Physics (PHYS2500J), Unit 4 Circuits:
1. DC and transient circuit

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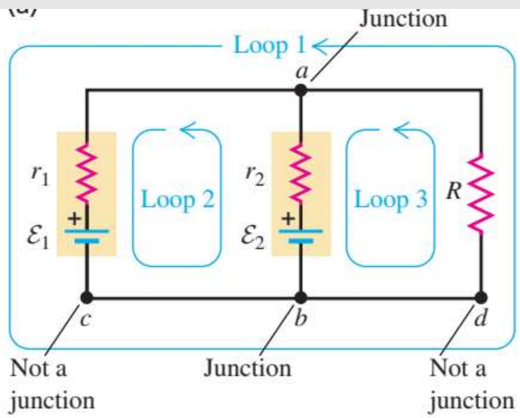
Contents

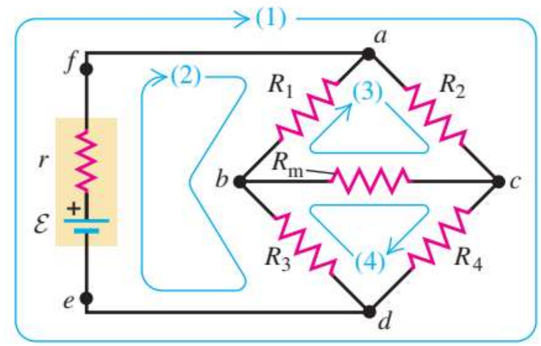
Kirchhoff's rules

1. Not all circuits can be simplified into serial-parallel combinations of resistors.

e.g. 1, multiple voltage sources.



e.g. 2, complicated networks



But they can always be abstracted into junctions and loops

Kirchhoff's rules for junctions and loops are based on electromagnetic fields but different.

基尔霍夫定律（电流定律和电压定律）

Kirchhoff's junction rule (valid at any junction): The sum of the currents into any junction ... $\sum I = 0$ ... equals zero. (26.5)

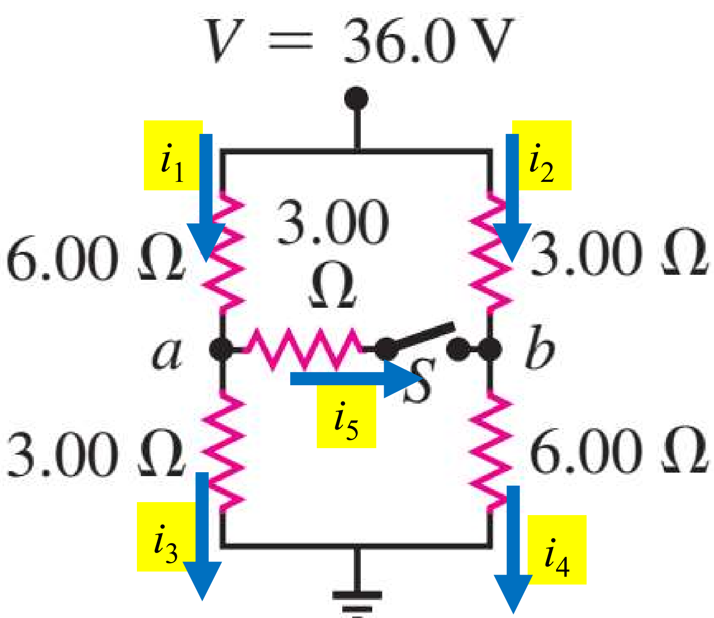
Continuity of current, junction can not be defined including one plate of capacitor.

Kirchhoff's loop rule (valid for any closed loop): The sum of the potential differences around any loop ... $\sum V = 0$ ... equals zero. (26.6)

Induced emf has to be included into either the inductive voltage drop or the emf part (generators)

Example problems

1. For the following circuit diagram, (a) What is the potential difference V_{ab}, the potential of point *a* relative to point *b*, when the switch S is open? (b) What is the current through S when it is closed? (c) What is the equivalent resistance when S is closed?



Hint: The power supply in the following circuit diagram is not shown explicitly. It is understood that the point at the top, labeled "36.0 V," is connected to the positive terminal of a 36.0-V battery having negligible internal resistance, and that the *ground* symbol at the bottom is connected to the negative terminal of the battery. The circuit is completed through the battery, even though it is not shown.

Kirchhoff's junction rule

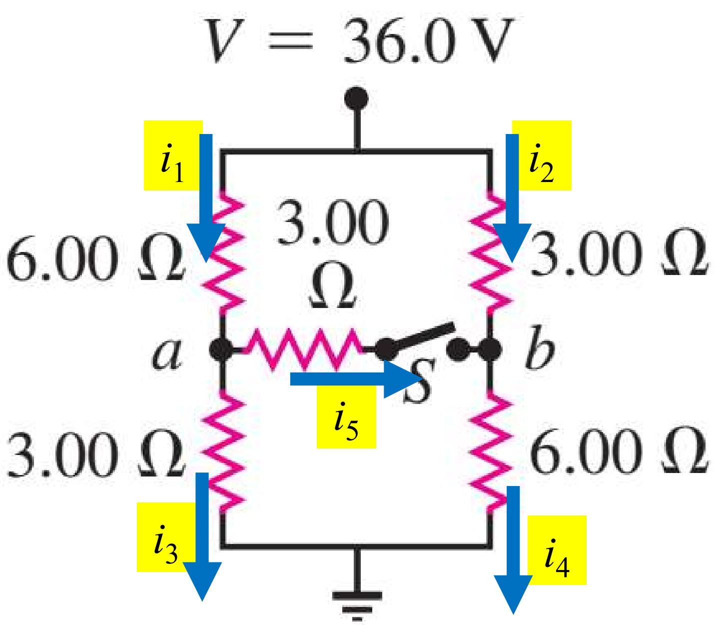
$$i_1 = i_3 + i_5$$ Junction *a*

$$i_4 = i_2 + i_5$$ Junction *b*

Kirchhoff's loop rule

$$6\ \Omega i_1 + 3\ \Omega i_5 - 3\ \Omega i_2 = 0$$
$$3\ \Omega i_3 - 6\ \Omega i_4 - 3\ \Omega i_5 = 0$$
$$6\ \Omega i_1 + 3\ \Omega i_3 = 36\ \text{V}$$

1. For the following circuit diagram, (a) What is the potential difference V_{ab}, the potential of point *a* relative to point *b*, when the switch S is open? (b) What is the current through S when it is closed? (c) What is the equivalent resistance when S is closed?



$$V_{ab} = 12\ \text{V}$$

$$i_5 = -12/7\ \text{A}$$

$$i_1 = i_4 = 24/7\ \text{A}$$

$$i_2 = i_3 = 36/7\ \text{A}$$

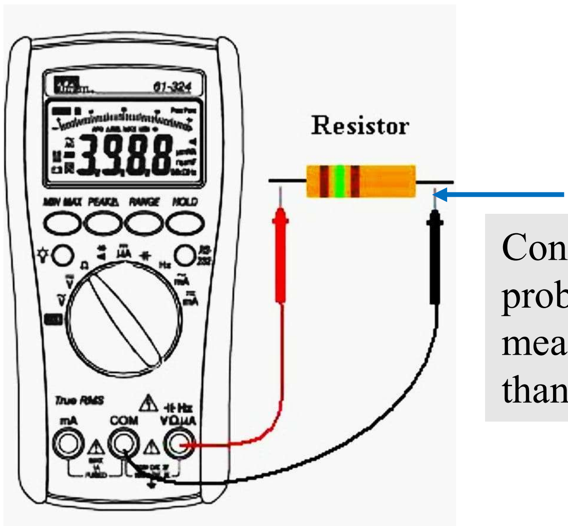
$$R = \frac{V}{i_1 + i_2}$$

$$R = 4.2\ \Omega$$

Contents

How to measure a small resistance?

1. For resistance of 10 Ohm to 1 Mohm, use a multimeter to measure the resistance

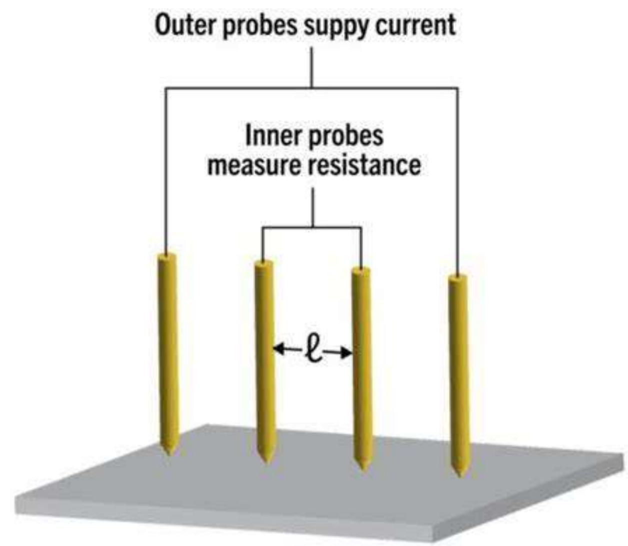


Contact resistance is a problem for resistance measurement of less than 1 Ohm

2. The multimeter supplies current *I* and measures voltage drop *V*, *R=V/I.*

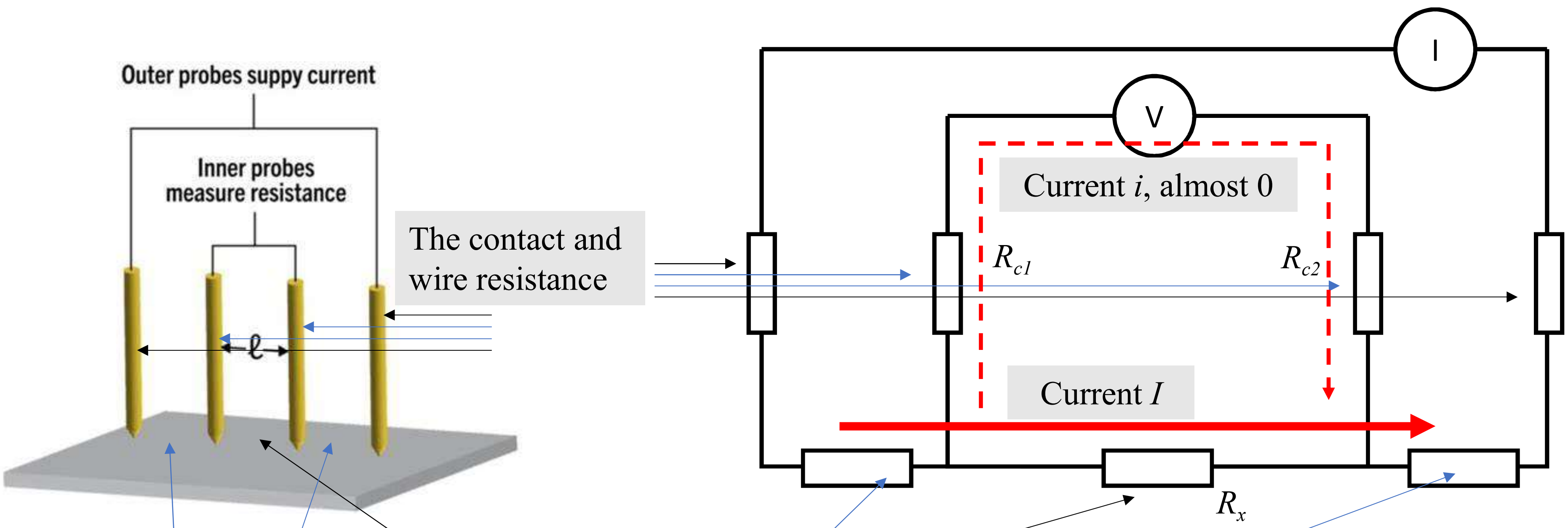
3. What is really measured is the sum of resistances of the resistor, wire and contacts.

4. For small resistance, a "4-point" method is used, where current *I* is fed and measured through the outer two wires and voltage *V* is measured through the inner two wires, *R=V/I*.



5. The contact resistance and wire resistance are still there!
But: a. the contact resistance related to the current probes are not picked up by the voltage probes; b. the contact resistance exist for the voltage probes but no current, thus no voltage drop is generated regarding to these resistance.

Circuit diagram



The part under measurement

The part not under measurement

Kirchhoff's loop rule

$$iR_{c1} + iR_{c2} - IR_x + V = 0$$

Neglect i

$$R_x = V/I$$

Contents

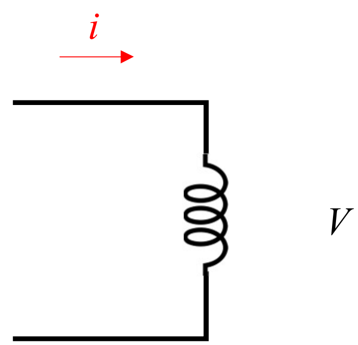
The basic circuit equations for a capacitor and an inductor

1. For resistors, the basic circuit equation (the relation between current and voltage)

$$V = IR$$

2. For inductors, the basic circuit equation

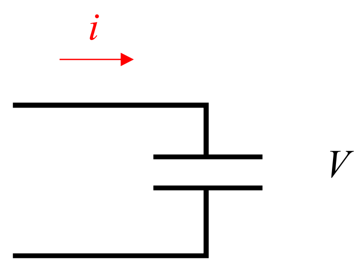
$$V = L\frac{\mathrm{d}i}{\mathrm{d}t}$$



3. For capacitors, the basic circuit equation

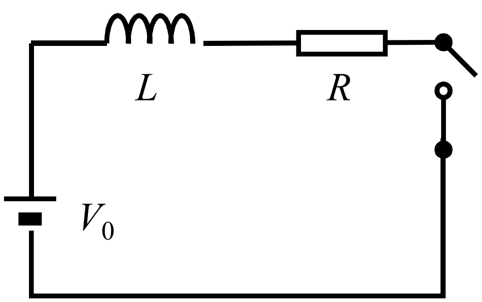
$$V = \frac{Q}{C}$$

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\mathrm{d}Q}{C\mathrm{d}t} = \frac{i}{C}$$



LR process: charging an inductor

At t=0, close the switch, charging the inductor (charging means pushing energy into)



The solution is

$$i = \frac{V}{R} - \frac{V}{R} e^{-Rt/L}$$

In a general form of

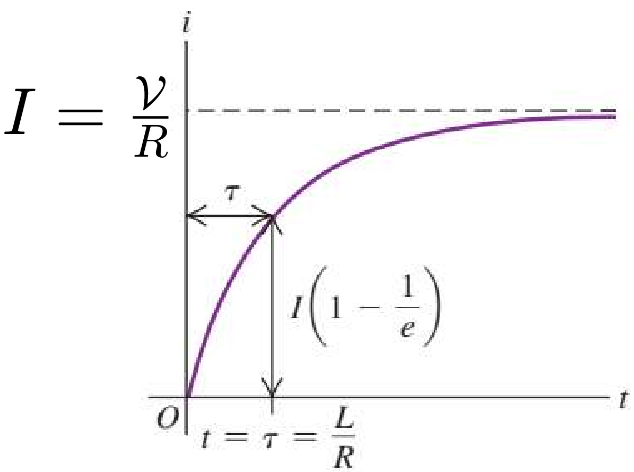
$$i = i_0 - \underline{i_1 e^{-t/\tau}}$$

Time related term

Stable value

A decaying term

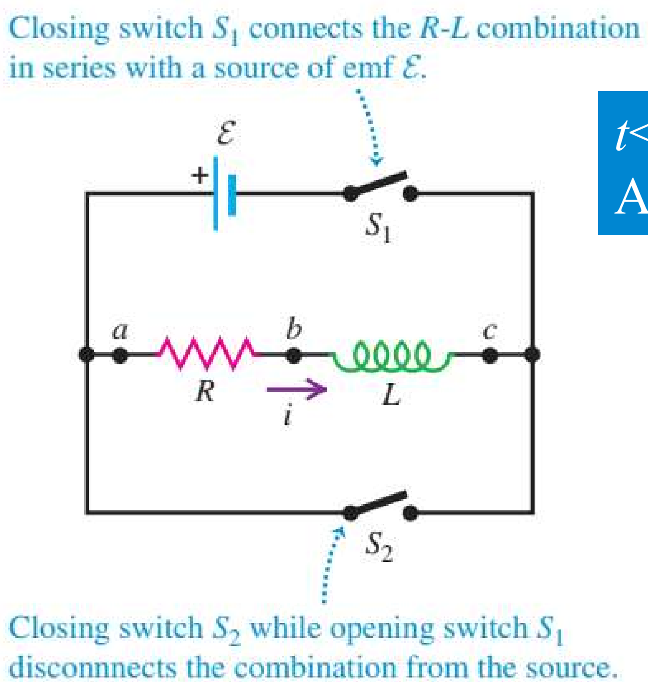
Time constant $\tau \equiv \frac{L}{R}$



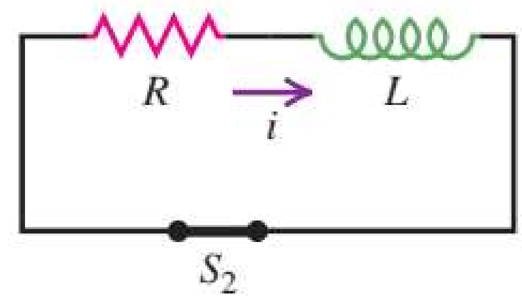
Initial value

$$i_0 - i_1$$

discharging an inductor



t<0, s1 kept close
At t=0, Open s1 when close s2



It forms such a circuit with a initial current V/R.

equation

$$iR + L\frac{\mathrm{d}i}{\mathrm{d}t} = 0$$

Initial condition

$$i(t=0) = \frac{V}{R}$$

Solution

$$i = \frac{V}{R} e^{-t/\tau}$$

$$\tau = \frac{L}{R}$$

A decaying current, energy is slowly dissipated on the resistor.

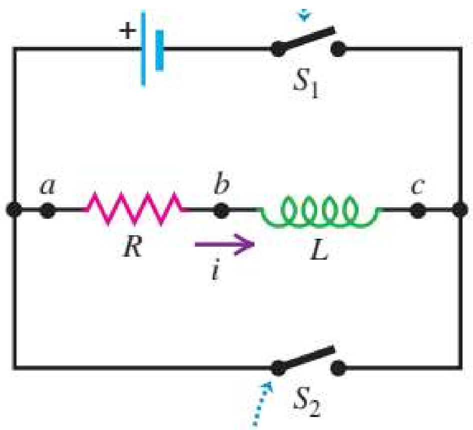
The time constant has the same form as charging process. It is the time constant for any change of LR circuit.

Time constant, initial value, and stable value

$$i = \frac{V}{R} e^{-t/\tau}$$

Time constant	$\tau = \frac{L}{R}$
Initial value	$\frac{V}{R}$
Final value (stable value)	0

discharging of an inductor may be dramatic

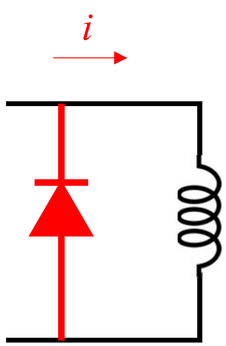


t<0, s1 kept close
At t=0, Open s1 but leave s2 open ???

$$i(t=0)=\frac{V}{R}$$
$$i(t=\delta t)=0$$

The change of current takes really short time, means a huge inductive voltage is generated.

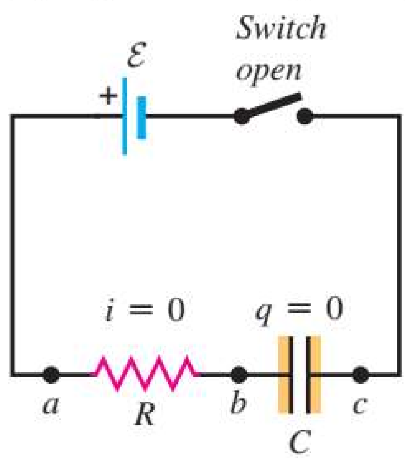
Where is it consumed? On switch 1, which means a huge voltage is applied on s1 so that the current can continue (making an arc)



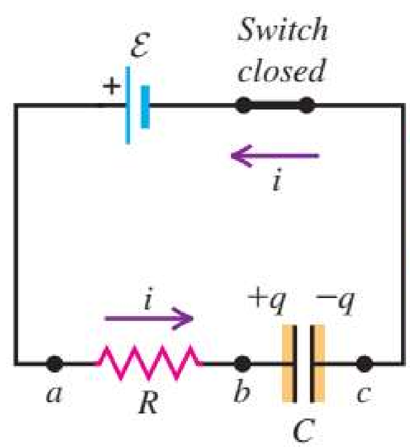
A flyback diode (free wheeling diode) for a (DC) inductor is a good idea to protect the circuit while does not affect the function of the circuit.

RC process: charging of a capacitor

(a) Capacitor initially uncharged



(b) Charging the capacitor



When the switch is closed, the charge on the capacitor increases over time while the current decreases.

equations

$$\mathcal{V} = iR + V_c$$

$$i = C\frac{\mathrm{d}V_c}{\mathrm{d}t}$$

Initial condition

$$V_c(t = 0) = 0$$

$$-RC\frac{\mathrm{d}(\mathcal{V} - V_c)}{\mathrm{d}t} = \mathcal{V} - V_c$$

Solution

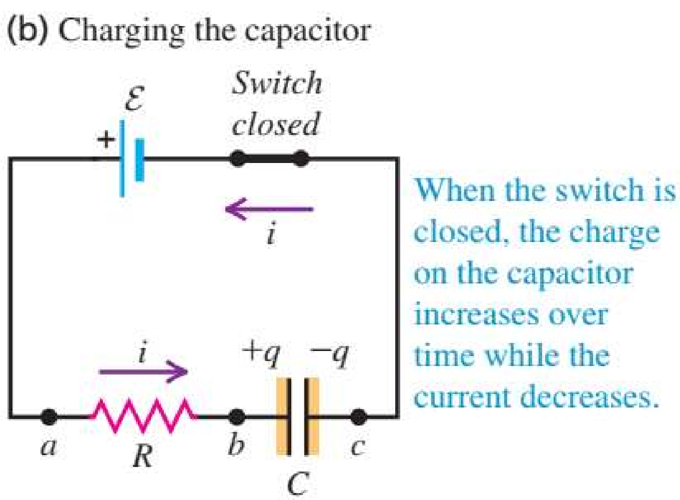
$$V_c = \mathcal{V} - \mathcal{V}e^{-t/RC}$$

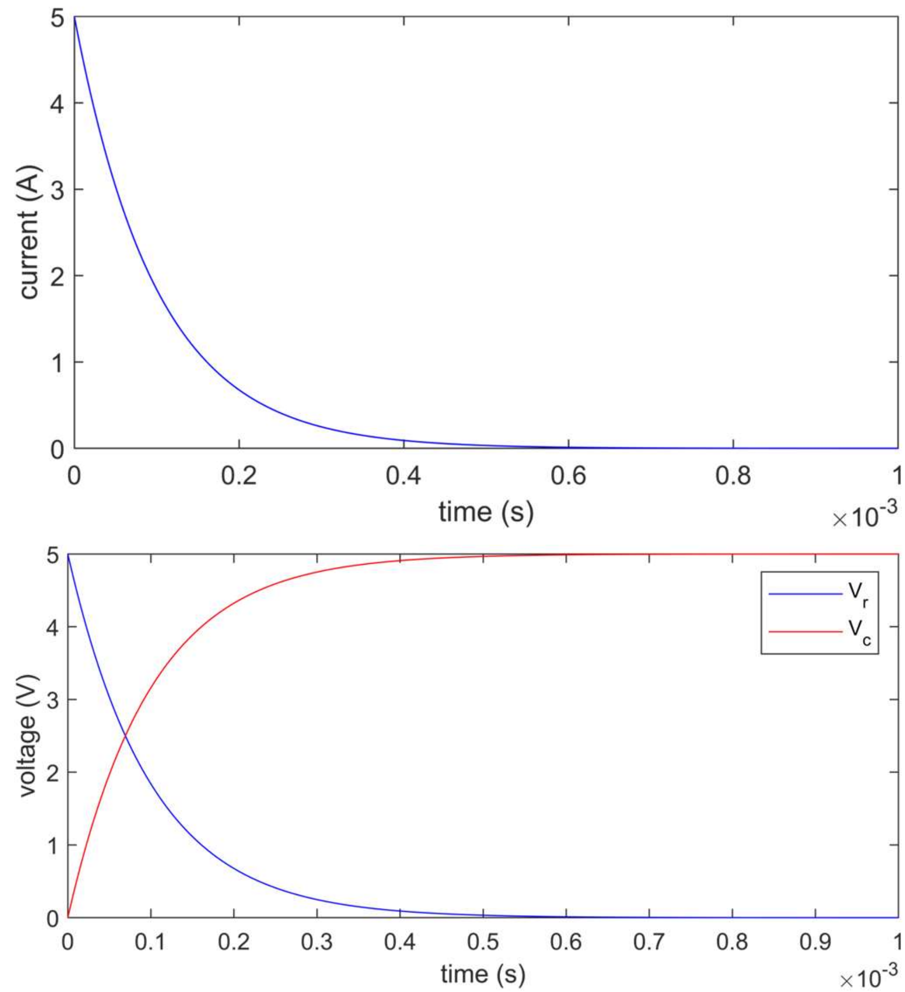
$$i = i_0 e^{-t/RC}$$

A time constant

$$i = i_0 e^{-t/RC}$$

$$i = i_0 e^{-t/\tau} \qquad \tau = RC$$





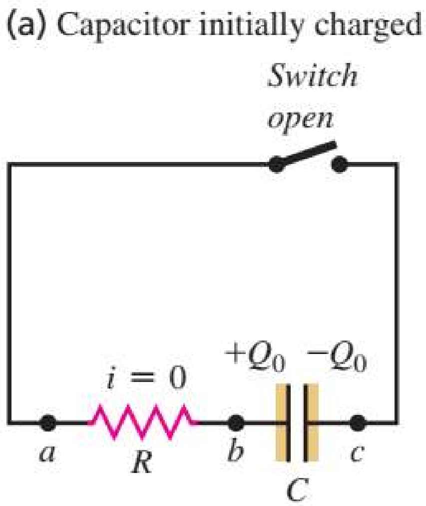
5V, 1Ohm, 100uF circuit

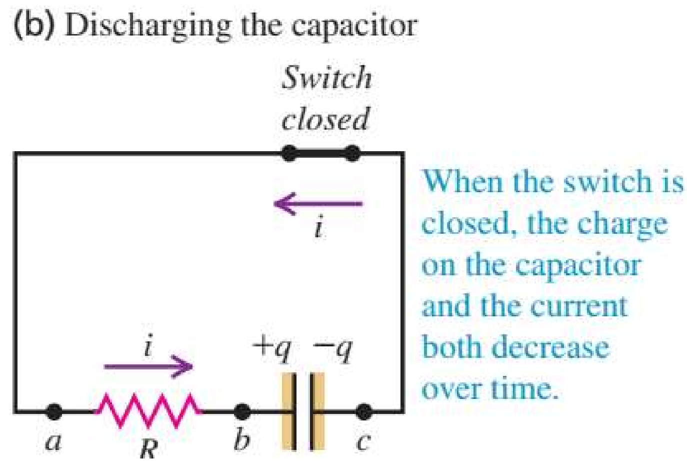
Time constant, initial value, and stable value

$$V_c = \mathcal{V} - \mathcal{V}e^{-t/RC}$$

Time constant	$\tau = RC$
Initial value	0
Final value (stable value)	$\mathcal{V}$

Discharge of a capacitor





Equations

$$C\frac{\mathrm{d}V_c}{\mathrm{d}t} = i$$

$$V_c = -iR$$

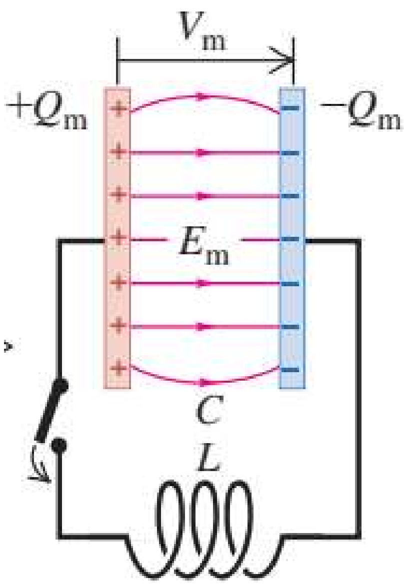
Solution

$$i = \frac{V_0}{R}e^{-t/\tau}, \ \tau = RC$$

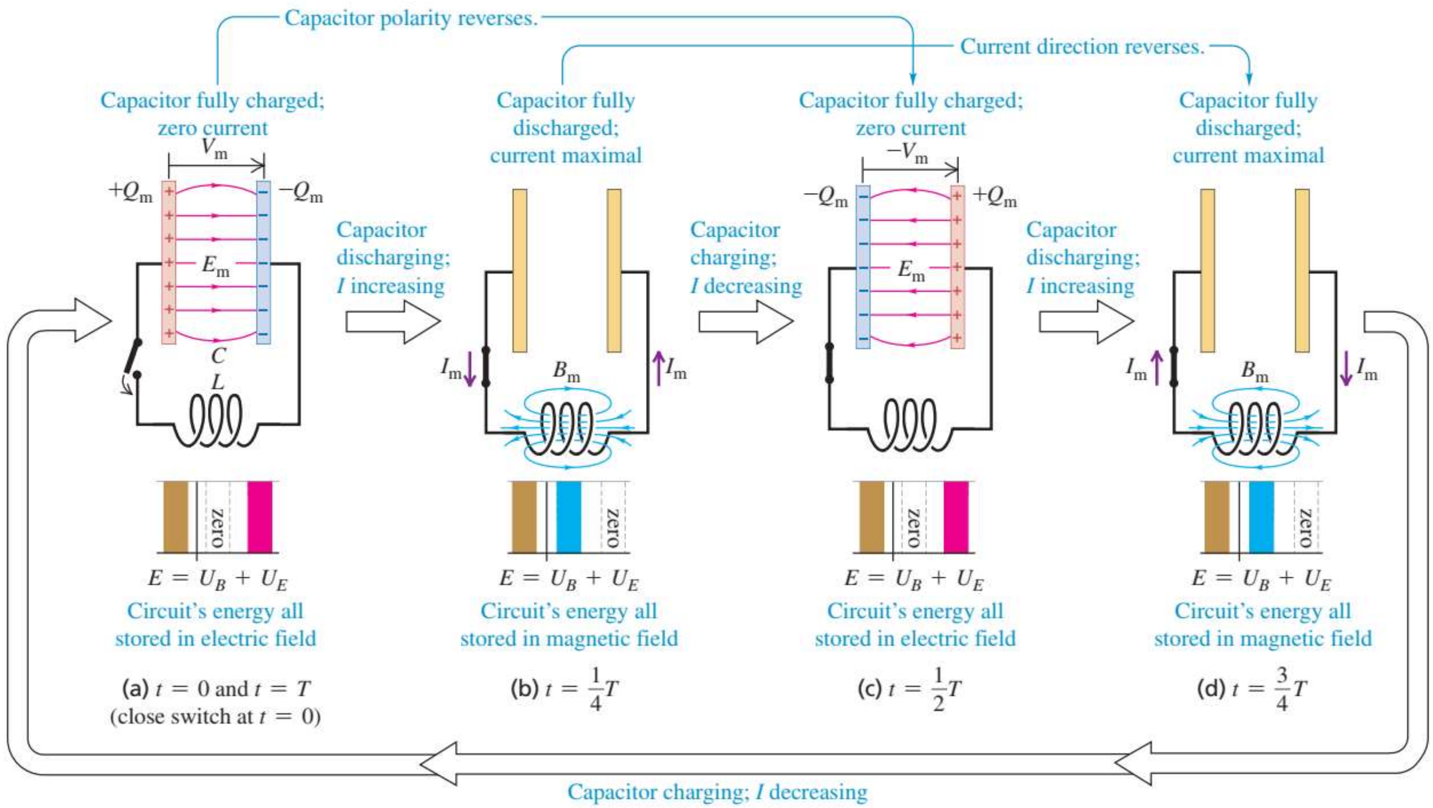
Contents

LC process

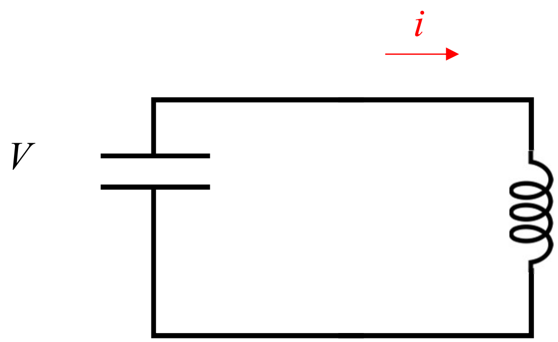
1. If the switch is closed at t=0, what will happen?



30.14 In an oscillating L-C circuit, the charge on the capacitor and the current through the inductor both vary sinusoidally with time. Energy is transferred between magnetic energy in the inductor (U_B) and electrical energy in the capacitor (U_E). As in simple harmonic motion, the total energy E remains constant. (Compare Fig. 14.14 in Section 14.3.)



LC process



Inductor equation

$$V = L\frac{\mathrm{d}i}{\mathrm{d}t}$$

Capacitor equation (notice the direction of the current)

$$i = -C\frac{\mathrm{d}V}{\mathrm{d}t}$$

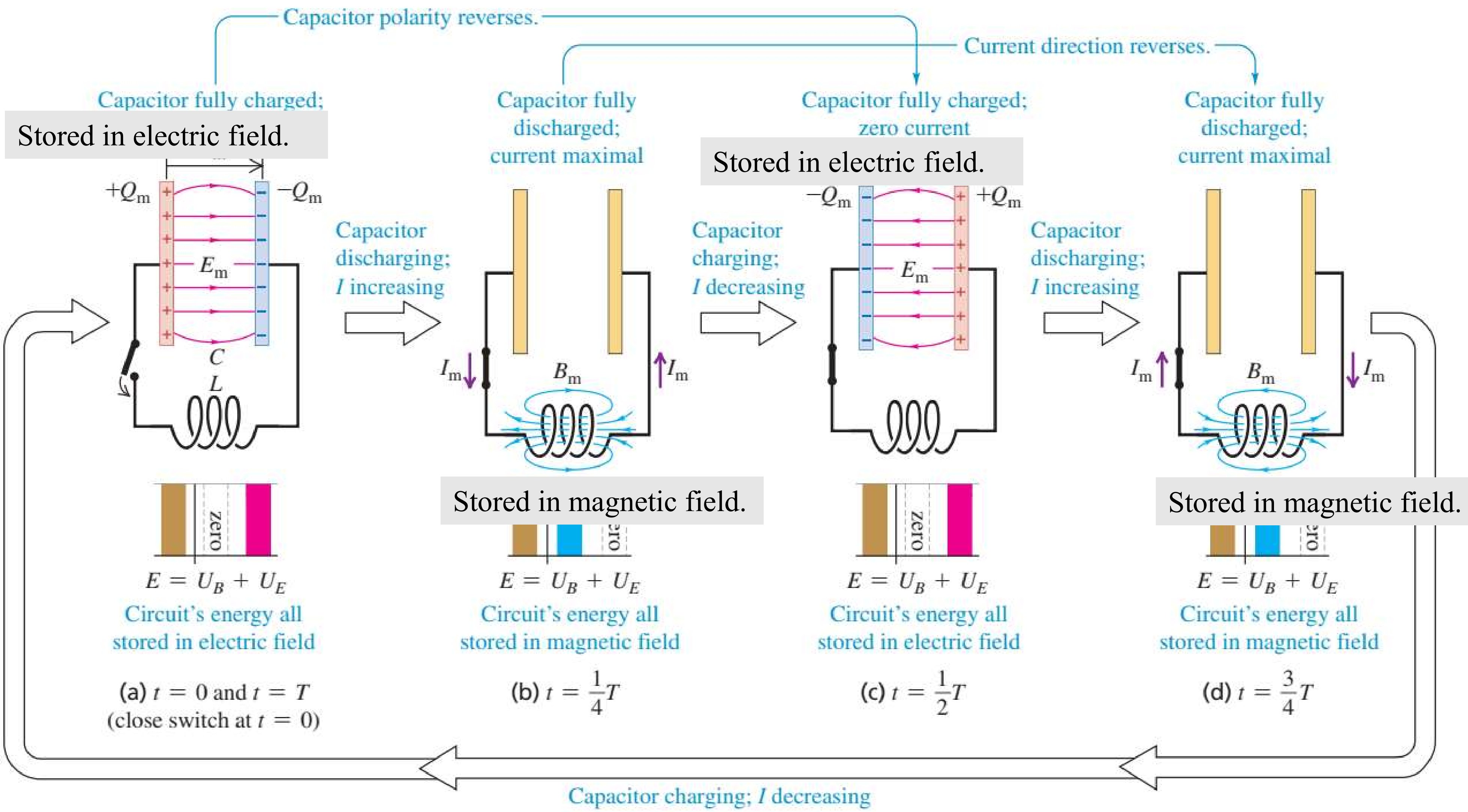
$$V = -LC\frac{\mathrm{d}^2V}{\mathrm{d}t^2} \qquad i = -LC\frac{\mathrm{d}^2i}{\mathrm{d}t^2}$$

Solution

$$V = V_0\cos(\omega t)$$
$$\omega = \sqrt{LC}$$

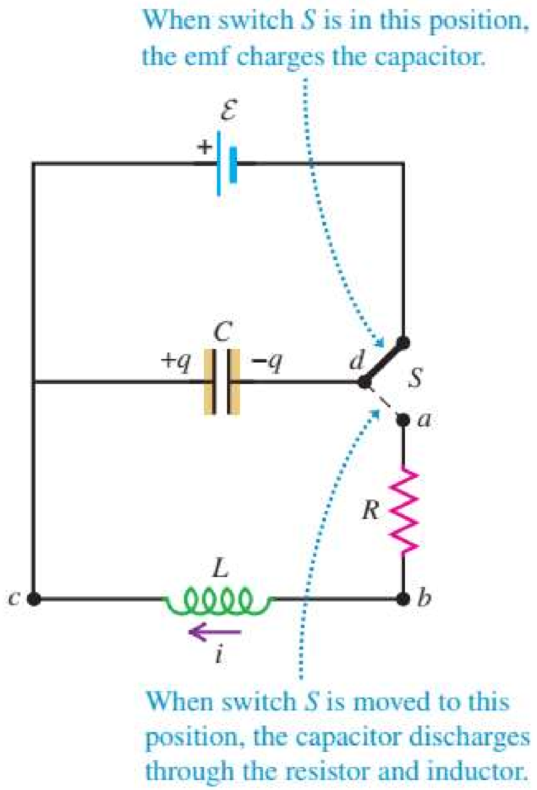
An LC oscillator, behaves like a spring oscillator

Energy of the system



LCR process

30.17 An *L-R-C* series circuit.



$$V_c = L\frac{\mathrm{d}i}{\mathrm{d}t} + iR$$

$$i = -\frac{\mathrm{d}Q}{\mathrm{d}t} = -C\frac{\mathrm{d}V_c}{\mathrm{d}t}$$

$$\frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{1}{LC}q = 0$$

$$q = Ae^{-\boxed{(R/2L)}t}\cos\left(\boxed{\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}}\,t + \phi\right)$$

1 / Damping time

Oscillation time

30.16 Graphs of capacitor charge as a function of time in an *L-R-C* series circuit with initial charge *Q*.