Physics (PHYS2500J), Unit 6 Wave optics:
2. Interference

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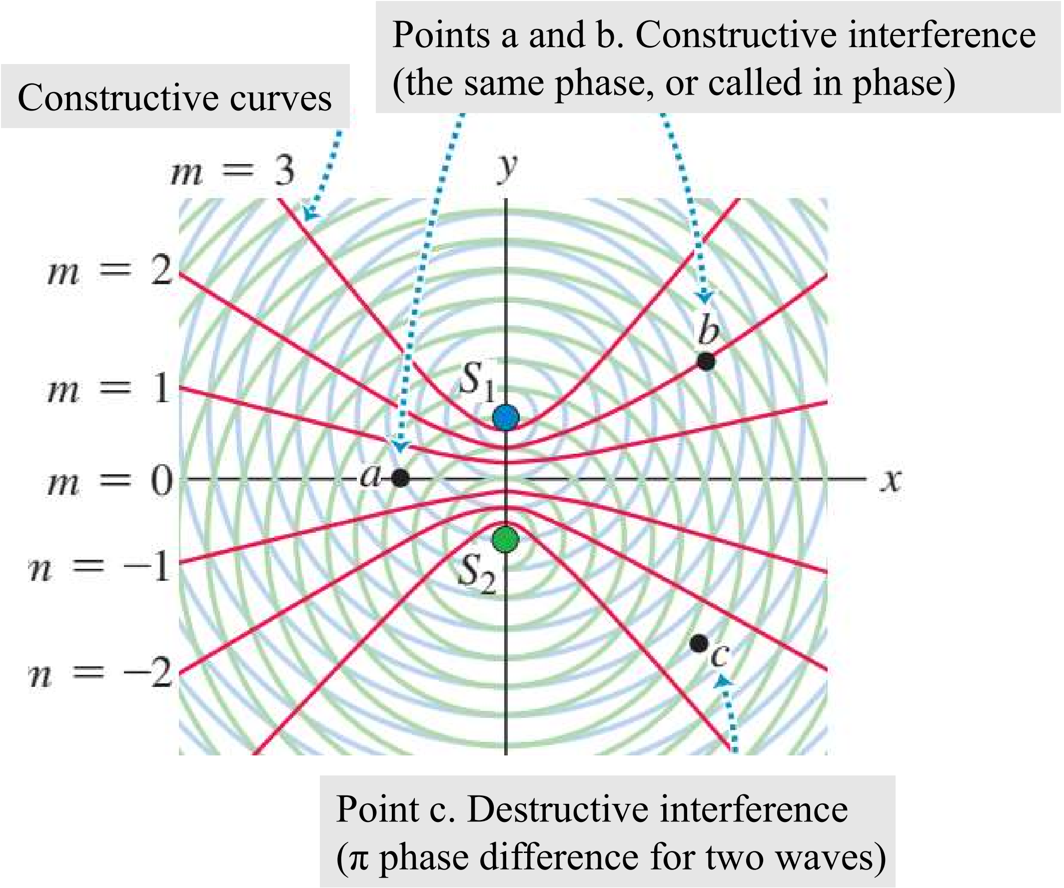
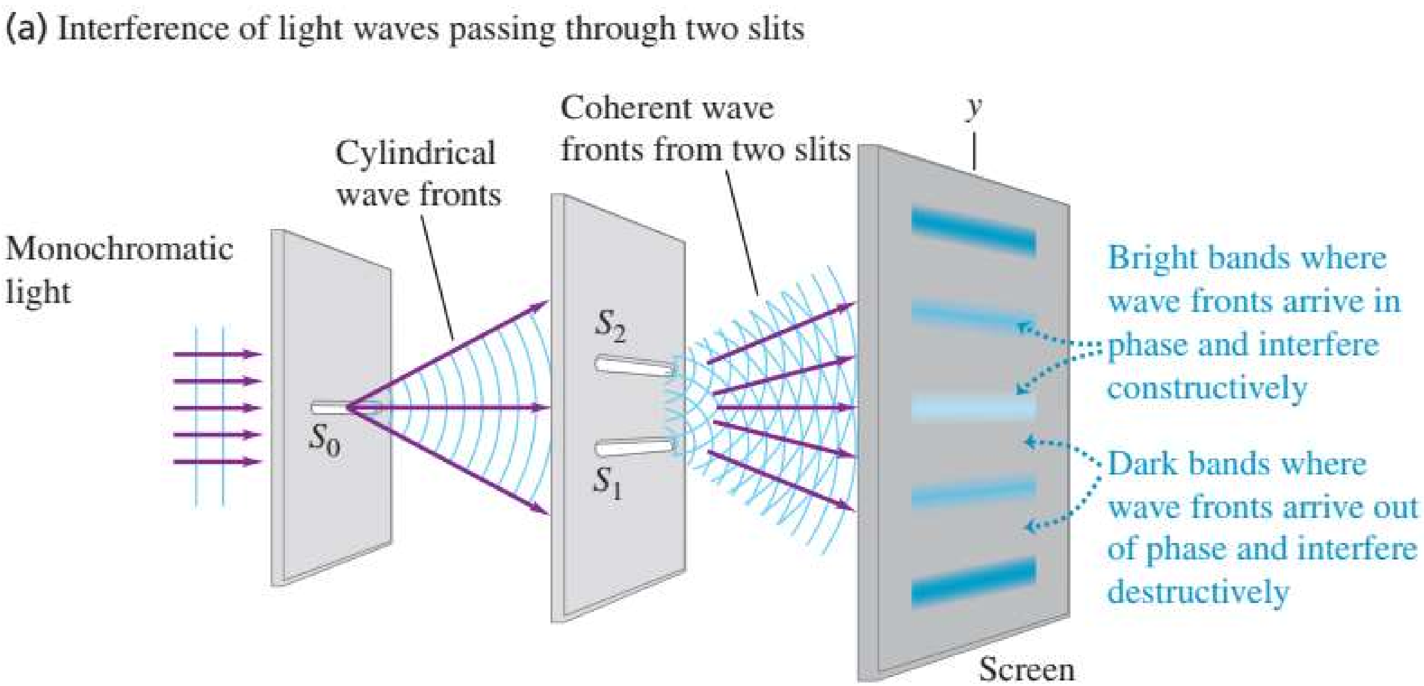
Principle of superposition:

When two or more waves overlap, the resultant displacement at any point and at any instant is found by adding the instantaneous displacements that would be produced at the point by the individual waves if each were present alone.



Young's experiment of double slits

1804, Thomas Young experimentally discovered the interference of light, through two slits following one generating the "coherent light source"

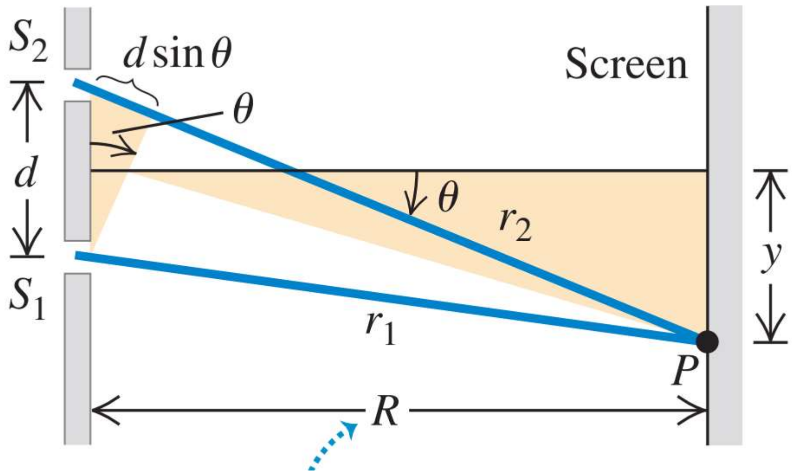


To explain the phenomenon with interference, one need to address on the question: what is the "displacement" here, with a "negative value"?

Now we know it is the electric field.

Young's experiment of double slits

Calculate the location of bright strips for Young's double slits experiment.



Assuming $d << R$ and $y << R$

To calculate the relative phase of the two waves. Start with optical path difference

$$\Delta l = d\sin\theta$$

When θ is small,

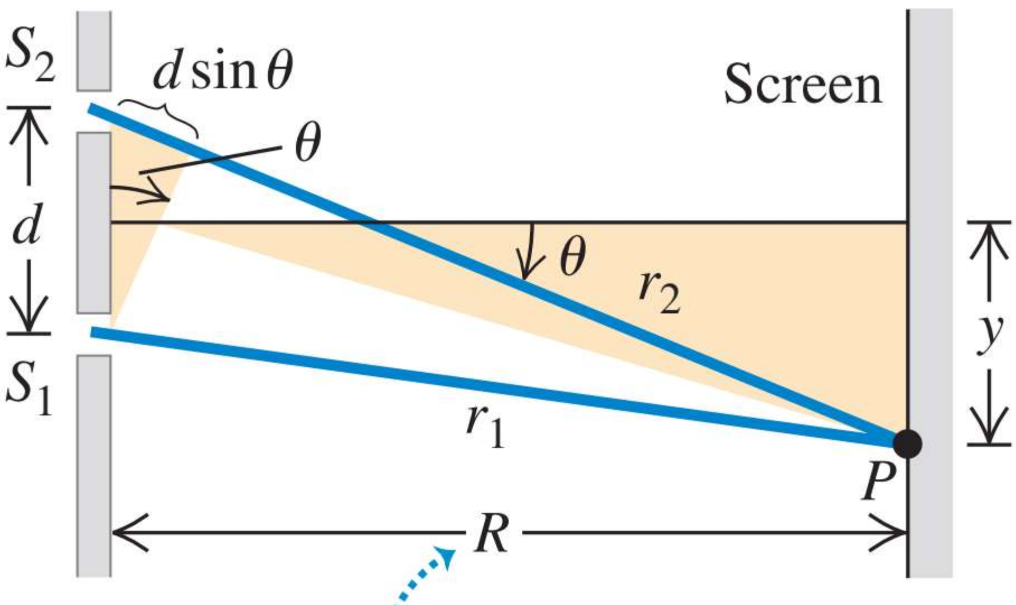
$$\sin\theta = \theta = \tan\theta = \frac{y}{R}$$

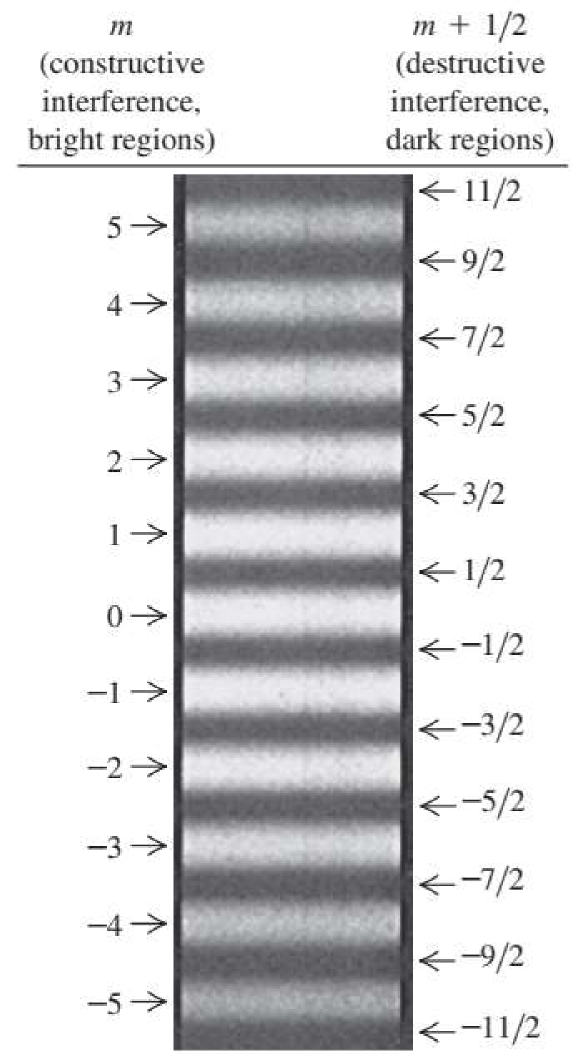
$$\Delta\phi = 2\pi\frac{\Delta l}{\lambda} = 2\pi\frac{dy}{\lambda R}$$

$\Delta\varphi = (2n+1)\pi$, dark strips; $\quad \frac{dy}{\lambda R} = \pm\frac{1}{2}, \pm1\frac{1}{2}, \pm2\frac{1}{2}, \ldots$

$\Delta\varphi = 2n\pi$, bright strips. $\quad \frac{dy}{\lambda R} = 0, \pm1, \pm2, \ldots$

Experimentally observed strips





Intensity and phase difference

The interference pattern is measured in intensity

$$I = \overline{|E \times H|} = n\overline{E^2}/\mu c$$

Based on the superposition principle of electric field,

$$I = n\overline{(\vec{E}_1 + \vec{E}_2)^2}/\mu c$$

If the two electric field vectors are in the same direction, and each can be written as sinusoidal wave.

$$\begin{aligned} I &= n\overline{(E_{10}\cos\omega t + E_{20}\cos(\omega t + \phi))^2}/\mu c \\ &= \frac{n}{\mu c}\left(\frac{E_{10}^2}{2} + \frac{E_{20}^2}{2} + E_{10}E_{20}\cos\phi\right) \\ &= I_{10} + I_{20} + 2\sqrt{I_{10}I_{20}}\cos\phi \end{aligned}$$

Where E_{10} and I_{10} are the electric field amplitude and intensity if only wave 1 exists, φ is the phase difference between the two wave at this location. φ is a function of location and maybe time.

Cases where interference does not happen in this way.

1. In non-linear medium where material property (ε) is a function of electric field intensity, which makes n a function of E instead of a constant.

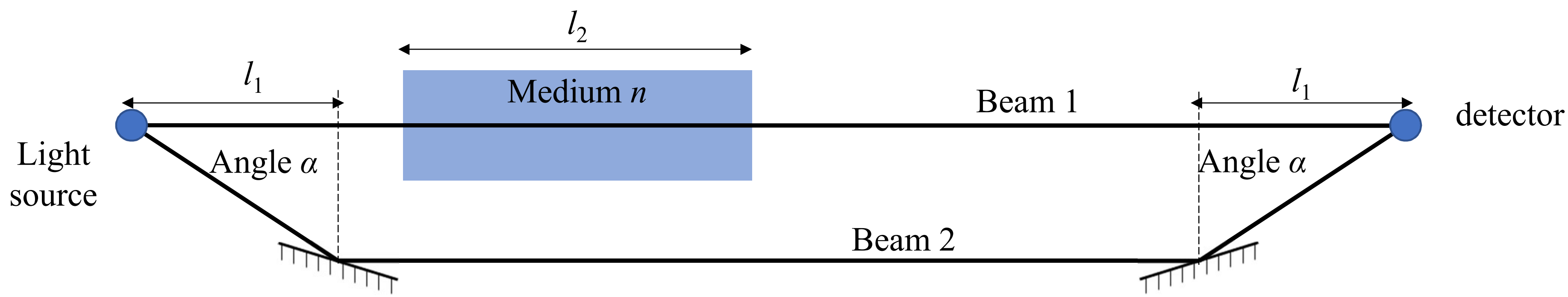
2. Where light sources of the two wave are not coherent, which makes φ a function of time.

Phase difference is the key to evaluate interference

1. To know the intensity after interference, the phase difference of the two wave is needed.

2. If the two wave have the same origin, or the phase relation at the starting point is known (e.g. plane wave on the same wave front), calculation of phase difference after the starting point is the key.

3. If the wave travels in medium, the wavelength is changed.



What is the phase difference between the two beams?

1. Beam 2 travels two longer paths

$$2\frac{l_1}{\lambda_0}\left(\frac{1}{\cos\alpha}-1\right)2\pi$$

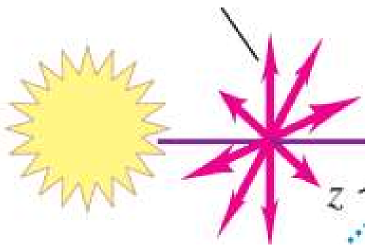
2. Beam 1 travels one section in medium

$$2\pi\left(\frac{l_2}{\lambda'}-\frac{l_2}{\lambda_0}\right)=(n-1)\frac{2l_2\pi}{\lambda_0}$$

Total phase difference being

$$2\pi\frac{2l_1\left(\frac{1}{\cos\alpha}-1\right)-(n-1)l_2}{\lambda_0}$$

The coherence (相干性) of sunlight



If the sunlight contains polarization in all directions, why didn't it just goes away because of destructive interference?

Superposition of "displacement" applies to coherent sources.

Two coherent source, which can interference has to 1. be in the same frequency; 2. have certain stable phase difference.

For non-coherent source, intensity or energy, instead of the displacement sums up.

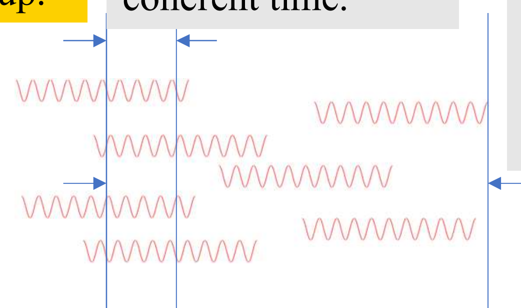
In short time, *E* have certain phase, coherent time.

In longer time scale, phase changes all the time.

Sun light, or light bulbs.

- Red light
- …
- Violet light (not for bulb)

Emitted by lots of atoms at random time.



Even for single color component, the light is not totally coherent, but just with a short time τ (coherent time 相干时间). Or coherent length （相干长度） $c\tau \sim$ µm

Mathematical description of coherence

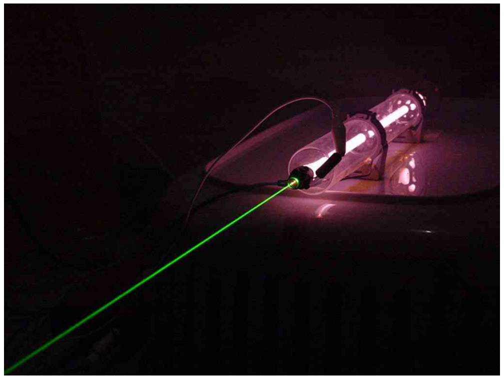
$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(\psi_1 - \psi_2)$$

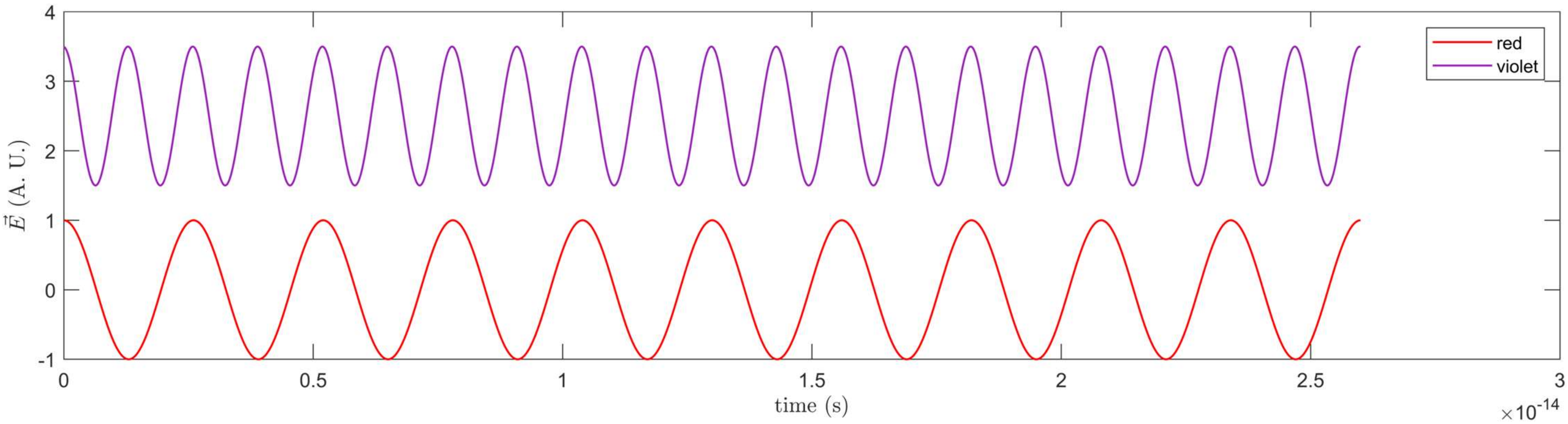
$$\psi_1(\vec{r}, t) = \phi_1(t) - \vec{k}_1 \cdot \vec{r}$$

1. φ can be time dependent. Random phase in short time reduces cross-over term back to 0.
2. If the two sources are coherent, namely φ_1 - φ_2 being constant over time. ψ_1-ψ_2 depends on space and leads to constructive or destructive coherence.

Different frequency sources are not coherent

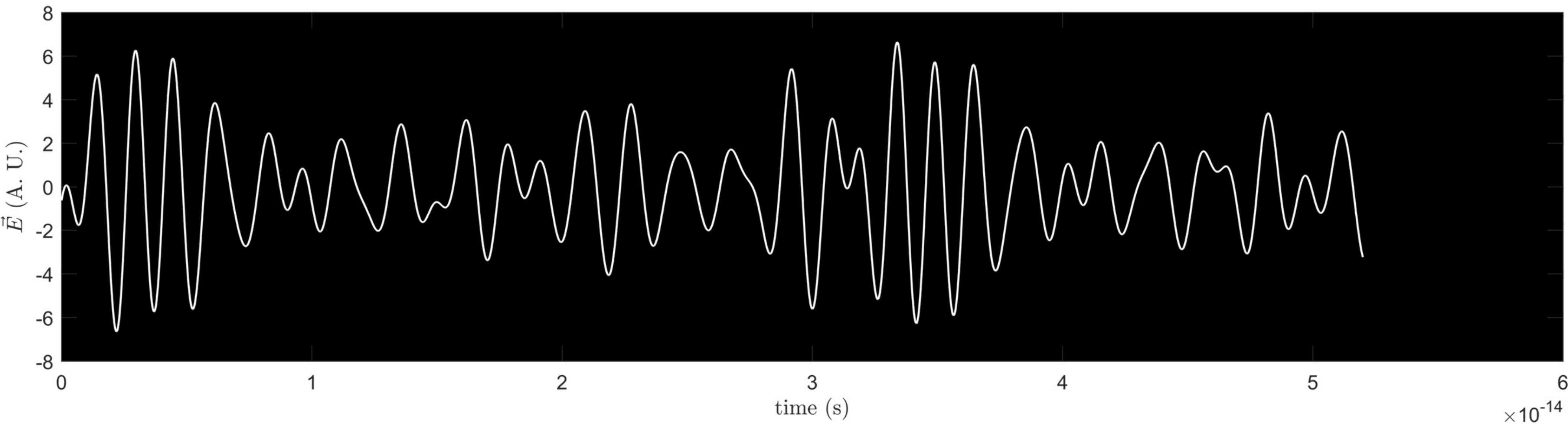
Helium-Neon laser: 632.8 nm with +/- 0.000001 nm. ppb level of precision Monochromatic source.





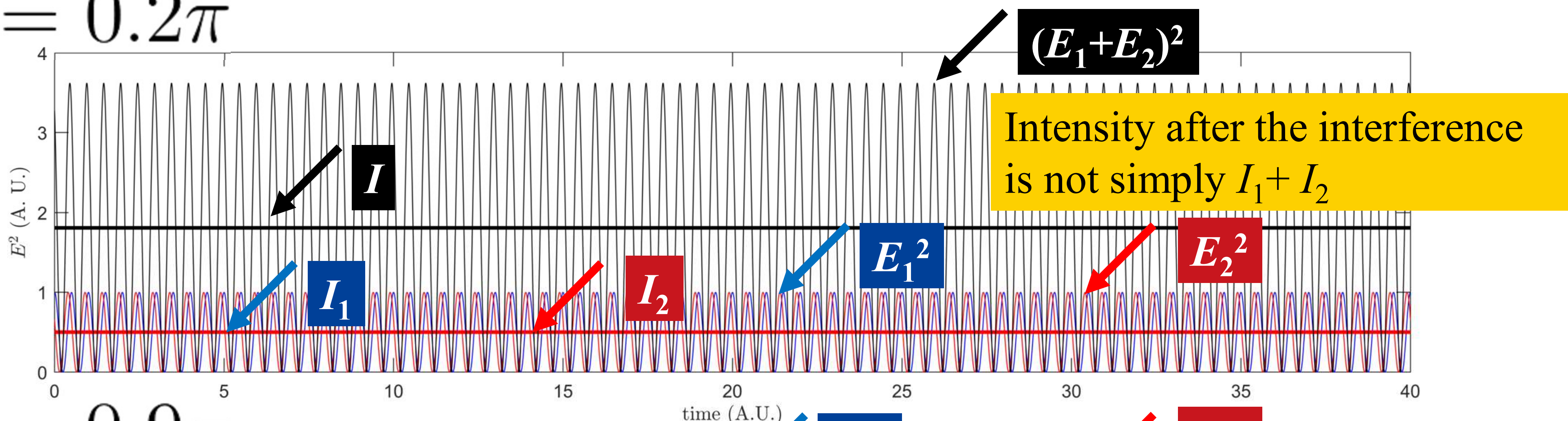
Monochromatic vs. white light

Composed of 12 different colors with wavelength 749 nm to 398 nm, each has the same intensity (1 in the arbitrary unit plot) and a random initial phase.

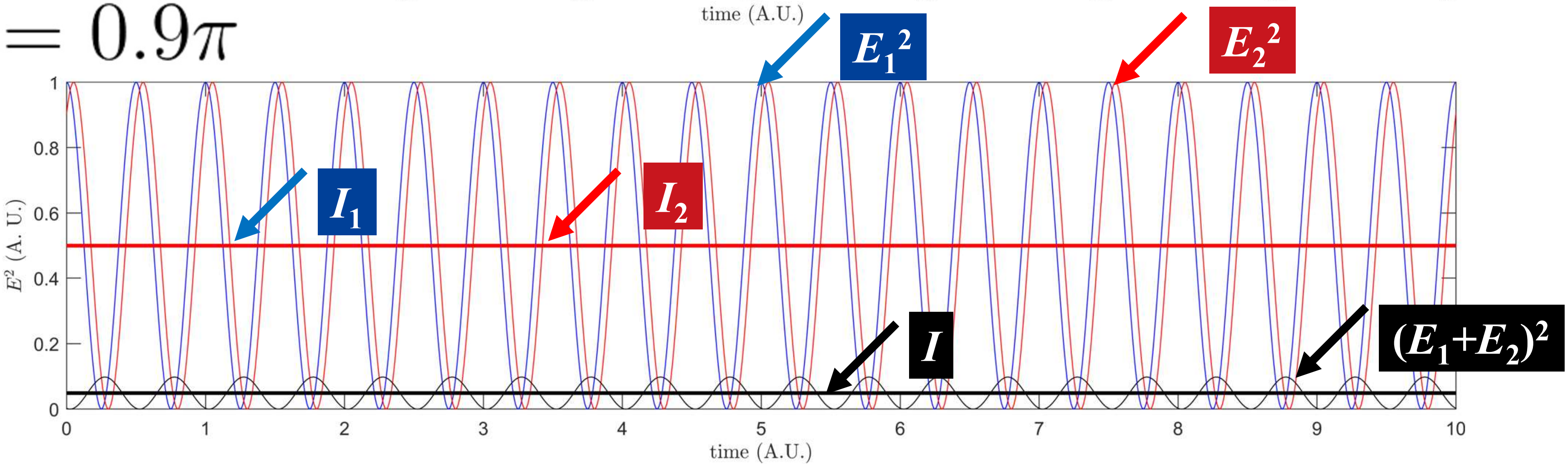


Coherent: constructive and destructive interference

$$\delta\phi = 0.2\pi$$

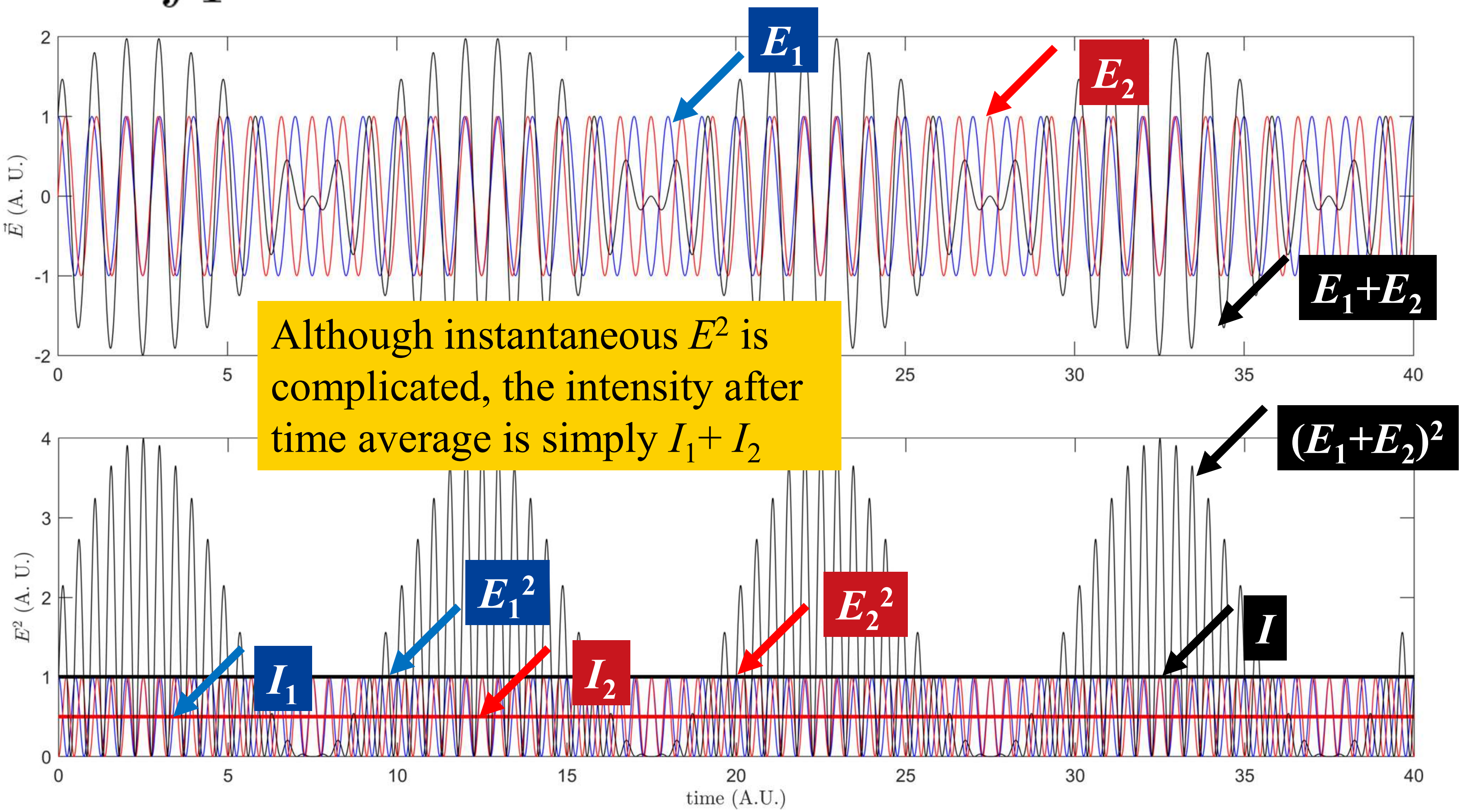


$$\delta\phi = 0.9\pi$$

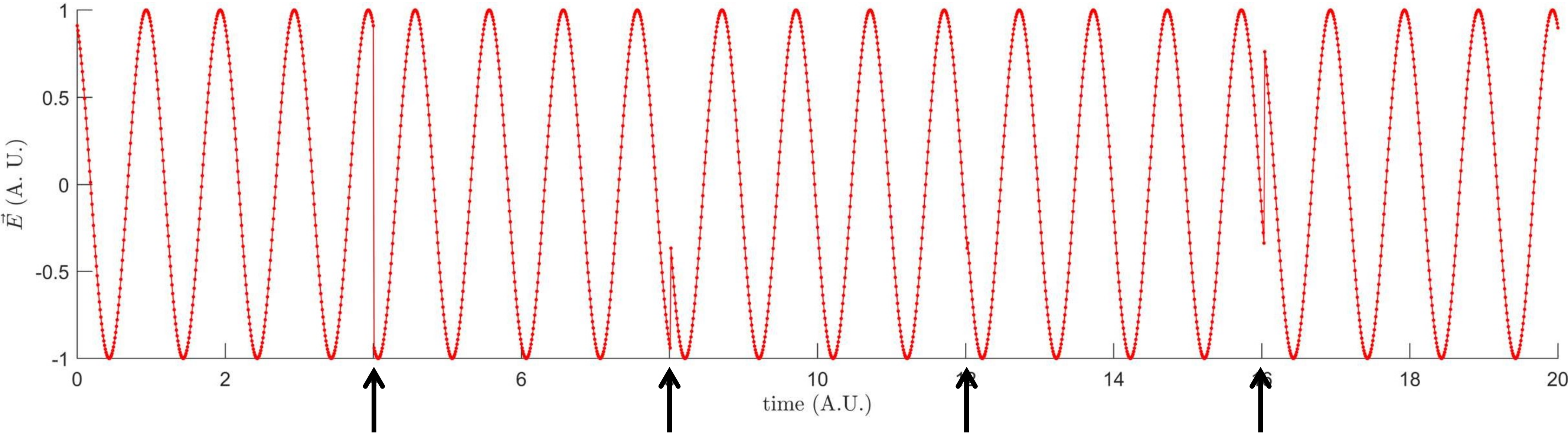


What is NOT coherent: 1. different frequency

$$f_2 = 1.1 f_1$$



What is NOT coherent: 2. discontinuous phase



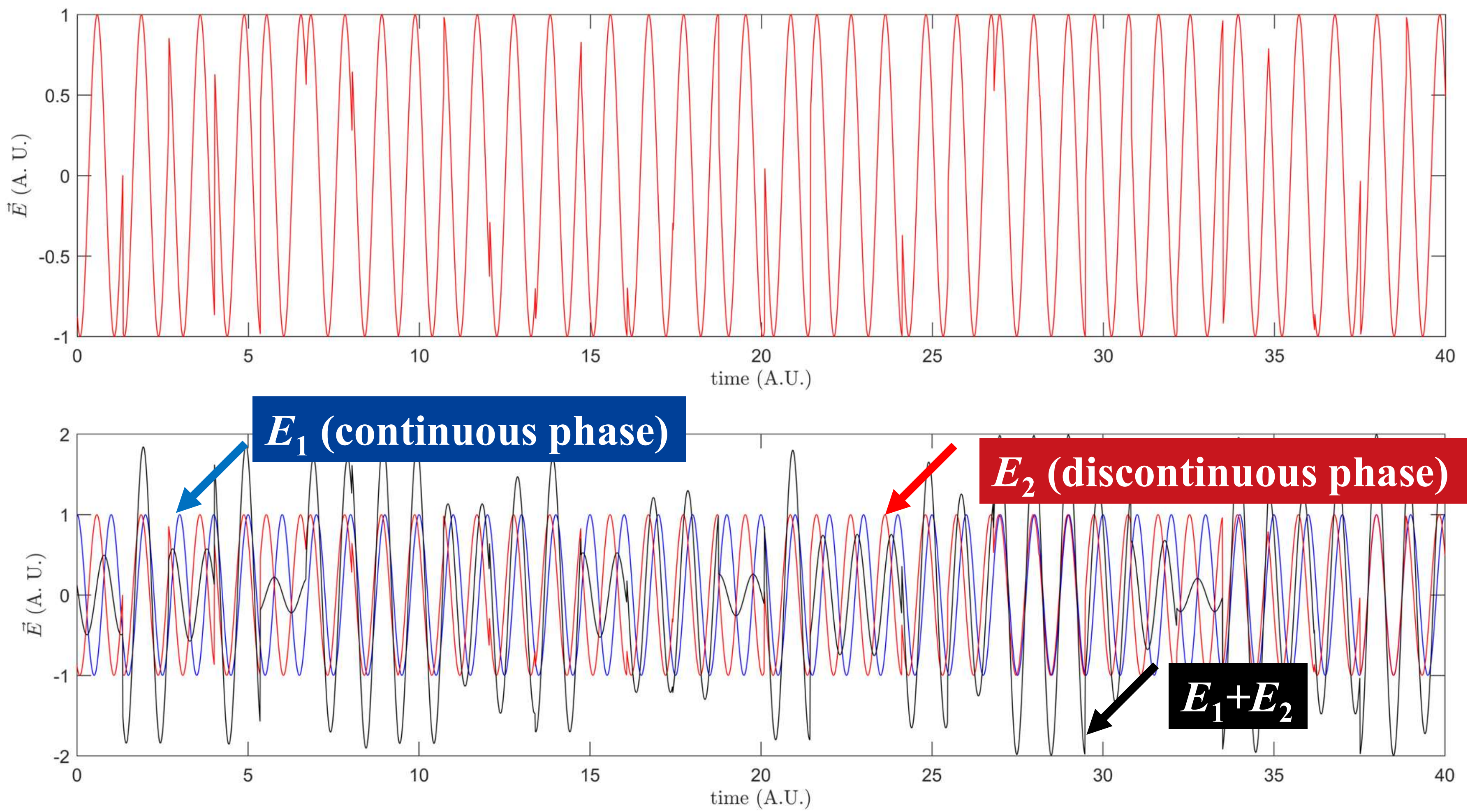
A source with discontinuous phase. (which is general for normal light sources, with coherent time of < 10 ns, due to the interrupt of other atoms.)

10 ns is a "high-end" number for a very thin gas light source.

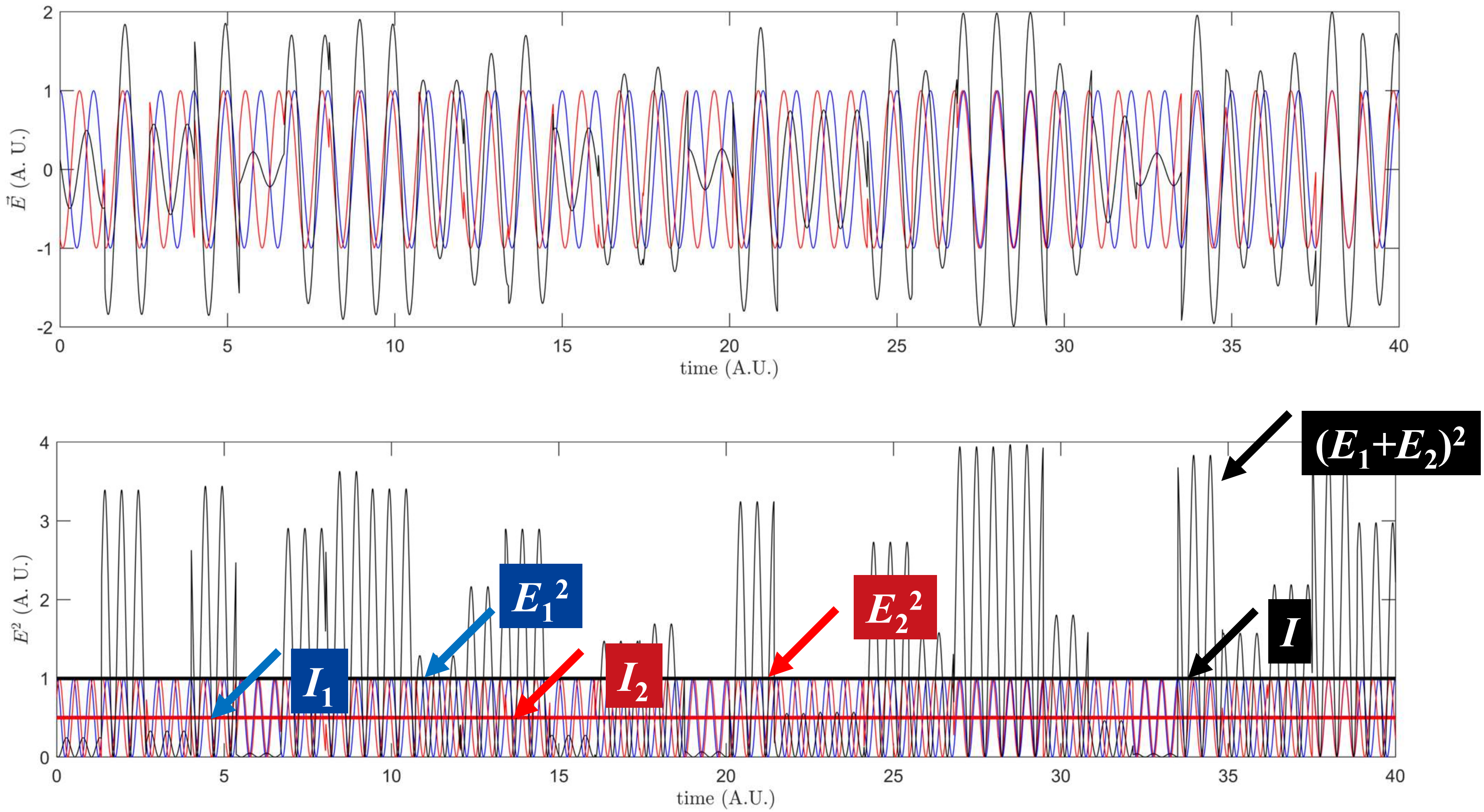
Coherent length: C*10 ns ~ 3 m

Sun and light bulb: coherent length of a few µms.

What is NOT coherent: 2. varying phase



What is NOT coherent: 2. varying phase



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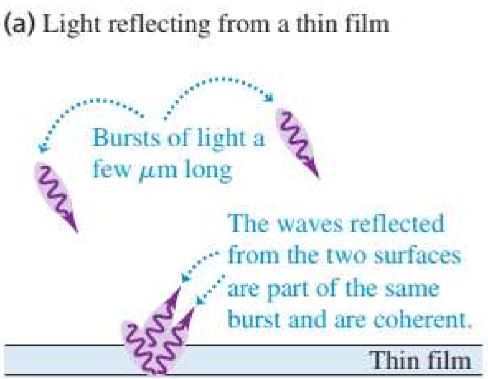
Colorful pattern found in thin transparent films

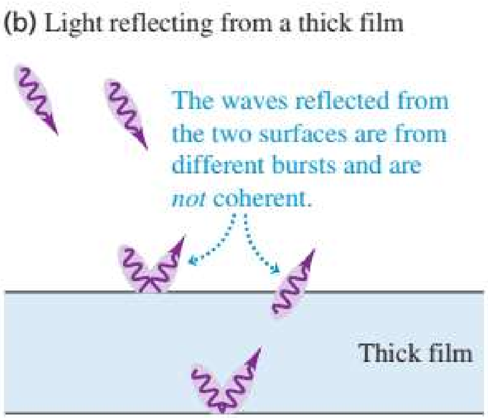
Oil film on water

Soap bubble film

1. Interference happens with two light sources: front and back reflection

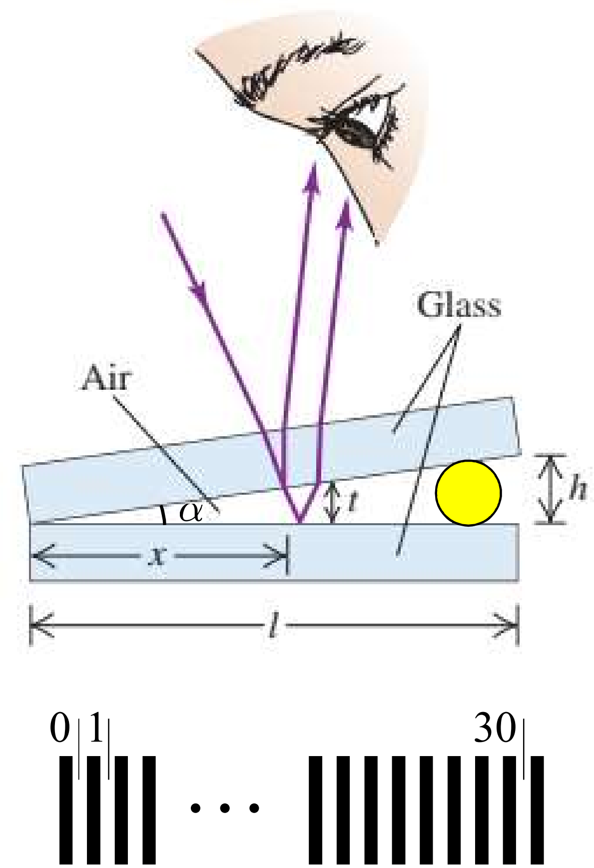


2. This type of interference happens only with thin enough film. Reflections from front and back side of a thick layer (like a piece of glass) does not satisfy the coherent condition.



Thin film interference

A metal wire of diameter h is placed between two glass plates. A sodium light (589.3 nm in air) is used to check the diameter of the wire. With almost normal incidence condition, it is found that distance between 30 bright stripes are 4.295 mm. l=28.88 mm. How much is h?



The phase difference between two reflected beam: $2\pi\frac{2t}{\lambda}$

From one bright stripe to the next, the difference of thickness $\frac{\lambda}{2}$

Total thickness difference for 30 stripes

$$\Delta t = 30 \times \frac{\lambda}{2} = 8839.5\ \mathrm{nm}$$

Tilting angle

$$\alpha = \frac{\Delta t}{4.295\ \mathrm{mm}} = 0.002058$$

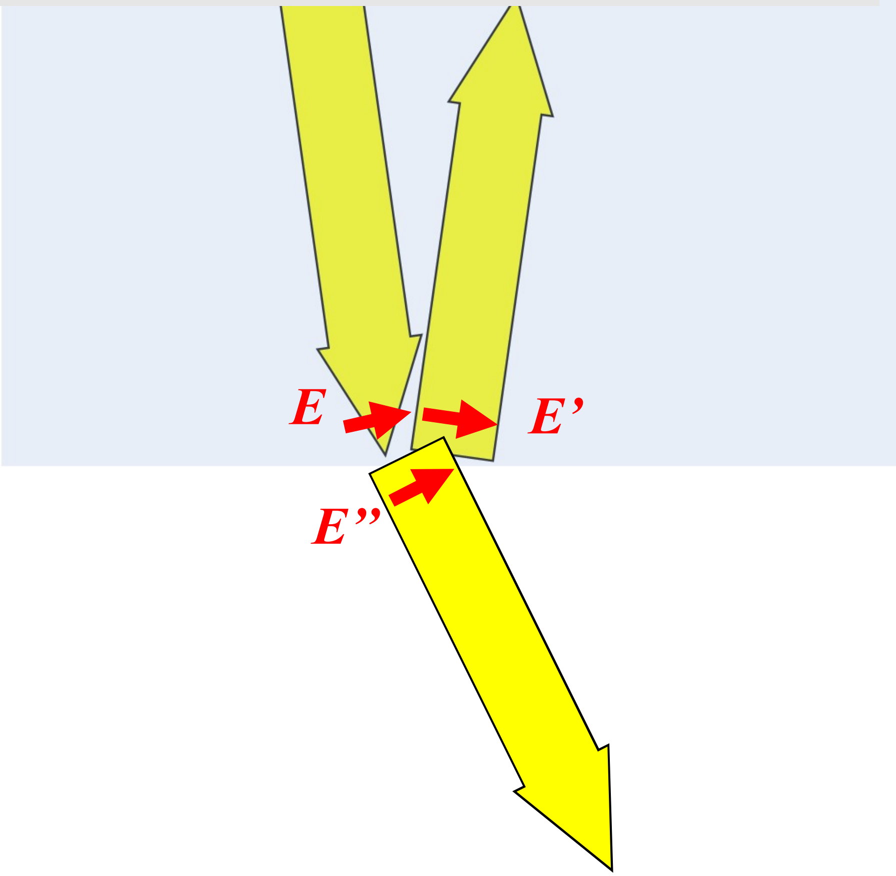
The wire thickness

$$h = l\alpha = 59.4\ \mu\mathrm{m}$$

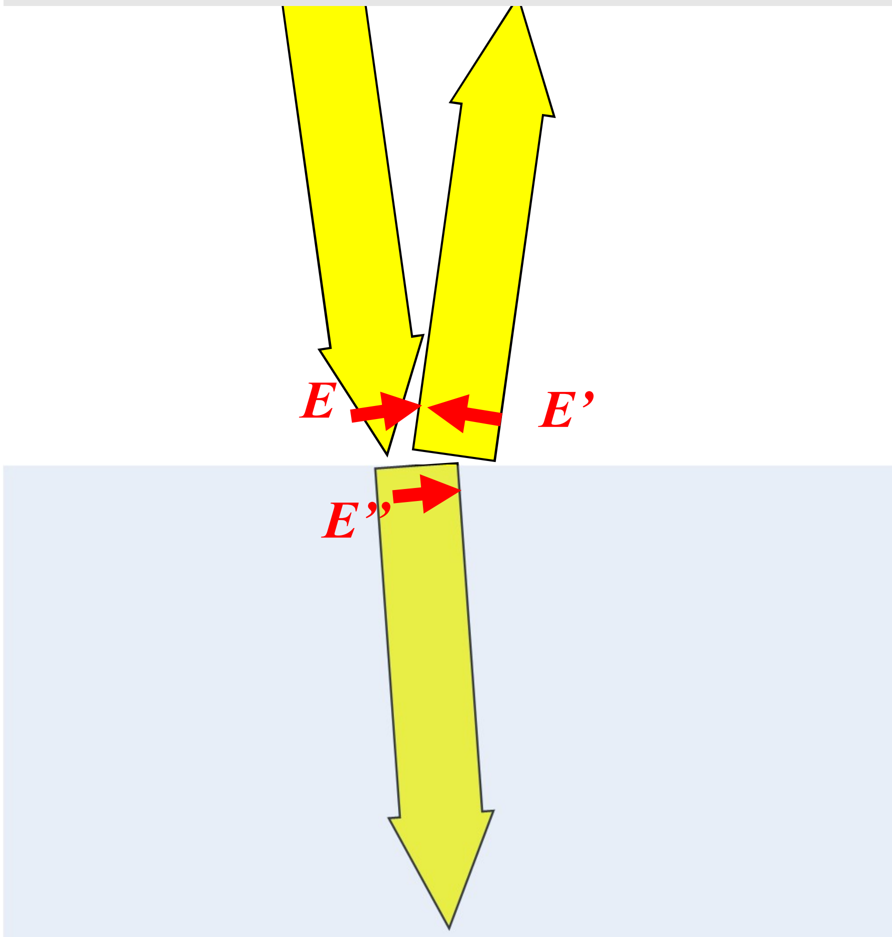
Interference can be used to measure very small distance, with the art of counting numbers.

π phase shift for reflection

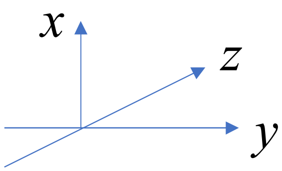
From water (denser medium) to air (less dense medium), phase of electric field is continuous for the incident and reflected beam.



From air (less dense medium) to water (denser medium), phase of electric field has π shift for the incident and reflected beam.



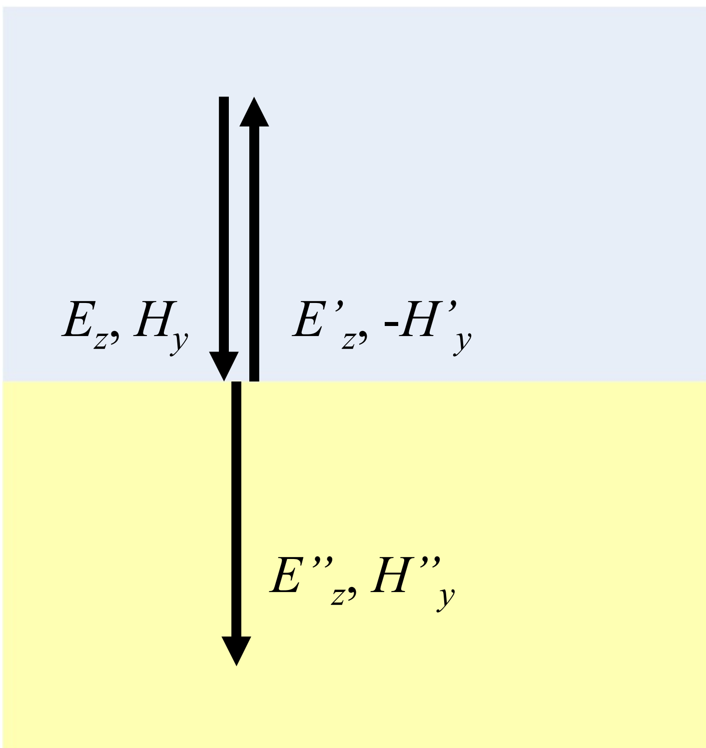
For normal incidence, proof of π phase shift



Boundary condition for electric and magnetic field. Note that the positive direction of H is reversed for reflected beam since the propagation direction is reversed.

$$E_z \cos\omega t + E'_z \cos\omega t = E''_z \cos\omega t$$

$$H_y \cos\omega t - H'_y \cos\omega t = H''_y \cos\omega t$$



Change H equations using the relation between E and H

$$E_z + E'_z = E''_z$$

$$\frac{n_1 E_z}{\mu_1 c} - \frac{n_1 E'_z}{\mu_1 c} = \frac{n_2 E''_z}{\mu_2 c}$$

since $\mu_1 \approx \mu_2$

finally

$$E'_z = E_z \frac{n_1 - n_2}{n_1 + n_2}$$

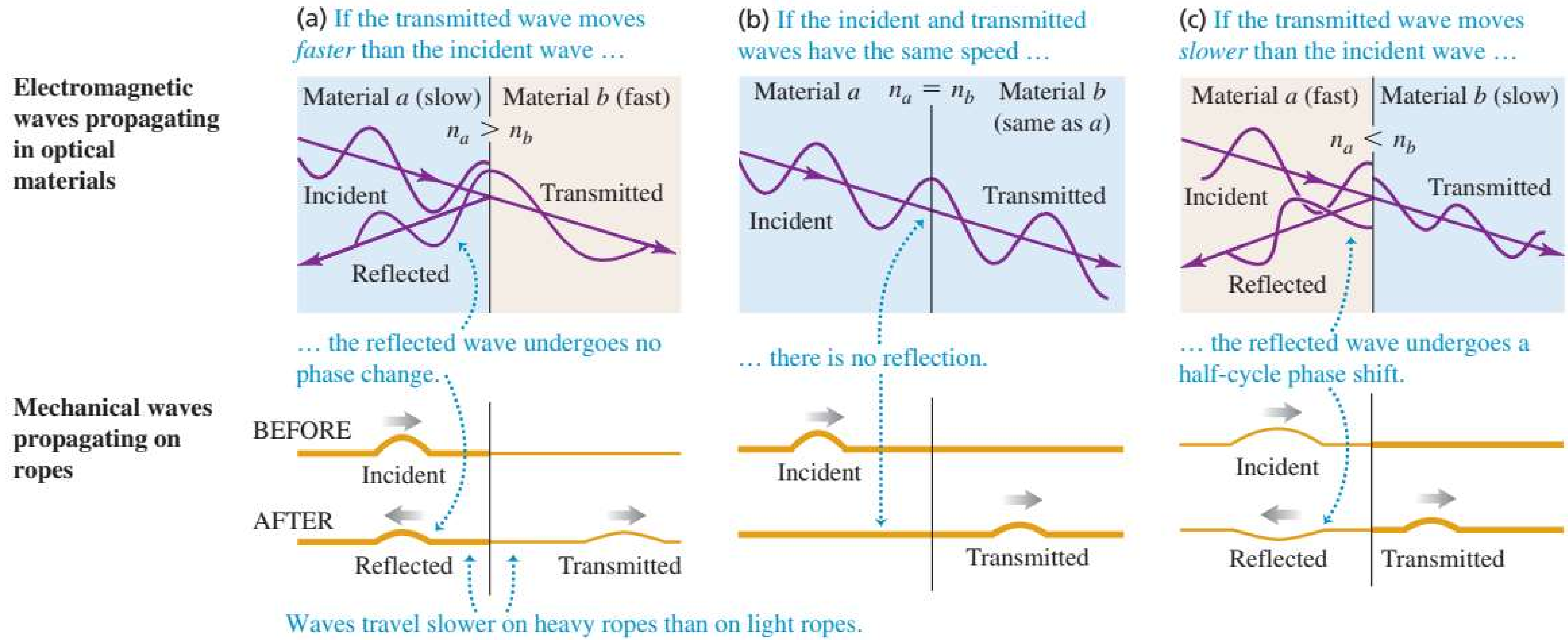
$n_1 > n_2$, water to air, E' in phase with E

$n_1 < n_2$, air to water, E' has π phase shift

Analogy in mechanical wave

Similar effects found in mechanical wave.

35.13 Upper figures: electromagnetic waves striking an interface between optical materials at normal incidence (shown as a small angle for clarity). Lower figures: mechanical wave pulses on ropes.



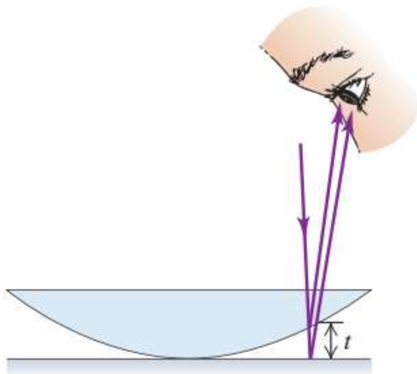
You can watch the video here:

https://www.zhihu.com/question/267529217

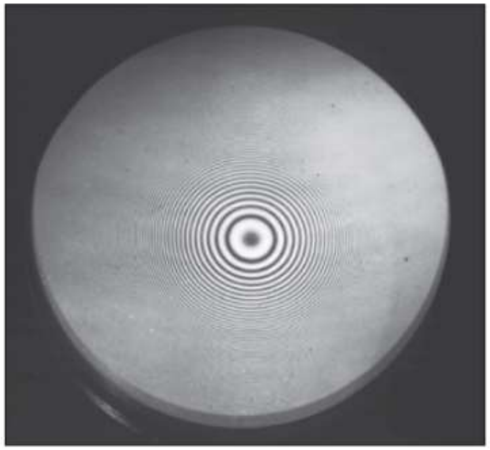
Applications of thin film interference: Newton's rings and testing of lens

Change the tilted flat surface into a convex lens, stripes changed into rings.

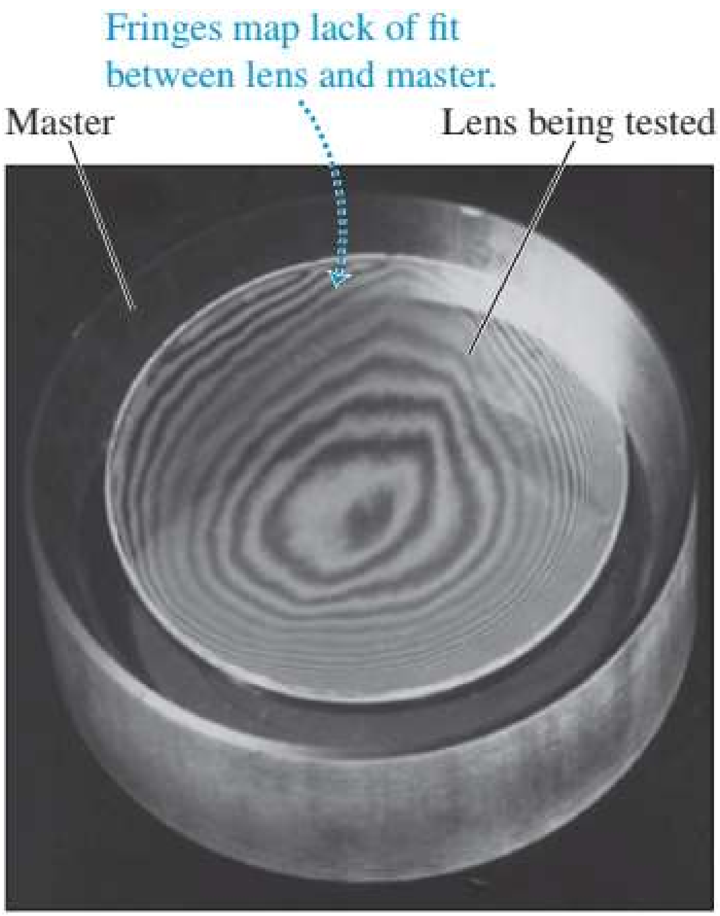
(a) A convex lens in contact with a glass plane



(b) Newton's rings: circular interference fringes

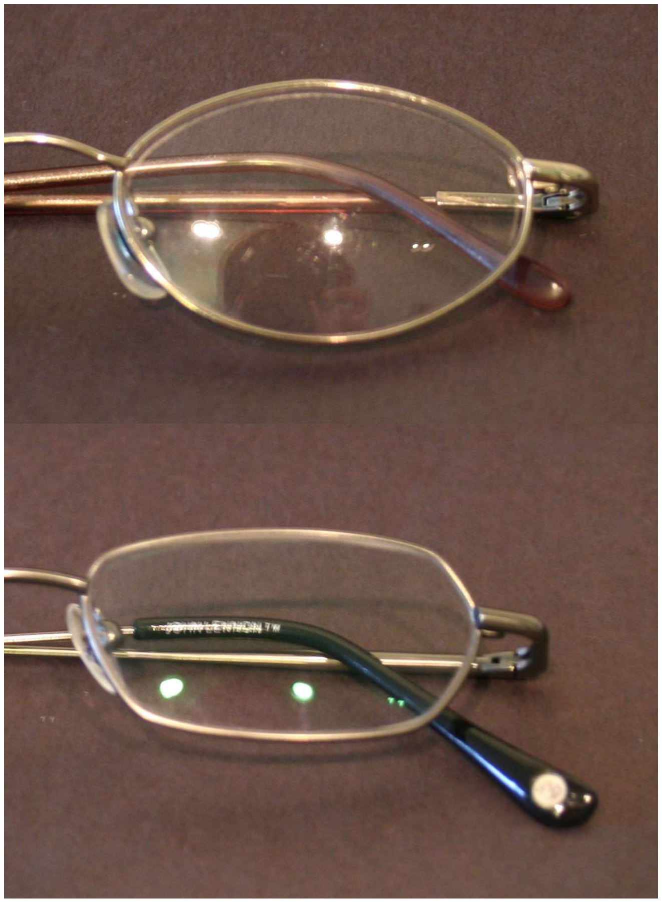


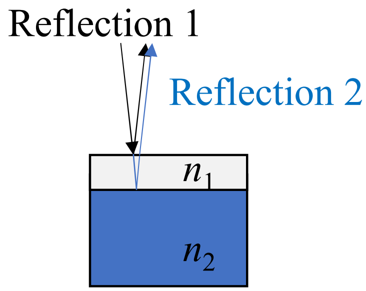
To check if a lens is well made (well made lens should have the same curve as the master), one can count the number of rings.



Anti-reflective coatings and reflective coatings

A thin layer of material coating can be applied to increase the transparency of a piece of glass. For visible lights centered around green light (550 nm), what is the thickness of the anti-reflective coating?





Reflection 1 has π phase lag

If $n_1<n_2$, reflection 2 also has π phase lag, and phase advance of

$$\frac{2n_1 t}{\lambda}2\pi$$

If $n_1>n_2$, reflection 2 has only phase advance of

$$\frac{2n_1 t}{\lambda}2\pi$$

Anti-reflection coating requires the two reflected beam destructively interference.

If $n_1<n_2$

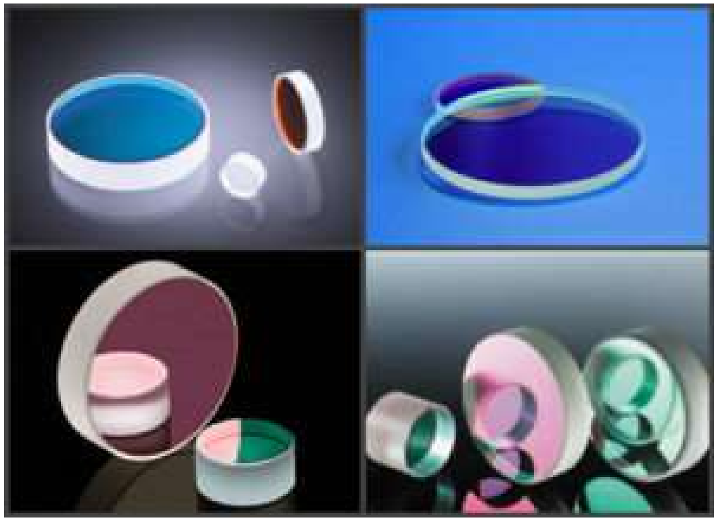
$$t=\frac{(2k+1)\lambda}{4n_1}$$

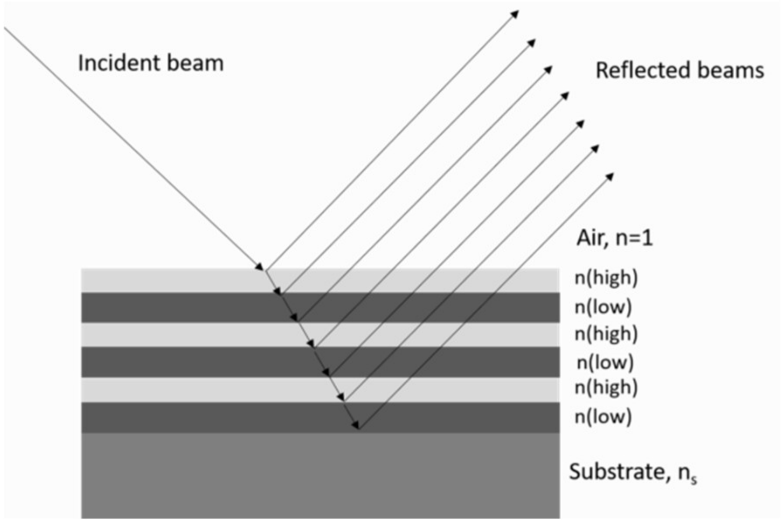
If $n_1>n_2$

$$t=\frac{k\lambda}{2n_1}$$

https://en.wikipedia.org/wiki/Anti-reflective_coating

Similarly, coatings can be used to enhance reflection. Multilayer reflection mirrors are made by this principle.





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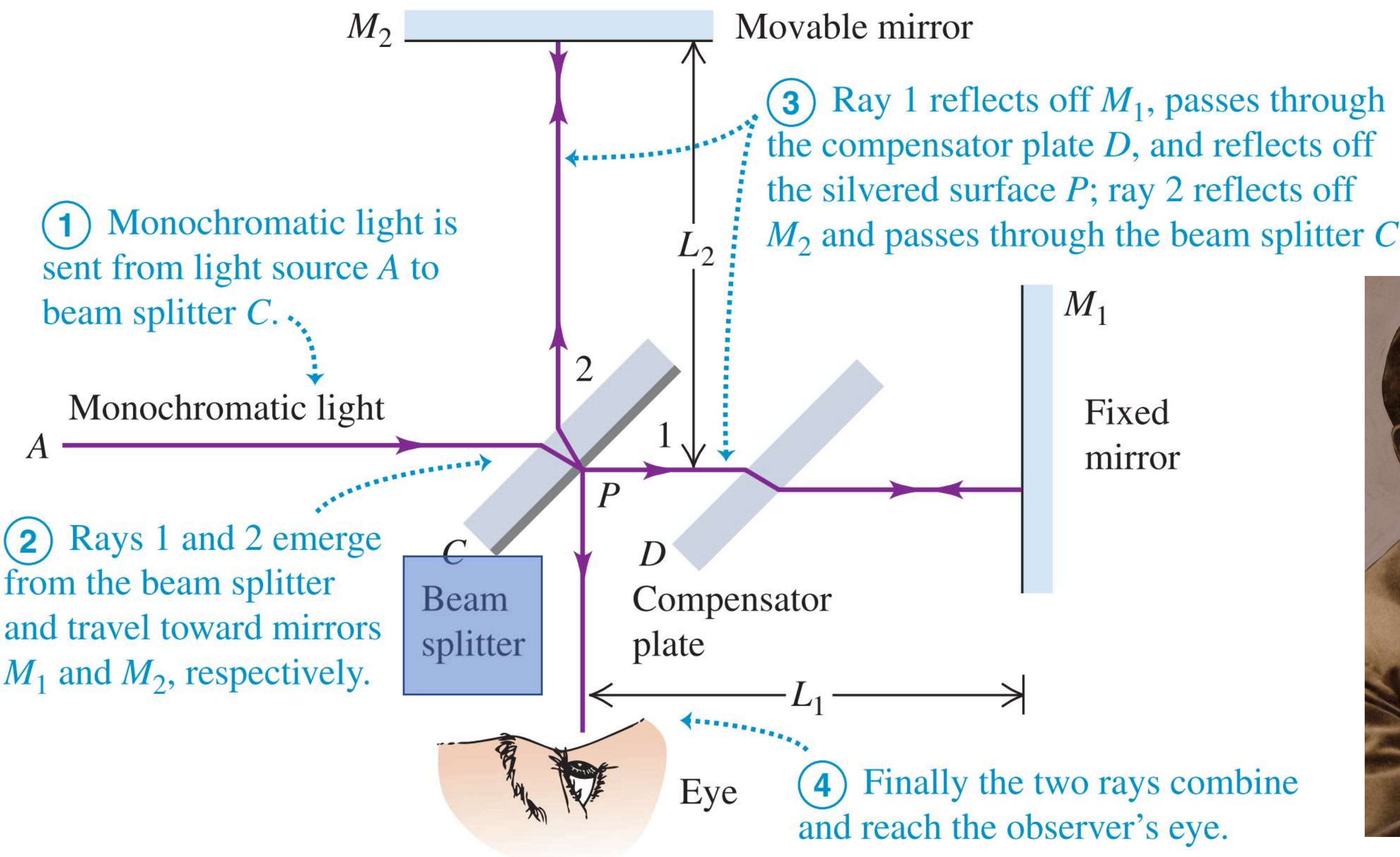
Michelson interferometer



Michelson interferometer: detailed pattern