Physics (PHYS2500J), Unit 1 Electrostatics: 4. Dielectric Material

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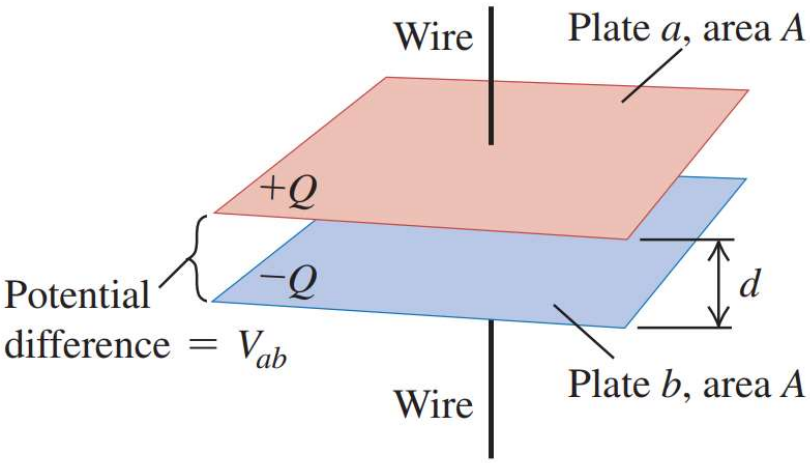
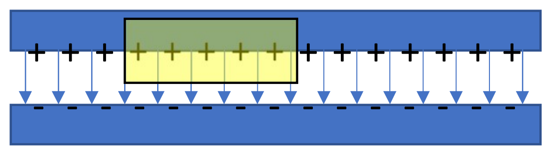
Capacitance: parallel conducting plate



What is the relation between the potential difference and the charge in the plates, if the charge on both plate is of the same quantity but opposite sign?

Using Gauss's law



$$E = \frac{\sigma_e}{\epsilon_0}$$

$$V = Ed = \frac{\sigma_e d}{\epsilon_0} = \frac{d}{\epsilon_0 S} Q$$

The definition of capacitance:

$$Q = CV$$

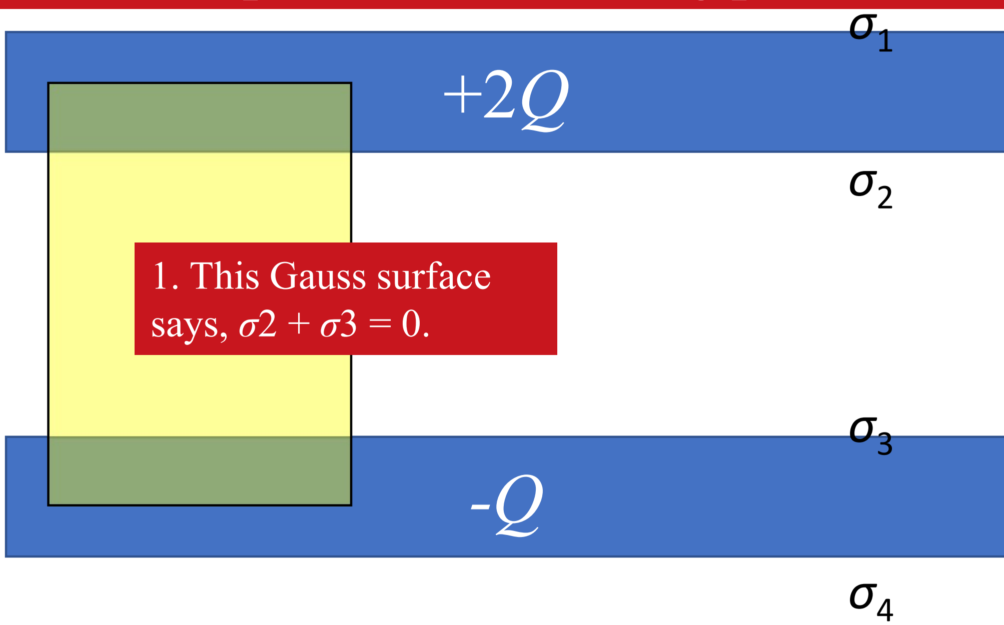
Capacitance of a parallel plate capacitor (no medium): $C = \frac{\epsilon_0 S}{d}$

What if the two plates have different charge?

Symbol in circuit diagram

Reason 1: in a circuit, you will mostly see the charge on one side of the capacitor is from the other side, moving in the form of current.

Capacitance: parallel conducting plate



2. Field in the metal is the field generated by the 4 surface charges. Now it can be separated into group 1: $\sigma 2$ $\sigma 3$ and group 2: $\sigma 1$ $\sigma 4$.

3. $\sigma 1 = \sigma 4$, so the field generated by the outer surface cancels each other inside the metal, too.

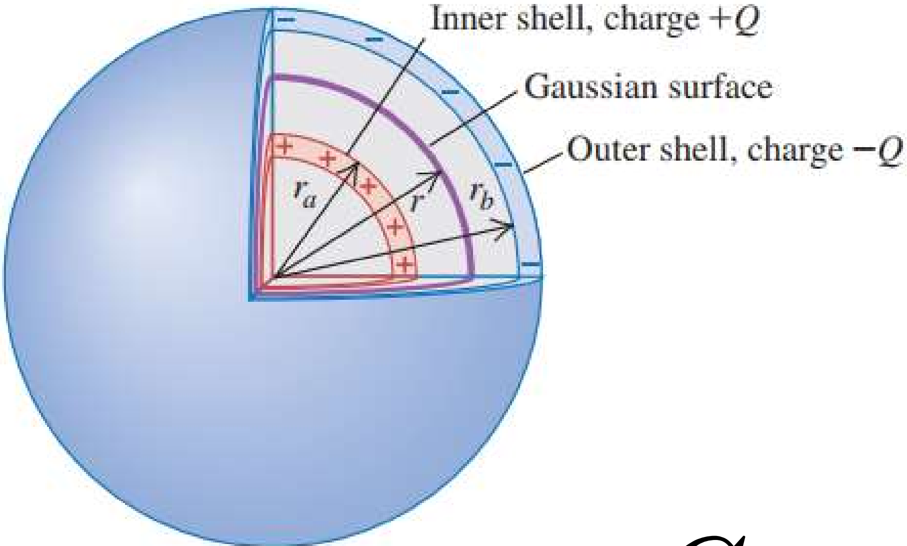
$$\sigma_2 = -\sigma_3 = \frac{3Q}{2S}$$

$$\sigma_1 = \sigma_4 = \frac{Q}{2S}$$

For capacitance, now we have actually 3 conductors involved, upper and lower plates, and ground. It is called partial capacitance.

The charge in the inner surface can still be considered a normal parallel plate capacitor and the rest is the charge in the capacitor made of the plates and the ground.

Capacitance of two concentric spheres



$$V = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

$$C = \frac{Q}{V} = \frac{4\pi\epsilon_0 r_a r_b}{r_b - r_a}$$

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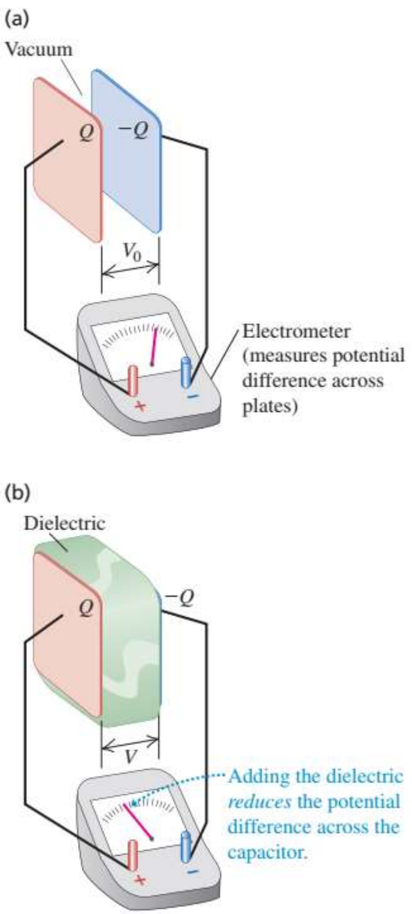
Experimental fact: capacitance increases with inserting dielectric

$$C_1 > C_0$$

Discovery of dielectric materials

24.14 Effect of a dielectric between the plates of a parallel-plate capacitor. **(a)** With a given charge, the potential difference is V_0. **(b)** With the same charge but with a dielectric between the plates, the potential difference V is smaller than V_0.



$$K = \frac{C}{C_0} \qquad \text{(definition of dielectric constant)}$$

TABLE 24.1 Values of Dielectric Constant K at 20°C

Material	K	Material	K
Vacuum	1	Polyvinyl chloride	3.18
Air (1 atm)	1.00059	Plexiglas®	3.40
Air (100 atm)	1.0548	Glass	5–10
Teflon	2.1	Neoprene	6.70
Polyethylene	2.25	Germanium	16
Benzene	2.28	Glycerin	42.5
Mica	3–6	Water	80.4
Mylar	3.1	Strontium titanate	310

A basic component of circuit: capacitors

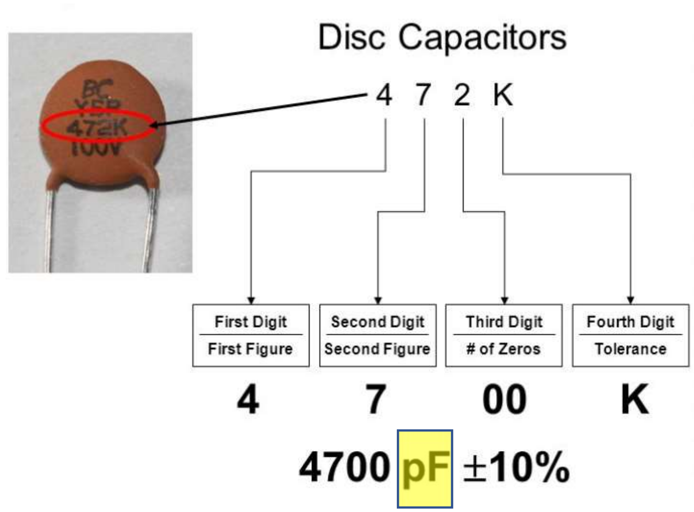
$$C = \frac{\epsilon_0 S}{d}$$ Higher ε

Ceramic capacitor: The ceramic capacitor is a type of capacitor that is used in many applications from audio to RF. Values range from a few picofarads to around 0.1 microfarads. Ceramic capacitor types are by far the most commonly used type of capacitor.

cheap and reliable, loss factor is particularly low

One very typically used signal filter.

Mica and Ceramics
- Low ESL and ESR
- Keep leads short
- Uses
  - High frequency filtering
  - Bypassing
  - decoupling

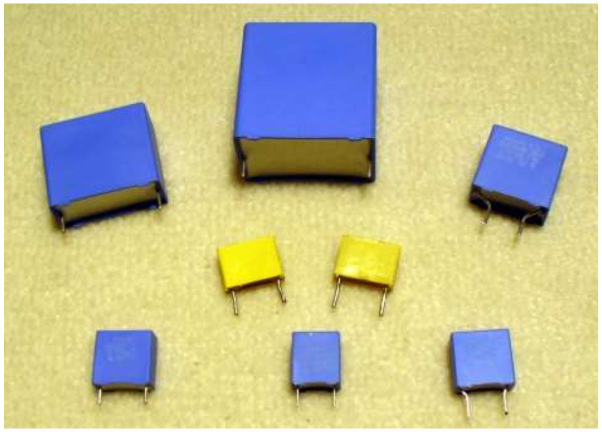


Code	Tolerance
A	±0.05%
B	±0.1%
C	±0.25%
D	±0.5%
F	±1%
G	±2%
J	±5%
K	±10%
M or NONE	±20%
N	±30%
Q	−10%, +30%
S	−20%, +50%
T	−10%, +50%
Z	−20%, +80%

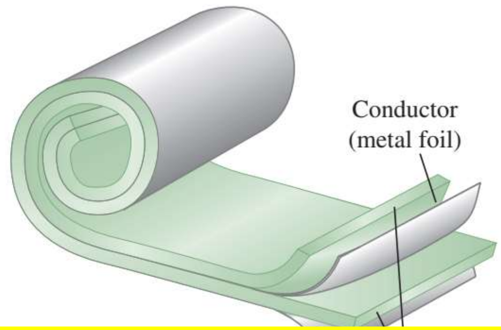
A basic component of circuit: capacitors

Polystyrene and Polypropylene (Metal film Caps.)

- Low ESR
- Very stable C – f characteristic
- Suitable for power bank



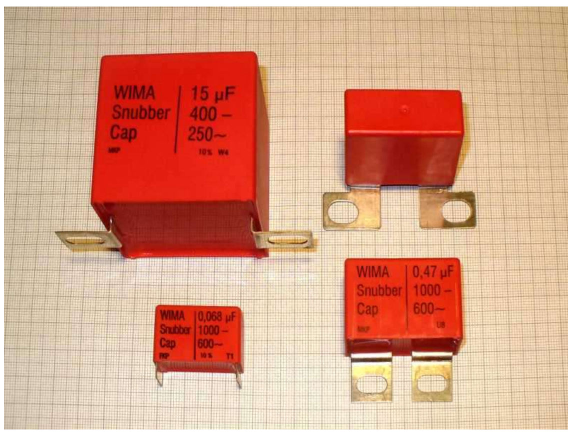
24.13 A common type of capacitor uses dielectric sheets to separate the conductors.



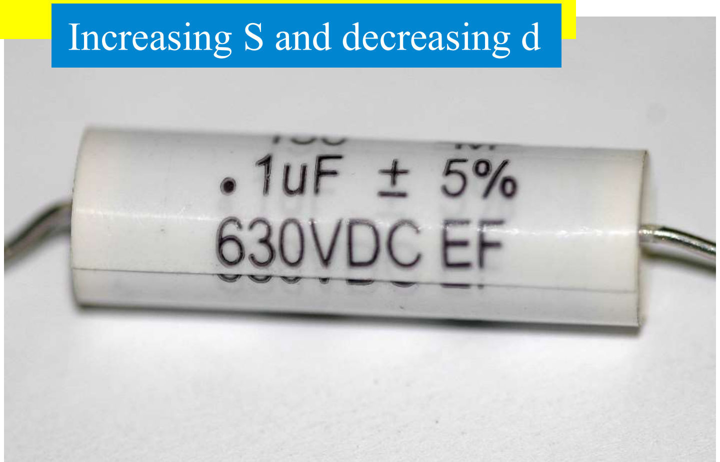
$$C = \frac{\epsilon_0 S}{d}$$

What is the key logic here?

Increasing S and decreasing d



Polyester film capacitor



A basic component of circuit: capacitors

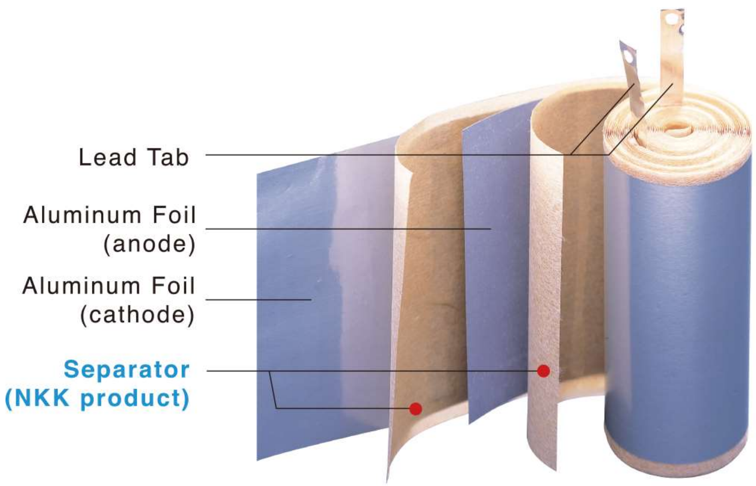
Electrolytic capacitor: Electrolytic capacitors are a type capacitor that is **polarized**.
They are able to offer high capacitance values – typically above 1µF and are most widely used for low-frequency applications – power supplies, decoupling and audio coupling applications as they have a frequency limit of around 100 kHz.

high capacitance values

$$C = \frac{\epsilon_0 S}{d}$$

What is the key logic here?

Also increasing S and decreasing d

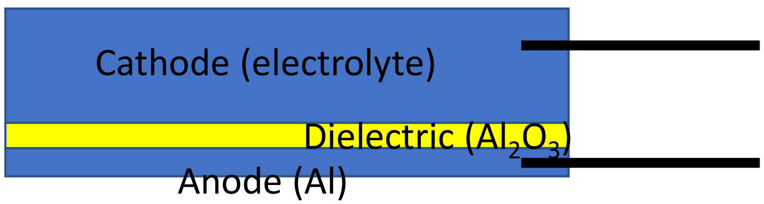


Al_2O_3 thin coating

Cathode forms with liquid (immersed surface area amplification)

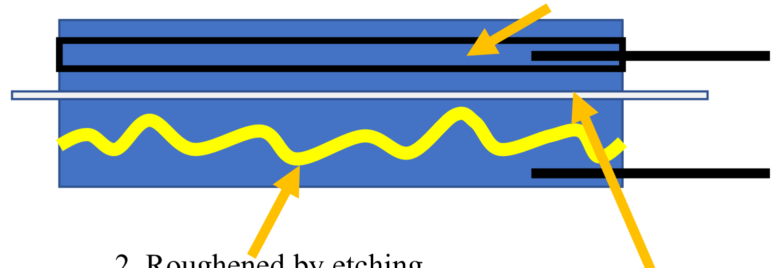
https://www.kodoshi.co.jp/english/product/capacitor_separator.html

A minimum structure



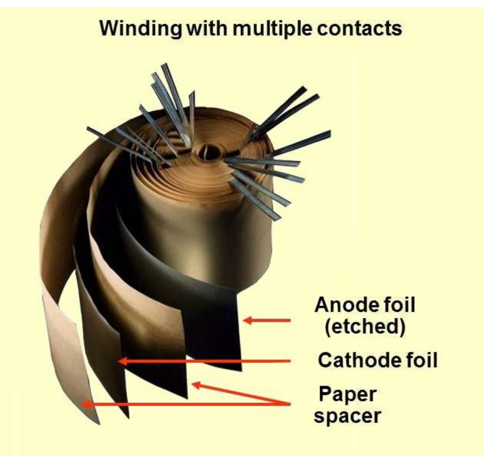
Improved structure

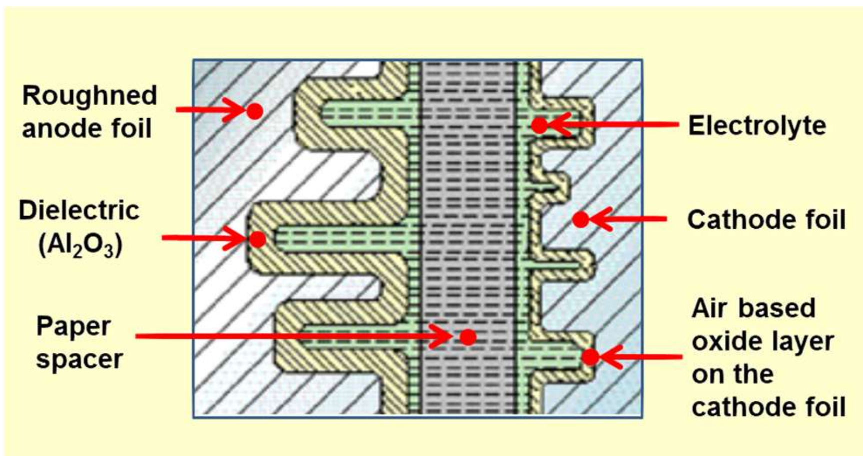
1. One more Al foil for less resistance on cathode

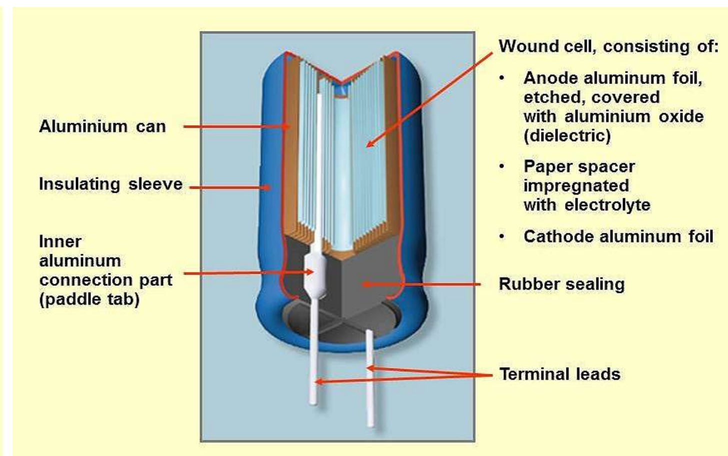


2. Roughened by etching, for larger surface area.

3. Paper spacer, avoid touching.

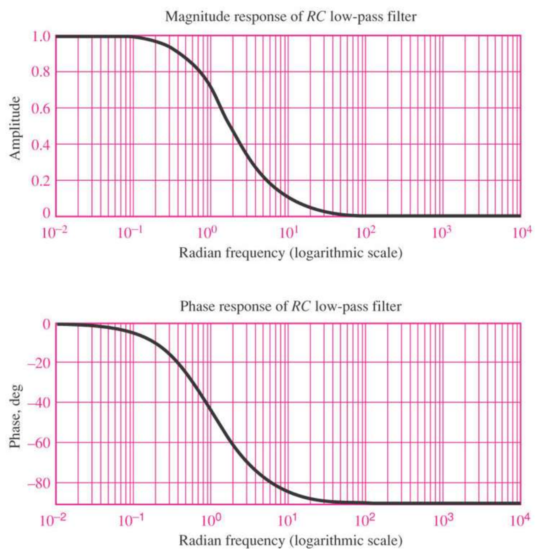






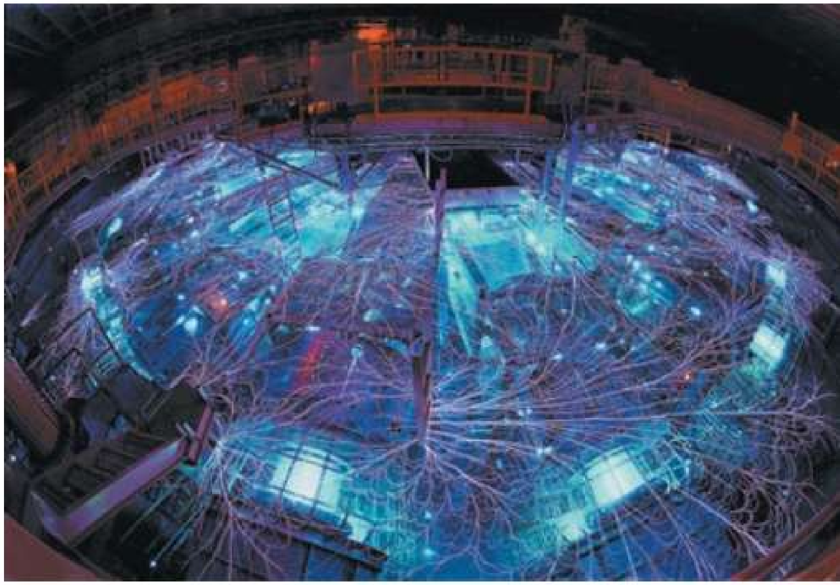
https://en.wikipedia.org/wiki/Aluminum_electrolytic_capacitor

What can we do with capacitors



Low pass filters

sensors



Discharge for high power

Motor capacitor: create a phase lag so the induction motor knows which direction to go.

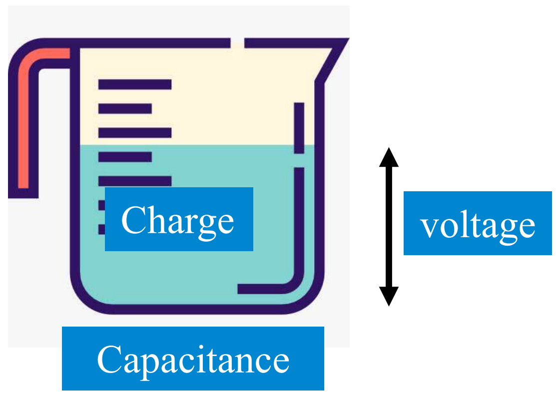
Limit for a capacitor

Is capacitance the amount of charge a capacitor can hold?

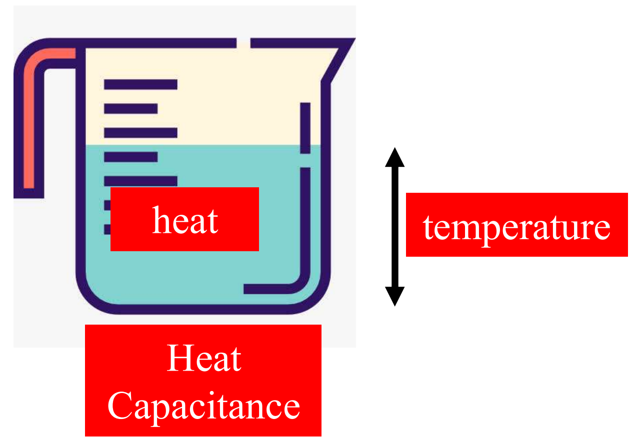
Capacitance is the amount of charge a capacitor holds at unit voltage.

$$Q = CV$$

It is more like the bottom area instead of the volume of a beaker



Similar to heat capacitance



Dielectric strength: the maximum field a material can withstand

TABLE 24.2 Dielectric Constant and Dielectric Strength of Some Insulating Materials

Material	Dielectric Constant, K	Dielectric Strength, E_m (V/m)
Polycarbonate	2.8	3×10^7
Polyester	3.3	6×10^7
Polypropylene	2.2	7×10^7
Polystyrene	2.6	2×10^7
Pyrex glass	4.7	1×10^7

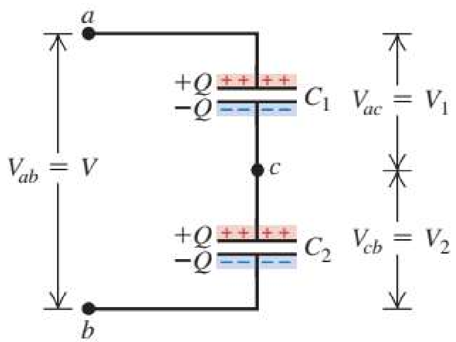
Connection of capacitors

24.8 A series connection of two capacitors.

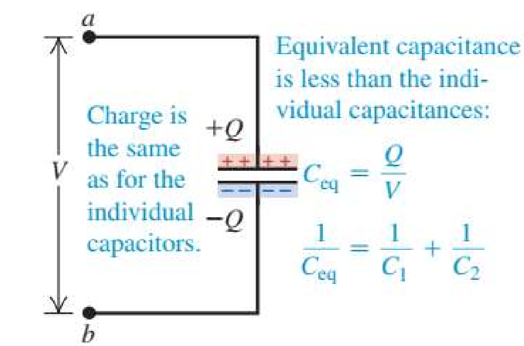
(a) Two capacitors in series

Capacitors in series:

- The capacitors have the same charge Q.
- Their potential differences add: $V_{ac} + V_{cb} = V_{ab}$.



(b) The equivalent single capacitor

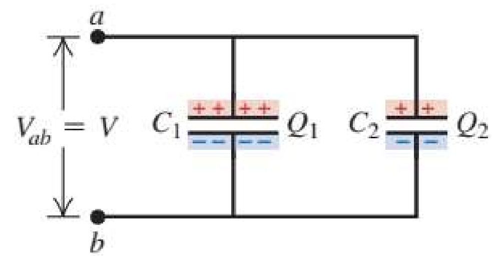


24.9 A parallel connection of two capacitors.

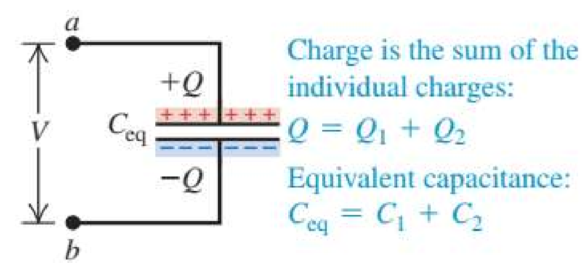
(a) Two capacitors in parallel

Capacitors in parallel:

- The capacitors have the same potential V.
- The charge on each capacitor depends on its capacitance: $Q_1 = C_1V$, $Q_2 = C_2V$.



(b) The equivalent single capacitor



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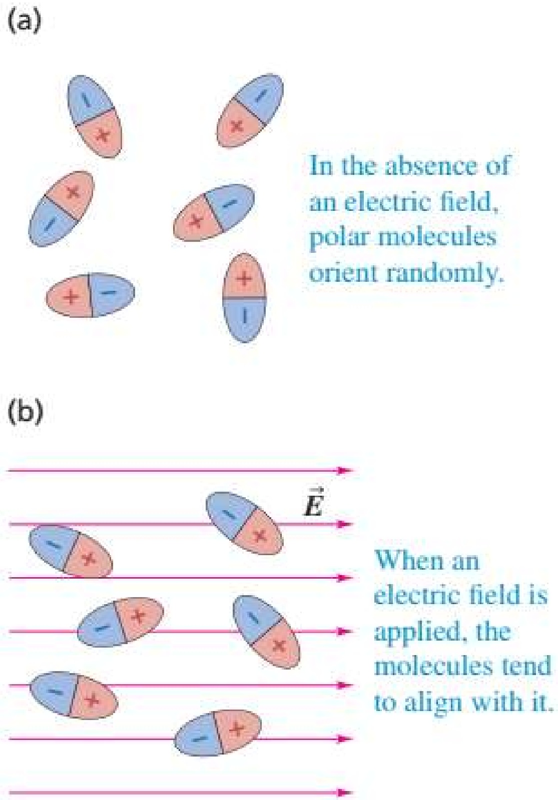
Dielectric materials

Orientational polarization



Electronic displacement polarization

Atomic (ionic) displacement polarization

New concept: Polarization

$$\vec{P} = \frac{\sum p_i}{V} = \underbrace{\frac{n}{V}p = Np}$$

If every dipole has the same moment.

Polarization is the density of dipole moment

Linear, isotropic, and homogeneous material

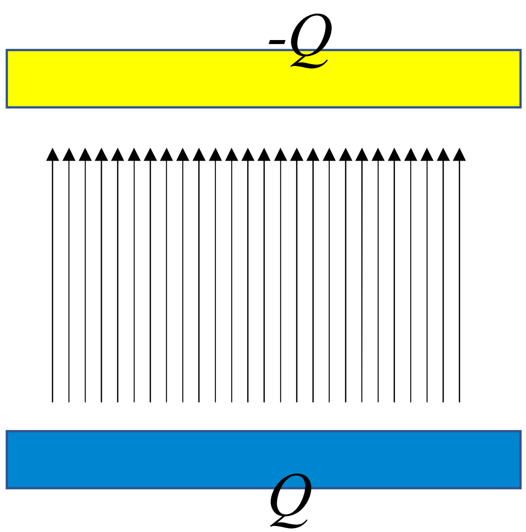
$$\vec{P} \propto \vec{E}$$

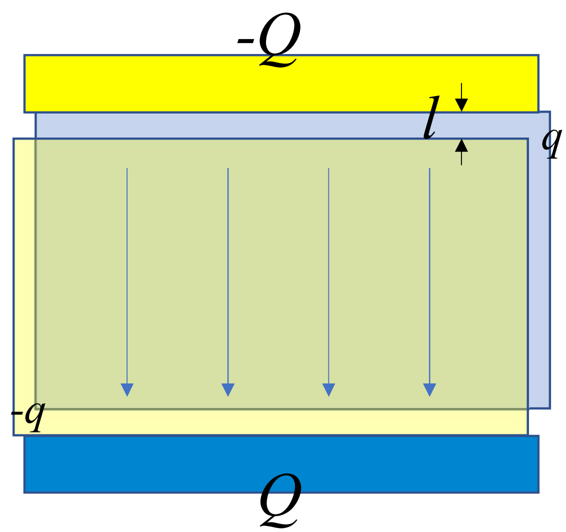
Dipole tends to align along the externally applied field.

$$\vec{P} = \chi_e \epsilon_0 \vec{E}$$

χ_e Electric susceptibility 极化率

How does dielectric material affect voltage





Vertical displacement of positive and negative charges (lateral just to make the figure clear)

$$q = nq_0 = NAlq_0 = PA$$

n number of dipoles, N volumetric density of dipoles

Surface charge density

$$\sigma_p = \vec{P} \cdot \hat{n}$$

Dot product just incase the two are not in-line.

Capacitance changed

Induced field

$$E_p = -\frac{\sigma_p}{\epsilon_0} = -\frac{P}{\epsilon_0} = -\chi E$$

If the amount of FREE charge is not changed:

$$E = E_f + E_p = E_f - \chi E$$

E_f: the field by free charge (charge on the metal, which can be conducted in the circuit), E_p: field by bounded charge, or polarization charge; E: total field

$$E_f \to E = \frac{1}{1+\chi} E_f$$

$$V_f \to \frac{1}{1+\chi} V_f$$

capacitance

$$\frac{Q}{V_f} \to (1+\chi)\frac{Q}{V_f}$$

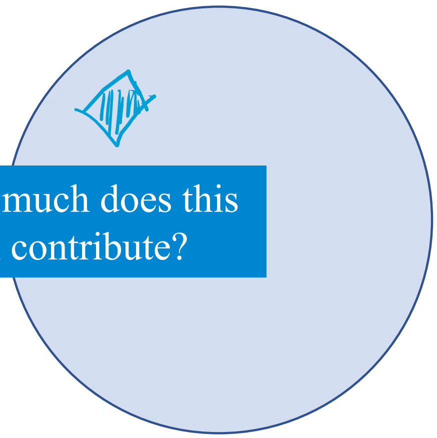
$$\epsilon_r \equiv 1 + \chi_e$$

Relative permittivity (dielectric constant)

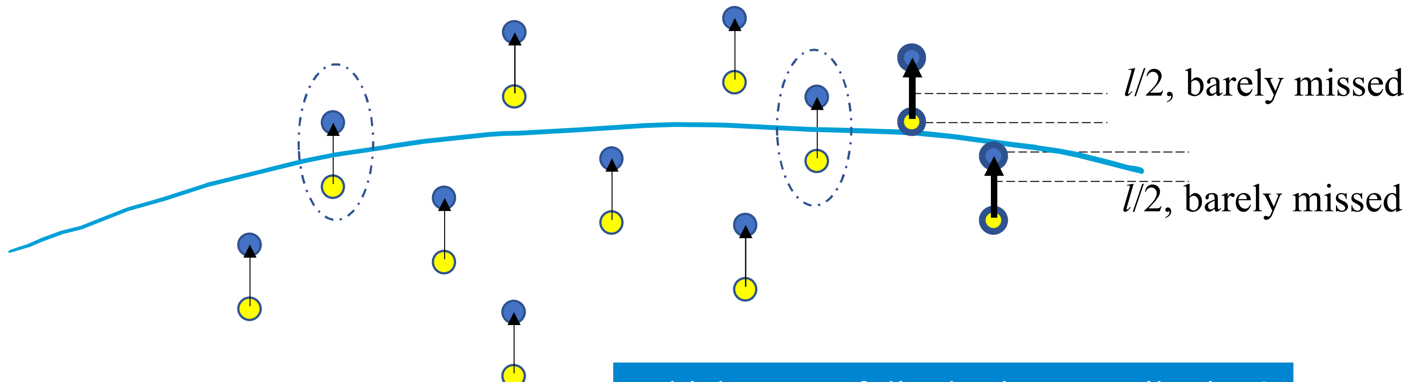
Polarization charge

A macroscopically small, microscopically large volume, which contains a lot of dipoles.
Vectors such as electric field, polarization can continuously vary in the volume.

How much polarization charge is contained in the volume then?



How much does this patch contribute?



Which parts of dipoles has contribution?
Positive or negative charge?

$$\mathrm{d}Q = -nq = -N(l\mathrm{d}A)q = -P\mathrm{d}A$$

Total charge

$$-\oint \vec{P} \cdot \mathrm{d}\vec{A}$$

If there is an angle?

$$\mathrm{d}Q = -nq = -N(l\mathrm{d}A\cos\theta)q = -\vec{P} \cdot \mathrm{d}\vec{A}$$

Using Gauss's theorem

Compare with Gauss's law for ***E*** field

$$Q = -\oint \vec{P}\cdot \mathrm{d}\vec{A}$$

$$\frac{Q}{\epsilon_0} = \oint \vec{E}\cdot \mathrm{d}\vec{A}$$

$$\frac{Q}{V} = -\frac{\oint \vec{P}\cdot \mathrm{d}\vec{A}}{V}$$

Gauss's theorem: the volumetric density of flux is divergence

$$\rho_p = -\nabla\cdot\vec{P}$$

$$\frac{\rho_e}{\epsilon_0} = \nabla\cdot\vec{E}$$

The dimension of E and P is different
A negative sign

boundary and volumetric charge density

$$\sigma = \vec{P} \cdot \vec{n}$$

volume charge density

$$\rho = -\nabla \cdot \vec{P}$$

What is the difference between charge in metal (called free charge) and polarization charge?

1. Free charge moves globally, polarization charge is bounded.

2. Free charge enters into the circuits.

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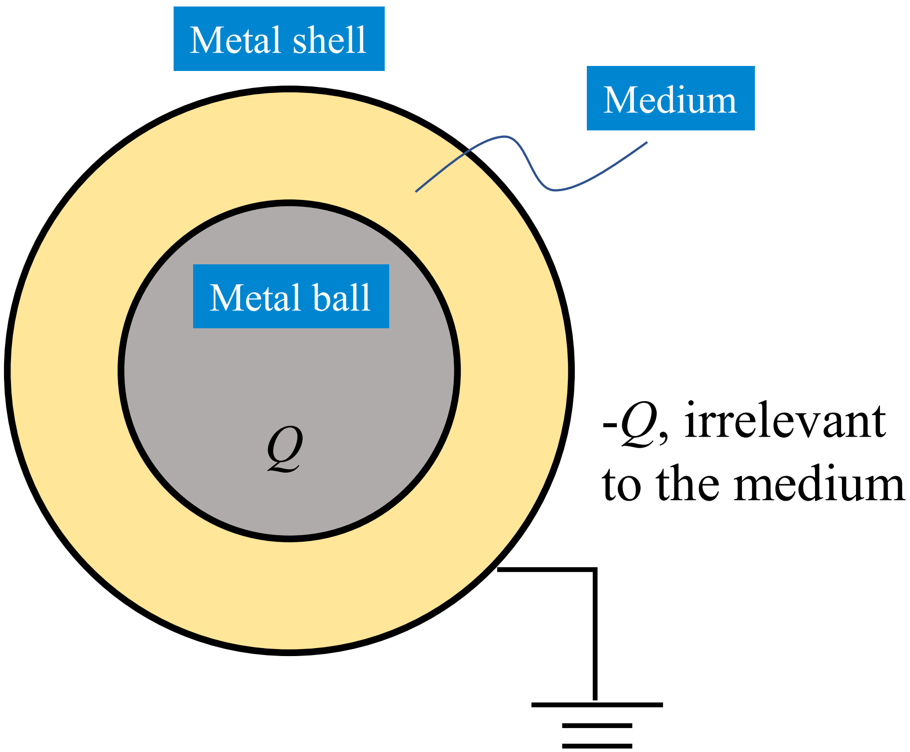
An easy way to describe free charge



Can any field describes free charge rather than total charge?

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} = \frac{\rho_f + \rho_p}{\epsilon_0}$$

$$\rho_p = -\nabla \cdot \vec{P}$$

$$\nabla \cdot \vec{E} = \frac{\rho_f}{\epsilon_0} - \frac{\nabla \cdot \vec{P}}{\epsilon_0}$$

Electric displacement vector

$$\nabla \cdot \vec{E} = \frac{\rho_f}{\epsilon_0} - \frac{\nabla \cdot \vec{P}}{\epsilon_0}$$

A new value

D has the same dimension as ***P***

$$\nabla \cdot \vec{D} = \nabla \cdot (\epsilon_0 \vec{E} + \vec{P}) = \rho_f$$

Is this the contribution of free charge or the total charge?

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

Displacement vector

Total field

polarization

Although ***D*** is written as a sum of ***E*** and ***P***, it is **NOT** the contribution of both electric field and the polarization charge.
It is a vector for simplicity in terms of "free charge".
E is the real field, as combination of both free and polarization charges.

The new physical variables

Polarization ***P***

What is the dimension of ***P***?

$$\dim\ P = \frac{\text{dipole moment}}{\text{volume}} = \frac{\text{A·s·m}}{\text{m}^3} = \frac{\text{A·s}}{\text{m}^2}$$

Electric displacement vector ***D***

What is the dimension of ***D***?

$$\dim\ D = \dim\ \epsilon_0 \cdot \dim\ E$$

Coulomb's law

$$\dim\ \epsilon_0 = \frac{\text{charge}^2}{\text{distance}^2}\frac{1}{\text{force}} = \frac{\text{A}^2\text{·s}^2}{\text{m}^2}\frac{\text{s}^2}{\text{kg·m}} = \frac{\text{A}^2\text{·s}^4}{\text{kg·m}^3}$$

or,

Capacitance equation

$$\frac{\epsilon_0 S}{d} = \frac{Q}{V}$$

$$\dim\ \epsilon_0 = \frac{\text{charge·distance}}{\text{voltage·distance}^2} = \frac{\text{charge}^2\text{·distance}}{\text{energy·distance}^2} = \frac{\text{A}^2\text{·s}^2\text{·m}}{\text{m}^2\text{·kg·m}^2/\text{s}^2} = \frac{\text{A}^2\text{·s}^4}{\text{kg·m}^3}$$

So, $$\dim\ D = \dim\ \epsilon_0 \cdot \dim\ E = \frac{\text{A}^2\text{·s}^4}{\text{kg·m}^4}\frac{\text{kg·m}}{\text{A·s}^3} = \frac{\text{A·s}}{\text{m}^3}$$

Material properties

Linear, homogeneous, isotropic material

$$\vec{P} = \chi_e \epsilon_0 \vec{E}$$

The electric susceptibility χ_e is a dimensionless number directly describes the material response to electric field.

Some other forms of the same property

$$\vec{D} = \epsilon \vec{E} = \epsilon_r \epsilon_0 \vec{E}$$

Dielectric constant (permittivity)

Relative permittivity

Dielectric constant of free space

Relation between χ_e and ε_r

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P} = \epsilon_0 \vec{E} + \chi_e \epsilon_0 \vec{E} = (1 + \chi_e) \epsilon_0 \vec{E}$$

$$\epsilon_r = 1 + \chi_e$$

Both are dimensionless.

$$\oint \vec{E} \cdot \mathrm{d}\vec{S} = \frac{Q}{\epsilon_0} \qquad \nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\oint \vec{D} \cdot \mathrm{d}\vec{S} = Q_f \qquad \nabla \cdot \vec{D} = \rho_f$$

$$\oint \vec{P} \cdot \mathrm{d}\vec{S} = -Q_p \qquad \nabla \cdot \vec{P} = -\rho_p$$

More complicated dielectric materials

Susceptibility may be

1. Non-homogeneous media: χ is a function of position;
2. Non-linear: χ is a function of E field intensity;
3. Anisotropic media: χ is a tensor instead of scalar, $\boldsymbol{P}$ is not parallel with $\boldsymbol{E}$;
4. Ferro-electric material: χ depends on E but not a single valued function, depending on the history.

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Interface between two dielectric media

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Which of the equations are valid all the time?

And which works in uniform medium but have problem across the boundary.

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$-\nabla \phi = \vec{E}$$

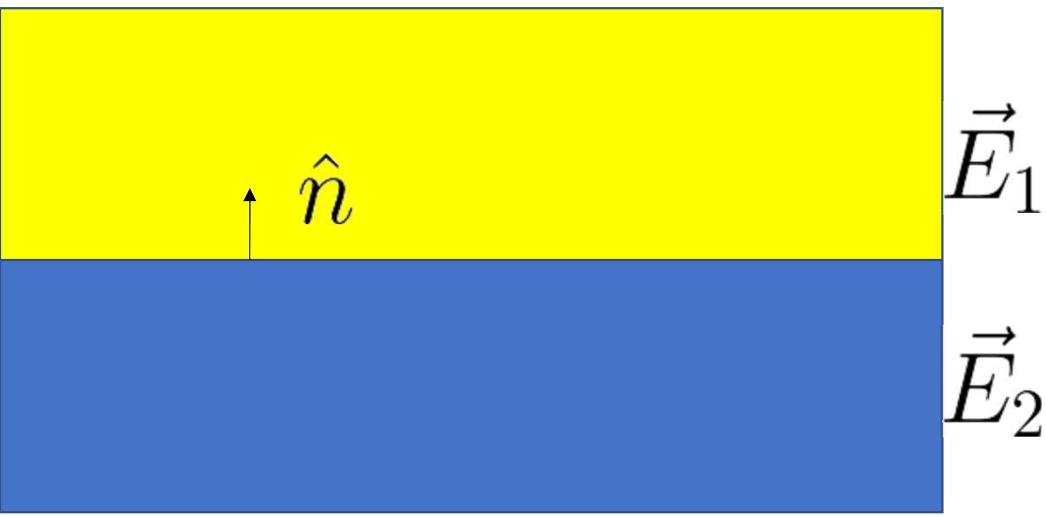
$$\vec{P} = \chi \epsilon_0 \vec{E}$$

$$\vec{D} = \epsilon \vec{E}$$

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

Boundary conditions

A boundary condition usually describes the normal or tangential component of the vector field.



What is the relationship between the tangential component of *E?*

$$\hat{n} \times \vec{E}_2 = \hat{n} \times \vec{E}_1$$

Can be proved with path integral in closed loop, or with potential on the boundary.

Boundary conditions

A boundary condition usually describes the normal or tangential component of the vector field.

What about the normal component?

$$\hat{n} \cdot (\vec{E}_1 - \vec{E}_2) = \sigma / \epsilon_0$$

A better description?

Surface polarization charge

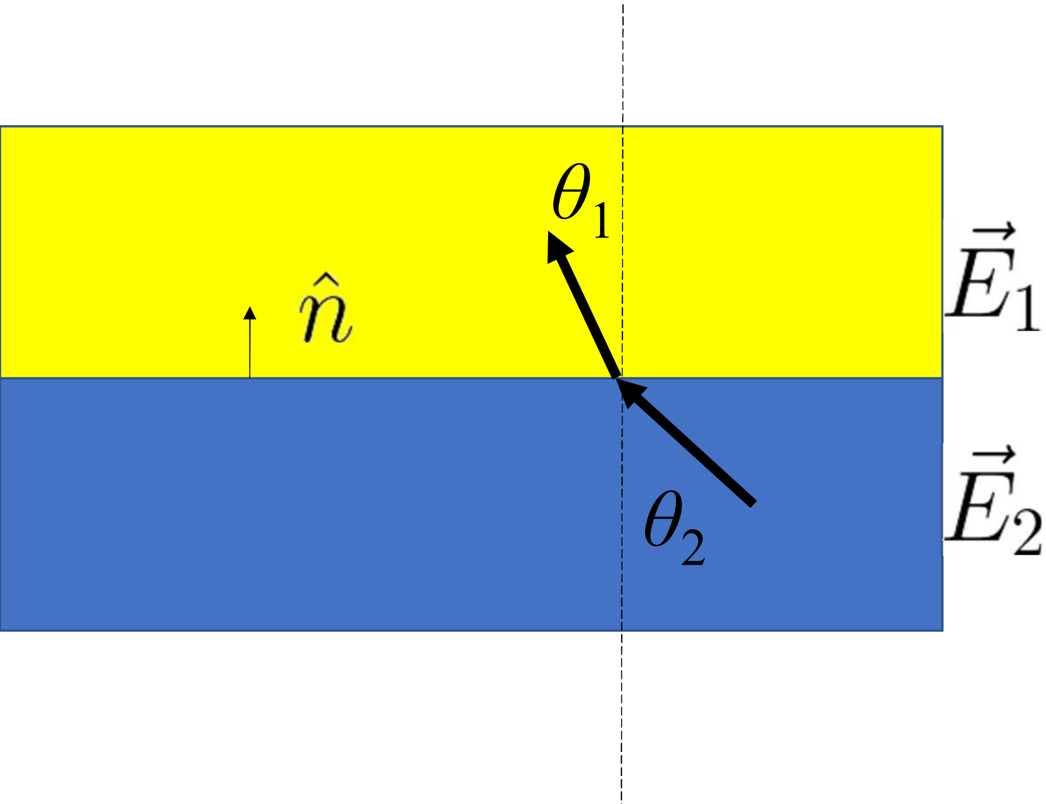
$$\sigma = \vec{P}_2 \cdot \hat{n} - \vec{P}_1 \cdot \hat{n}$$

$$\hat{n} \cdot [(\epsilon_0 \vec{E}_1 + \vec{P}_1) - (\epsilon_0 \vec{E}_2 + \vec{P}_2)] = 0$$

$$\hat{n} \cdot \vec{D}_1 = \hat{n} \cdot \vec{D}_2$$

Boundary conditions

A boundary condition usually describes the normal or tangential component of the vector field.



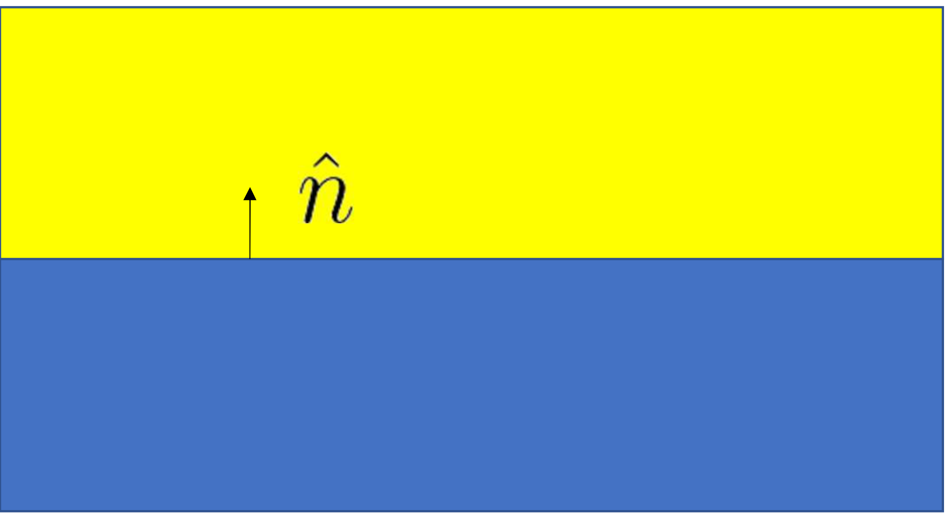
$$E_1 \sin\theta_1 = E_2 \sin\theta_2$$

$$\epsilon_1 E_1 \cos\theta_1 = \epsilon_2 E_2 \cos\theta_2$$

$$\tan\theta_1 = \frac{\epsilon_1}{\epsilon_2} \tan\theta_2$$

Boundary conditions

A boundary condition usually describes the normal or tangential component of the vector field.



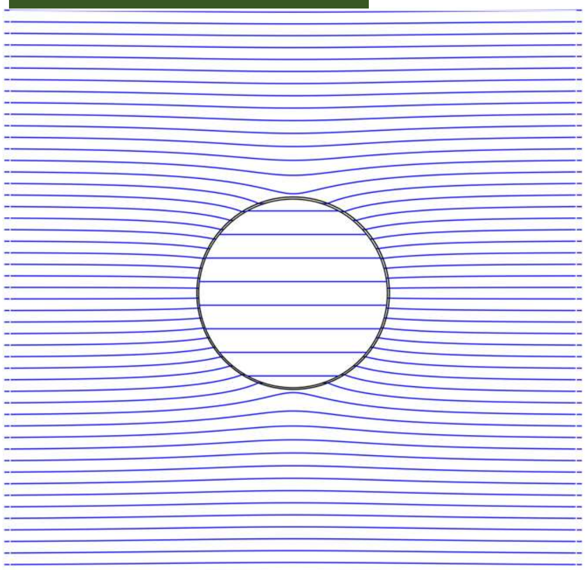
$$\phi_1 \qquad\qquad \phi_1 = \phi_2$$

$$\phi_2 \qquad\qquad \epsilon_1 \frac{\partial \phi_1}{\partial n} = \epsilon_2 \frac{\partial \phi_2}{\partial n}$$

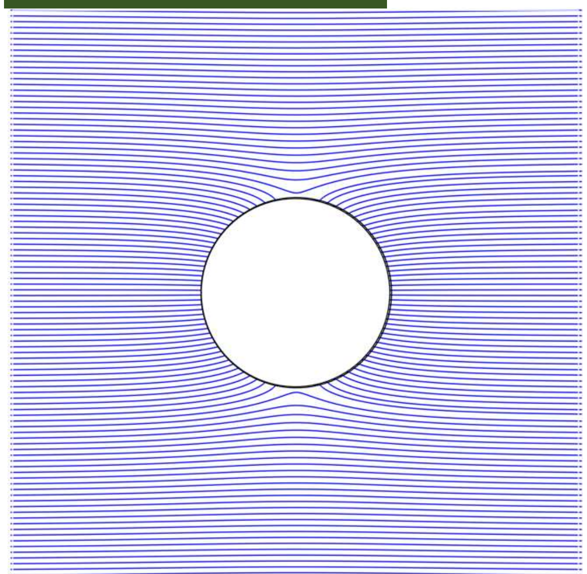
Unless there is free charge on the surface

$$\hat{n} \cdot (\vec{D}_1 - \vec{D}_2) = \rho_f$$

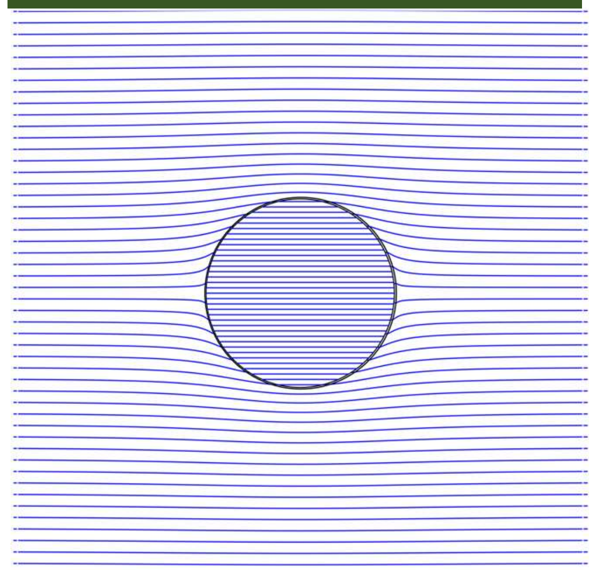
Dielectric ball



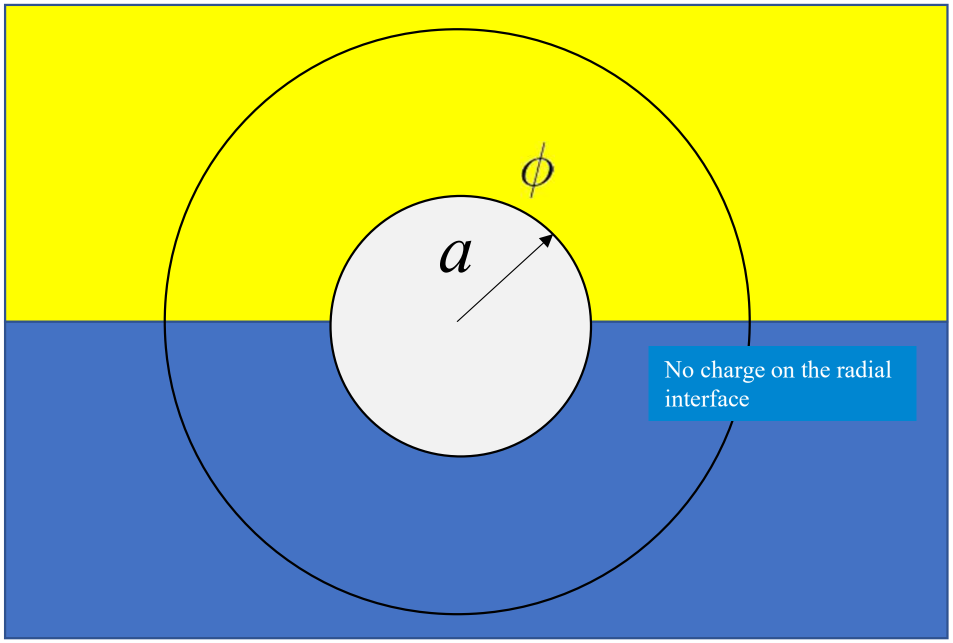
Conducting ball



Dielectric material with a hole



Example



ε_1

ε_2

If the potential of the ball is known to be φ, what is the field like?

Symmetry of the problem?

Volumetric charge?

Surface charge?

Starting with the simplest case.

$$\epsilon_0 \oint \vec{E} \cdot \mathrm{d}\vec{S} = Q_f + Q_p$$

Field

$$\vec{E} = \frac{Q_f + Q_p}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\phi = \frac{Q_f + Q_p}{4\pi\epsilon_0 a}$$

$$\vec{E} = \frac{\phi a}{r^2}\hat{r}$$

Charge

$$\sigma = \vec{P} \cdot (-\hat{r}) = -\chi_e \epsilon_0 \phi / a$$

$$Q_p = -2\pi a(\chi_1 + \chi_2)\epsilon_0 \phi$$

$$Q_f = 4\pi a \epsilon_0 \phi + 2\pi a(\chi_1 + \chi_2)\epsilon_0 \phi$$