Homework 4

24.12 •• A cylindrical capacitor has an inner conductor of radius 2.2 mm and an outer conductor of radius 3.5 mm. The two conductors are separated by vacuum, and the entire capacitor is 2.8 m long. (a) What is the capacitance per unit length? (b) The potential of the inner conductor is 350 mV higher than that of the outer conductor. Find the charge (magnitude and sign) on both conductors.

24.11 •• A spherical capacitor contains a charge of 3.30 nC when connected to a potential difference of 220 V. If its plates are separated by vacuum and the inner radius of the outer shell is 4.00 cm, calculate: (a) the capacitance; (b) the radius of the inner sphere; (c) the electric field just outside the surface of the inner sphere.

24.12

(a) $$C = \frac{2\pi\varepsilon_0 L}{\ln\frac{R_B}{R_A}} = \frac{2\pi \cdot 8.854\times10^{-12}\cdot 1}{\ln\frac{3.5}{2.2}} = \boxed{1.198\times10^{-10}\,F}$$

(b) $$q = C\cdot h\cdot V = \boxed{1.18\times10^{-10}\,C}$$

The charge of inner conductor is positive, and the charge of outer conductor is negative as the potential of the inner conductor is higher than that of outer conductor.

24.11

(a) $$C = \frac{Q}{V} = \frac{3.3\times10^{-9}C}{220V} = \boxed{1.5\times10^{-11}F}$$

(b) $$C = \frac{4\pi\varepsilon_0\cdot 0.04\cdot R_A}{0.04-R_A} = 1.5\times10^{-11}$$

$$0.16\pi\varepsilon_0 R_A = 6\times10^{-13} - 1.5\times10^{-11}R_A$$

$$1.945\times10^{-11}R_A = 6\times10^{-13}$$

$$\Rightarrow \boxed{R_A = 3.08\,cm}$$

(c) According to the Gauss's Law:

$$\oiint \vec{E}\cdot d\vec{A} = \frac{Q}{\varepsilon_0}$$

$$\vec{E}\cdot 4\pi(0.0308)^2 = \frac{3.3\times10^{-9}C}{8.854\times10^{-12}F/m}\,\hat{a}_R$$

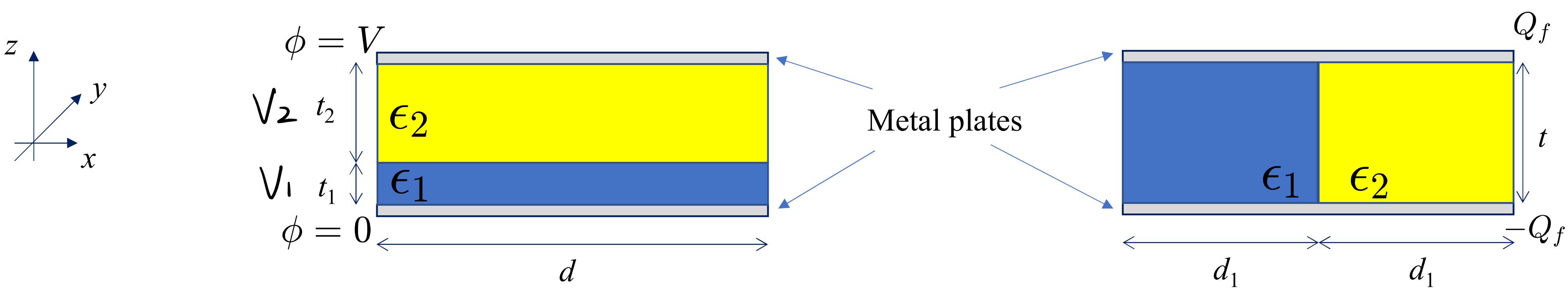
$$\therefore \boxed{\vec{E} = 3.13\times10^4 (N/C)}$$

Homework 4

Problem 3

For the following two scenarios, please calculate the electric field $E_z(x,z)$ of each case. The metal plates are considered much larger in x and y directions than in z direction, so that the end effects can be neglected. The dimension in y direction is 1 m.

For case 1, the potential difference V is given, and for case 2, the charge on the metal plates Q_f and $-Q_f$ are given.



case 1

case 2

Case 1: Set the left bottom as origin.

Two capacitors are connected in series.

$$Q_0 = CV = \frac{\varepsilon_0 S V}{\frac{t_1}{\varepsilon_1}+\frac{t_2}{\varepsilon_2}} \Rightarrow \sigma_0 = \frac{\varepsilon_0 V}{\frac{t_1}{\varepsilon_1}+\frac{t_2}{\varepsilon_2}} = \frac{\varepsilon_0\varepsilon_1\varepsilon_2 V}{t_1\varepsilon_2+t_2\varepsilon_1}$$

$$V_2 = E_2 d_2 = \frac{Q_0}{C_2}$$

$$E_2 = \frac{\sigma_0-\sigma_2'}{\varepsilon_0} \quad E_1 = \frac{\sigma_0-\sigma_1'}{\varepsilon_0}$$

where $\sigma_2' = \sigma_0\frac{\varepsilon_2-1}{\varepsilon_2}$, $\sigma_1' = \sigma_0\frac{\varepsilon_1-1}{\varepsilon_1}$

$$\therefore E_2 = \frac{\sigma_0}{\varepsilon_0\varepsilon_2} = \frac{V\varepsilon_1}{t_1\varepsilon_2+t_2\varepsilon_1} \quad \text{direction: } -\hat{a}_z$$

$$E_1 = \frac{\sigma_0}{\varepsilon_0\varepsilon_1} = \frac{V\varepsilon_2}{t_1\varepsilon_2+t_2\varepsilon_1} \quad \text{direction: } -\hat{a}_z$$

To sum up.

$$\vec{E}_z(x,z) = \begin{cases} -\frac{V\varepsilon_2}{t_1\varepsilon_2+t_2\varepsilon_1}\hat{a}_z & 0<x<d \text{ and } 0<z<t_1 \\ -\frac{V\varepsilon_1}{t_1\varepsilon_2+t_2\varepsilon_1}\hat{a}_z & 0<x<d \text{ and } t_1<z<t_2 \\ 0 & \text{otherwise} \end{cases}$$

Case 2: Set the left bottom as origin.

Two capacitors are connected in parallel.

$$C_{eq} = C_1 + C_2 = \frac{\varepsilon_0\varepsilon_1 d_1 \cdot l}{t} + \frac{\varepsilon_0\varepsilon_2 d_1 \cdot l}{t} = \frac{\varepsilon_0 d_1}{t}(\varepsilon_1+\varepsilon_2)$$

$$E = \frac{U}{t} = \frac{1}{t}\frac{Q_f}{C_{eq}} = \frac{Q_f}{\varepsilon_0 d_1(\varepsilon_1+\varepsilon_2)} \quad \text{direction: } -\hat{a}_z$$

To sum up

$$\vec{E}_z(x,z) = \begin{cases} -\frac{Q_f}{\varepsilon_0 d_1(\varepsilon_1+\varepsilon_2)}\hat{a}_z & (0<x<2d_1 \text{ and } 0<z<t) \\ 0 & \text{otherwise} \end{cases}$$