Home work: hand in required

22.1 • A flat sheet of paper of area 0.250 m^2 is oriented so that the normal to the sheet is at an angle of 60° to a uniform electric field of magnitude 14 N/C. (a) Find the magnitude of the electric flux through the sheet. (b) Does the answer to part (a) depend on the shape of the sheet? Why or why not? (c) For what angle ϕ between the normal to the sheet and the electric field is the magnitude of the flux through the sheet (i) largest and (ii) smallest? Explain your answers.

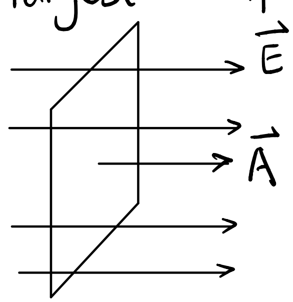
(a) $\Phi = E \cdot A \cdot \cos\phi = 14\,N/C \cdot 0.25\,m^2 \cdot \frac{1}{2}$

$= 1.75\,(V \cdot m)$

(b) No. Flux only depends on E field intensity, area and the angle between them. It doesn't depend on the shape or any other factors.

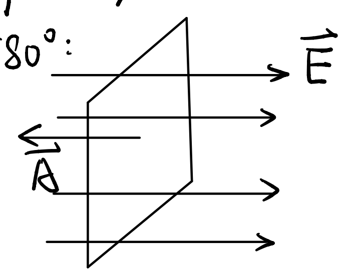
(c) (i) largest when $|\cos\phi| = 1 \Rightarrow \phi = 0°/180°$

$\phi = 0$:



$\Phi = 3.5\,(V \cdot m)$

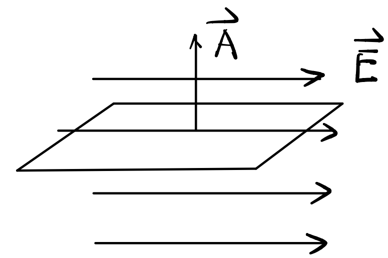
$\phi = 180°$:



$\Phi = -3.5\,(V \cdot m)$

(ii) smallest when $|\cos\phi| = 0 \Rightarrow \phi = 90°$

$\phi = 90°$: $\Phi = 0\,(V \cdot m)$



Home work: hand in required

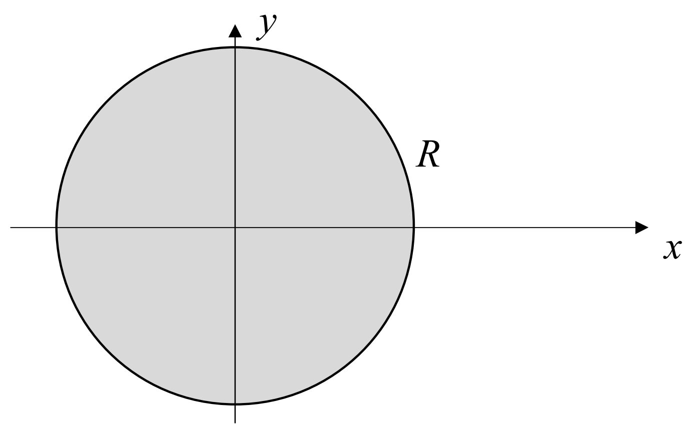
1. Please work out the field of a dipole of $P=Ql_0$, assuming the dipole is along positive z direction and located at origin. Write the field $E(x, y, z)$ as a vector at point $(x, 0, 0)$ and $(0, 0, z)$. What is the field ~~is~~ if the distance between the field point and the dipole is much larger than l_0?
2. Please write down the force and torque that the dipole experiences in arbitrary homogeneous field (arbitrary direction and magnitude).
3. Considering a circular plane with homogeneous charge distribution as follows, please use Coulomb's law to calculate the field at point (0,0,z) generated by the plane, and answer the following questions:

   3.1 what else necessary quantities do you need to deal with the problem?

   3.2 can you please vary the value of R as 2z, 5z, 10z, 100z, and compare the field at these assumptions?

   3.3 How large is R that you consider the plane close enough to infinite?

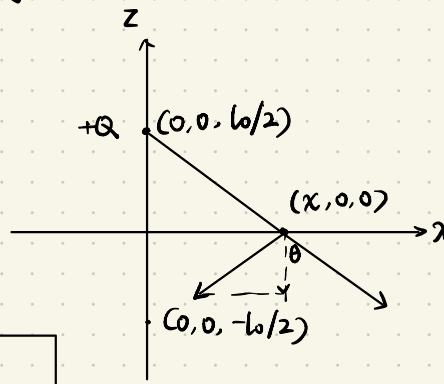
   3.4 Can you prove the field value of infinite plane we learned during the class (from Gauss's law) using the result you get from this problem? (you can either use a mathematical limit, or use a large number of R to have an approximate proof).



1. According to the definition of dipole

There is a charge $+Q$ at $(0,0,l_0/2)$ and a charge $-Q$ at $(0,0,-l_0/2)$

For point $(x,0,0)$



$$E=\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{x^2+l_0^2/4}\cdot 2\cos\theta$$

$$=\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{x^2+l_0^2/4}\cdot\frac{l_0/2}{\sqrt{x^2+l_0^2/4}}\cdot 2$$

$$=\frac{Q\cdot l_0}{4\pi\varepsilon_0(x^2+l_0^2/4)^{\frac{3}{2}}}$$

$$\boxed{\vec{E}=\frac{-Q\cdot l_0}{4\pi\varepsilon_0(x^2+l_0^2/4)^{\frac{3}{2}}}\hat{a}_z \quad \text{when } l_0\to\infty\ \vec{E}\to\vec{0}}$$

For point $(0,0,z)$

$$\boxed{\vec{E}=\frac{Q}{4\pi\varepsilon_0}\left(\frac{(z-l_0/2)\hat{a}_z}{|z-l_0/2|^3}-\frac{(z+l_0/2)\hat{a}_z}{|z+l_0/2|^3}\right) \quad \text{when } l_0\to\infty\ \vec{E}\to\vec{0}}$$

2. Force: $\vec{F}=(-Q+Q)\cdot\vec{E}=\boxed{\vec{0}}$

Torque: $\vec{\tau}=\vec{p}\times\vec{E}$

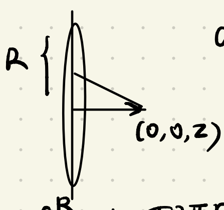
where $\vec{p}=(0,0,l_0)$, $\vec{E}=(E_x,E_y,E_z)$

$$\therefore\vec{\tau}=\begin{vmatrix}\hat{a}_x & \hat{a}_y & \hat{a}_z\\ 0 & 0 & l_0\\ E_x & E_y & E_z\end{vmatrix}=-l_0E_y\hat{a}_x+l_0E_x\hat{a}_y$$

$$=\boxed{(-l_0E_y,\ l_0E_x,\ 0)}$$

3. (1) $\boxed{\text{The charge density } \sigma \text{ of the field, Radius } R}$

(2) General Case:



$$dQ=\sigma\cdot dS$$

$$=\sigma\cdot 2\pi r\,dr$$

$$E=\int_{disk}\frac{1}{4\pi\varepsilon_0}\frac{dQ}{r^2+z^2}$$

$$=\int_0^R\frac{1}{4\pi\varepsilon_0}\frac{\sigma 2\pi r}{r^2+z^2}\cdot\frac{z}{\sqrt{r^2+z^2}}dr$$

$$=\frac{\sigma}{2\varepsilon_0}\int_0^R\frac{rz}{(r^2+z^2)^{\frac{3}{2}}}dr=\frac{\sigma}{2\varepsilon_0}\left(-\frac{z}{\sqrt{r^2+z^2}}\right)\Bigg|_0^R$$

$$=\frac{\sigma}{2\varepsilon_0}\left(-\frac{z}{\sqrt{R^2+z^2}}+1\right)$$

and the direction is the same is $(0,0,z)$

when

$R=2z$ $E=(1-\frac{1}{\sqrt{5}})\frac{\sigma}{2\varepsilon_0}$

$R=5z$ $E=(1-\frac{1}{\sqrt{26}})\frac{\sigma}{2\varepsilon_0}$

$R=10z$ $E=(1-\frac{1}{\sqrt{101}})\frac{\sigma}{2\varepsilon_0}$

$R=100z$ $E=(1-\frac{1}{\sqrt{10001}})\frac{\sigma}{2\varepsilon_0}$

(3) when $R=1000z$, $E=(1-\frac{1}{\sqrt{1000001}})\frac{\sigma}{2\varepsilon_0}\approx 0.999\frac{\sigma}{2\varepsilon_0}$

where 0.999 is quite close to 1. So $\boxed{R=1000z \text{ is large enough.}}$

(4) $\boxed{R\to\infty \quad E=\frac{\sigma}{2\varepsilon_0}\text{, which can be seen as an infinite plane. Proved.}}$

Home work: hand in required

4. You know the electric potential in a static electric field as

$$\phi(x, y, z) = 3x + 2y \ (\mathrm{V/m})$$

Then, can you write down the field in vector form?

5. You know the electric potential in a static electric field as

$$\phi(x, y, z) = x^2 + y^2 + z^2 \ (\mathrm{V/m})$$

Then, can you write down the field in vector form?
What kind of charge distribution is involved in this problem?

4. $\vec{E} = -\vec{\nabla}\phi = -\left(\frac{\partial}{\partial x}(3x+2y), \frac{\partial}{\partial y}(3x+2y), \frac{\partial}{\partial z}(3x+2y)\right)$

$= -3\hat{a}_x - 2\hat{a}_y$

5. $\vec{E} = -\vec{\nabla}\phi = -\left(\frac{\partial}{\partial x}(x^2+y^2+z^2), \frac{\partial}{\partial y}(x^2+y^2+z^2), \frac{\partial}{\partial z}(x^2+y^2+z^2)\right)$

$= -2x\hat{a}_x - 2y\hat{a}_y - 2z\hat{a}_z$

$\nabla^2\phi = -\frac{\rho(r)}{\varepsilon_0} \Rightarrow \rho(r) = -2\varepsilon_0\hat{a}_x - 2\varepsilon_0\hat{a}_y - 2\varepsilon_0\hat{a}_z$

$\Rightarrow$ uniform charge distribution