Physics (PHYS2500J), Unit 1 Electrostatics: 6. Steady current

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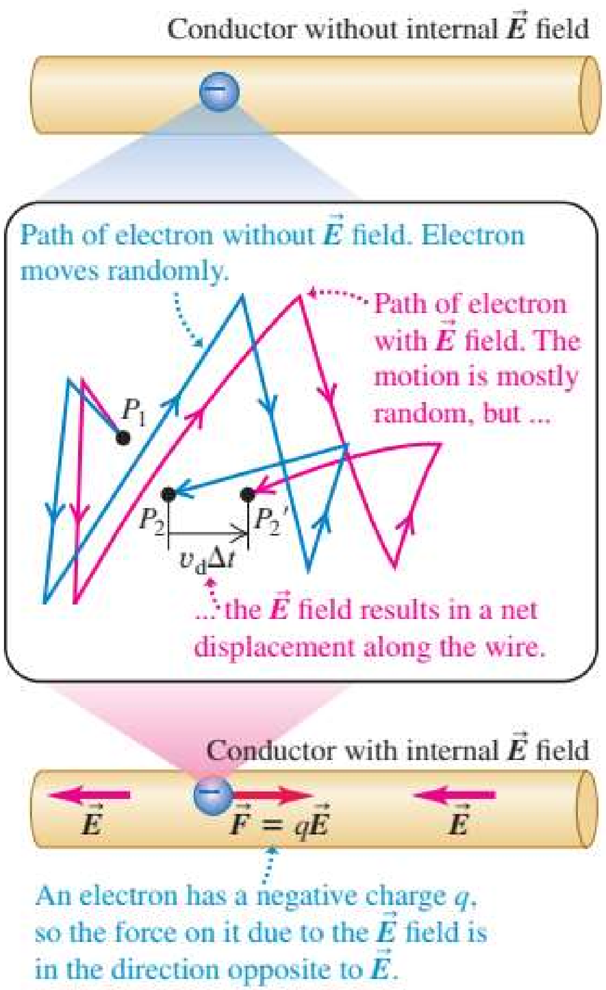
Fall 2023

Contents

Motion of electrons



Velocity of electrons

Thermal motion:

$$v_{rms} = \sqrt{\frac{3k_B T}{m_e}} = 1.17 \times 10^5 \text{ m} \cdot \text{s}^{-1}$$

$T = 300$ K

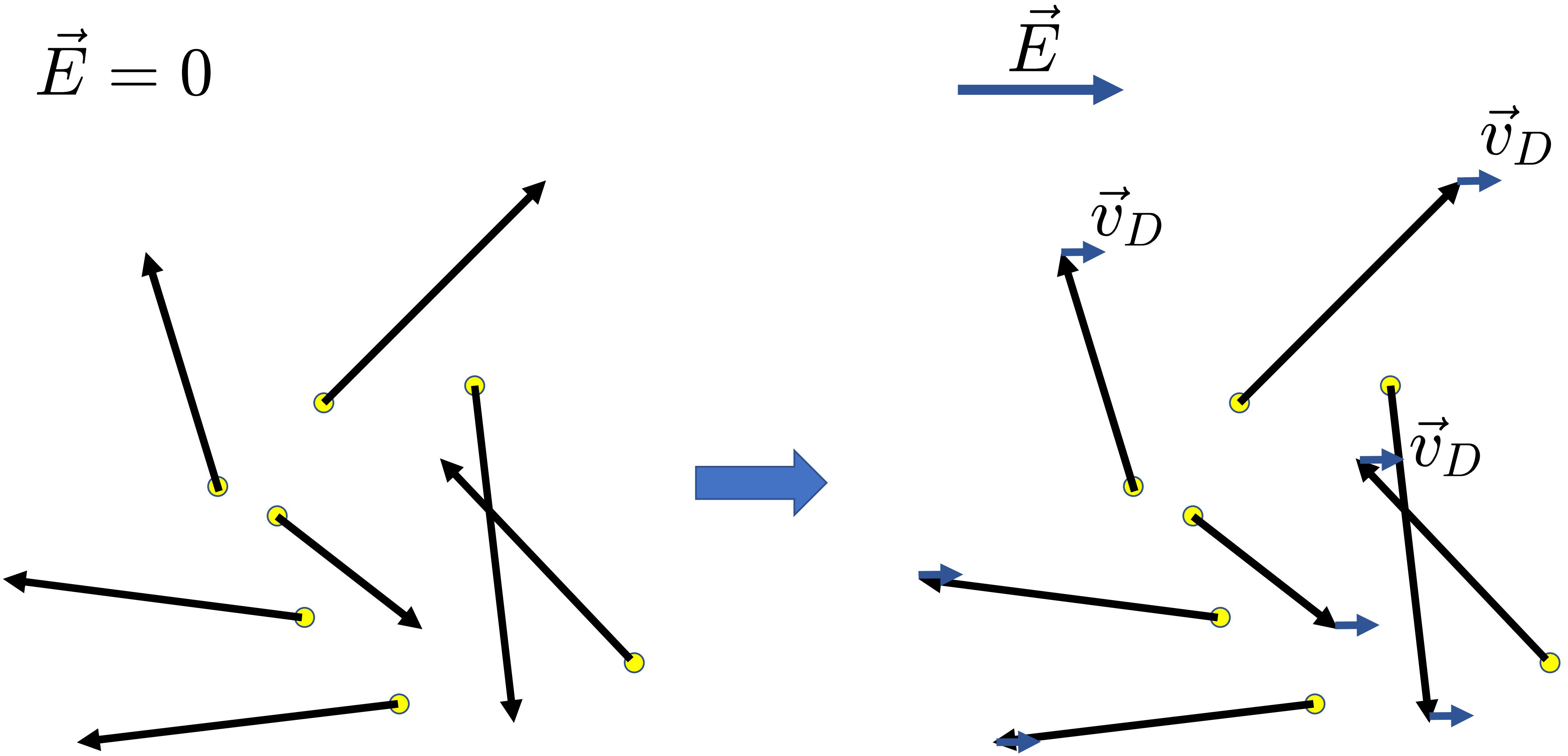
Fermi velocity of typical metal is in the order of 10^6 m/s

If an Electric field is applied: Drift motion

What is the velocity of drift motion?

Keep this a question to be answered.

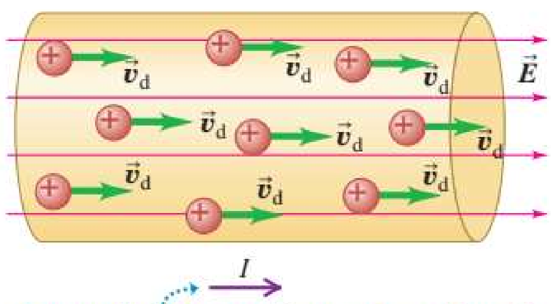
A more proper picture



The drift velocity is much smaller than the random velocity of electrons.

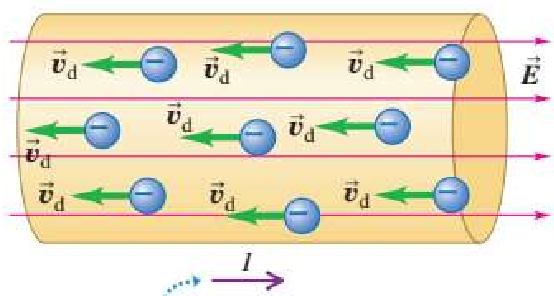
Current and electric field

(a)



A **conventional current** is treated as a flow of positive charges, regardless of whether the free charges in the conductor are positive, negative, or both.

(b)



In a metallic conductor, the moving charges are electrons — but the *current* still points in the direction positive charges would flow.

The random velocity, though large, is not pictured for simplicity.

Although the conducting **Charge Carriers** have negative charge, the current is always in the same direction with applied ***E*** field

Definition of current

The net charge passes through an area in unit time is the current through the area.

$$I = \frac{\mathrm{d}Q}{\mathrm{d}t}$$

In SI unit systems, the unit for current A is a base unit.

Ohm's law

Most common drift motion of charged particles are driven by electric field / voltage.

Ohm experimentally found that

$$V = IR$$

R is a property of **an object** relies on

material

Cross section

length

Ohm's law

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$$V = IR \qquad R = \frac{\rho l}{A}$$

Length of the path (series connection)

Cross section of the path (parallel connection)

TABLE 25.1 Resistivities at Room Temperature (20°C)

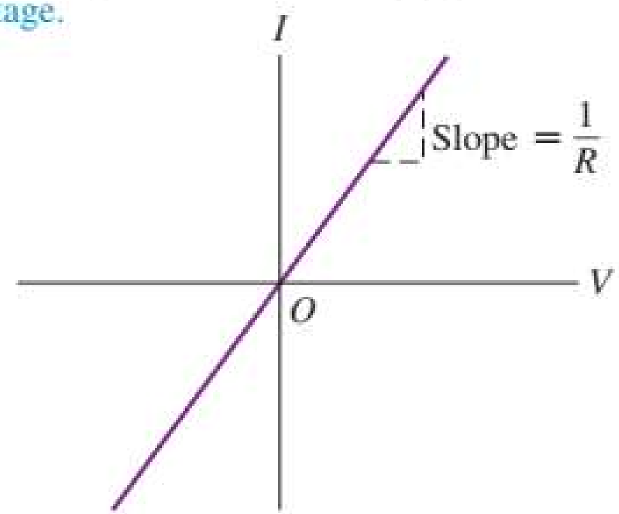
	Substance	$\rho\ (\Omega \cdot m)$	Substance	$\rho\ (\Omega \cdot m)$
Conductors			**Semiconductors**	
Metals	Silver	1.47×10^{-8}	Pure carbon (graphite)	3.5×10^{-5}
	Copper	1.72×10^{-8}	Pure germanium	0.60
	Gold	2.44×10^{-8}	Pure silicon	2300
	Aluminum	2.75×10^{-8}	**Insulators**	
	Tungsten	5.25×10^{-8}	Amber	5×10^{14}
	Steel	20×10^{-8}	Glass	10^{10}–10^{14}
	Lead	22×10^{-8}	Lucite	$>10^{13}$
	Mercury	95×10^{-8}	Mica	10^{11}–10^{15}
Alloys	Manganin (Cu 84%, Mn 12%, Ni 4%)	44×10^{-8}	Quartz (fused)	75×10^{16}
	Constantan (Cu 60%, Ni 40%)	49×10^{-8}	Sulfur	10^{15}
	Nichrome	100×10^{-8}	Teflon	$>10^{13}$
			Wood	10^{8}–10^{11}

It is not totally proper to call it a "law". It is the property of most materials.

Linearity is convenient but not the whole picture

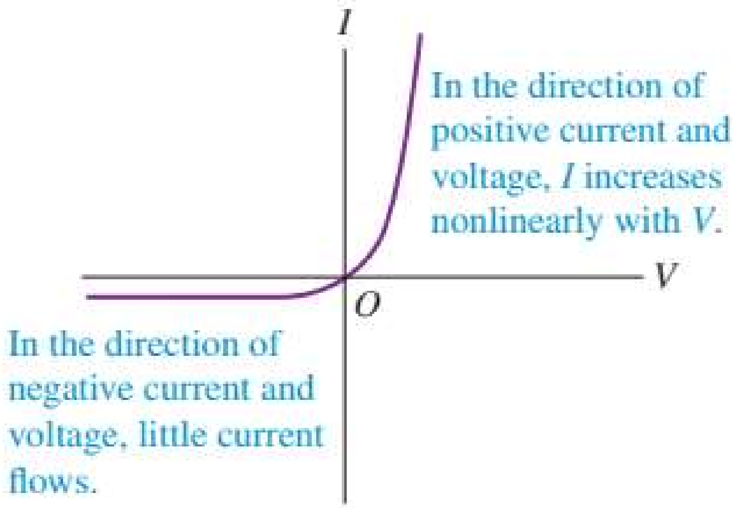
(a)

Ohmic resistor (e.g., typical metal wire): At a given temperature, current is proportional to voltage.



Some material does not follow the "law" of Ohm's

Semiconductor diode: a nonohmic resistor



Good electric conductors are also (generally) good thermal conductors

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Two mechanisms for heat conduction in solid material:

Electrons and phonons

diamond: 3320 W/(mk)

TABLE 25.1 Resistivities at Room Temperature (20°C)

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			Wood	10^{8}–10^{11}

TABLE 17.5 Thermal Conductivities

Substance	k (W/m · K)
Metals	
Aluminum	205.0
Brass	109.0
Copper	385.0
Lead	34.7
Mercury	8.3
Silver	406.0
Steel	50.2
Solids (representative values)	
Brick, insulating	0.15
Brick, red	0.6
Concrete	0.8
Cork	0.04
Felt	0.04
Fiberglass	0.04
Glass	0.8
Ice	1.6
Rock wool	0.04
Styrofoam	0.027
Wood	0.12–0.04
Gases	
Air	0.024
Argon	0.016
Helium	0.14
Hydrogen	0.14
Oxygen	0.023

The unit of resistivity ρ

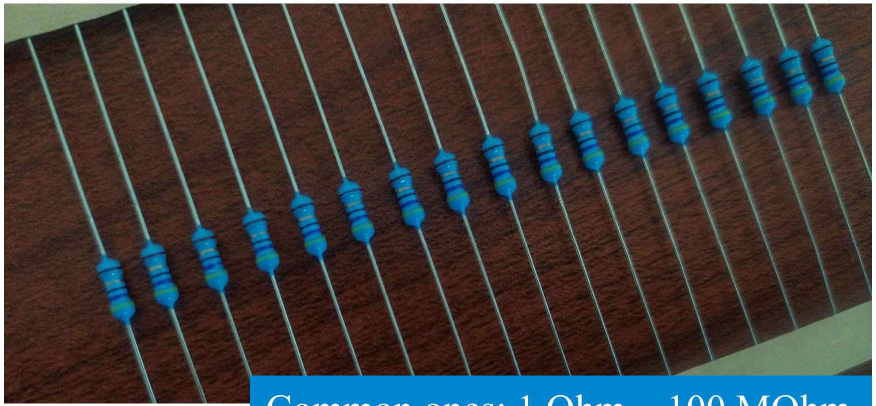
$$R = \frac{\rho l}{A}$$

$$\Omega = \frac{[\dim\ \rho]\cdot \mathrm{m}}{\mathrm{m}^2}$$

$$\boxed{\dim\ \rho = \Omega \cdot m}$$

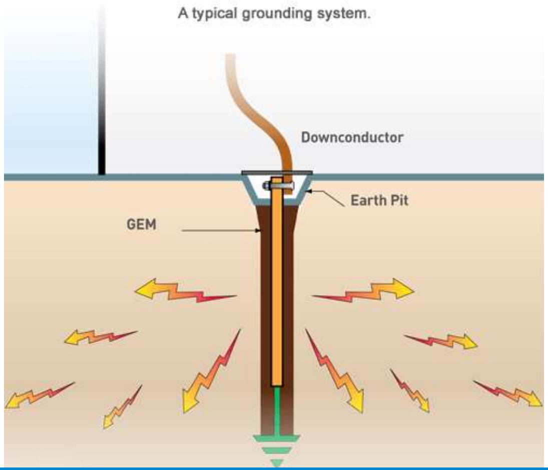
What is the resistance of the following objects

~10 mOhm



Common ones: 1 Ohm ~ 100 MOhm

100 kOhm ~ 2 Mohm, depending on measurement condition (sweaty, wound, dry skin,…)



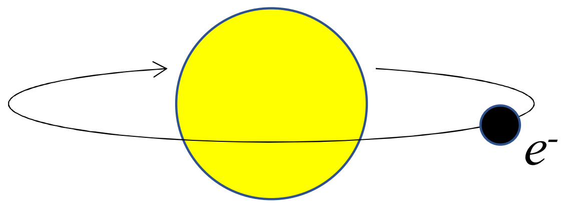
5 – 10 Ohm is considered a good grounding connection.

Contents

Contents

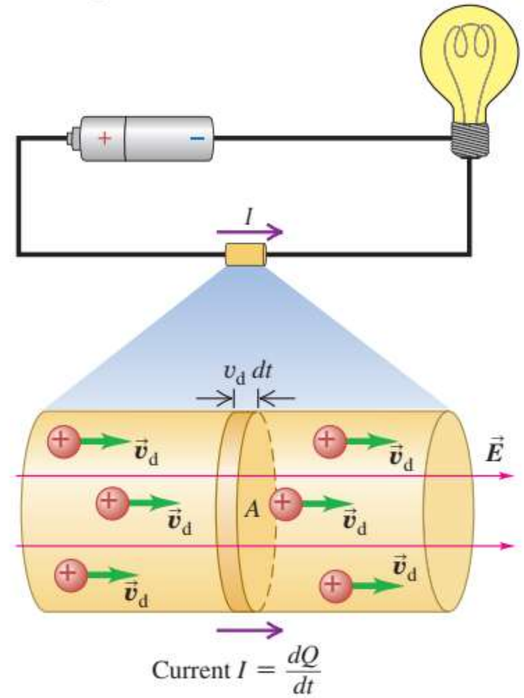
Current in two examples

What are the currents in the following two case?



Angular speed ω

Hint: The net charge passes through an area in unit time is the current through the area.



$$I = \frac{Q}{t} = \frac{-e\omega t/2\pi}{t} = \frac{-e\omega}{2\pi}$$

Volume passed during dt

$$\mathrm{d}q = \boxed{Av\mathrm{d}t}Nq$$

$$I = \frac{\mathrm{d}q}{\mathrm{d}t} = ANqv$$

Current denstity

$$I = \frac{\mathrm{d}q}{\mathrm{d}t} = ANqv$$

Relating a microscopic picture with the macroscopic quantity.

Area gives a hint:
Current I is a flux.

$$I = \int \vec{J} \cdot \mathrm{d}\vec{S}$$

Current is the flux of a vector field J, current density.

Current density field counts for a volume larger than microscopic scale, so many charge carriers are involved; but smaller than macroscopic scale, so that the charge carriers behavior in the region is considered homogeneous.

The unit of current density is $\mathrm{A/m^2}$

Comparing the two above

$$\vec{J} = Nq\vec{v}$$

Multiple types of charge carriers (or multiple velocity)

$$\vec{J} = \sum_i N_i q_i \vec{v}_i$$

$$U = IR$$

$$EL = I\frac{\rho L}{S}$$

$$E = \rho\frac{I}{S} = \rho J$$

Microscopic form of Ohm’s law

$$\vec{E} = \rho\vec{J}$$

Or, $$\vec{J} = \sigma\vec{E}$$

Electric conductivity $$\sigma = \frac{1}{\rho}$$

The resistivity of copper

$$1.8 \times 10^{-8}\ \Omega \cdot \mathrm{m} \;/\; 1.8 \times 10^{-6}\ \Omega \cdot \mathrm{cm}$$

Scatter center in materials

Perfect metal has no resistance.

This is a conclusion of quantum mechanics, considering electrons as wave, the propagation of the wave is not affected by a perfect lattice.

Defects, impurities, and thermal vibrations of lattice are “scatter centers” which are responsible for the resistance.

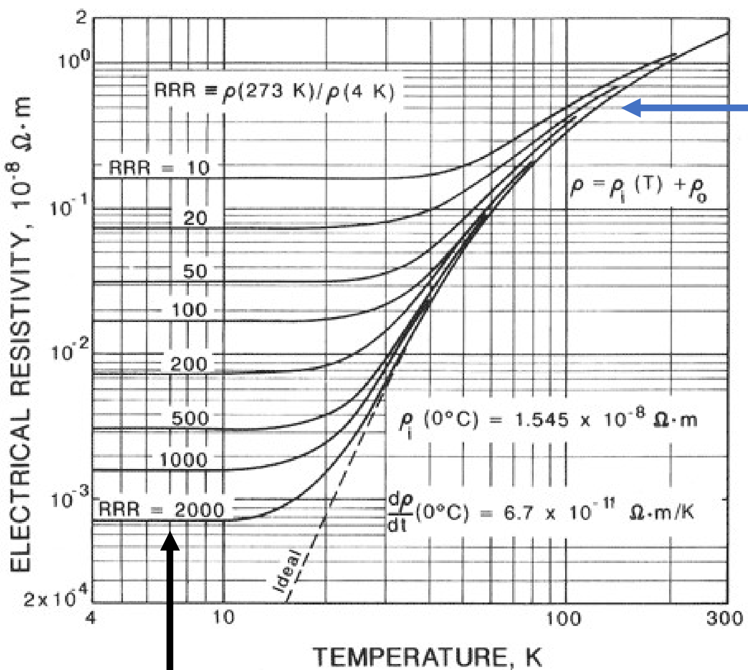
The resistivity of metal increases with temperature

$$\rho(T) = (1 + \alpha\frac{T}{T_0})\rho(T_0)$$

TABLE 25.2 Temperature Coefficients of Resistivity (Approximate Values Near Room Temperature)

Material	α [(°C)$^{-1}$]	Material	α [(°C)$^{-1}$]
Aluminum	0.0039	Lead	0.0043
Brass	0.0020	Manganin	0.00000
Carbon (graphite)	−0.0005	Mercury	0.00088
Constantan	0.00001	Nichrome	0.0004
Copper	0.00393	Silver	0.0038
Iron	0.0050	Tungsten	0.0045

Residual resistivity ratio (RRR, triple R value)



Good copper and not so good ones do not differ much at higher temperatures since the major contribution to resistance comes from thermal vibrations of the lattice.

High RRR valued copper are expensive for:

1. High purity (low density of impurity scatter centers)
2. Annealing heat treatments (low density of defects)

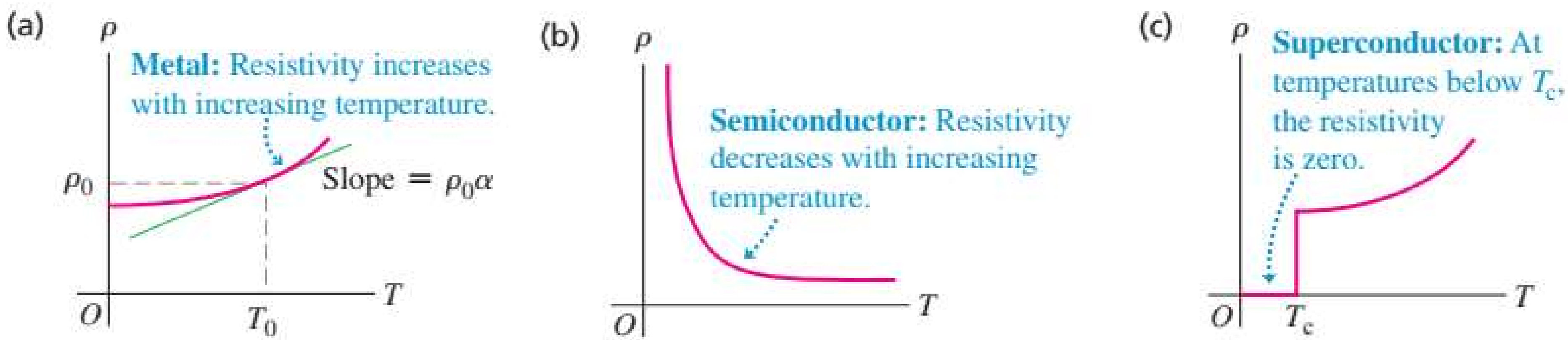
Cryogenic resistivity of materials

Three guesses for resistivity near 0 K before He was liquified.

1. Goes down to 0, following the trend;
2. Stop decreasing at some value;
3. Will increase since electrons are “frozen”.

25.6 Variation of resistivity ρ with absolute temperature T for (a) a normal metal, (b) a semiconductor, and (c) a superconductor. In (a) the linear approximation to ρ as a function of T is shown as a green line; the approximation agrees exactly at $T = T_0$, where $\rho = \rho_0$.

Experiments supported 2 and 3, with a huge surprise of superconductivity.



Relaxation time of electrons in copper

Assume electrons can fly freely in between two collisions with the lattice, and the time of "free fly" is called relaxation time, τ.

$$a\tau = v_{max}$$

$$a = \frac{qE}{m}$$

$$J = Nqv = Nqv_{max}/2$$

$$J = (Nq\frac{q}{m}\tau/2)E$$

$$J = \frac{1}{\rho}E$$

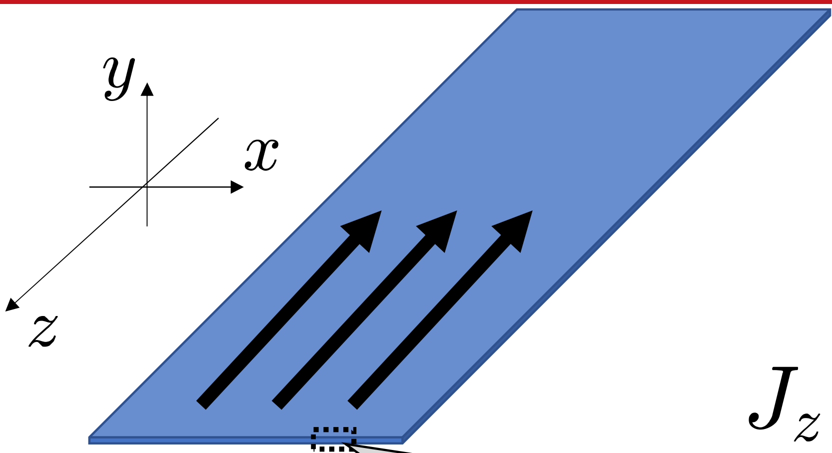
$$\tau = \frac{2m}{Nq^2\rho}$$

The resistivity of copper $1.8 \times 10^{-8}\ \Omega \cdot \mathrm{m}$

τ is about 10^{-14} s, 10 fs

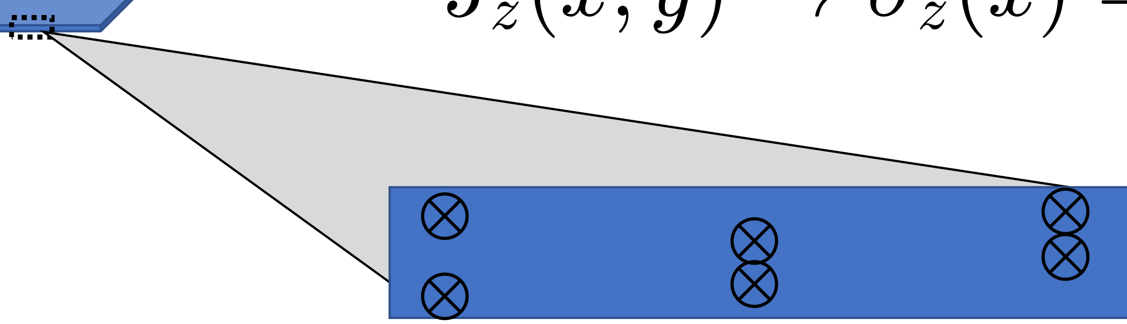
Contents

2D current density (sheet current density)



Consider current as distributed on a surface.
E.g. current in a Cu foil.

$$J_z(x,y) \to \sigma_z(x) = \int \mathrm{d}y J_z(x,y)$$



Valid when the distribution of current in the thickness direction is neglected compared with the distribution in the width direction.

Divergence of current density field

$$\oiint \vec{J} \cdot \mathrm{d}\vec{S} = ?$$

Divergence of current density field

Gauss's theorem

$$\oiint \vec{J} \cdot \mathrm{d}\vec{S} = -\frac{\mathrm{d}Q}{\mathrm{d}t} \quad \Rightarrow \quad \nabla \cdot \vec{J} = -\frac{\mathrm{d}\rho}{\mathrm{d}t}$$

Charge conservation:
The continuity of current condition

Where current distribution is not time dependent (steady current)

$$\oiint \vec{J} \cdot \mathrm{d}\vec{S} = 0 \qquad \nabla \cdot \vec{J} = 0$$

Another time scale of current

Current continuity condition

$$\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t} \longrightarrow \nabla \cdot \vec{J} = 0$$

$$-\frac{\partial \rho_f}{\partial t} = \nabla \cdot \vec{J} = \sigma \nabla \cdot \vec{E} = \frac{\sigma}{\epsilon} \rho_f$$

Free charge, since polarization charge does not obey Ohm's law

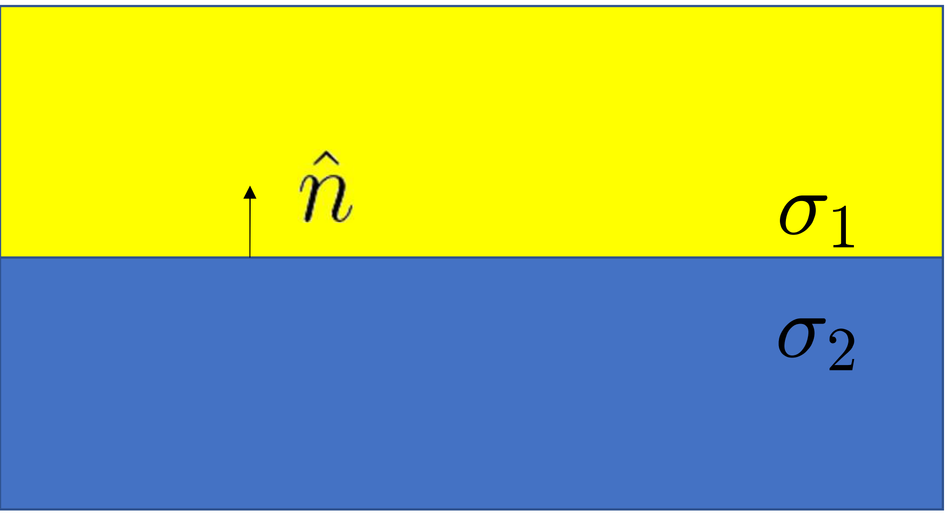
Free charge, so use ε instead of ε_0

$$-\frac{\mathrm{d}\rho}{\rho} = \frac{\sigma \mathrm{d}t}{\epsilon}$$

$$\rho(t) = \rho(0) \exp(-t/\tau), \tau = \epsilon/\sigma$$

Relaxation time (time required for charge to redistribute, in the order of 10^{-19} s

Boundary conditions for current density



$\vec{E}_1$

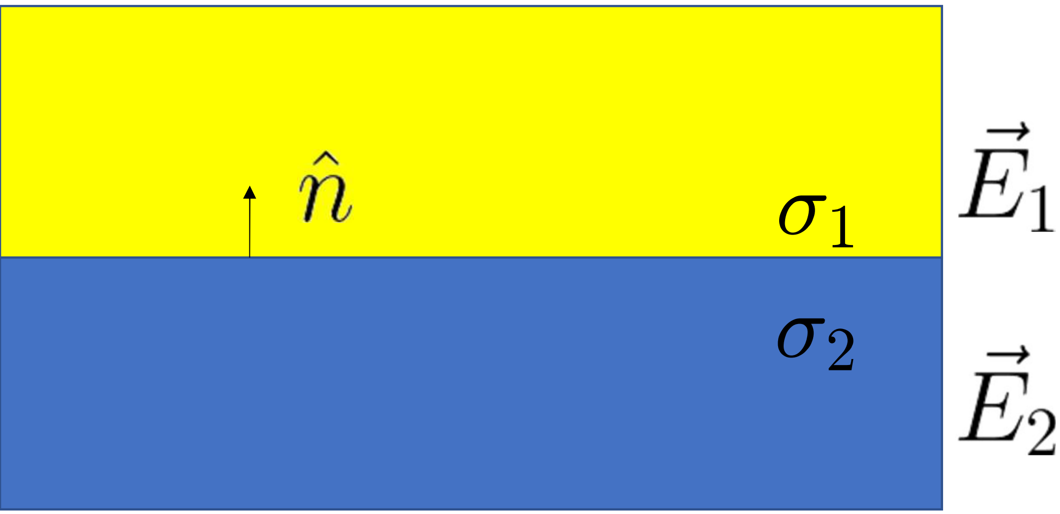
$\vec{E}_2$

Conservative field:

$$\hat{n} \times \vec{E}_2 = \hat{n} \times \vec{E}_1$$

E_n?

Boundary conditions for current density



Conservation field:

$$\hat{n} \times \vec{E}_2 = \hat{n} \times \vec{E}_1$$

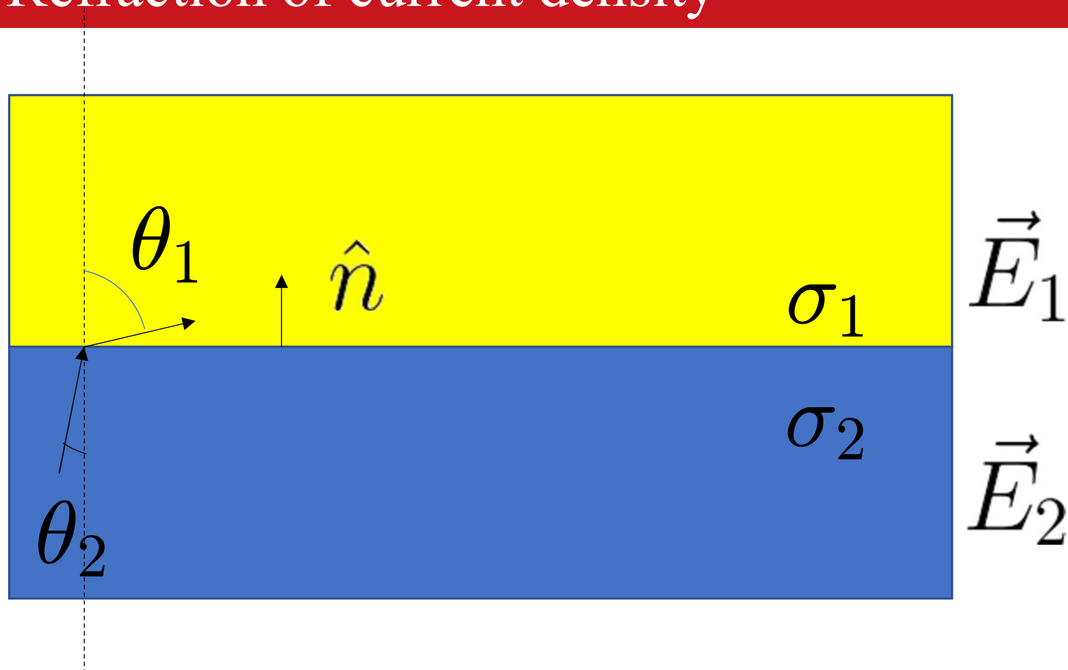
Continuity of current

$$\sigma_1 \vec{E}_1 \cdot \hat{n} = \sigma_2 \vec{E}_2 \cdot \hat{n}$$

1. Electric field can exist in the metal now, since current is present.

2. Surface charge is required to mediate the field.

Refraction of current density



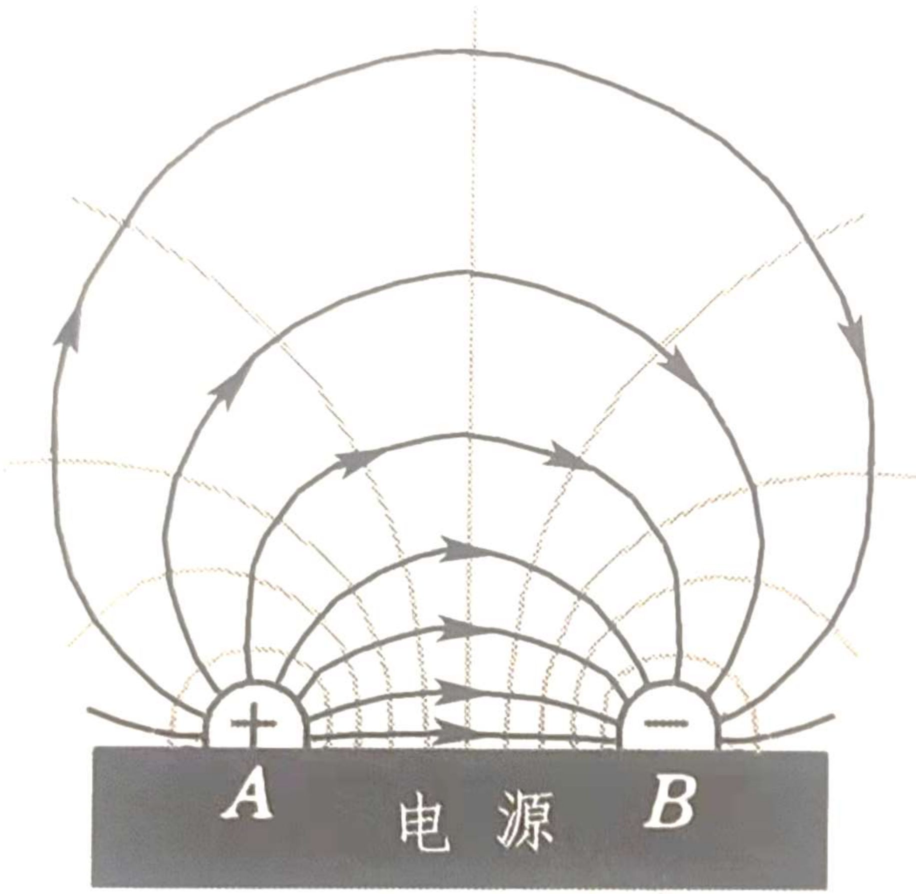
$$E_1 \sin\theta_1 = E_2 \sin\theta_2$$
$$\sigma_1 E_1 \cos\theta_1 = \sigma_2 E_2 \cos\theta_2$$

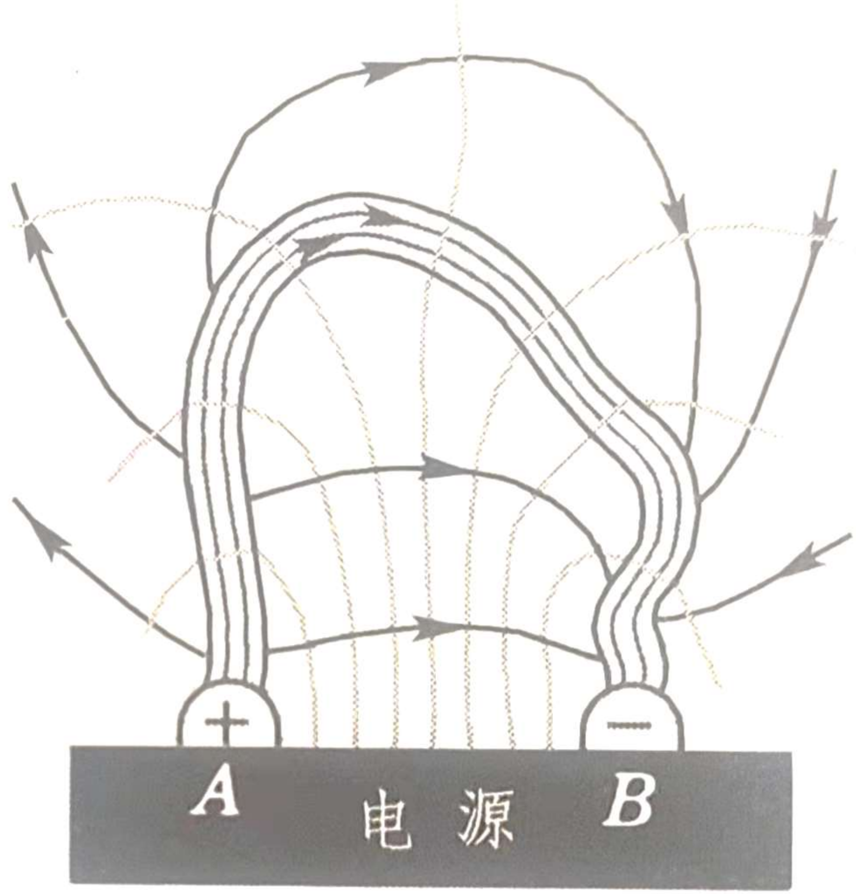
$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\sigma_1}{\sigma_2}$$

When σ_1>>1

$\tan\theta_1$ → infinity. No field goes into material 1, but distribute on the surface of it. Current stays inside material 1. Almost no normal component of current.

Electric field for a section of wire

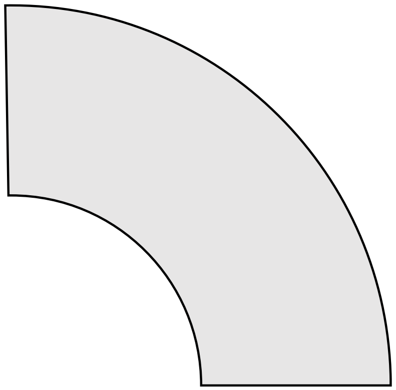




赵凯华，陈熙谋《新概念物理——电磁学》，第二版，p79

Contents

Example



Please calculate the resistance of the following piece of copper:

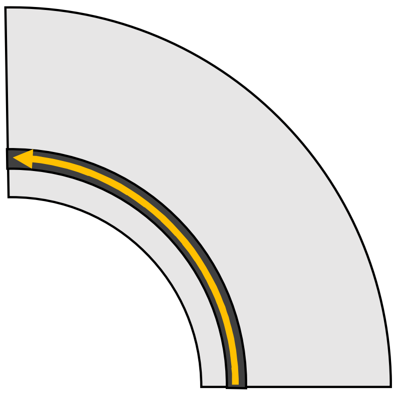
An (π/2) arc shaped copper conductor， with inner and outer radius *a* and *b*, thickness *h*.

The resistance in axial, radial and azimuthal directions.

Axial (z)

$$R = \frac{\rho L}{S} = \frac{4\rho h}{\pi(b^2 - a^2)}$$

Example



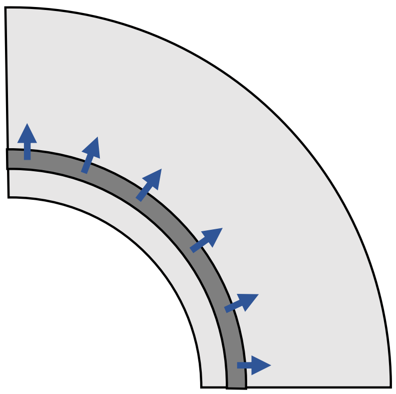
Azimuthal (θ): resistance of the element

$$\delta R = \frac{\rho L}{S} = \frac{\pi \rho r/2}{\delta r h}$$

The whole conductor is considered the **parallel** connection of many arc elements.

$$R = \frac{1}{\int (1/\delta R)} = \frac{1}{\frac{2h}{\pi\rho}\int \frac{\mathrm{d}r}{r}} = \frac{\pi\rho}{2h\ln(b/a)}$$

Example



Radial (r): resistance of the element

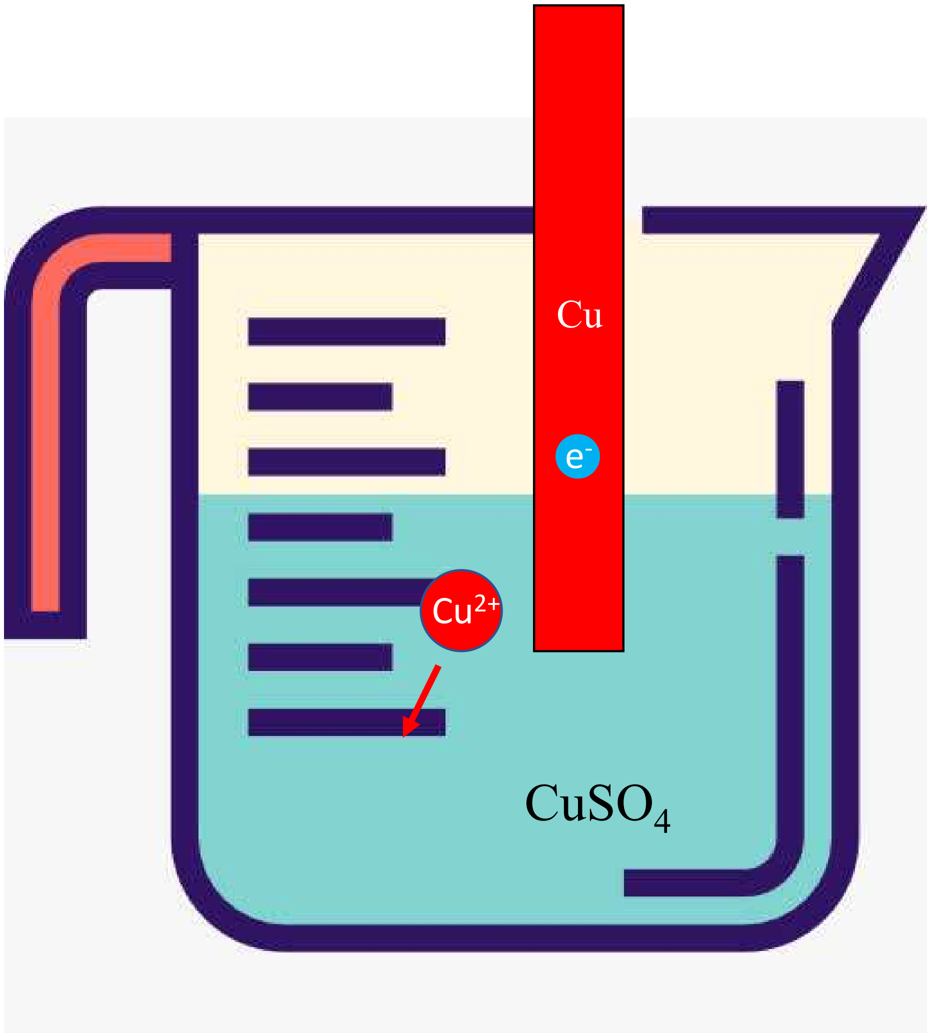
$$\delta R = \frac{\rho L}{S} = \frac{\rho \mathrm{d}r}{\pi r h/2}$$

The whole conductor is considered the **series** connection of many arc elements.

$$R = \int \delta R = \frac{2\rho}{\pi h}\int \frac{\mathrm{d}r}{r} = \frac{2\rho \ln(b/a)}{\pi h}$$

Contents

What is happening in a battery?



Tiny amount of copper dissolved into the solution

$Cu = Cu^{2+} + 2e^{-};$

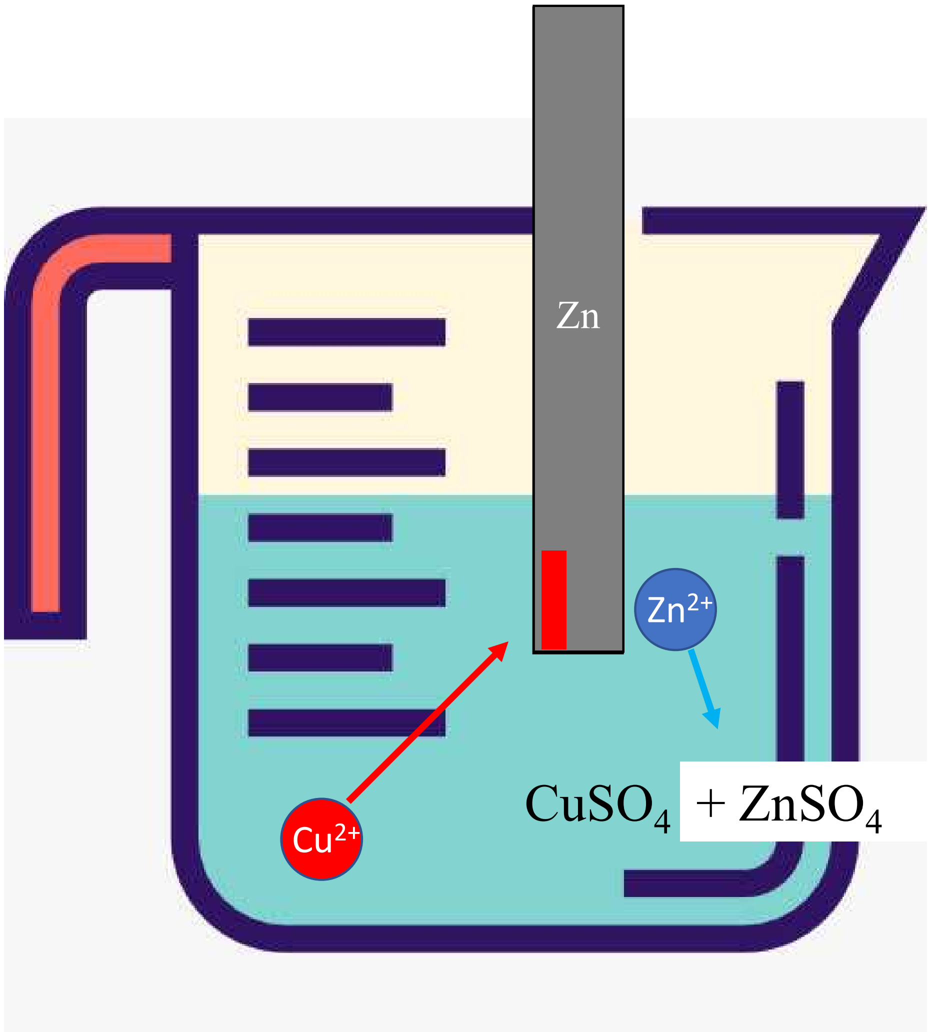
By natural diffusion, Cu^{2+} swims away

The Cu rod is left with some negative charge.

New Cu^{2+} does not want to swim away, a balance is reached:

Voltage ←→ Density gradient ←→ Reaction

What is happening in a battery?



Similar things happen for

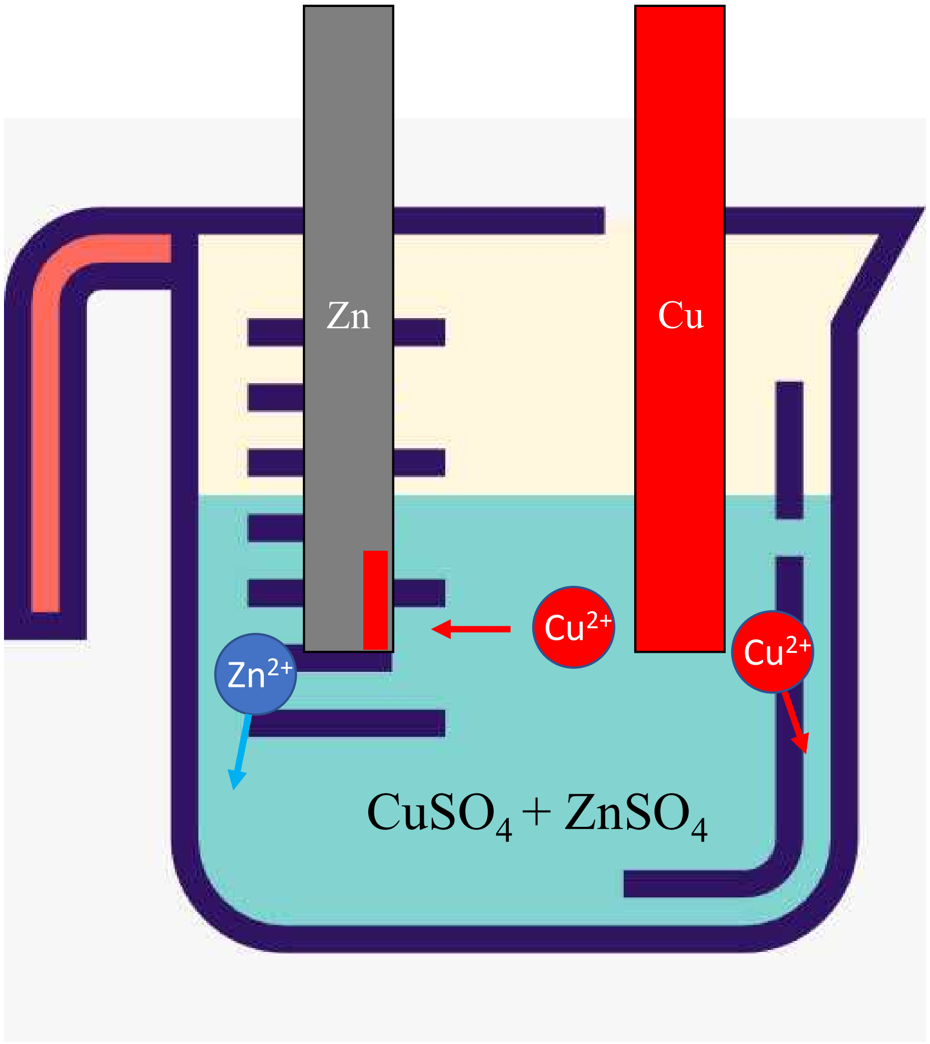
$$Zn = Zn^{2+} + 2e^-;$$

But Zn is more active than copper.

Assume the balance is established (in fact it is not a balanced state) for Zn^{2+}, which means a higher voltage than that of copper. Cu^{2+} is attracted. High density for Cu^{2+}. Reaction happens in favor of Cu.

$$Zn + Cu^{2+} = Zn^{2+} + Cu$$

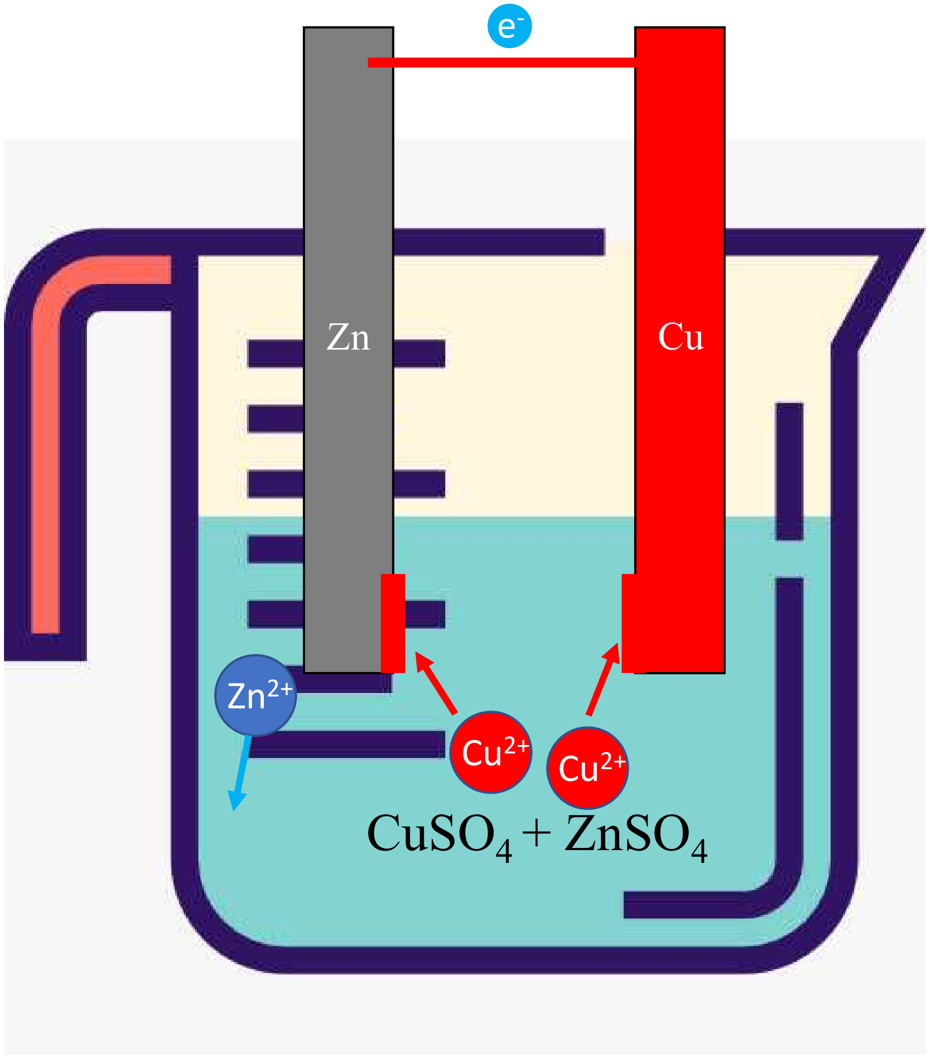
What is happening in a battery?



If both a copper rod and a zinc rod exist, Cu^{2+} will swim to zinc (as if the copper rod is not there), since Zn has lower electric potential.

If the two are connected by a metal wire,

What is happening in a battery?

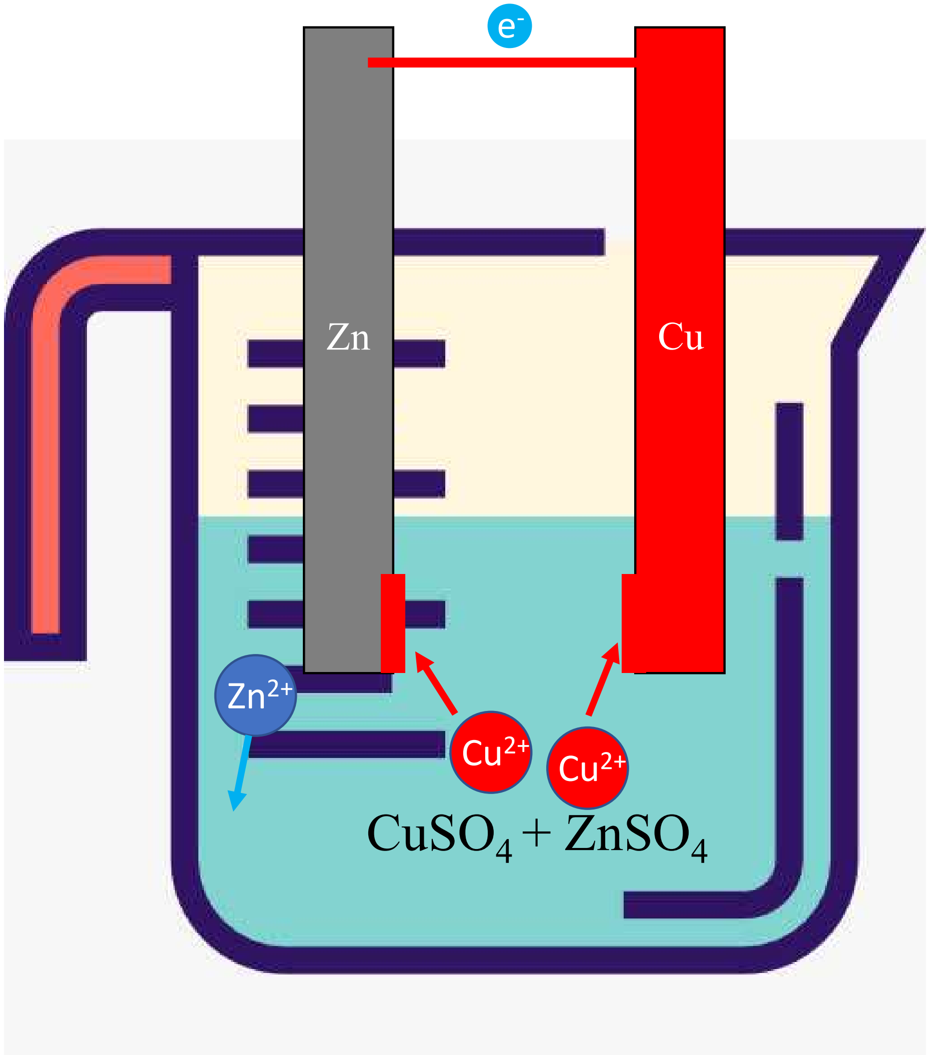


If both a copper rod and a zinc rod exist, Cu^{2+} will swim to zinc (as if the copper rod is not there), since Zn has lower electric potential.

If the two are connected by a metal wire, electrons will move to Cu through the wire.

Deposition of Cu happens on both metal rod since they have the same electric potential now.

What is happening in a battery?



For a battery, the interesting question is:

How much is the current through the wire?

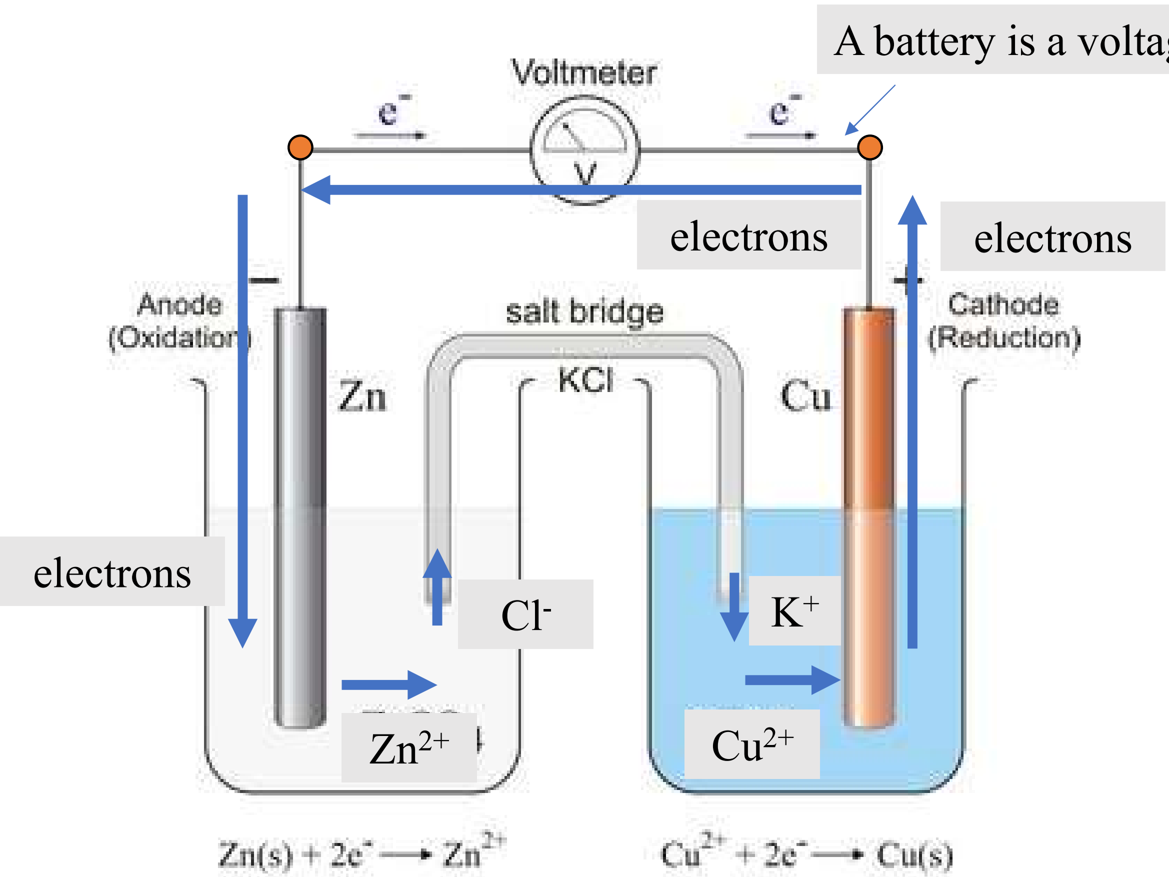
Charge conservation:

$$I = \frac{\mathrm{d}Q}{\mathrm{d}t} = 2 \times \text{Copper deposition rate(on Copper rod)}$$

All the copper grown on Zn are wasted!

What is happening in a battery?

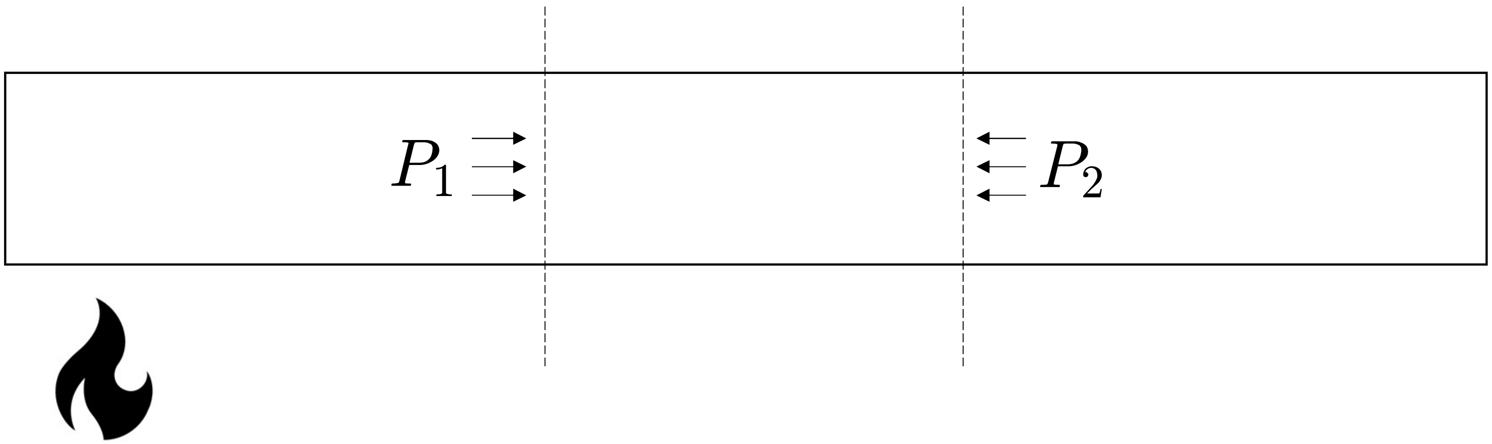
A battery is a voltage source.



$Zn(s)\,|\,ZnSO_4(aq)\,||\,CuSO_4(aq)\,|\,Cu(s)$

The path of (positive) charge:
current

Balance of gas



Steady state: $P_1 = P_2$

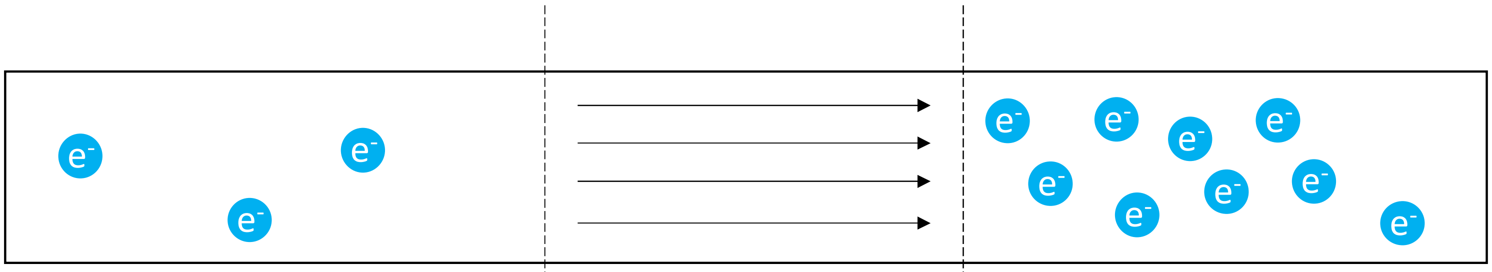
$$P = \frac{n}{V} RT \equiv NRT$$

$$T_1 > T_2$$

$\Rightarrow$ $N_1 < N_2$

When case is changed into electric problems

Electric field (potential difference) is established



One more question to be answered:
why only one type of charge moves?

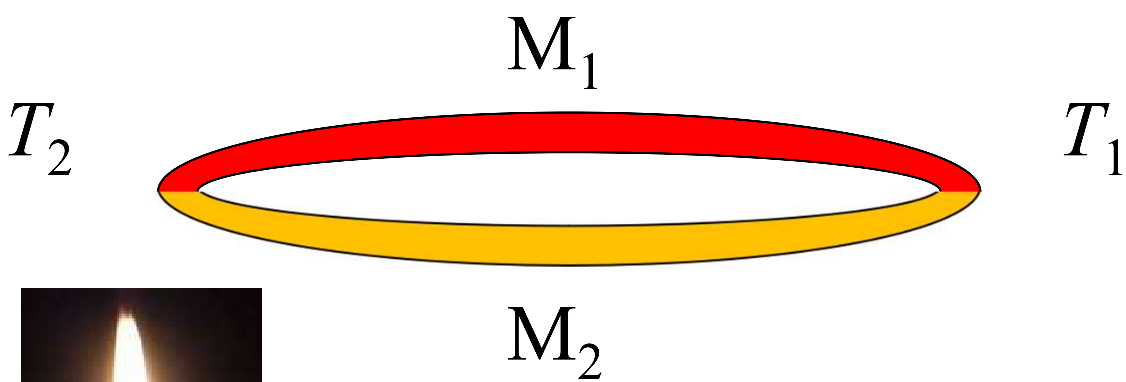
Thomson effect: the voltage is called thermal voltage

$$\mathcal{V} = \sigma \Delta T$$

Small effect, 10^{-5} V/K for Bi @ R.T.

$$\mathcal{V} = \int \sigma(T)\mathrm{d}T = \int \sigma(T)\frac{\mathrm{d}T}{\mathrm{d}x}\mathrm{d}x$$

Can we use temperature difference to make a power source?



If M_1 and M_2 are the same metal, no circulation current.

If M_1 and M_2 are not the same metal, circulation current appears, which can be utilized as an energy source.

Seebeck effect

What is common for the (electric) power supplies discussed above?

Some effect (force) moves the charges against static electric field.

Nonelectrostatic forces: qK
Nonelectrostatic field: K

Ohm's law changes into: $\rho\vec{J} = \vec{E}$ $\longrightarrow$ $\rho\vec{J} = \vec{E} + \vec{K}$

$\int \vec{E} \cdot \mathrm{d}\vec{l} = V$ is defined a voltage DROP.

$\int \vec{K} \cdot \mathrm{d}\vec{l} = \mathcal{V}$ is defined electromotive force (emf).

The integral is usually inside the power source, or over the whole circuit.

Strict definition: work done by the power source when unit charge go through it.

$$\int \vec{K} \cdot \mathrm{d}\vec{l} = \mathcal{V}$$

Dimension:

$$\frac{[Energy](\mathrm{J})}{[Charge](\mathrm{C})} = [Voltage](\mathrm{V})$$

Concept:

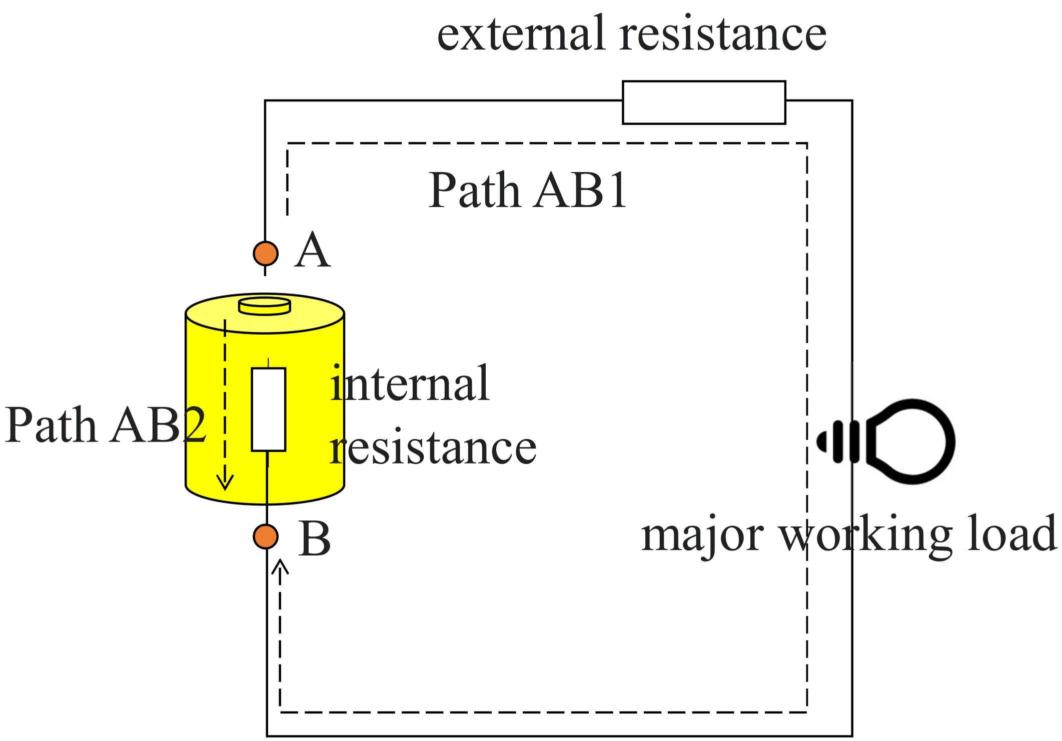
the "voltage" generated by the energy source.

" " because

1. In lots of cases, the force "lifting" the may not be electrical.

2. In some cases, it may involve a nonconservative electric field.

An example of electromotive force (emf)



$$V_o = \int_{AB1} \vec{E} \cdot \mathrm{d}\vec{l}$$

$$\rho_i \vec{J} = \vec{K} + \vec{E}$$

$$-\frac{\rho_i l_i}{A_i}(JA_i) = -Kl_i + El_i$$

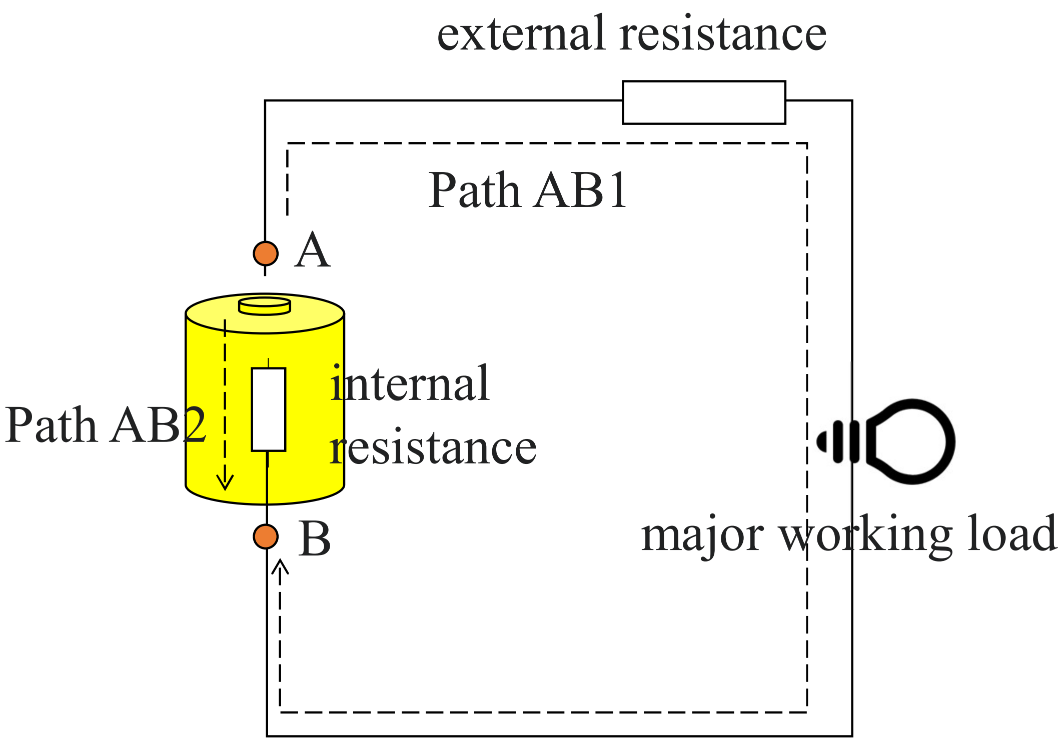
$$Kl_i = \mathcal{V} = V_o + IR_i$$

Open loop (I=0) condition:
terminal voltage = emf

Energy conservation:

$$\mathcal{V}\mathrm{d}Q = V_o\mathrm{d}Q + I^2R_i\mathrm{d}t$$

$$I\mathrm{d}t = \mathrm{d}Q$$



$$\oint \vec{E} \cdot \mathrm{d}\vec{l} = 0$$

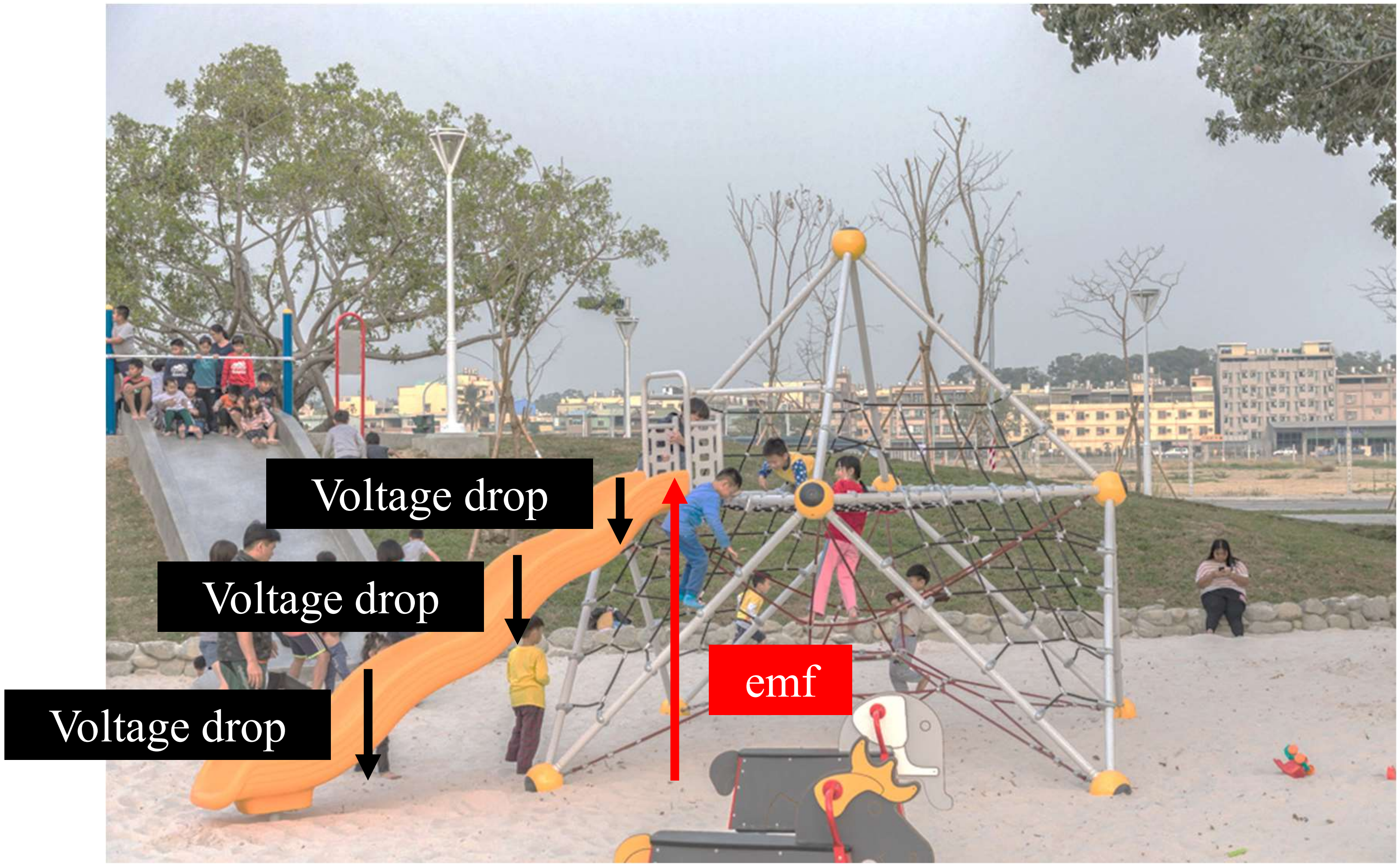
$$\oint \vec{K} \cdot \mathrm{d}\vec{l} = \int_{AB2} \vec{K} \cdot \mathrm{d}\vec{l} = K l_i = \mathcal{V}$$

Closed loop integral of static ***E*** is 0, electric circuit helps to convert energy, but electric potential energy does not build up or consume with circular motion of charge.

Potential drop in a conservative field

“Voltage” drop:

$$g\Delta h$$



Electromotive force

Things that bring charge to move against static electrical force:

25.4 Part of the electric circuit that includes this light bulb passes through a beaker with a solution of sodium chloride. The current in the solution is carried by both positive charges (Na^+ ions) and negative charges (Cl^- ions).

- temperature
- density difference
- Different fermi level (contact)
- Lorentz force
- Induction electric field

Capacitor bank: not really an emf, but can be convenient for analyzing and understanding.