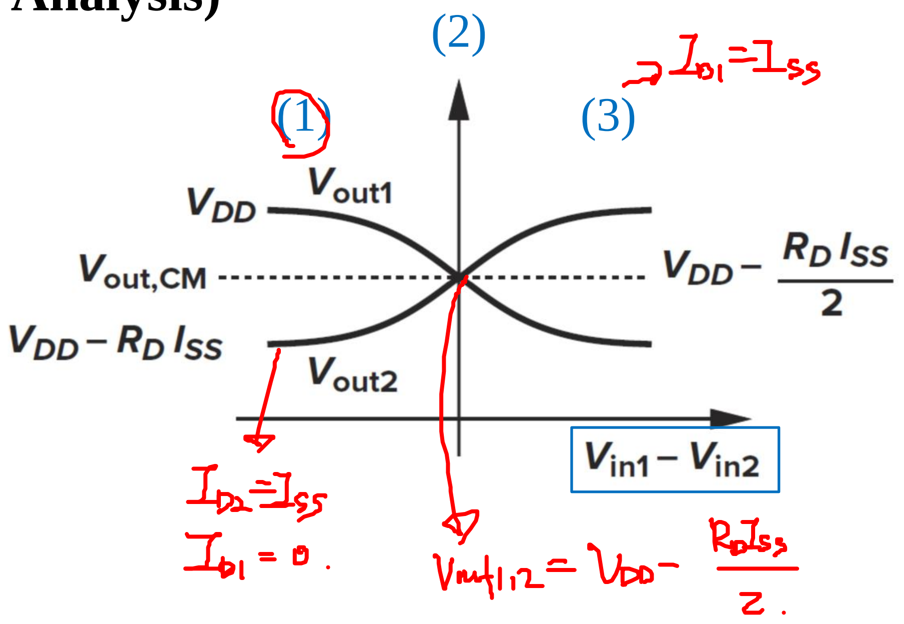
$$V_{out,CM} = V_{DD} - \frac{R_D I_{SS}}{2}$$

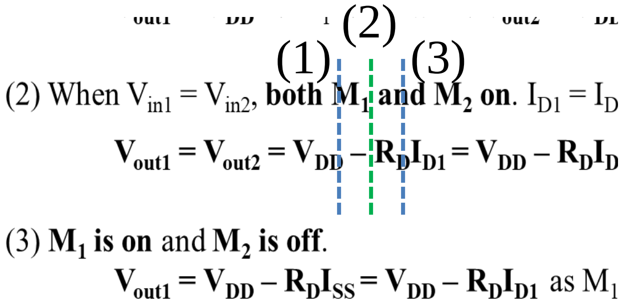
A simple modification yields an elegant, versatile topology. In a differential pair, the sources of M_1 and M_2 are **tied to a constant current source** rather than to ground. The sum of the transistor currents ($I_{D1} + I_{D2}$) is equal to the tail current I_{SS}, and are independent of $V_{in,CM}$.

Thus, if $V_{in1} = V_{in2}$, the bias current of each transistor equals $I_{SS}/2$ and the output common-mode level is $V_{DD} - R_D \times I_{SS}/2$.

Differential-Mode (Qualitative Analysis)







(1) **M_1 is off** and **M_2 is on** by assuming that V_{TH} is higher than the min. point

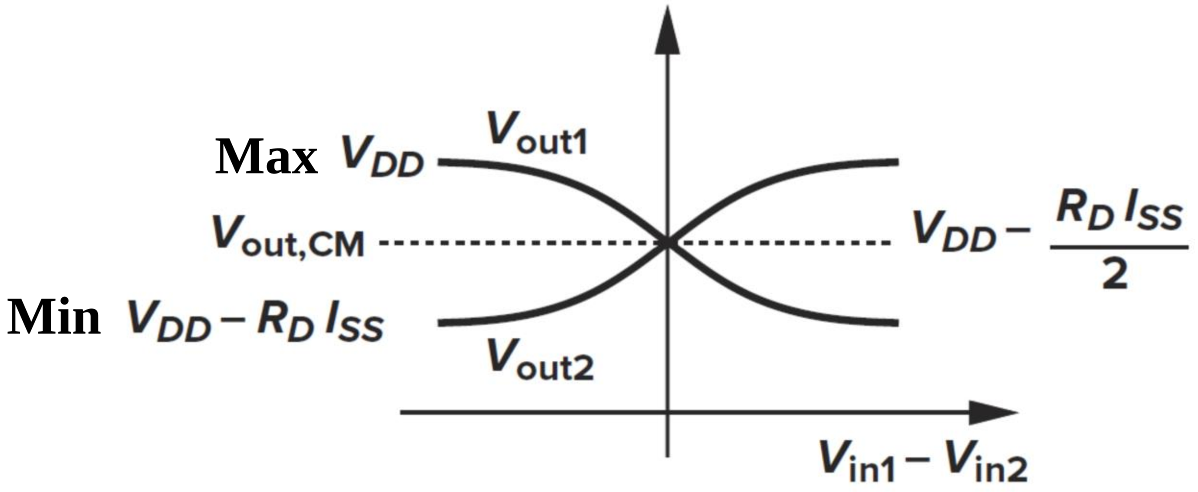
$$\mathbf{V_{out1} = V_{DD}} \text{ as } M_1 \text{ is off while } \mathbf{V_{out2} = V_{DD} - R_D I_{SS} = V_{DD} - R_D I_{D2}}$$

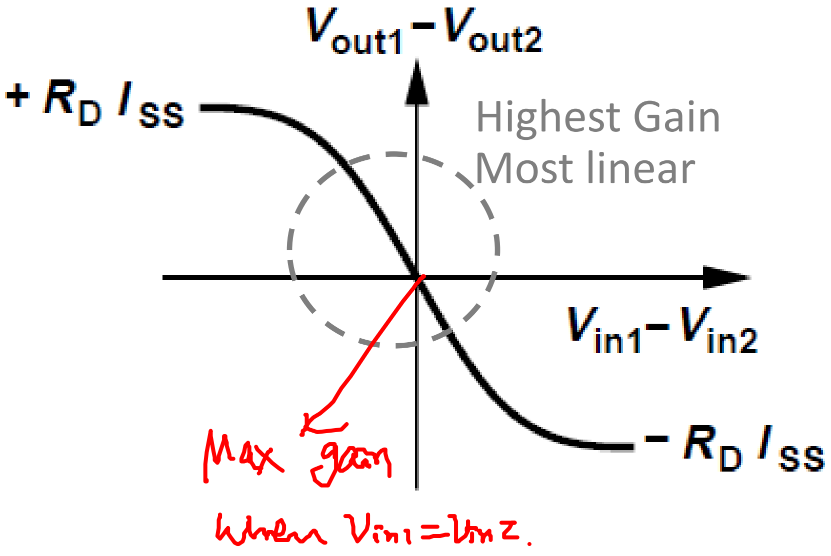
(2) When $V_{in1} = V_{in2}$, **both M_1 and M_2 on**. $I_{D1} = I_{D2} = \frac{1}{2} I_{SS}$

$$\mathbf{V_{out1} = V_{out2} = V_{DD} - R_D I_{D1} = V_{DD} - R_D I_{D2} = V_{DD} - \frac{1}{2} R_D I_{SS}}$$

(3) **M_1 is on** and **M_2 is off**.

$$\mathbf{V_{out1} = V_{DD} - R_D I_{SS} = V_{DD} - R_D I_{D1}} \text{ as } M_1 \text{ is off while } \mathbf{V_{out2} = V_{DD}}$$





(1) Maximum and minimum levels at the output are well-defined, V_{DD} and $V_{DD} - R_D I_{SS}$, and independent of the input CM level.

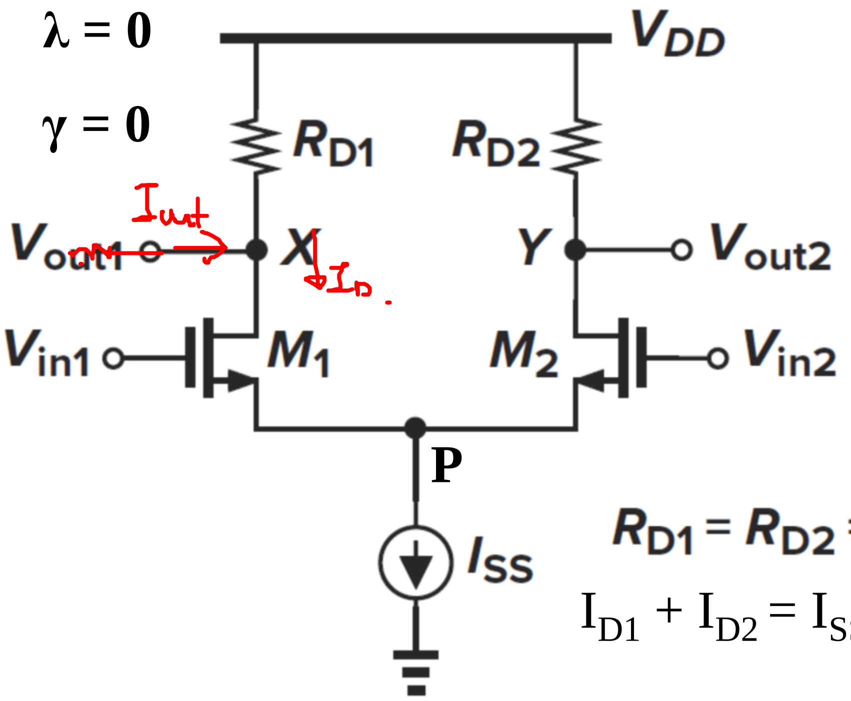
(2) The **small signal gain**, the slope of V_{out1}-V_{out2} *vs* V_{in1}-V_{in2}, is **maximum for $V_{in1} = V_{in2}$**.

Differential-Mode (Quantitative Analysis)

(1) DC Analysis

$\lambda = 0$

$\gamma = 0$



$R_{D1} = R_{D2} = R_D$

$I_{D1} + I_{D2} = I_{SS}$

$V_{out1} = V_{DD} - R_{D1} I_{D1}; V_{out2} = V_{DD} - R_{D2} I_{D2}$

Thus, $\mathbf{V_{out1} - V_{out2}} = R_{D2} I_{D2} - R_{D1} I_{D1} = \boldsymbol{R_D(I_{D2} - I_{D1})}$

Assuming that the circuit is symmetric, and M_1 and M_2 are in a **saturation region**,

$V_G = V_{in1}; V_S = V_P; V_{GS} = V_{in1} - V_P$

$\rightarrow V_P = V_{in1} - V_{GS1}$

And similarly, $V_P = V_{in2} - V_{GS2}$

Thus, $\mathbf{V_{in1} - V_{in2} = V_{GS1} - V_{GS2}}$

As $I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH})^2$, $V_{GS} = \sqrt{\frac{2I_D}{\mu_n C_{ox}\frac{W}{L}}} + V_{TH}$ and put it into V_{GS1} and V_{GS2}

$$V_{in1} - V_{in2} = V_{GS1} - V_{GS2} \text{ where } V_{GS} = \sqrt{\frac{2I_D}{\mu_n C_{ox}\frac{W}{L}}} + V_{TH}$$

$$= \sqrt{\frac{2I_{D1}}{\mu_n C_{ox}\frac{W}{L}}} - \sqrt{\frac{2I_{D2}}{\mu_n C_{ox}\frac{W}{L}}}$$

To find $I_{D1} – I_{D2}$, take square of the equation above

$$(V_{in1} - V_{in2})^2 = \frac{2I_{D1}}{\mu_n C_{ox}\frac{W}{L}} + \frac{2I_{D2}}{\mu_n C_{ox}\frac{W}{L}} - 2\frac{\sqrt{4I_{D1}I_{D2}}}{\mu_n C_{ox}\frac{W}{L}} = \frac{2}{\mu_n C_{ox}\frac{W}{L}}\left(I_{SS} - 2\sqrt{I_{D1}I_{D2}}\right)$$

$$\rightarrow \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in1} - V_{in2})^2 = I_{SS} - 2\sqrt{I_{D1}I_{D2}}$$

$$\rightarrow \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in1} - V_{in2})^2 - I_{SS} = -2\sqrt{I_{D1}I_{D2}}$$ take square of it again

$$\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in1}-V_{in2})^2 - I_{SS} = -2\sqrt{I_{D1}I_{D2}}$$ take square of it again

$$\Rightarrow \frac{1}{4}\left(\mu_n C_{ox}\frac{W}{L}\right)^2 (V_{in1}-V_{in2})^4 + {I_{SS}}^2 - \mu_n C_{ox}\frac{W}{L}(V_{in1}-V_{in2})^2 I_{SS} = 4I_{D1}I_{D2}$$

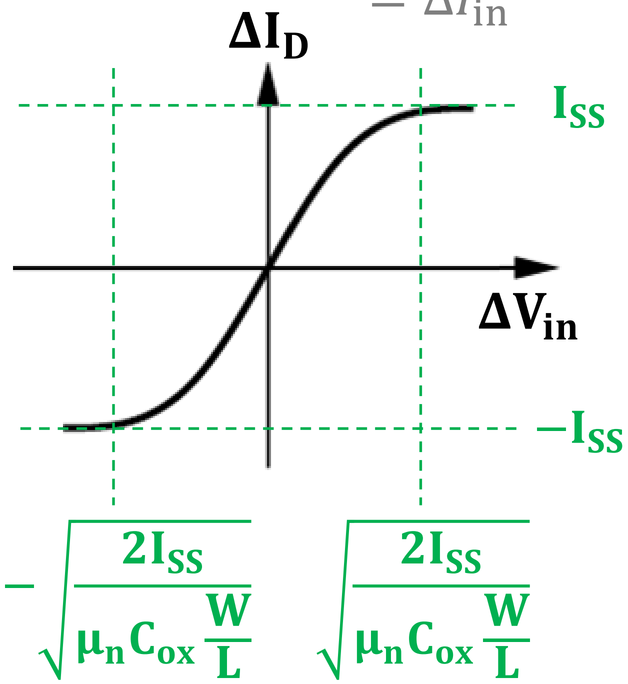
Where $4I_{D1}I_{D2} = \underbrace{\boxed{(I_{D1}+I_{D2})^2}}_{= I_{SS}} - (I_{D1}-I_{D2})^2$ from a quadratic equation

$$\frac{1}{4}\left(\mu_n C_{ox}\frac{W}{L}\right)^2 (\underbrace{\boxed{V_{in1}-V_{in2}}}_{=\Delta V_{in}})^4 + \cancel{I_{SS}}^2 - \mu_n C_{ox}\frac{W}{L}(\underbrace{\boxed{V_{in1}-V_{in2}}}_{=\Delta V_{in}})^2 I_{SS} = \cancel{I_{SS}}^2 - (\underbrace{\boxed{I_{D1}-I_{D2}}}_{=\Delta I_{in}})^2$$

$$\mathbf{I_{D1} - I_{D2} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\Delta V_{in}\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - {\Delta V_{in}}^2}}$$

If $V_{in1} = V_{in2}$, $I_{D1} - I_{D2} = 0$

If $|V_{in1} - V_{in2}|$ increases from zero, $I_{D1} - I_{D2}$ also increases



$\Delta I_D = G_m \Delta V_{in}$.

$$G_m = \frac{\partial \Delta I_D}{\partial \Delta V_{in}} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\frac{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - 2\Delta V_{in}^2}{\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - \Delta V_{in}^2}}$$

as I_D is I_{out} in the circuit

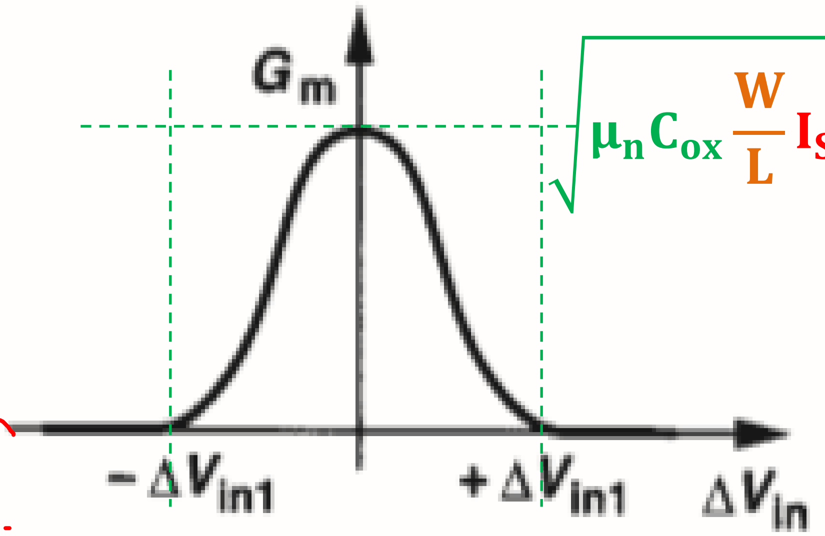
At $\Delta V_{in} = 0$, $G_m = \sqrt{\mu_n C_{ox}\frac{W}{L} I_{SS}}$

As $V_{out1} - V_{out2} = R_D \Delta I_D = R_D G_m \Delta V_{in}$ $\Rightarrow A_v = \frac{\Delta V_{out}}{\Delta V_{in}}$

ΔV_{out}

$\rightarrow |A_v| = \sqrt{\mu_n C_{ox}\frac{W}{L} I_{SS} R_D}$ $= G_m R_D$.

$$or\ A_{DM} = \frac{V_{out1} - V_{out2}}{\Delta V_{in}} = -g_{m1,2} R_D$$



$$\sqrt{\mu_n C_{ox}\frac{W}{L} I_{SS}} = g_{m1,2}$$

$$-\sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}} \qquad \sqrt{\frac{2I_{SS}}{\mu_n C_{ox}\frac{W}{L}}}$$

- Larger I_{SS} leads to higher G_m and wider input range.
- Smaller W/L leads to lower G_m but wider input range.

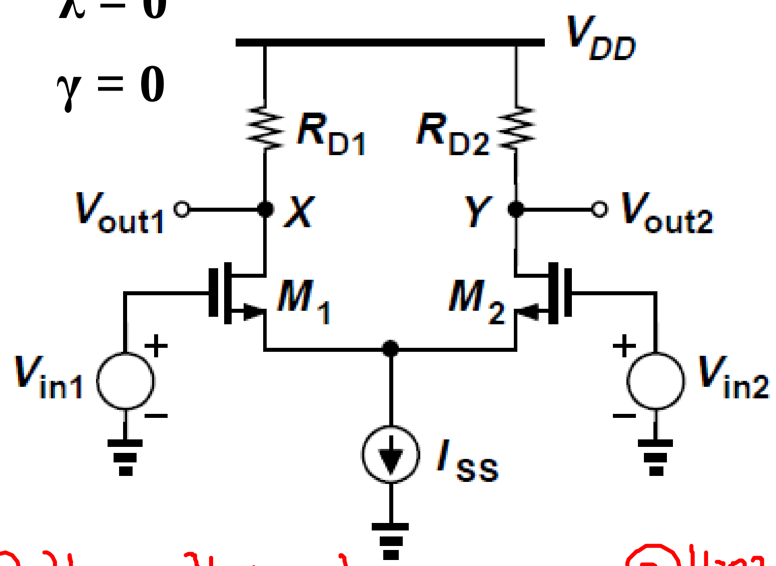
(2) Small-signal analysis – Method 1. Superposition

two inputs v_{in1}, v_{in2}.

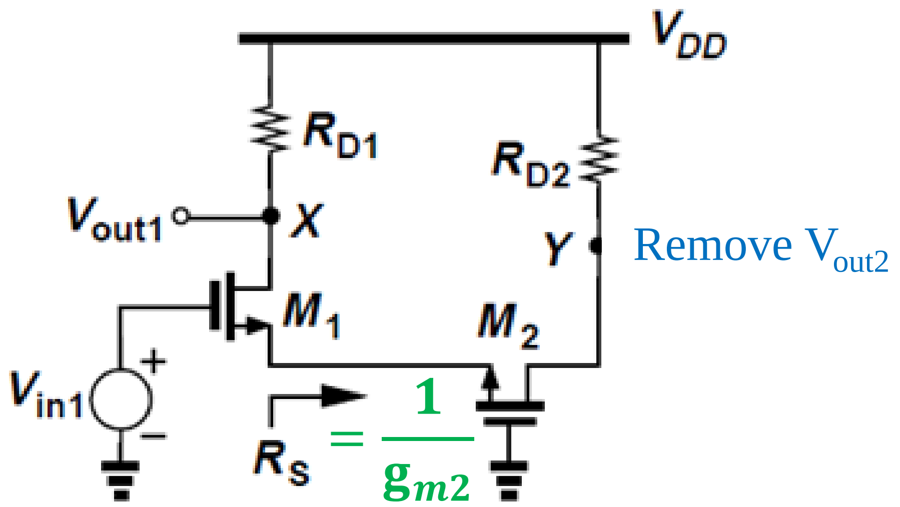
two outputs v_{out1}, v_{out2}.

$\lambda = 0$

$\gamma = 0$



Using a superposition method



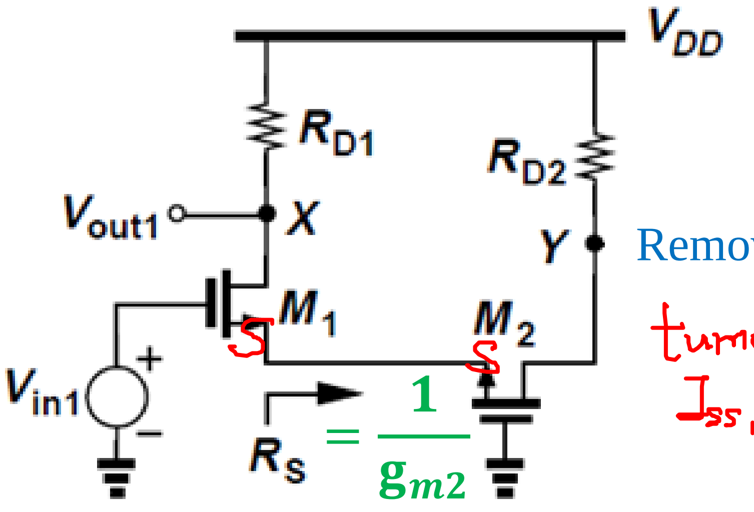
① v_{in1} — v_{out1}, v_{out2} ⇒ $v_X - v_Y$.

② v_{in2} — v_{out1}, v_{out2} ⇒ $v_X - v_Y$.

⇒ by superposition $v_X - v_Y = ① + ②$.

$$\upsilon_{out1}(v_X) = -\frac{R_D}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}}\upsilon_{in1}$$

Using a superposition method

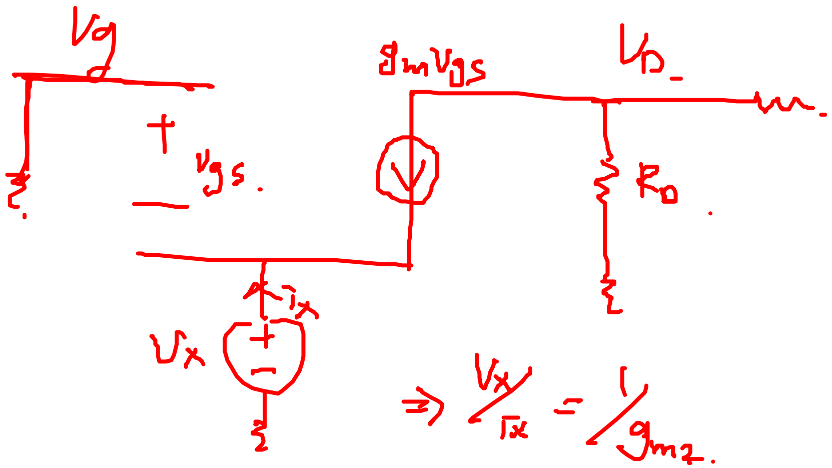


$R_S = \dfrac{1}{g_{m2}}$

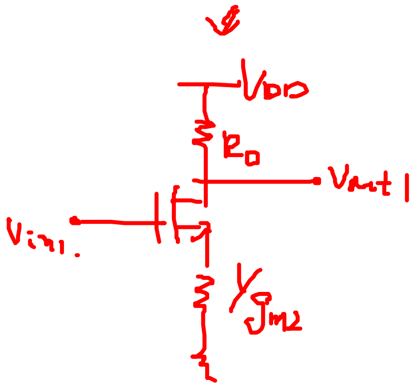
Remove V_{out2}

turned off I_{SS}, V_{in2}.

R_{Th} ⇒

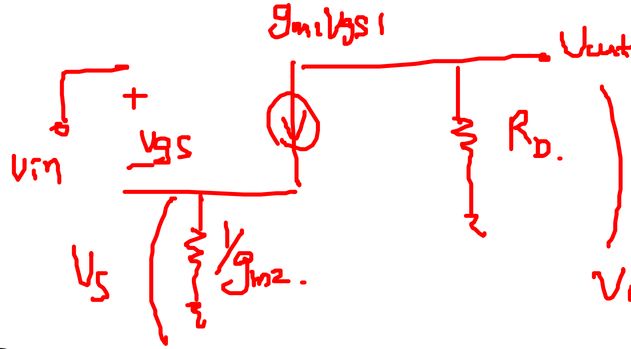


$\Rightarrow \dfrac{V_x}{I_x} = \dfrac{1}{g_{m2}}$



CS stage with source degeneration.

⇒



(1) $-V_{in} + V_{gs} + V_S = 0$

$V_S = \dfrac{g_{m1}}{g_{m2}} V_{gs1}$

$V_{gs1} = \dfrac{1}{1 + \dfrac{g_{m1}}{g_{m2}}} \times V_{in}$

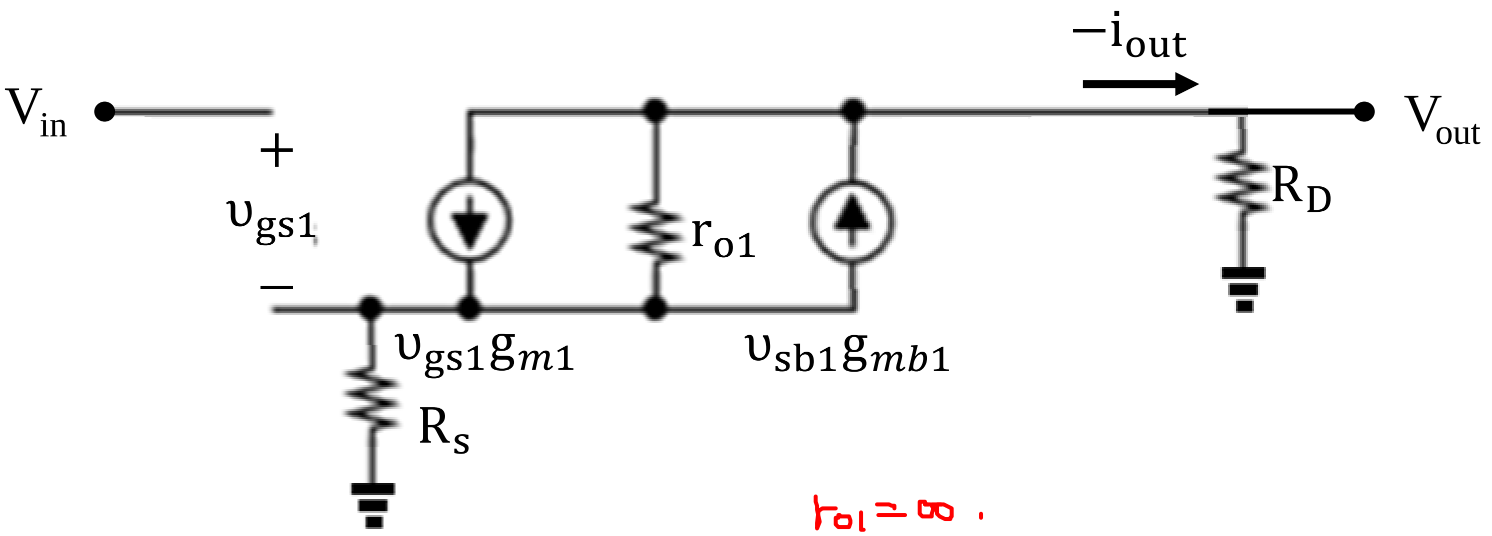
$V_{out1} = -R_D g_{m1} V_{gs1}$ ⇒ $A_v =$

$$v_{out1}(v_X) = -\frac{R_D}{\dfrac{1}{g_{m1}} + \dfrac{1}{g_{m2}}} v_{in1}$$

Recall: Gain of degenerated CS stage

$\lambda \neq 0$

$\gamma \neq 0$



$r_{o1} = \infty$.

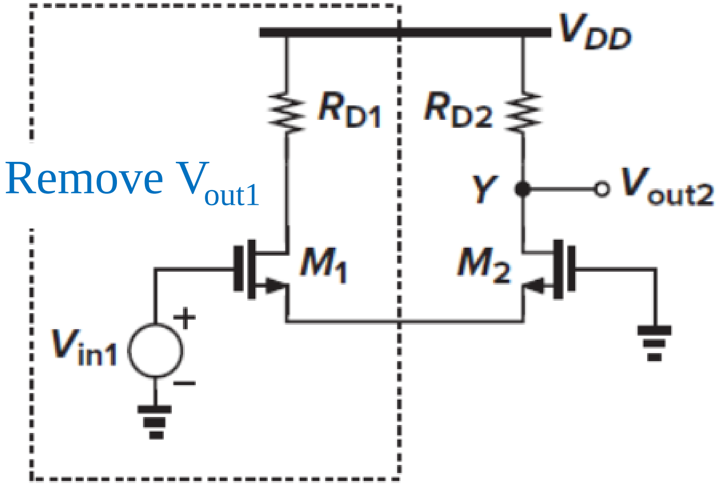
$g_{mb1} = 0$

$R_S = 1/g_{m2}$.

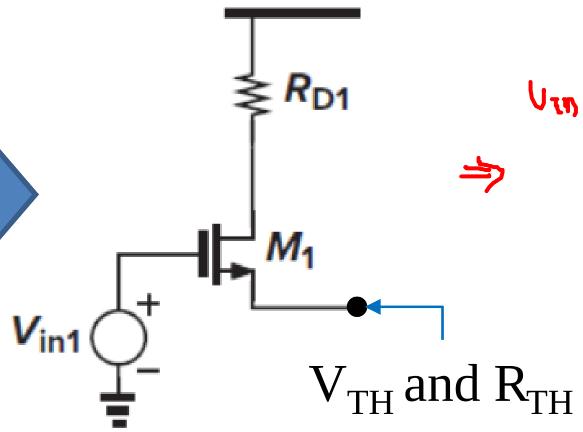
$$A_v = \frac{\upsilon_{out}}{\upsilon_{in}} = \frac{R_{out}(-i_{out})}{v_{in}} = -G_m R_{out}$$

$$= \frac{-g_{m1}\cancel{r_{o1}}}{R_S + r_{o1} + (g_{m1} + g_{mb1})r_{o1}R_S} \cdot \frac{[R_S + r_{o1} + (g_{m1} + g_{mb1})r_{o1}R_S]R_D}{[\cancel{R_S} + \underset{1}{\cancel{r_{o1}}} + (g_{m1} + \cancel{g_{mb1}})\cancel{r_{o1}}R_S] + \cancel{R_D}}$$

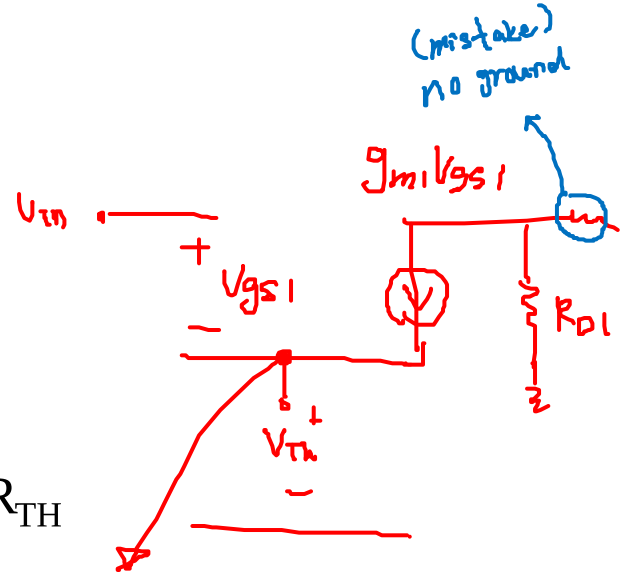
Using a superposition method



Find Thevenin Eq. circuit



⇒

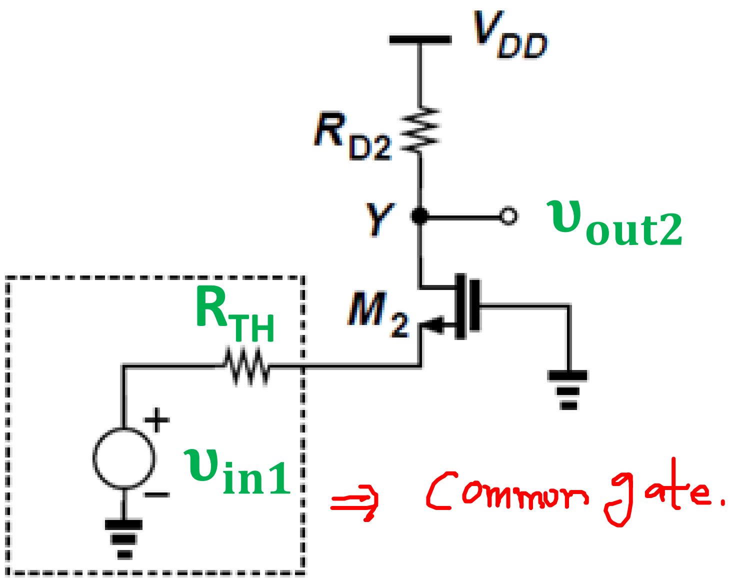


by KCL $g_{m1}V_{gs1} = 0$.

$V_{gs1} = 0$.

$V_{in1} = V_{th}$

Using the small-signal model, we can find $\mathbf{V_{TH}} = V_{in1}$ and $\mathbf{R_{TH}} = 1/g_{m1}$



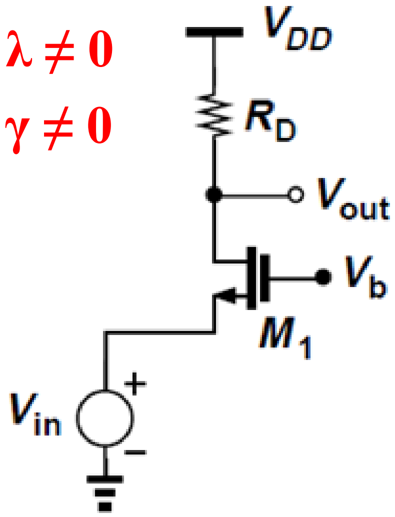
From the stage above, we get v_{out2}

$$\boldsymbol{v_{out2}}\ (\boldsymbol{v_Y}) = \frac{\boldsymbol{R_D}}{\frac{\mathbf{1}}{\mathbf{g_{m1}}} + \frac{\mathbf{1}}{\mathbf{g_{m2}}}} \boldsymbol{v_{in1}}$$

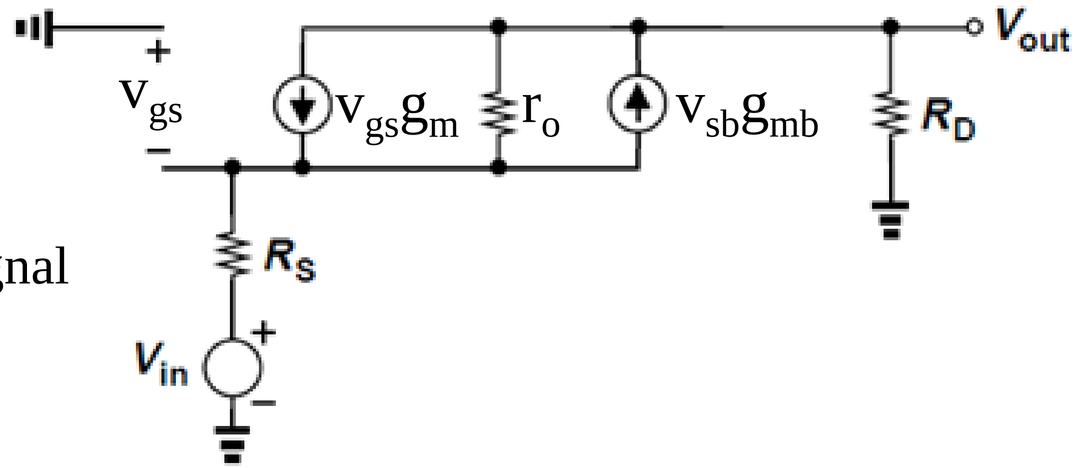
Recall: Common gate

$\lambda \neq 0$

$\gamma \neq 0$



Small Signal
$+ R_S$

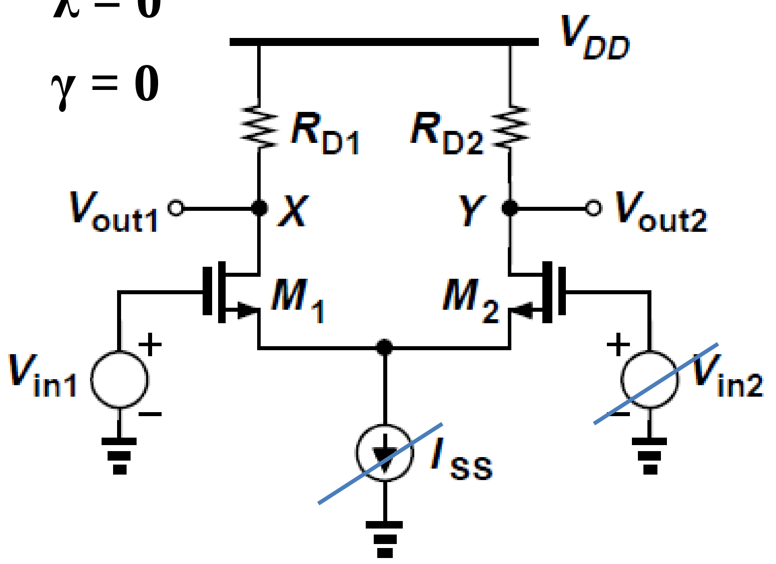


$$A_v = \frac{(g_m + g_{mb})r_o + 1}{r_o + R_S + (g_m + g_{mb})r_oR_S + R_D} R_D$$

$$\begin{cases} r_{o1} = \infty \\ g_{mb} = 0 \\ R_S = 1/g_{m1} \end{cases}$$

$\lambda = 0$

$\gamma = 0$



$R_{D1} = R_{D2} = R_D$

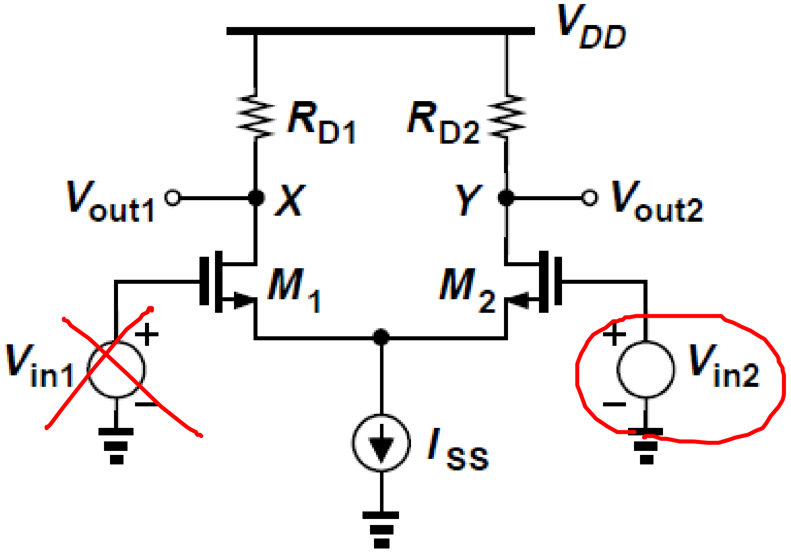
$g_{m1} = g_{m2} = g_m$.

$$v_{out1}(v_X) = -\frac{R_D}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}} v_{in1} \longrightarrow v_{out2}(v_Y) = \frac{R_D}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}} v_{in1}$$

And thus, **the overall voltage gain for v_{in1} is $v_X - v_Y$**

$$= -\frac{2R_D}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}} v_{in1} = -g_m R_D v_{in1}$$

if $g_{m1} = g_{m2} = g_m$



$$\upsilon_{out1}-\upsilon_{out2}=-\frac{2R_D}{\frac{1}{g_{m1}}+\frac{1}{g_{m2}}}\upsilon_{in1}=-g_mR_D\boldsymbol{v_{in1}}$$

$V_{out2}-V_{in2} \Rightarrow$ CS stage

$V_{out1}-V_{in2} \Rightarrow$ CG stage.

By the virtue of symmetry, the effect of v_{in2} at X and Y is identical to that of v_{in1} **except for a change in the polarities**.

superposition.

$$\upsilon_{out1}-\upsilon_{out2}=\frac{2R_D}{\frac{1}{g_{m1}}+\frac{1}{g_{m2}}}\upsilon_{in2}={+}g_mR_D\boldsymbol{v_{in2}}$$

$V_{out1}-V_{out2}=-g_mR_DV_{in1}+g_mR_DV_{in2}.$

$$\mathbf{A_{DM}}=\frac{\boldsymbol{\upsilon_{out1}-\upsilon_{out2}}}{\boldsymbol{\upsilon_{in1}-\upsilon_{in2}}}=\boldsymbol{-g_mR_D}$$

Example 1 Calculate the A_{DM} of the differential pair below if the biasing conditions of M_1 and M_2 are the same.

$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{th})^2$

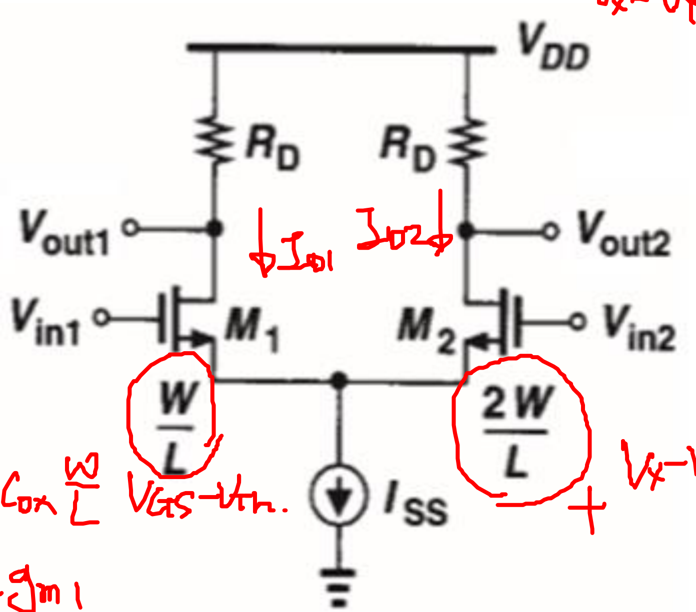
$I_{D1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1(V_{GS}-V_{th})^2$

$I_{D2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2$ ''

$g_m = \sqrt{2\mu_n C_{ox}\frac{W}{L}I_D}$.

$g_{m1} = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_1 I_{D1}} = \mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{th})$.

$g_{m2} = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_2 I_{D2}} = 2\times g_{m1}$



$V_X - V_Y = \frac{-2R_D}{\frac{1}{g_{m1}}+\frac{1}{g_{m2}}}v_{in1}$

$= \frac{-2R_D}{\frac{1}{g_{m1}}+\frac{1}{2g_{m1}}}v_{in1}$ ①

$+ \quad V_X - V_Y = \frac{2R_D}{\frac{1}{g_{m1}}+\frac{1}{2g_{m1}}}v_{in2}$ ②

$V_X - V_Y = ① + ②$

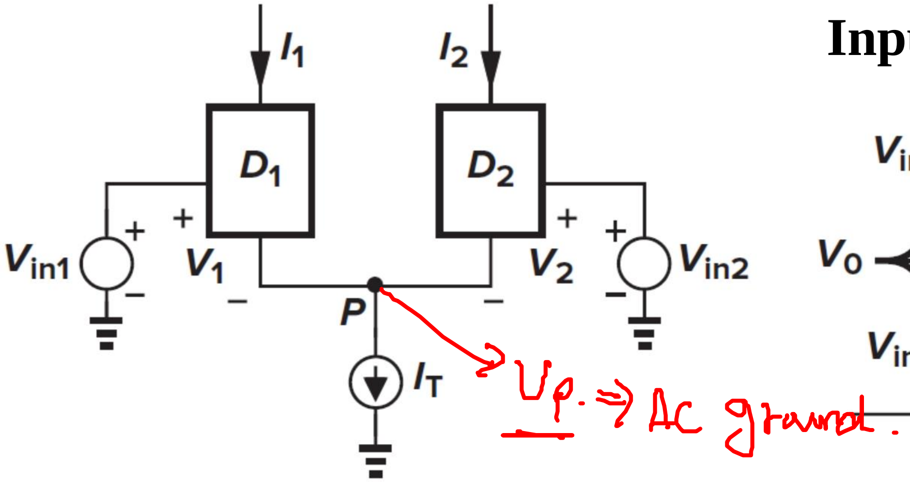
$$\upsilon_{out1}-\upsilon_{out2} = -\frac{2R_D}{\frac{1}{g_{m1}}+\frac{1}{2g_{m1}}}\upsilon_{in1} = -\frac{4}{3}g_{m1}R_D\upsilon_{in1}$$

$$\upsilon_{out1}-\upsilon_{out2} = \frac{2R_D}{\frac{1}{g_{m1}}+\frac{1}{2g_{m1}}}\upsilon_{in2} = \frac{4}{3}g_{m1}R_D\upsilon_{in2}$$

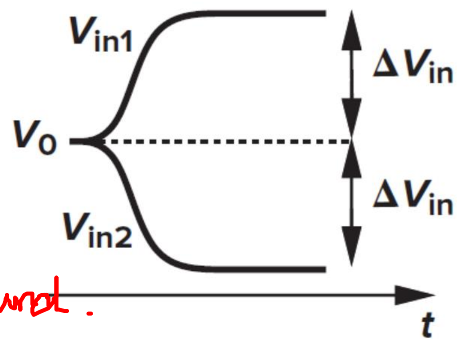
$$\mathbf{A_{DM}} = \frac{\upsilon_{out1}-\upsilon_{out2}}{\upsilon_{in1}-\upsilon_{in2}} = -\frac{4}{3}g_{m1}R_D$$

(2) Small-signal analysis – Method 2. Half circuit

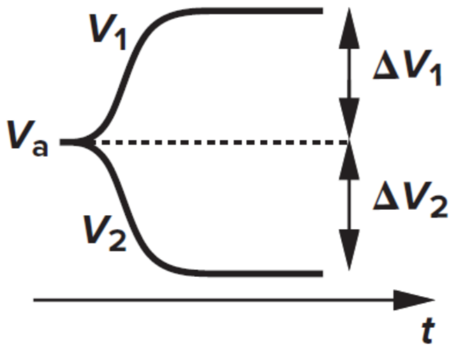
If a fully-symmetric differential pair senses differential inputs, then the concept of ‘half circuit’ can be applied.



Input V_G changes

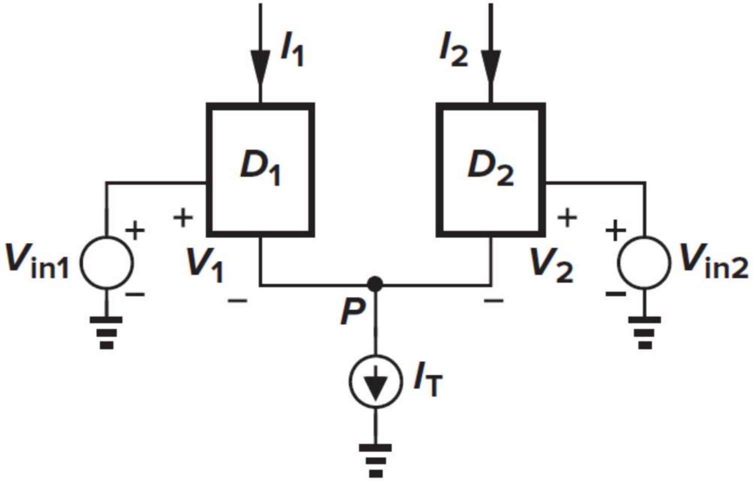


Corresponding V_{GS} changes

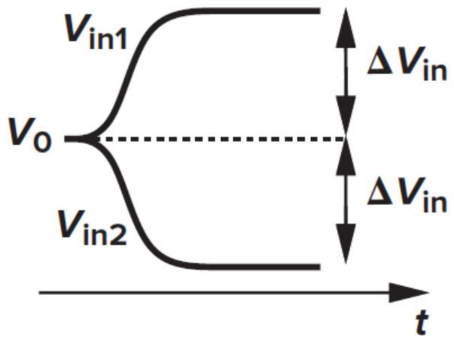


Lemma. D_1 and D_2 represent any three terminal active device. Suppose V_{in1} and V_{in2} change differentially, the former from V_0 to $V_0 + \Delta V_{in}$ and the latter from V_0 to $V_0 - \Delta V_{in}$. Then, if the circuit remains linear, **V_P does not change**. Assume $\lambda = 0$.

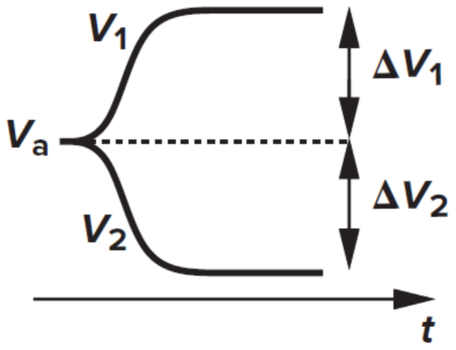
***Lemma proof**



Input V_G changes



Corresponding V_{GS} changes



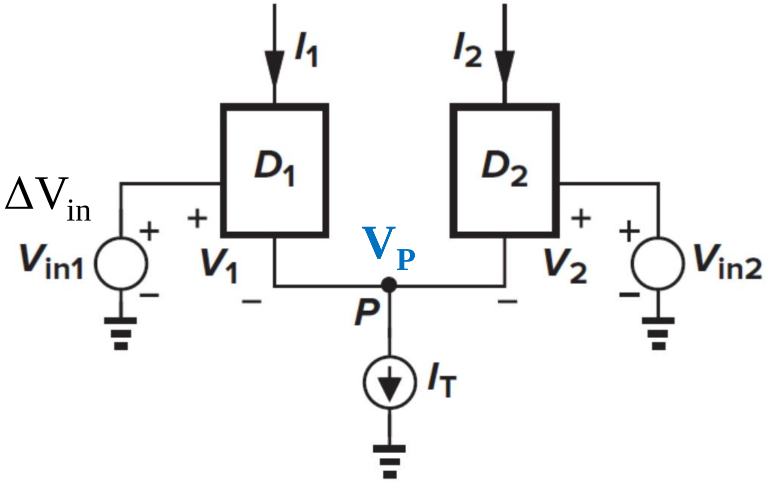
V_1 (V_{GS1}) = $\mathbf{V_a} + \Delta V_1$, V_2 (V_{GS2}) = $\mathbf{V_a} - \Delta V_2$. Then, the output currents change by $g_m \Delta V_1$ (ΔI_1) and $-\, g_m \Delta V_2$ (ΔI_2).

Since the current is constant $I_1 + I_2 = I_T$, the sum of changes $g_m \Delta V_1 - g_m \Delta V_2 = 0$

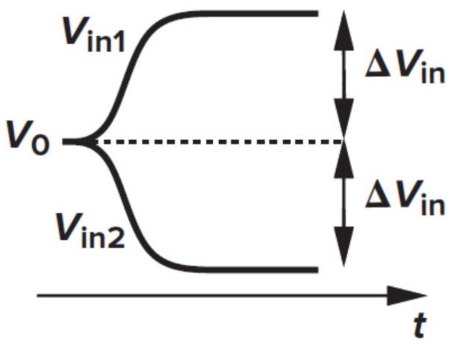
$$\rightarrow \mathbf{\Delta V_1 = \Delta V_2}\ .$$

***Lemma proof (continue)**

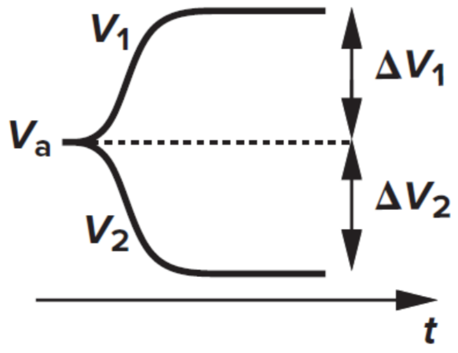
$V_{in1} = V_0 + \Delta V_{in}$



Input V_G changes



Corresponding V_{GS} changes



V_G V_S V_{GS} Common

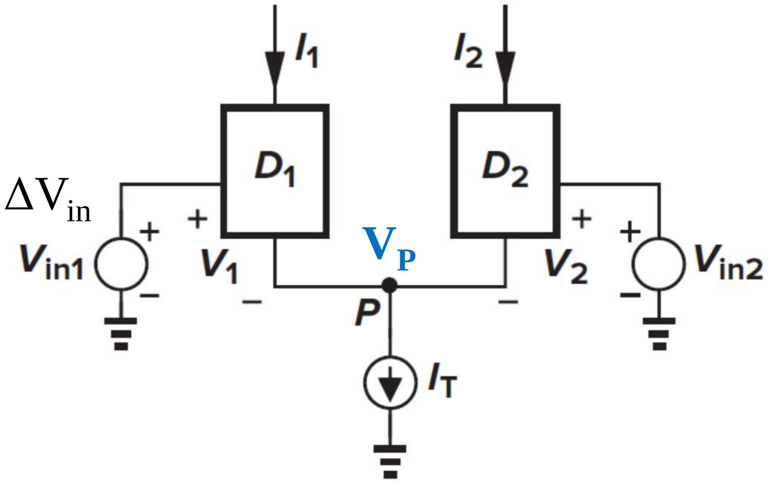
Because $V_{in1} - V_p = V_1$ and $V_{in2} - V_p = V_2$, we get $V_{in1} - V_1 = V_{in2} - V_2$

From the bias condition we have, (i) $V_{in1} = V_0 + \Delta V_{in}$; (ii) $V_{in2} = V_0 - \Delta V_{in}$; (iii) $V_1 = V_a + \Delta V_1$; and (iv) $V_2 = V_a - \Delta V_2$. Insert these into the equation in green $V_{in1} - V_1 = V_{in2} - V_2$

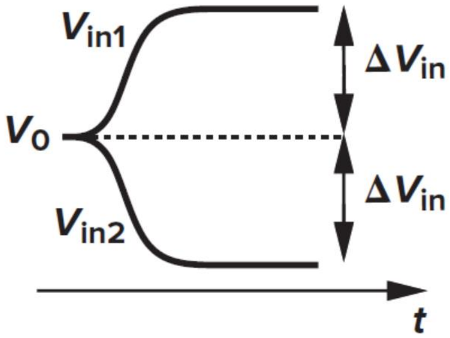
we get $V_0 + \Delta V_{in} - (V_a + \Delta V_1) = V_0 - \Delta V_{in} - (V_a - \Delta V_2)$.

***Lemma proof (continue)**

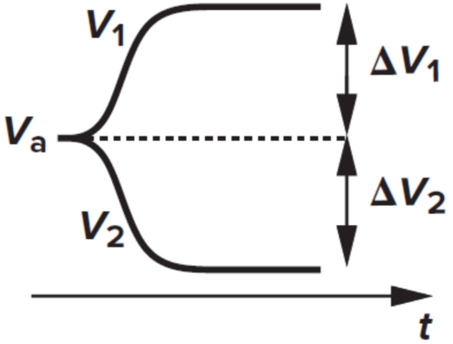
$V_{in1} = V_0 + \Delta V_{in}$



Input V_G changes



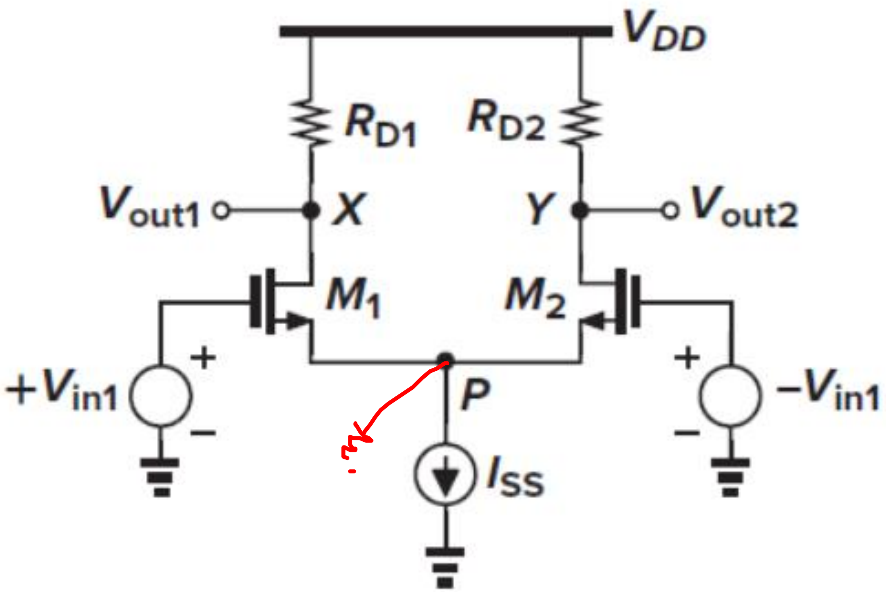
Corresponding V_{GS} changes

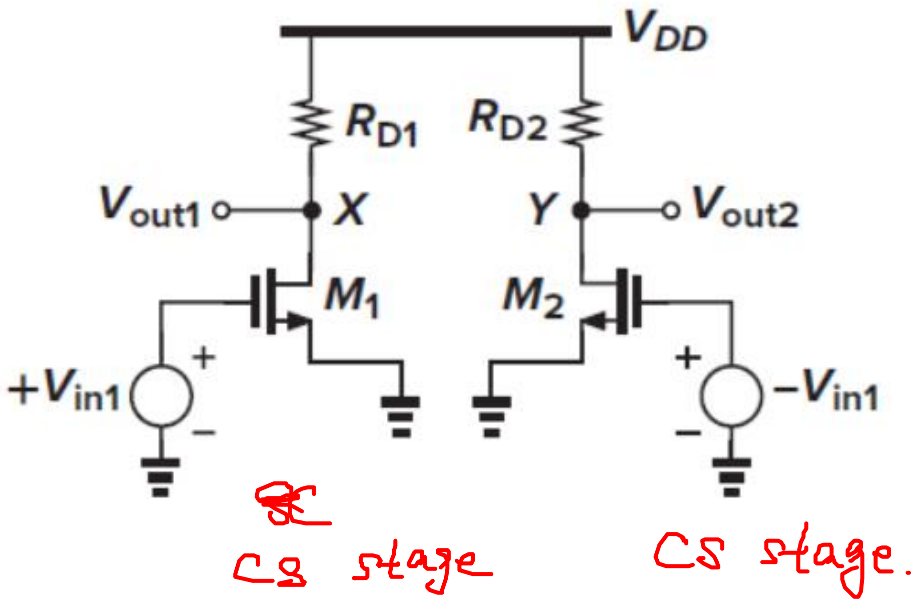


From $\cancel{V_0} + \Delta V_{in} - (\cancel{V_a} + \Delta V_1) = \cancel{V_0} - \Delta V_{in} - (\cancel{V_a} - \Delta V_2)$,

$2\Delta V_{in} = \Delta V_1 + \Delta V_2 = 2\Delta V_1$. In other words, if V_{in1} and V_{in2} change by $+\Delta V_{in}$ and $-\Delta V_{in}$, respectively, then V_1 and V_2 change by the same values. Since $V_P = V_{in1} - V_1$, and since V_1 exhibits the same change as V_{in1}, **V_P does not change**.

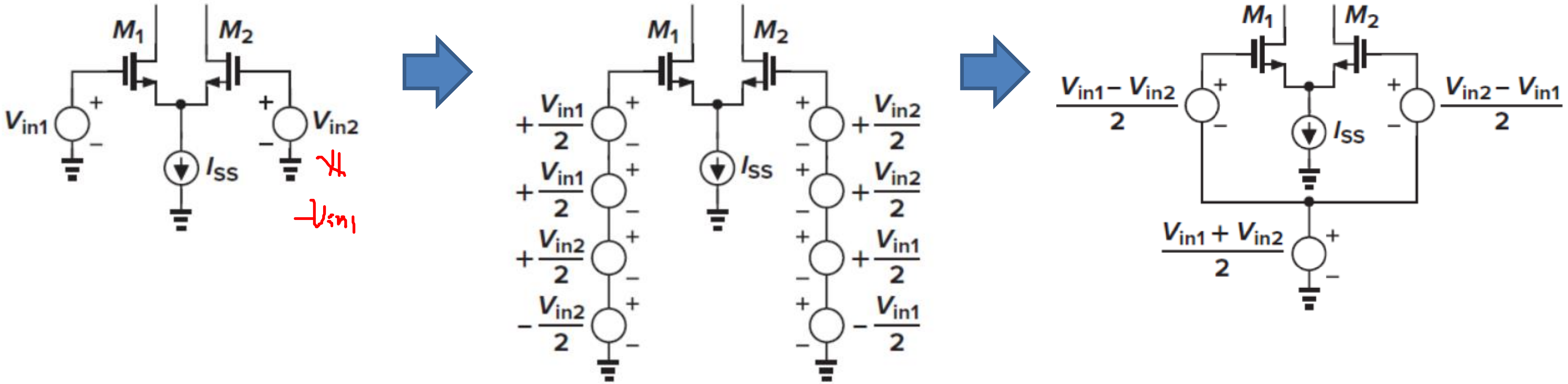
Since V_P experiences no change, node P can be considered **ac ground**, and the circuit can be decomposed into **two separate halves**.





We can write $V_X/V_{in1} = -g_mR_D$ and $V_Y /(-V_{in1}) = -g_mR_D$. Thus, $\mathbf{(V_X - V_Y)/(2V_{in1}) = -g_mR_D}$.

If two inputs are **not fully differential**, we can adjust inputs as shown below.

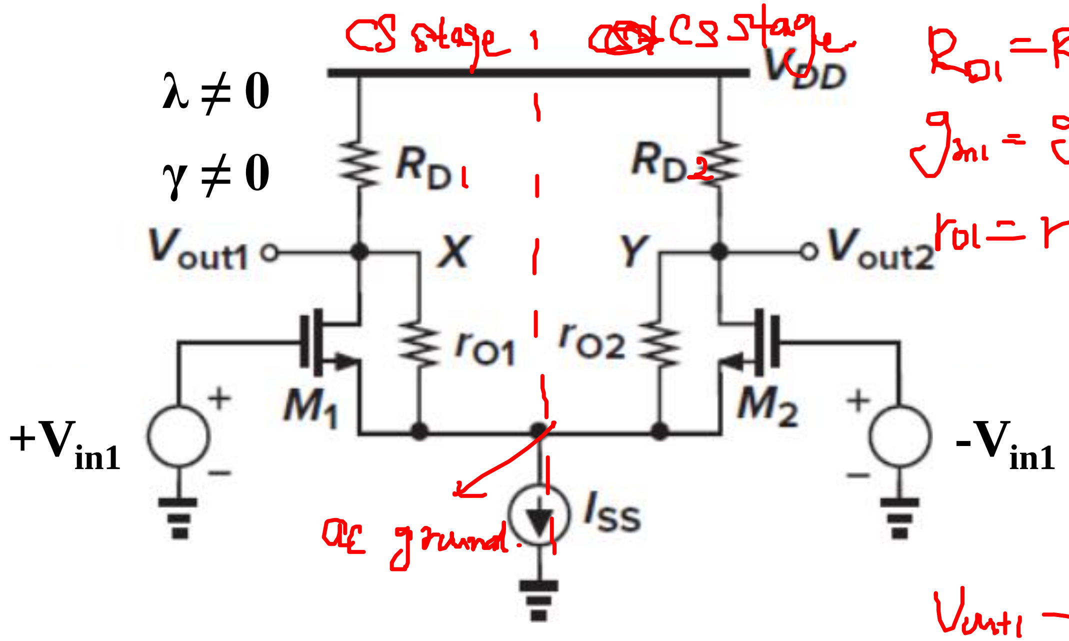


the circuit senses **a combination of a differential input and a common-mode variation**. Therefore, the effect of each type of input can be **computed by superposition**, with the half-circuit concept applied to the differential-mode operation.

Example 2 Calculate the differential gain of the circuit below.

M1.

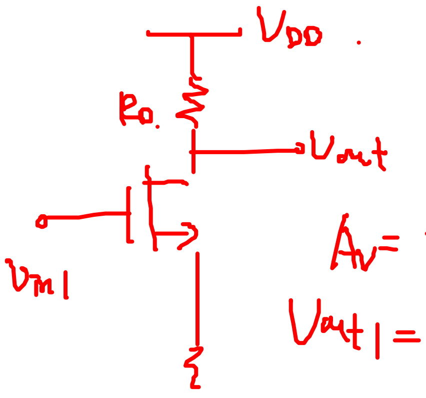
Differential-Mode Response, Input is symmetry



$R_{D1} = R_{D2} = R_D$

$g_{m1} = g_{m2} = g_m$

$r_{o1} = r_{o2} = r_o$.



$A_v = -g_m(R_D \parallel r_o)$

$V_{out1} = -g_m (R_D \parallel r_o) V_{in1}$

$V_{out2} = g_m (R_D \parallel r_o) V_{in1}$.

$V_{out1} - V_{out2} = -g_m (R_D \parallel r_o)(V_{in1} + V_{in1})$

$$\mathbf{A_{DM}} = \frac{\boldsymbol{\upsilon}_{\mathbf{out1}} - \boldsymbol{\upsilon}_{\mathbf{out2}}}{\mathbf{2}\boldsymbol{\upsilon}_{\boldsymbol{in1}}} = -\mathbf{g_m}(\mathbf{R_D} \parallel \mathbf{r_o})$$

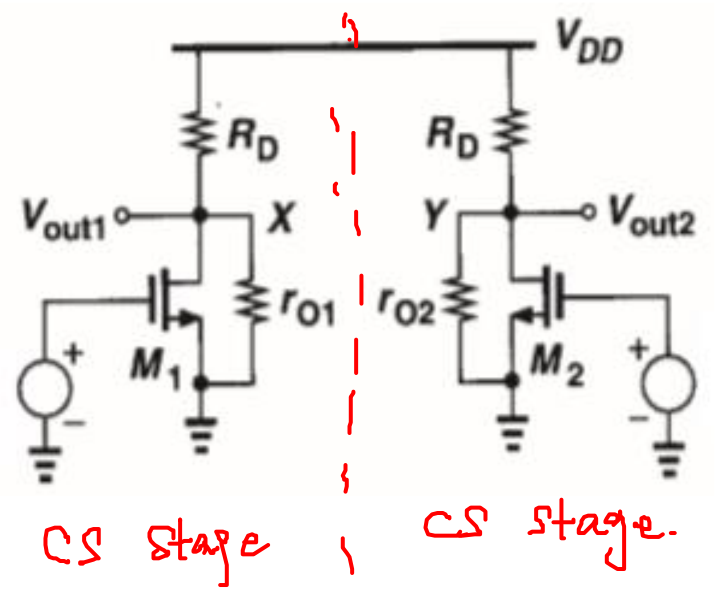
Example 2 Calculate the differential gain of the circuit below.

Differential-Mode Response, **Input is asymmetry**

$\lambda \neq 0$

$\gamma \neq 0$

$$\frac{\upsilon_{in1} - \upsilon_{in2}}{2}$$



$$\frac{\upsilon_{in2} - \upsilon_{in1}}{2}$$

$$V_{out1} = -g_m(R_D \| r_o)\frac{V_{in1} - V_{in2}}{2}.$$

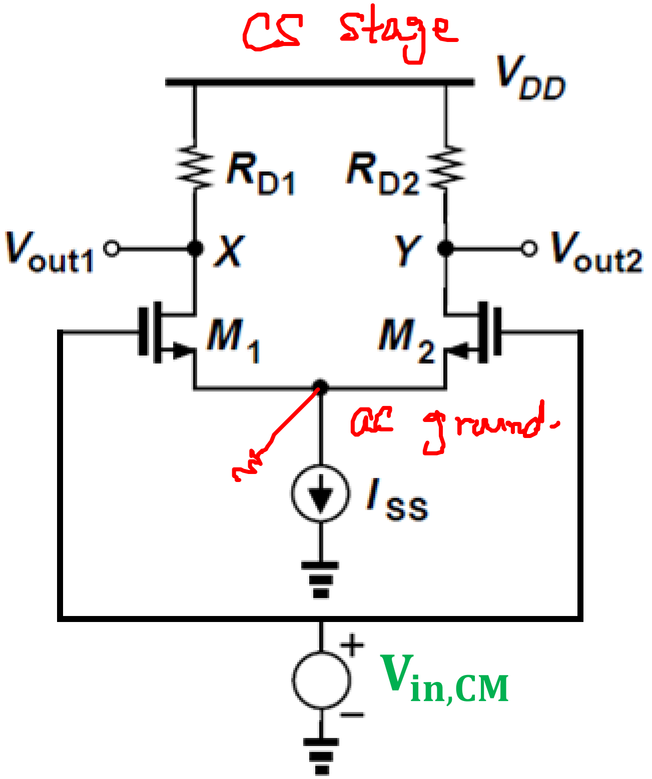
$$V_{out2} = +g_m(R_D \| r_o)\frac{-V_{in2} + V_{in1}}{2}$$

$$V_{out1} - V_{out2} = -g_m(R_D \| r_o)(V_{in1} - V_{in2})$$

$$\mathbf{A_{DM}} = \frac{\boldsymbol{\upsilon_{out1} - \upsilon_{out2}}}{\boldsymbol{\upsilon_{in1} - \upsilon_{in2}}} = -\mathbf{g_m}(\mathbf{R_D} \| \mathbf{r_o})$$

Example 2 Calculate the differential gain of the circuit below.

Common-Mode Response (ideal)



$V_{out1} = -g_m (R_D \| r_o) V_{in,CM}.$

$V_{out2} = -g_m (R_D \| r_o) V_{in,CM}.$

gain = 0.

If the circuit is fully symmetric, the current drawn by M_1 and M_2 is $I_{SS}/2$ and is independent of $V_{in,CM}$. This means that V_{out1} and V_{out2} do not change by $V_{in,CM}$, leading to zero gain.

$$\mathbf{A_{CM-DM} = \frac{\upsilon_{out1} - \upsilon_{out2}}{\upsilon_{in,CM}} = 0}$$

Common-Mode Response (Non-ideal)

An example in previous slide shows an idealized case of a common-mode response. In reality, neither is the circuit fully symmetric nor does the current source exhibit an infinite output impedance. As a result, **a fraction of the input CM variation appears at the output**.

R_{SS}

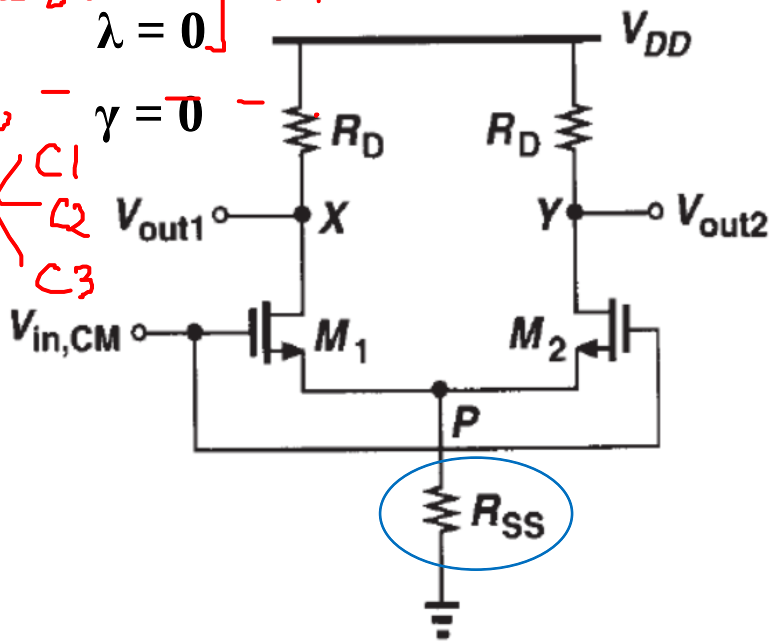
Case 1. Finite output impedance and symmetric circuit

diff input: sym / circuit sym — diff → Lemma / ac ground; asym — common: ideal $A_V = 0$; non-ideal: C1, C2, C3

A_{CM}

$\lambda = 0$

$\gamma = 0$

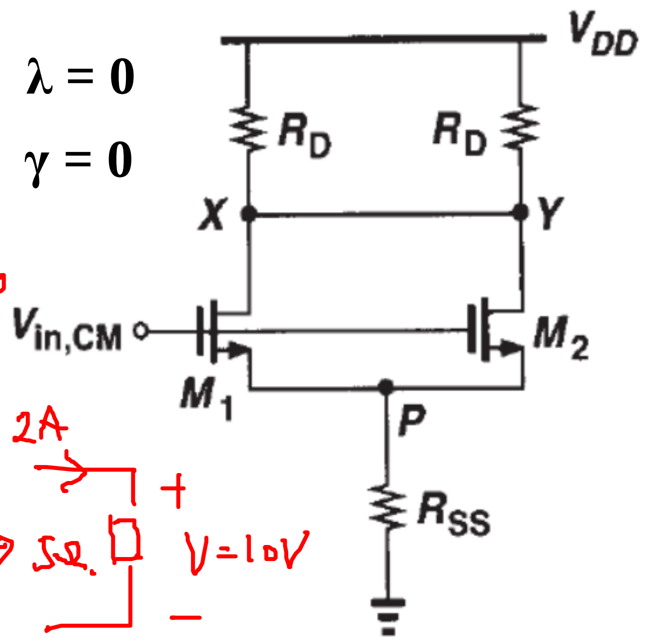


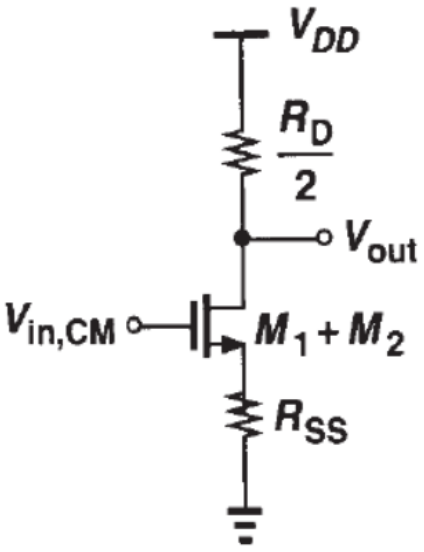
Circuit is not symmetry
diff input

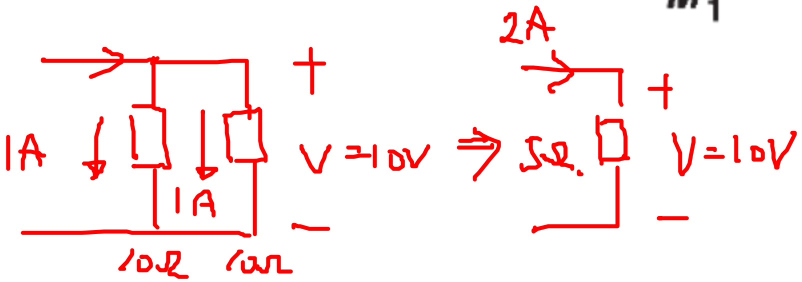
$\lambda = 0$

$\gamma = 0$

$\delta V_{in,CM} \to \Delta I_D \to \Delta V_P$







V_P is now affected by changes in $V_{in,CM}$ due to R_{SS}. However, due to the symmetry, V_X remains equal to V_Y. Thus, we can view the circuit as two parallel transistor circuits.

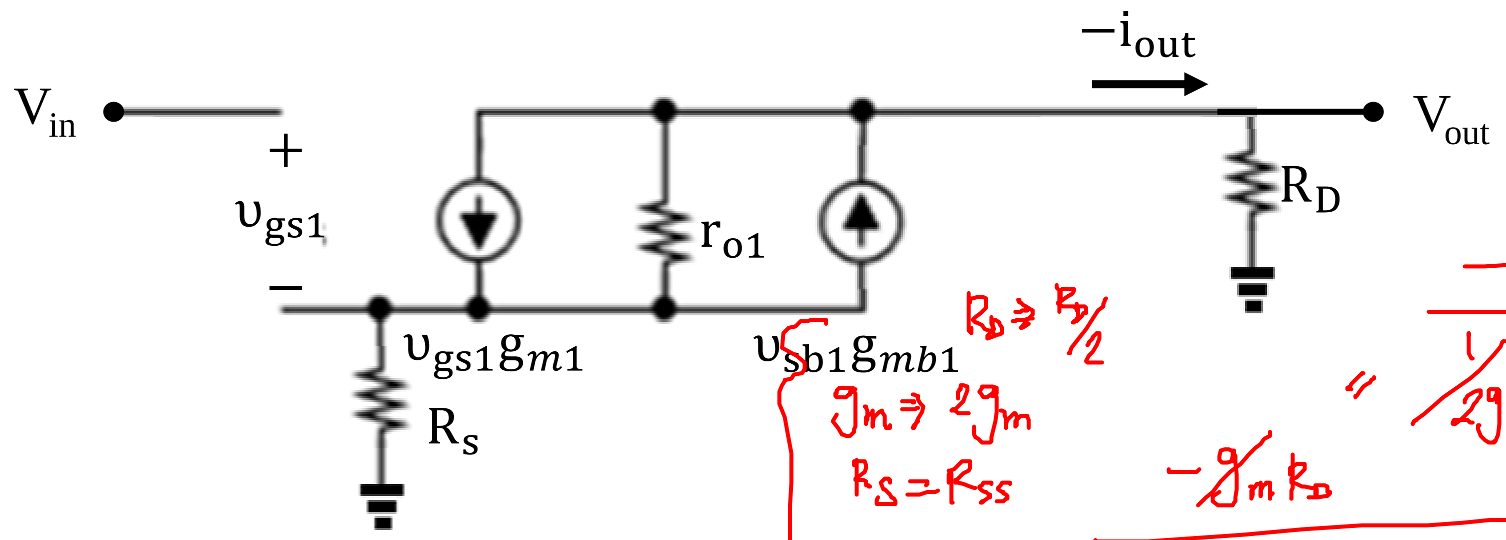
$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH})^2 \times 2 \text{ and the same for the transconductance } \mathbf{g_m \times 2}$$

From small-signal model, we get $A_{v,CM} = -\dfrac{R_D/2}{1/2g_m + R_{SS}}$

Recall: Gain of degenerated CS stage

$\lambda \neq 0$

$\gamma \neq 0$



$R_D \Rightarrow R_D/2$

$g_m \Rightarrow 2g_m$

$R_S = R_{SS}$

$r_{o1} = \infty$

$g_{mb} = 0$

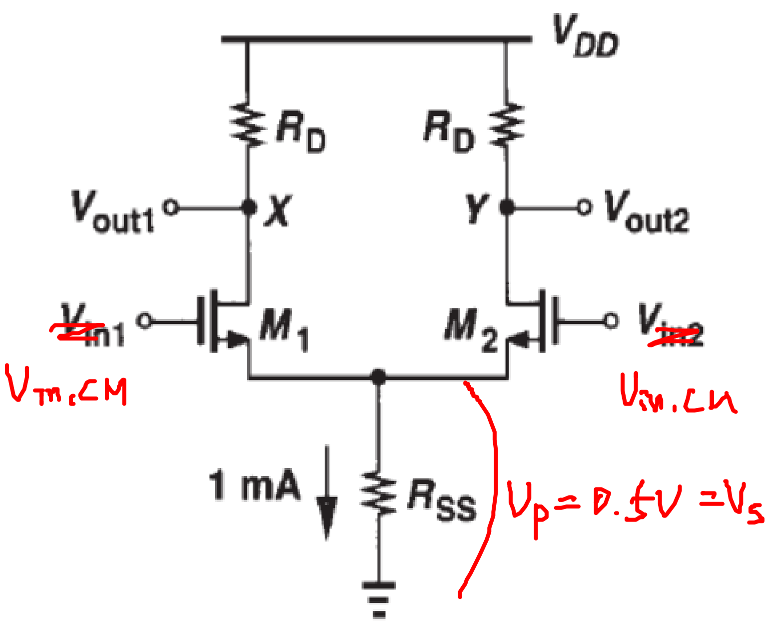
$$\frac{-\cancel{g_m} R_D}{\frac{1}{g_m} + \cancel{g_m} R_S} = \frac{-R_D/2}{\frac{1}{2g_m} + R_{SS}}$$

$$A_v = \frac{\upsilon_{out}}{\upsilon_{in}} = \frac{R_{out}(-i_{out})}{v_{in}} = -G_m R_{out}$$

$$= \frac{-g_{m1} \cancel{r_{o1}}}{\cancel{R_S + r_{o1} + (g_{m1} + g_{mb1}) r_{o1} R_S}} \cdot \frac{\cancel{[R_S + r_{o1} + (g_{m1} + g_{mb1}) r_{o1} R_S]} R_D}{[\cancel{R_S} + \cancel{r_{o1}} + (g_{m1} + \cancel{g_{mb1}}) \cancel{r_{o1}} R_S] + \cancel{R_D}}$$

1

Example 3 The circuit uses a resistor rather than a current source to define a tail current of 1 mA. Assume that $(W/L)_{1,2} = 25/0.5$, $\mu_n C_{ox} = 50\ \mu A/V^2$, $V_{TH} = 0.6$ V, $\lambda = \gamma = 0$, and $V_{DD} = 3$ V.



$I_{D1} = I_{D2} = 0.5mA.$

$= \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{th})^2$

$V_G = V_{in,CM}$, $V_S = 0.5V$

$\Rightarrow V_{in,CM} = 1.73V.$

$R_{SS} = 500\Omega.$

(a) What is the required input CM voltage for which R_{SS} sustains 0.5 V?
(b) Calculate R_D for a differential gain of 5. Assume there is a constant current 1 mA at R_{SS}.
(c) What happens at the output if the input CM level is 50 mV higher than the value calculated in (a)?

Example 3 The circuit uses a resistor rather than a current source to define a tail current of 1 mA. Assume that $(W/L)_{1,2} = 25/0.5$, $\mu_n C_{ox} = 50\ \mu A/V^2$, $V_{TH} = 0.6$ V, $\lambda = \gamma = 0$, and $V_{DD} = 3$ V.

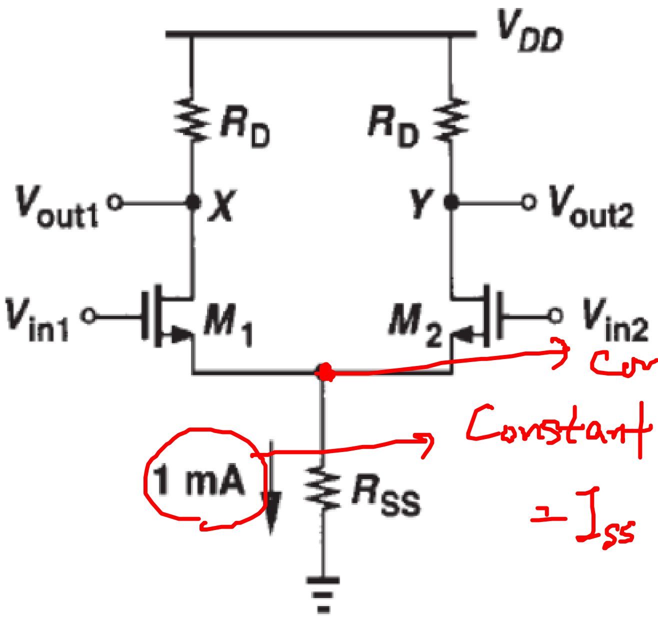
half circuit.

$$|-g_m(R_D \| r_o)| = 5$$

$$g_m = \sqrt{2 \mu_n C_{ox} \frac{W}{L} I_D}.$$

$$= 0.00158. \qquad r_o = \infty.$$

$$R_D = 3.16 k\Omega.$$



constant voltage.

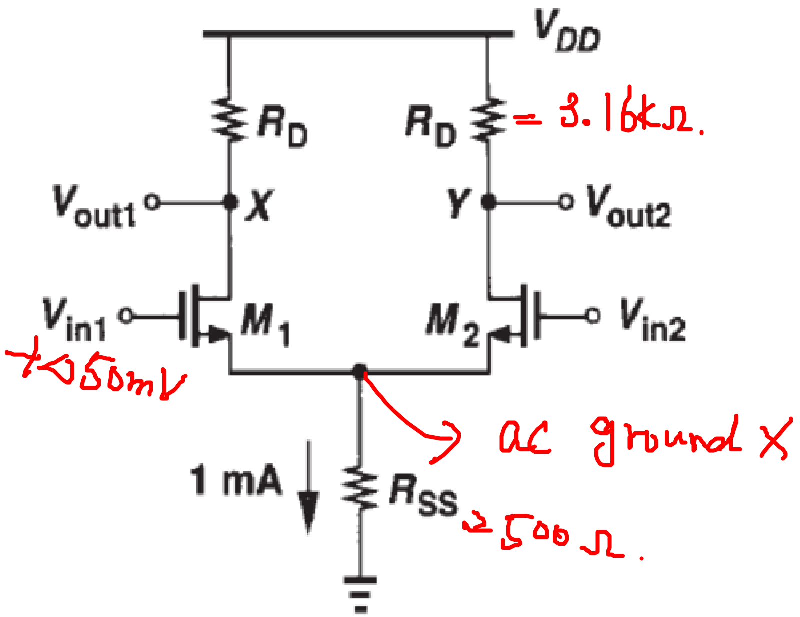
Constant current $= I_{SS}$

(a) What is the required input CM voltage for which R_{SS} sustains 0.5 V?

(b) Calculate R_D for a differential gain of 5. Assume there is a constant current 1 mA at R_{SS}.

(c) What happens at the output if the input CM level is 50 mV higher than the value calculated in (a)?

Example 3 The circuit uses a resistor rather than a current source to define a tail current of 1 mA. Assume that $(W/L)_{1,2} = 25/0.5$, $\mu_n C_{ox} = 50\ \mu A/V^2$, $V_{TH} = 0.6$ V, $\lambda = \gamma = 0$, and $V_{DD} = 3$ V.



$$A_v = \frac{-R_D/2}{\frac{1}{2g_m} + R_{SS}} = \frac{-3.16k\Omega/2}{\frac{1}{2\times 0.0058} + 500} = -1.94$$

$$\Delta 50mV \Rightarrow 50mV \times -1.94 = \underline{-97\ mV}$$

$$V_{DS} = V_{GS} - V_{TH}$$

$\underline{0.143\,V}$ → near the edge of the saturation.

(a) What is the required input CM voltage for which R_{SS} sustains 0.5 V?

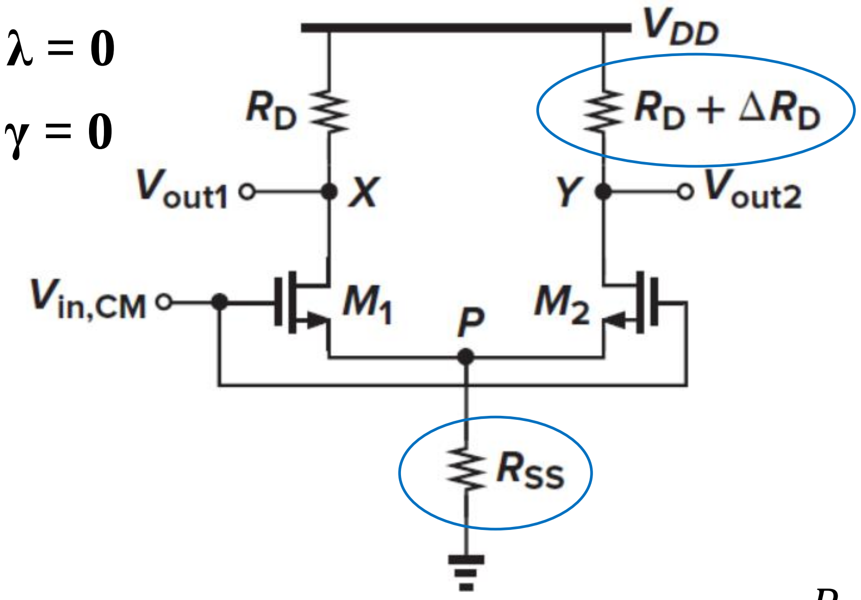
(b) Calculate R_D for a differential gain of 5. Assume there is a constant current 1 mA at R_{SS}.

(c) What happens at the output if the input CM level is 50 mV higher than the value calculated in (a)?

Case 2. Finite output impedance and asymmetric circuit

λ = 0

γ = 0



As $V_{in,CM}$ increases, output will change according to $\frac{V_{out}}{V_{in}}$ $A_{v,CM} = -\dfrac{R_D/2}{1/2g_m + R_{SS}}$

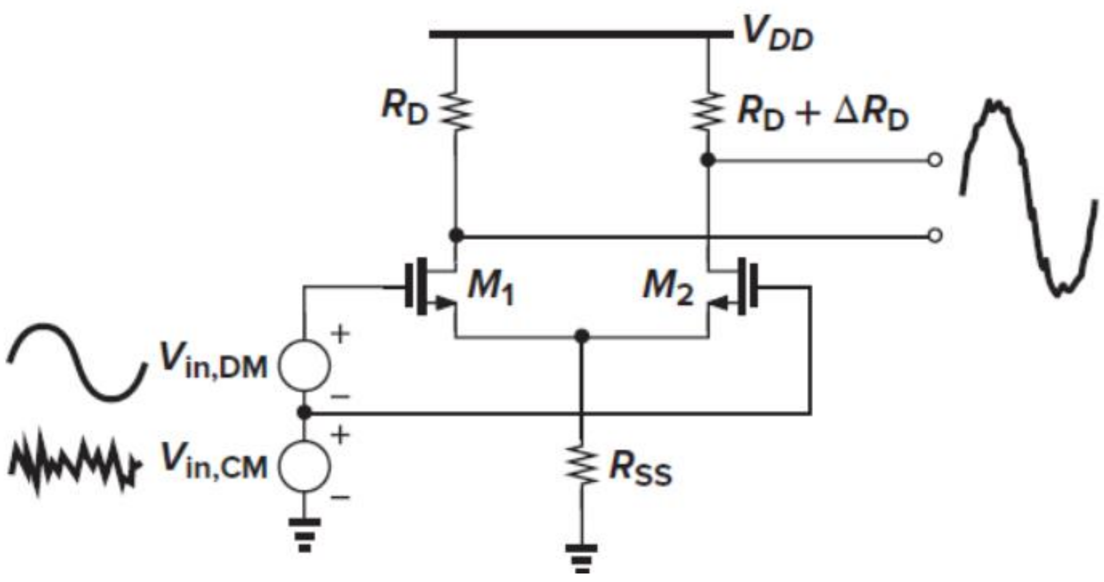
However, due to asymmetry in R_D,

× Δg_m.

$$\Delta V_X = -\Delta V_{in,CM}\frac{g_m R_D}{1 + 2g_m R_{SS}}$$

$$\Delta V_Y = -\Delta V_{in,CM}\frac{g_m (R_D + \Delta R_D)}{1 + 2g_m R_{SS}}$$

Common-mode input introduces differential component at the output



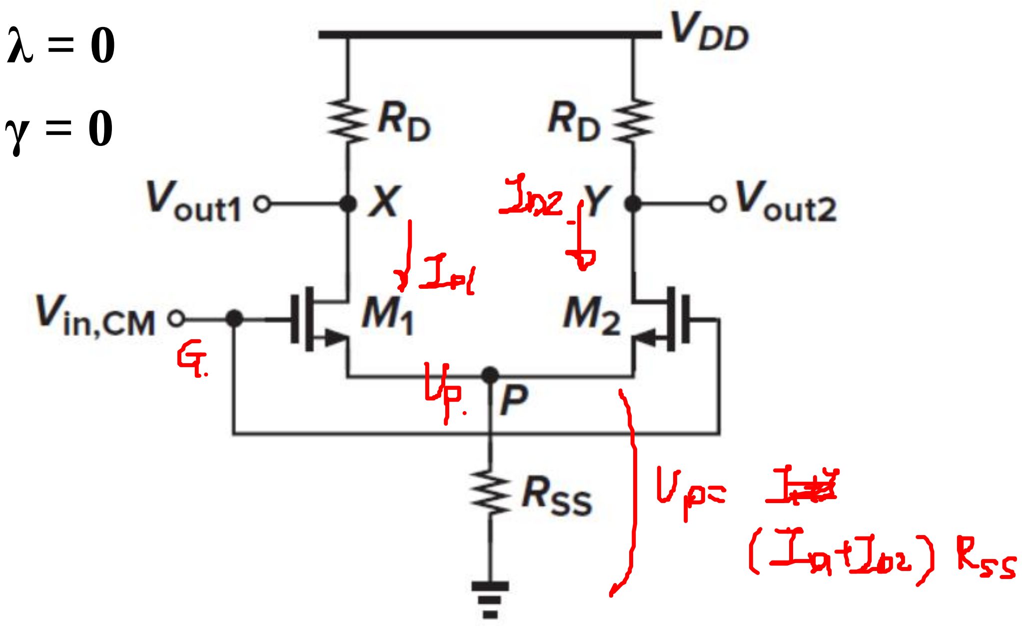
A common-mode change at the input introduces a differential component at the output → **common-mode to differential conversion**.

If the input of a differential pair includes both a differential signal and common-mode noise, the circuit corrupts the amplified differential signal by the input CM change.

Case 3. Asymmetry resulting from mismatches between M_1 and M_2

$\lambda = 0$

$\gamma = 0$



Due to dimension and threshold voltage mismatches, the two transistors carry slightly different currents and exhibit unequal transconductances. As $I_D = g_m V_{GS}$, $I_{D1} = g_{m1}(V_{in,CM} - V_P)$ and $I_{D2} = g_{m2}(V_{in,CM} - V_P)$.

Because $(I_{D1} + I_{D2})R_{SS} = V_P \rightarrow \mathbf{V_P = (g_{m1} + g_{m2})(V_{in,CM} - V_P)R_{SS}}$

$$V_P = \frac{(g_{m1} + g_{m2})R_{SS}}{(g_{m1} + g_{m2})R_{SS} + 1} V_{in,CM}$$

Output voltage (V_{out1} or V_X) becomes

$$\boldsymbol{V_X} = \cancel{\boldsymbol{V_{DD}}} - g_{m1}(V_{in,CM} - V_P)R_D = \cancel{\boldsymbol{V_{DD}}} + \frac{\boldsymbol{-g_{m1}}}{(g_{m1}+g_{m2})\boldsymbol{R_{SS}} + \boldsymbol{1}}\boldsymbol{R_D V_{in,CM}}$$

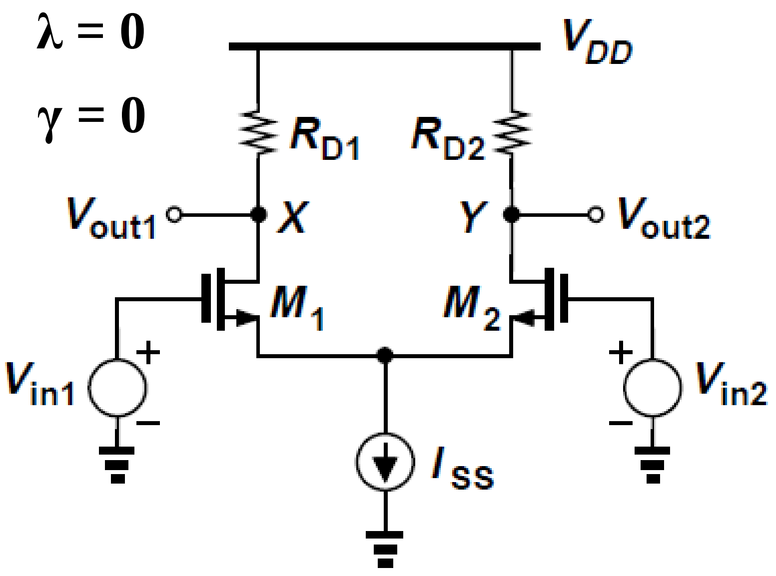
Similarly, $\boldsymbol{V_Y} = \cancel{\boldsymbol{V_{DD}}} + \frac{\boldsymbol{-g_{m2}}}{\boldsymbol{(g_{m1}+g_{m2})R_{SS}+1}}\boldsymbol{R_D V_{in,CM}}$

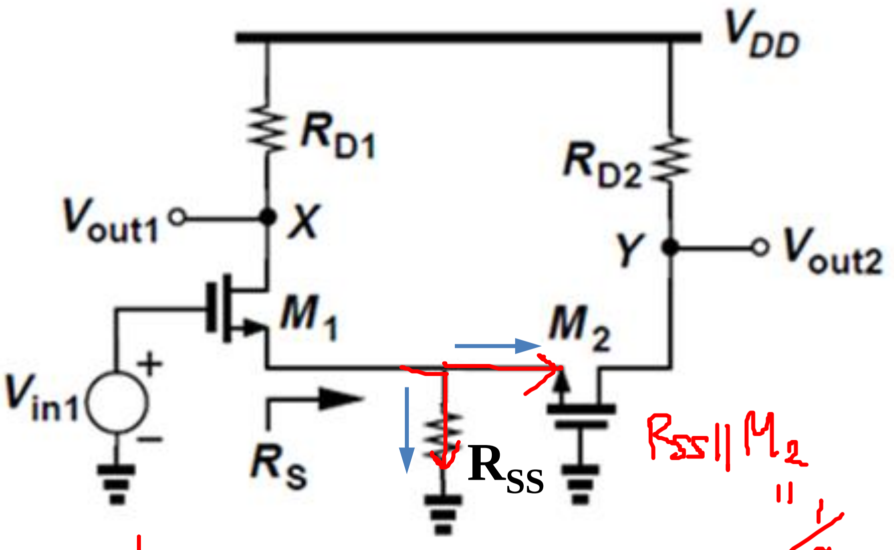
Finally, we get $V_X - V_Y = -\frac{g_{m1}-g_{m2}}{(g_{m1}+g_{m2})R_{SS}+1} R_D V_{in,CM}$ or, $\boldsymbol{A_{CM-DM}} = -\frac{\boldsymbol{\Delta g_m}}{\boldsymbol{(g_{m1}+g_{m2})R_{SS}+1}}\boldsymbol{R_D}$

$$A_{CM-DM} = -\frac{g_m \Delta R_D}{(g_{m1}+g_{m2})R_{SS}+1}$$

Differential-Mode (Small-Signal, Superposition)

half-circuit (X)





CS stage with source degeneration

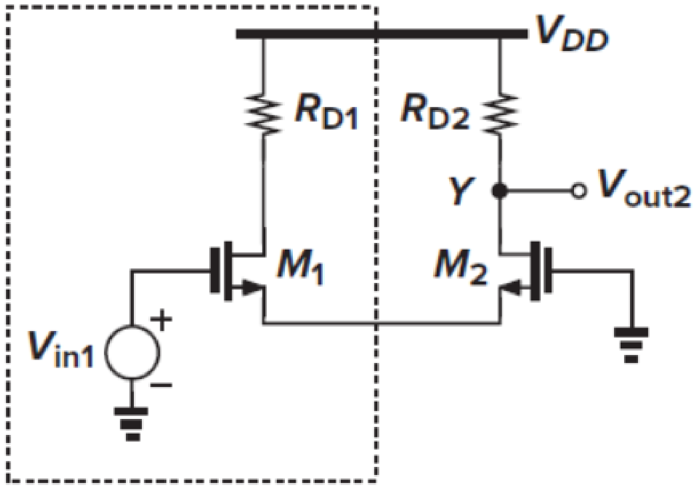
$$R_S = \frac{1}{g_{m2}} \parallel R_{SS}$$

Referring to DM without R_{SS}, we can get v_{out1}

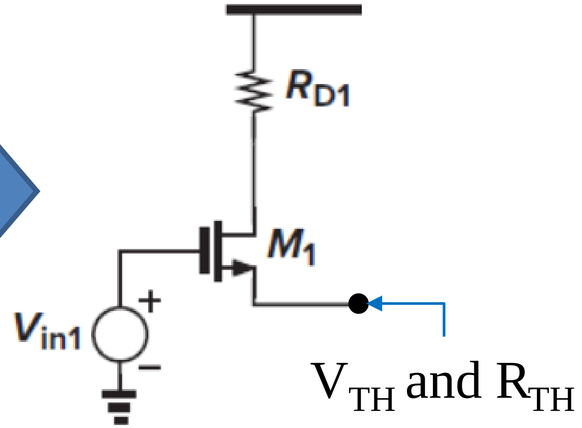
$$\boxed{v_{out1}(v_X) = -\frac{R_D}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}} v_{in1}}$$

$$v_{out1} = -\frac{R_D}{\frac{1}{g_{m1}} + \left(\frac{1}{g_{m2}} \parallel R_{SS}\right)} v_{in1}$$

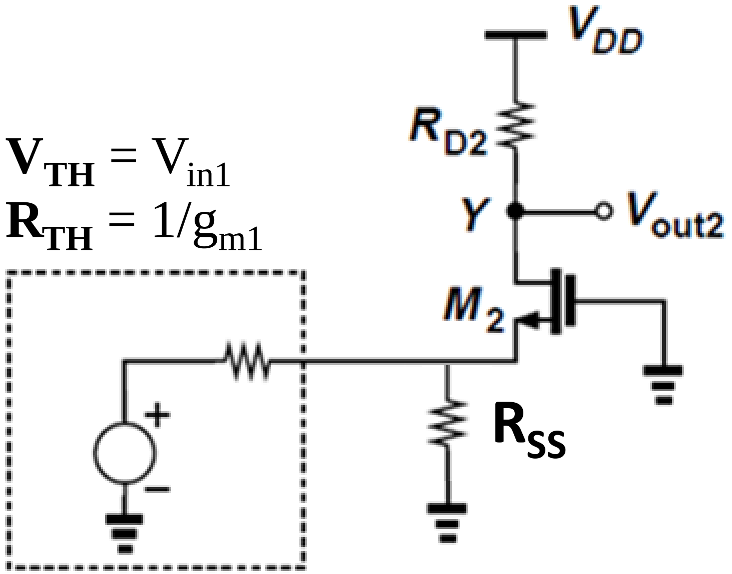
As we saw previously



Find Thevenin Eq. circuit



$\mathbf{V_{TH}} = V_{in1}$
$\mathbf{R_{TH}} = 1/g_{m1}$

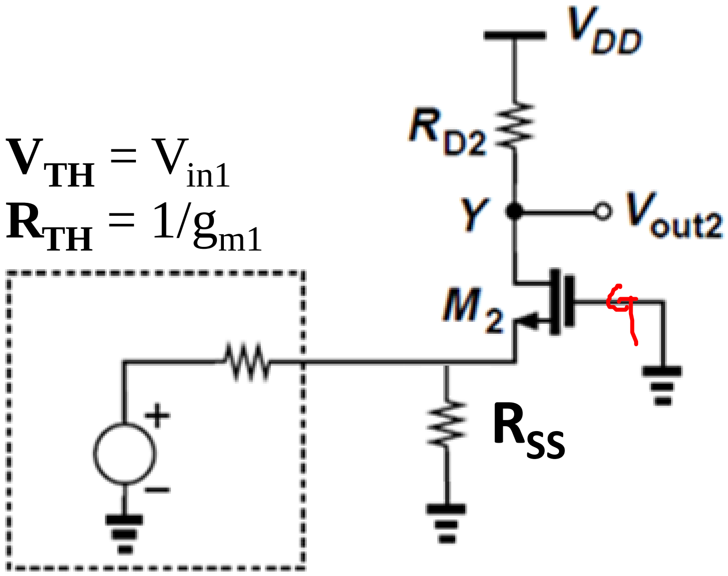


$$G_m = \left| \frac{g_{m1} g_{m2} R_{SS}}{1 + g_{m1} R_{SS} + g_{m2} R_{SS}} \right|$$

$$R_{out} = R_D$$

$$or\ v_{out2} = \frac{\dfrac{R_{SS}}{R_{SS} + \dfrac{1}{g_{m2}}} R_D}{\dfrac{1}{g_{m1}} + \left(\dfrac{1}{g_{m2}} \parallel R_{SS} \right)} v_{in1}$$

$V_{TH} = V_{in1}$
$R_{TH} = 1/g_{m1}$

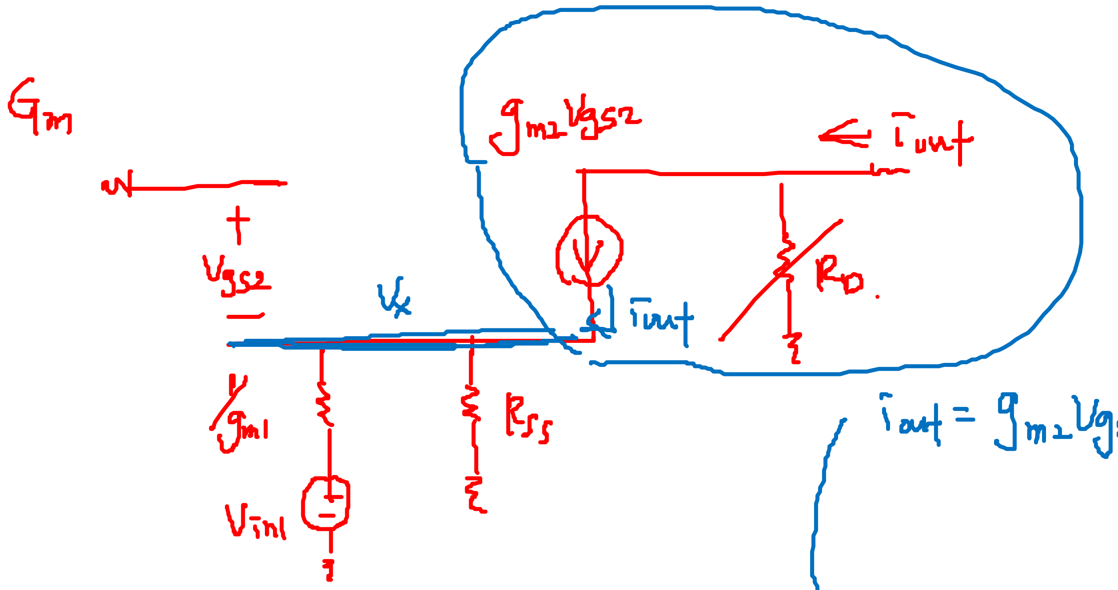


$$G_m = \left| \frac{-\,g_{m1}g_{m2}R_{SS}}{1 + g_{m1}R_{SS} + g_{m2}R_{SS}} \right|$$

$$R_{out} = R_D$$

$$or\ v_{out2} = \frac{\dfrac{R_{SS}}{R_{SS} + \dfrac{1}{g_{m2}}} R_D}{\dfrac{1}{g_{m1}} + \left(\dfrac{1}{g_{m2}} \parallel R_{SS}\right)} v_{in1}$$

⇒



$i_{out} = g_{m2}V_{gs2}$

KCL at node X

① $i_{out} = \dfrac{V_x - V_{in1}}{1/g_{m1}} + \dfrac{V_x}{R_{SS}}$

② $V_x = -V_{gs2} = -\dfrac{i_{out}}{g_{m2}}$.

$$G_m = \frac{i_{out}}{v_{in1}} = \frac{-g_{m1}g_{m2}R_{SS}}{1 + g_{m1}R_{SS} + g_{m2}R_{SS}}$$

$R_{out} = R_D$.

$V_{out2} = -G_m R_{out}$.

From V_{in1} we have

$$① \quad \upsilon_{out1} - \upsilon_{out2} = -\frac{(g_{m1} + 2R_{SS}g_{m1}g_{m2})R_D}{1 + (g_{m1} + g_{m2})R_{SS}}\upsilon_{in1} \Leftarrow -\underbrace{\frac{g_{m1}R_D + g_{m1}g_{m2}R_{SS}R_D}{1+(g_{m1}+g_{m2})R_{SS}}\upsilon_{in1}}_{V_{out1}} - \underbrace{\frac{g_{m1}g_{m2}R_{SS}R_D}{\text{''}}\upsilon_{in1}}_{V_{out2}}$$

From V_{in2} we have

$$② \quad \upsilon_{out1} - \upsilon_{out2} = \frac{(g_{m2} + 2R_{SS}g_{m1}g_{m2})R_D}{1 + (g_{m1} + g_{m2})R_{SS}}\upsilon_{in2}$$

$\frac{g_{m1}+g_{m2}}{2}$ = mean of g_m.

$$\frac{\upsilon_{out1} - \upsilon_{out2}}{v_{in1} - v_{in2}} \approx -\frac{(g_{m1} + g_{m2}) + 4R_{SS}g_{m1}g_{m2})}{1 + (g_{m1} + g_{m2})R_{SS}}\boldsymbol{R_D}/\boldsymbol{2}$$

by superposition $V_{out1} - V_{out2} = ① + ② = -\frac{(g_{m1}+2g_{m1}g_{m2}R_{SS})R_D}{1+g_{m1}R_{SS}+g_{m2}R_{SS}}v_{in1} - \frac{(g_{m2}+2g_{m1}g_{m2}R_{SS})R_D}{\text{''}}(v_{in2})$

Anom

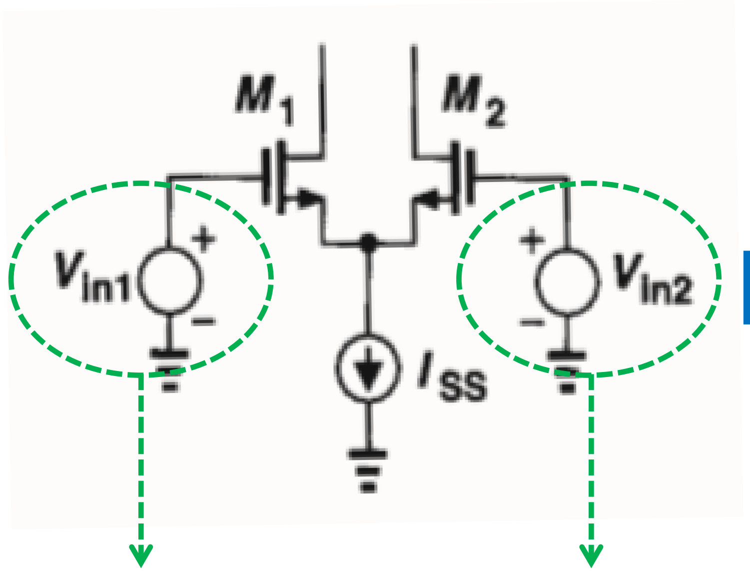
For meaningful comparison of differential circuits, the undesirable differential component produced by CM variation must be normalized to the wanted differential output.

Common-mode rejection ratio, or CMRR, is defined as

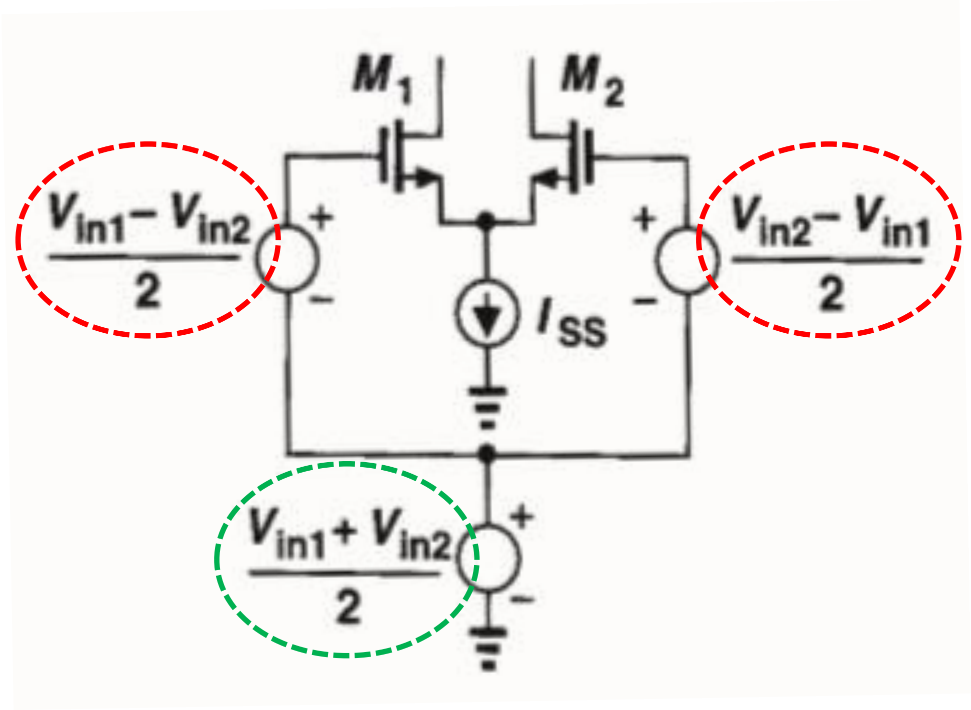
$$\boldsymbol{CMRR = \left|\frac{A_{DM}}{A_{CM-DM}}\right|}$$

$$\frac{A_{DM} = \left|\dfrac{(g_{m1} + g_{m2} + 4R_{SS}g_{m1}g_{m2})}{1 + (g_{m1} + g_{m2})R_{SS}} R_D/2\right|}{A_{CM-DM} = \left|\dfrac{g_{m1} - g_{m2}}{(g_{m1}+g_{m2})R_{SS} + 1} R_D\right|} = \frac{(g_{m1} + g_{m2} + 4R_{SS}g_{m1}g_{m2})}{2(g_{m1} - g_{m2})}$$

Common-Mode + Differential-Mode



Not necessarily fully differential



$$A_{DM} = \frac{\upsilon_{out1} - \upsilon_{out2}}{v_{in1} - v_{in2}}$$

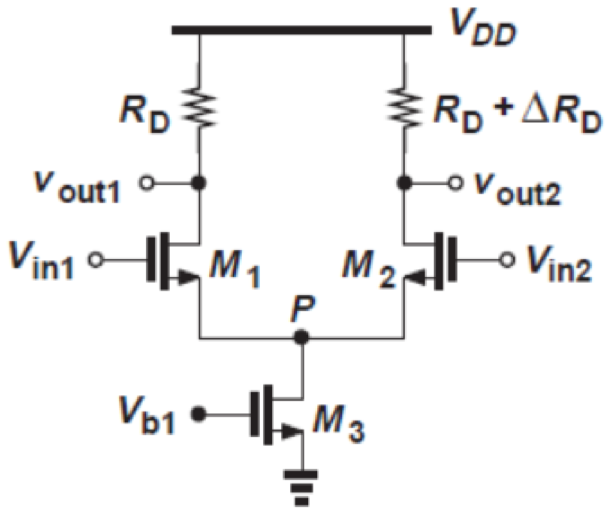
$$A_{CM} = \frac{\upsilon_{out,CM}}{\upsilon_{in,CM}}$$

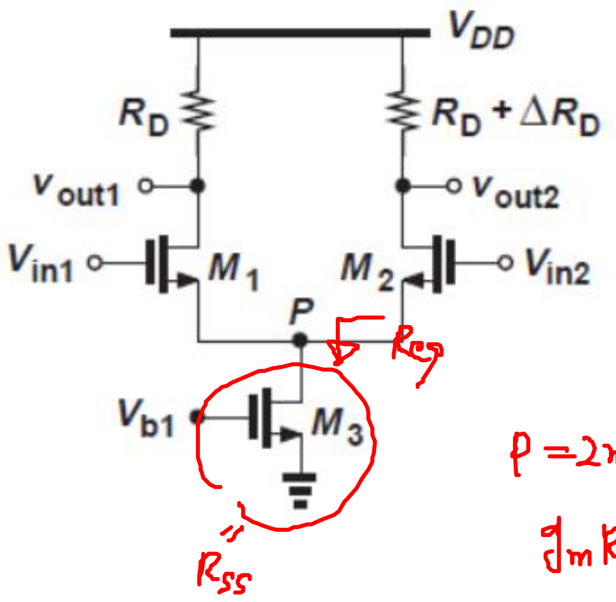
$$A_{CM-DM} = \frac{\upsilon_{out1} - \upsilon_{out2}}{\upsilon_{in,CM}}$$

$$CMRR = \left|\frac{A_{DM}}{A_{CM-DM}}\right|$$

Example 4 The differential pair shown below must achieve a CMRR of 60 dB (= 1000). Assume a power budget of 2 mW ($V_{DD} \times I_{total}$), a nominal differential voltage gain of 5, and neglect channel-length modulation in M_1 and M_2, compute the minimum required λ for M_3. Assume the virtual ground concept applies at node P for the differential voltage gain.

Use parameters W/L for both M_1 and M_2 = 10/0.18, $\mu_n C_{ox} = 100\ \mu A/V^2$, V_{DD} = 1.8 V, ΔR/R = 2%, and $I_{D1} = I_{D2} = I_{D3}/2$.





$$A_{DM} = \frac{v_{out1} - v_{out2}}{v_{in1} - v_{in2}} = -g_m(R_D \parallel r_o)$$

$\lambda = 0.2352$

$$A_{CM-DM} = -\frac{\Delta g_m}{(g_{m1} + g_{m2})R_{SS} + 1} R_D$$

$P = 2mW = V_{DD} \times I_{total} = 1.8 \times I_{total} \Rightarrow I_{total} = I_{D3} = \frac{2mW}{1.8V} = 1.11 \times 10^{-3}$ A.

$g_m R_D = 5 \qquad g_m = \sqrt{2\mu_n C_{ox}\frac{W}{L}I_{D3}/2} = 0.00248$

$R_D = 2016.13\ \Omega.$

CMRR = 1000.

Slides Case 2

$$\Delta V_X = \frac{-g_m R_D}{1 + 2g_m R_{SS}} \Delta V_{in,CM}$$

$$\Delta V_Y = \frac{-g_m (R_D + \Delta R_D)}{1 + 2g_m R_{SS}} \Delta V_{in,CM}$$

$$\Rightarrow \frac{\Delta V_X - \Delta V_Y}{\Delta V_{in,CM}} = \frac{g_m \Delta R_D}{1 + 2g_m R_{SS}} \cancel{\Delta V_{in,CM}}.$$

$R_{SS} = r_{O3} = \frac{1}{\lambda I_{D3}}$

$$CMRR = \frac{g_m R_D}{\frac{g_m \Delta R_D}{1 + 2g_m \frac{1}{\lambda I_{D3}}}} = 1000.$$