Question 1. MOSFET DC Biasing

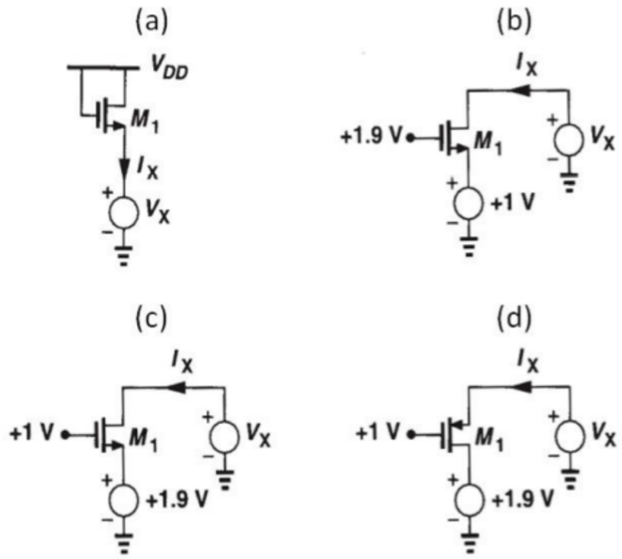
[40 points] Use the drain current equations below. Don't consider channel-length modulation and body effect. Assuming $\mu_n = 350 \times 10^{-4} m^2/V/s$, $\mu_p = 350 \times 10^{-4} m^2/V/s$, $V_{TH} = 0.7V$ (NMOS), $V_{TH} = -0.8V$ (PMOS), $W_{drawn}/L_{drawn} = 20\mu m/2\mu m$, $t_{ox} = 9 \times 10^{-9} m$, $L_D = 0.08\mu m$, sketch I_X of M_1 as a function of V_X increasing from 0V to $V_{DD} = 5V$.

(1) $$I_D = \mu_n C_{ox} \frac{W}{L_{eff}} \left[(V_{GS} - V_{TH}) V_{DS} - \frac{1}{2} V_{DS}^2 \right] \text{ (NMOS in triode region)}$$

(2) $$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L_{eff}} (V_{GS} - V_{TH})^2 \text{ (NMOS in saturation region)}$$

(3) $$I_D = \mu_p C_{ox} \frac{W}{L_{eff}} \left[(V_{SG} - |V_{TH}|) V_{SD} - \frac{1}{2} V_{SD}^2 \right] \text{ (PMOS in triode region)}$$

(4) $$I_D = \frac{1}{2} \mu_p C_{ox} \frac{W}{L_{eff}} (V_{SG} - |V_{TH}|)^2 \text{ (PMOS in saturation region)}$$



$$C_{ox} = \varepsilon_{ox}/t_{ox} = 8.85 \times 10^{-12} \times 3.9 / t_{ox} = \frac{8.85 \times 10^{-12} \times 3.9}{9 \times 10^{-9}} = 3.835 \times 10^{-3}\ F/m^2$$

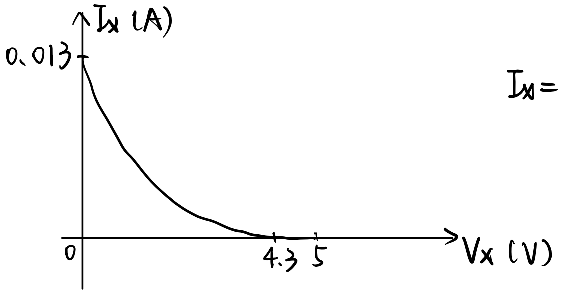
$$L_{eff} = L_{drawn} - 2L_D = 2 - 2 \times 0.08 = 1.84\,\mu m$$

(a) $V_{GS} = V_{DS} = 5 - V_X$

$5 - V_X < V_{TN} = 0.7V \Rightarrow V_X > 4.3V$, NMOS in cutoff, $I_X = 0$

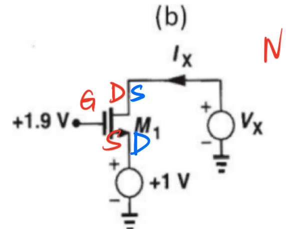
When $V_X < 4.3V$, $V_{DS} > V_{GS} - V_{TN}$, NMOS in saturation

$$I_X = \frac{1}{2} \mu_n C_{ox} \frac{W}{L_{eff}} (V_{GS} - V_{TN})^2$$
$$= \frac{1}{2} \times 350 \times 10^{-4} \times 3.835 \times 10^{-3} \times \frac{20}{1.84} (5 - V_X - 0.7)^2$$
$$= 7.29 \times 10^{-4} (4.3 - V_X)^2$$



$$I_x = \begin{cases} 7.29\times10^{-4}(4.3-V_x)^2 & 0\le V_x<4.3\,V \\ 0 & 4.3\,V\le V_x\le 5\,V \end{cases}$$

(b) ① $V_x > 1\ V$



$V_{GS} = 0.9\ V,\ V_{DS} = V_x - 1$

$V_{GS} > V_{TN} \Rightarrow$ on

$V_{DS} > V_{GS} - V_{TN} \Rightarrow V_x - 1 > 0.9 - 0.7 \Rightarrow V_x > 1.2\,V$, saturation

(1) When $V_x > 1.2V$, saturation

$$I_x = \frac{1}{2}\mu_n C_{ox}\frac{W}{L_{eff}}(V_{GS}-V_{TN})^2$$
$$= \frac{1}{2}\times350\times10^{-4}\times3.835\times10^{-3}\times\frac{20}{1.84}\times0.2^2$$
$$= 2.92\times10^{-5}\ A$$

(2) When $1\,V < V_x < 1.2V$, triode

$$I_x = \mu_n C_{ox}\frac{W}{L_{eff}}\left[(V_{GS}-V_{TN})V_{DS}-\frac{1}{2}V_{DS}^2\right]$$
$$= 350\times10^{-4}\times3.835\times10^{-3}\times\frac{20}{1.84}\left[0.2(V_x-1)-\frac{1}{2}(V_x-1)^2\right]$$
$$= 1.46\times10^{-3}(V_x-1)(0.7-0.5V_x)$$

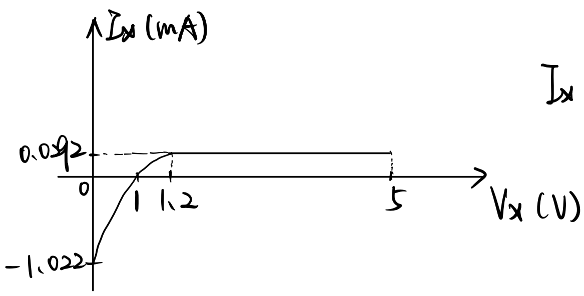
② $V_x < 1\ V$

$V_{GS} = 1.9 - V_x,\ V_{DS} = 1 - V_x$

$V_{GS} > V_{TN} \Rightarrow$ on

$V_{DS} < V_{GS} - V_{TN} = 1.9 - V_x - 0.7 = 1.2 - V_x \Rightarrow$ triode

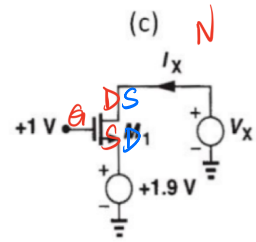
$$I_x = -\mu_n C_{ox}\frac{W}{L_{eff}}\left[(V_{GS}-V_{TN})V_{DS}-\frac{1}{2}V_{DS}^2\right]$$
$$= -350\times10^{-4}\times3.835\times10^{-3}\times\frac{20}{1.84}\left[(1.2-V_x)(1-V_x)-\frac{1}{2}(1-V_x)^2\right]$$
$$= 1.46\times10^{-3}(V_x-1)(0.7-0.5V_x)$$



$$I_x = \begin{cases} 1.46\times10^{-3}(V_x-1)(0.7-0.5V_x) & 0\le V_x<1.2V \\ 2.92\times10^{-5}\ A & 1.2V\le V_x\le 5V \end{cases}$$

(c) ① $V_x > 1.9V$

$V_{GS} = -0.9V < V_{TN} \Rightarrow$ off, $I_x = 0$



② $V_x < 1.9V$

$V_{GS} = 1 - V_x$, $V_{DS} = 1.9 - V_x$

$V_{GS} = 1 - V_x < V_{TN} \Rightarrow V_x > 1 - V_{TN} = 0.3\ V \Rightarrow$ off, $I_x = 0$

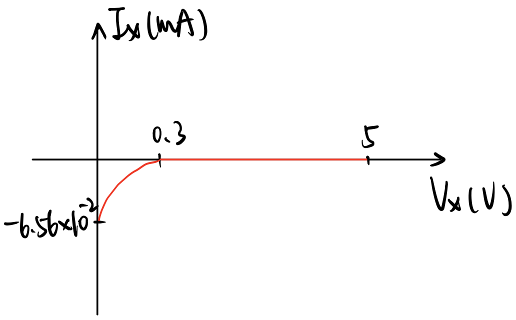
(1) $0.3V < V_x < 1.9V$, $I_x = 0$

(2) $0 < V_x < 0.3V$, $V_{DS} > V_{GS} - V_{TN} = 1 - V_x - 0.7 = 0.3 - V_x \Rightarrow$ saturation

$$I_x = -\frac{1}{2}\mu_n C_{ox}\frac{W}{L_{eff}}(V_{GS} - V_{TN})^2$$

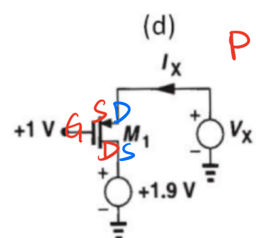
$$= -\frac{1}{2} \times 350 \times 10^{-4} \times 3.835 \times 10^{-3} \times \frac{20}{1.84}(0.3 - V_x)^2$$

$$= -7.29 \times 10^{-4}(0.3 - V_x)^2$$



$$I_x = \begin{cases} -7.29 \times 10^{-4}(0.3 - V_x)^2 & 0 \le V_x \le 0.3V \\ 0 & 0.3V < V_x \le 5V \end{cases}$$

(d) ① $V_x > 1.9V$



$V_{SG} = V_x - 1$, $V_{SD} = V_x - 1.9 \Rightarrow V_{SG} > |V_{TP}| \Rightarrow$ on

$V_{SD} < V_{SG} - |V_{TP}| = V_x - 1 - 0.8 = V_x - 1.8 \Rightarrow$ triode

$$I_x = \mu_p C_{ox}\frac{W}{L_{eff}}\left[(V_{SG} - |V_{TP}|)V_{SD} - \frac{1}{2}V_{SD}^2\right]$$

$$= 350 \times 10^{-4} \times 3.835 \times 10^{-3} \times \frac{20}{1.84}\left[(V_x - 1.8)(V_x - 1.9) - \frac{1}{2}(V_x - 1.9)^2\right]$$

$$= 1.46 \times 10^{-3}(V_x - 1.9)(0.5V_x - 0.85)$$

② $V_x < 1.9V$

$V_{SG} = 1.9 - 1 = 0.9V$, $V_{SD} = 1.9 - V_x \Rightarrow V_{SG} > |V_{TP}| \Rightarrow$ on

$V_{SD} > V_{SG} - |V_{TP}| \Rightarrow 1.9 - V_x > 0.9 - 0.8 = 0.1 \Rightarrow V_x < 1.8\ V$, saturation

(1) $V_X < 1.8V \Rightarrow$ saturation

$$I_X = -\frac{1}{2}\mu_p C_{ox}\frac{W}{L_{eff}}(V_{SG} - |V_{TP}|)^2$$

$$= -\frac{1}{2}\times 350\times 10^{-4}\times 3.835\times 10^{-3}\times\frac{20}{1.84}(0.9-0.8)^2$$

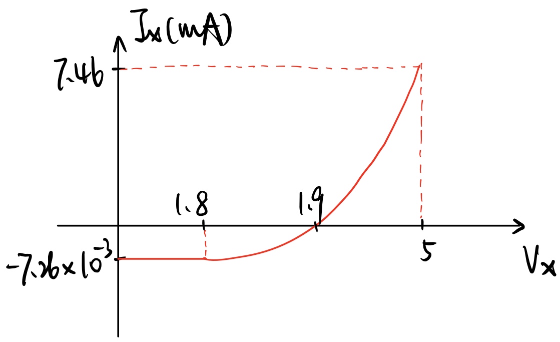
$$= -7.29\times 10^{-6}\ A$$

(2) $1.8V < V_X < 1.9V \Rightarrow$ triode

$$I_X = -\mu_p C_{ox}\frac{W}{L_{eff}}\left[(V_{SG}-|V_{TP}|)V_{SD} - \frac{1}{2}V_{SD}^2\right]$$

$$= -350\times 10^{-4}\times 3.835\times 10^{-3}\times\frac{20}{1.84}\left[0.1(1.9-V_X) - \frac{1}{2}(1.9-V_X)^2\right]$$

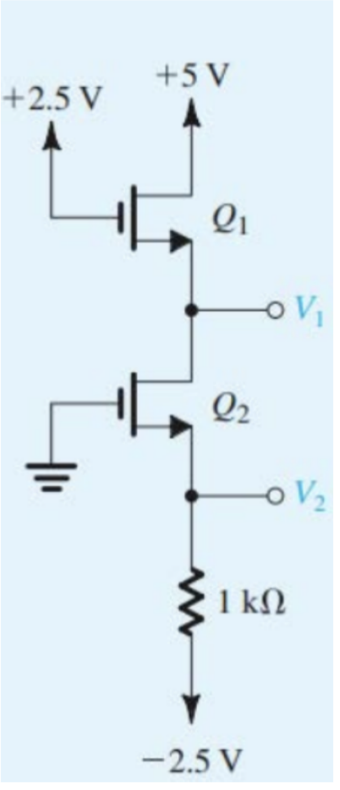
$$= 1.46\times 10^{-3}(V_X - 1.9)(0.5V_X - 0.85)$$



$$I_X = \begin{cases} -7.26\times 10^{-6}\ A & 0\le V_X < 1.8V \\ 1.46\times 10^{-3}(V_X-1.9)(0.5V_X-0.85) & 1.8V\le V_X\le 5V \end{cases}$$

Question 2. Combination of MOSFET

[10 points] For the circuit below, find the labeled node voltages. The NMOS transistor has $V_{TH} = 0.9V$, $k_n = \mu_n C_{ox}(W/L) = 1.5mA/V^2$.



$V_{GS1} = 2.5 - V_1$, $V_{DS1} = 5 - V_1$, $V_{GS2} = -V_2$, $V_{DS2} = V_1 - V_2$ $\Rightarrow$ $V_{DS1} > V_{GS1} - V_{TH}$

Assume Q_1, Q_2 both in saturation

$$\begin{cases} I_D = \frac{1}{2} k_n (V_{GS1} - V_{TH})^2 = \frac{1}{2} k_n (2.5 - V_1 - 0.9)^2 = \frac{1}{2} k_n (1.6 - V_1)^2 \\ I_D = \frac{1}{2} k_n (V_{GS2} - V_{TH})^2 = \frac{1}{2} k_n (-V_2 - 0.9)^2 \\ V_2 + 2.5 = 10^3 \cdot I_D \end{cases}$$

$\Rightarrow V_2 = 1.37\,V$ or $-1.84\,V$ $\Rightarrow V_2 = -1.84\,V$ since $V_{GS2} > V_{TH}$ in saturation

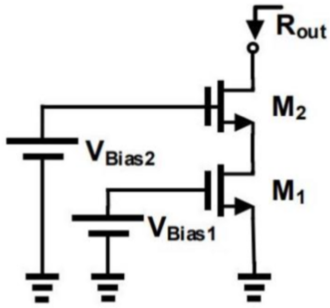
$\Rightarrow V_1 = 0.66\,V$ or $2.54\,V$ $\Rightarrow V_1 = 0.66\,V$ since $V_{GS1} > V_{TH}$ in saturation

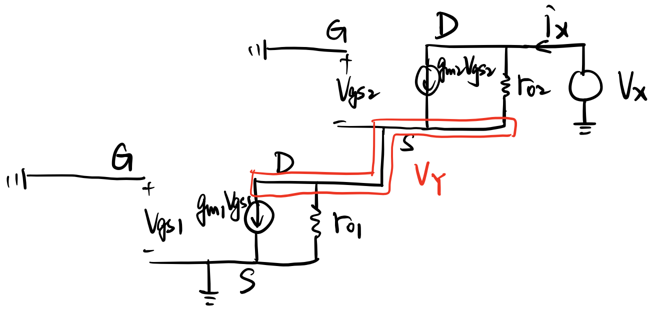
$V_{DS2} = 0.66 + 1.84 = 2.5V$, $V_{GS2} = 1.84\,V$ $\Rightarrow V_{DS2} > V_{GS2} - V_{TH}$ $\Rightarrow$ Assumption satisfied.

So $\boxed{V_1 = 0.66V,\ V_2 = -1.84\,V}$.

Question 3. Small Signal of of MOSFET

[10 points] The circuit shown below is a MOSFET cascode amplifier. Draw the small signal model and derive R_{out} for the amplifier. Assume transistors M_1 and M_2 are in saturation and include r_O in your calculation.





$$\begin{cases} V_{gs1} = 0 \quad , \quad V_{gs2} = -V_Y \\ \dfrac{V_Y}{r_{o1}} - i_x = 0 \\ V_Y = V_x - r_{o2}(i_x - g_{m2}V_{gs2}) \end{cases}$$

$$\Rightarrow \quad r_{o1} i_x = V_x - r_{o2} i_x - g_{m2} V_Y r_{o2}$$

$$r_{o1} i_x = V_x - r_{o2} i_x - g_{m2} r_{o1} r_{o2} i_x$$

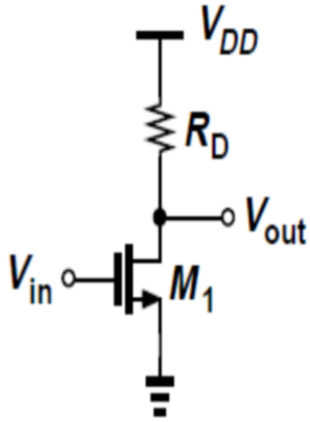
$$V_x = (r_{o1} + r_{o2} + g_{m2} r_{o1} r_{o2}) i_x$$

$$R_{out} = \frac{V_x}{i_x} = r_{o1} + r_{o2} + g_{m2} r_{o1} r_{o2}$$

So $\boxed{R_{out} = r_{o1} + r_{o2} + g_{m2} r_{o1} r_{o2}}$.

Question 4. Common-Source with Resistive Load

[20 points] Assume $\lambda = 0$ and $\gamma = 0$. For $V_{DD} = 5V$, $V_{in} = 0.9$ V + small signal, $R_D = 15k\Omega$ and $L_{drawn} = 2\mu m$, find out the value W_{drawn} to obtain a voltage gain $|A_v| > 10$ and V_{OUT} (the DC biasing voltage at the output) close to 2.5 V as much as possible.



DC: $V_{GS} = V_{IN} = 0.9\,V$, $V_{TN} = 0.7\,V$, $V_{DS} = V_{OUT} = 2.5\,V$

Assume saturation region.

$$\frac{V_{DD} - V_{OUT}}{R_D} = I_D = \frac{1}{2}\mu_n C_{ox} \frac{W_{draw}}{L_{draw} - 2L_D} (V_{GS} - V_{TN})^2$$

$$\Rightarrow W_{drawn} = \frac{2(V_{DD} - V_{OUT})(L_{drawn} - 2L_D)}{R_D \mu_n C_{ox} (V_{GS} - V_{TN})^2}$$

$$= \frac{2 \times (5 - 2.5)(2 - 2 \times 0.08)}{15 \times 10^3 \times 350 \times 10^{-4} \times \frac{8.85 \times 10^{-12} \times 3.9}{9 \times 10^{-9}} \times (0.9 - 0.7)^2}$$

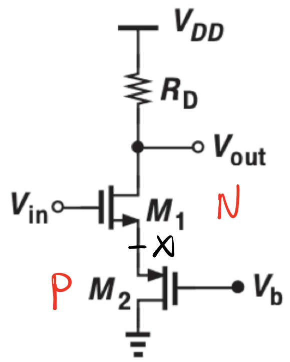
$$= 114.24\,\mu m.$$

AC: $$|A_V| = \left|\frac{V_{out}}{V_{in}}\right| = g_m R_D = \mu_n C_{ox} \frac{W_{drawn}}{L_{draw} - 2L_D} (V_{GS} - V_{TN}) R_D > 10$$

$$W_{drawn} > \frac{10\,(L_{drawn} - 2L_D)}{\mu_n C_{ox} R_D (V_{GS} - V_{TN})} = \frac{10\,(2 - 2 \times 0.08)}{350 \times 10^{-4} \times \frac{8.85 \times 10^{-12} \times 3.9}{9 \times 10^{-9}} \times (0.9 - 0.7) \times 15 \times 10^3} = 45.69\,\mu m$$

So $\boxed{W_{drawn} = 114.24\,\mu m}$.

Question 5. CS Stage DC & AC Analysis
[20 points] In the circuit below, assume $\lambda = \gamma = 0$, $\mu_n C_{ox}\left(\frac{W}{L}\right)_1 = 1mA/V^2$, $\mu_p C_{ox}\left(\frac{W}{L}\right)_2 = 5mA/V^2$, $V_{TN} = 0.7V$, $V_{TP} = -1V$, $I_{D1} = I_{D2} = 0.5mA$, $V_{DD} = 5V$, $V_b = 1V$.
(a) Calculate V_{IN} and R_D so that M_1 is at the edge of saturation.
(b) Derive the voltage gain $A_v = \frac{v_{out}}{v_{in}}$ of the circuit.



(a) M_2: $V_D < V_G + |V_{TP}|$ $\Rightarrow$ M_2 always in saturation.

$$I_{D2} = \frac{1}{2}\mu_p C_{ox}\frac{W}{L}(V_x - V_b - |V_{TP}|)^2$$
$$= \frac{1}{2}\cdot 5m(V_x - 1 - 1)^2$$
$$= 0.5mA$$

$\Rightarrow$ $V_x = 1.55V$ or $2.45V$

For M_2 to be in saturation, $V_x > V_b + |V_{TP}| = 1 + 1 = 2V$

$\therefore$ $V_x = 2.45V$

$$I_{D1} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in} - V_x - V_{TN})^2$$
$$= \frac{1}{2}\cdot 1m(V_{in} - 2.45 - 0.7)^2$$
$$= 0.5mA$$

$\Rightarrow$ $V_{in} = 4.15V$ or $2.15V$

For M_1 to be in saturation, $V_{in} > V_x + V_{TN} = 2.45 + 0.7 = 3.15V$

$\therefore$ $V_{in} = 4.15V$

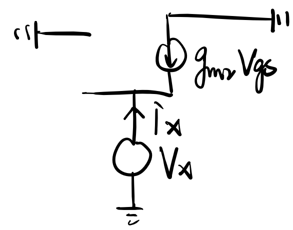
M_1 is at the edge of saturation: $V_{out} = V_{in} - V_{TN} = 3.45V$

$$\therefore R_D = \frac{V_{DD} - V_{out}}{I_{D1}} = \frac{5-3.45}{0.5m} = 3.1\,k\Omega$$

$$\therefore \boxed{V_{IN} = 4.15V,\quad R_D = 3.1\,k\Omega}$$

(b)

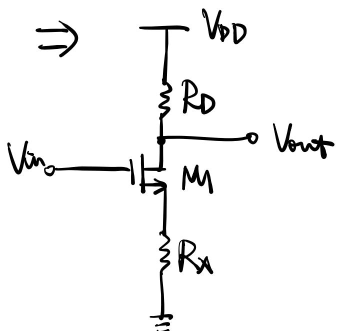
For M_2:

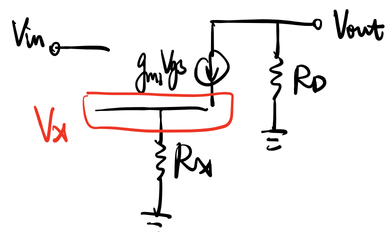


$$\begin{cases} V_{gs} = -V_x \\ i_x = -g_{m2} V_{gs} \end{cases}$$

$$\Rightarrow R_x = \frac{V_x}{i_x} = \frac{1}{g_{m2}}$$

$\Rightarrow$





$$\begin{cases} V_{out} = -g_{m1} V_{gs} R_D & ① \\ V_x = g_{m1} V_{gs} R_x & ② \\ V_{gs} = V_{in} - V_x & ③ \end{cases}$$

②、③: $$V_x = \frac{g_{m1} R_x}{g_{m1} R_x + 1} V_{in} \quad ④$$

①、③、④: $$A_v = \frac{V_{out}}{V_{in}} = -g_{m1} R_D \left(1 - \frac{g_{m1} R_x}{g_{m1} R_x + 1}\right) = -\frac{g_{m1} g_{m2} R_D}{g_{m1} + g_{m2}}$$

$$\therefore \boxed{A_v = -\frac{g_{m1} g_{m2} R_D}{g_{m1} + g_{m2}}}$$