ECE3110J/VE311 Electronic Circuits

MOS Single-Stage Amplifiers

Design of Analog CMOS Integrated Circuits, Chapter 3
Fundamentals of Microelectronics, Chapter 7

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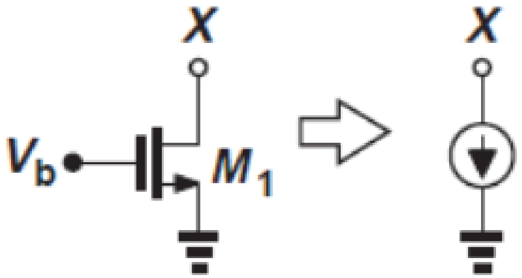
Amplification is an essential function in most analog and many digital circuits. We amplify an analog or digital signal because it may be too small to drive a load, overcome the noise of a subsequent stage, or provide logical levels to a digital circuit.

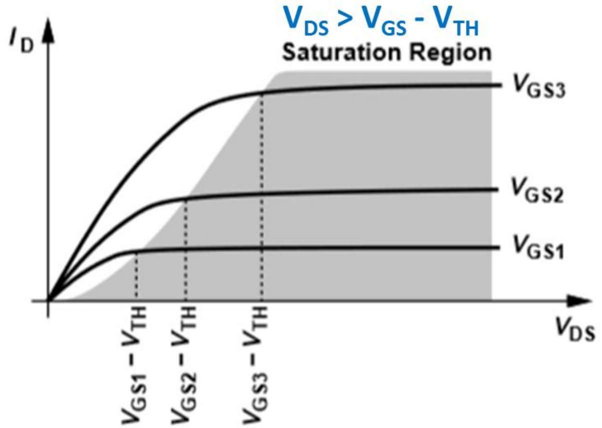
In this Chapter, we study the low-frequency behavior of single-stage MOS amplifiers. An important part of a designer's job is to **use proper approximations so as to create a simple mental picture of a complicated circuit**.

There are four types of amplifiers: (1) Common-source, (2) common-gate topology, (3) source follower, and (4) cascode configurations.

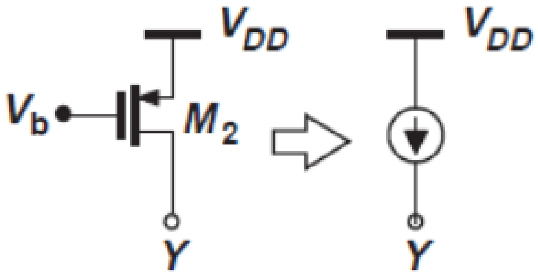
MOSFET Current Source

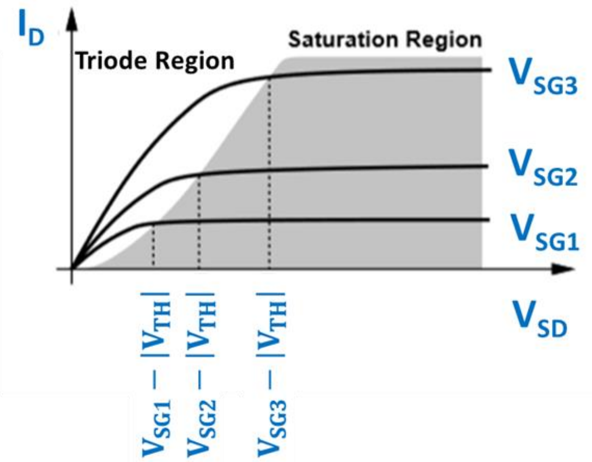
MOS transistors **operating in saturation** can act as current sources. If $\lambda = 0$, these currents remain **independent of V_X or V_Y** (so long as the transistors are in saturation).





In saturation, I_D is constant and is independent of V_D (V_{DS})



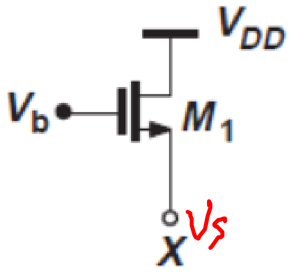


In saturation, I_D is constant and is independent of V_D (V_{SD})

However, configurations below **do not operate as current sources** because **variation of V_X or V_Y directly changes the gate-source voltage** of each transistor, thus changing the drain current considerably.

NMOS

V_{GS}

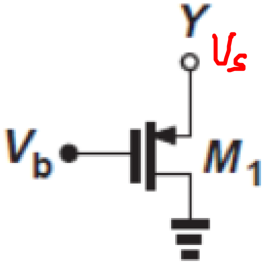


V_X changes → V_{GS} is affected → changing I_D

$I_D(sat) = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TH})^2$

$\mu_p C_{ox}$

PMOS



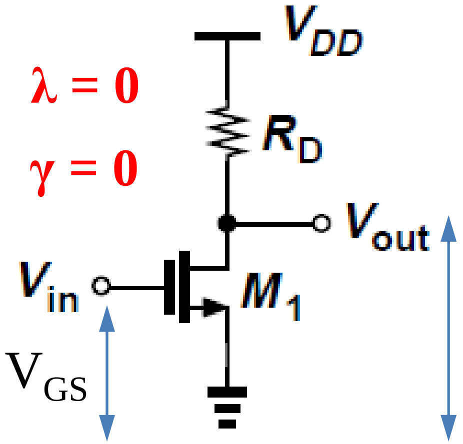
V_Y changes → V_{SG} is affected → changing I_D

Common-Source with Resistive Load

By virtue of its transconductance, a MOSFET **converts variations in its V_{GS} to a small-signal I_D**, which can pass through a resistor to generate an output voltage.

$\lambda = 0$

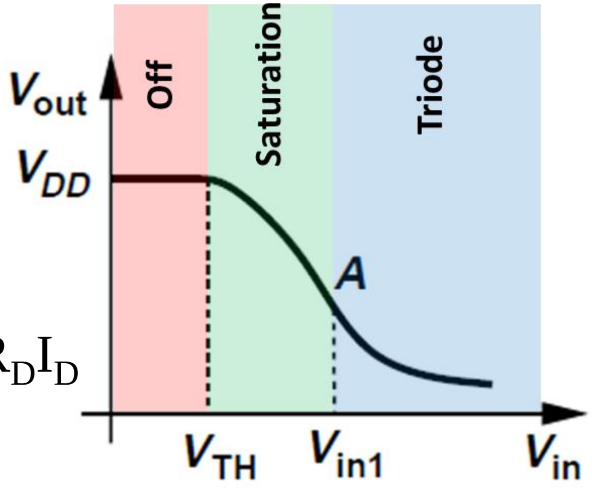
$\gamma = 0$



$V_{in} = V_{GS}$

$V_{out} = V_{DD} - R_D I_D$

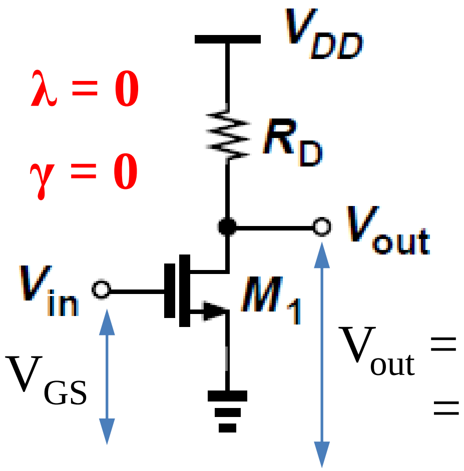
$= V_{DS}$



Large Signal Model

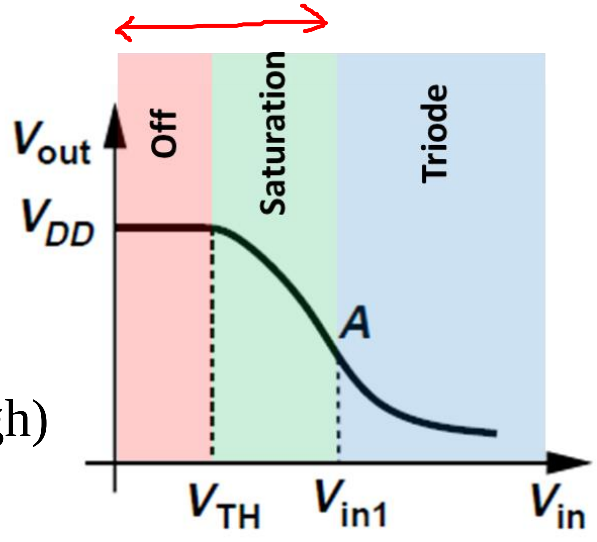
(1) $V_{in} < V_{TH} \rightarrow M_1$ Off

$\mathbf{V_{out} = V_{DD}}$



$V_{in} = V_{GS}$

$V_{out} = V_{DD}$ - $R_D I_D$ (V_{DD} is large enough)
$= V_{DS} > V_{GS}\ (V_{in}) - V_{TH}$



$V_{GS}\ (V_{in})\uparrow\ I_D\uparrow$

$V_{DS} = V_{GS} - V_{TH}$
$= V_{in} - V_{TH}. \Rightarrow V_{in} = V_{DS} + V_{TH}$

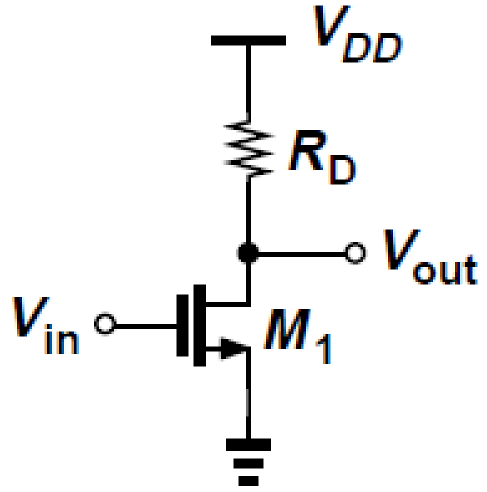
(2) $V_{in1} > V_{in} > V_{TH} \rightarrow M_1$ in Saturation until V_{in} (V_{GS}) exceeds V_{out} (V_{DS}) by V_{TH}

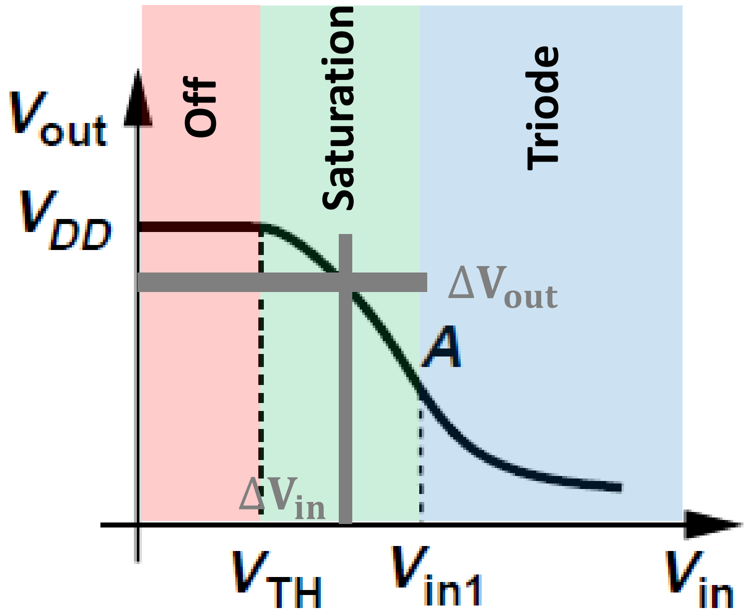
$V_{GS} = V_{in}\uparrow \Rightarrow I_D\uparrow$

$$V_{out} = V_{DD} - R_D \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in} - V_{TH})^2$$

(3) $V_{in} > V_{in1}$, i.e. $V_{in} > V_{out} + V_{TH} \rightarrow M_1$ in Triode

$$V_{out} = V_{DD} - R_D \mu_n C_{ox}\frac{W}{L}\left[(V_{in} - V_{TH})V_{out} - \frac{1}{2}{V_{out}}^2\right]$$





In Saturation, $V_{out} = V_{DD} - R_D \overbrace{\left(\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in} - V_{TH})^2\right)}^{I_D}$

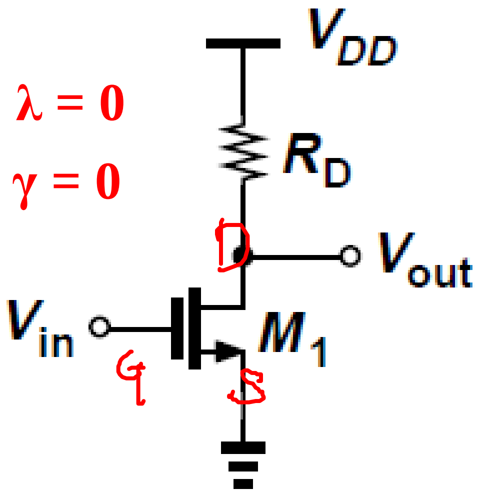
$$A_v = \frac{\partial V_{out}}{\partial V_{in}} = -R_D \overbrace{\left[\mu_n C_{ox}\frac{W}{L}(V_{in} - V_{TH})\right]}^{g_m} = -g_m R_D$$

V_{gs} increases by ΔV_{in} → $\mathbf{I_d}$ **increases** by $\Delta V_{in} \cdot g_m$ → $\mathbf{V_{out}}$ **decreases** by $\Delta V_{in} \cdot (g_m \cdot R_D)$

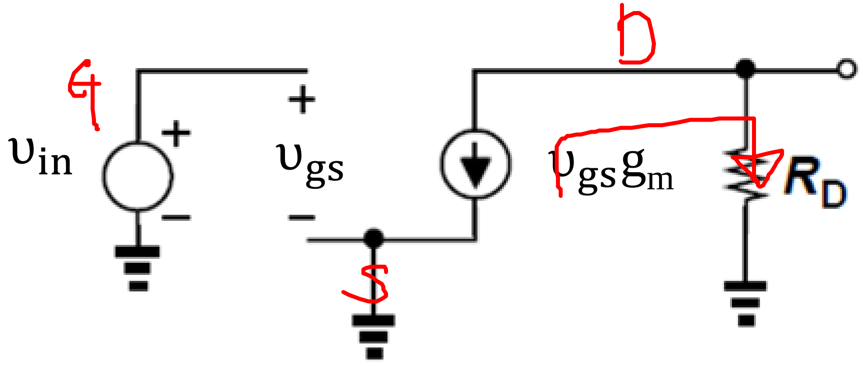
Small Signal Model

$\lambda = 0$

$\gamma = 0$



$\Rightarrow$



$= -g_m R_D v_{gs}$

$v_{gs} = v_{in}$

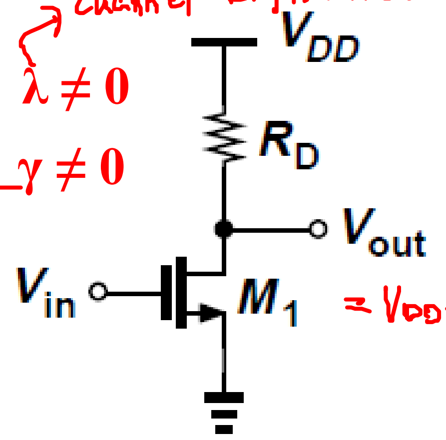
$$A_v = \frac{\upsilon_{out}}{\upsilon_{in}} = -g_m R_D$$

Small-signal analysis leads to the same result as DC analysis

Large Signal Model with $\lambda \neq 0$ and $\gamma \neq 0$

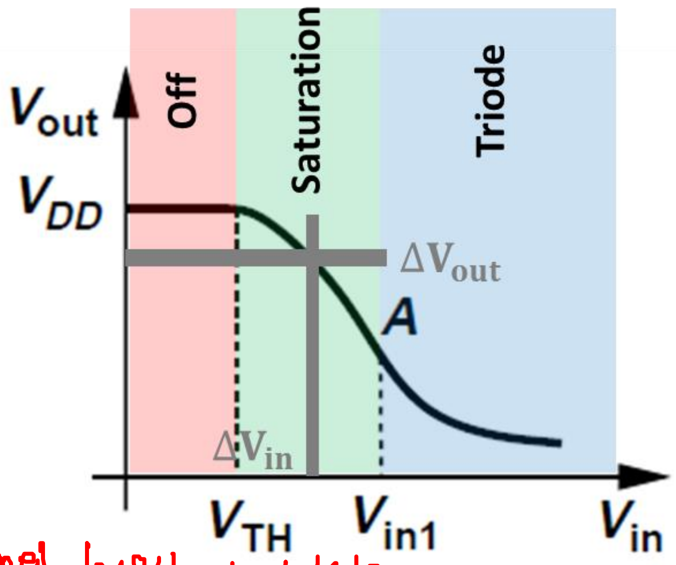
channel length modulation → $\lambda \neq 0$

Body effect → $\gamma \neq 0$



$= V_{DD} - R_D I_D$.

$r_o = \dfrac{1}{I_D \lambda}$.



I_D with Channel length modulation.

$$A_v = \frac{\partial V_{out}}{\partial V_{in}} = \frac{\partial\left[V_{DD} - \left(R_D \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in}-V_{TH})^2(1+\lambda V_{out})\right)\right]}{\partial V_{in}}$$

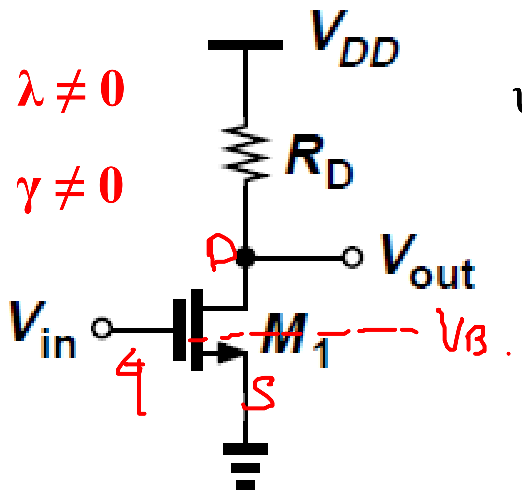
$$= -R_D \underbrace{\boxed{\mu_n C_{ox}\frac{W}{L}(V_{in}-V_{TH})(1+\lambda V_{out})}}_{= g_m} - R_D \underbrace{\boxed{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in}-V_{TH})^2}}_{\approx I_D}\lambda \underbrace{\boxed{\frac{\partial V_{out}}{\partial V_{in}}}}_{= A_v}$$

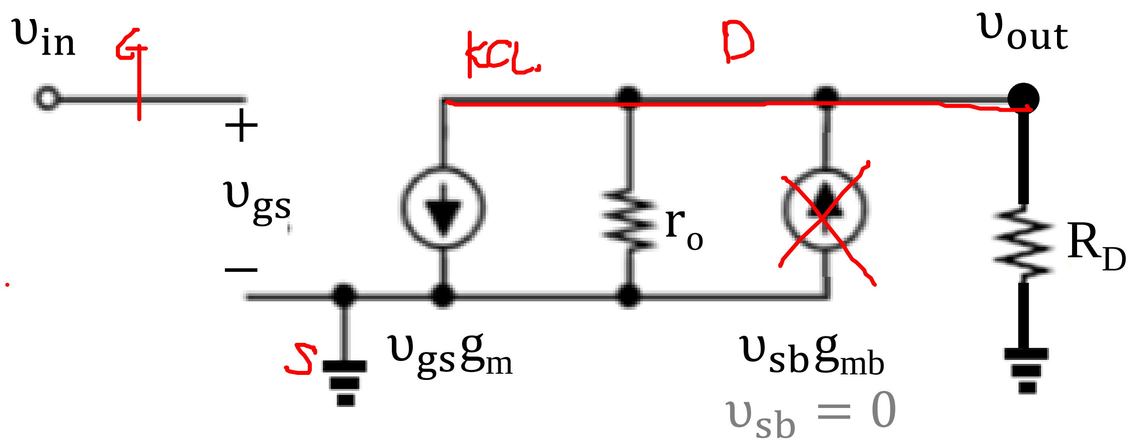
$$\mathbf{A_v} = \frac{-g_m R_D}{1 + R_D \underbrace{\boxed{I_D\lambda}}_{= 1/r_o}} = -g_m \frac{1}{\frac{1}{R_D} + \frac{1}{r_o}} = -\boldsymbol{g_m}(\mathbf{R_D} \parallel \mathbf{r_o})$$

Small Signal Model with $\lambda \neq 0$ and $\gamma \neq 0$

$\lambda \neq 0$

$\gamma \neq 0$





$$A_v = \frac{\upsilon_{out}}{\upsilon_{in}} = -g_m(R_D \parallel r_o)$$

by KCL.

$$v_{out}\left(\frac{1}{r_o} + \frac{1}{R_D}\right) = g_m v_{in}$$

$$g_m \cancel{v_{gs}}^{\,v_{in}} + \frac{v_{out}}{r_o} + \frac{v_{out}}{R_D} = 0, \quad v_{gs} = v_{in}$$

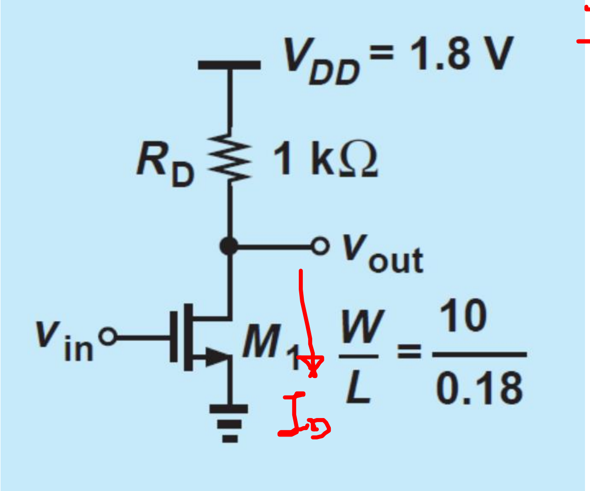
$$A_v = \frac{v_{out}}{v_{in}} = \frac{\cancel{g_m}}{\frac{1}{r_o} + \frac{1}{R_D}} = r_o \parallel R_D$$

Small-signal analysis leads to the same result as DC analysis.

Example 3.1 Calculate the small-signal voltage gain of the CS stage if $I_D = 1$ mA, $\mu_n C_{ox} = 100\ \mu A/V^2$, $V_{TH} = 0.5$ V, and $\lambda = 0$. Verify that M_1 operates in saturation.

$A_v = -g_m R_D$.

$= -3.33..$



$I_D(sat) = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH})^2$

$1mA = \frac{1}{2}\, 100\times 10^{-6} \times \frac{10}{0.18}\,(V_{GS} - 0.5)^2$

$V_{GS} = 1.1V$

$V_{out} = V_{DD} - R_D I_D$

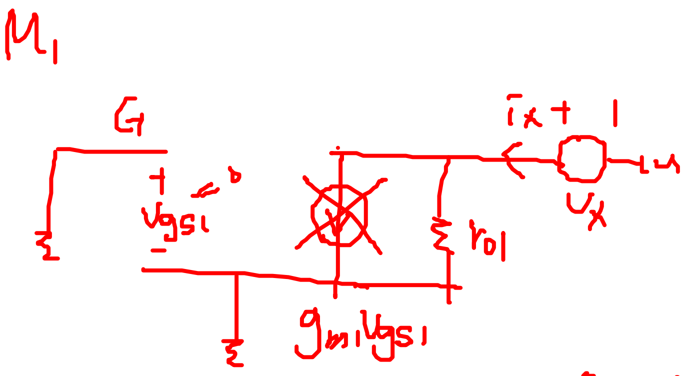
$= 1.8V - 1k \times 1mA = \underline{0.8V = V_{DS}}$

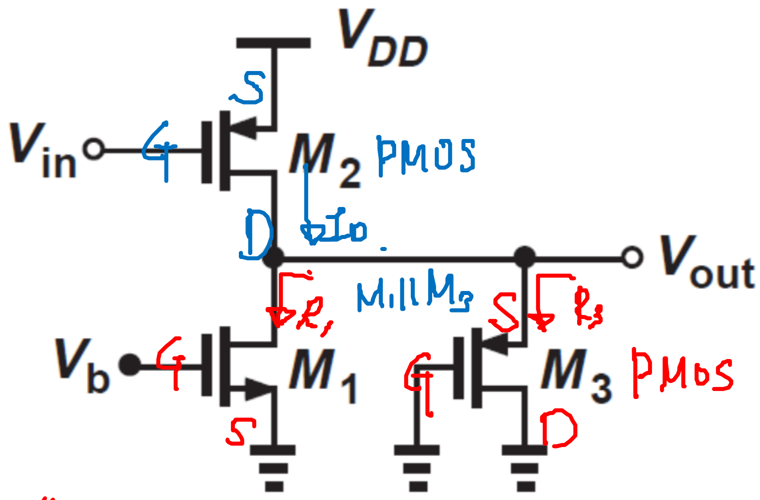
$V_{DS} > V_{GS} - V_{TH}$.

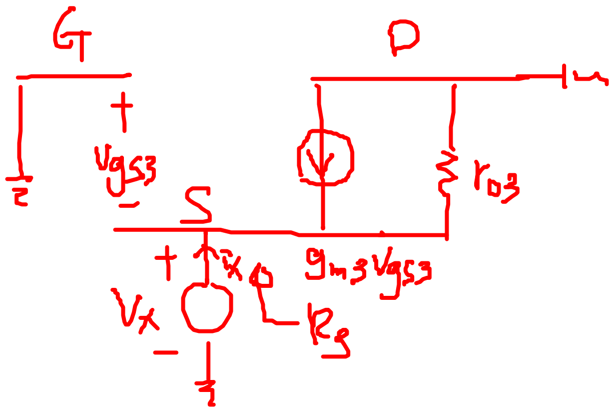
$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \mu_n C_{ox}\frac{W}{L'}(V_{GS} - V_{TH}) = \sqrt{2\mu_n C_{ox}\frac{W}{L'} I_D} = \frac{2ID}{V_{GS} - V_{TH}}$$

$= 0.0033...$

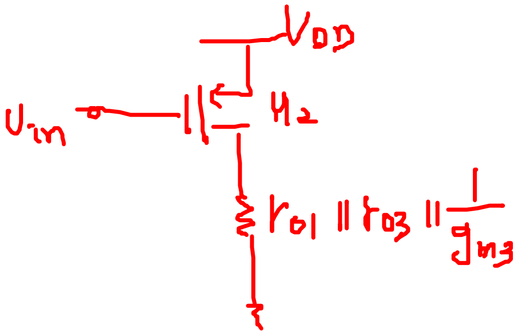
Example 3.2 If $\lambda \neq 0$, determine the voltage gain of the stages. No body effect.







$R_1 = r_{o1}$



$\Rightarrow -g_m(r_o \| R_D)$

$A_v = -g_{m2}\left(r_{o2} \| r_{o1} \| r_{o3} \| \frac{1}{g_{m3}}\right)$

$i_x = -g_{m3} V_{gs3} + \frac{V_x}{r_{o3}}$

$V_{gs3} = -V_x$

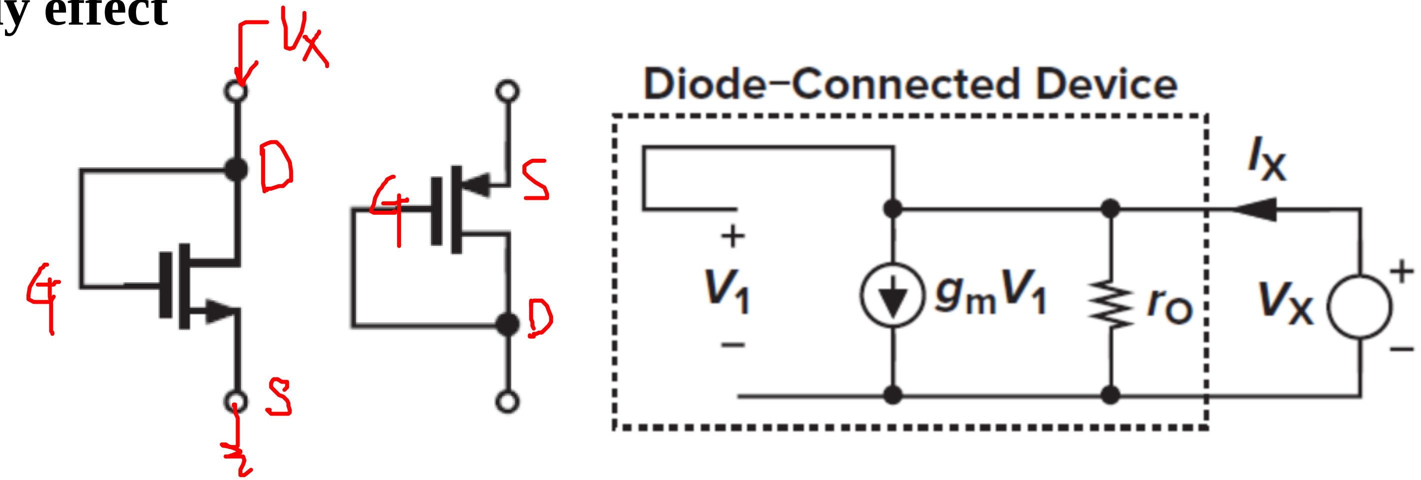
$i_x = g_{m3} V_x + \frac{V_x}{r_{o3}}$

$R_3 = V_x / i_x = \frac{1}{g_{m3} + 1/r_{o3}} = \frac{1}{g_{m3}} \| r_{o3}$

Common-Source with Diode-Connected Load

In many CMOS technologies, it is difficult to fabricate resistors with tightly-controlled values or a reasonable physical size. A MOSFET can operate as **a small-signal resistor** if its gate and drain are shorted, called **a diode-connected** device.

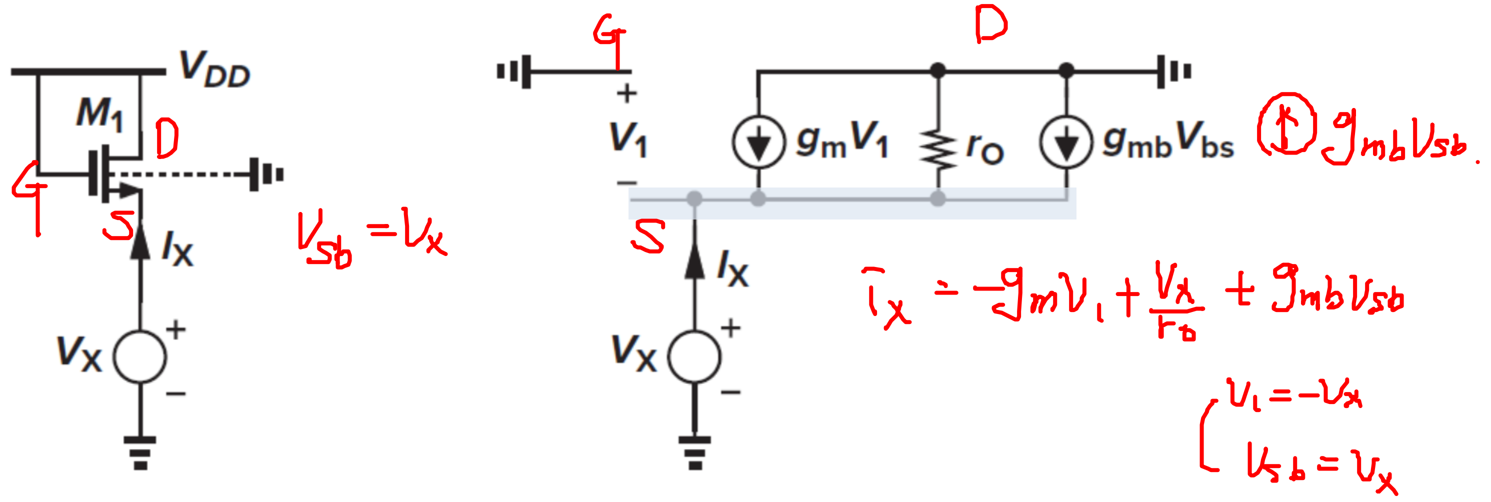
Without the body effect



From the small-signal model, **impedance of the diode-connected MOSFET** is

$$1/g_m \| r_o \approx 1/g_m$$

With the body effect



From the circuit above, $V_1 = -V_X$ and $V_{bs} = -V_X$ and $I_X = {}^{V_X}/_{r_O} + (g_m+g_{mb})V_X$

Thus, $^{\mathbf{V_X}}/_{\mathbf{I_X}} = \dfrac{1}{g_m+g_{mb}+r_O^{-1}} = \dfrac{1}{g_m+g_{mb}} \| r_O \approx \dfrac{\mathbf{1}}{\boldsymbol{g_m}\cancel{+}\boldsymbol{g_{mb}}}$

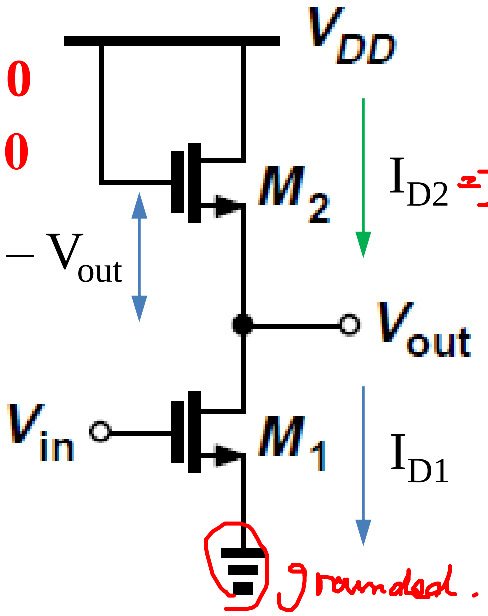
CS Stage with diode-connected load

(1) Large Signal Model

$\lambda = 0$

$\gamma \neq 0$

$V_{GS2} = V_{DD} - V_{out}$



$I_{D2} = I_{D1}.$

I_{D1}

$\frac{d}{dV_{in}}$

When M_1 is in saturation

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \overbrace{\boxed{(V_{in}-V_{TH1})^2}}^{=V_{GS1}-V_{TH1}} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 \overbrace{\boxed{[V_{DD}-V_{out}-V_{TH2}]^2}}^{=V_{GS2}-V_{TH2}}$$

$$\sqrt{\left(\frac{W}{L}\right)_1}(V_{in}-V_{TH1}) = \sqrt{\left(\frac{W}{L}\right)_2}(V_{DD}-V_{out}-V_{TH2})$$

Variable as V_{GS2} is dependent on V_{out}

$$\sqrt{\left(\frac{W}{L}\right)_1} = \sqrt{\left(\frac{W}{L}\right)_2}\left(-\frac{\partial V_{out}}{\partial V_{in}} - \frac{\partial V_{TH2}}{\partial V_{in}}\right)$$

η

$$\sqrt{\left(\frac{W}{L}\right)_1} = \sqrt{\left(\frac{W}{L}\right)_2}\left(-\frac{\partial V_{out}}{\partial V_{in}} - \frac{\partial V_{TH2}}{\partial V_{out}}\frac{\partial V_{out}}{\partial V_{in}}\right)$$

$V_{out} = V_{SB}$

$$= \eta = \frac{\gamma}{2\sqrt{2\Phi_F + V_{SB}}} \text{ (Part 2 Chapter 2)}$$

$$A_v = \frac{\partial V_{out}}{\partial V_{in}} = -\sqrt{\frac{(W/L)_1}{(W/L)_2}}\frac{1}{1+\eta}$$

Recall: Modified threshold voltage with body effect

$$V_{TH} = V_{TH0} + \gamma\left(\sqrt{|2\Phi_F + V_{SB}|} - \sqrt{|2\Phi_F|}\right) \qquad \Phi_F = \frac{kT}{q}\ln\frac{N_{sub}}{n_i} \qquad \gamma = \frac{\sqrt{2q\epsilon_{Si}N_{sub}}}{C_{ox}}$$

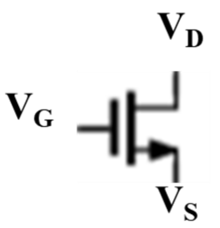
For the transconductance,

$$g_{mb} = \frac{\partial I_D}{\partial V_{BS}}\left(= -\frac{\partial I_D}{\partial V_{SB}}\right) = \frac{\partial I_D}{\partial V_{TH}} \cdot \frac{\partial V_{TH}}{\partial V_{BS}} \qquad I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L'}(V_{GS} - V_{TH})^2$$

$$= +\mu_n C_{ox}\frac{W}{L'}(V_{GS} - V_{TH}) \cdot \boxed{\frac{\partial V_{TH}}{\partial V_{BS}}} = \boxed{\mu_n C_{ox}\frac{W}{L'}(V_{GS} - V_{TH})} \cdot \boxed{\frac{\gamma}{2}\frac{1}{\sqrt{|2\Phi_F + V_{SB}|}}}$$

$$= +\frac{\partial V_{TH}}{\partial V_{SB}} = \eta \qquad = \mathbf{g_m} \qquad = \boldsymbol{\eta}$$

$$= g_m \cdot \eta$$

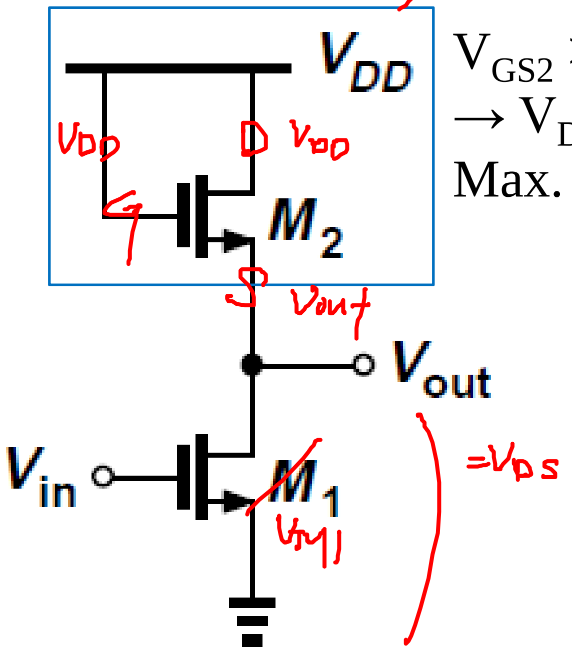


$\Delta I_D = \Delta V_{GS} \times g_m$ $\quad$ $\Delta I_D = \Delta V_{BS}\ (> 0) \times g_{mb}$

- V_{GS} positively increases, I_D increases.
- V_{BS} positively increases, i.e. V_{SB} negatively increases, V_{TH} decreases and thus I_D increases.
- Or, V_{SB} leads to changes in V_{TH} and thus I_D.

$V_{DS} > V_{GS} - V_{TH2} \Rightarrow V_{DD} - \cancel{V_{out}} > V_{DD} - \cancel{V_{out}} - V_{TH2}$

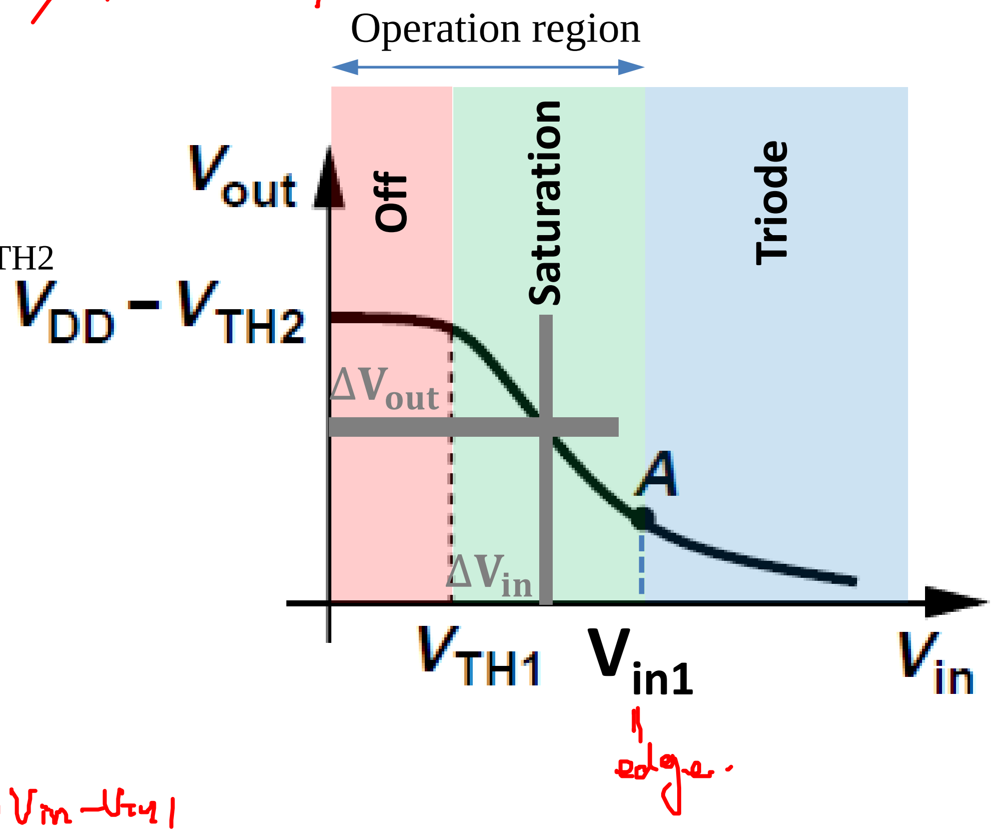
Always ON



$V_{GS2} > V_{TH2}$

$\rightarrow V_{DD} - V_{out} > V_{TH2}$

Max. voltage drop = V_{TH2}



(1) $V_{in} < V_{TH1}$, i.e. M_1 is off

$\mathbf{V_{out} = V_{DD} - V_{TH2}}$

$V_{out} > V_{in} - V_{TH1}$

$V_{DS} > V_{GS} - V_{TH1}$

(2) $V_{in1} > V_{in} > V_{TH1}$, i.e. M_1 in saturation

$\mathbf{V_{out} = V_{in1} - V_{TH1}}$

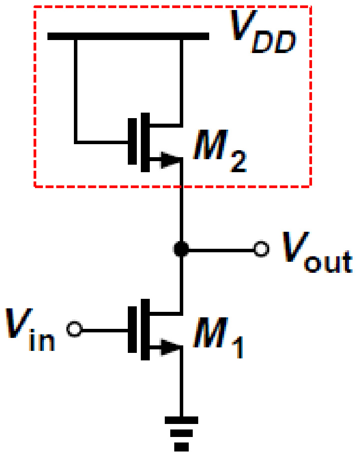
(3) $V_{in} > V_{out} + V_{TH1}$, i.e. M_1 in triode $\rightarrow V_{DS} < \overset{V_{GS}}{\cancel{V_{in}}} - V_{TH1}$ $\quad V_{out} < V_{in} - V_{TH1}$

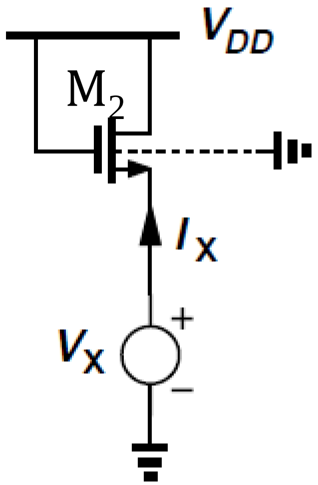
V_{out} further drops as M_1 goes into the triode region ($V_{out} = V_{DS}$)

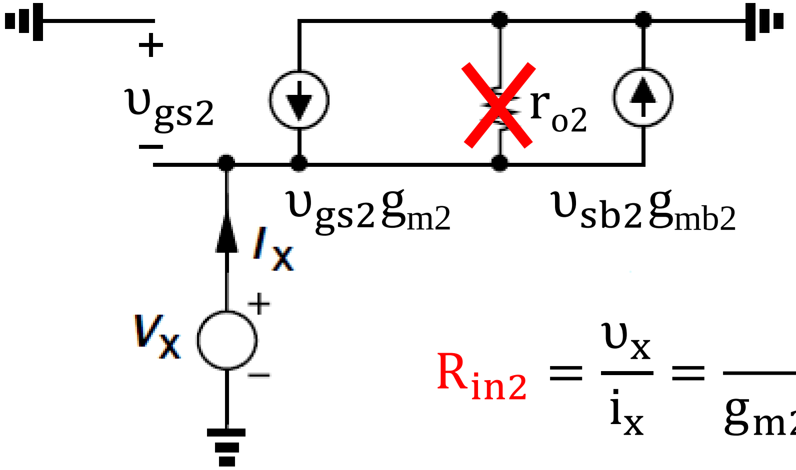
(2) Small-Signal Model

$\lambda = 0$

$\gamma \neq 0$





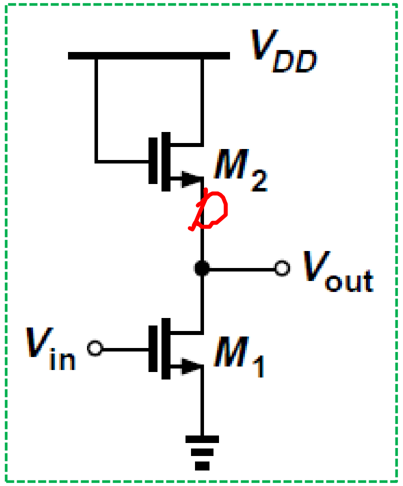


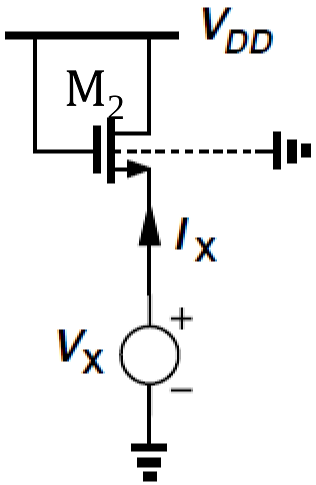
$$R_{in2} = \frac{\upsilon_x}{i_x} = \frac{1}{g_{m2} + g_{mb2}}$$

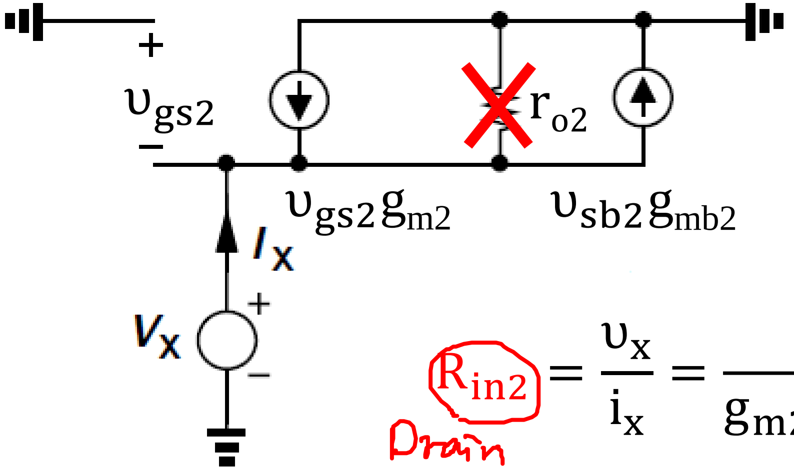
(2) Small-Signal Model

$\lambda = 0$

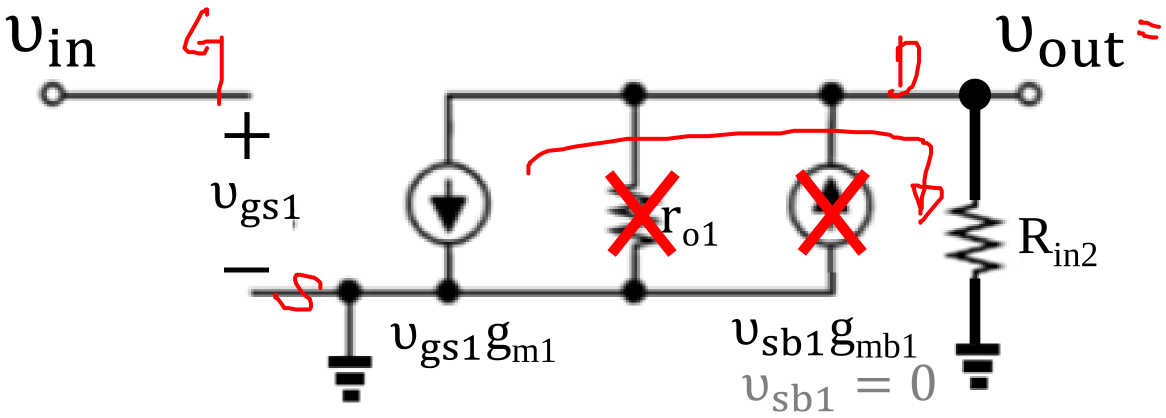
$\gamma \neq 0$







$$\underset{\text{Drain}}{R_{in2}} = \frac{\upsilon_x}{i_x} = \frac{1}{g_{m2} + g_{mb2}}$$



$$\upsilon_{out} = -g_{m1} v_{gs} R_{in2}.$$

$$A_v = \frac{\upsilon_{out}}{\upsilon_{in}} = \frac{-g_{m1}}{g_{m2} + \underset{= \eta g_{m2}}{g_{mb2}}} = -\frac{g_{m1}}{(1+\eta) g_{m2}}.$$

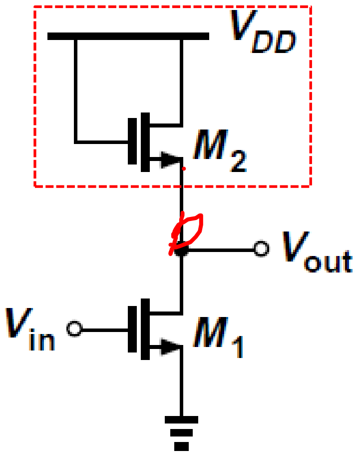
$$= -\sqrt{\frac{(W/L)_1}{(W/L)_2}} \frac{1}{1+\eta}$$

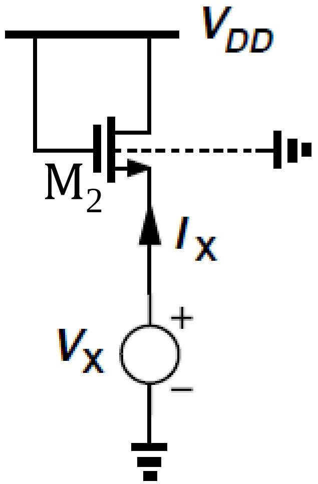
Small-signal analysis leads to the same result as DC analysis.

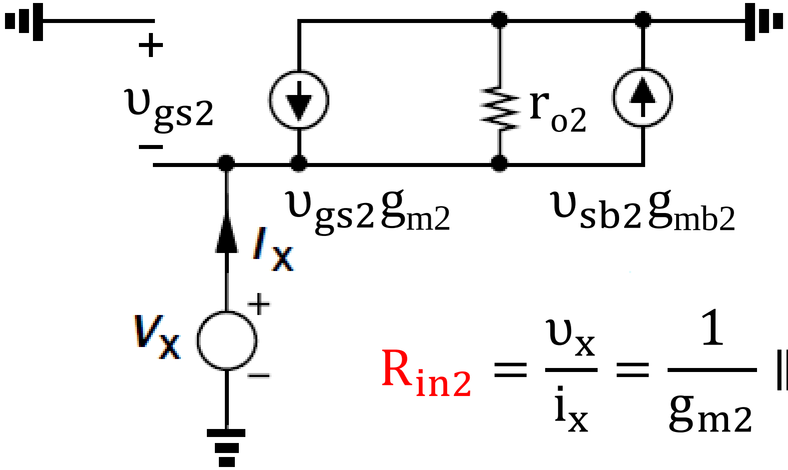
(2) Small-Signal Model

$\lambda \neq 0$

$\gamma \neq 0$



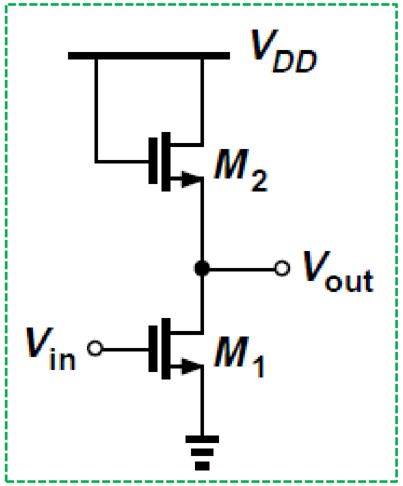


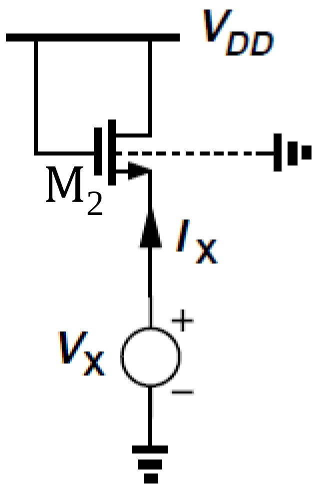


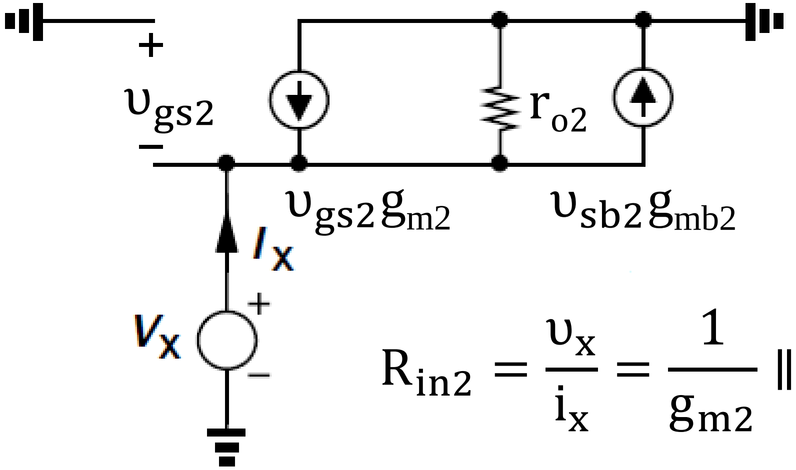
$$R_{in2} = \frac{\upsilon_x}{i_x} = \frac{1}{g_{m2}} \parallel \frac{1}{g_{mb2}} \parallel r_{o2}$$

(2) Small-Signal Model

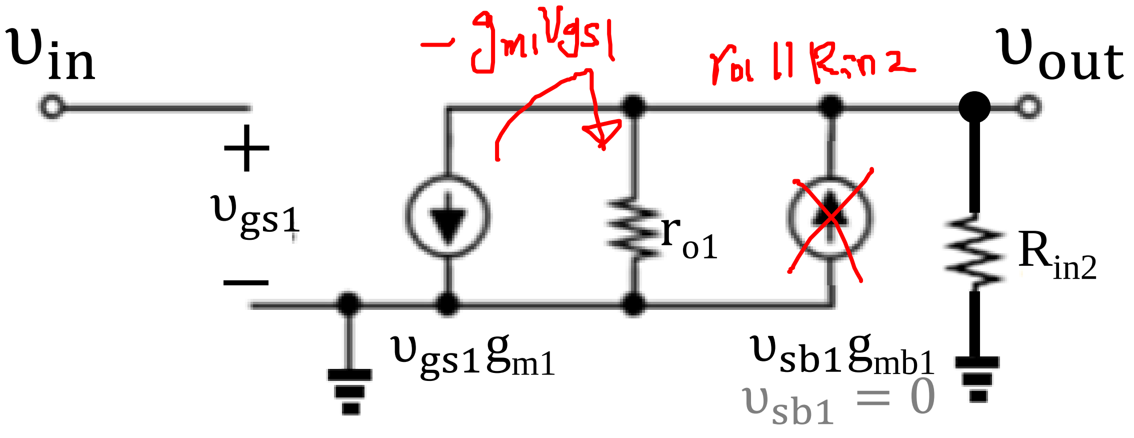
$\lambda \neq 0$
$\gamma \neq 0$







$$R_{in2} = \frac{\upsilon_x}{i_x} = \frac{1}{g_{m2}} \parallel \frac{1}{g_{mb2}} \parallel r_{o2}$$



$$A_v = \frac{\upsilon_{out}}{\upsilon_{in}}$$

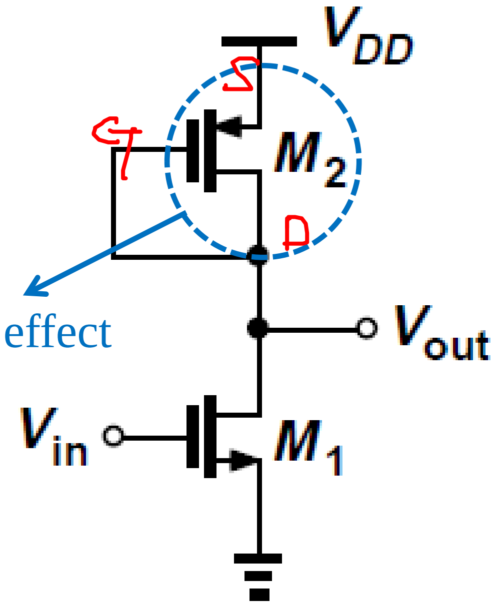
R_{in2} $\quad r_o >> 1/gm$

$$= -g_{m1}\left(\frac{1}{g_{m2}} \parallel \frac{1}{g_{mb2}} \parallel r_{o2} \parallel r_{o1}\right)$$

CS Stage with diode-connected load – PMOS

$\lambda \neq 0$

$\gamma \neq 0$



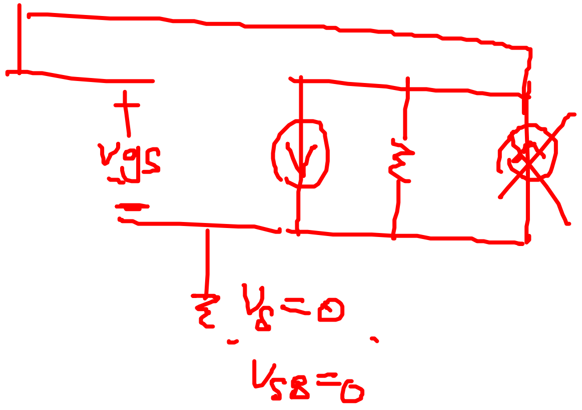
$R_{eq} < 20$

~~10 || 100 || 100~~

20 || 200 || 2000

$r_o >> 1/gm$

$$A_v = \frac{\upsilon_{out}}{\upsilon_{in}} = -g_{m1}\left(\frac{1}{g_{m2}} \parallel r_{o2} \parallel r_{o1}\right) \approx -\frac{g_{m1}}{g_{m2}} = -\sqrt{\frac{\mu_n (W/L)_1}{\mu_p (W/L)_2}} = -\frac{V_{SG2} - V_{TH2}}{V_{GS1} - V_{TH1}}$$

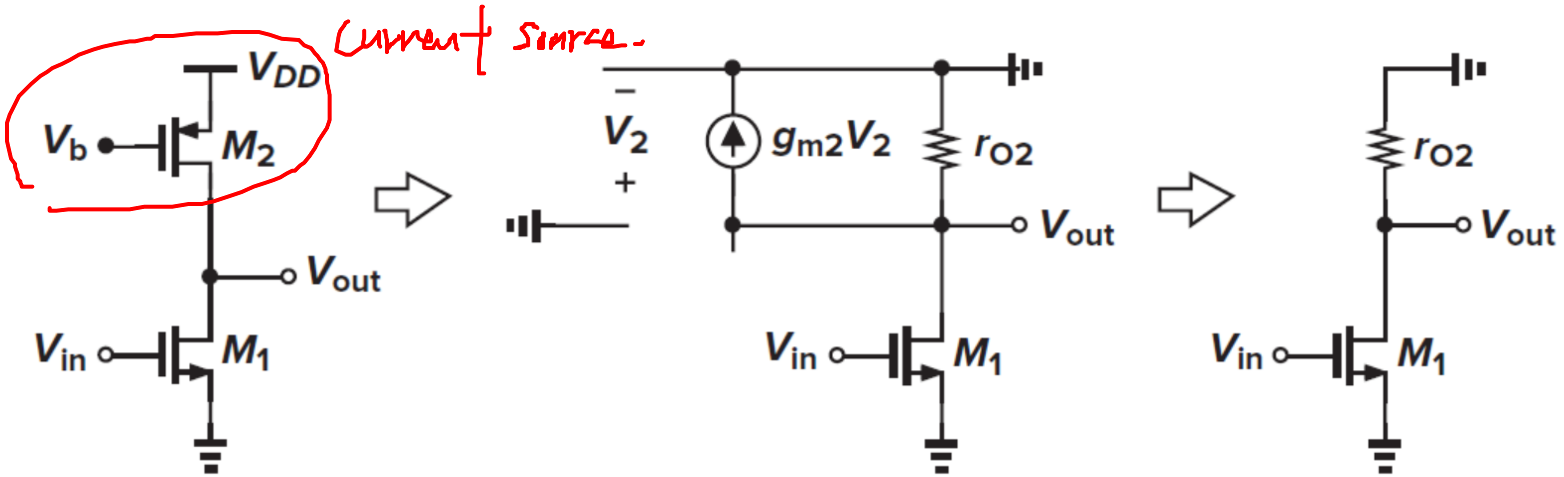


$\frac{1}{g_m}$, $\frac{1}{g_m} \parallel \frac{1}{g_{mb}}$, $\frac{1}{g_m} \parallel \frac{1}{g_{mb}} \parallel r_o$

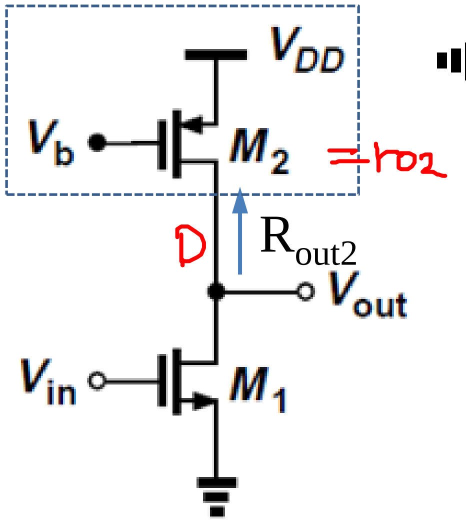
$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \mu_n C_{ox} \frac{W}{L'}(V_{GS} - V_{TH}) = \sqrt{2\mu_n C_{ox} \frac{W}{L'} I_D}$$

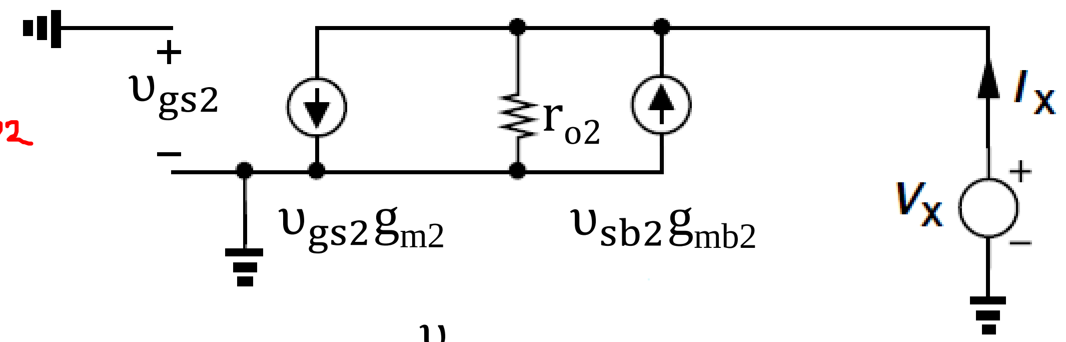
- For $A_v = 10$, $(W/L)_1 >> (W/L)_2 \rightarrow$ **Disproportionally large transistor**
- For $A_v = 10$, $(V_{SG2} - V_{TH2}) = 10 \times (V_{GS1} - V_{TH1}) \rightarrow$ **Limited output swing**

Common-Source with Current-Source Load

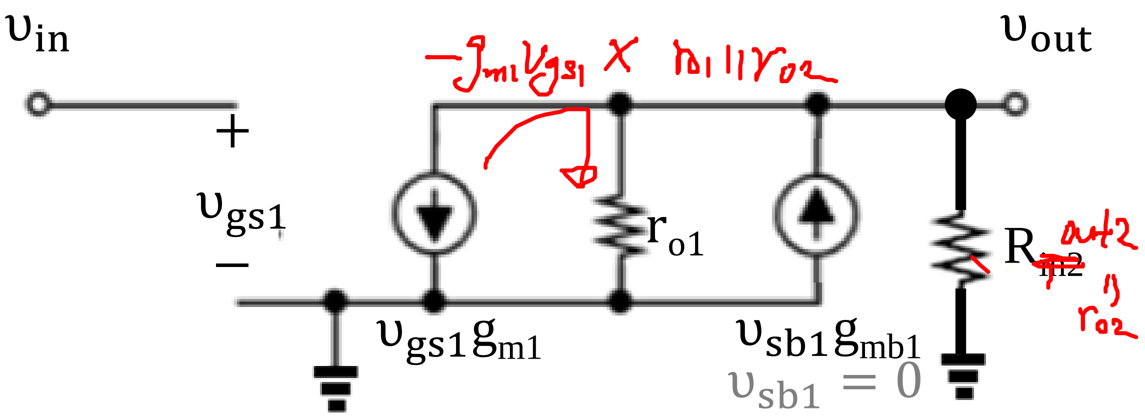


In applications requiring **a large voltage gain in a single stage**, the relationship $A_V = -g_m R_D$ suggests that we should increase the load impedance of the CS stage. With a resistor or diode-connected load, however, increasing the load resistance translates to **a large DC drop** across the load, thereby **limiting the output voltage swing**. A more practical approach is to **replace the load with a device that does not obey Ohm's law**, e.g., a current source.



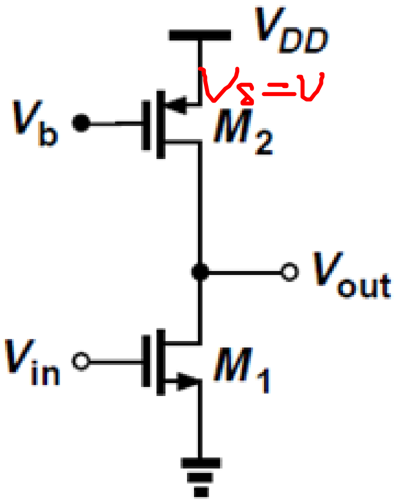


$$R_{out2} = \frac{\upsilon_x}{i_x} = r_{o2}$$



$$A_v = \frac{\upsilon_{out}}{\upsilon_{in}} = -\mathbf{g}_{m1}(\mathbf{r}_{o2} \parallel \mathbf{r}_{o1})$$

- To achieve high A_v, the output swing is severely limited in the CS stages with resistive load and diode-connected load.
- Here $\mathbf{V_{out,\,max}} = \mathbf{V_{DD}} - (\mathbf{V_{SG2}} - \mathbf{V_{TH2}})$, which can be quite close to V_{DD}.



$V_S = V_{DD}$
$V_G = V_b$
$V_D = V_{out}$

$V_{SD} = V_{SG} - |V_{TH2}|$

$V_{DD} - V_{out} = V_{SG} - V_{TH2}$

$$A_v = \frac{\upsilon_{out}}{\upsilon_{in}}$$
$$= -g_{m1}(r_{o2} \parallel r_{o1}) \qquad r_o \approx \frac{1}{\lambda I_D} \qquad \lambda \propto \frac{1}{L}$$

Sat. $V_{DS} > V_{GS} - V_{TH1}$
$V_{out} > V_{GS} - V_{TH1}$

triode $V_{out} < V_{GS} - V_{TH1}$

M_1 is off

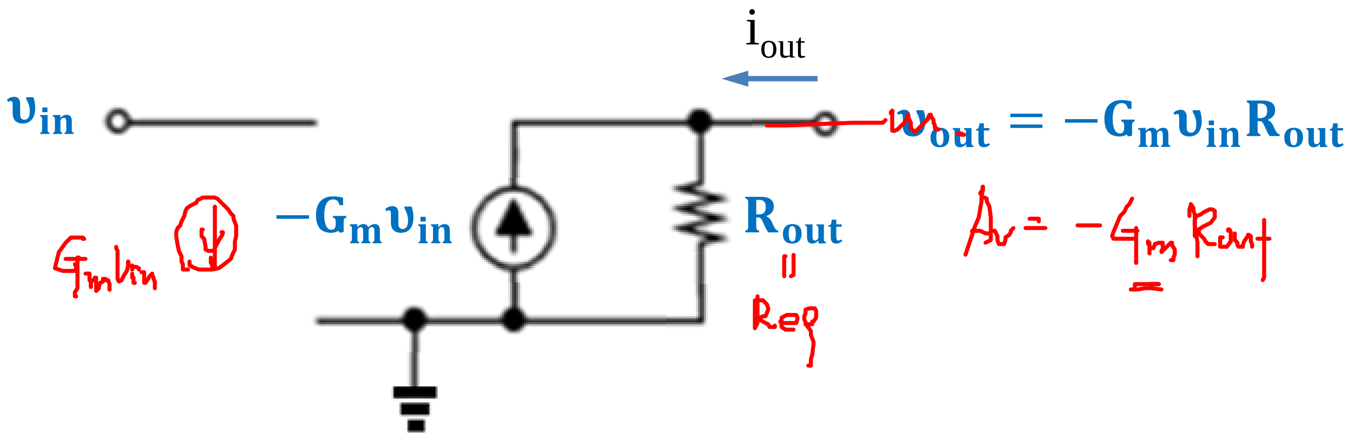
- $V_{out,\ max} = V_{DD} - (V_{SG2} - |V_{TH2}|)$
- $V_{out,\ min} = (V_{GS1} - V_{TH1})$
- For high g_{m1} and small $(V_{GS1} - V_{TH1})$, W of M_1 needs to be large.
- For high r_{o1} and r_{o2}, L of M_1 and M_2 need to be large and L of M_1 and M_2 needs to be increased proportionally. The cost is the **large parasitic drain junction capacitance** at the output.

Common-Source with Source Degeneration

In some applications, the nonlinear dependence of the drain current upon the overdrive voltage introduces excessive nonlinearity, making it desirable to “soften” the device characteristics → Source degeneration.

Transconductance of circuit G_m g_m

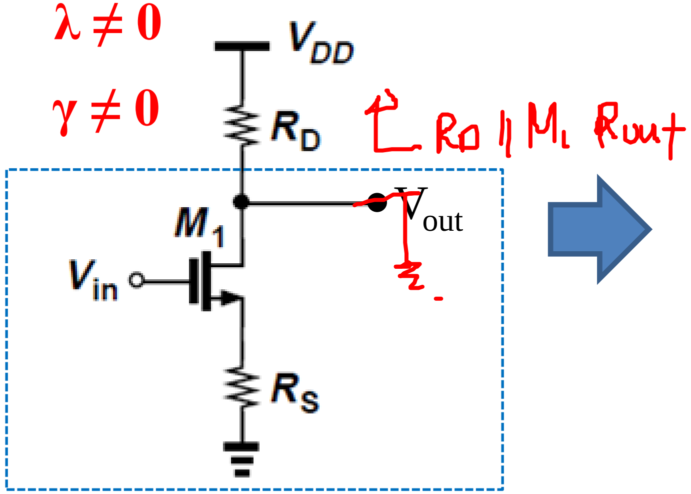
We defined the transconductance of a transistor g_m. This concept can be generalized to circuits as well. The **V_{out} is set to zero by shorting the output node to ground**, and the “**short circuit transconductance**” of the circuit is defined as $\mathbf{G_m = i_{out}/\upsilon_{in}}$

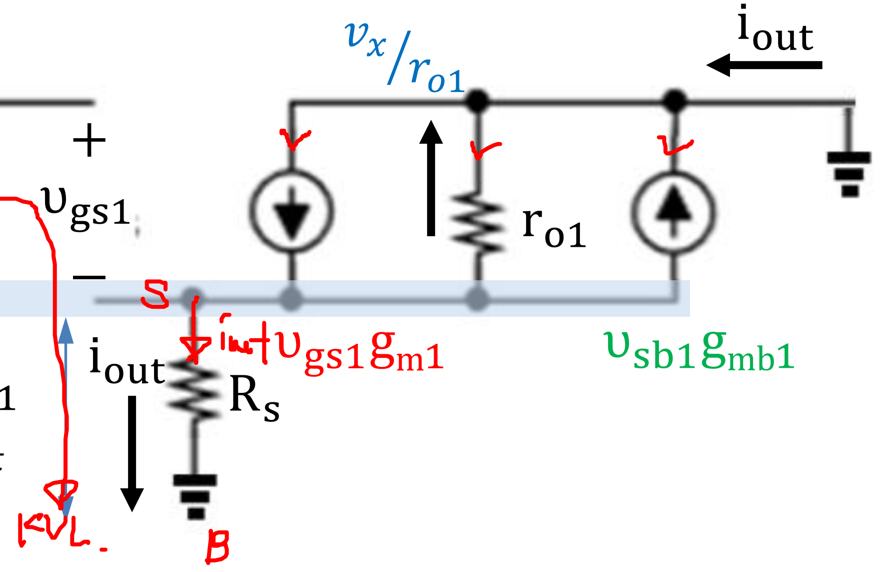


Equivalent transconductance

$\lambda \neq 0$

$\gamma \neq 0$





$\upsilon_x = \upsilon_{sb1}$
$= R_s i_{out}$

KCL $\quad i_{out} = v_{gs1}g_{m1} - \overset{R_s i_{out}}{v_x}/r_{o1} - \overset{R_s i_{out}}{v_{sb1}}g_{mb1}$ ⇒ eq with i_{out} and v_{in}

KVL $\quad -v_{in} + \upsilon_{gs1} + i_{out}R_s = 0 \rightarrow \boldsymbol{\upsilon_{gs1} = v_{in} - i_{out}R_s}$

$$G_m = \frac{i_o}{\upsilon_{in}} = \frac{g_{m1}r_{o1}}{R_S + r_{o1} + (g_{m1} + g_{mb1})r_{o1}R_S} \approx \cancel{\frac{1}{R_S}}$$

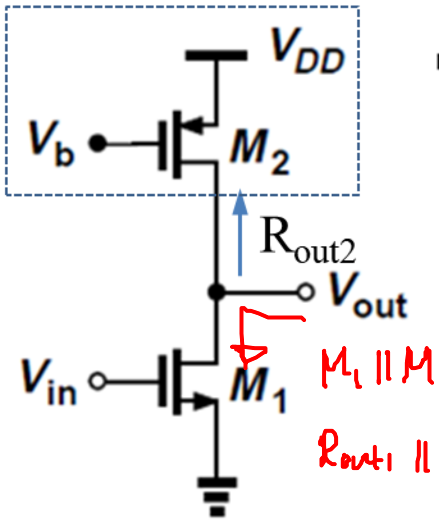
If $(g_{m1} + g_{mb1})r_{o1}R_S >> r_{o1}$ and R_S

Output Resistance

Another important consequence of source degeneration is the increase in the output resistance of the stage. How to calculate R_{out}? $\boldsymbol{\upsilon_{in}}$ **is grounded** and υ_{out} connected to υ_{test}.

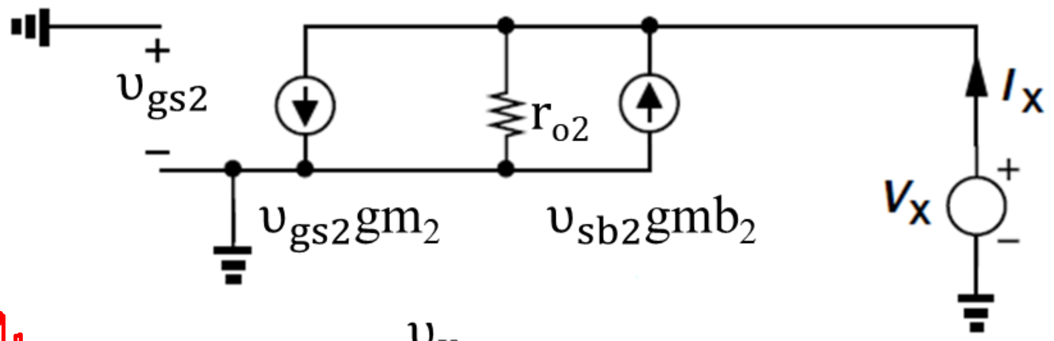
$$\mathbf{R_{out}} = \boldsymbol{\upsilon_{test}} / \mathbf{i_{test}}$$

e.g.



$M_1 \parallel M_2$

$R_{out1} \parallel R_{out2}$

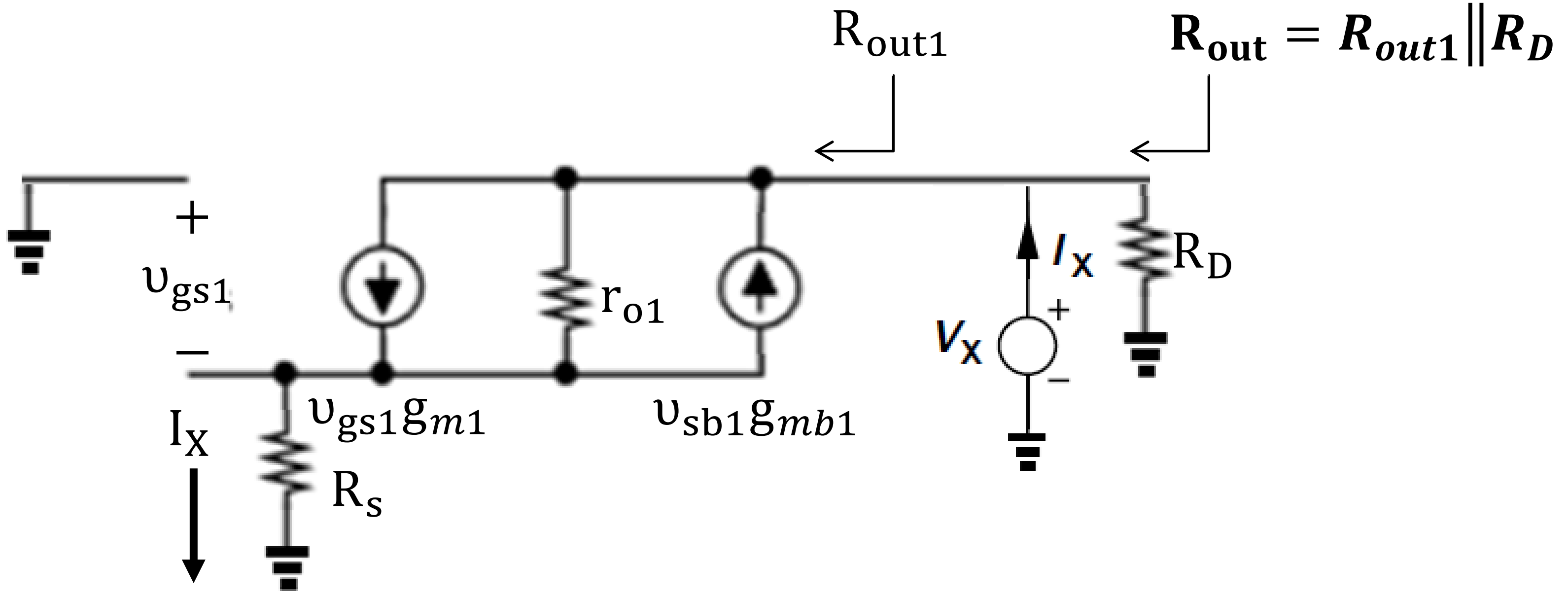


$$R_{out2} = \frac{\upsilon_x}{i_x} = r_{o2}$$

Output Resistance

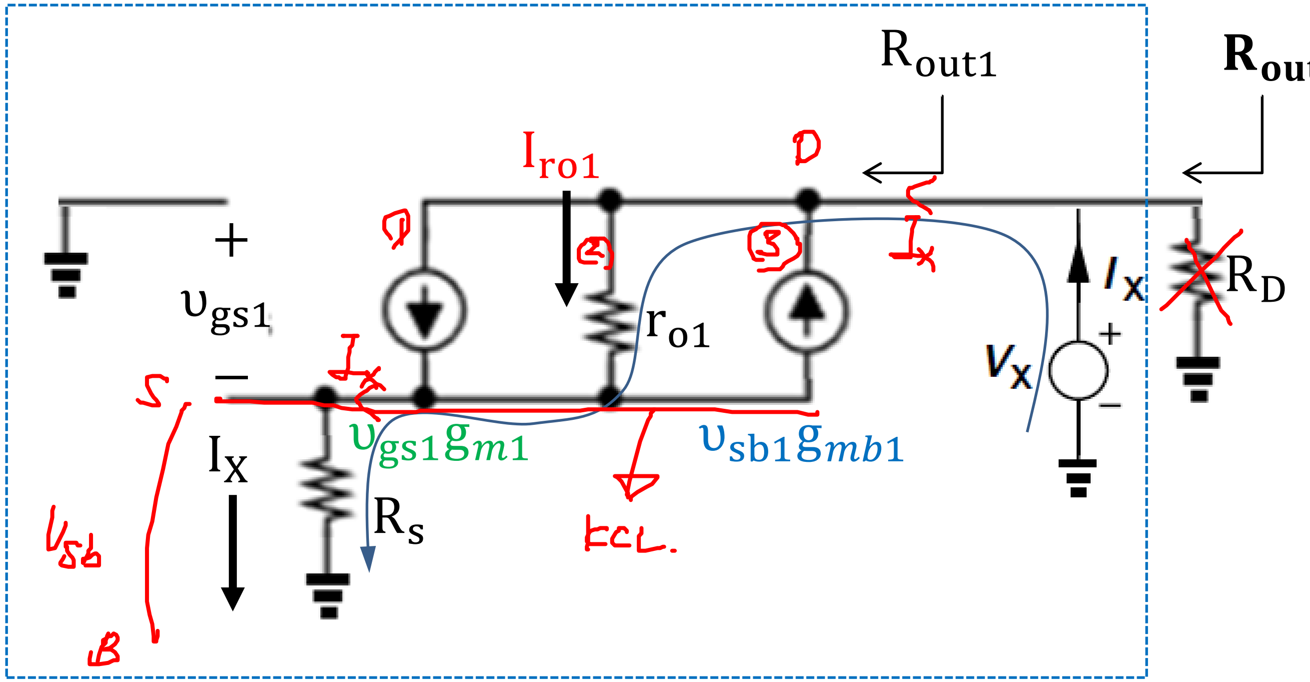
$\lambda \neq 0$

$\gamma \neq 0$



Output Resistance

$\lambda \neq 0$
$\gamma \neq 0$



from source.

By KVL, $-V_X + I_{ro1} r_{o1} + I_x R_s = 0$

By KCL, $I_{ro1} = I_x + v_{sb1} g_{mb1} - v_{gs1} g_{m1}$ where $v_{sb} = -v_{gs1} = I_x R_s$

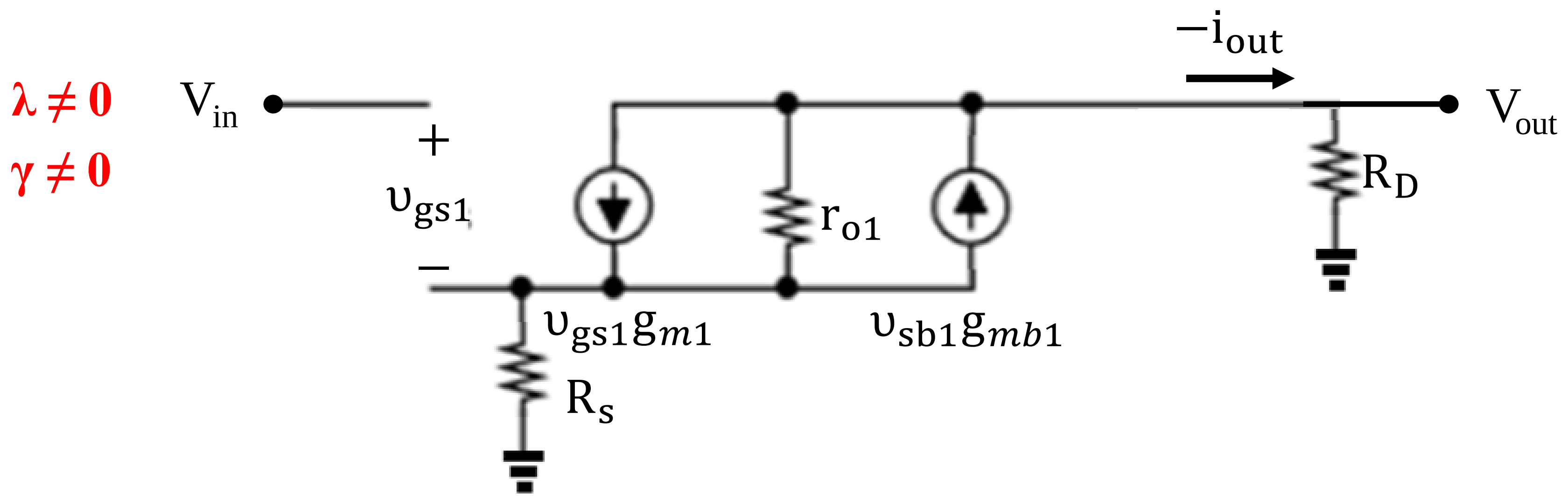
$[r_{o1} + R_s + (g_{mb1} + g_{m1}) r_{o1} R_s] I_x = V_x \rightarrow \boldsymbol{R_{out1}} = \boldsymbol{r_{o1} + R_s + (g_{mb1} + g_{m1}) r_{o1} R_s}$

V_x / I_x

$$\mathbf{R_{out}} = R_{out1} \parallel R_D = [r_{o1} + R_S + (g_{m1} + g_{mb1}) r_{o1} R_S] \parallel R_D \approx R_D$$

If $(g_{m1} + g_{mb1}) r_{o1} R_S >> R_D$

Gain of degenerated CS stage



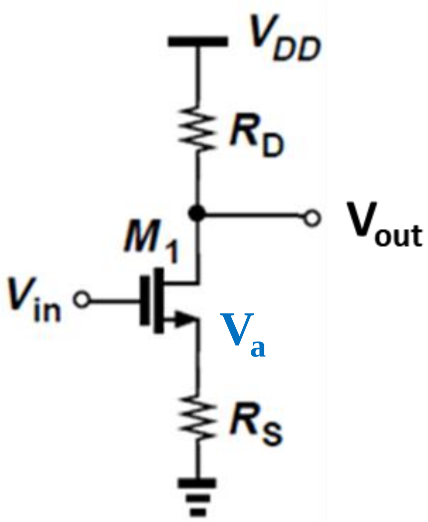
$$A_v = \frac{\upsilon_{out}}{\upsilon_{in}} = \frac{R_{out}(-i_{out})}{v_{in}} = -G_m R_{out}$$

$$= \frac{-g_{m1}r_{o1}}{\cancel{R_S + r_{o1} + (g_{m1} + g_{mb1})r_{o1}R_S}} \cdot \frac{\cancel{[R_S + r_{o1} + (g_{m1} + g_{mb1})r_{o1}R_S]}R_D}{[R_S + r_{o1} + (g_{m1} + g_{mb1})r_{o1}R_S] + R_D}$$

$$\approx -\frac{R_D}{R_S}$$

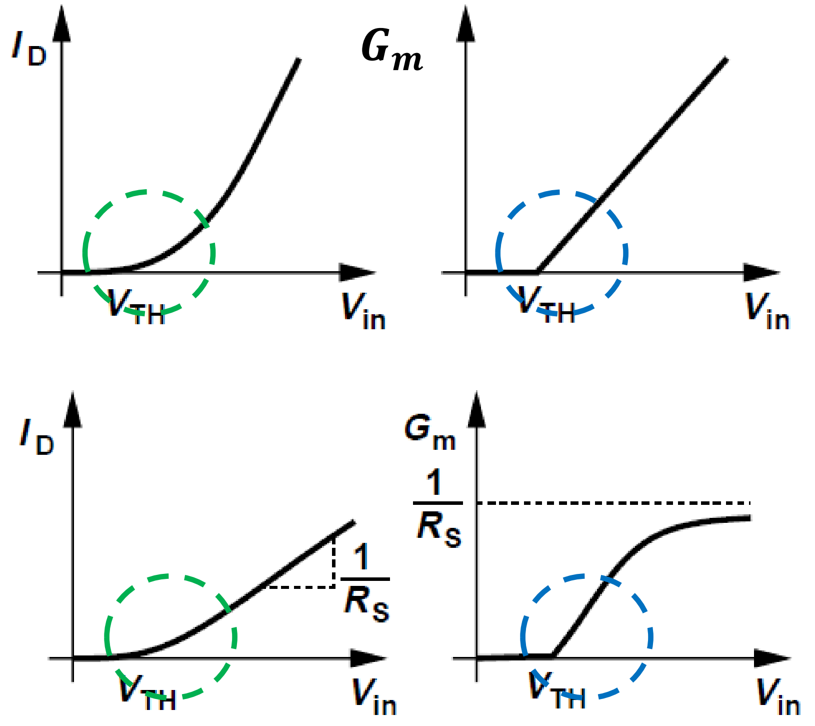
If $(g_{m1} + g_{mb1})r_{o1}$, the intrinsic gain, is large.

Large Signal Analysis



$R_S = 0$

$R_S \neq 0$



$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \mu_n C_{ox} \frac{W}{L'} (V_{GS} - V_{TH})$$

$$g_{mb} = -g_m \cdot \eta$$

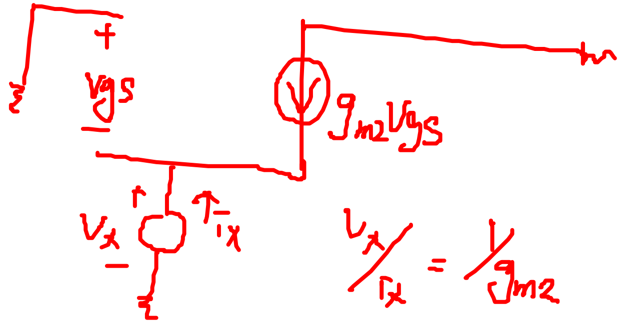
$G_m = g_{m1}$

$$G_m = \frac{i_o}{\upsilon_{in}} = \frac{g_{m1} r_{o1}}{\cancel{R}_S + r_{o1} + (g_{m1} + \cancel{g}_{mb1}) r_{o1} \cancel{R}_S} \approx \frac{1}{R_S}$$

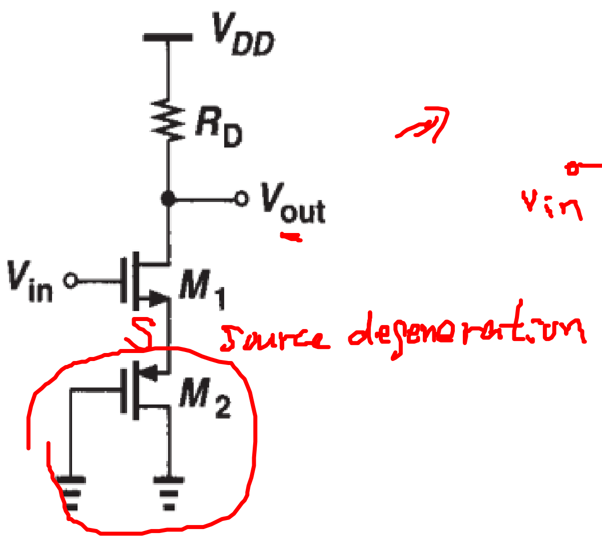
- At low V_{in} (g_m small), turn-on behavior of $R_S \neq 0$ is similar to that of $R_S = 0$.
- At large V_{in} (g_m large), the effect of R_s, i.e. degradation, becomes more significant.
- $V_{in} = 0$ V → M_1 off, no current flowing → $V_a = 0$ V and $V_{out} = V_{DD}$

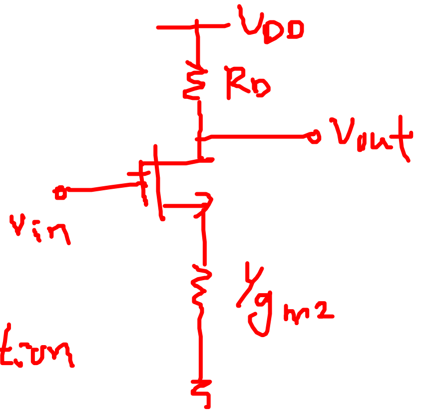
Example 3.3 Assuming $\lambda = \gamma = 0$, calculate the small signal voltage gain of the circuit below.

M_2

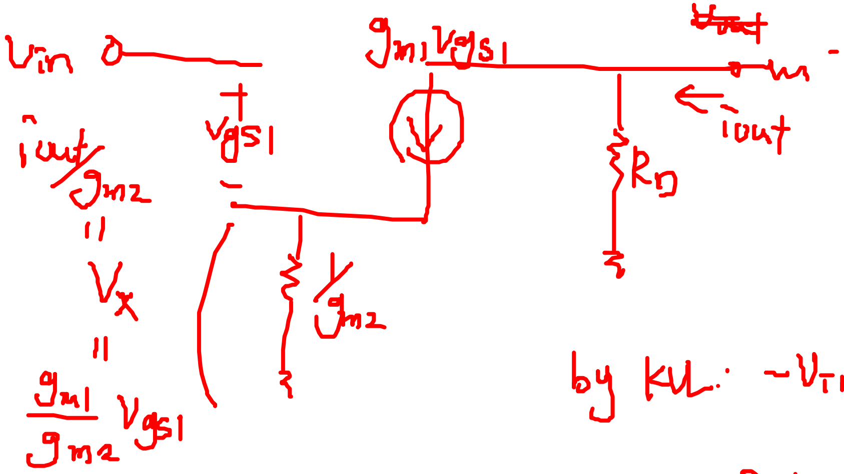


$V_x/i_x = 1/g_{m2}$





$R_{out} = R_D$



$i_{out} = g_{m1} V_{gs1}$

$V_{gs1} = \frac{i_{out}}{g_{m1}}$

by KVL: $-V_{in} + V_{gs1} + \frac{g_{m1}}{g_{m2}} V_{gs1} = 0$

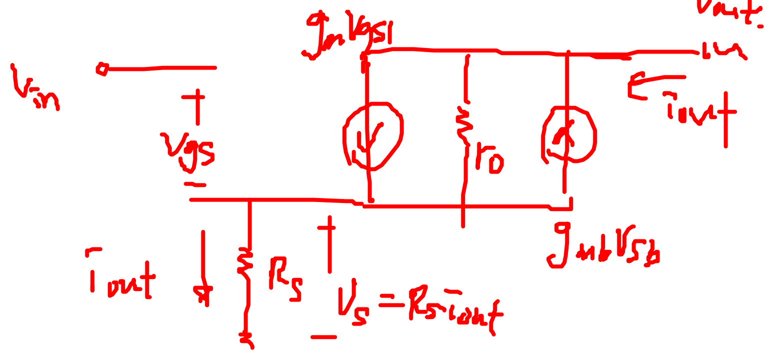
$G_m = i_{out}/V_{in} = g_{m1} \| g_{m2}$

$$A_v = -G_m R_{out}$$

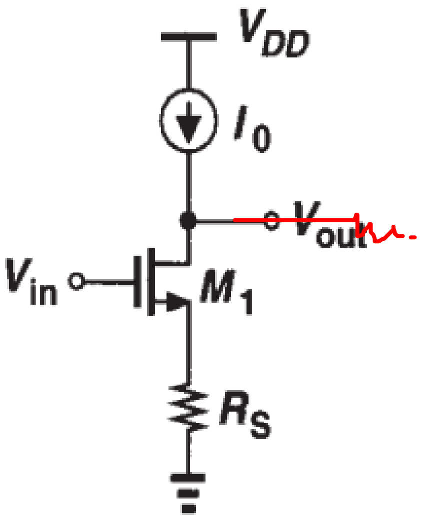
$$G_m = \frac{1}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}}$$

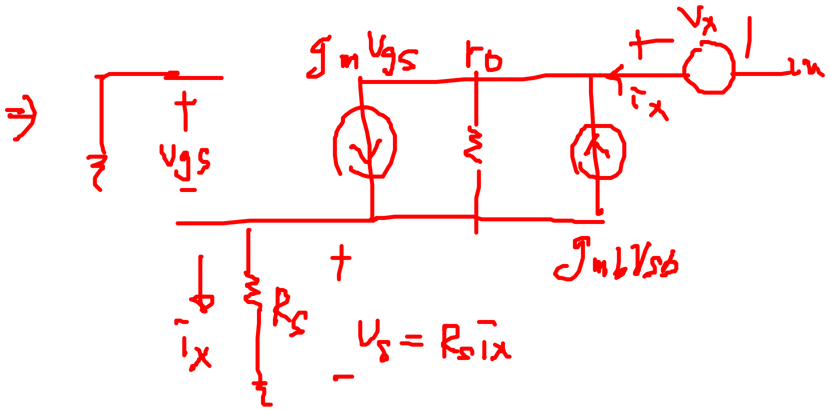
$$R_{out} = R_D$$

Example 3.4 Calculate the small signal voltage gain of the circuit below.



Vout. $\lambda \neq \gamma \neq 0$





Source: $i_{out} = g_m V_{gs} + \frac{-V_s}{r_o} - g_{mb} V_{sb}$

KVL: $-V_{in} + V_{gs} + V_s = 0 \Rightarrow V_{gs} = V_{in} - R_s i_{out}$ $\Rightarrow \frac{i_{out}}{V_{in}} = G_m$.

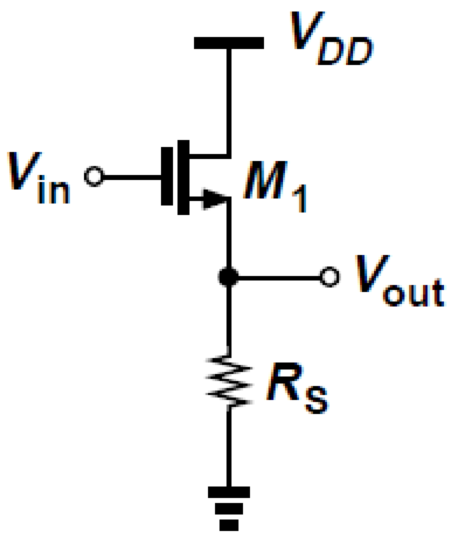
$$A_v = -G_m R_{out} = -g_{m1} r_{o1}$$

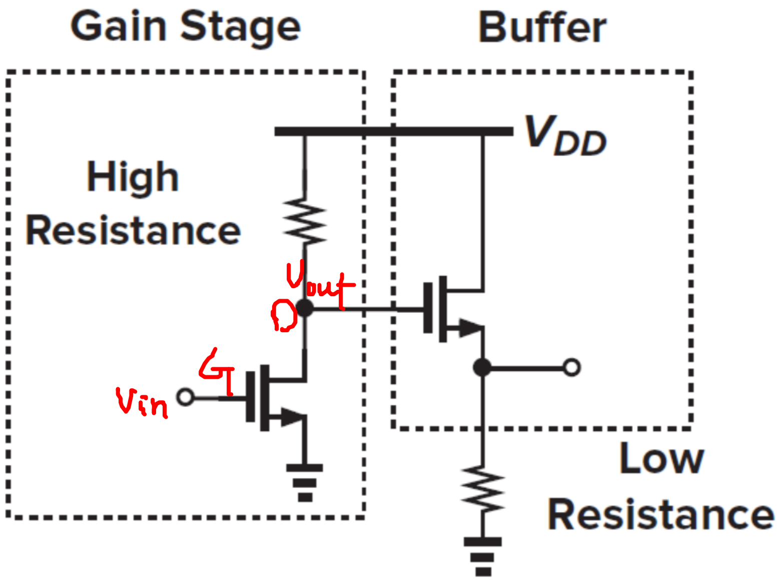
$$G_m = \frac{+g_{m1} r_{o1}}{r_{o1} + R_S + (g_{m1} + g_{mb1}) r_{o1} R_S}$$

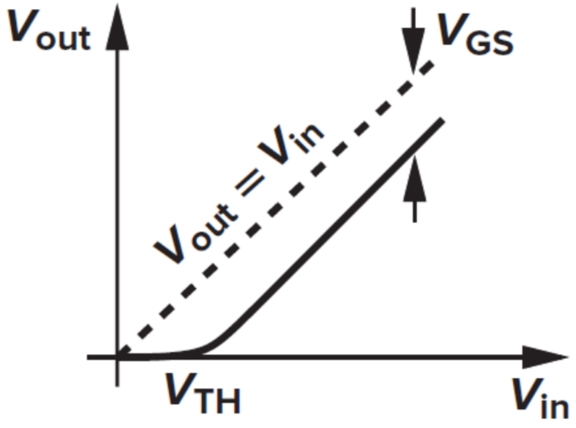
$$R_{out} = r_{o1} + R_S + (g_{m1} + g_{mb1}) r_{o1} R_S$$

Source Follower

The source follower senses the signal at the gate, while **presenting a high input impedance**, and **drives the load at the source**, allowing the source potential to **“follow” the gate voltage**. The source follower can be used to drive a low resistance without degrading the voltage gain of a CS stage, i.e. buffer.

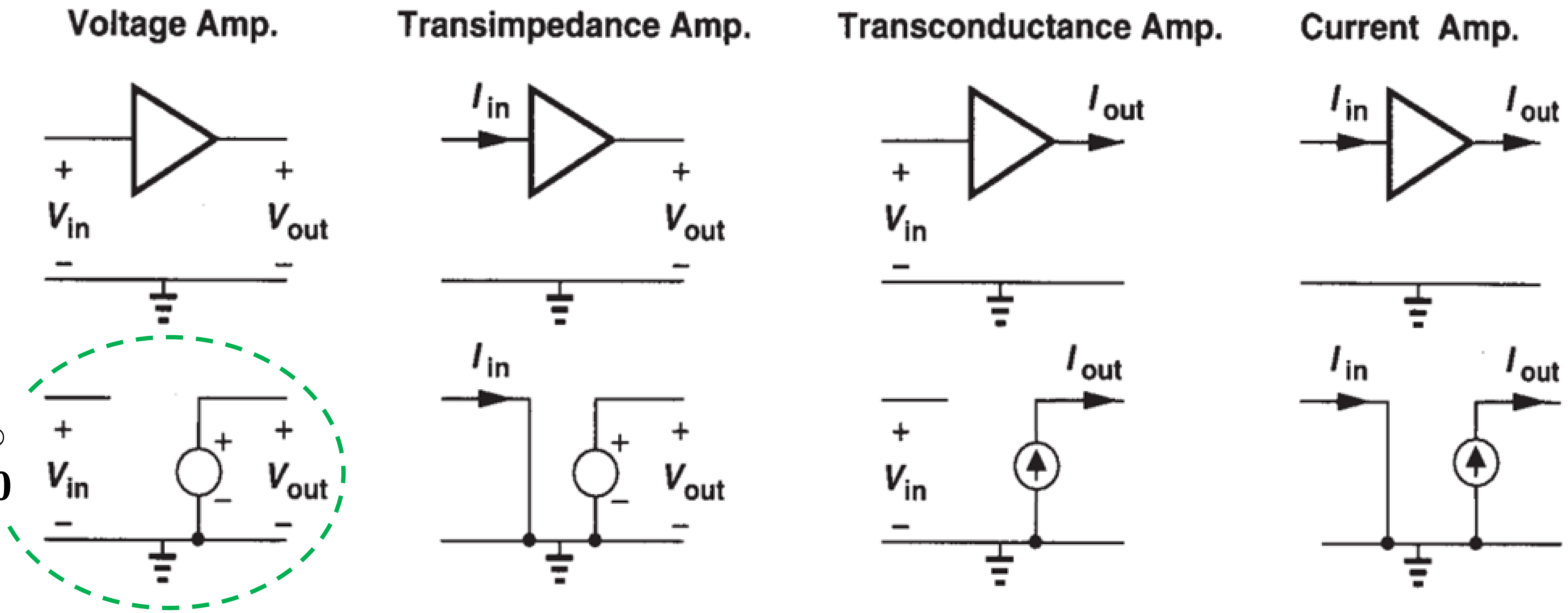






No gain, but V_{out} follows V_{in} (source)

***Ideal Amplifier**

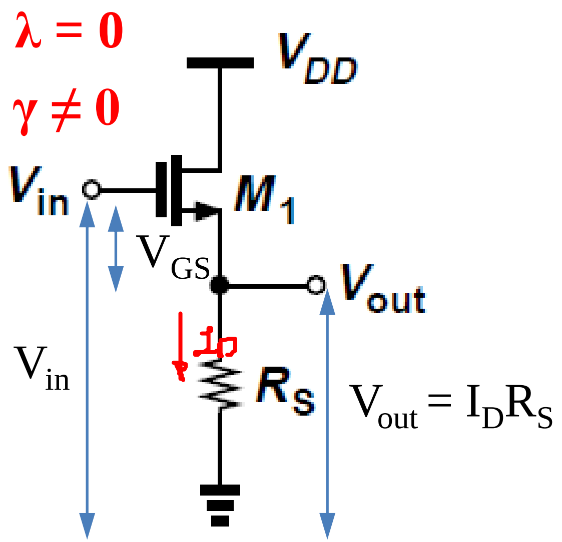


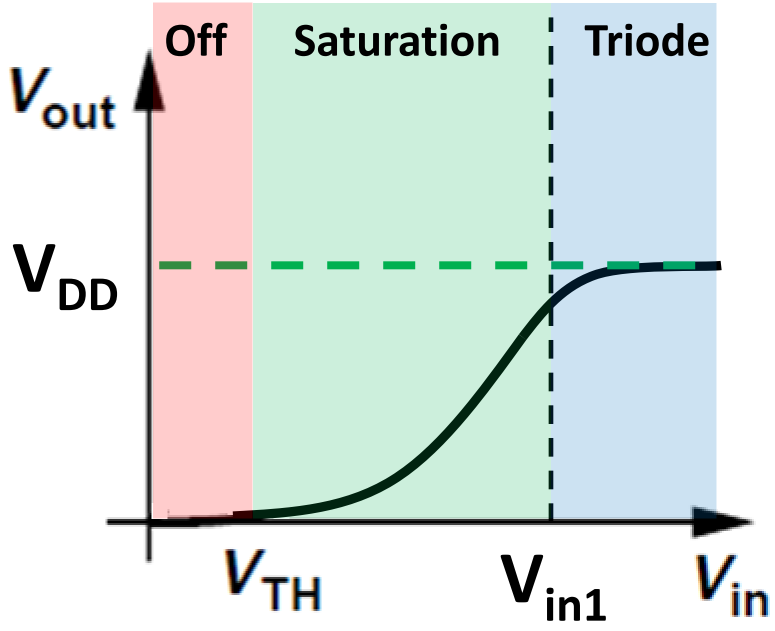
$R_{in} = \infty$
$R_{out} = 0$

CS + Source Follower

For driving a low impedance load, source follower, as a buffer, provides **no gain** but **large input impedance** and **low output impedance**.

(1) Large Signal Analysis





(1) M_1 Off when $V_{in} < V_{TH}$

$$V_{out} = 0$$

$V_{DS} < V_{GS} - V_{TH} \Rightarrow V_{DD} - \cancel{V_{out}} < V_{in1} - \cancel{V_{out}} - V_{TH}.$

$V_{in1} > V_{DD} + V_{TH}.$

(2) M_1 in Saturation when $V_{in1} > V_{in} > V_{TH}$

$$R_S \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in} - V_{out} - V_{TH})^2 = V_{out}$$

$(\underbrace{V_{GS}}_{V_{in}-V_{out}} - V_{TH} = V_{in} - V_{out} - V_{TH})$

(3) M_1 in Triode when $V_{in} > V_{in1}$

$$R_S \mu_n C_{ox}\frac{W}{L}\left[(V_{in} - V_{out} - V_{TH})(V_{DD} - V_{out}) - \frac{1}{2}(V_{DD} - V_{out})^2\right] = V_{out}$$

$(V_{DS} = V_{DD} - V_{out})$

In Saturation

$$R_S \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in}-V_{out}-V_{TH})^2 = V_{out}$$

$$\frac{d}{dV_{in}} \quad R_S \frac{1}{2}\mu_n C_{ox}\frac{W}{L}2(V_{in}-V_{out}-V_{TH})\left(1-\frac{\partial V_{out}}{\partial V_{in}}-\frac{\partial V_{TH}}{\partial V_{in}}\right) = \frac{\partial V_{out}}{\partial V_{in}}$$

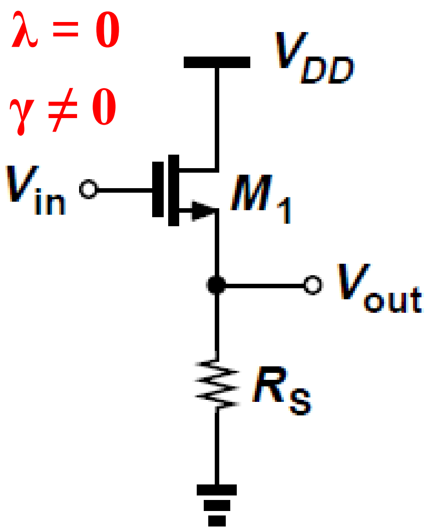
$$R_S \underbrace{\boxed{\mu_n C_{ox}\frac{W}{L}(V_{in}-V_{out}-V_{TH})}}_{=g_m}\left(1-\frac{\partial V_{out}}{\partial V_{in}}-\underbrace{\frac{\partial V_{TH}}{\partial V_{out}}}_{=\eta=\frac{\gamma}{2\sqrt{2\Phi_F+V_{SB}}}}\frac{\partial V_{out}}{\partial V_{in}}\right) = \frac{\partial V_{out}}{\partial V_{in}}$$

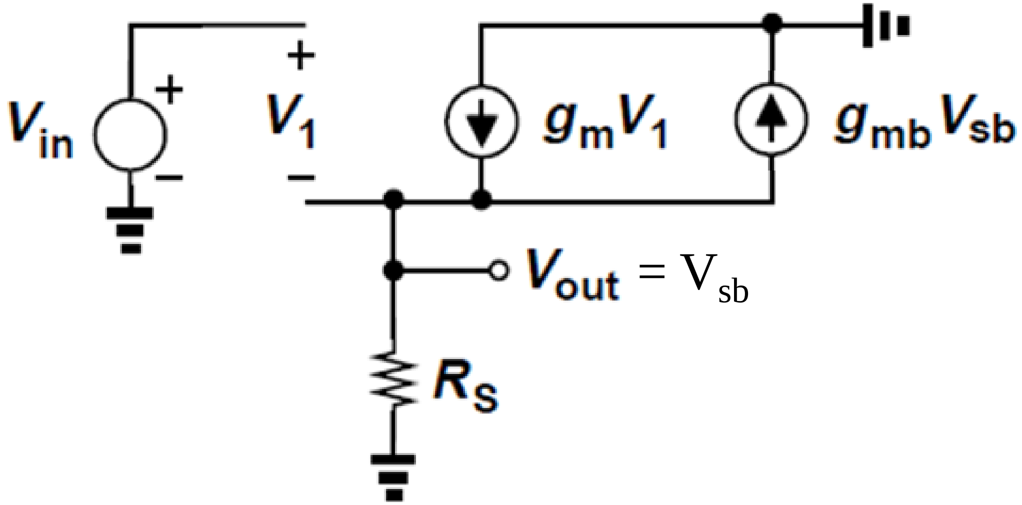
$$A_v = \frac{g_m R_S}{1+g_m R_S(1+\eta)} = \frac{g_m R_S}{1+(g_m+g_{mb})R_S} \approx \frac{1}{1+\eta} \quad \text{If } (g_m+g_{mb})R_S >> 1$$

→ smaller than 1.

$\eta > 0$

(2) Small Signal Analysis

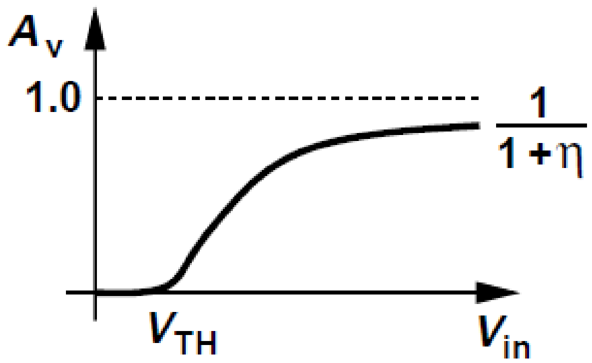




By KVL, $-V_{in} + V_1 + V_{out} = 0 \rightarrow \mathbf{V_1 = V_{in} - V_{out}}$

By KCL, $g_m V_1 - g_{mb} V_{sb} = {V_{out}}/{R_s}$

$$\rightarrow \boldsymbol{g_m(V_{in} - V_{out}) - g_{mb}V_{out} = {V_{out}}/{R_s}}$$

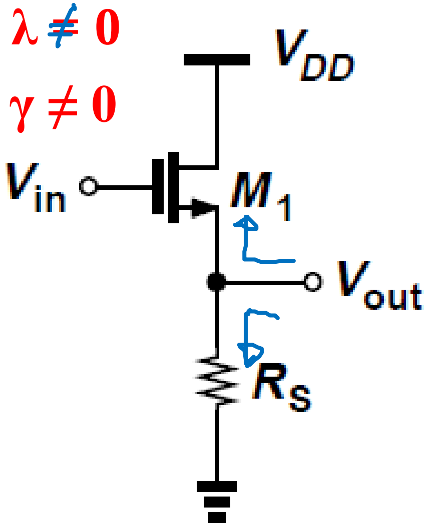


We get $A_v = \frac{V_{out}}{V_{in}} = \frac{g_m R_s}{1+g_m R_s + g_{mb} R_s} = \frac{1}{1+\eta}$ If $(g_m + g_{mb})R_S >> 1$

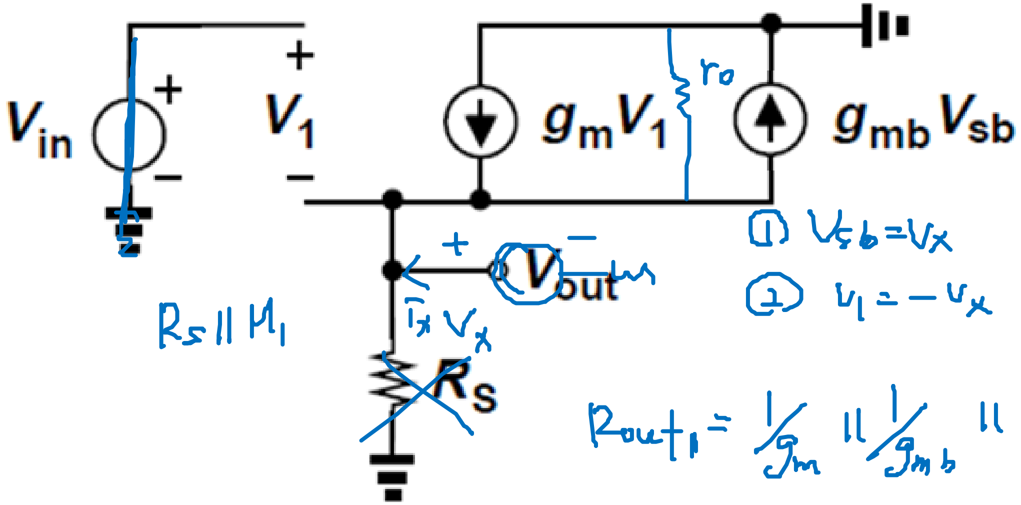
(2) Small Signal Analysis (Alternative) $= G_m$

$\lambda \neq 0$

$\gamma \neq 0$



$i_x + g_m v_1 = \frac{v_x}{r_o} + g_m V_{sb}$.



① $V_{sb} = V_x$

② $V_1 = -V_x$

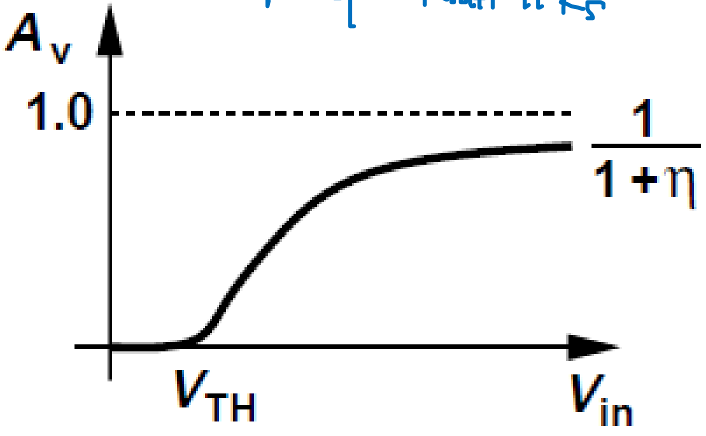
$R_{out1} = 1/g_m \parallel 1/g_{mb} \parallel r_o$

$R_{out} = R_{out1} \parallel R_S$

$$G_m = -g_m$$

$$R_{out} = R_S \parallel \left(\frac{1}{g_m + g_{mb}}\right)$$

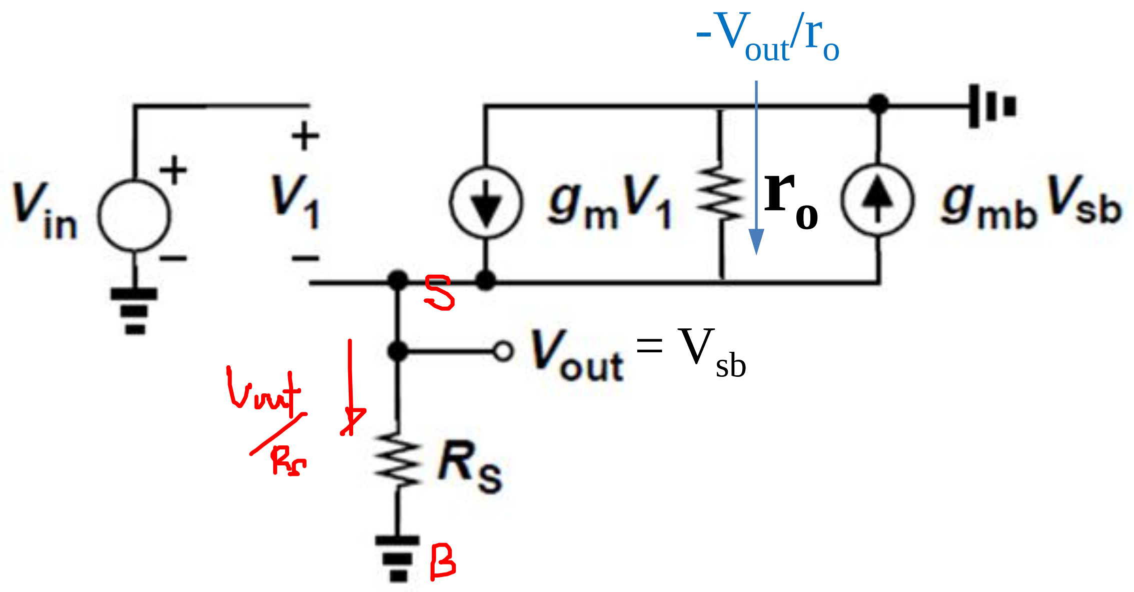
$$A_v = \frac{g_m R_S}{1 + (g_m + g_{mb})R_S} \approx \frac{1}{1+\eta}$$



(2) Small Signal Analysis

$\lambda \neq 0$

$\gamma \neq 0$



By KVL, $-V_{in} + V_1 + V_{out} = 0 \rightarrow \mathbf{V_1 = V_{in} - V_{out}}$

By KCL, $g_m V_1 - g_{mb} V_{sb} - {}^{V_{out}}/_{r_o} = {}^{V_{out}}/_{R_S}$

at S

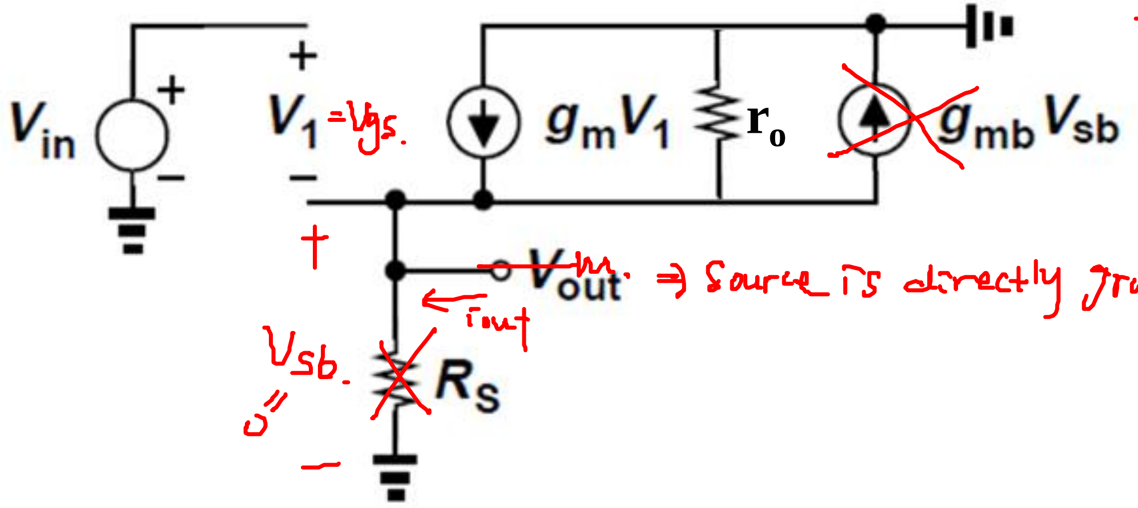
$$\rightarrow \boldsymbol{g_m(V_{in} - V_{out}) - g_{mb} V_{out} - {}^{1}/_{r_o} V_{out} = {}^{V_{out}}/_{R_s}}$$

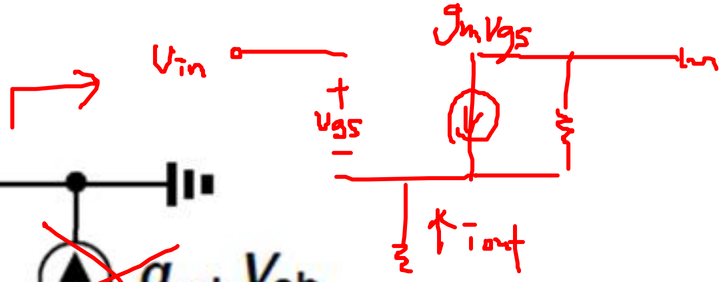
We get $A_v = \dfrac{V_{out}}{V_{in}} = \dfrac{g_m}{g_m + g_{mb} + {}^{1}/_{R_S} + {}^{1}/_{r_o}} = \dfrac{1}{1+\eta}$ If $(\overset{>1}{g_m} + \overset{>1}{g_{mb}})\overset{>1}{r_o}\overset{>1}{R_S} >> r_o$ and R_S

(2) Small Signal Analysis (Alternative)

$\lambda \neq 0$

$\gamma \neq 0$





⇒ Source is directly grounded. $V_{gs} = V_{in}$

$g_m V_{gs} = -i_{out}$

$\underline{\dfrac{i_{out}}{V_{in}} = -g_m}$

$$G_m = -g_m$$

$$R_{out} = r_o \parallel R_S \parallel \left(\frac{1}{g_m + g_{mb}}\right) = \frac{1}{g_m} \parallel \frac{1}{g_{mb}}$$

$$a \parallel b = \frac{1}{1/a + 1/b}$$

$$A_v = \frac{g_m r_o R_S}{r_o + R_S + (g_m + g_{mb}) r_o R_S} \approx \frac{1}{1+\eta} < 1$$

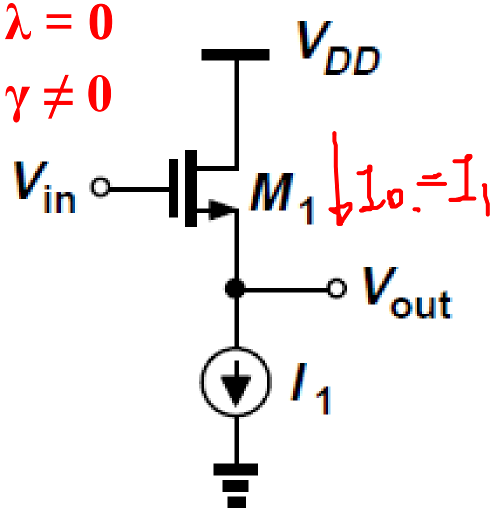
If $(g_m + g_{mb}) r_o R_S >> r_o$ and R_S

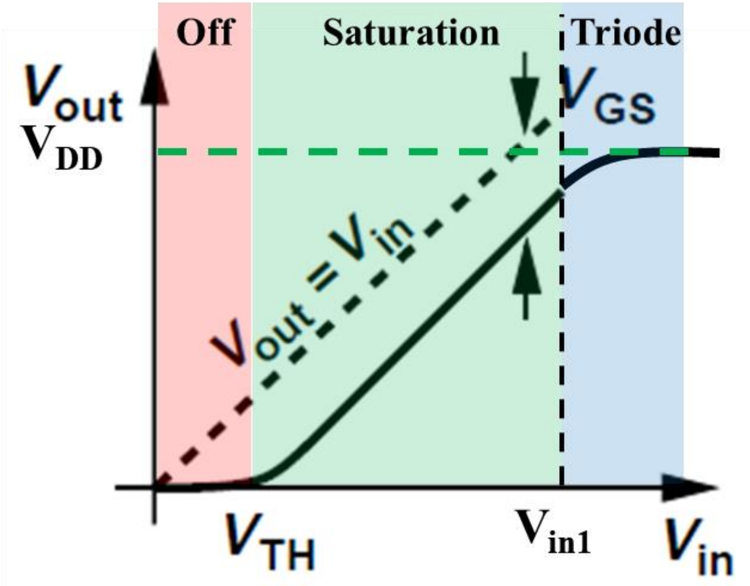
Source Follower with Current Source

(1) Large Signal Analysis

$\lambda = 0$

$\gamma \neq 0$





$\frac{d}{dV_{in}}$

$$\left[\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(\underbrace{V_{in} - V_{out}}_{\substack{\\}}\!\!\overset{= V_{GS}}{} - V_{TH})^2 = I_1\right] \longrightarrow \frac{1}{2}\mu_n C_{ox}\frac{W}{L}2(V_{in} - V_{out} - V_{TH})\left(1 - \frac{\partial V_{out}}{\partial V_{in}} - \frac{\partial V_{TH}}{\partial V_{in}}\right) = 0$$

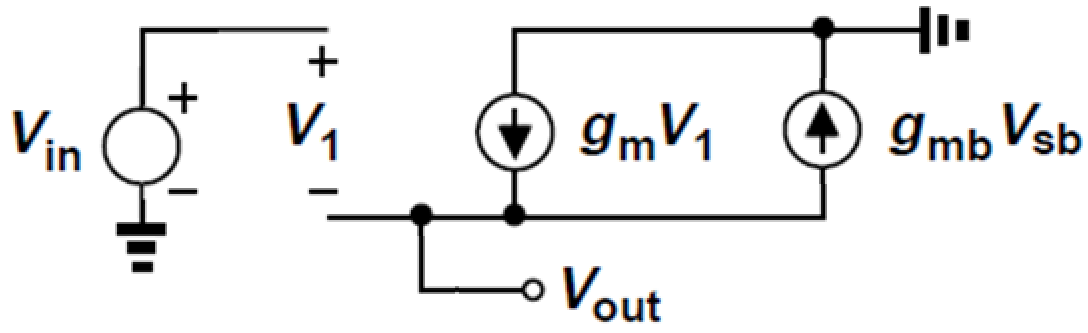
$$\underbrace{\mu_n C_{ox}\frac{W}{L}(V_{in} - V_{out} - V_{TH})}_{= g_m}\left(1 - \frac{\partial V_{out}}{\partial V_{in}} - \underbrace{\frac{\partial V_{TH}}{\partial V_{out}}}_{= \eta}\frac{\partial V_{out}}{\partial V_{in}}\right) = 0$$

$$\mathbf{A_v} = \frac{\mathbf{1}}{\mathbf{1 + \eta}} > 0$$

If $\gamma = 0$, $A_v = 1$

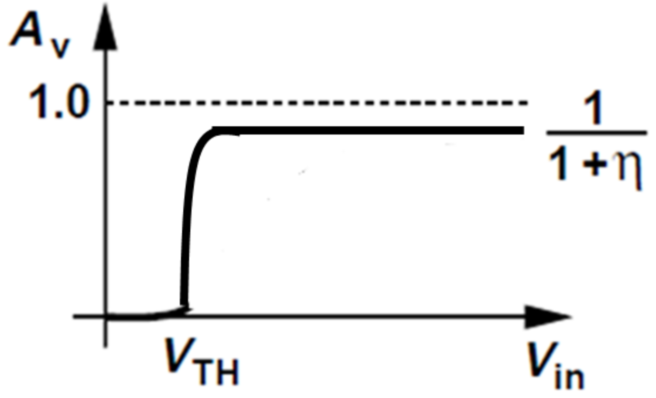
(2) Small Signal Analysis

$\lambda = 0$
$\gamma \neq 0$



By KVL, $-V_{in} + V_1 + V_{out} = 0 \rightarrow \mathbf{V_1 = V_{in} - V_{out}}$
By KCL, $g_m V_1 - g_{mb} V_{sb} = 0 \rightarrow \boldsymbol{g_m(V_{in} - V_{out}) - g_{mb} V_{out} = 0}$

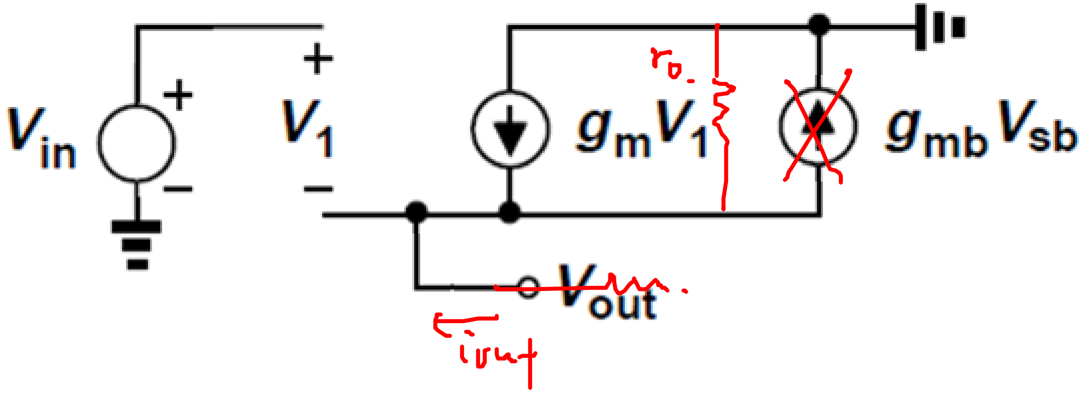
We get $A_v = \dfrac{V_{out}}{V_{in}} = \dfrac{g_m}{g_m + g_{mb}} = \dfrac{1}{1+\eta}$



(2) Small Signal Analysis (Alternative)

$\lambda = 0$

$\gamma \neq 0$

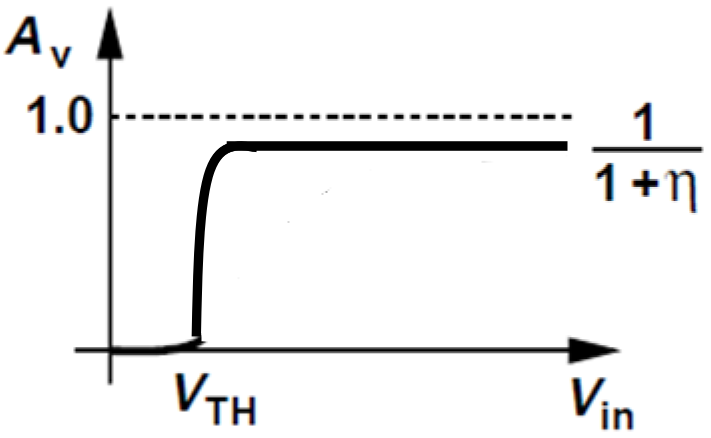


$$G_m = -g_m$$

$$R_{out} = \frac{1}{g_m + g_{mb}} \parallel r_o.$$

$$A_v = \frac{1}{1+\eta}$$

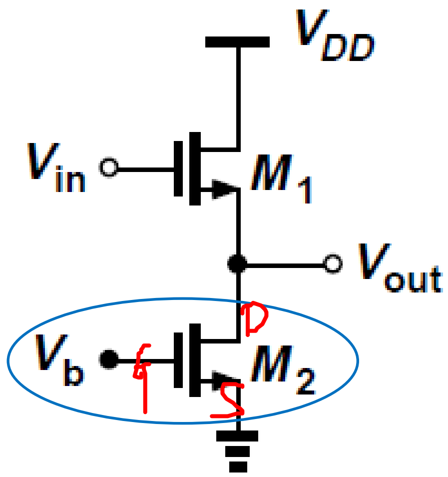
If $\gamma = 0$, $A_v = 1$.

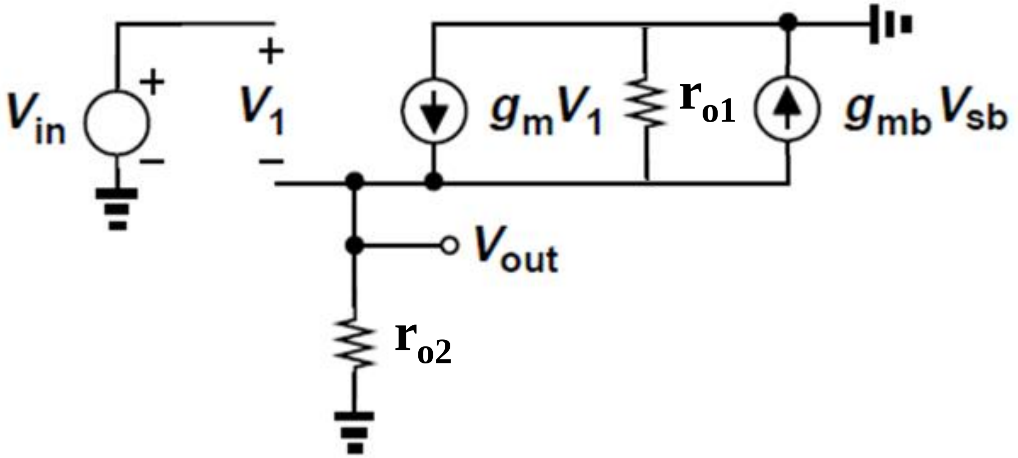


Source Follower with Current Source

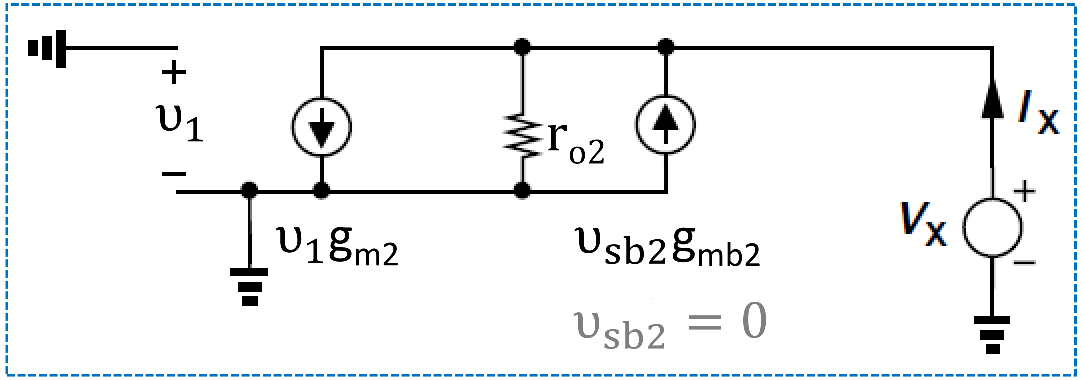
$\lambda \neq 0$

$\gamma \neq 0$

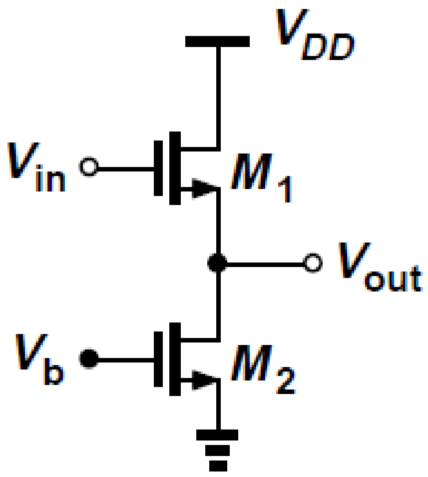


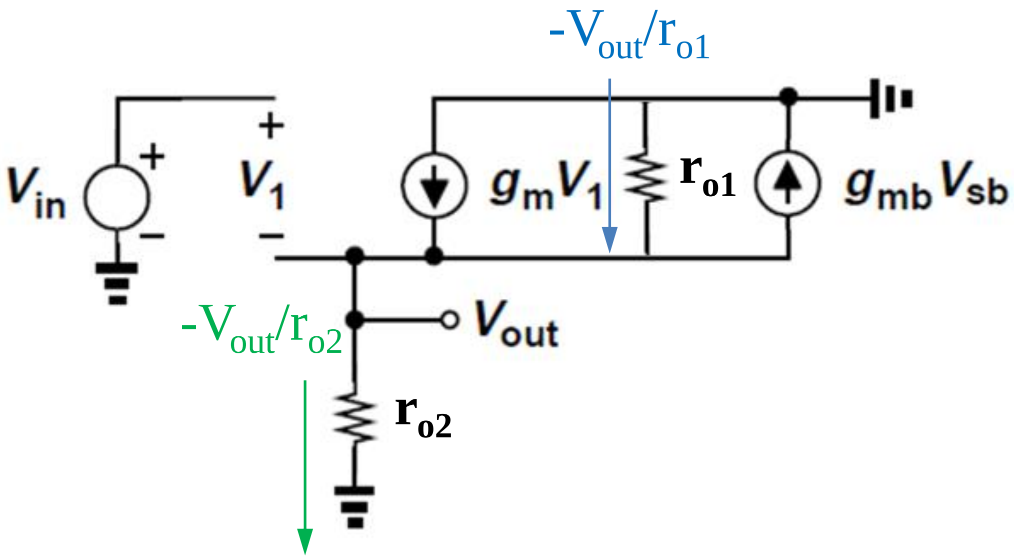


Recall:



$$R_{in2} = \frac{\upsilon_x}{i_x} = r_{o2}$$





By KVL, $-V_{in} + V_1 + V_{out} = 0 \rightarrow \mathbf{V_1 = V_{in} - V_{out}}$

By KCL, $g_m V_1 - g_{mb} V_{sb} - {}^{V_{out}}/_{r_{o1}} = {}^{V_{out}}/_{r_{o2}}$

$$\rightarrow \boldsymbol{g_m(V_{in} - V_{out}) - g_{mb} V_{out} - {}^{V_{out}}/_{r_{o1}} = {}^{V_{out}}/_{r_{o2}}}$$

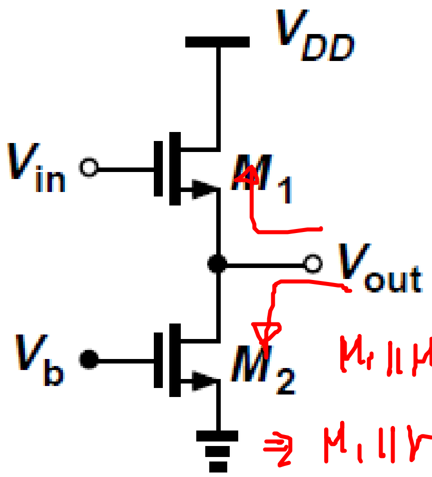
We get $A_v = \dfrac{V_{out}}{V_{in}} = \dfrac{g_m}{g_m + g_{mb} + {}^{1}/_{r_{o1}} + {}^{1}/_{r_{o2}}}$

If r_{o1} and r_{o2} large, A_v is linear.

Alternative method

$\lambda \neq 0$

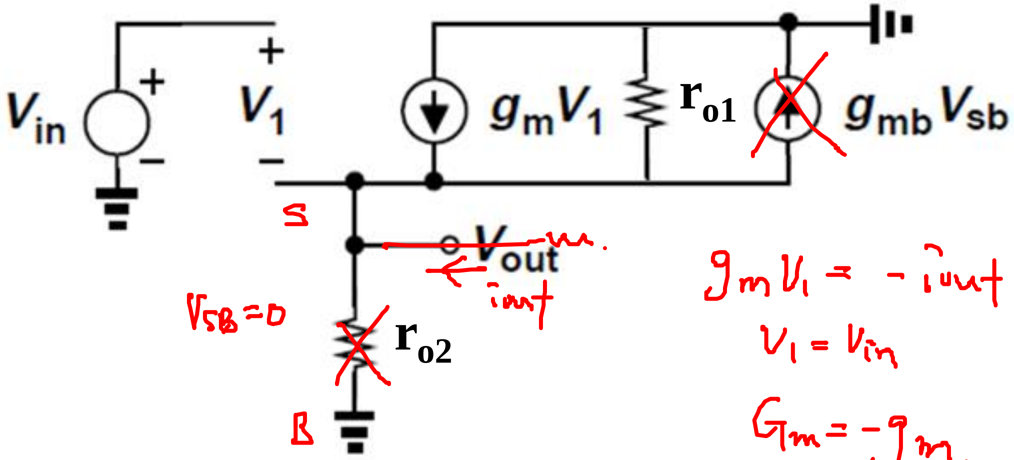
$\gamma \neq 0$



$M_1 \parallel M_2$

$\Rightarrow M_1 \parallel r_{o2}$

$M_1 = 1/g_m \parallel 1/g_{mb} \parallel r_{o1}$



$g_m V_1 = -i_{out}$

$V_1 = V_{in}$

$G_m = -g_m.$

$G_m = \cancel{g_{m1}}$ $-g_m.$

Source.

$1/g_m$, $1/g_{mb}$

$$R_{out} = r_{o1} \parallel r_{o2} \parallel \boxed{\left(\frac{1}{g_{m1} + g_{mb1}}\right)} = \frac{1}{g_{m1}} \parallel \frac{1}{g_{mb1}}$$

$$A_v = \frac{g_{m1} r_{o1} r_{o2}}{r_{o1} + r_{o2} + (g_{m1} + g_{mb1}) r_{o1} r_{o2}}$$

If r_{o1} and r_{o2} large, A_v is linear.

Example 3.5 Suppose that in the source follower of Fig. 3.37(a), $(W/L)_1 = 20/0.5$, $I_1 = 200\ \mu A$, $V_{TH0} = 0.6$ V, $2\varnothing_F = 0.7$ V, $V_{DD} = 1.2$ V, $\mu_n C_{ox} = 50\ \mu A/V^2$, and $\gamma = 0.4\ V^{1/2}$.

$$V_{TH} = V_{TH0} + \gamma\left(\sqrt{|2\Phi_F + \underset{\underset{V_{out}}{\parallel}}{V_{SB}}|} - \sqrt{|2\Phi_F|}\right)$$

$I_D = I_1 = 200\mu A.$

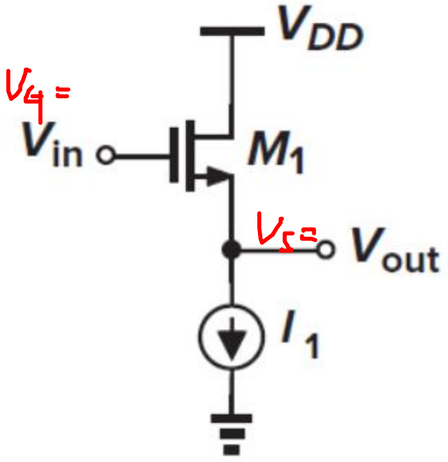
$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH})^2 = 200\mu A$

$\frac{1}{2}\times 50\times 10^{-6}\times\frac{20}{0.5}\times(\underset{\underset{1.2V}{\parallel}}{V_{in}} - V_{out} - 0.6)^2 = 200\mu A$

V_{out} without body effect $= 0.153V.$

$V_{TH} = 0.6 + 0.4\left(\sqrt{0.7 + 0.153} - \sqrt{0.7}\right) = 0.635V.$

re-calculate: $V_{out} = 0.118V.$



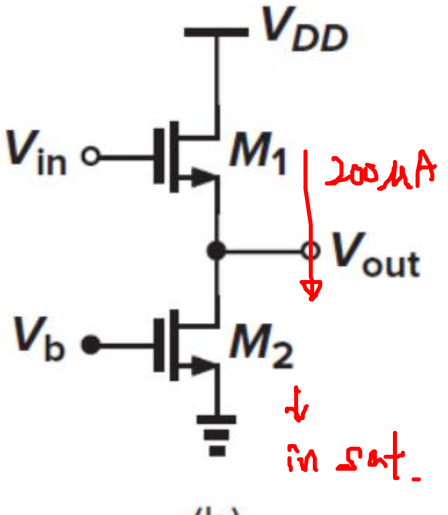
(a)

(a) Calculate V_{out} for $V_{in} = 1.2$ V.

Example 3.5 Suppose that in the source follower of Fig. 3.37(a), $(W/L)_1 = 20/0.5$, $I_1 = 200\ \mu A$, $V_{TH0} = 0.6\ V$, $2\varnothing_F = 0.7\ V$, $V_{DD} = 1.2\ V$, $\mu_n C_{ox} = 50\ \mu A/V^2$, and $\gamma = 0.4\ V^{1/2}$.

$$V_{TH} = V_{TH0} + \gamma\left(\sqrt{|2\Phi_F + V_{SB}|} - \sqrt{|2\Phi_F|}\right)$$

$V_{out} = 0.118 V$



(b)

$$I_{D-M_2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (\underbrace{V_{GS}-V_{TH}}_{=V_{out}})^2 = 200\mu A.$$

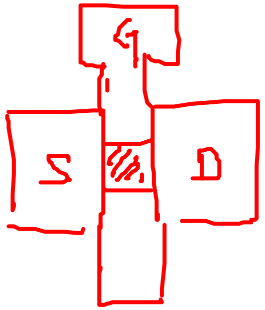
$V_{DS} \geq V_b - V_{TH}$.

$V_{out} \geq V_b - V_{TH} = V_{GS} - V_{TH}$.

$\left(\frac{W}{L}\right)_2 = 574.54$

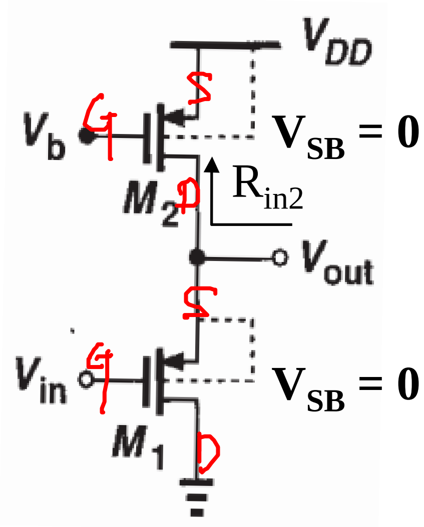
(b) If I_1 is implemented as M_2 in Fig. 3.37(b), find the minimum value of $(W/L)_2$ for which M_2 remains saturated when $V_{in} = 1.2$ V.

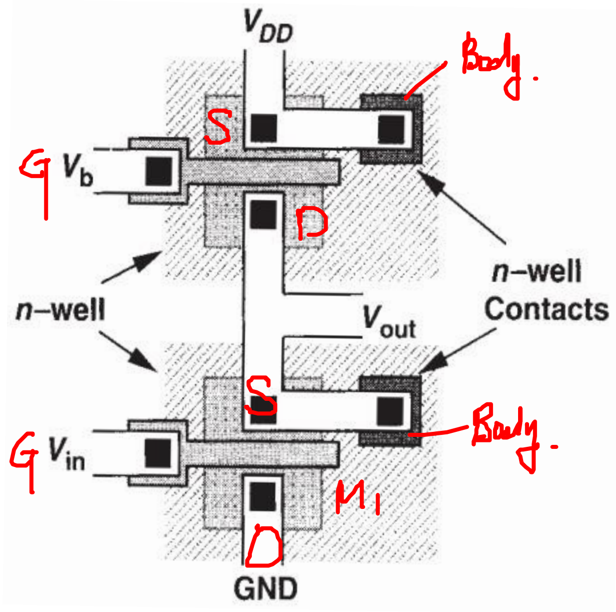
PMOS Source Follower with Current Source ($V_{SB} = 0$)



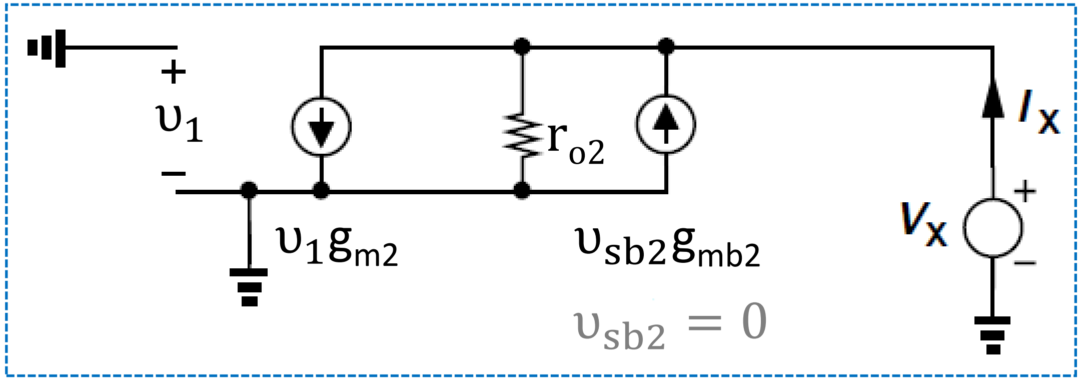
$\lambda \neq 0$

$\gamma \neq 0$





M_2 PMOS

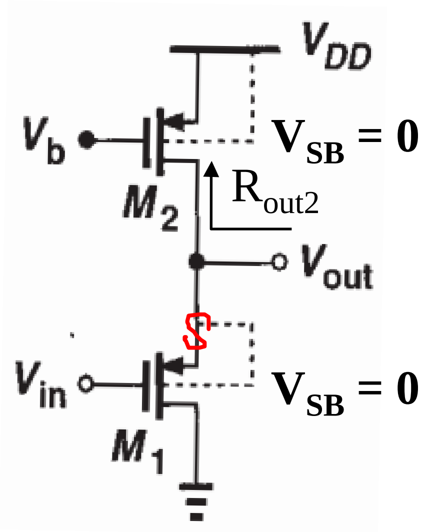


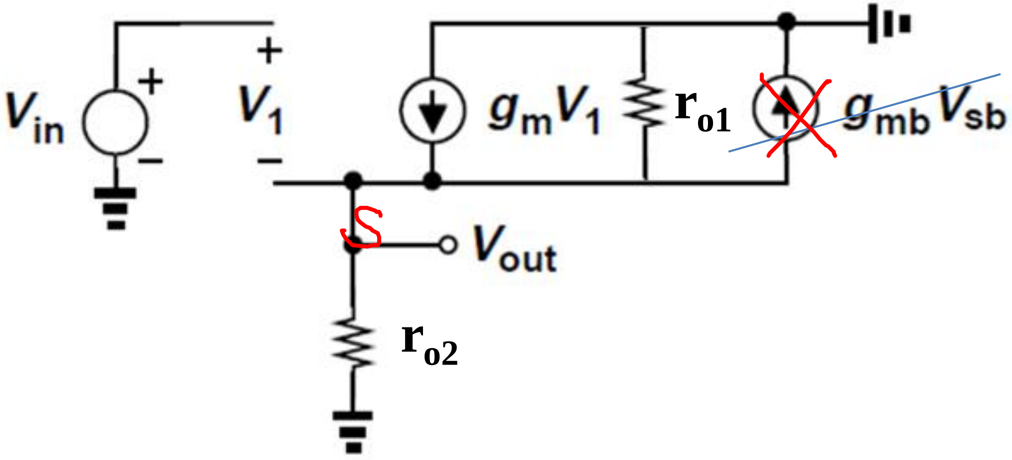
$$R_{in2} = \frac{\upsilon_x}{i_x} = r_{o2}$$

PMOS Source Follower with Current Source ($V_{SB} = 0$)

$\lambda \neq 0$

$\gamma \neq 0$





By KVL, $-V_{in} + V_1 + V_{out} = 0 \rightarrow \mathbf{V_1 = V_{in} - V_{out}}$

By KCL, $g_m V_1 - {}^{V_{out}}/_{r_{o1}} = {}^{V_{out}}/_{r_{o2}}$

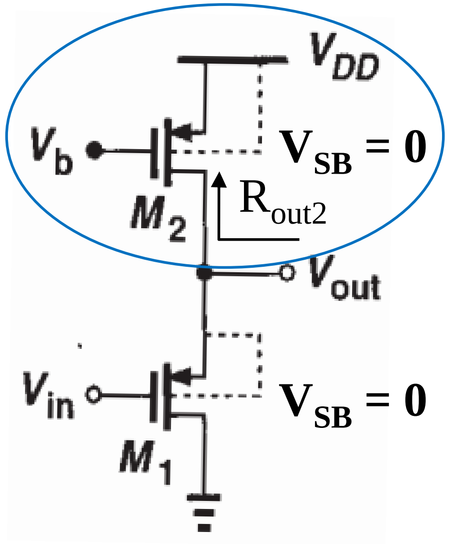
$$\rightarrow \boldsymbol{g_m(V_{in} - V_{out}) - {}^{V_{out}}/_{r_{o1}} = {}^{V_{out}}/_{r_{o2}}}$$

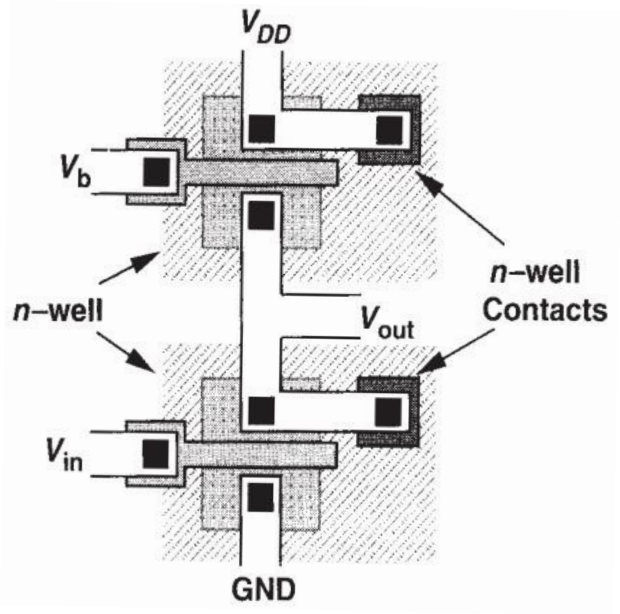
We get $A_v = \dfrac{V_{out}}{V_{in}} = \dfrac{g_m}{g_m + {}^1/_{r_{o1}} + {}^1/_{r_{o2}}}$

PMOS Source Follower with Current Source ($V_{SB} = 0$) (Alternative)

$\lambda \neq 0$

$\gamma \neq 0$





$$g_m = \mu_p C_{ox} \frac{W}{L} (V_{gs} - V_{TH})$$

r_{o1}

r_{o2}

$$G_m = -g_{m1}$$

$$R_{out} = r_{o1} \parallel r_{o2} \parallel \frac{1}{g_{m1}\downarrow}$$

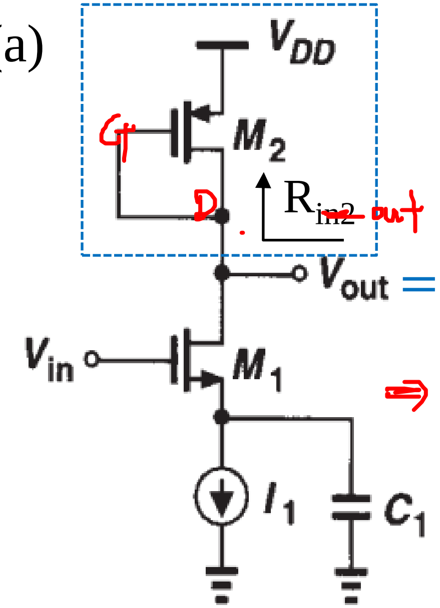
$$A_v = \frac{g_{m1} r_{o1} r_{o2}}{r_{o1} + r_{o2} + g_{m1} r_{o1} r_{o2}}$$

- The sacrifice here is the higher output impedance due to smaller mobility of holes relative to electrons.

$$g_m\downarrow = \underline{\mu_p} C_{ox} \frac{W}{L} (V_{gs} - V_{TH})$$ $\mu_p < \mu_n.$, $m_h > m_e$

Example 3.6 (a) Calculate the voltage gain if C_1 acts as an ac short. (b) What relationship among the V_{GS} of M_1-M_3 guarantees that M_1 is saturated?

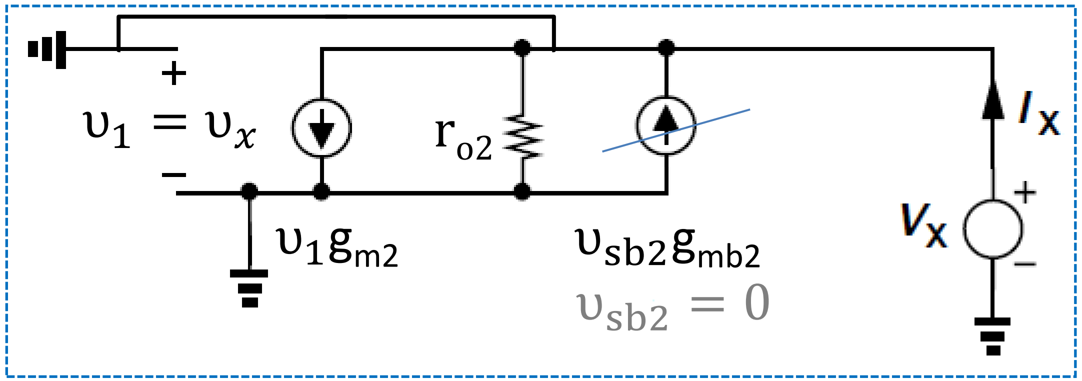
(a)



$V_{out} = \mathbf{V_{DD} - V_{SG2}}$

$\Rightarrow$ CS stage.

M$_2$ PMOS



$$\mathbf{R}_{\cancel{in2}\ out} = \frac{v_x}{r_{o2}} + v_x g_{m2} = i_x \rightarrow \frac{\upsilon_x}{i_x} = \mathbf{r_{o2}} \left\| {}^{\mathbf{1}}/_{\boldsymbol{g_{m2}}} \right.$$

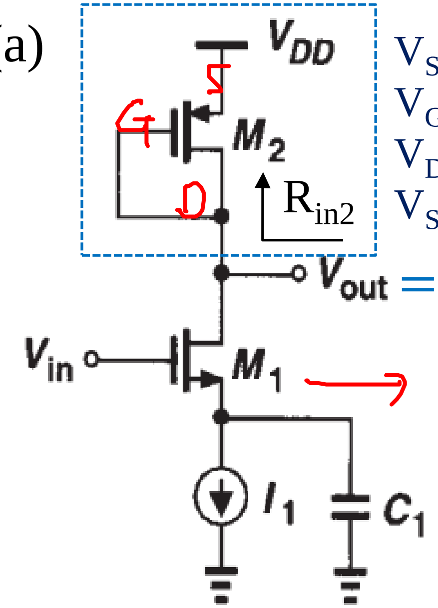
$$\mathbf{V_{in} \leq V_{DD} - V_{SG2} + V_{TH1}}$$

$$G_m = -g_{m1}$$

$$R_{out} = r_{o1} \parallel r_{o2} \parallel \frac{1}{g_{m2}}$$

Example 3.6 (a) Calculate the voltage gain if C_1 acts as an ac short. (b) What relationship among the V_{GS} of M_1-M_3 guarantees that M_1 is saturated?

(a)



$V_S = V_{DD}$
$V_G = V_{out}$
$V_D = V_{out}$
$V_{SG} = V_{DD} - V_{out} \Rightarrow$

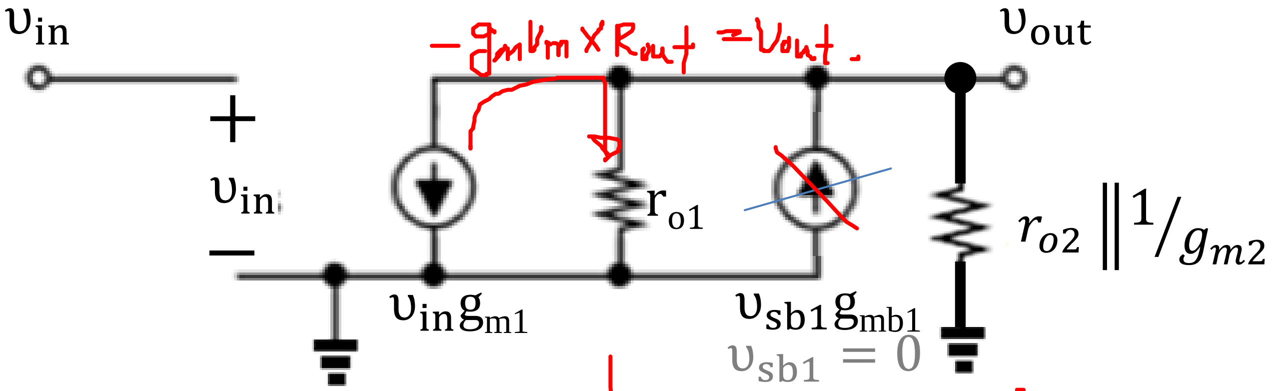
$V_{out} = V_{DD} - V_{SG2}$

$\rightarrow V_{DS} \geq V_{GS} - V_{TH}.$

$V_{out} \geq V_{in} - V_{TH1}.$

$V_{DD} - V_{SG} \geq V_{in} - V_{TH1}.$

$$V_{in} \leq V_{DD} - V_{SG2} + V_{TH1}$$



I_1 open and C_1 short

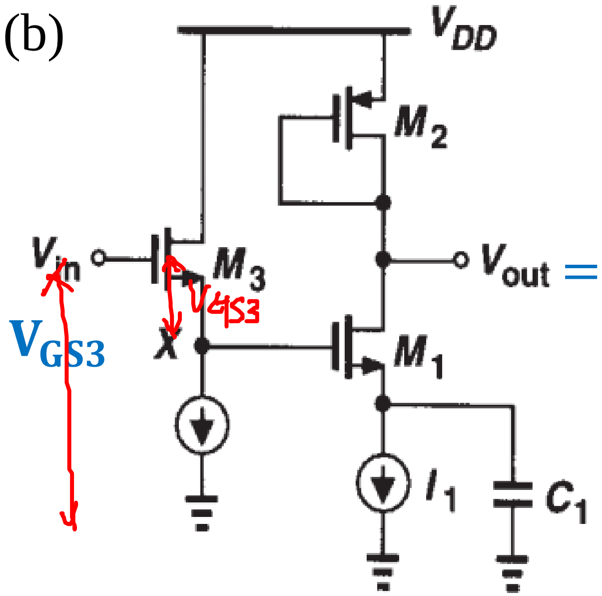
$r_{o1} \| r_{o2} \| 1/g_{m2}$

$$\frac{v_{out}}{r_{in2}} + v_{in} g_{m1} + \frac{v_{out}}{r_{o1}} = 0 \rightarrow (r_{o1} \| r_{o2} \| \frac{1}{g_{m2}}) V_{out} = -g_{m1} v_{in}$$

$$A_v = -g_{m1}(r_{o1} \| r_{o2} \| \frac{1}{g_{m2}})$$

Example 3.6 (a) Calculate the voltage gain if C_1 acts as an ac short. (b) What relationship among the V_{GS} of M_1-M_3 guarantees that M_1 is saturated?

(b)



$V_{out} = \mathbf{V_{DD} - V_{SG2}}$

$\mathbf{V_X = V_{in} - V_{GS3}}$

$V_{DS} \geq V_{GS} - V_{TH1}$

$V_{DD} - V_{SG1} \geq V_{in} - V_{GS3} - V_{TH1}.$

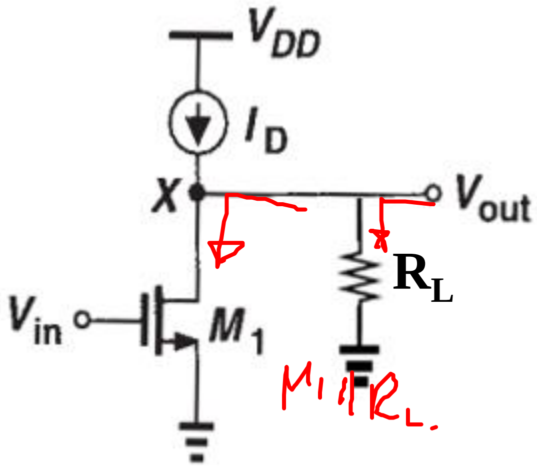
MOSFET Saturation Condition:

$V_{out}(V_{DS}) \geq V_X(V_{GS}) - V_{TH1} \rightarrow$

$V_{DD} - V_{SG2} \geq V_{in} - V_{GS3} - V_{TH1}$

$$\mathbf{V_{in} - V_{GS3} \leq V_{DD} - V_{SG2} + V_{TH1}}$$

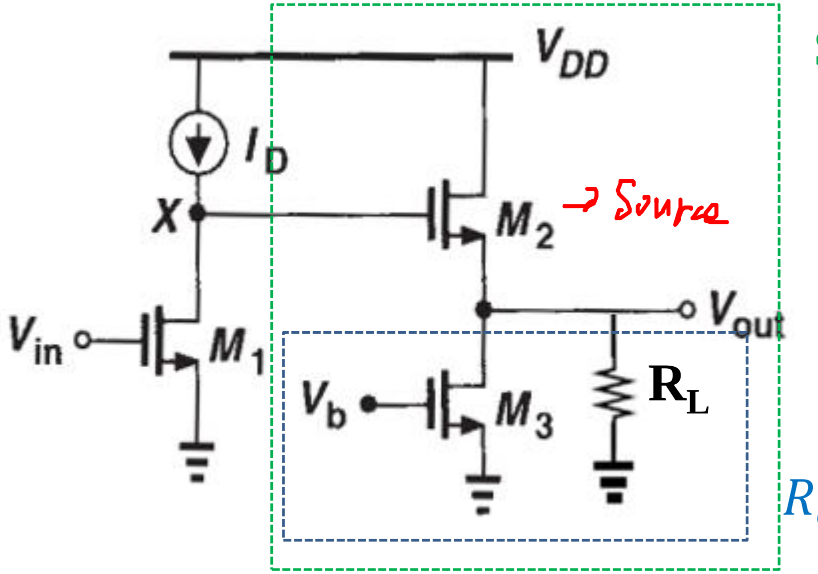
CS + Source Follower



Small-signal Model

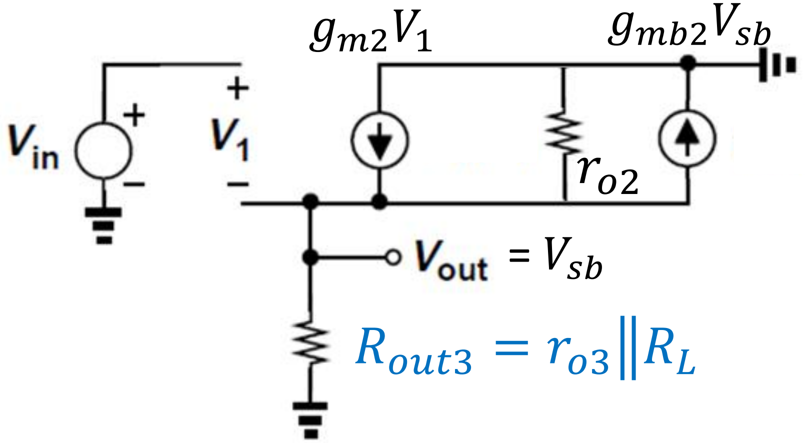
The gain of the circuit above is $A_v = -g_{m1}(r_{o1} \parallel R_L)$

CS + Source Follower



Source Follower

$R_{out3} = r_{o3} \| R_L$



$R_{out3} = r_{o3} \| R_L$

By KVL, $-V_{in} + V_1 + V_{out} = 0 \rightarrow \mathbf{V_1 = V_{in} - V_{out}}$

By KCL, $g_{m2}V_1 - g_{mb2}V_{sb} - {V_{out}}/{r_{o2}} = {V_{out}}/{r_{o3}\|R_L}$

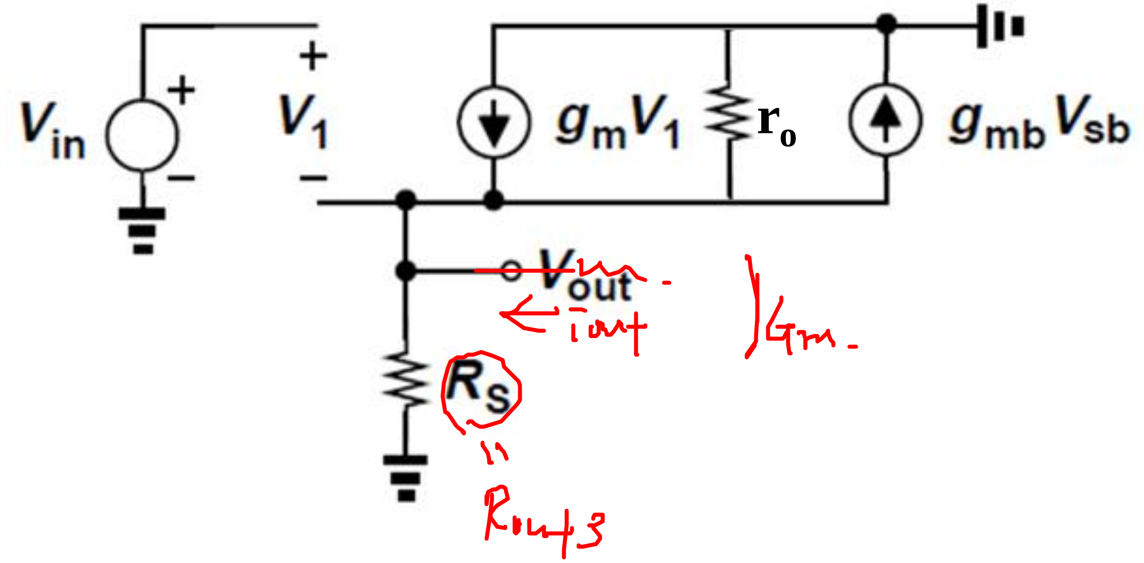
$$\rightarrow \boldsymbol{g_{m2}(V_{in} - V_{out}) - g_{mb2}V_{out} - {V_{out}}/{r_{o2}} = {V_{out}}/{r_{o3}\|R_L}}$$

$$\boldsymbol{A_{v2}} = \frac{V_{out}}{V_{in}} = \frac{g_{m2}}{g_{m2} + g_{mb2} + {1}/{r_{o2}} + {1}/{r_{o3}\|R_L}} = \mathbf{g_{m2}}\left(\mathbf{r_{o2}} \parallel \frac{\mathbf{1}}{\mathbf{g_{m2} + g_{mb2}}} \parallel \mathbf{r_{o3}} \parallel \mathbf{R_L}\right)$$

Recall: Source follow small signal analysis

$\lambda \neq 0$

$\gamma \neq 0$

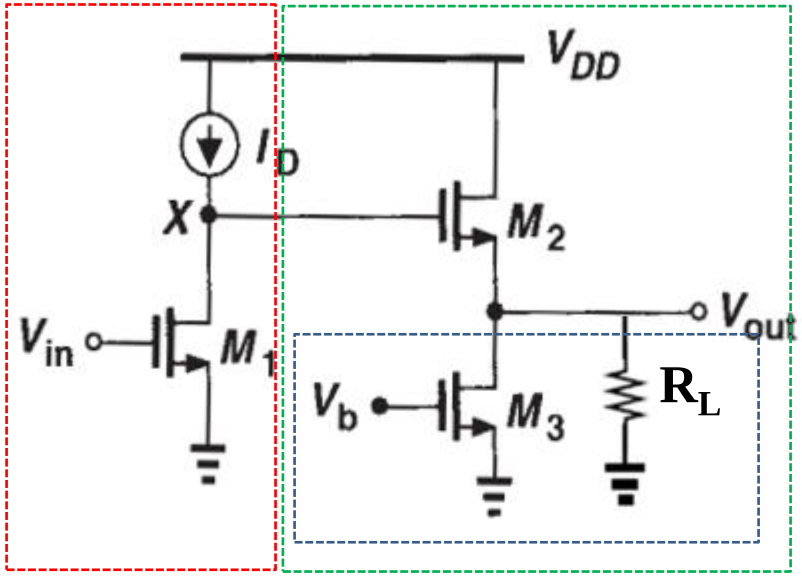


$$G_m = -g_m$$

$$R_{out} = r_o \parallel R_S \parallel \left(\frac{1}{g_m + g_{mb}}\right)$$

$$\mathbf{A_v = g_m(r_o \parallel R_S \parallel \left(\frac{1}{g_m + g_{mb}}\right))}$$

CS + Source Follower

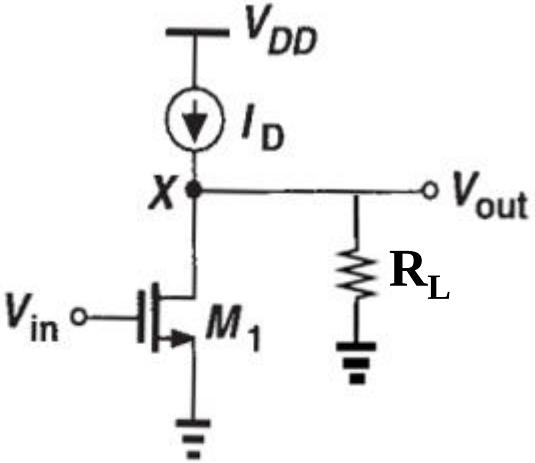


$R_{out3} = r_{o3} \| R_L$

Source Follower $\quad \boldsymbol{A_{v2} = g_{m2}\left(r_{o2} \parallel \frac{1}{g_{m2} + g_{mb2}} \parallel r_{o3} \parallel R_L\right)}$

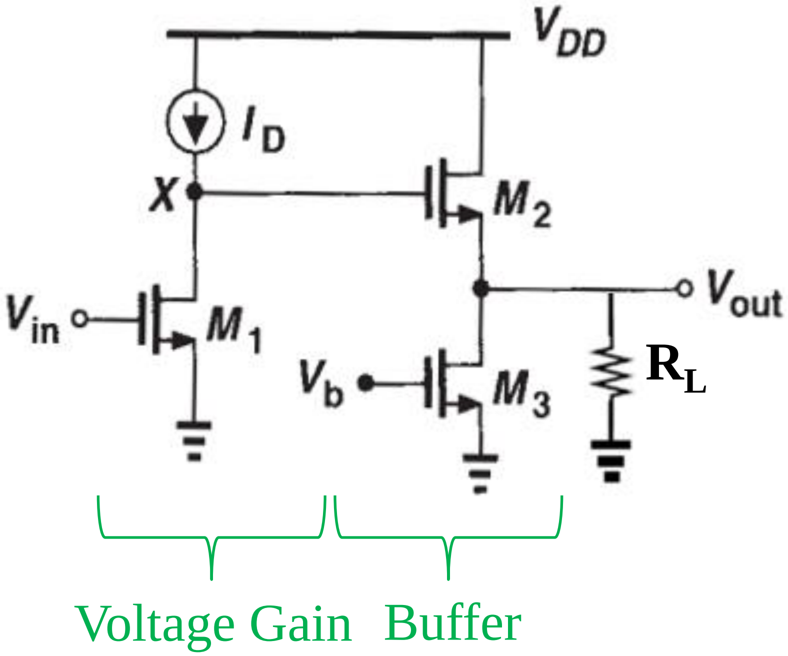
CS stage: $\quad \boldsymbol{A_{v1}} = -g_{m1} r_{o1}$

Total gain $\boldsymbol{A_v} = A_{v1} \times A_{v2} = -\boldsymbol{g_{m1} r_{o1}} \times \boldsymbol{g_{m2}\left(r_{o2} \parallel \frac{1}{g_{m2}+g_{mb2}} \parallel r_{o3} \parallel R_L\right)}$



$$A_v = -g_{m1}(r_{o1} \parallel R_L)$$

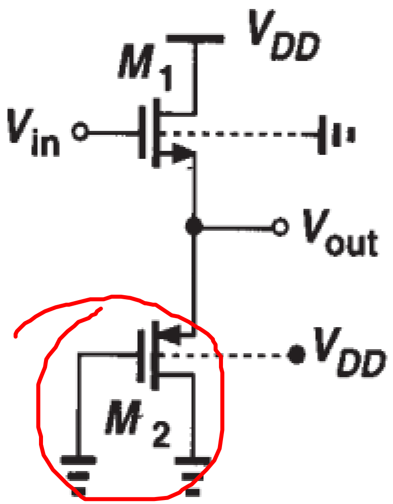
- Voltage gain severely reduced when R_L very small



$$A_v = -g_{m1}r_{o1} \times g_{m2}\left(r_{o2} \parallel \frac{1}{g_{m2} + g_{mb2}} \parallel r_{o3} \parallel R_L\right)$$

- Voltage gain maintained when R_L very small

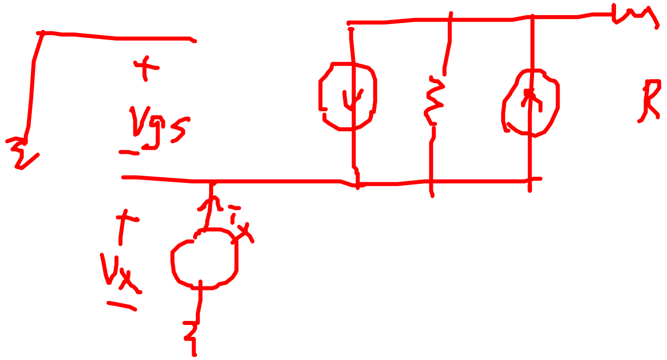
Example 3.7 Calculate the small signal voltage gain of the circuit below.



$$G_m = -g_{m1}$$

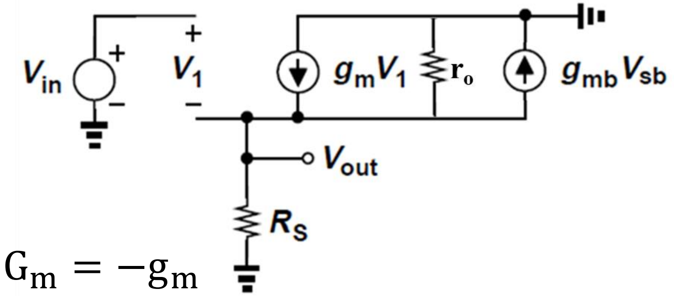
$$R_{out} = \frac{1}{g_{m1} + g_{mb1}} \parallel r_{o1} \parallel \frac{1}{g_{m2} + g_{mb2}} \parallel r_{o2}$$

$$A_v = -G_m R_{out}$$



$R_{out} = 1/g_{m2} \parallel 1/g_{mb2} \parallel r_{o2}$

$\lambda \neq 0$
$\gamma \neq 0$



$$G_m = -g_m$$

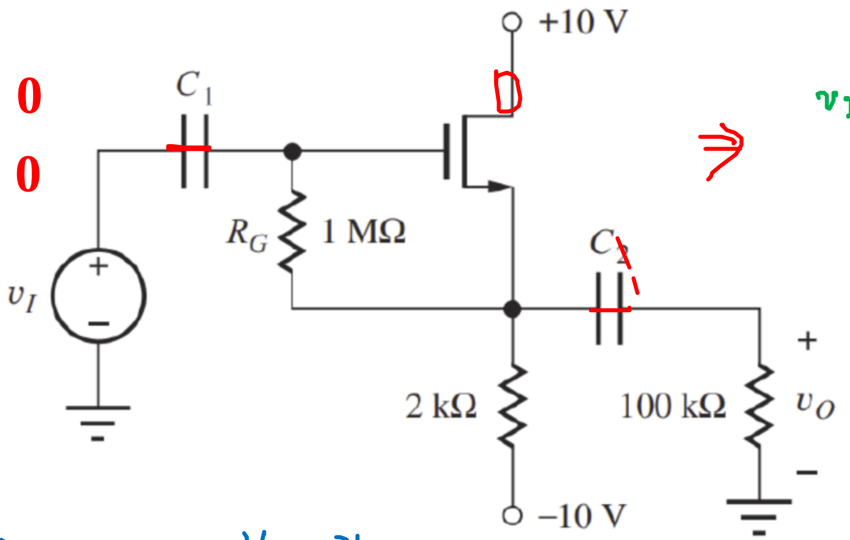
$$R_{out} = r_o \parallel R_S \parallel \left(\frac{1}{g_m + g_{mb}}\right)$$

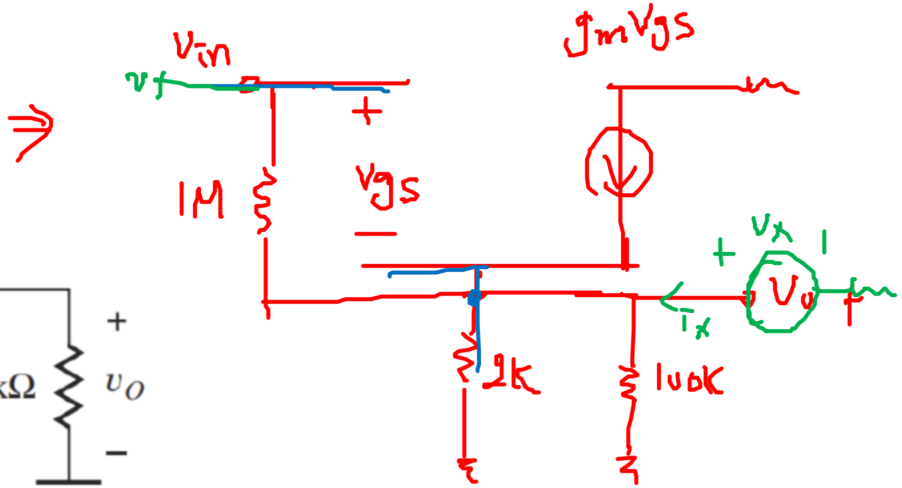
$$\mathbf{A_v = g_m\left(r_o \parallel \overset{R_{out}}{\cancel{R_S}} \parallel \left(\frac{1}{g_m + g_{mb}}\right)\right)}$$

Example 3.8 Assume that the FET is operating with $g_m = 3.54$ mS. Draw the small-signal model and find A_v, R_{in}, and R_{out} for the amplifier.

$\lambda = 0$

$\gamma = 0$





Gm.

by KCL: $\frac{V_{in}}{1M} + g_m V_{gs} + i_{out} = 0$, $V_{gs} = V_{in}$

$G_m = \frac{i_{out}}{V_{in}} = -\left(g_m + \frac{1}{1M}\right) = -0.003541$

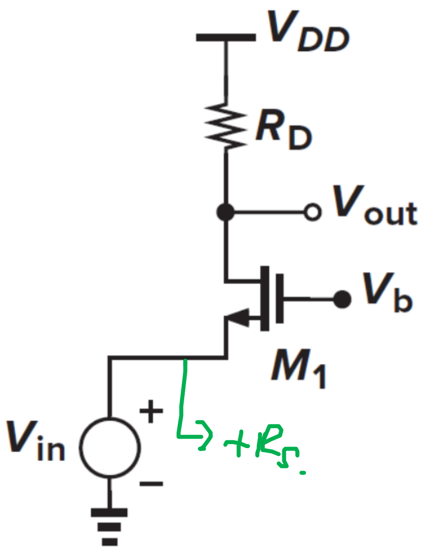
$R_{out} = 1M \parallel 2k \parallel 100k \parallel \frac{1}{g_m}$

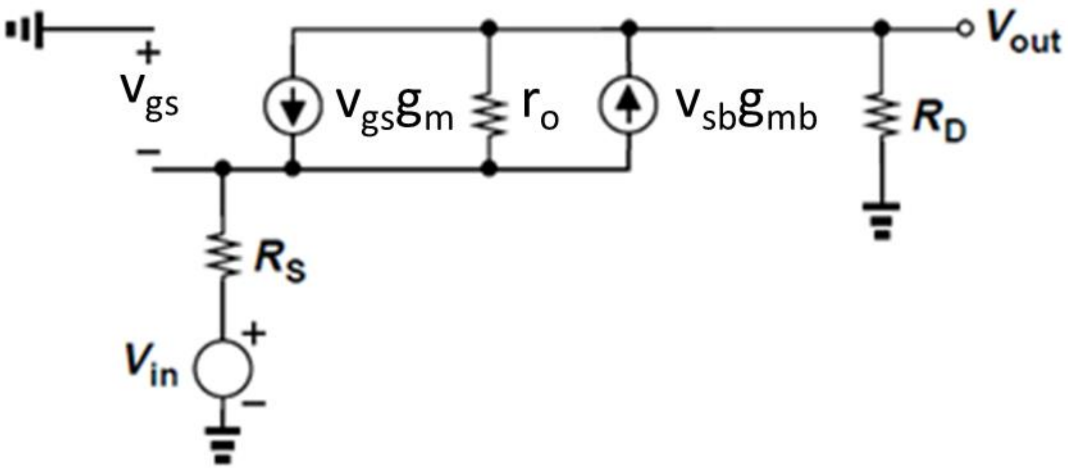
$= 246.85$

$A_v = -0.874$

$R_{in} = 7.94 M\Omega$

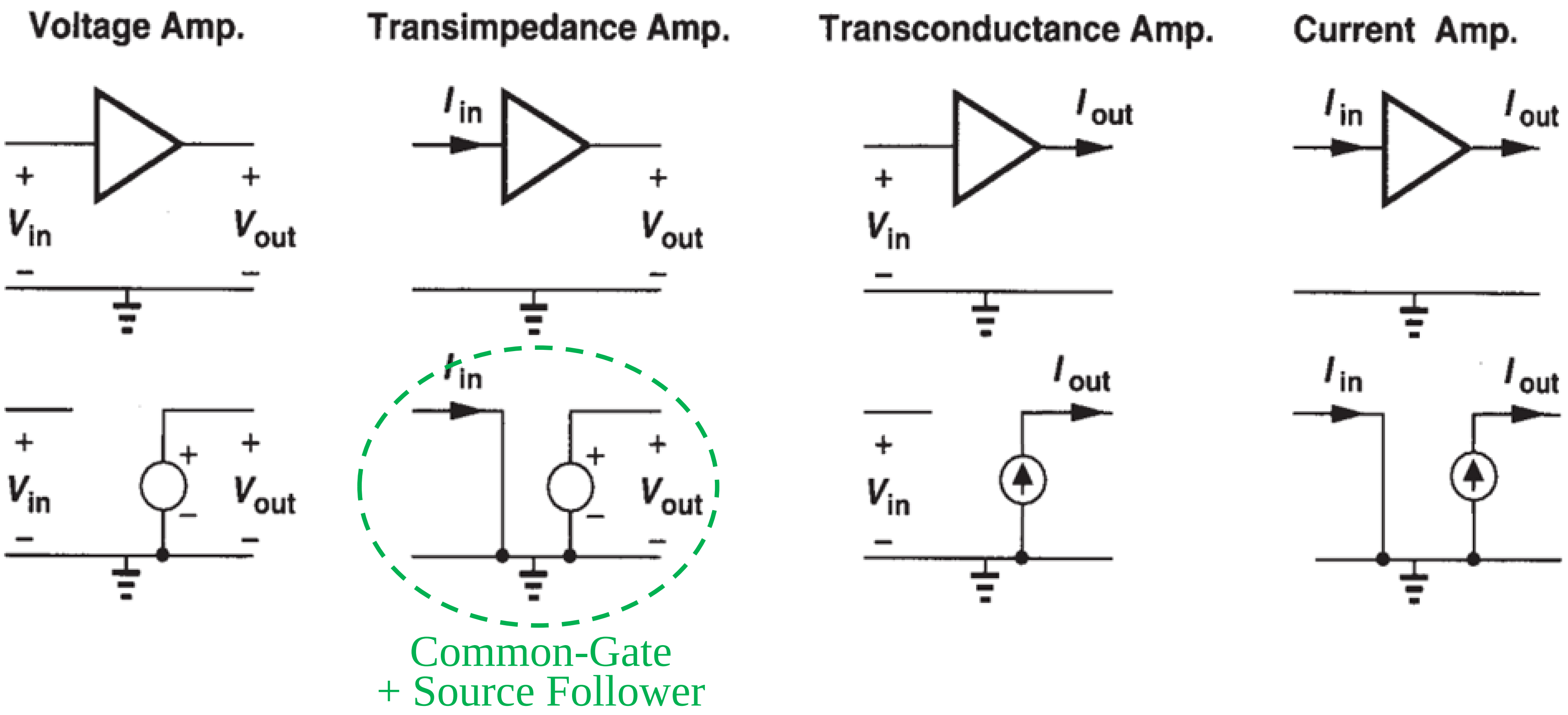
Common-Gate Stage





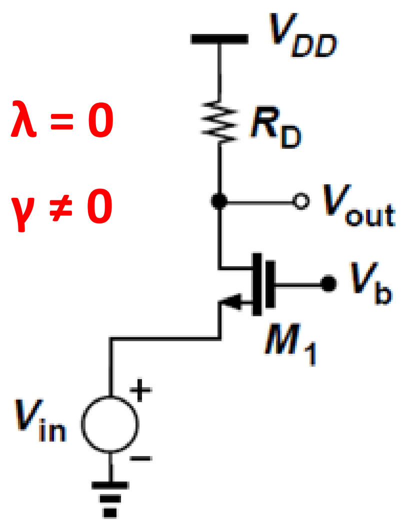
In common-source amplifiers and source followers, the input signal is applied to the gate of a MOSFET. It is also possible to **apply the signal to the source terminal**. **A common-gate (CG) stage** senses the input at the source and produces the output at the drain.

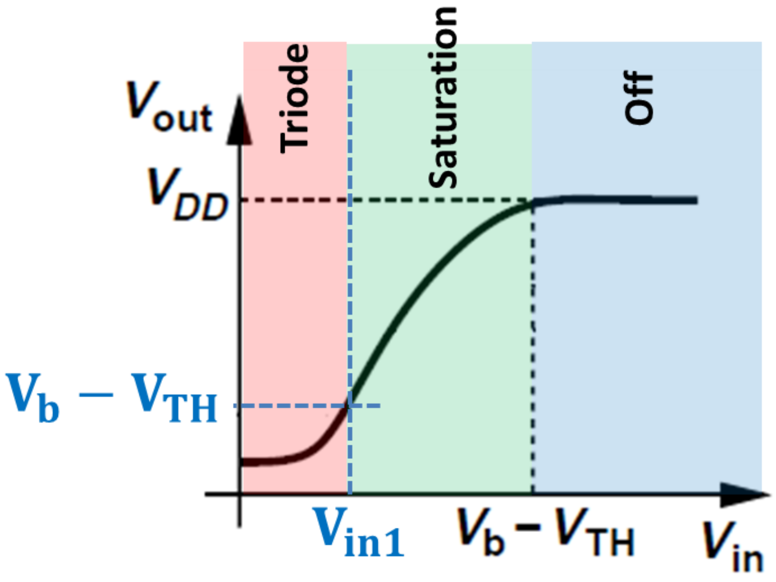
***Ideal Amplifier**



- For converting and amplifying small-signal current to voltages, common-gate provides **low input impedance** and **moderate gain**, but relatively **large output impedance**.

(1) Large-Signal Analysis





(1) M_1 Off when $V_b - V_{in}$ (V_{GS}) < V_{TH}

$$V_{out} = V_{DD}$$

(2) M_1 in Saturation ($V_{DS} > V_{GS} - V_{TH}$, or $V_{out} > V_b - V_{TH}$) if V_{in} decreases

$$V_{out} = V_{DD} - R_D I_D = V_{DD} - R_D \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_b - V_{in} - V_{TH})^2$$

M_1 in Saturation ($V_{DS} > V_{GS} – V_{TH}$, or $V_{out} > V_b – V_{TH}$) if V_{in} decreases

$$V_{out} = V_{DD} - R_D \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_b - V_{in} - V_{TH})^2$$

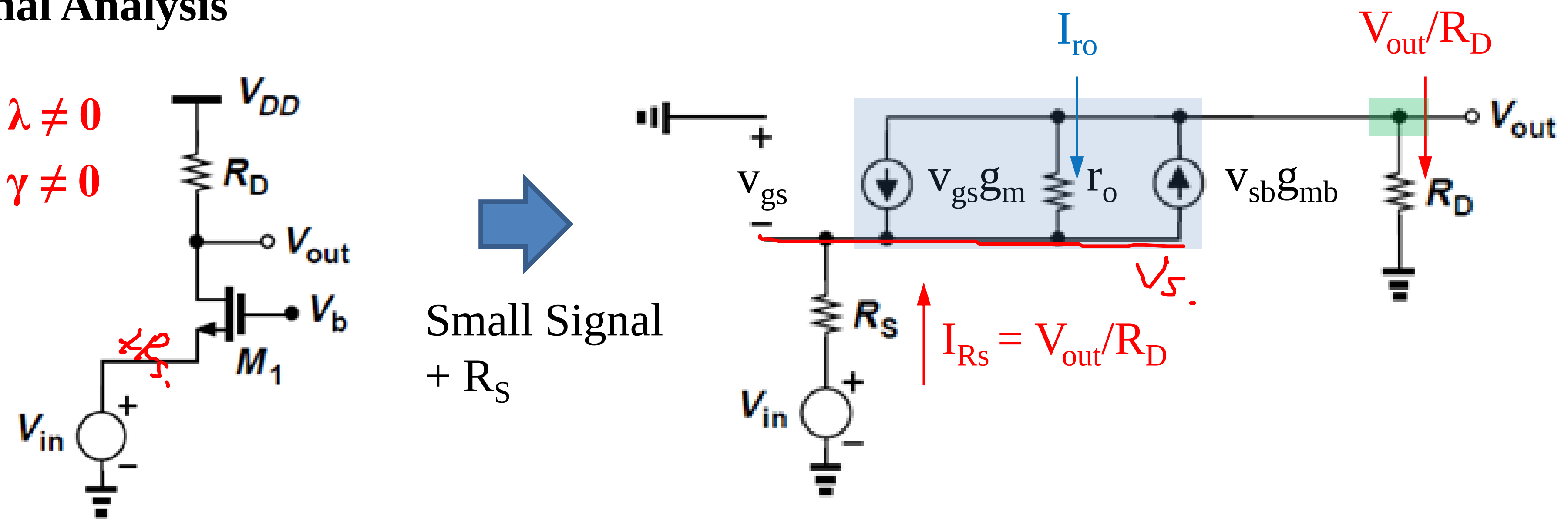
$$\frac{\partial V_{out}}{\partial V_{in}} = -R_D \frac{1}{2} \mu_n C_{ox} \frac{W}{L} 2(V_b - V_{in} - V_{TH}) \left(-1 - \frac{\partial V_{TH}}{\partial V_{in}}\right)$$

$$= R_D \underbrace{\boxed{\mu_n C_{ox} \frac{W}{L} (V_b - V_{in} - V_{TH})}}_{= g_m} \left(1 + \underbrace{\boxed{\frac{\partial V_{TH}}{\partial V_{in}}}}_{= \eta = \frac{\gamma}{2\sqrt{2\Phi_F + V_{SB}}}}\right)$$

$$\mathbf{A_v = \frac{\partial V_{out}}{\partial V_{in}} = R_D g_m (1 + \eta)}$$

- g_m is a function of I_D and η is a function of V_{SB}
- A_v is not quite linear

(2) Small-Signal Analysis



By KVL, $V_{gs} - I_{Rs}R_s + V_{in} = 0 \rightarrow v_{gs} = (\mathrm{V_{out}}/R_D)R_s - V_{in}$

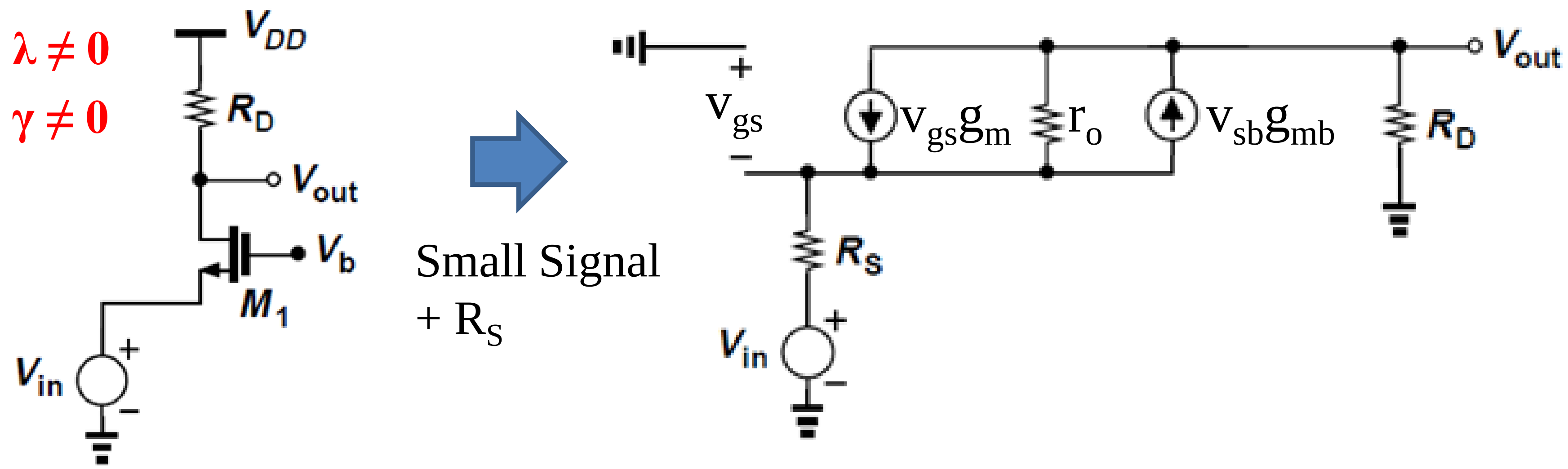
By KCL at the green node, $-v_{gs}g_m - I_{ro} + v_{sb}g_{mb} = {}^{V_{out}}/_{R_D} \rightarrow I_{ro} = -v_{gs}g_m + v_{sb}g_{mb} - {}^{V_{out}}/_{R_D}$

$v_{sb} = -(\mathrm{V_{out}}/R_D)R_s + V_{in} = -v_{gs}$

By KVL, $-V_{out} + r_o * I_{ro} + v_{sb} = 0 \rightarrow -V_{out} + r_o\left(-v_{gs}g_m - v_{gs}g_{mb} - {}^{V_{out}}/_{R_D}\right) - v_{gs} = 0$

$$A_v = \frac{(g_m + g_{mb})r_oR_D + R_D}{r_o + R_S + R_D + (g_m + g_{mb})r_oR_S} \approx R_Dg_m(1+\eta) \quad \text{If } R_S = 0 \text{ and } r_o = \infty$$

(2) Small-Signal Analysis



$$G_m = -\frac{(g_m + g_{mb})r_o + 1}{r_o + R_S + (g_m + g_{mb})r_o R_S}$$

$$R_{out} = R_D \parallel [r_o + R_S + (g_m + g_{mb})r_o R_S]$$

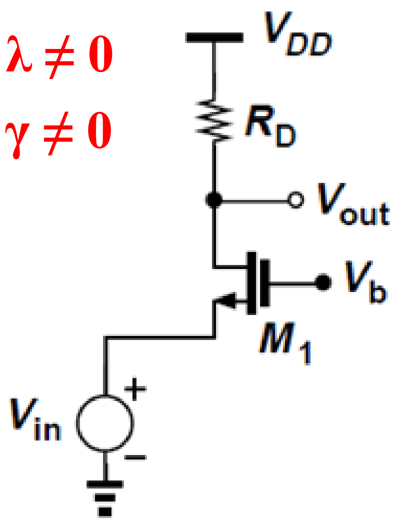
$$A_v = \frac{(g_m + g_{mb})r_o + 1}{r_o + R_S + (g_m + g_{mb})r_o R_S + R_D} R_D \approx R_D g_m(1 + \eta)$$

If $R_S = 0$ and $r_o = \infty$

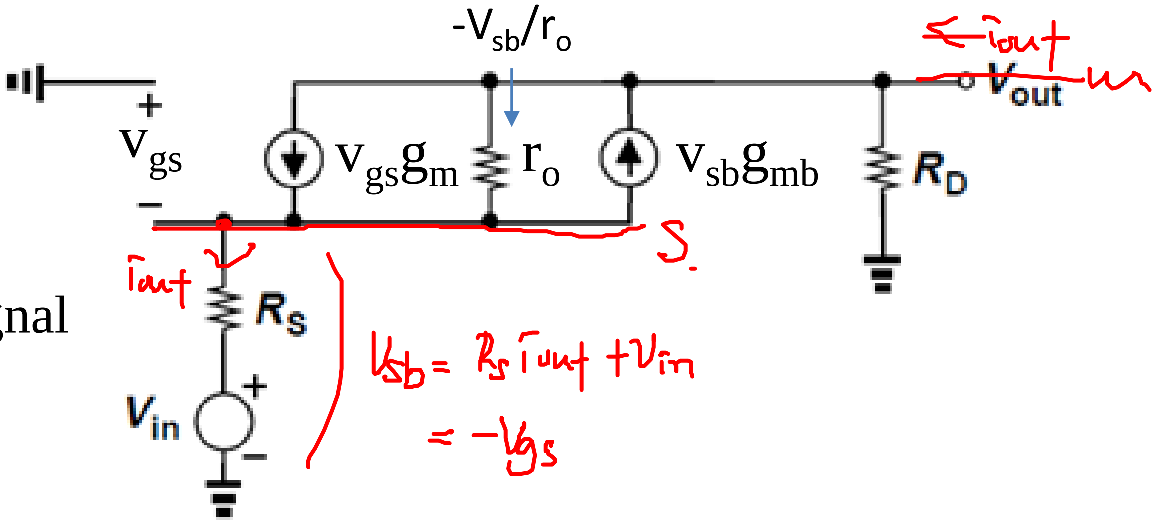
(2) Small-Signal Analysis

$\lambda \neq 0$

$\gamma \neq 0$



Small Signal + R_S



$V_{sb} = R_S i_{out} + V_{in}$

$= -V_{gs}$

$$G_m = \frac{-[(g_m + g_{mb})r_o + 1]}{r_o + R_S + (g_m + g_{mb})r_o R_S}$$

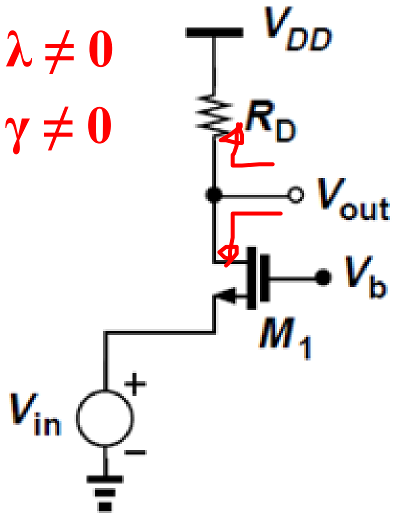
by KCL at S, $\hat{i}_{out} = V_{gs} g_m + \frac{-V_{sb}}{r_o} - V_{sb} g_{mb}.$

$$i_{out}\left[r_o + (g_m + g_{mb})r_o R_S + R_S\right] = -\left[(g_m + g_{mb})r_o + 1\right]V_{in}$$

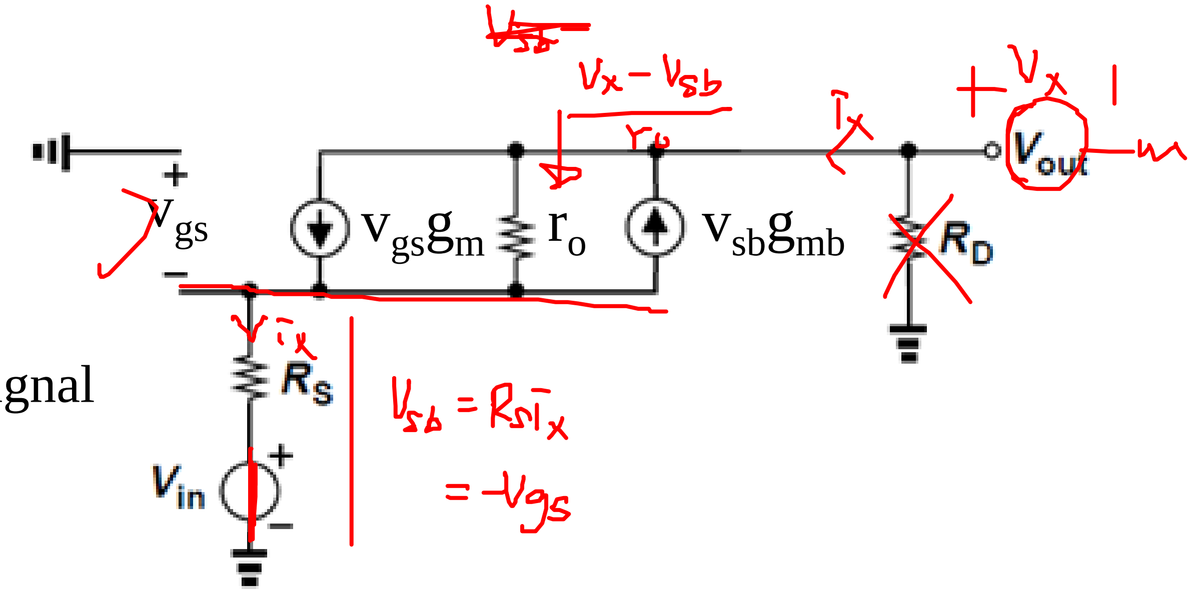
(2) Small-Signal Analysis

$\lambda \neq 0$

$\gamma \neq 0$



Small Signal
$+ R_S$



$V_{sb} = R_S i_x$

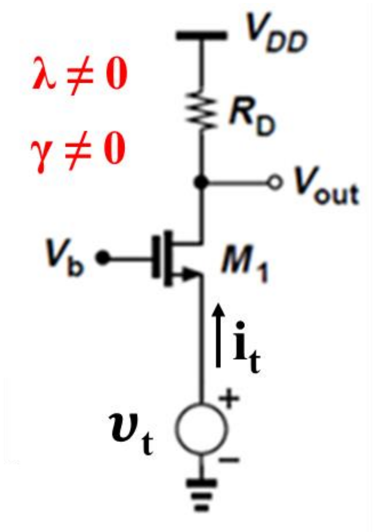
$= -V_{gs}$

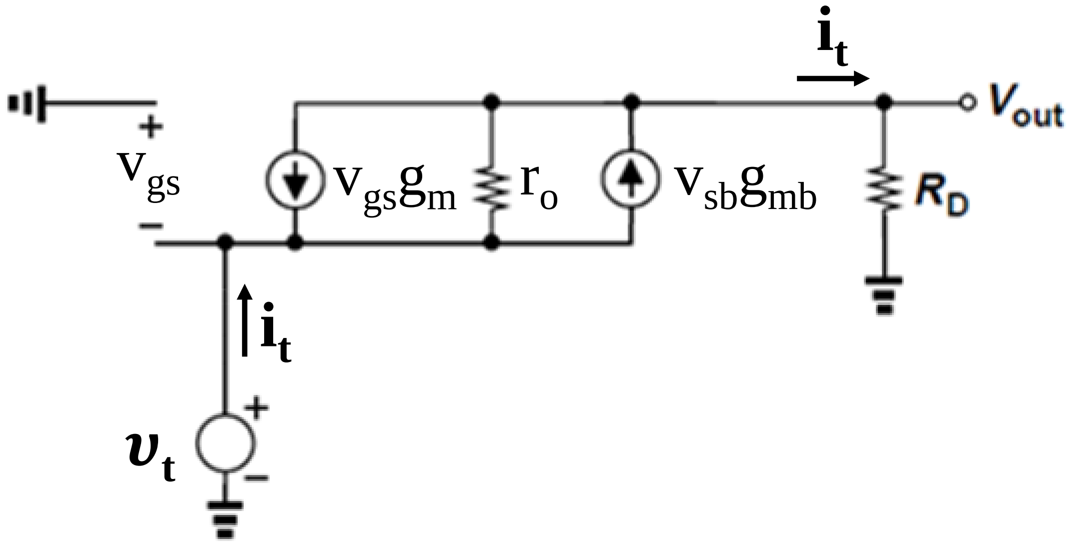
$R_{out} = R_D \parallel [r_o + R_S + (g_m + g_{mb})r_o R_S]$

by KCL: $i_x = g_m V_{gs} + \dfrac{V_x - V_{sb}}{r_o} - g_{mb} V_{sb}$

$$\frac{V_x}{i_x} = \left[(g_m + g_{mb}) R_S r_o + r_o + R_S \right] \parallel R_D .$$

Input Impedance of CG Stage





$$-V_{gs} = v_t = V_{sb}, and\ \upsilon_{out} = R_D i_t$$

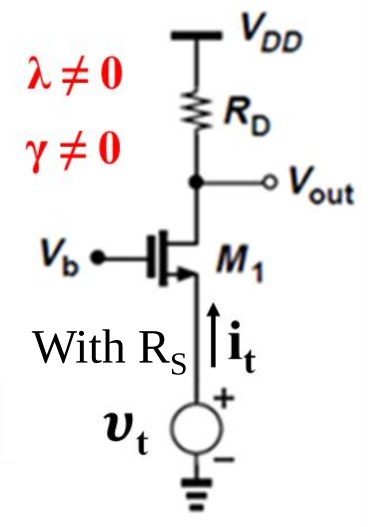
$$i_t = -\upsilon_{gs} g_m + \upsilon_{sb} g_{mb} - \frac{\upsilon_{out} - \upsilon_t}{r_o} \rightarrow i_t = \upsilon_t g_m + \upsilon_t g_{mb} - \frac{R_D i_t - \upsilon_t}{r_o}$$

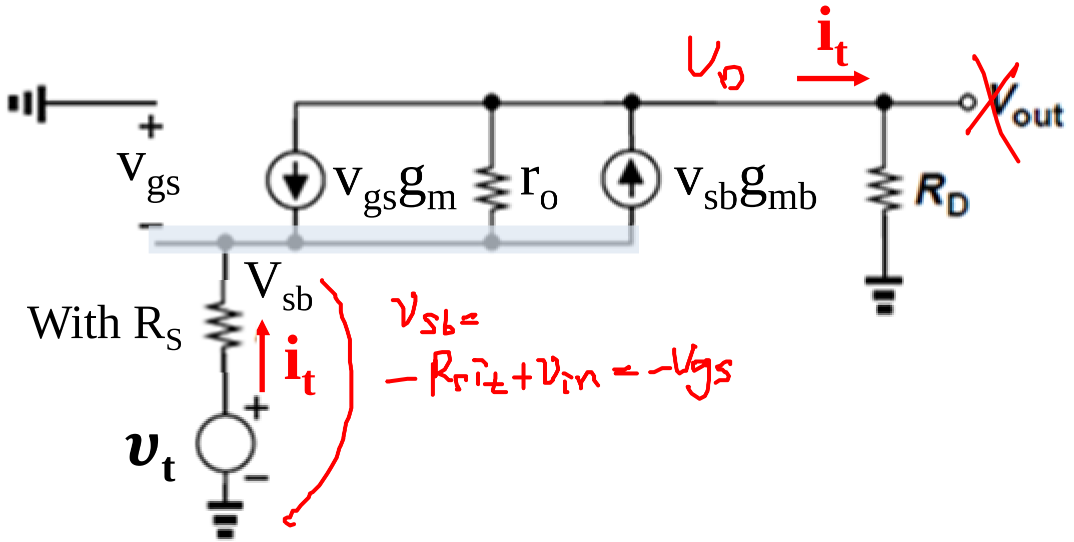
$$R_{in} = \frac{R_D + r_o}{1 + (g_m + g_{mb}) r_o}$$

If $R_D = 0$: $R_{in} = r_o \parallel \frac{1}{g_m} \parallel \frac{1}{g_{mb}}$

If $R_D = \infty$: $R_{in} = \infty$

Input Impedance of CG Stage





$$-V_{gs} = V_{sb} = v_t - R_s i_t, \text{ and } \overset{V_D}{\cancel{v_{out}}} = R_D i_t$$

$$i_t = -v_{gs} g_m + v_{sb} g_{mb} - \frac{\underset{V_D}{\cancel{v_{out}}} - \boxed{(v_t - R_s i_t)}}{r_o} \quad = V_{sb}$$

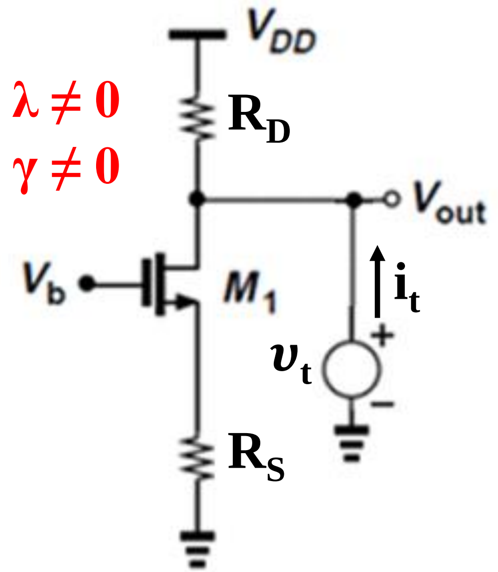
KCL at Source.

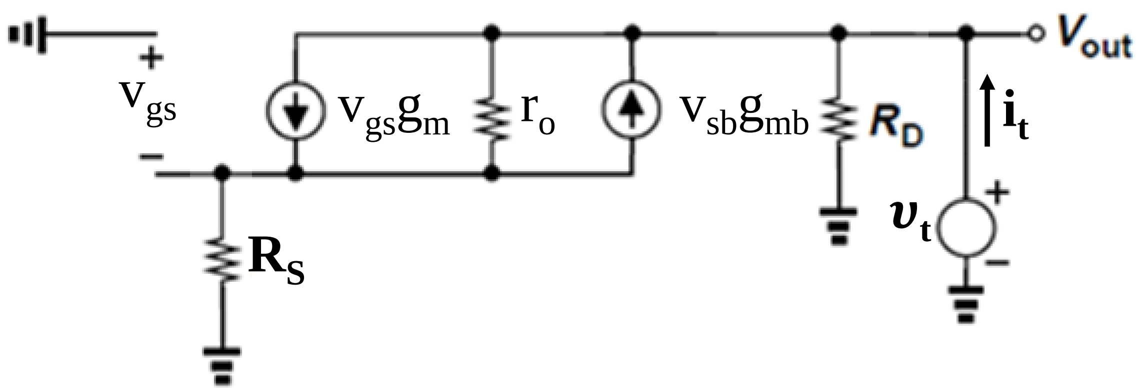
$$\rightarrow i_t = (v_t - R_s i_t) g_m + (v_t - R_s i_t) g_{mb} - \frac{R_D i_t - (v_t - R_s i_t)}{r_o}$$

$$R_{in} = \frac{R_D + r_o + R_S + R_S r_o (g_m + g_{mb})}{1 + (g_m + g_{mb}) r_o}$$

If $R_S = 0$ $\quad R_{in} = \dfrac{R_D + r_o}{1 + (g_m + g_{mb}) r_o}$

Output Impedance of CG Stage

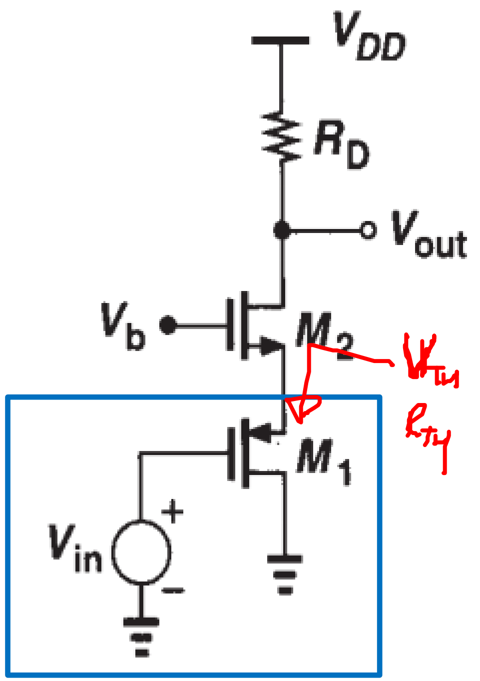


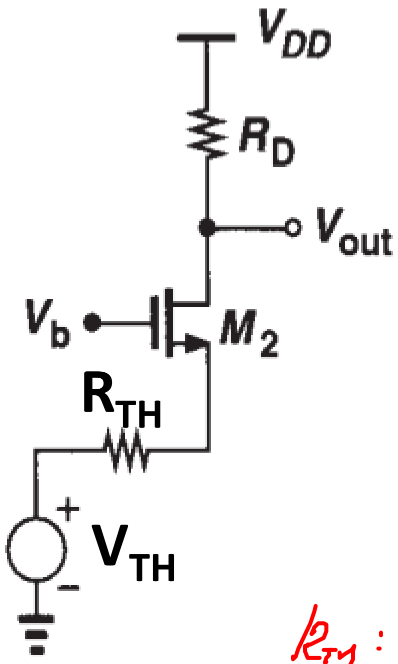


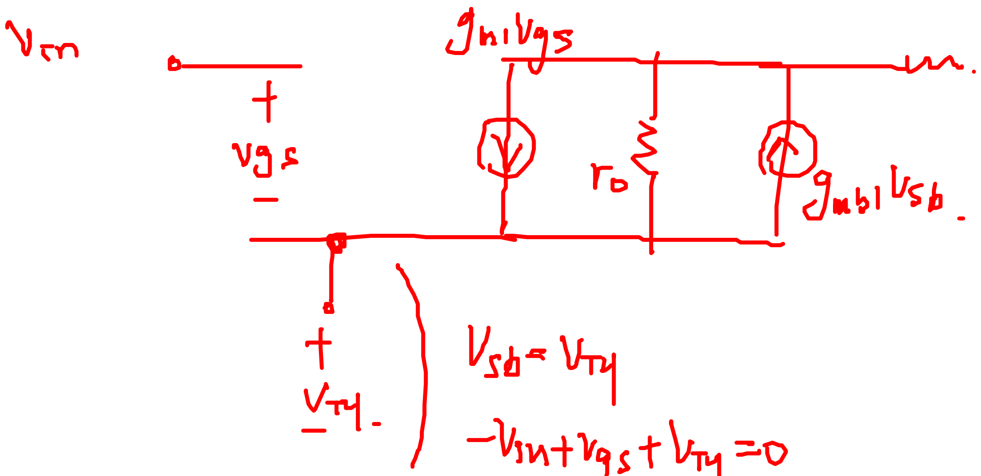
*Small-signal model the same as CS with source degradation (Slide 31)

$$R_{out} = [R_S + r_o + (g_m + g_{mb})r_oR_S] \parallel R_D$$

Example 3.9 Calculate the small-signal voltage gain of the circuit below. ($\lambda \neq 0$, $\gamma \neq 0$)







$V_{sb} = V_{TH}$

$-V_{in} + v_{gs} + V_{TH} = 0$

$R_{TH}: 1/g_{m1} \parallel 1/g_{mb1} \parallel r_{o1}$

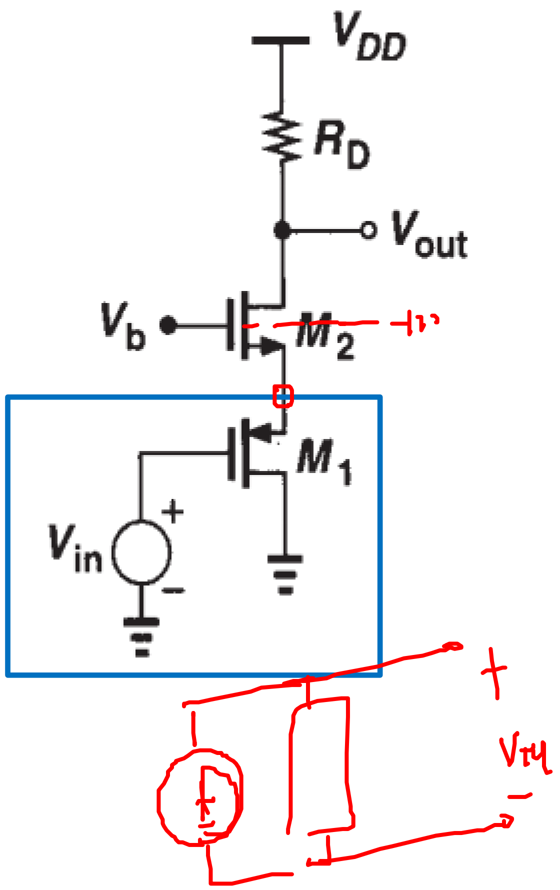
$V_{gs} = V_{in} - V_{TH}$.

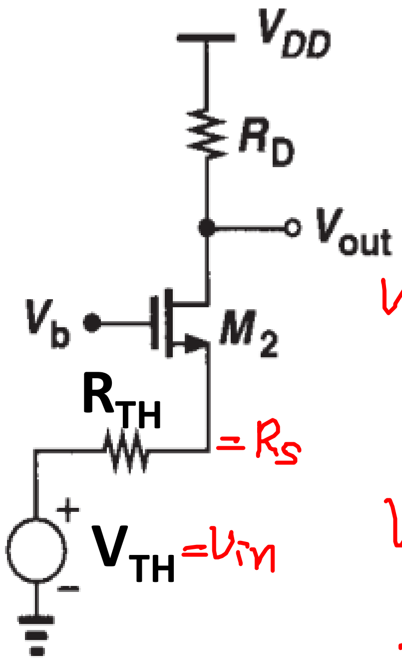
by KCL at S: $g_{m1}V_{gs} + \frac{-V_{sb}}{r_{o1}} - g_{mb1}V_{sb} = 0$

$V_{TH} = \left(g_{m1} / g_{m1} + g_{mb1} + r_{o1} \right) \times V_{in}$

$$A_v = \frac{(g_m + g_{mb})r_o + 1}{r_o + R_S + (g_m + g_{mb})r_o R_S + R_D} R_D$$

Example 3.9 Calculate the small-signal voltage gain of the circuit below. ($\lambda \neq 0$, $\gamma \neq 0$)





$A_v = \dfrac{V_{out}}{v_{in}}$

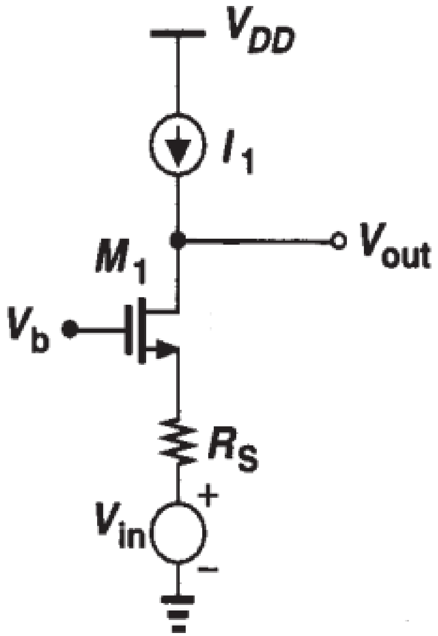
$V_{out} = A_v \times V_{in}$

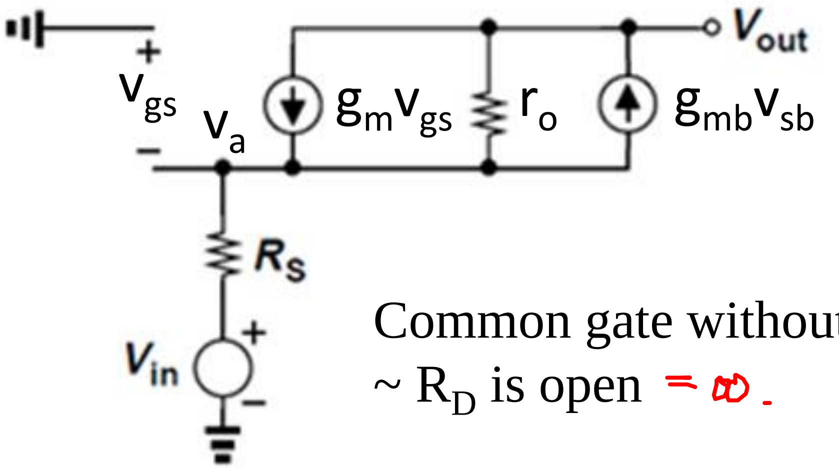
V_{TH}.

$$\frac{V_{out}}{v_{in}} = \frac{(g_m + g_{mb})r_o + 1}{r_o + R_{TH} + (g_m + g_{mb})r_o R_{TH} + R_D} \times R_D \times \frac{g_{m1}}{g_{m1} + g_{mb1} + 1/r_{o1}} \times \cancel{V_{in}}$$

$$A_v = \frac{(g_m + g_{mb})r_o + 1}{r_o + \cancel{R_S}^{\,R_{TH}} + (g_m + g_{mb})r_o \cancel{R_S}^{\,R_{TH}} + R_D} R_D$$

Example 3.10 Calculate the small-signal voltage gain of the circuit below. ($\lambda \neq 0$, $\gamma \neq 0$)





Common gate without R_D

~ R_D is open $= \infty$.

Common gate gain

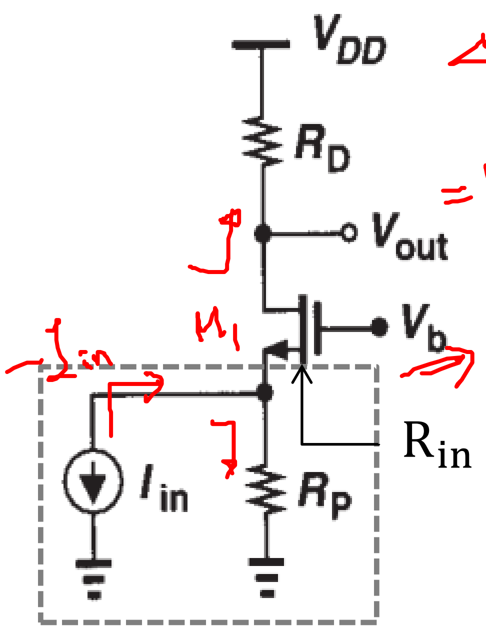
$$A_v = \frac{[(g_m + g_{mb})r_o + 1]\cancel{R_D}}{r_o + R_S + (g_m + g_{mb})r_o R_S + \cancel{R_D}}$$

R_D .

Common gate gain

$$\mathbf{A_v = (g_m + g_{mb})r_o + 1}$$

Example 3.11 Calculate the small-signal transimpedance (V_{out}/i_{in}) gain of the circuit below. ($\lambda \neq 0$, $\gamma \neq 0$)



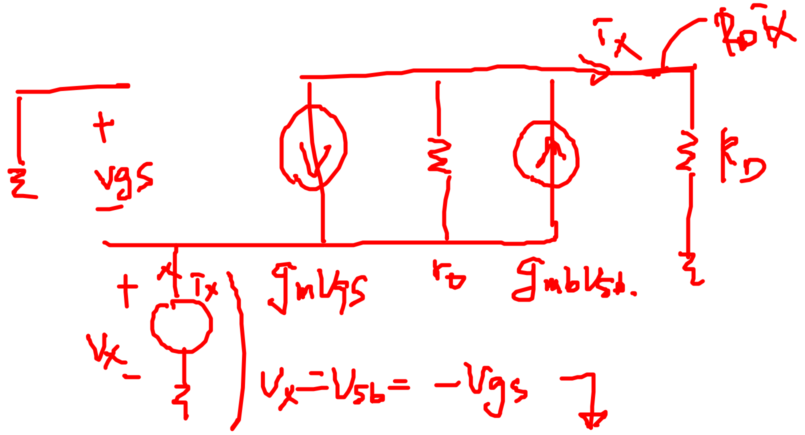
Current Source

$\Delta V_{out} = R_D \Delta I_D$.

$= V_{DD} - R_D I_D$.

$R_{ep} = M_1 \| R_P$.

$$R_{in} = \frac{R_D + r_o}{1 + (g_m + g_{mb})r_o}$$



$V_x = V_{sb} = -V_{gs}$

$$i_x = -g_m V_{gs} + \frac{V_x - R_D i_x}{r_o} + g_{mb} V_{sb}.$$

$$V_x / i_x = \frac{1 + R_D/r_o}{g_m + g_{mb} + 1/r_o}$$

$$-I_D = -I_{in} \frac{R_P}{R_{in} + R_P}$$

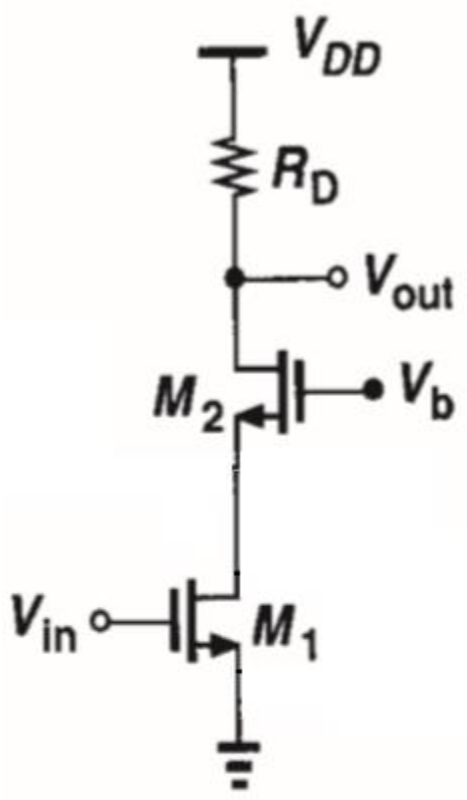
$$\Delta V_{out} = R_D \times \frac{R_P}{R_{in} + R_P} \Delta I_{in}$$

$$-I_D = -i_{in} \frac{R_P}{R_{in} + R_P} \cancel{R_D}$$

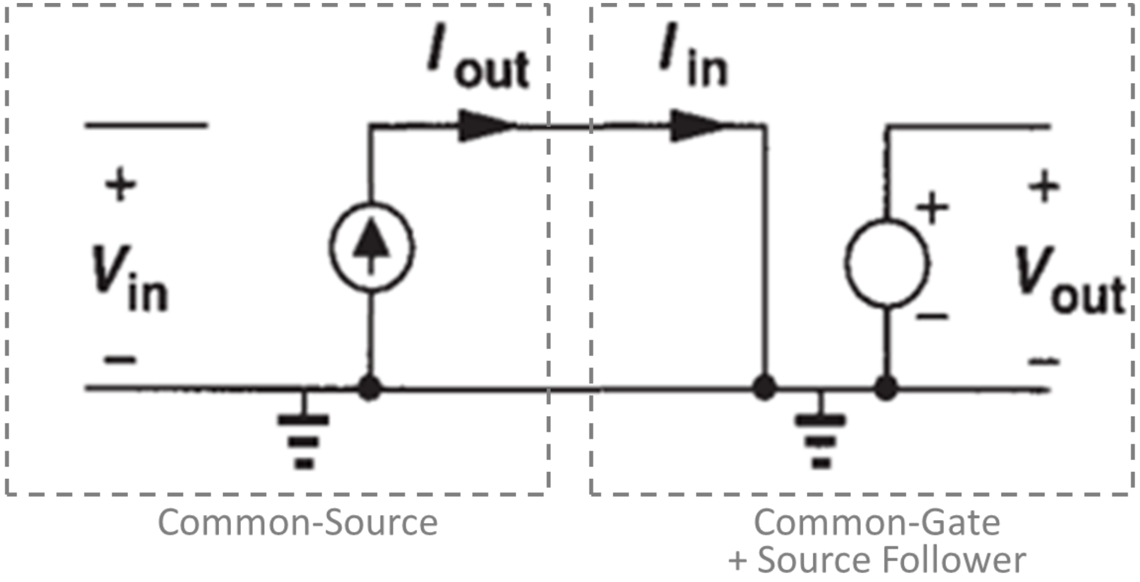
$$\frac{v_{out}}{i_{in}} = +\frac{R_P}{R_{in} + R_P} R_D = \frac{+R_P R_D [1 + (g_m + g_{mb}) r_o]}{R_D + r_o + R_P + (g_m + g_{mb}) r_o R_P}$$

Cascode

The input signal of a common-gate stage may be a current. We also know that a transistor in a common-source arrangement converts a voltage signal to a current signal. **The cascade of a CS stage and a CG stage is called a cascode topology**, providing many useful properties.



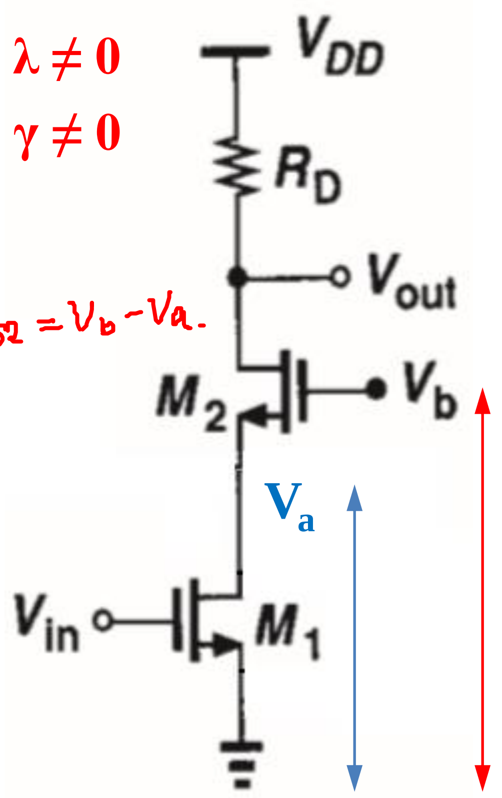
***Ideal Amplifier**



CS + CG with Resistive Load

$\lambda \neq 0$

$\gamma \neq 0$



$V_{GS2} = V_b - V_a.$

M_1 generates a small-signal drain current proportional to the small-signal input voltage, V_{in}, and M_2 simply routes the current to R_D. We call **M_1 the input device** and **M_2 the cascode device**.

$\Rightarrow V_a = V_b - V_{GS2}.$

since $V_{GS2} = V_b - V_a$

$V_{DS} \geq V_{GS} - V_{TH1}$

For M_1 to be in Saturation, $V_a \geqq V_{in} - V_{TH1} \rightarrow V_b - V_{GS2} \geqq V_{in} - V_{TH1}$

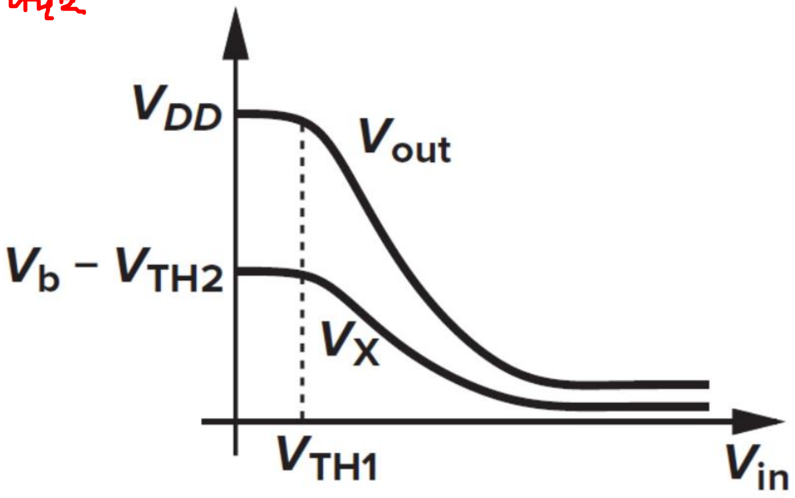
Thus, $V_b \geqq V_{in} - V_{TH1} + V_{GS2}$

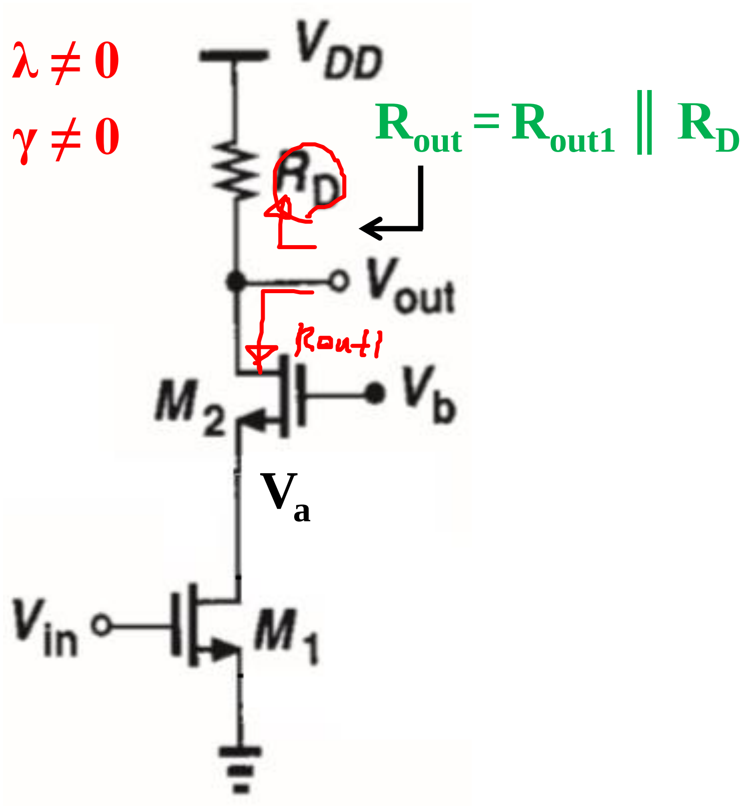
overdrive voltage of M_1

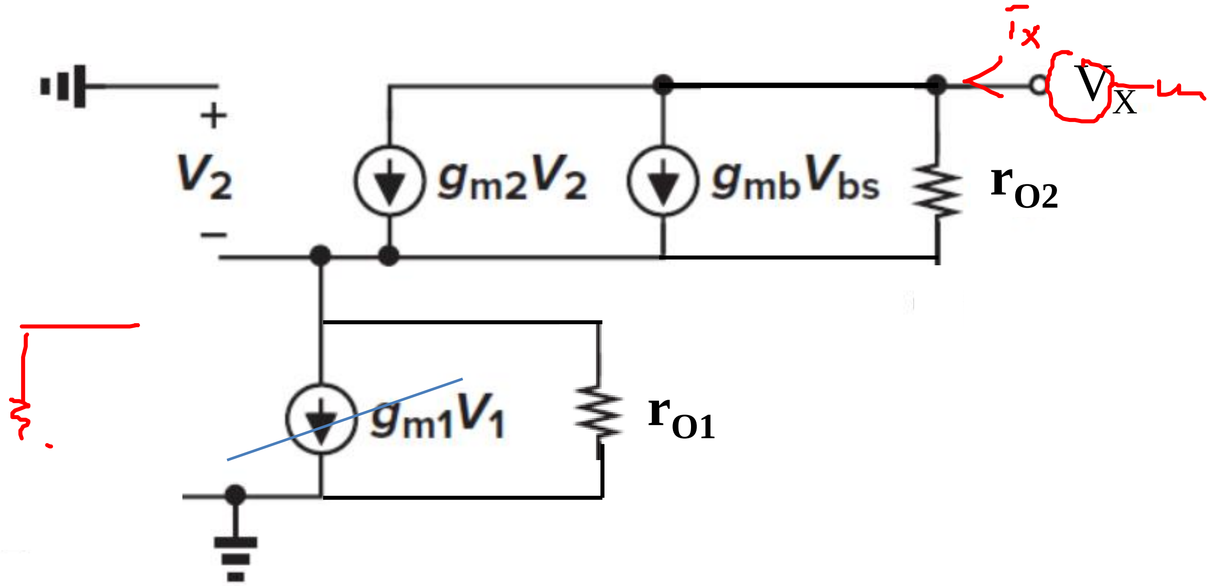
For M_2 to be in Saturation, $V_{out} \geqq V_b - V_{TH2} \geqq (V_{in} - V_{TH1}) + (V_{GS2} - V_{TH2})$

$V_{DS} \geq V_{GS} - V_{TH2}$

$$\rightarrow \boldsymbol{V_{DD} \geqq V_{out}}$$
$$\geqq (\mathbf{V_{in}} - \mathbf{V_{TH1}}) + (\mathbf{V_{GS2}} - \mathbf{V_{TH2}})$$
$$\geqq \mathbf{V_{ov1}} + \mathbf{V_{ov2}}$$



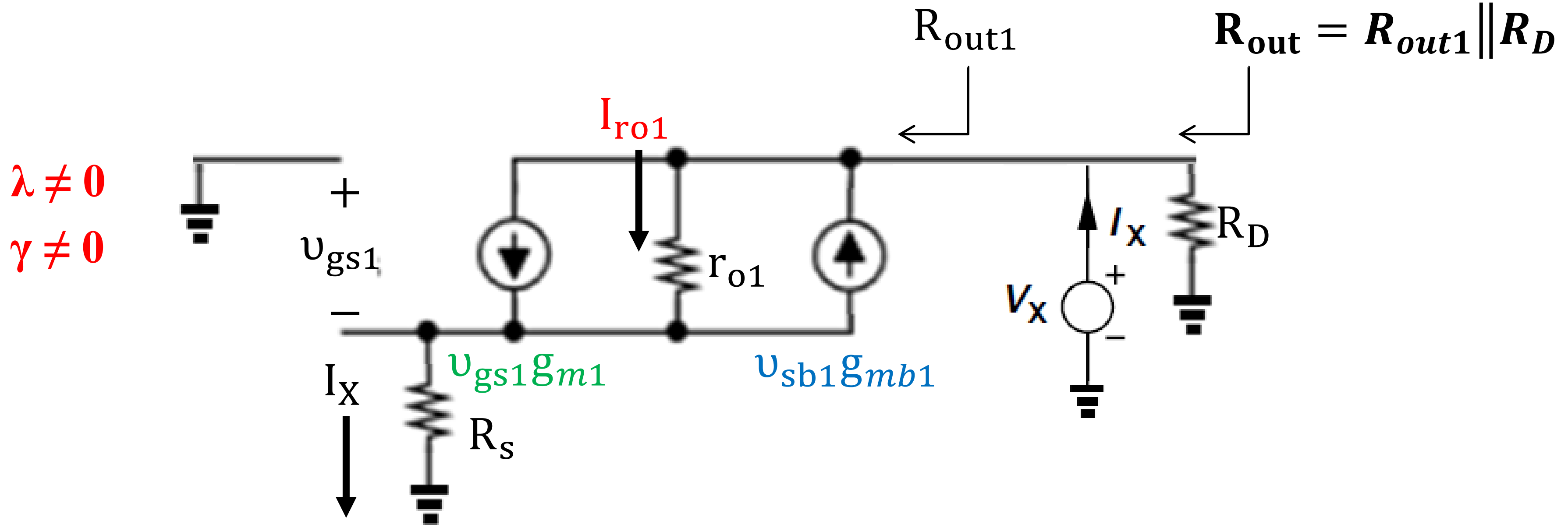




Small-Signal model for R_{out1} shows that the circuit can be viewed as a CS stage with a degeneration resistor of r_{O1}.

$$\mathbf{R}_{\boldsymbol{out1}} = [r_{o2} + r_{o1} + (g_{m2} + g_{mb})r_{o2}r_{o1}] \approx (g_{m2} + g_{mb})r_{o2}r_{o1}$$
$$\mathbf{R}_{\mathbf{out}} = R_{out1} \parallel R_D = (g_{m2} + g_{mb})r_{o2}r_{o1} \parallel R_D$$

Recall: Rout of CS with a source degeneration



By KVL, $-V_X + I_{ro1} r_{o1} + I_x R_s = 0$

By KCL, $I_{ro1} = I_x + v_{sb1} g_{mb1} - v_{gs1} g_{m1}$ where $v_{sb} = -v_{gs1} = I_x R_s$

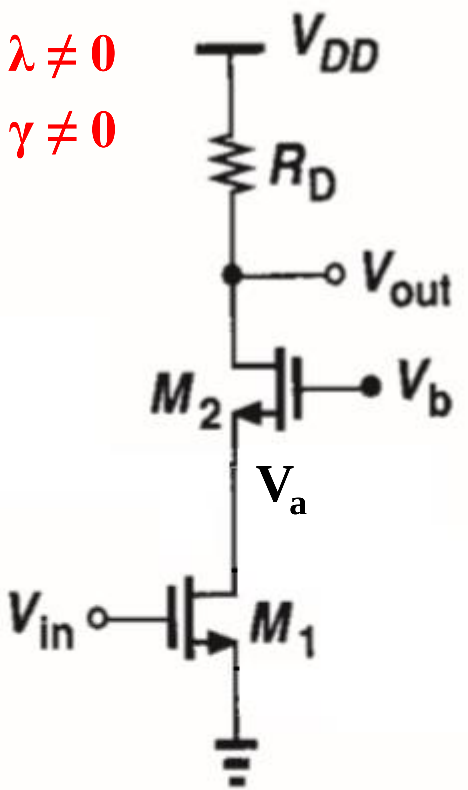
$[r_{o1} + R_s + (g_{mb1} + g_{m1}) r_{o1} R_s] I_x = V_x \rightarrow \boxed{\boldsymbol{R_{out1} = r_{o1} + R_s + (g_{mb1} + g_{m1}) r_{o1} R_s}}$

$$\mathbf{R_{out}} = R_{out1} \parallel R_D = [r_{o1} + R_S + (g_{m1} + g_{mb1}) r_{o1} R_S] \parallel R_D \approx R_D$$

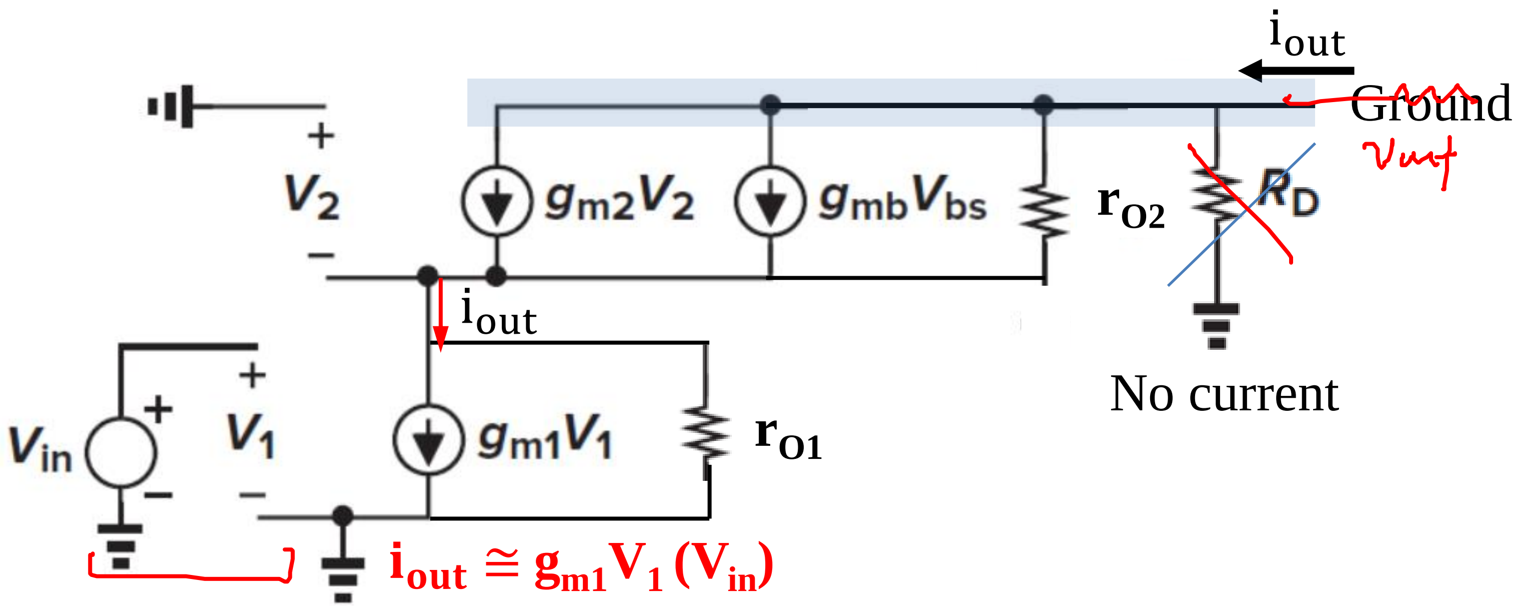
If $(g_{m1} + g_{mb1}) r_{o1} R_S >> R_D$

$\lambda \neq 0$

$\gamma \neq 0$



To calculate G_m, $\boldsymbol{\upsilon_{out}}$ **is grounded and** $\mathbf{G_m = i_{out}/\upsilon_{in}}$



$\mathbf{i_{out} \cong g_{m1} V_1 \, (V_{in})}$

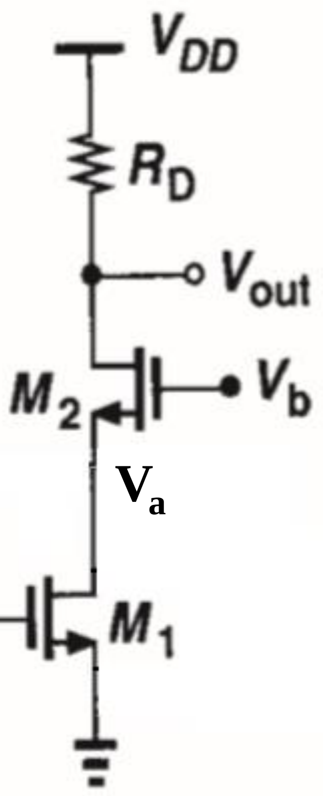
$V_{in} = V_1$

$\rightarrow i_{out} = g_{m1} V_{in}$

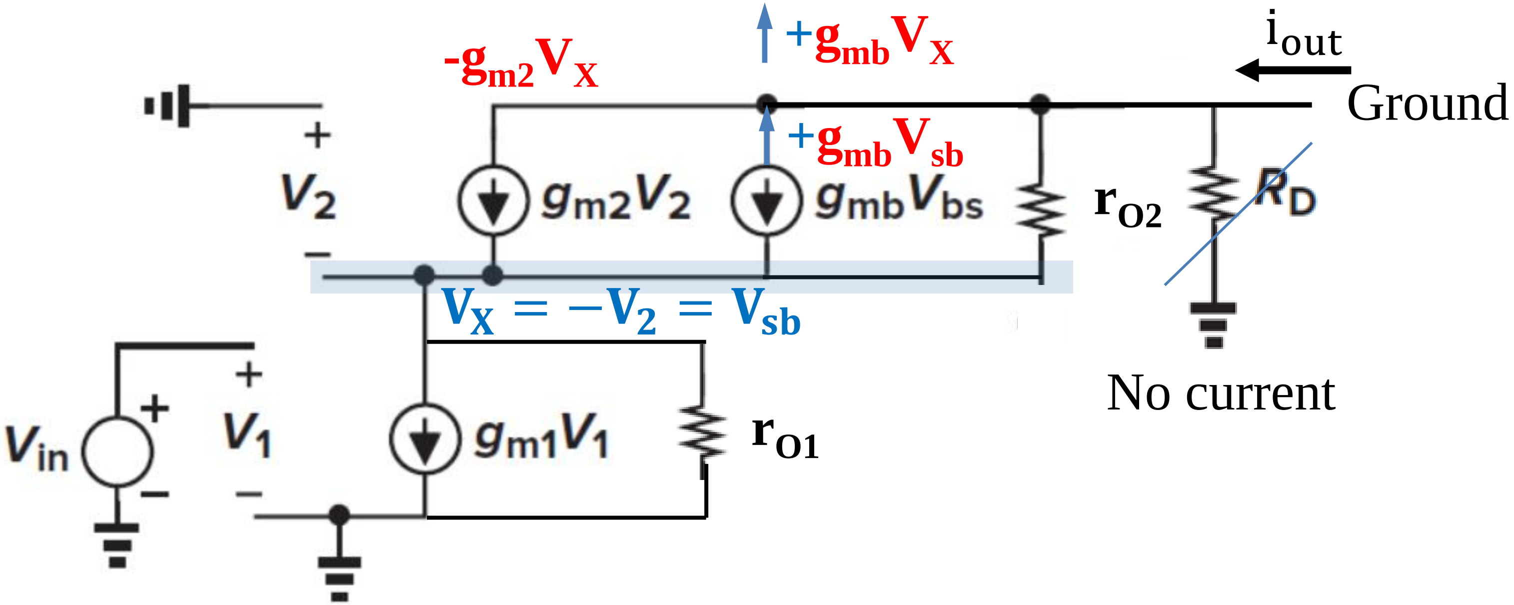
$\mathbf{G_m} \cong$ approx. $\boldsymbol{g_{m1}}$

$\boldsymbol{A_v} = \mathbf{G_m R_{out1}} = g_{m1}(g_{m2} + g_{mb}) r_{o2} r_{o1}$

$\lambda \neq 0$
$\gamma \neq 0$



To calculate G_m, $\boldsymbol{\upsilon_{out}}$ **is grounded and** $\mathbf{G_m} = \mathbf{i_{out}/\upsilon_{in}}$



By KCL at the node VX, $g_{m1}V_1 + \frac{V_X}{r_{O1}} = g_{m2}V_2 - g_{mb}V_{sb} - \frac{V_X}{r_{O2}} = 0$

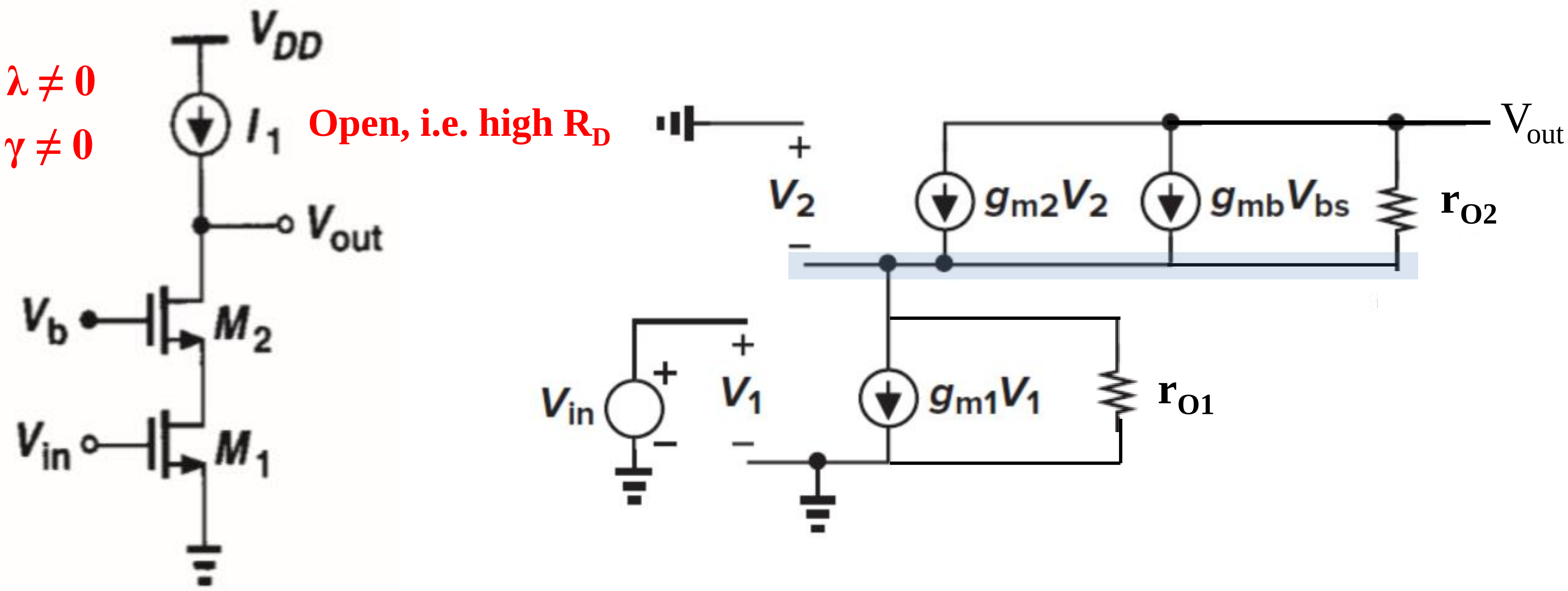
Since $V_X = -V_2 = V_{sb}$, we get $g_{m1}V_{in} + \frac{V_X}{r_{O1}} + g_{m2}V_X + g_{mb}V_X + \frac{V_X}{r_{O2}} = 0$ (1)

And, $i_{out} = g_{m2}V_2 - g_{mb}V_{sb} - \frac{V_X}{r_{O2}} = -g_{m2}V_X - g_{mb}V_X - \frac{V_X}{r_{O2}}$ (2)

From (1) and (2) $\frac{i_{out}}{V_{in}} = \frac{g_{m1}r_{O1}}{r_{O1} + 1/[(g_{m2}+g_{mb}) + 1/r_{O2}]} = \frac{g_{m1}\cancel{r_{o1}}}{\cancel{r_{o1}} + \left(r_{o2} \parallel \overset{a}{\frac{1}{g_{m2}+g_{mb2}}}\right)} \cong \mathbf{g_{m1}}$

$\frac{1}{g_{m2}+g_{mb2}} > a$

CS + CG with Ideal Current Source Load



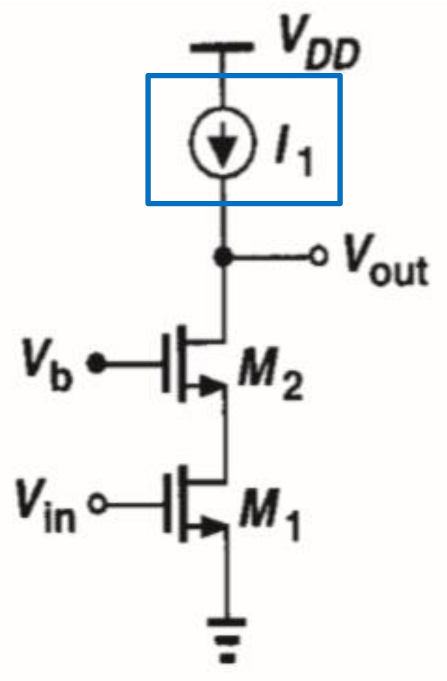
$$\mathbf{G_m} = \frac{g_{m1} r_{o1}}{r_{o1} + \left(r_{o2} \parallel \dfrac{1}{g_{m2} + g_{mb2}} \right)} \quad or \cong \mathbf{g_{m1}}$$

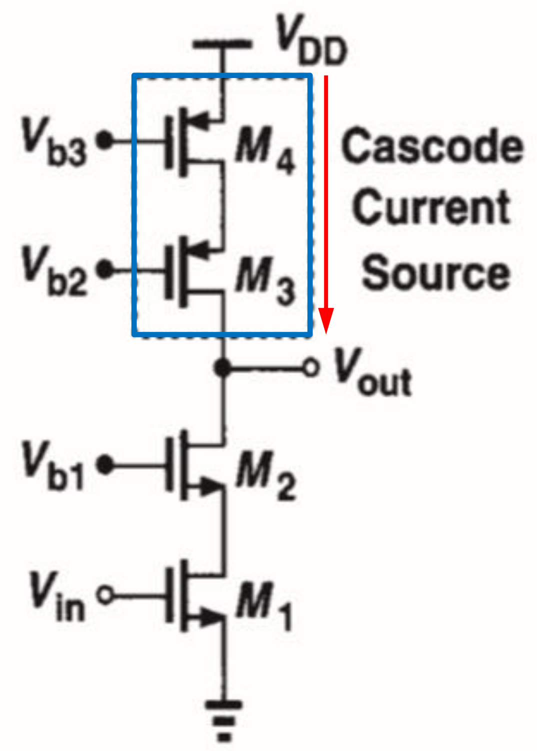
$$\mathbf{R_{out}} = [r_{o2} + r_{o1} + (g_{m2} + g_{mb}) r_{o2} r_{o1}] \parallel \cancel{R_D} \approx (g_{m2} + g_{mb}) r_{o2} r_{o1} \text{ as } R_D \text{ is infinite.}$$

$$\boldsymbol{A_v} = \mathbf{G_m R_{out}}$$

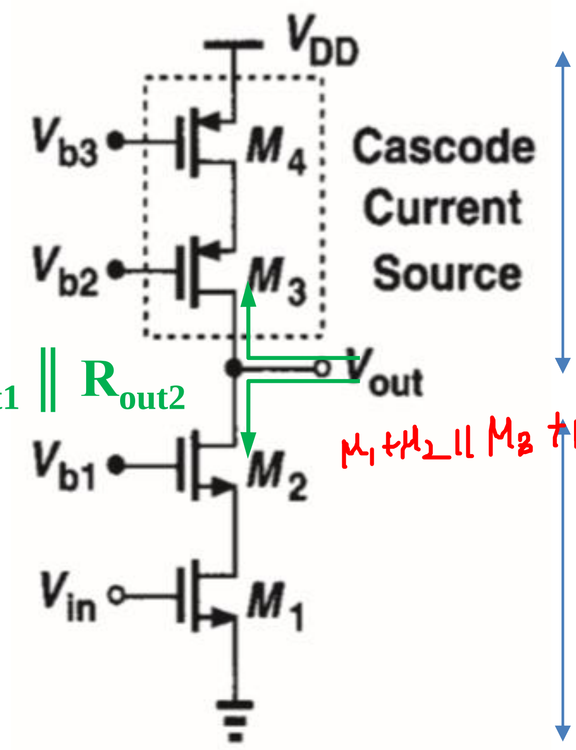
CS + CG with Cascode Current Source Load

A cascode structure need not operate as an amplifier. Another popular application of this topology is in **building constant current sources**. The **high output impedance** yields a current source closer to the ideal.





$R_{out} = R_{out1} \parallel R_{out2}$

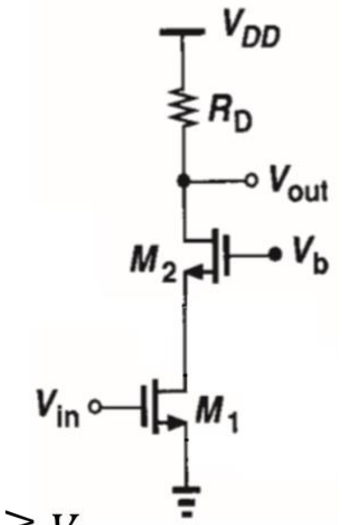


$M_1 + M_2 \parallel M_3 + M_4$

Output Voltage Swing is limited to

$$\mathbf{V_{DD} - V_{ov3} - V_{ov4} \geqq V_{out}}$$

$$\mathbf{V_{out} \geqq V_{ov1} + V_{ov2}}$$



$\rightarrow V_{DD} \geqq V_{out}$
$\geqq (V_{in} - V_{TH1}) + (V_{GS2} - V_{TH2})$
$\geqq V_{ov1} + V_{ov2}$

$$R_{out} = [r_{o1} + r_{o2} + (g_{m2} + g_{mb2})r_{o2}r_{o1}] \parallel [r_{o3} + r_{o4} + (g_{m3} + g_{mb3})r_{o3}r_{o4}]$$

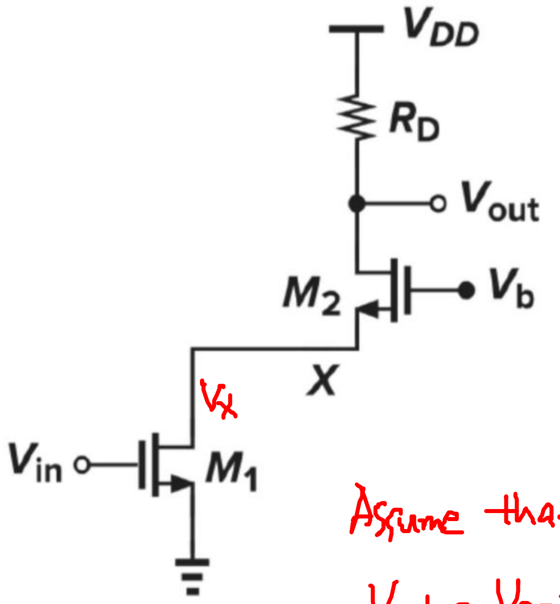
$$G_m = g_{m1}\frac{r_{o1}}{r_{o1} + \left(r_{o2} \parallel \dfrac{1}{g_{m2} + g_{mb2}}\right)} \cong \boldsymbol{g_{m1}}$$

$$A_v = G_m R_{out}$$

Example 3.12 In the circuit below, assume $\mu_n C_{ox}(\frac{W}{L})_1 = \mu_n C_{ox}(\frac{W}{L})_2 = 1\ mA/V^2$, $I_{D1} = I_{D2} = 0.55$ mA, $V_{DD} = 3.1$ V, $V_{Th} = 1$ V, $\lambda = 0.1$, and $R_D = 2k\Omega$. Neglect body effect.

(a) Calculate V_b and V_{in} (DC) such that M_1 is exactly at saturation ($V_{DS} = V_{GS} - V_{Th}$)

(b) Draw small signal model and calculate small signal gain A_v.



$V_{DS} = V_{GS} - V_{Th}$. $\Rightarrow$ $\underline{V_X = V_{in} - V_{Th}}$.

$V_D = V_X$

$V_G = V_{in}$

$I_{D1} = \frac{1}{2}\mu_n C_{ox}\left[\frac{W}{L}\right]_1 (\underbrace{V_{GS} - V_{TH}}_{= V_{DS} = V_X})^2 (1 + \lambda \overset{V_X}{V_{DS}}) = 0.55\ mA.$

$\Rightarrow$ Put the values, Solve $\boxed{V_X = 1V}$. $V_X = V_{in} - V_{Th}$.

$\boxed{V_{in} = 2V}$.

Assume that M_2 in saturation.

$V_{out} = V_{DD} - R_D I_{D2} = 3.1 - 2k \times 0.55mA = 2V$.

$V_{DS} \geq V_{GS} - V_{Th}$. $\Rightarrow$ $V_{out} - \cancel{V_X} \geq V_b - \cancel{V_X} - V_{Th}$. $\Rightarrow$ $\underline{V_b \leq 3V}$. $\boxed{V_b = 3V}$.

$I_{D2}(sat) = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_{GS} - V_{Th})^2 (1 + \lambda V_{DS}) \Rightarrow 0.55mA$.

Example 3.12 In the circuit below, assume $\mu_n C_{ox}(\frac{W}{L})_1 = \mu_n C_{ox}(\frac{W}{L})_2 = 1\ mA/V^2$, $I_{D1} = I_{D2} = 0.55$ mA, $V_{DD} = 3.1$ V, $V_{Th} = 1$ V, $\lambda = 0.1$, and $R_D = 2k\Omega$. Neglect body effect.

(a) Calculate V_b and V_{in} (DC) such that M_1 is exactly at saturation ($V_{DS} = V_{GS} - V_{Th}$)

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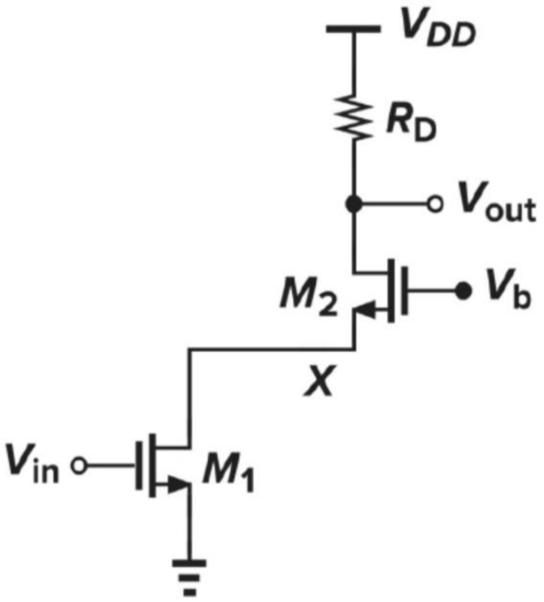


$A_v = -G_m R_{out}$

$G_m \cong g_{m1}$

$R_{out} = [r_{o1} + r_{o2} + (g_{m2} + \cancel{g_{mb2}}) r_{o1} r_{o2}] \| R_D \Rightarrow 4M \| 2K$

$\simeq 1990\Omega.$

$g_{m1} = \frac{2I}{V_{GS1} - V_{TH}} = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{th})(1 + \lambda V_{DS}) = 1.1\times10^{-3}$

$g_{m2} = 1.1\times10^{-3}$

$r_{o1} = r_{o2} = \frac{1}{I\lambda} = 18.18K.$

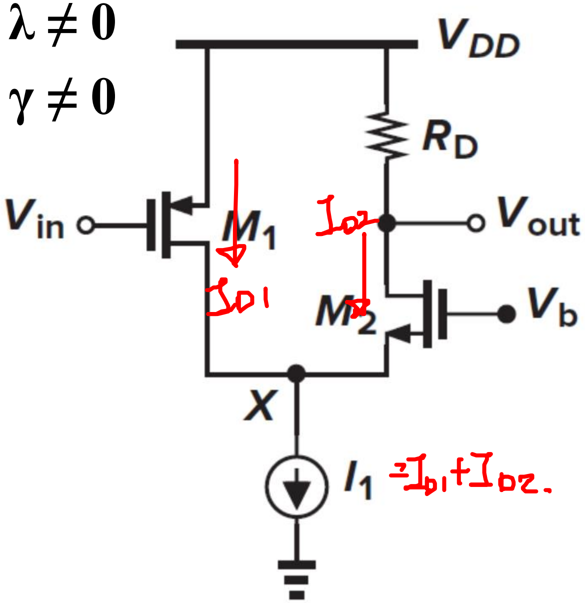
$A_v = -G_m R_{out} = \underline{-2.2}$

Folded Cascode

The idea behind the cascode structure is to convert the input voltage to a current and apply the result to a common-gate stage. However, the input device and the cascode device need not be of the same type.

$\lambda \neq 0$

$\gamma \neq 0$



$|I_{D1}| + I_{D2}$ is equal to I_1 and is constant

$I_1 = I_{D2}$.

If $V_{SG} < |V_{TH1}|$, i.e. $V_{DD} - V_{in} < |V_{TH1}|$, M_1 is off and M_2 carries I_1

$$V_{out} = V_{DD} - I_1 R_D$$

If $V_{SG} > |V_{TH1}|$, i.e. $V_{DD} - V_{in} > |V_{TH1}|$, M_1 is on. $I_{D2} = I_1 - I_{D1}$.

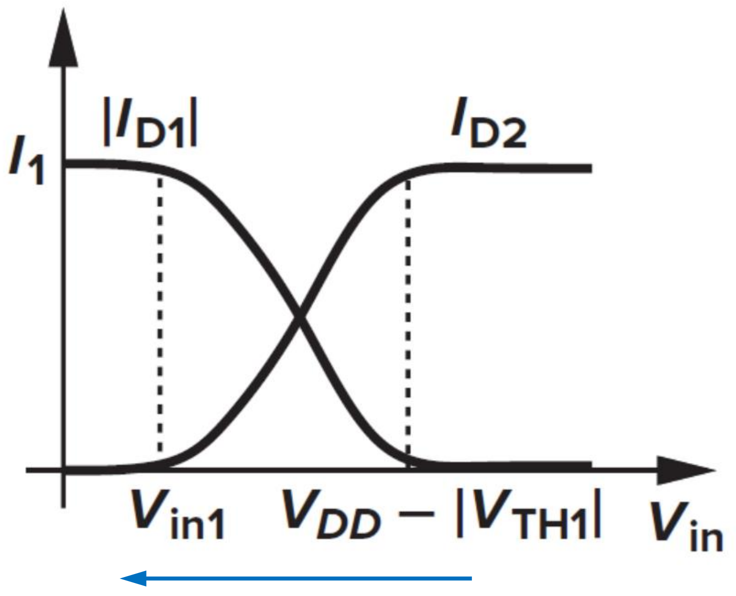
And if M_1 is in saturation. $I_{D2} = I_1 - \underbrace{\frac{1}{2}\mu_P C_{ox}(\frac{W}{L})(V_{DD} - V_{in}\downarrow - |V_{TH1}|)^2}_{I_{D1}\uparrow}$

$I_{D2}\downarrow$

V_{in} drops and I_{D2} decreases further, and finally $I_{D1} = I_1 = \frac{1}{2}\mu_P C_{ox}(\frac{W}{L})(V_{DD} - V_{in1} - |V_{TH1}|)^2$

$I_{D1} = I_1 = \frac{1}{2}\mu_P C_{ox}(\frac{W}{L})\,(V_{DD} - V_{in} - |V_{TH1}|)^2$

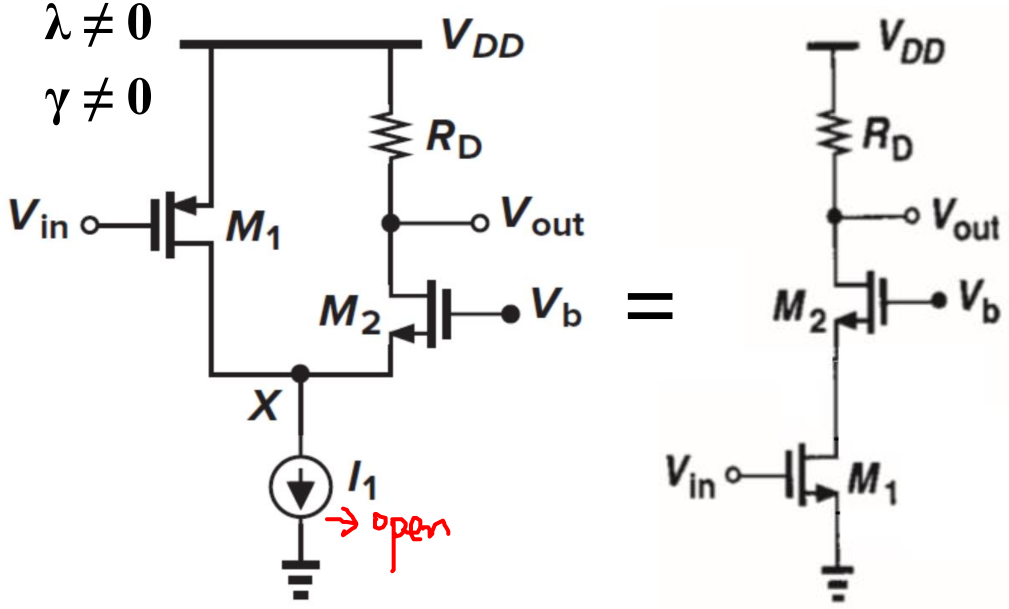
Thus, $V_{in1} = V_{DD} - \sqrt{\frac{2I_1}{\mu_P C_{ox}\left(\frac{W}{L}\right)}} - |V_{TH1}|$

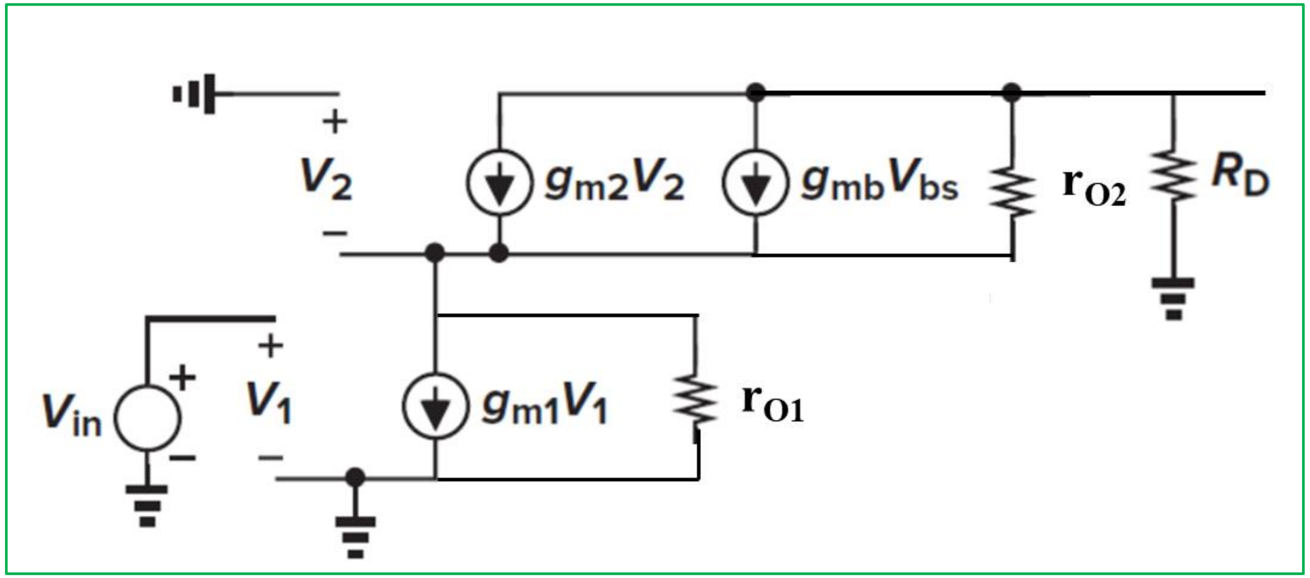


As V_{in} decreases I_{D1} goes into the saturation

$\lambda \neq 0$

$\gamma \neq 0$





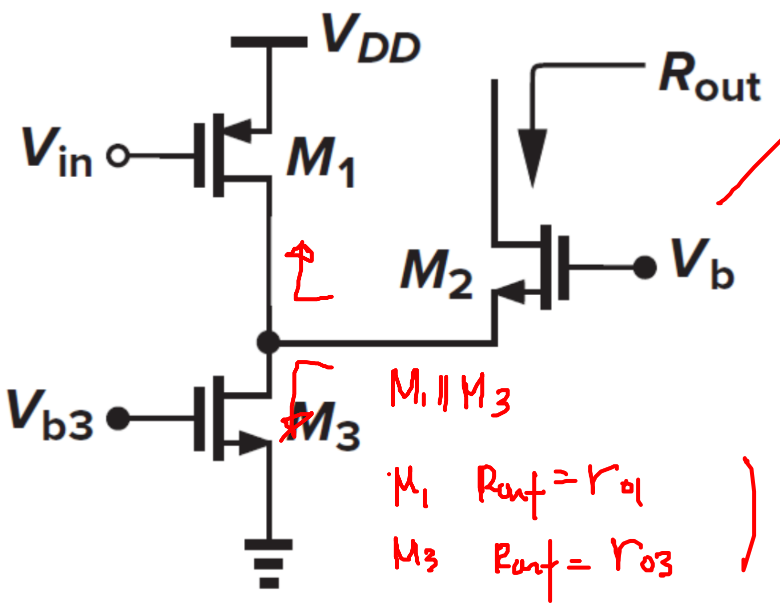
$$\mathbf{G_m} = \frac{g_{m1} r_{o1}}{r_{o1} + \left(r_{o2} \parallel \frac{1}{g_{m2} + g_{mb2}}\right)} \; or \cong \boldsymbol{g_{m1}}$$

$$\mathbf{R}_{\boldsymbol{out}} = [r_{o2} + r_{o1} + (g_{m2} + g_{mb}) r_{o2} r_{o1}] \parallel R_D \approx (g_{m2} + g_{mb}) r_{o2} r_{o1} \parallel R_D$$

$$\boldsymbol{A_v} = \mathbf{G_m R_{out}}$$

Example 3.13 Calculate the output impedance of the folded cascode where M_3 operates as the bias current source.

CS stages with source degeneration.



$R_{out} = r_{o2} + (r_{o1} \| r_{o3}) + (g_{m2} + g_{mb2}) r_{o2} (r_{o1} \| r_{o3})$

$M_1 \| M_3$

M_1 $R_{out} = r_{o1}$

M_3 $R_{out} = r_{o3}$

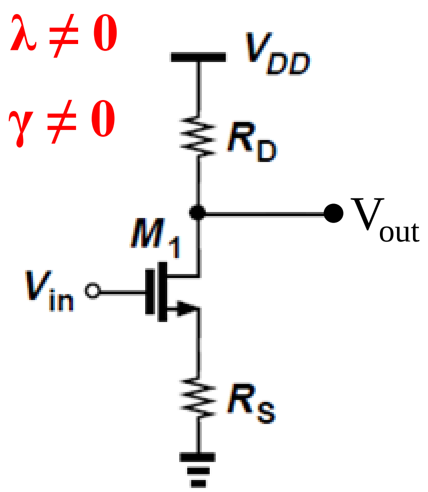
$R_S = r_{o1} \| r_{o3}$.

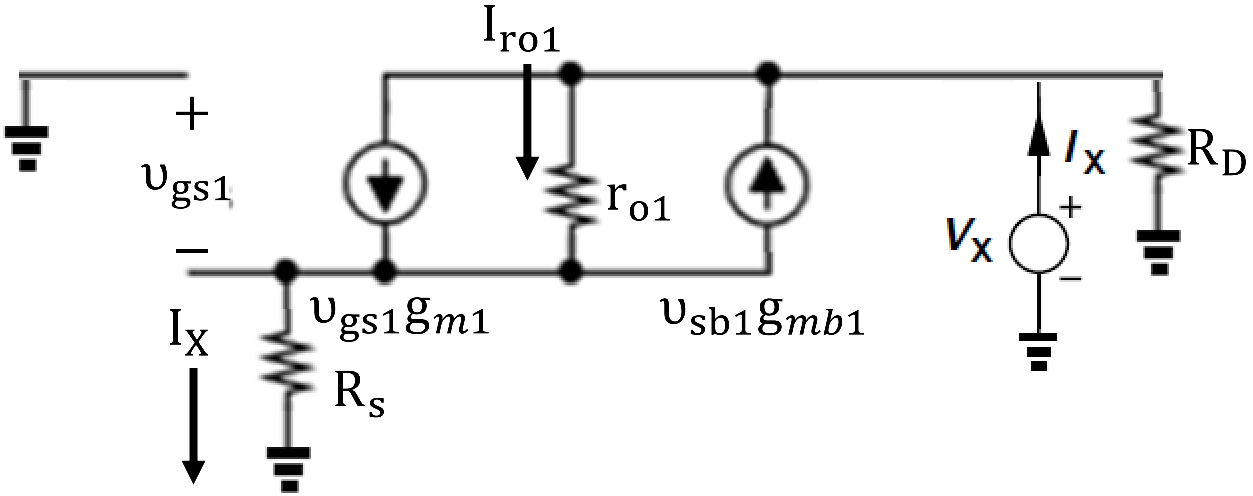
Recall: CS stage with source degeneration

$R_{out} = \mathrm{R_{out1}} \parallel \cancel{R_D} = [\mathrm{r_{o1}} + \cancel{\mathrm{R_S}} + (\mathrm{g}_{m1} + \mathrm{g}_{mb1})\mathrm{r_{o1}R_S}] \parallel \cancel{R_D} \approx R_D$

$$R_{out} = [1 + (g_{m2} + g_{mb2}) r_{O2}](r_{O1} \| r_{O3}) + r_{O2}$$

Recall: CS stage with source degeneration





$$\mathbf{R_{out}} = \mathrm{R_{out1}} \parallel R_D = [\mathrm{r_{o1}} + \mathrm{R_S} + (\mathrm{g}_{m1} + \mathrm{g}_{mb1})\mathrm{r_{o1}}\mathrm{R_S}] \parallel R_D \approx R_D$$