Question 1. Source Follower + CS

[40 points] In this question, we will study the source follower cascaded by a CS stage. Neglect body effect and channel length modulation.

(a) First we carry out a dc analysis on the CS stage. In the CS stage circuit shown in Figure 1, assume $\mu_n C_{ox}\left(\frac{W}{L}\right)_1 = \mu_p C_{ox}\left(\frac{W}{L}\right)_2 = 1\text{mA/V}^2$, $V_{TN} = 0.7\text{V}$, $V_{TP} = -1\text{V}$, $V_{DD} = 2.5\text{V}$. Calculate I_1 and V_{OUT} such that M_1 operates in saturation with $V_{GS1} = 0.8\text{V}$. What is V_{IN} if M_1 is at the edge of saturation?

(b) Then we do small signal analysis on the CS stage. In the circuit shown in Figure 1, derive the small signal voltage gain $A_v = \frac{v_{out}}{v_{in}}$. What is the maximum dc level of V_{in} for which M1 remains saturated? Express V_{IN} by V_{DD}, gate-source voltages and threshold voltages of M_1 and M_2.

(c) To accommodate an input dc level close to V_{DD}, we further cascade it with a source follower as shown in Figure 2. Derive the small signal gain $\frac{v_{out}}{v_{in}}$. If $V_{IN} = V_{DD}$, what relationship between the gate-source voltages of M_2 and M_3 guarantees that M_1 is saturated?

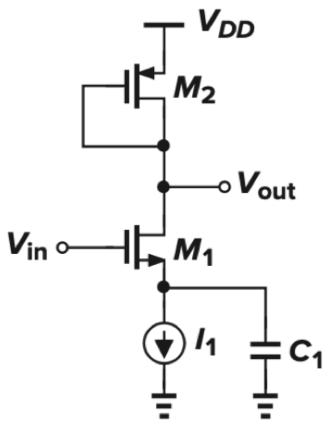


FIGURE 1. CS Stage

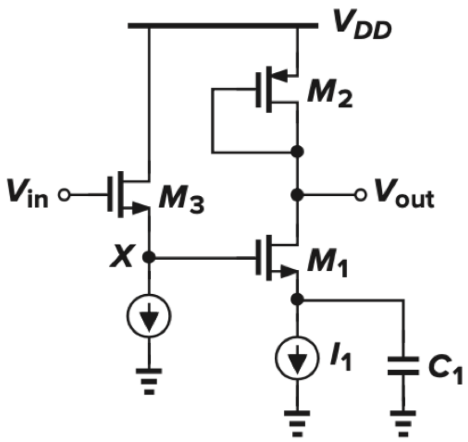


FIGURE 2. Source Follower + CS

(a) Suppose M_2 is on : $V_{SG2} > |V_{TP}|$. By observation, $V_{SD2} > V_{SG2} - |V_{TP}|$

$\Rightarrow M_2$ is in saturation.

For M_1: $I_1 = I_{D1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{GS1} - V_{TN})^2 = \frac{1}{2}\cdot 1\text{m}\cdot(0.8-0.7)^2 = 5\times10^{-3}\text{mA}$

For M_2: $I_{D2} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - V_{out} - |V_{TP}|)^2 = \frac{1}{2}\cdot 1\text{m}\cdot(2.5 - V_{out} - 1)^2 = 5\times10^{-3}\text{mA}$

$\Rightarrow V_{out} = 1.4\text{V}$ or 1.6V

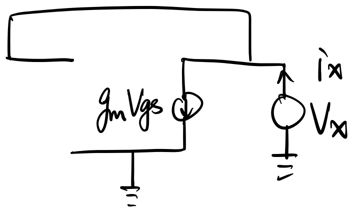
Since $V_{SG2} = V_{DD} - V_{out} = 2.5 - V_{out} > |V_{TP}| = 1\text{V} \Rightarrow V_{out} = 1.4\text{V}$

$\therefore \boxed{I_1 = 5\times10^{-3}\text{mA},\ V_{OUT} = 1.4\text{V}}$

If M_1 is at the edge of saturation, $V_{D1} = V_{G1} - V_{TN}$

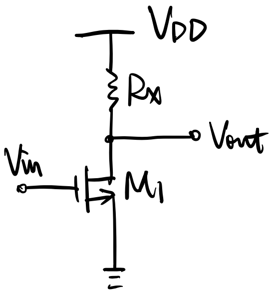
$\Rightarrow V_{out} = V_{in} - 0.7 \Rightarrow \boxed{V_{in} = 1.4 + 0.7 = 2.1\text{V}}$

(b) For M_2:

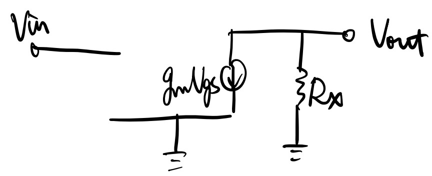


$$\begin{cases} V_{gs} = V_x \\ i_x = g_{m2} V_{gs} \end{cases}$$

$$\Rightarrow R_x = \frac{V_x}{i_x} = \frac{1}{g_{m2}}$$



For M_1:



$$\begin{cases} V_{gs} = V_{in} \\ g_{m1} V_{gs} + \dfrac{V_{out}}{R_x} = 0 \end{cases}$$

$$\Rightarrow \boxed{A_v = \frac{V_{out}}{V_{in}} = -g_{m1} R_x = -\frac{g_{m1}}{g_{m2}}}$$

For M_1 to be in saturation,

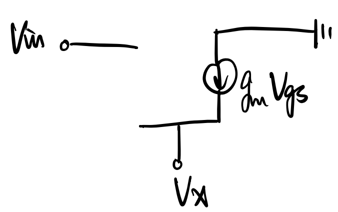
$$V_{DS1} \geq V_{GS1} - V_{TH1} \Rightarrow V_{out} \geq V_{in} - V_{TH1}$$

$$V_{in,max} = V_{out} + V_{TH1}$$

$$\because V_{out} = V_{DD} - |V_{GS2}|$$

$$\therefore \boxed{V_{in,max} = V_{DD} - |V_{GS2}| + V_{TH1}}$$

(c) For M_3 source follower part,



$$\begin{cases} V_{gs} = V_{in} - V_x \\ g_m V_{gs} = 0 \end{cases}$$

$$\Rightarrow A_{v1} = \frac{V_x}{V_{in}} = 1$$

$$\Rightarrow \boxed{A_v = \frac{V_{out}}{V_{in}} = \frac{V_{out}}{V_x} \cdot \frac{V_x}{V_{in}} = -\frac{g_{m1}}{g_{m2}}}$$

If $V_{in} = V_{DD}$, then $V_x = V_{DD} - V_{GS3}$.

For M_1 to be in saturation, $V_{DS1} \geq V_{GS1} - V_{TH1} \Rightarrow V_{out} \geq V_x - V_{TH1}$

$\Rightarrow V_{DD} - |V_{GS2}| \geq V_{DD} - V_{GS3} - V_{TH1}$

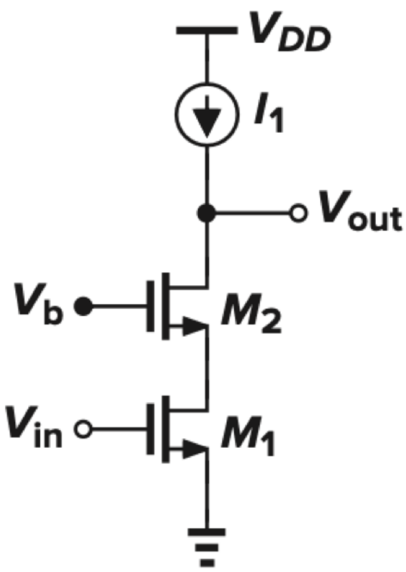
$\Rightarrow \boxed{V_{GS3} + V_{TH1} \geq |V_{GS2}|}$

Question 2. Cascode Amplifier

[30 points] In the following cascode amplifier, $\mu_n C_{ox}\left(\frac{W}{L}\right)_1 = \mu_n C_{ox}\left(\frac{W}{L}\right)_2 = 1\text{mA/V}^2$, $V_{TH} = 0.7\text{V}$, $\lambda = 0.1$, $I_1 = 1\text{mA}$, $V_b = 3\text{V}$. No body effect.

(a) Calculate V_{IN} and V_{OUT} such that M_1 operates at the edge of saturation.

(b) Draw small signal model and calculate the small signal gain $A_v = \frac{v_{out}}{v_{in}}$.



(a) For M_1: $I_{D1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in} - V_{TH})^2 (1 + \lambda V_x) = I_1$ ①

Since M_1 operates at the edge of saturation, $V_{DS1} = V_{GS1} - V_{TH}$

$\Rightarrow V_x = V_{in} - V_{TH}$ ②

①.②: $\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 V_x^2 (1 + \lambda V_x) = I_1$

$\frac{1}{2} \cdot 1\text{m} \cdot V_x^2 (1 + 0.1 V_x) = 1\text{mA}$

$\Rightarrow V_x = 1.33\text{ V}$ (rule out negative results for $V_x = V_{in} - V_{TH} > 0$)

$V_{in} = V_x + V_{TH} = 1.33 + 0.7 = 2.03\text{ V}$

Assume M_2 in saturation, $V_{out} \geq V_b - V_{TH} = 2.3\,V$

$$I_{D2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_b - V_x - V_{TH})^2(1+\lambda(V_{out}-V_x))$$

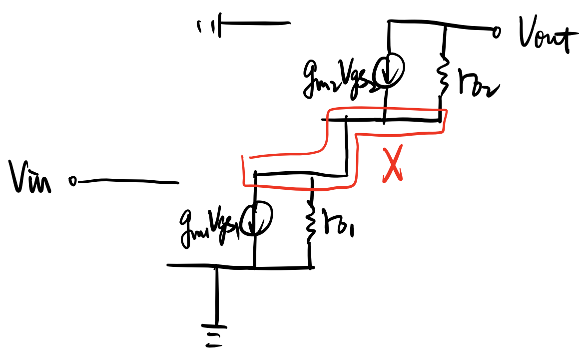
$$= \frac{1}{2}\cdot 1m\cdot (3-1.33-0.7)^2(1+0.1(V_{out}-1.33))$$

$$= 1\,mA$$

$\Rightarrow V_{out} = 12.59\,V$ (assumption satisfied)

$\therefore \boxed{V_{IN} = 2.03\,V,\ V_{OUT} = 12.59\,V}$

(b)



$$\begin{cases} V_{gs1} = V_{in} & ① \\ V_{gs2} = -V_x & ② \\ g_{m1}V_{gs1} + \frac{V_x}{r_{o1}} - g_{m2}V_{gs2} + \frac{V_x - V_{out}}{r_{o2}} = 0 & ③ \\ g_{m2}V_{gs2} + \frac{V_{out}-V_x}{r_{o2}} = 0 & ④ \end{cases}$$

$\Rightarrow g_{m1}V_{in} + \frac{V_x}{r_{o1}} = 0 \Rightarrow V_x = -g_{m1}r_{o1}V_{in}$

④: $-g_{m2}V_x + \frac{V_{out}-V_x}{r_{o2}} = 0$

$$V_{out} = V_x(1+g_{m2}r_{o2}) = -g_{m1}r_{o1}(1+g_{m2}r_{o2})V_{in}$$

$$\Rightarrow A_v = \frac{V_{out}}{V_{in}} = -g_{m1}r_{o1}(1+g_{m2}r_{o2})$$

$\because r_{o1} = r_{o2} = \frac{1}{\lambda I_D} = \frac{1}{0.1\times 1m} = 10\,k\Omega$

$g_{m1} = g_{m2} = \sqrt{2\mu_n C_{ox}\frac{W}{L}I_D} = 1.41\,m\Omega^{-1}$

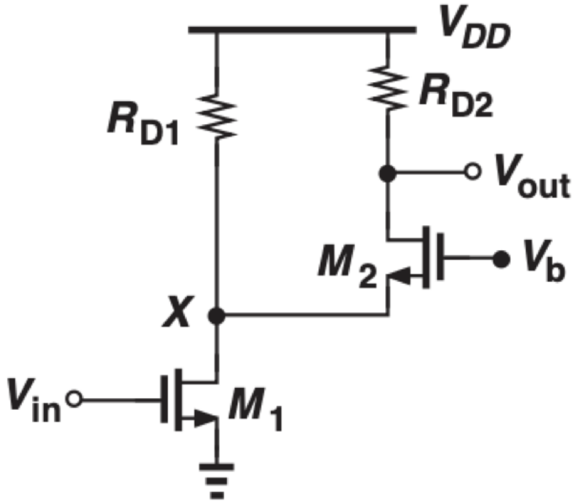
$\therefore \boxed{A_v = -g_{m1}r_{o1}(1+g_{m2}r_{o2}) = -213}$

Question 3. CS + CG

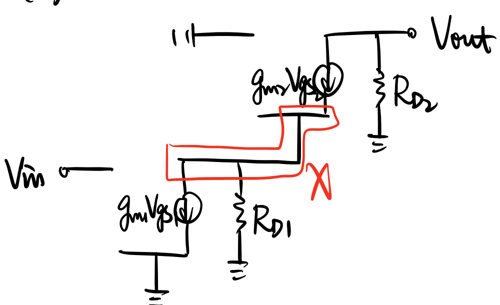
[30 points] In the following circuit, a common-source stage (M_1 and R_{D1}) is followed by a common-gate stage (M_2 and R_{D2}). Assume $\lambda = 0$.

(a) Derive the voltage gain $A_v = \frac{v_{out}}{v_{in}}$.

(b) Simplify the result obtained in (a) if $R_{D1} \to \infty$.



(a)



$$\begin{cases} V_{gs_1} = V_{in} & ① \\ V_{gs_2} = -V_X & ② \\ g_{m1}V_{gs_1} + \frac{V_X}{R_{D1}} - g_{m2}V_{gs_2} = 0 & ③ \\ g_{m2}V_{gs_2} + \frac{V_{out}}{R_{D2}} = 0 & ④ \end{cases}$$

$$\Rightarrow g_{m1}V_{in} + \frac{V_X}{R_{D1}} + g_{m2}V_X = 0 \Rightarrow V_X = -\frac{g_{m1}}{\frac{1}{R_{D1}} + g_{m2}} V_{in} = -g_{m1}\left(R_{D1} \,\|\, \frac{1}{g_{m2}}\right) V_{in}$$

④: $-g_{m2}V_X + \frac{V_{out}}{R_{D2}} = 0$

$$\Rightarrow V_{out} = g_{m2}R_{D2}V_X = -g_{m1}g_{m2}R_{D2}\left(R_{D1} \,\|\, \frac{1}{g_{m2}}\right) V_{in}$$

$\therefore \boxed{A_v = -g_{m1}g_{m2}R_{D2}\left(R_{D1} \,\|\, \frac{1}{g_{m2}}\right)}$

(b) $R_{D1} \to \infty$: $A_v = -g_{m1}g_{m2}R_{D2} \cdot \frac{1}{g_{m2}} = \boxed{-g_{m1}R_{D2}}$