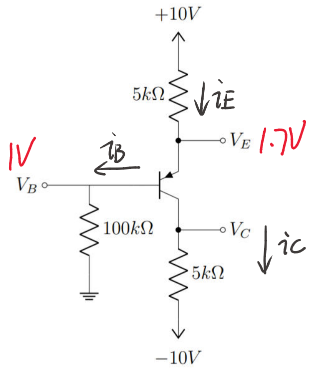
HW2 Solution

Q1



$$i_E = \frac{10 - V_E}{5k}$$

$$= \frac{10 - 1.7}{5k}$$

$$= 1.66 \times 10^{-3} A$$

$$i_B = \frac{V_B - 0}{100k} = \frac{1}{100k} = 1 \times 10^{-5} A$$

$\because i_E = i_B + i_C$, $i_C = \alpha i_E$

$\therefore i_E = i_B + \alpha i_E$

$$\Rightarrow \alpha = 1 - \frac{i_B}{i_E} = 1 - \frac{1 \times 10^{-5}}{1.66 \times 10^{-3}} = 0.994$$

$\because i_E = i_B + i_C$, $i_C = \beta i_B$

$$\therefore i_E = i_B + \beta i_B \Rightarrow \beta = \frac{i_E}{i_B} - 1 = \frac{1.66 \times 10^{-3}}{1 \times 10^{-5}} - 1 = 165$$

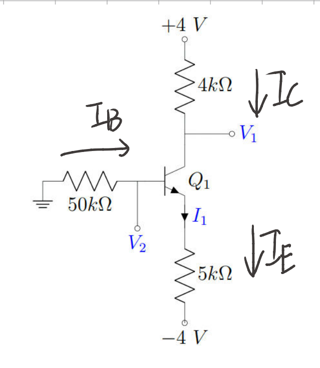
$$\because i_C = i_E - i_B = 1.66 \times 10^{-3} - 1 \times 10^{-5} = 1.65 \times 10^{-5} A$$

$$i_C = \frac{V_C + 10}{5k}$$

$$\therefore 1.65 \times 10^{-5} = \frac{V_C + 10}{5k}$$

$$\Rightarrow V_C = -1.75V$$

Q2



β is huge $\Rightarrow I_B = 0$

$\Rightarrow V_B = 0 = V_2$

$V_{BE} = 0.8V$

$\Rightarrow V_E = V_B - 0.8 = -0.8V$

$\therefore I_E = \dfrac{V_E + 4}{5k} = \dfrac{3.2}{5k} = 6.4 \times 10^{-4} A$

$\Rightarrow I_1 = 6.4 \times 10^{-4} A$

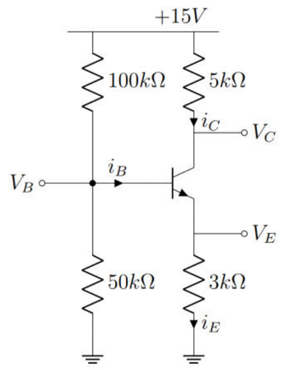
$\because I_B = 0,\ I_E = I_B + I_C$

$\therefore I_E = I_C = 6.4 \times 10^{-4} A$

$\therefore \dfrac{4 - V_1}{4k} = 6.4 \times 10^{-4}$

$\Rightarrow V_1 = 1.44V$

Q3

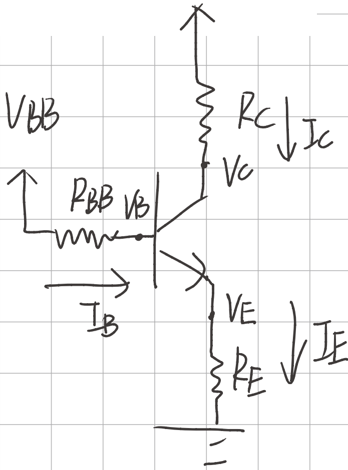


Use Thevenin's Theorem

$$V_{BB} = 15 \times \frac{50}{100+50} = 5V$$

$$R_{BB} = 100 \parallel 50 = 33.3k\Omega$$

$$\begin{cases} V_{BB} = V_B + I_B R_{BB} \\ V_B = V_{BE} + V_E \\ V_E = R_E I_E \end{cases}$$



$$\Rightarrow V_{BB} = I_B R_{BB} + V_{BE} + R_E I_E$$

$$\because I_B = \frac{I_E}{\beta+1}$$

$$\therefore V_{BB} = \frac{I_E}{\beta+1} \cdot R_{BB} + V_{BE} + R_E I_E$$

$$\Rightarrow I_E = \frac{V_{BB} - V_{BE}}{\frac{R_{BB}}{\beta+1} + R_E}$$

$$= \frac{5-0.7}{\frac{33.3k}{100+1} + 3k}$$

$$= 1.29mA$$

$$\Rightarrow I_B = \frac{I_E}{\beta+1} = \frac{1.29}{100+1} = 0.0128mA$$

$$V_B = V_{BE} + I_E R_E = 0.7 + 1.29 \times 3 = 4.57V$$

$$I_C = I_E - I_B = 1.29 - 0.0128 = 1.28mA$$

$$V_C = 15 - I_C R_C = 15 - 1.28 \times 5 = 8.6V$$

Q4

(a) $$I_c = I_s\left(e^{\frac{qV_{BE}}{kT}} - 1\right)\left(1 + \frac{V_{CE}}{V_{AF}}\right)$$

$$= 5\times10^{-6}\left(e^{\frac{0.5}{0.026}} - 1\right)\left(1 + \frac{1}{200}\right)$$

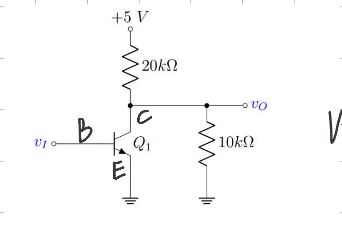
$$= 1.13 \times 10^{-7} A$$

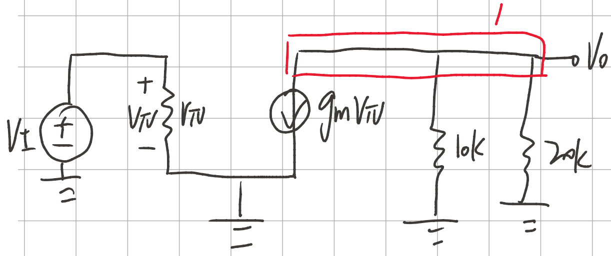
$$g_m = \frac{I_c}{\frac{kT}{q}} = \frac{1.13\times10^{-7}}{0.026} = 4.34\times10^{-6}\ A/V$$

$$r_o = \frac{V_{AF}}{I_C} = \frac{200}{1.13\times10^{-7}} = 1.77\times10^{9}\ \Omega$$

Q5

$$I_C = 0.4mA \Rightarrow g_m = \frac{I_C}{V_T} = \frac{0.4\times10^{-3}}{0.025875} = 0.015$$





KVL: $-V_I + V_\pi = 0 \Rightarrow V_I = V_\pi$

KCL: $g_m V_\pi + \frac{V_o}{10k} + \frac{V_o}{20k} = 0$

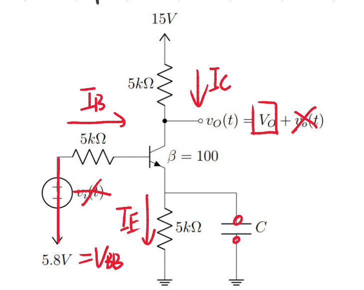
$$\frac{3V_o}{20k} = -g_m V_I$$

$$\Rightarrow \frac{V_o}{V_I} = -g_m \cdot \frac{20k}{3} = -100$$

$$\Rightarrow A_v = -100$$

Q6

Step 1: DC Analysis



To do:

① turn off small-signal voltage, replace it with a short circuit

② replace capacitor with an open circuit

③ Check $V_{CE} \geq V_{BE}$

$$\begin{cases} V_{BB} = V_B + I_B R_B \\ V_B = V_{BE} + V_E \\ V_E = I_E R_E \\ I_E = (\beta+1) I_B \end{cases}$$

$\Rightarrow V_{BB} = I_B R_B + V_{BE} + I_E R_E$

$\Rightarrow 5.8 = I_B \cdot 5 + 0.7 + (\beta+1) I_B \cdot 5$

$\Rightarrow I_B = 0.01\,mA$

$\therefore I_C = \beta I_B = 100 \times 0.01 = 1\,mA$

$I_E = I_B + I_C = 1.01\,mA$

$V_C = 15 - I_C R_C = 15 - 1\times 5 = 10V$

$V_E = I_E R_E = 1.01 \times 5 = 5.05V$

$\Rightarrow V_{CE} = V_C - V_E = 10 - 5.05 = 4.95V \geq V_{BE}$

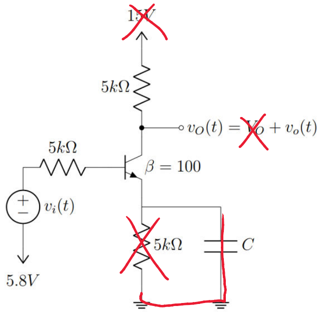
$\Rightarrow$ forward active

Step 2: Calculate g_m, r_π

$$g_m = \frac{I_C}{V_T} = \frac{1\times 10^{-3}}{0.025} = 0.04 A/V$$

$$r_\pi = \frac{V_T}{I_B} = \frac{0.025}{0.01\times 10^{-3}} = 2.5k$$

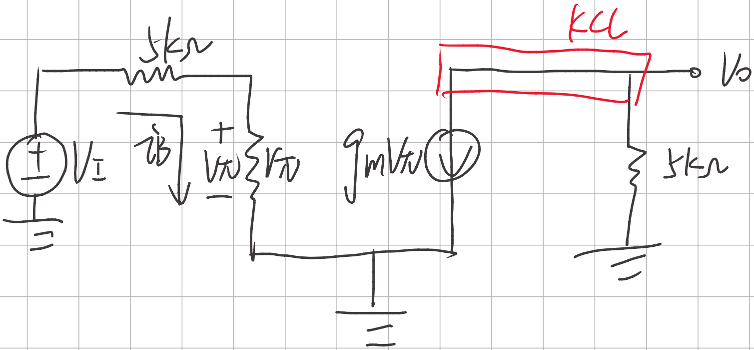
Step 3/4/5: Small signal model



To do:

① capacitor $\rightarrow$ short circuit

② Set all D.C. sources to 0



$$KVL: \; -V_I + i_B \cdot 5k + i_B r_\pi = 0$$

$$\Rightarrow i_B = \frac{V_I}{5k + r_\pi}$$

$$\Rightarrow V_\pi = i_B r_\pi = \frac{r_\pi}{5k + r_\pi} \cdot V_I$$

$$= \frac{2.5k}{2.5k + 5k} V_I = \frac{1}{3} V_I$$

$$KCL: \quad g_m V_\pi + \frac{V_o}{5k} = 0$$

$$\Rightarrow g_m \cdot \frac{1}{3} V_I = -\frac{V_o}{5k}$$

$$\Rightarrow A_V = \frac{V_o}{V_I} = -\frac{1}{3} g_m \cdot 5k$$

$$= -\frac{1}{3} \times 0.04 \times 5k$$

$$= -66.7$$