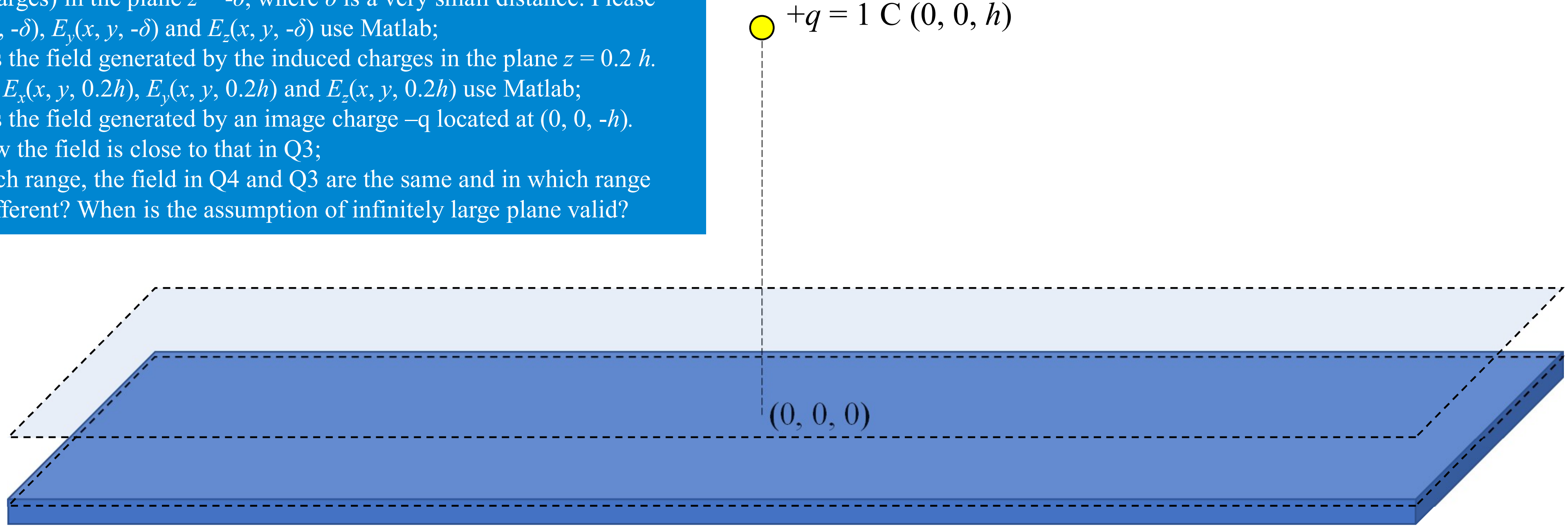
Problem with Matlab

Please finish this work by modifying the example code. You can ask your fellow students' help if you are not familiar with Matlab.
Submit the homework in the form of pdf (less than 2 pages of A4 paper). Q1 just requires an equation and a plot; Q2 to Q4 requires 3 2-D plots each; Q5 can be answered in any form you think proper (less than half a page).

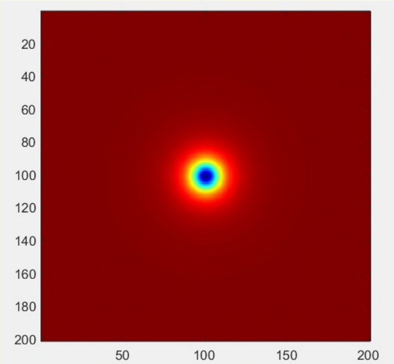
A charge q is located at (0, 0, h), and a large metal plate is located so that the top of the plate is at plane $z = 0$.

Q1. what is the distribution of the induced charge on the metal plate? Please try to write down the equation based on Gauss's law and make a plot in Matlab of $\sigma(x, y)$;
Q2. what is the field generated by these charges (the positive charge q and the induced charges) in the plane $z = -\delta$, where δ is a very small distance. Please plot $E_x(x, y, -\delta)$, $E_y(x, y, -\delta)$ and $E_z(x, y, -\delta)$ use Matlab;
Q3. what is the field generated by the induced charges in the plane $z = 0.2\,h$. Please plot $E_x(x, y, 0.2h)$, $E_y(x, y, 0.2h)$ and $E_z(x, y, 0.2h)$ use Matlab;
Q4. what is the field generated by an image charge –q located at (0, 0, $-h$). Please show the field is close to that in Q3;
Q5. In which range, the field in Q4 and Q3 are the same and in which range they are different? When is the assumption of infinitely large plane valid?

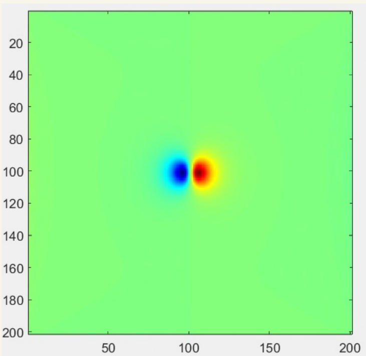


Question 1

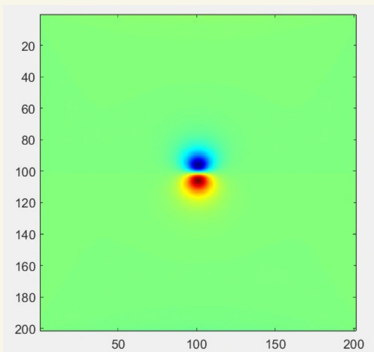
1. Equation: $\sigma_e(x,y) = -\frac{1}{2\pi}\frac{qh}{(x^2+y^2+h^2)^{\frac{3}{2}}}$.



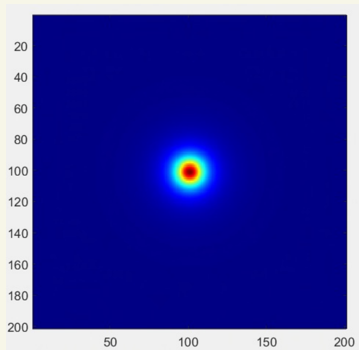
2. $\vec{E}_x$:



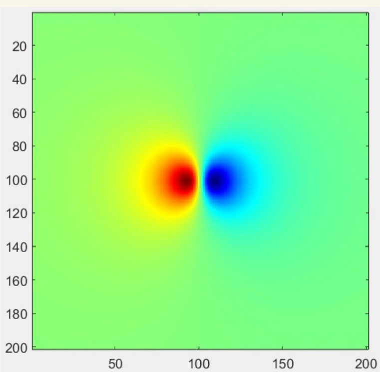
$\vec{E}_y$:



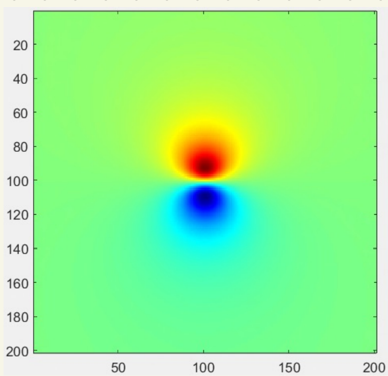
$\vec{E}_z$:



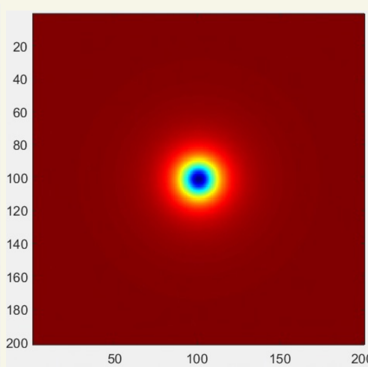
3. $\vec{E}_x$:



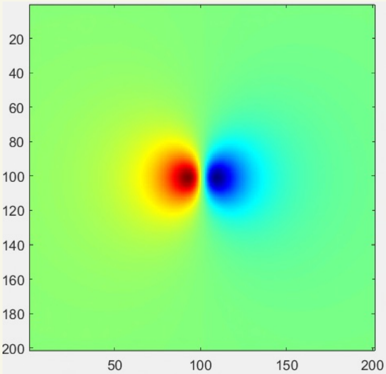
$\vec{E}_y$:



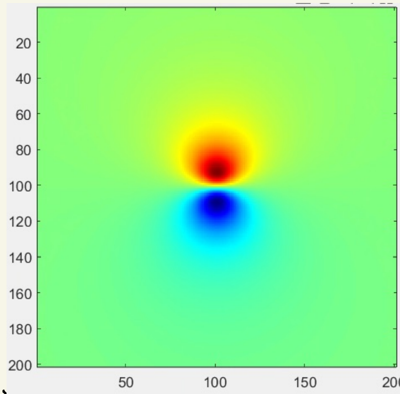
$\vec{E}_z$:



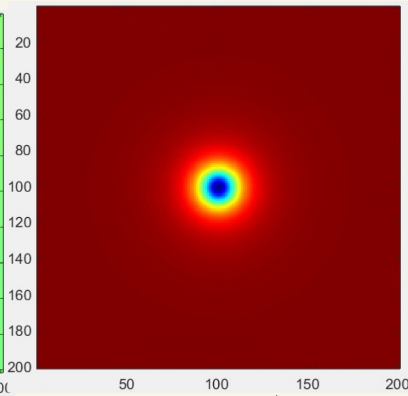
4. $\vec{E}_x$:



$\vec{E}_y$:



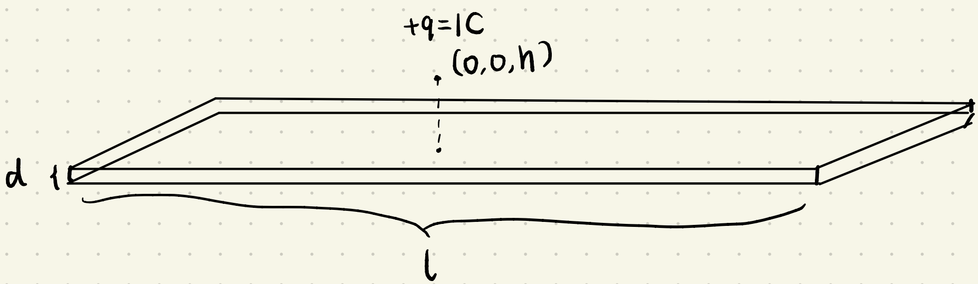
$\vec{E}_z$:



5. Only in our interested region, i.e., $h>0$, are fields in Q3 and Q4 identical. This is because Q3 calculates the electric field directly and Q4 applies the image charge method. And the image charge method is valid only when we focus on the region above the metal board; The assumption of infinite large plate is valid only when

① The thickness of the board $d \to 0$.

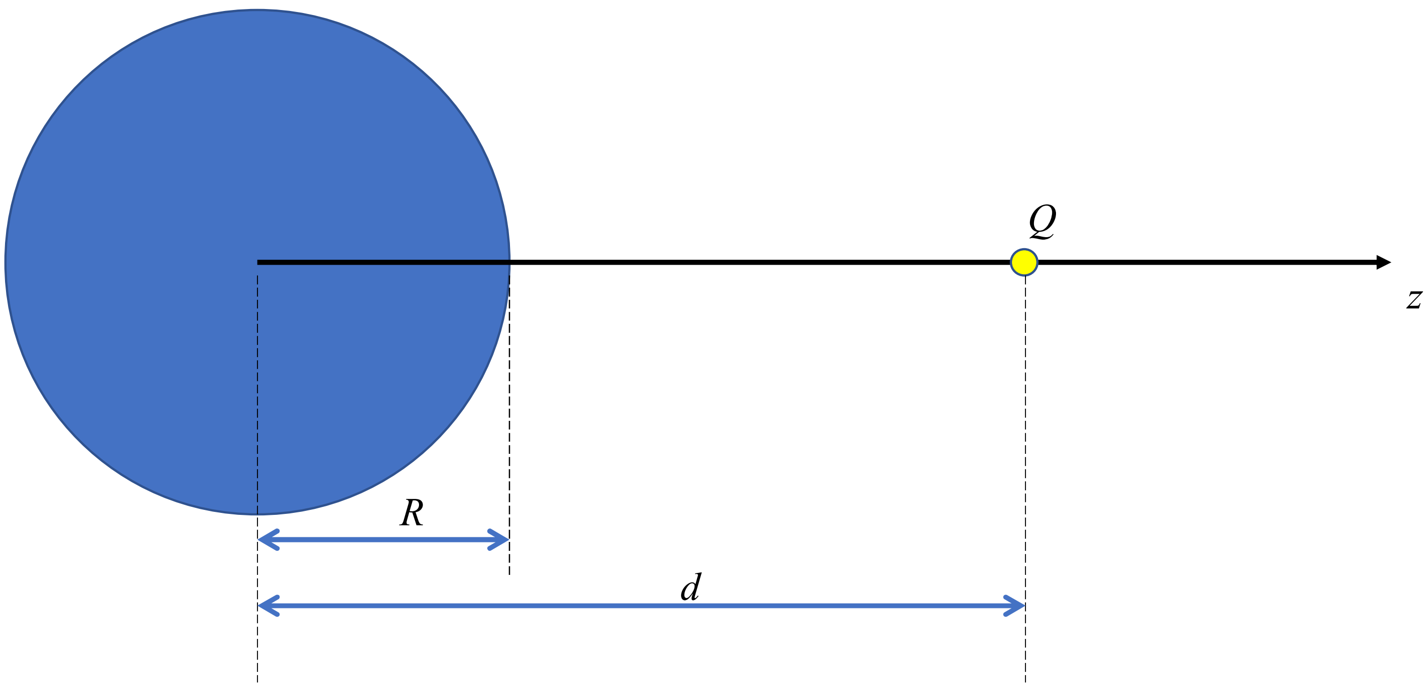
② The length of the board $l \gg h$.



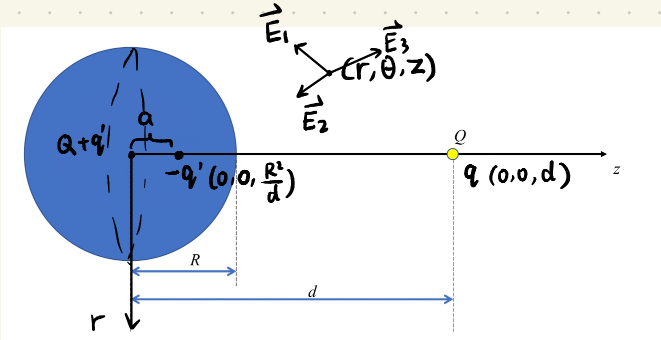
Problem 2

A metal sphere (radius R) contains charge Q.
A point charge q is located with distance d away from the center of the sphere ($d > R$).

Please calculate the field at any point (r, θ, z) with the origin located at the center of the sphere.



Question 2



We apply the method of image charge: (Because the sphere is equipotential.)

If the sphere is grounded, the system can be seen as $-q'$ and q

Where $q' = \frac{qR}{d}$, $a = \frac{R^2}{d}$ (Our interested region is outside the sphere.)

However, the metal sphere contains charge Q

$\Rightarrow$ There is still a point charge $Q+q'$ at the center of the sphere.

Situation 1: $r^2+z^2 > R^2$ for the point (r,θ,z), we apply the superposition principle.

point charge q: $\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{q(r\cdot\hat{a}_r+(z-d)\cdot\hat{a}_z)}{(r^2+(z-d)^2)^{\frac{3}{2}}}$

point charge $-q'$: $\vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\frac{-\frac{qR}{d}(r\cdot\hat{a}_r+(z-\frac{R^2}{d})\cdot\hat{a}_z)}{(r^2+(z-\frac{R^2}{d})^2)^{\frac{3}{2}}}$

point charge $Q+q'$ $\vec{E}_3 = \frac{1}{4\pi\varepsilon_0}\frac{(Q+\frac{qR}{d})(r\cdot\hat{a}_r+z\cdot\hat{a}_z)}{(r^2+z^2)^{\frac{3}{2}}}$

According to the superposition principle:

$$\boxed{\begin{aligned}\vec{E} &= \vec{E}_1+\vec{E}_2+\vec{E}_3\\ &= \frac{1}{4\pi\varepsilon_0}\Bigg[\left(\frac{q}{(r^2+(z-d)^2)^{\frac{3}{2}}}-\frac{\frac{qR}{d}}{(r^2+(z-\frac{R^2}{d})^2)^{\frac{3}{2}}}+\frac{(Q+\frac{qR}{d})}{(r^2+z^2)^{\frac{3}{2}}}\right)r\,\hat{a}_r\\ &\quad+\left(\frac{q(z-d)}{(r^2+(z-d)^2)^{\frac{3}{2}}}-\frac{\frac{qR}{d}(z-\frac{R^2}{d})}{(r^2+(z-\frac{R^2}{d})^2)^{\frac{3}{2}}}+\frac{(Q+\frac{qR}{d})z}{(r^2+z^2)^{\frac{3}{2}}}\right)\hat{a}_z\Bigg]\end{aligned}}$$

Situation 2: $r^2 + z^2 < R^2$.

We no longer need to consider charge q, as the metal sphere is equipotential.

In the sphere, we choose the gaussian surface of radius $\sqrt{r^2+z^2}$

$$\oint \vec{E} \cdot d\vec{A} = E \cdot 4\pi (r^2+z^2) = \frac{Q(r^2+z^2)^{\frac{3}{2}}}{\varepsilon_0 R^3}$$

$$\therefore E = \frac{Q\sqrt{r^2+z^2}}{4\pi\varepsilon_0 R^3}$$

$$\vec{E} = \frac{Q\sqrt{r^2+z^2}}{4\pi\varepsilon_0 R^3} \cdot \frac{r}{\sqrt{r^2+z^2}} \hat{a}_R + \frac{Q\sqrt{r^2+z^2}}{4\pi\varepsilon_0 R^3} \cdot \frac{z}{\sqrt{r^2+z^2}} \hat{a}_z$$

$$= \boxed{\frac{Qr}{4\pi\varepsilon_0 R^3} \hat{a}_R + \frac{Qz}{4\pi\varepsilon_0 R^3} \hat{a}_z}$$