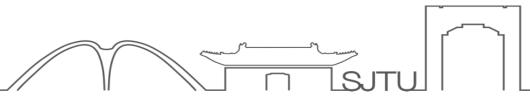


ECE3110J/VE311 Electronic Circuits

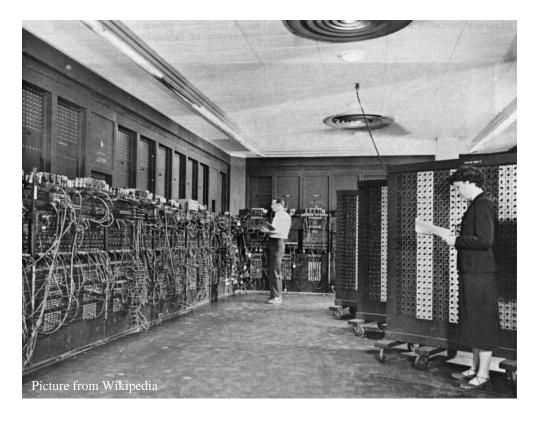
Bipolar Junction Transistor (BJT)

Microelectronic Circuit Design, Chapter 5 Fundamentals of Microelectronics, Chapter 4

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Before the invention of transistors



ENIAC is composed of

- **20,000** vacuum tubes
- 7,200 crystal diodes
- 1,500 relays
- 70,000 resistors
- 10,000 capacitors
- Approximately 5,000,000 hand-soldered joints

Consumed 150 kW of electricity, occupied 167 m², weighed over 27 tones

From Vacuum tube to Transistor

Instead of controlling an electron in vacuum, transistor controls it in solid materials



The original triode vacuum tube, the Audion, invented by Lee de Forest in 1906. (Image courtesy of Gregory F. Maxwell.)



A replica of the first transistor created in 1947.



In 1947, the world's first transistor was created, which was the beginning of the end for the vacuum tube. The transistor could replicate all the functions of tubes, like switching and amplification, but was made out of semiconductor materials.

BJT and Nobel Prize



Photo from the Nobel Foundation archive. William Bradford Shockley



John Bardeen



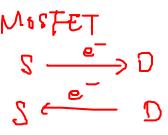
Photo from the Nobel
Foundation archive.

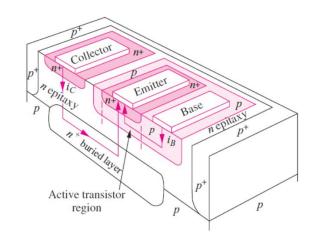
Walter Houser
Brattain

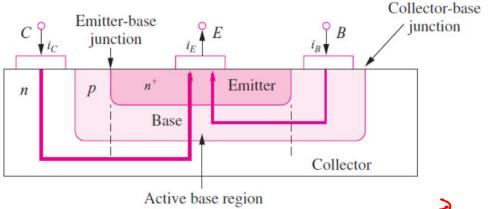
Amoy Xiamen

The transistors replaced the vacuum tubes and made a dramatic change in the world of electronics. Bardeen and Brattain together with William Shockley received the Nobel Prize in Physics in 1956 "for their researches on semiconductors and their discovery of the transistor effect.".

Device Schematics



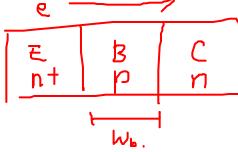




BJT has n+-p-n or p+-n-p structure.

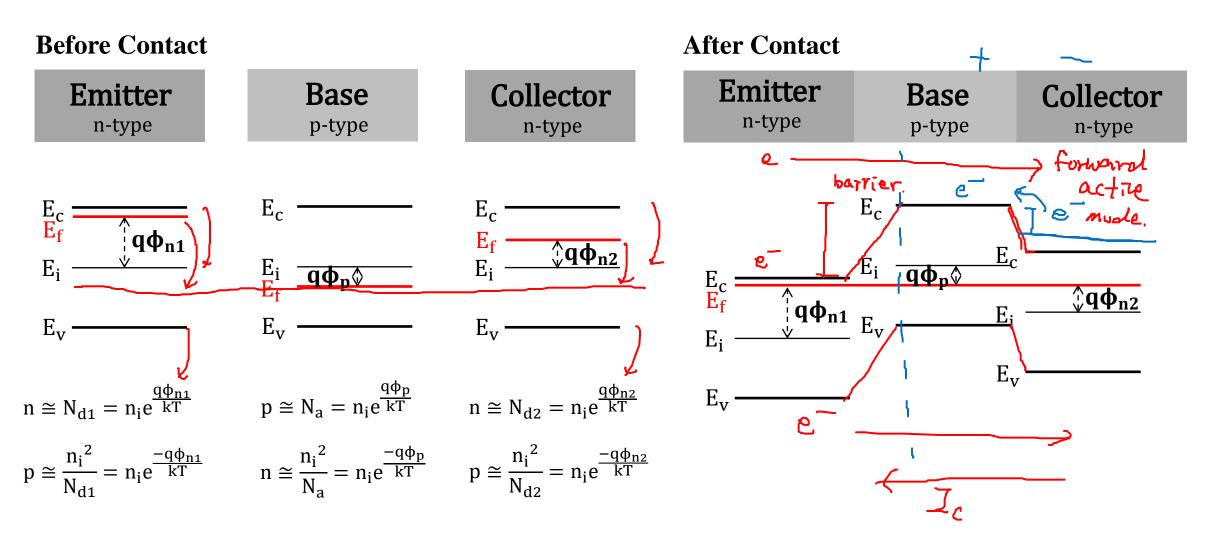
e.g. n⁺-p-n structure

- Emitter: heavily doped n-type
- Base: p-region
- Collector: lower n-region Prain
- Electron diffusion length (L_n) in base $\gg W_b$
- Emitter doping $N_{de} \gg$ base doping N_{ab} $(I_n \approx I)$



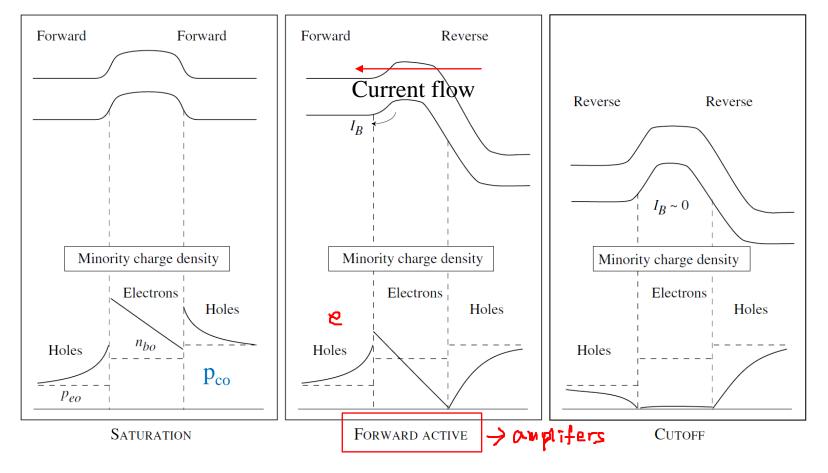


*Energy Band of BJT



How can we make the current flow in BJT from C to E, i.e. electrons flow from E to C

Modes in BJT



Saturation mode - both EBJ and BCJ are forward biased, for switching Cutoff mode -both EBJ and BCJ are reverse biased.

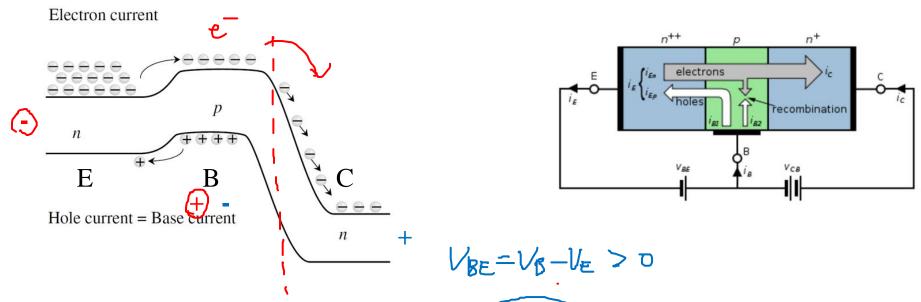
Forward active mode - EBJ is forward biased, BCJ is reverse biased, for amplifiers

(Forward) Active Mode

EBJ (emitter base junction) – forward bias — reduce barrier

BCJ (base collector junction) – reverse bias / Zero (built - in Putertail)

*Note: voltage notation



EB diode (n⁺-p) is forward biased $(V_{BE} > 0)$ or $V_{EB} < 0$)

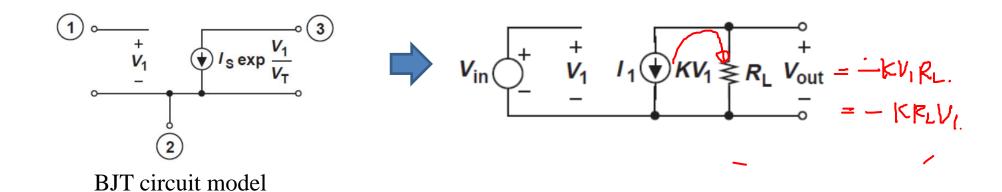
→ diffusion of electrons into the p-side

BC diode (p-n) is reversed biased $(V_{BC} < 0)$ or $V_{CB} > 0$)

 \rightarrow transfer of injected electrons into the n-side

BJT as Amplifier

In its simplest form, the bipolar transistor can be viewed as a voltage-dependent current source. We will see how a current source can form an amplifier and hence why bipolar devices are useful and interesting.

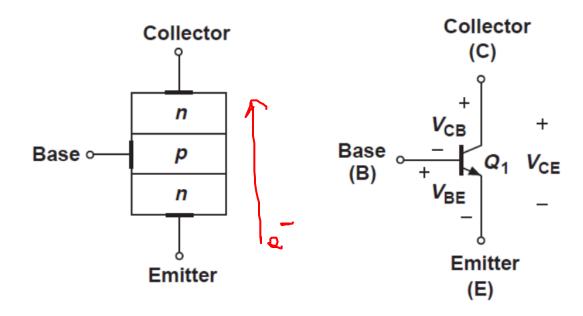


Since $V_1 = V_{in}$ and $V_{out} = -R_L I_1$, we have $V_{out} = \bigcirc KR_L V_{in}$. If $KR_L > 1$, then the circuit is an (inverting) amplifier.

The amplification factor, or voltage gain, of the circuit is $A_v = \frac{V_{out}}{V_{in}} = -KR_L$

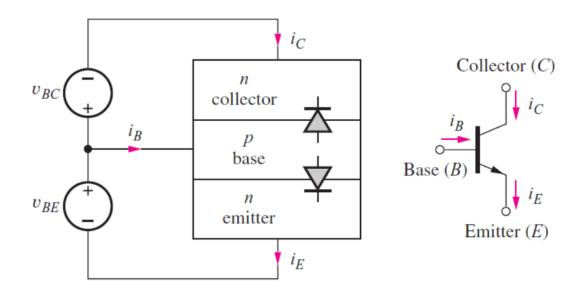
Structure of BJT

The BJT consists of three doped regions forming a sandwich. Below is an example comprising of a p layer sandwiched between two n regions and called an npn BJT.



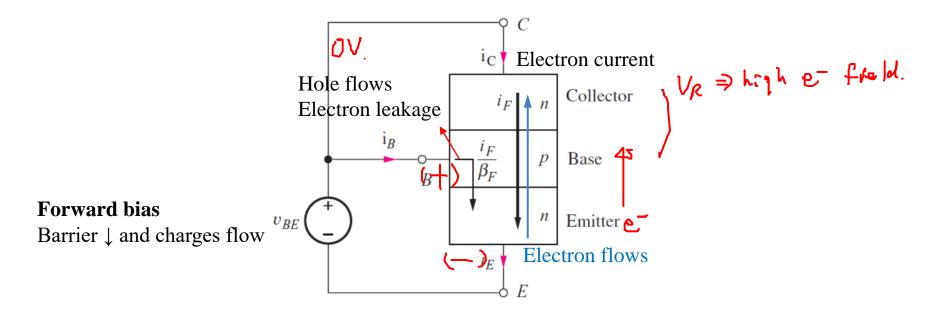
There are three terminal voltages V_E , V_B , and V_C , and consequently the voltage differences between terminals V_{BE} , V_{CB} , and V_{CE} . Among various biasing conditions, for amplification of BJT, there is only one biasing condition. $V_{BE} > 0$ $V_{BC} < 0$

Transport Model for NPN BJT



There are also three terminal currents, collector current i_C , emitter current i_E , and base current i_B . The base-emitter voltage v_{BE} and the base-collector voltage v_{BC} applied to the two pn junctions determine the magnitude of these three currents in the BJT and are defined as positive when they forward-bias their respective pn junctions.

Forward Characteristics

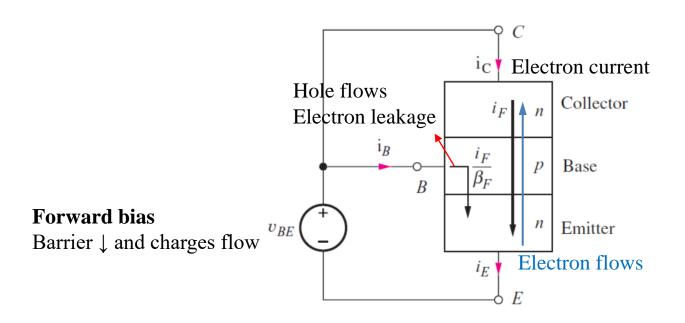


The base-emitter voltage establishes emitter current i_E , which equals the total current crossing the base-emitter junction.

$$i_E = i_C + i_B$$

$$i_C = i_F = I_S [e^{v_{BE}/v_T} - 1], I_S \text{ is the transistor saturation current } 10^{-18} \sim 10^{-9} \text{ [A]}.$$

$$i_B = {}^{i_F}/_{\beta_F} = {}^{I_S}/_{\beta_F} [e^{v_{BE}/v_T} - 1], \beta_F \text{ is Forward common-emitter current gain } 10 \sim 500.$$



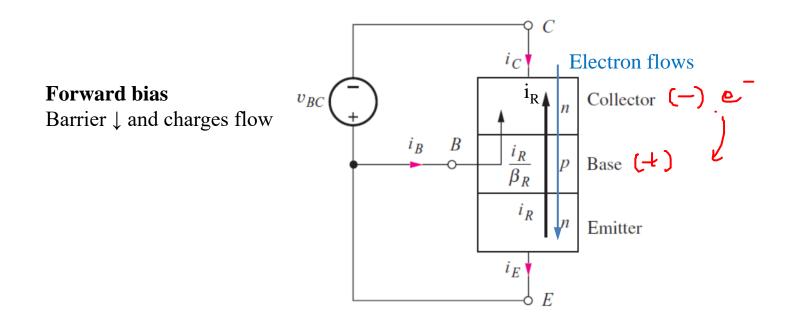
By KCL,
$$i_E = i_C + i_B$$

Therefore, $i_E = I_S \left[e^{v_{BE}/v_T} - 1 \right] + {I_S}/{\beta_F} \left[e^{v_{BE}/v_T} - 1 \right] = \left[I_S + {I_S \over \beta_F} \right] \left[e^{v_{BE}/v_T} - 1 \right]$

By defining $\alpha_F = \frac{\beta_F}{\beta_F + 1}$, $i_E = \frac{I_S}{\alpha_F} \left[e^{v_{BE}} / v_T - 1 \right]$, α_F is the forward common-base current gain in the range of $0.95 \sim 1.0$

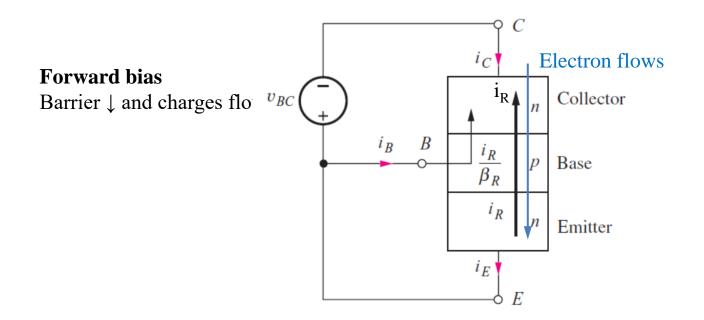
Finally,
$$\frac{i_C}{i_B} = \beta_F$$
, $\frac{i_C}{i_E} = \alpha_F$

Reverse Characteristics

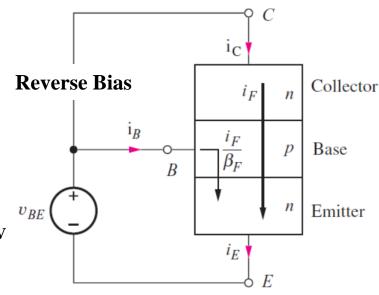


The base-collector voltage establishes the collector current i_C , now crossing the base-collector junction. The largest portion of the collector current, the reverse-transport current i_R , enters the emitter, travels completely across the narrow base region, and exits the collector terminal. Current i_R has a form identical to i_F :

$$i_{R} = -i_{E} = I_{S}[e^{v_{BC}/V_{T}} - 1]$$

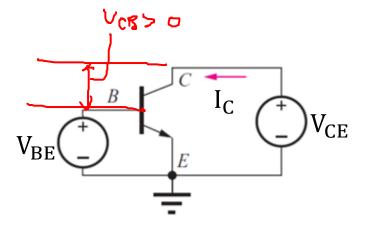


A fraction of the current i_R must also be supplied as base current through the base terminal: $i_B = {}^{i_R}/_{\beta_R} = {}^{I_S}/_{\beta_R} \left[e^{v_{BC}}/_{V_T} - 1 \right], \ \beta_R \text{ is the reverse common-emitter current gain.}$ $i_C = \left(-\frac{I_S}{\alpha_R} \right) \left[e^{v_{BC}}/_{V_T} - 1 \right], \ \alpha_R \text{ is the reverse common-base current gain in the range of } 0 \sim 0.95.$ The bias condition for the Forward active mode of BJT (a normal BJT operation mode) is $v_{BE} > 0$ and $v_{BC} < 0$.



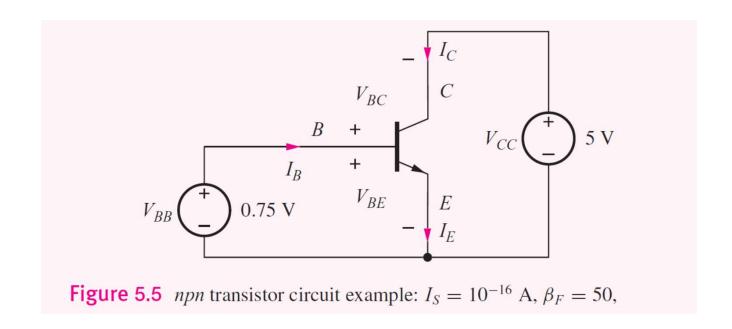
Forward bias

Barrier ↓ and charges flow



$$\mathbf{i}_{C} = \mathbf{i}_{F} = I_{S} \left[e^{v_{BE}/v_{T}} - \mathbf{1} \right]
i_{B} = {}^{i_{F}}/_{\beta_{F}} = {}^{I_{S}}/_{\beta_{F}} \left[e^{v_{BE}/v_{T}} - 1 \right]
i_{E} = I_{S} \left[e^{v_{BE}/v_{T}} - 1 \right] + {}^{I_{S}}/_{\beta_{F}} \left[e^{v_{BE}/v_{T}} - 1 \right] = \left(I_{S} + \frac{I_{S}}{\beta_{F}} \right) \left[e^{v_{BE}/v_{T}} - 1 \right]$$

Example 1. Find the terminal voltages and currents



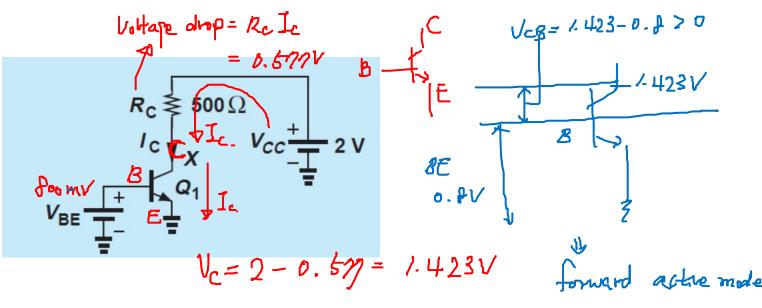
$$V_{BE} = 0.75 \text{ V}$$
 and $V_{BC} = -4.25 \text{ V}$ thus Forward active

$$i_{C} = i_{F} = I_{S} \left[e^{v_{BE}/V_{T}} - 1 \right] = 0.00107 \text{ A}$$
 $i_{B} = {}^{i_{F}}/\beta_{F} = {}^{I_{S}}/\beta_{F} \left[e^{v_{BE}/V_{T}} - 1 \right] = 2.14 \times 10^{-5} \text{ A}$
 $i_{E} = i_{C} + i_{B} = 0.00109 \text{ A}$

Example 2. Consider the circuit shown below where $I_S = 5 \times 10^{-17}$ A and $V_{BE} = 800$ mV. Assume $\beta = 100$. (a) Determine the transistor terminal currents and voltages and verify that the device indeed operates in the active mode. (b) Determine the maximum value of R_C that permits

operation in the active mode.

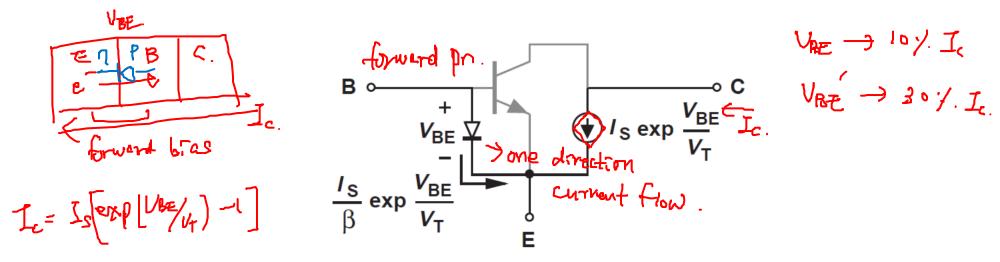
$$V_{g} = 0.8V$$
 $V_{E} = 0.V$
 $V_{C} = ?$
 $I_{C} = I_{S} \left(\frac{e^{2}}{e^{2}} \right) - 1$
 $= 6 \times 10^{-11} \left[\frac{e^{2}}{6.686} \right] - 1$
 $= 0.00 / 153 A$



Large-Signal Model of analysis -> brasing

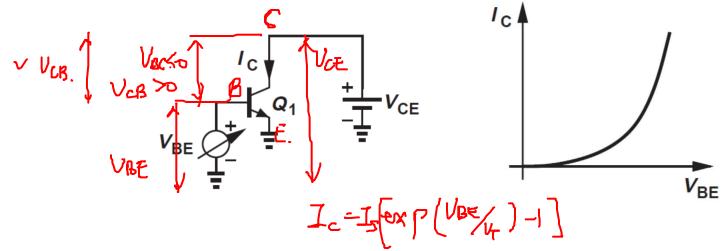
Large signal model can be used for **arbitrarily large voltage and current changes** in the BJT as long as the device operates **in the active mode**.

Since the B-E junction is forward-biased in the active mode, we can place a diode between B-E. Moreover, since I_C flowing into the emitter depends on only V_{BE} , we add a voltage-controlled current source between the collector and the emitter.



Large-signal model of BJT

IV characteristics – Transfer characteristic

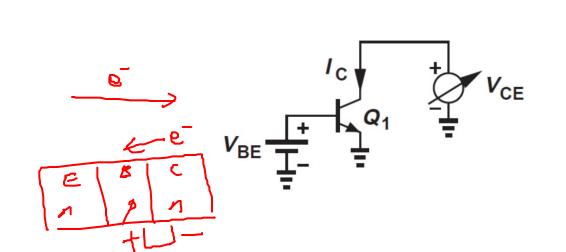


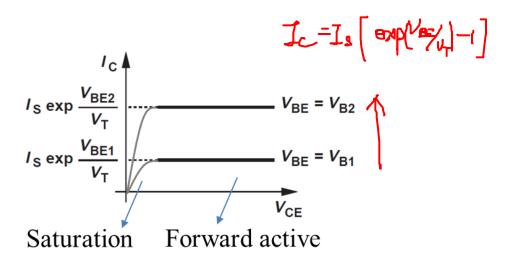
Transfer characteristic plots I_C vs V_{BE} with the assumption that the V_{CE} is constant and no lower than the V_{BE} . If $V_{CE} > V_{BE}$, then $v_{CB} > 0$ or $v_{BC} < 0$, meaning that v_{CB} is reverse biased. Thus, the BJT is in the forward active mode.

 I_C is independent of V_{CE} ; thus, different values of V_{CE} do not alter the characteristic.

$$i_{C} = I_{S}[e^{v_{BE}/V_{T}} - 1]$$

IV characteristics – Output characteristic





Output characteristic shows I_C for a given V_{BE} but with V_{CE} varying.

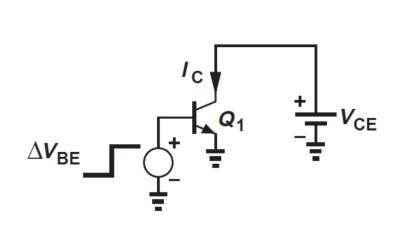
 $V_{CE} < V_{BE}$, then $v_{CB} < 0$ or $v_{BC} > 0$, meaning that v_{CB} is forward biased. Both diodes (B-E and C-B) are on, which is called as saturation.

 $V_{CE} > V_{BE}$, then $v_{CB} > 0$ or $v_{BC} < 0$, meaning that v_{CB} is **reverse biased** \rightarrow Forward active. If V_{BE} increases I_C increases because $i_C = I_S[e^{v_{BE}/V_T} - 1]$

Transconductance

The BJT acts as a voltage-dependent current source when operating in the forward active **region**. What is the measure of the **goodness** of a voltage-dependent current source?

The transistor becomes a better amplifying device by producing larger changes in I_C in response to a given signal level (V_{BE}) applied (I_C/V_{BE}), i.e. a better voltage-dependent current source or voltage-to-current converter.

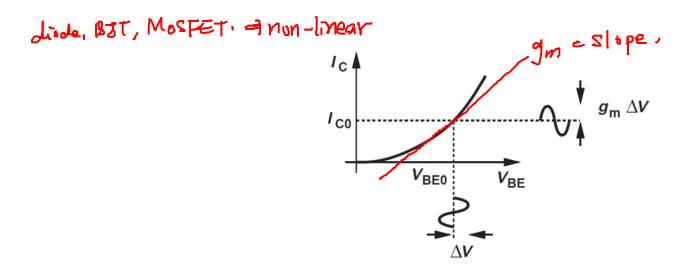


$$dI_{c} = \int m dV_{c}$$

$$g_{m} = \frac{dI_{C}}{dV_{BE}} = \frac{d}{dV_{BE}} \left(I_{S}exp\frac{V_{BE}}{V_{T}}\right) = \frac{1}{V_{T}}I_{S}exp\frac{V_{BE}}{V_{T}}$$

$$= \frac{I_{C}}{V_{T}}$$

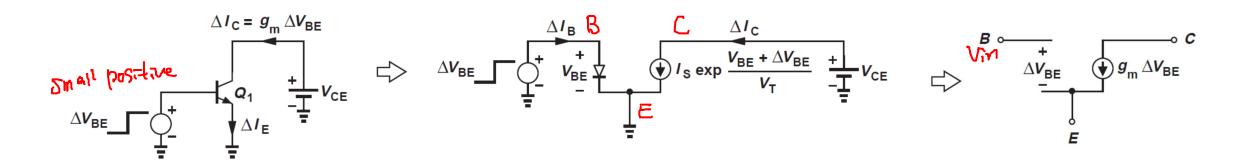
 $g_m = dI_C/dV_{BE}$ simply represents the slope of I_C-V_{BE} characteristic at a given I_{C0} , and the corresponding V_{BE0} . In other words, if V_{BE} experiences a small perturbation $\pm \Delta V$ around V_{BE0} , then the I_C displays a change of $\pm g_m \Delta V$ around I_{C0} .



The transconductance is **fundamentally a function of the I**_C rather than the I_B. For example, if I_C remains constant but β varies, then g_m does not change but I_B does.

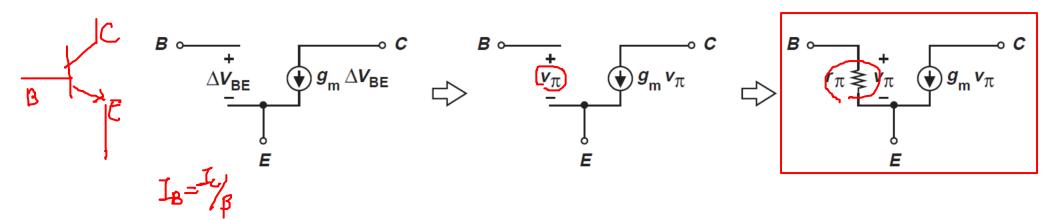
Small-Signal Model ? Small changes (ac)

To find a small-signal model of the BJT, we **perturb the voltage difference** between every two terminals while the third terminal remains at a constant potential. And we **determine the changes in the currents** flowing through all terminals, and **represent the results by proper circuit elements.**



(1) Begin with a change in V_{BE} while the V_{CE} is constant. By the definition of the transconductance g_m , $\Delta I_C = g_m \Delta V_{BE} \rightarrow a$ voltage-controlled current source

As
$$I_C = I_S[e^{V_{BE}/V_T} - 1]$$
, $\Delta I_C = I_S[e^{V_{BE} + \Delta v_{BE}/V_T} - 1]$



For simplicity, we denote V_{BE} by v_{π} and the change in the I_C by $g_m v_{\pi}$. The change in V_{BE} creates another change in I_B as well.

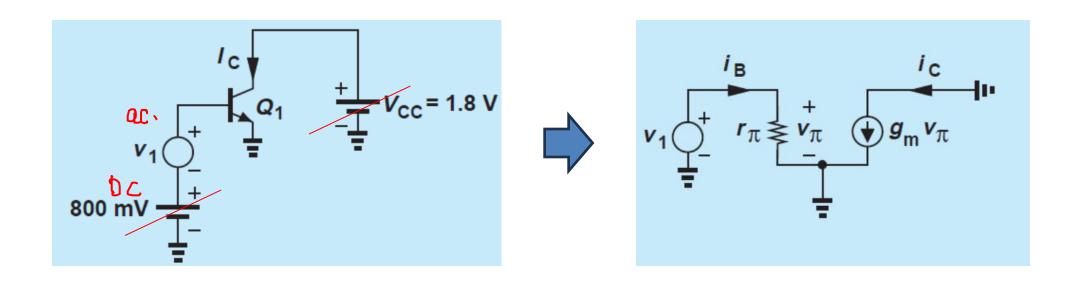
$$\Delta I_{B} = \frac{\Delta I_{C}}{\beta} = \frac{g_{m}}{\beta} \Delta V_{BE} \rightarrow r_{\pi} = \frac{\Delta V_{BE}}{\Delta I_{B}} = \frac{\beta}{g_{m}}$$

This represents a small signal model of the B-E diode as a resistance.

On the contrary, V_{CE} and V_{BC} do not result in any change of the small signal result as I_{C} and I_{B} are the function of V_{BE} . The simple small-signal model serves as a powerful tool in the analysis and design of BJT circuits.

Note that small-signal analysis deals with **only small changes** in voltages and currents in a circuit around their quiescent values. Thus, **all constant sources**, **i.e.**, **voltage and current sources that do not vary with time**, **must be set to zero for small-signal analysis**.

e.g., the supply voltage is constant and, while establishing proper bias points, plays no role in the response to small signals. We therefore ground all constant voltage sources and open all constant current sources while constructing the small-signal equivalent circuit.



Example 3. $I_S = 3 \times 10^{-16}$ A, $\beta = 100$, $V_T = 0.026$ V (a) When $v_1 = 0$, verify that the transistor operates in the active mode. (b) Determine the voltage gain of the circuit if the v_1 changes by 1 mV (c) suppose we raise R_C to 200 and V_{CC} to 3.6 V. Verify that the device operates in the active mode

and compute the voltage gain.

$$R_{\text{C}} = 100 \Omega$$

$$V_{1} = 0$$

$$V_{1} = 0$$

$$V_{1} = 0$$

$$V_{2} = 1.8 \text{ V}$$

$$V_{3} = 0$$

$$V_{4} = 0$$

$$V_{5} = 0$$

$$V_{5} = 0$$

$$V_{7} = 0$$

(a)
$$V_{g} = 0.8V$$
 $I_{c} = I_{s} \left[\exp \left(\frac{V_{be}}{V_{T}} \right) - 1 \right]$
 $V_{E} = 0V$ $= 3 \times 10^{-16} \left[\exp \left(\frac{0.8}{V_{T}} \right) - 1 \right]$
 $= (6.92 \text{ mA})$
 $V_{c} = I_{c} R_{c} = 100 \times 6.92 \times 10^{-3} = 0.692 V$
 $V_{c} = V_{cE} = V_{ow} = 1.8 - 0.692 = 1.108 V$
 $V_{cE} = V_{cE} = 0.002 = 0.002 = 0.002 = 0.002 V$

$$|b| V_{RE} = 0.80/V$$

$$I_{C} = 0.00719...$$

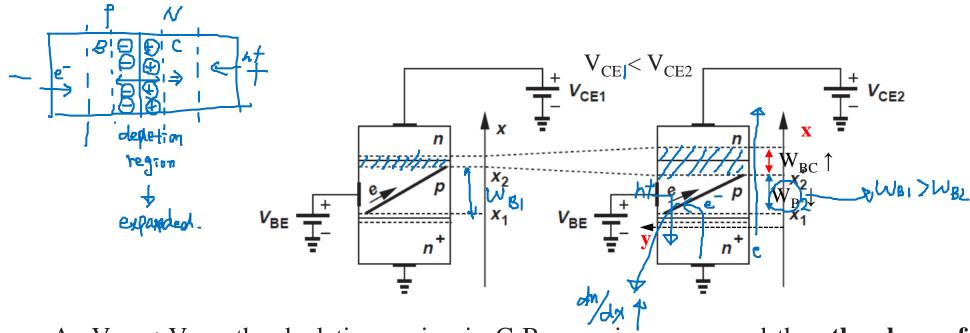
$$V_{C} = V_{CE} = V_{OUT} = 1.P - 0.71P = 1.0PV$$

$$V_{R_{C}} = 100 \times I_{C} = 0.719V$$

$$A_{V} = \frac{0.001 - 1.100}{0.80 - 0.8} = 27$$

Early Effect

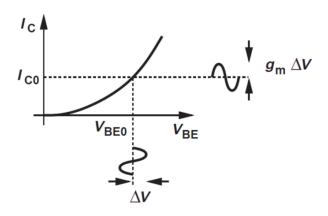
Example 3 points to an important trend if $\mathbf{R}_{\mathbf{C}} \to \infty$ would the gain also grows indefinitely? The **Early effect** translates to a nonideality in the device which **limits the gain of amplifiers**.

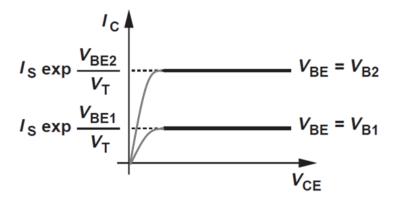


As $V_{CE} < V_{CE2}$, the depletion region in C-B areas increases, and thus the slope of the profile increases. Equivalently, the effective W_B decreases, thereby increasing I_C . Early effect indicates that the V_{CE} does affect the I_C .

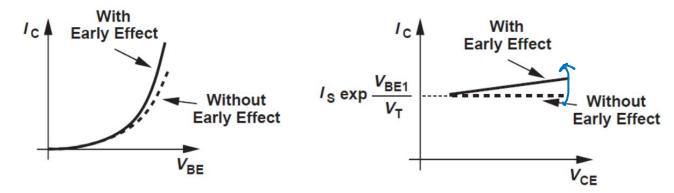
With the Early effect,
$$I_C \approx \left(I_S exp \frac{V_{BE}}{V_T}\right) \left(1 + \frac{V_{CE}}{V_A}\right) V_A = \text{Early voltage}$$

Without the Early effect, $i_C = I_S[e^{v_{BE}/V_T} - 1]$





With the Early effect, $I_C \approx \left(I_S exp \frac{V_{BE}}{V_T}\right) \left(1 + \frac{V_{CE}}{V_A}\right)$

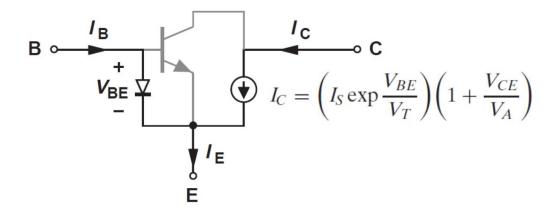


With the Early effect,
$$I_C \approx \left(I_S exp \frac{V_{BE}}{V_T}\right) \left(1 + \frac{V_{CE}}{V_A}\right)$$

$$\frac{\delta I_C}{\delta V_{CE}} = I_S \left(exp \frac{V_{BE}}{V_T}\right) \left(\frac{1}{V_A}\right) \approx \frac{I_C}{V_A}$$
If $\mathbf{V_A} \gg \mathbf{V_{CE}}$, we can see that $I_C \approx \left(I_S exp \frac{V_{BE}}{V_T}\right) \left(1 + \frac{V_{CE}}{V_A}\right)$ and thus $I_C \approx \left(I_S exp \frac{V_{BE}}{V_T}\right)$

The variation of I_C with V_{CE} reveals that the BJT in fact does not operate as an ideal current source, requiring modification of the perspective.

Large-Signal and Small-Signal Models with Early Effect



The presence of Early effect alters the BJT's large signal model as follows:

$$I_C = \left(I_S \exp \frac{V_{BE}}{V_T}\right) \left(1 + \frac{V_{CE}}{V_A}\right)$$

$$I_B = \frac{1}{\beta} \left(I_S \exp \frac{V_{BE}}{V_T} \right)$$

$$I_E = I_C + I_B$$
.

For the small-signal model, g_m and r_{π} are expressed as below. Thus, the controlled current source remains unchanged.

$$g_{m} = \frac{dI_{C}}{dV_{BE}} = \frac{d}{dV_{BE}} \left(I_{S} exp \frac{V_{BE}}{V_{T}} \right) \left(1 + \frac{V_{CE}}{V_{A}} \right) = \frac{1}{V_{T}} I_{S} exp \frac{V_{BE}}{V_{T}} \left(1 + \frac{V_{CE}}{V_{A}} \right)$$

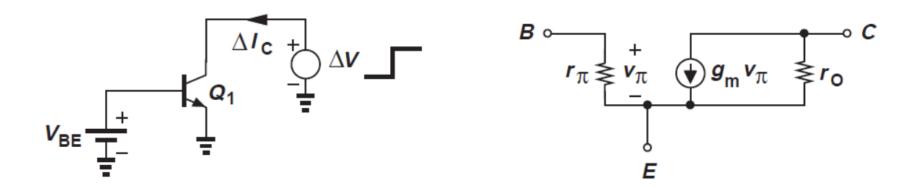
$$= \frac{I_{C}}{V_{T}}$$

$$r_{\pi} = \frac{\beta}{g_{m}} = \beta \frac{V_{T}}{I_{C}}$$

Considering that the I_C does vary with V_{CE} , we apply a voltage change at the collector and measure the resulting current change

$$I_{C} + \Delta I_{C} = \left(I_{S}exp\frac{V_{BE}}{V_{T}}\right)\left(1 + \frac{V_{CE} + \Delta V_{CE}}{V_{A}}\right)$$

$$Thus, \quad \Delta I_{C} = \left(I_{S}exp\frac{V_{BE}}{V_{T}}\right)\frac{\Delta V_{CE}}{V_{A}}$$

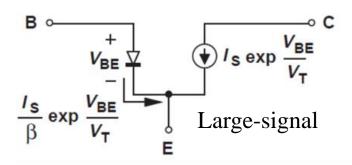


Since the voltage and current change correspond to the same two terminals, they satisfy Ohm's law, yielding an equivalent resistor r_0

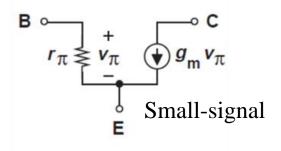
$$\Delta I_{C} = \left(I_{S}exp\frac{V_{BE}}{V_{T}}\right)\frac{\Delta V_{CE}}{V_{A}} \rightarrow r_{O} = \frac{\Delta V_{CE}}{\Delta I_{C}} = \frac{V_{A}}{I_{S}exp\frac{V_{BE}}{V_{T}}} \approx \frac{V_{A}}{I_{C}}$$

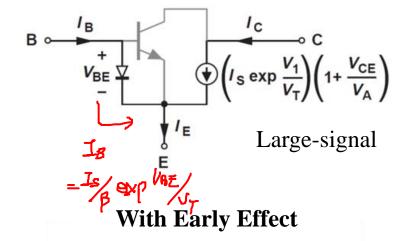
The small-signal model contains only one extra element r_0 to represent the Early effect. This output resistance r_0 plays a critical role in high gain amplifiers

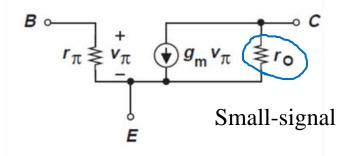
Summary



Without Early Effect

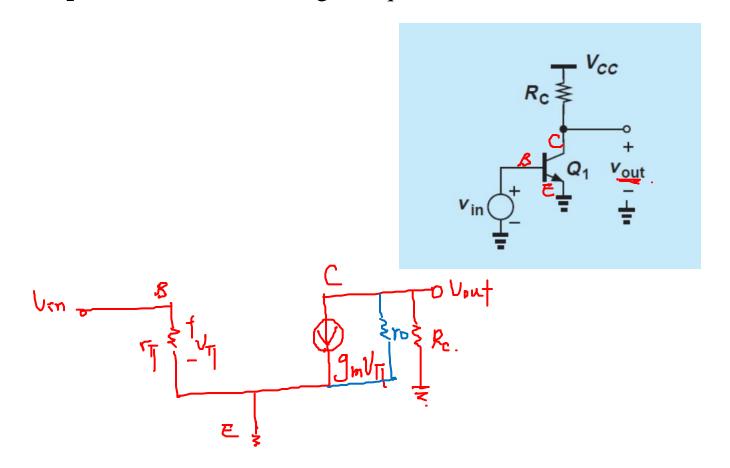




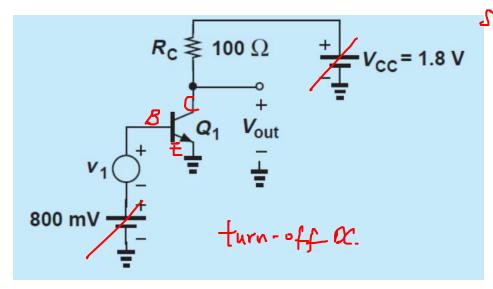


We must create **proper DC voltages and currents at the device terminals to accomplish two goals**: (1) guarantee **operation in the active mode** (e.g. for npn BJT $V_{BE} > 0$, $V_{CE} \ge 0$); (2) establish the I_C that yields the required values for the **small signal parameters** \mathbf{g}_{m} , \mathbf{r}_{O} , and \mathbf{r}_{π} .

Example 4 Draw the small-signal equivalent circuit. Please include the Early effect.



Example 3. $I_S = 3 \times 10^{-16}$ A, $\beta = 100$, $V_T = 0.026$ V (a) When $v_1 = 0$, verify that the transistor operates in the active mode. (b) Determine the voltage gain of the circuit if the v_1 changes by 1 mV (c) suppose we raise R_C to 200 and V_{CC} to 3.6 V. Verify that the device operates in the active mode and compute the voltage gain.



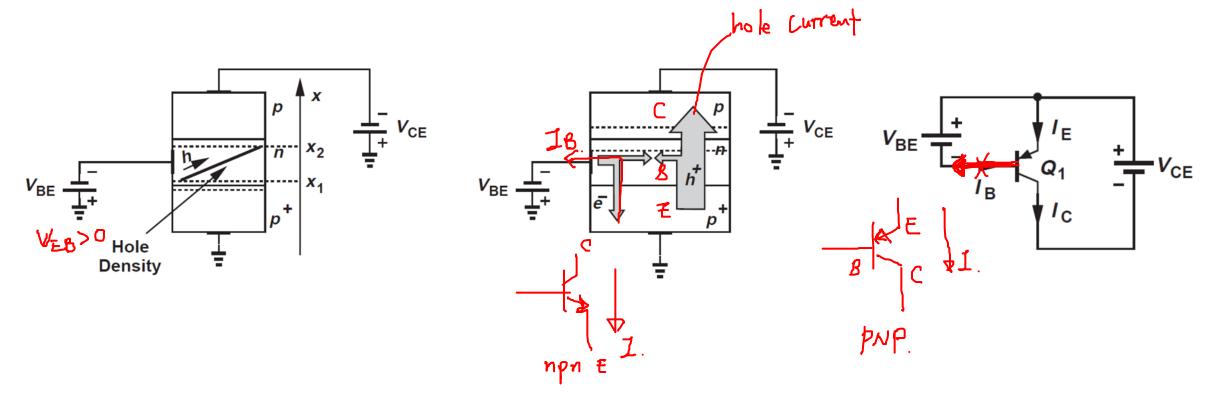
(c)
$$A_V = -9mRc$$

 $= -0.266 - x200 = |-53.2|$

rnall-Signal mode

$$V_{in} = -\frac{1}{2} \sum_{k=1}^{N} V_{in}$$
 $V_{in} = -\frac{1}{2} \sum_{k=1}^{N} V_{in}$
 $V_{in} = -\frac{1}{2} \sum_{k=1}^{N} V_{in}$

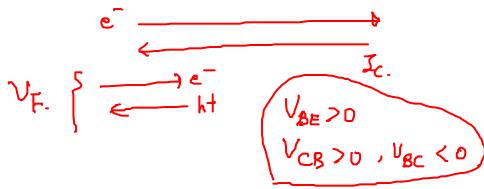
PNP BJT - hale current.



Operation of PNP in the active region requires forward biasing the base-emitter junction and reverse-biasing the collector junction. Thus, $V_{BE} < 0$ (Forward) and $V_{BC} > 0$ (Reverse). All of the operation principles and equations described for npn transistors apply to pnp devices as well.







$$i_{C} = i_{F} = I_{S} \left[e^{v_{BE}/V_{T}} - 1 \right]$$
 $i_{B} = {}^{i_{F}}/\beta_{F} = {}^{I_{S}}/\beta_{F} \left[e^{v_{BE}/V_{T}} - 1 \right]$
 $i_{E} = \left(I_{S} + \frac{I_{S}}{\beta_{F}} \right) \left[e^{v_{BE}/V_{T}} - 1 \right]$
 $= I_{C} + I_{A}$

$$i_{C} = i_{F} = I_{S} \left[e^{v_{EB}/v_{T}} - 1 \right]$$

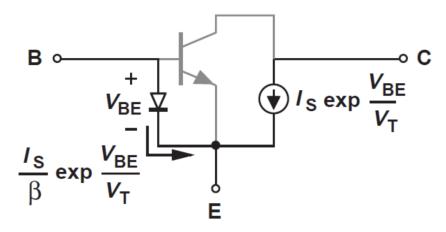
$$i_{B} = {^{i_{F}}/\beta_{F}} = {^{I_{S}}/\beta_{F}} \left[e^{v_{EB}/v_{T}} - 1 \right]$$

$$i_{E} = \left(I_{S} + \frac{I_{S}}{\beta_{F}} \right) \left[e^{v_{EB}/v_{T}} - 1 \right]$$

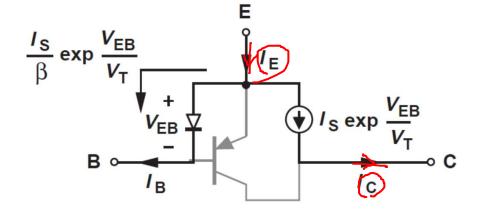
$$= \boxed{1}$$

PNP BJT Large Signal model DC analy 57.5.





Large-signal model of NPN BJT



Large-signal model of PNP BJT

The difference between the npn and pnp equations relates to the base-emitter voltage that appears in the exponent, and the direction of current flow.

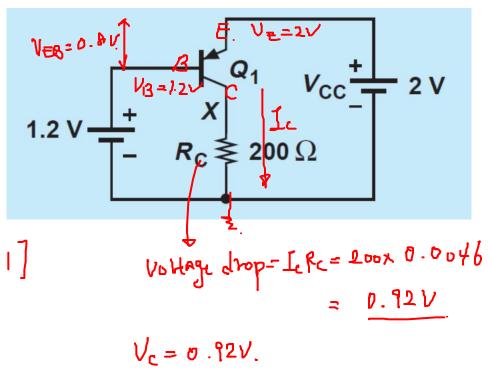
Also, the Early effect can be included as $I_C = \left(I_S \exp \frac{V_{EB}}{V_T}\right) \left(1 + \frac{V_{EC}}{V_A}\right)$

Example 5. Determine the terminal currents of Q_1 and verify operation in the forward active region. Assume $I_S = 2 \times 10^{-16}$ A and $\beta = 50$ where $V_A = \infty$ and $V_T = 0.026$ V.

$$V_{g} = 1.2V$$
 $V_{\overline{e}} = 2V$
 $V_{\overline{e}B} = 0.2V$

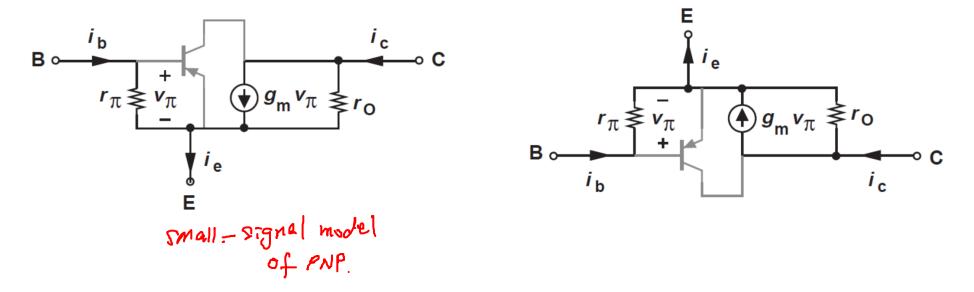
$$I_{c}=I_{s}[exp(\frac{1}{2})-1]$$

$$= 2x10^{-16}[exp(\frac{1}{2})-1]$$

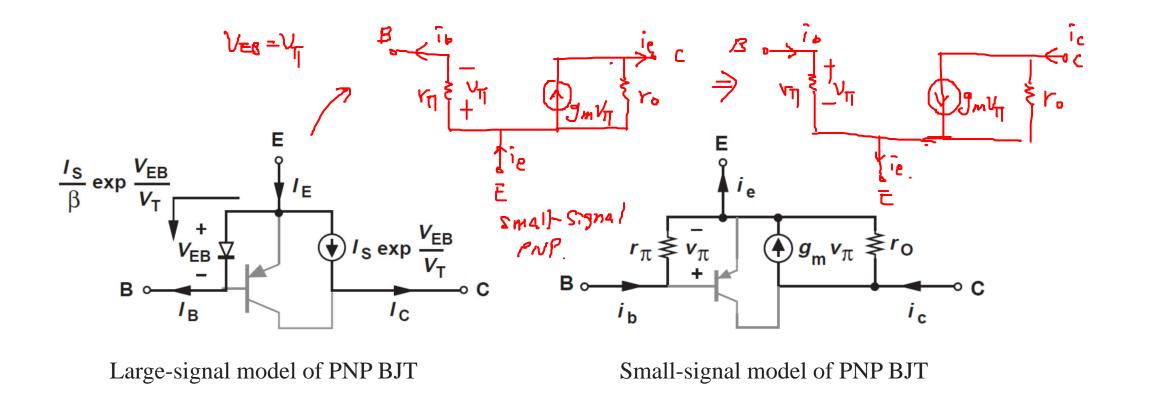


$$V_{c} = 0.92V$$
.
 $V_{Sc} = 1.2 - 0.92 > 0$. $V_{cs} < 0$.
 $V_{Es} > 0$

PNP BJT Small Signal model



Since the small-signal model represents changes in the voltages and currents, we expect npn and pnp transistors to have similar models. The small signal model of the pnp transistor is indeed identical to that of the npn device.



The direction of currents in the **large-signal model** shows how currents could flow when the pnp transistor is **properly DC biased**.

The direction of currents in the **small-signal model** shows how the ac currents flow when ac voltage **across base-emitter increases**.

Example 6 Draw the small-signal equivalent circuits for the topologies shown below.

