Physics (PHYS2500J), Unit 2 Magnetostatics: 2. Magnetic materials

Xiao-Fen Li
Associate Professor, SJTU

Fall 2023

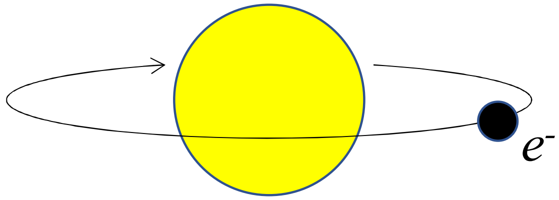
Contents

Question: what is the origin of magnetism in materials

We know the magnetic field generation and interaction of moving charged particles, and that of current. What about a simple magnet? A piece of iron needle?

Magnetic properties of materials come from microscopic motion of e^-

1. Ampere suggested the magnetic material are composed of little current loops.

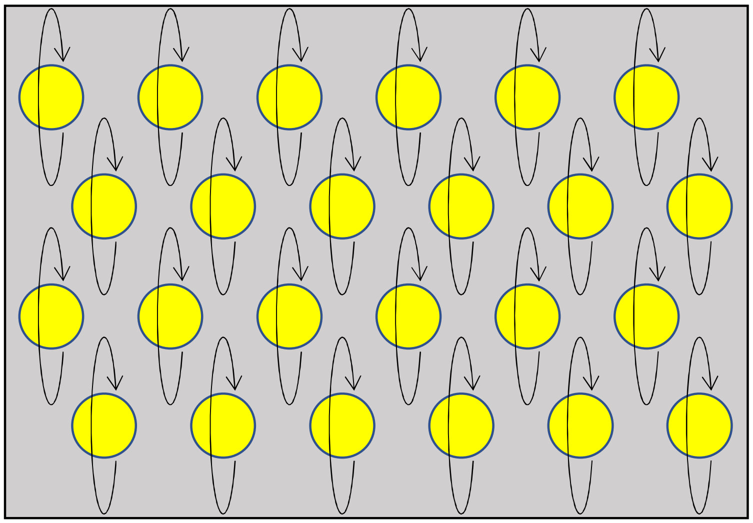


2. Now we know, the current corresponding to the orbital motion of electron is as shown in the figure.

Angular speed ω

$$I = \frac{Q}{t} = \frac{-e\omega t/2\pi}{t} = \frac{-e\omega}{2\pi}$$

3. A permanent magnet looks like the following:



The contribution of each small molecular current is measured by moment

1. The magnetic moment of one atom contains the moment of all its electrons.

$$\vec{m} = \sum (\vec{m}_l + \vec{m}_s)$$

Spin of electrons also generates magnetic moment

Orbital motion of electrons

All electrons involved

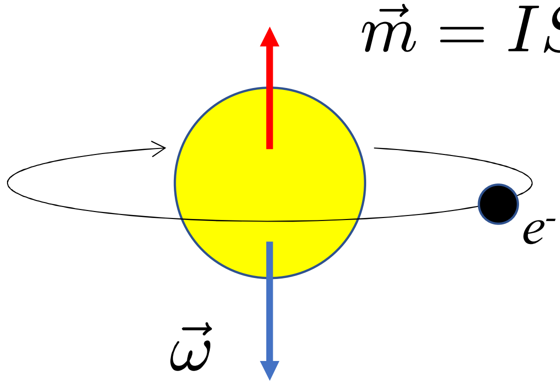
2. Take the orbital motion as an example, moment is calculated using the current and the area of the loop.

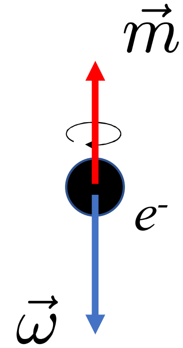
$$\vec{m} = I\vec{S} = \frac{-e\vec{\omega}}{2\pi}\pi R^2 = -\frac{eR^2\vec{\omega}}{2}$$

Current and moment

1. Direction of angular speed is opposite to direction of moment (electron carries negative charge);
2. Spin can not be described in classical picture, but it contributes to the moment, too.

$$\vec{m} = I\vec{S} = -\frac{e\vec{\omega}}{2\pi}S$$

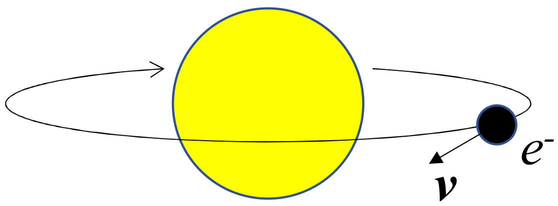




Larmor precession*

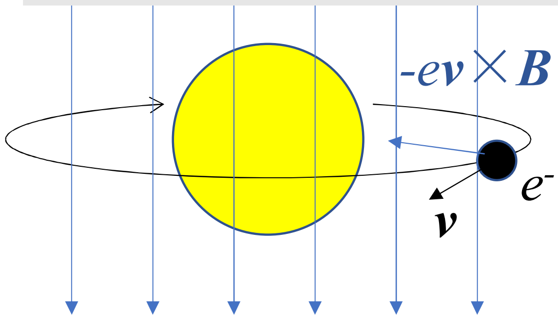
* Not required by the class.

1. Balance established for orbital motion.



$$m_e R\omega_0^2 = \frac{Qe}{4\pi\epsilon_0 R^2}$$

2. When magnetic field is applied to the atom, will ω increase or decrease?



Angular speed increases, an extra moment is induced.

The induced moment is reverse to applied field.

3. The quantitative description:

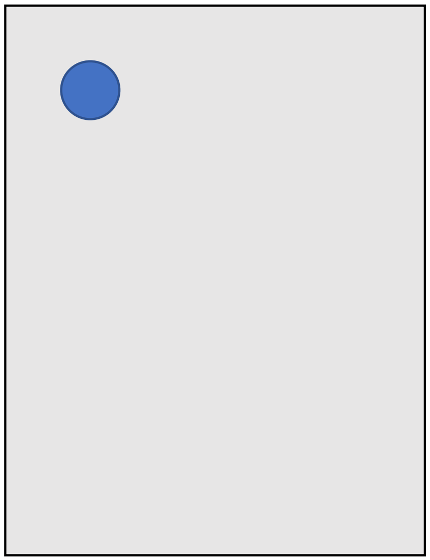
$$m_e R(\omega_0 + \delta\omega)^2 = \frac{Qe}{4\pi\epsilon_0 R^2} + eR(\omega_0 + \delta\omega)B$$

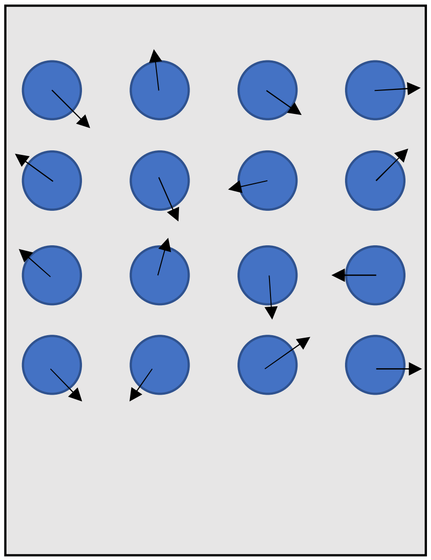
If $\delta\omega << \omega_0$

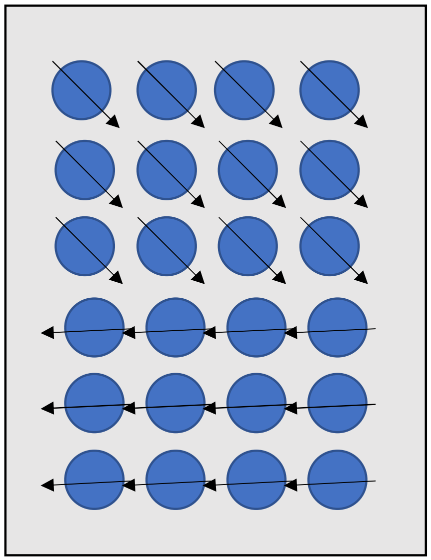
$$\delta\omega = \frac{eB}{2m_e}$$

Contents

Three types of material







1. The atom contains no net moment, since the contributions from all electrons cancels one another.

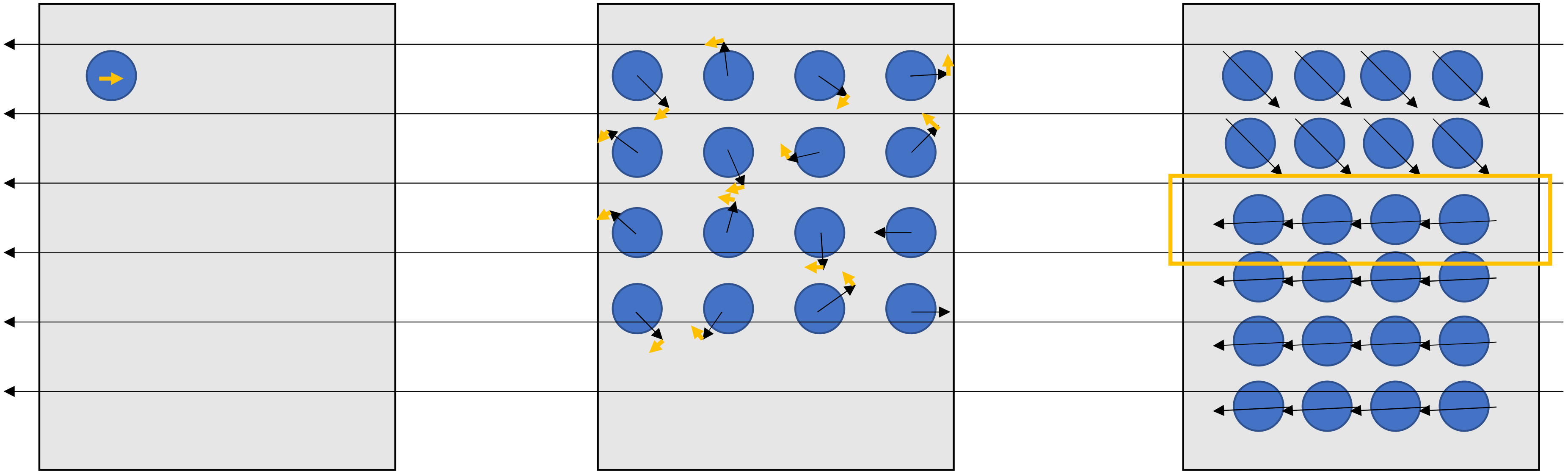
2. The atoms contain net moment. But thermal motion makes the direction of moment random. The material shows no magnetic properties.

3. The interactions between moments of neighboring atoms are strong, so that domains (inside which moments are aligned) are formed. However, the domains are randomly aligned. No macroscopic moment.

The three types of material shows no macroscopic magnetic properties.

When external magnetic field is applied.

Apply magnetic field to the left

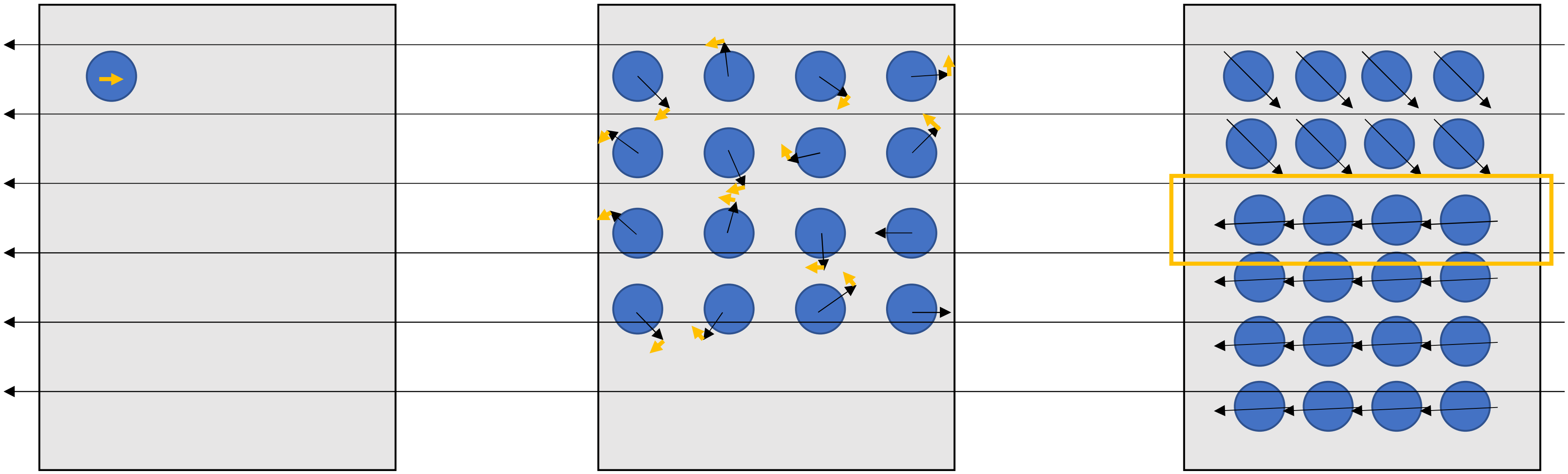


Reverse moment induced. (Larmor precession, universal to any atoms).

Moments align themselves with externally applied magnetic flux. (Reverse moment also induced but not comparable to the former factor)

The domain which is in the same direction with field is growing bigger.

Three types of materials



Diamagnetism:

E.g. Highly oriented pyrolytic graphite, water, etc.

Paramagnetism:

E.g. O_2

Ferromagnetism:

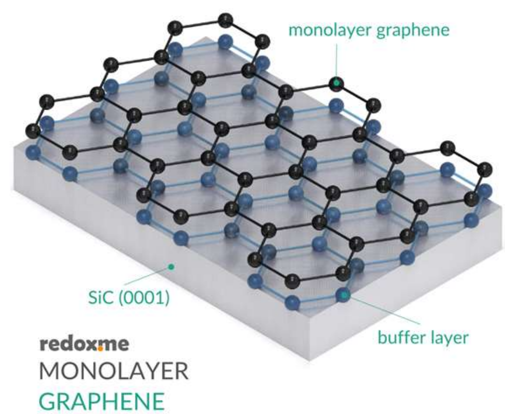
What we usually call magnetic materials. Such as Fe, Co, Ni, Gd, Fe_3O_4, etc.

Diamagnetic Levitation (introduction)

Video:

https://www.kjmagnetics.com/blog.asp?p=diamagnetic-levitation

Andre Geim



Get back to the question

Energy of a magnetic moment in magnetic field (understand)

Formerly, from the work done by Lorentz force torque on magnetic moment, we derived the energy of moment in magnetic field.

$$\mathcal{E} = -\vec{m} \cdot \vec{B}$$

1. If *m* is not affected by *B*, the moment will align itself to magnetic field. (paramagnetic and ferromagnetic materials).

$$\mathcal{E} = -mB$$

2. If field is not homogeneous, the moment will move to the location where the magnitude of field is largest. (magnetic attraction happened to a nail).

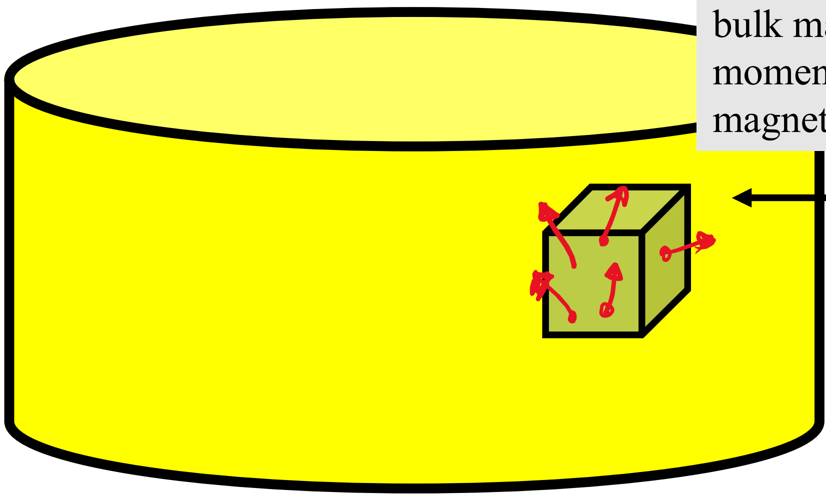
3. If *m* is a function of *B*, and unfortunately, $m = -kB$, which is the case for diamagnetic material, what happens?

The small moment tries its best to go to the place where the magnitude of *B* is small. (diamagnetic levitation)

Contents

Question: what is the magnetic field generated by a permanent magnet?

A new density variable: magnetization *M*



1. Consider a small volume *Ω* in a bulk magnetic material. The total moment and the density of magnetic moment.

Sum for multiple types of magnetic moments.

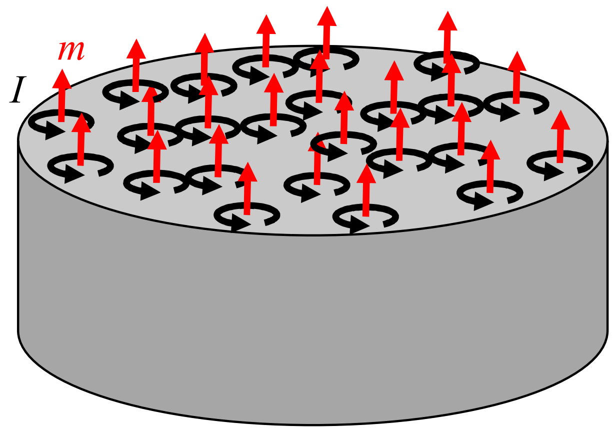
$$\vec{M} = \lim_{V=0} \frac{\sum_i n_i \vec{m}_i}{V}$$

$$\vec{M} = N\vec{m} = NI\vec{S}$$

Analogy to polarization vector *P*

$$\vec{P} = \lim_{V=0} \frac{\sum_i n_i \vec{p}_i}{V}$$

Surface current: the equivalence of a permanent magnet and a coil



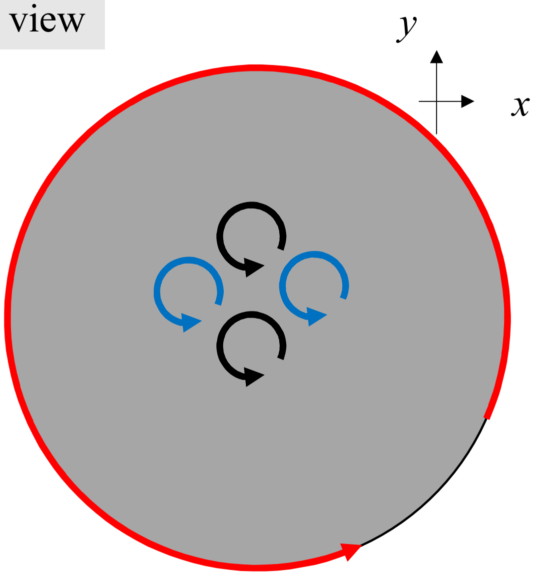
1. Homogeneously magnetized bulk, no macroscopic current density inside the material.

$$\vec{J}(\vec{r}) = 0, \ \vec{r} \in \Omega$$

J_x is 0 since the two black current loop cancels each other.

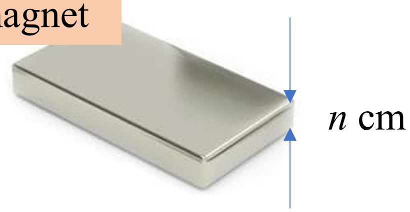
J_y is 0 since the two blue current loop cancels each other.

Top view



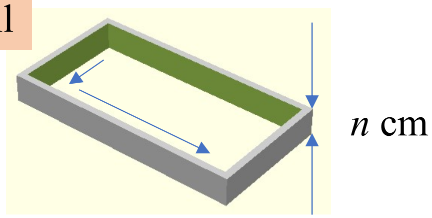
2. Only a surface current will appear.

A permanent magnet

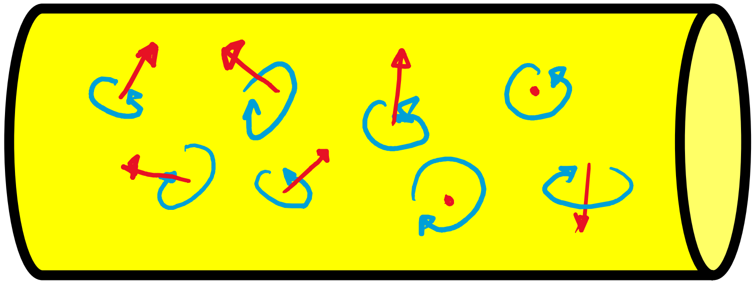


=

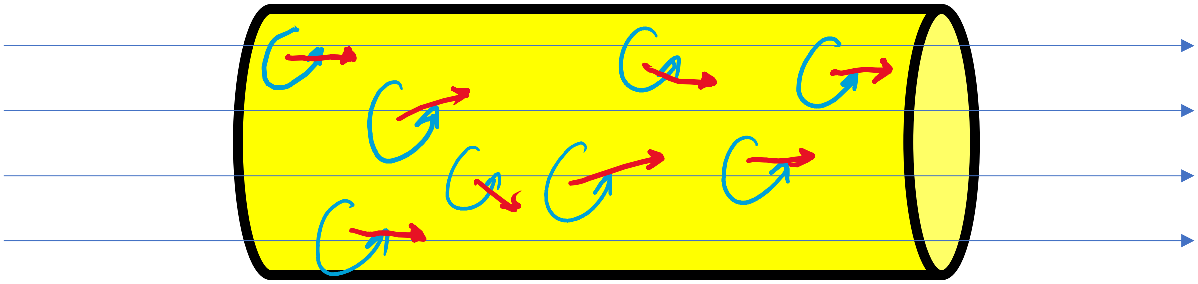
A loop coil



Paramagnetism from molecular current point of view



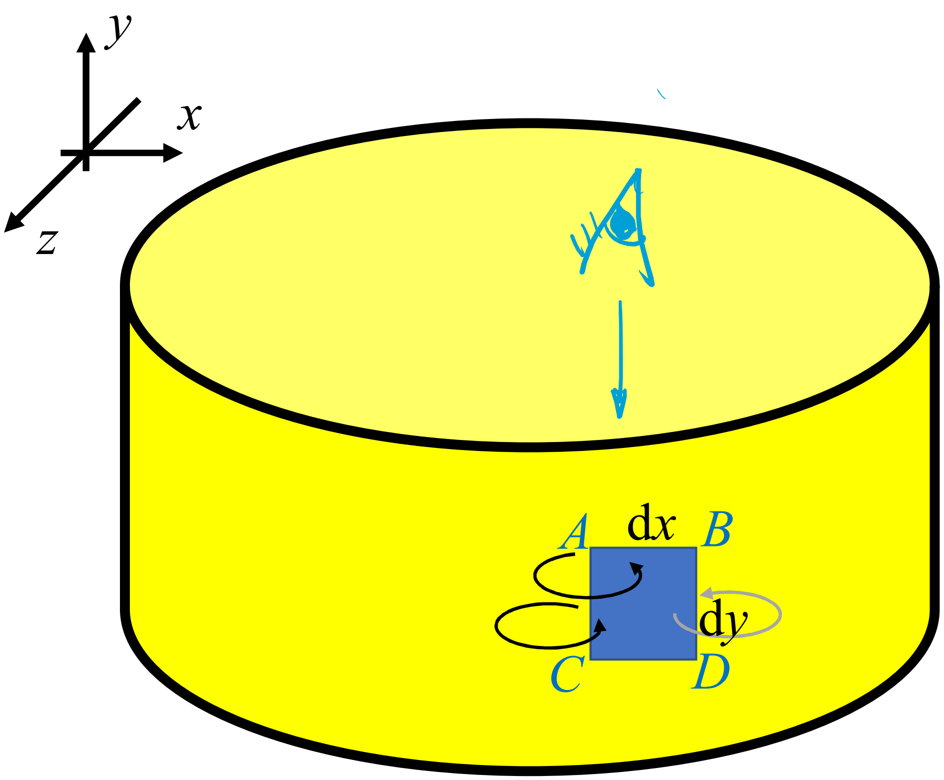
Molecular current, which is random oriented



Alignment under external field (paramagnetism).

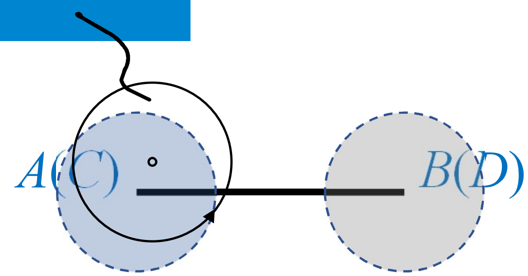
Quantitative relation between *J* and *M* *

* Not required by the class.



Black moments contributes current by *I* each, gray one contribute by -*I*

Example moment $m=SI$



The area of *I* contribution

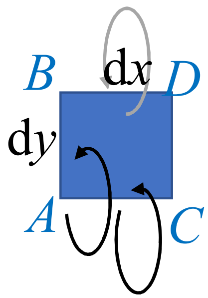
The area of -*I* contribution

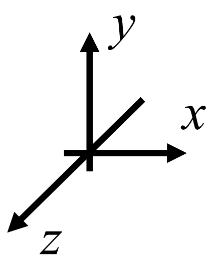
$$J=\frac{(n_{+}-n_{-})I}{\mathrm{d}x\mathrm{d}y}=\frac{(\mathrm{d}ySN(x)-\mathrm{d}ySN(x+\mathrm{d}x))I}{\mathrm{d}x\mathrm{d}y}$$

$$J=\frac{M(x)-M(x+\mathrm{d}x)}{\mathrm{d}x}$$

Or, $$-J_z=-\frac{\partial M_y}{\partial x}$$

General form of J





When M contains only y component,

$$-J_z = -\frac{\partial M_y}{\partial x}$$

When M contains only x component

$$-J_z = -\frac{\partial(-M_x)}{\partial y}$$

Or,

$$J_z = \frac{\partial M_y}{\partial x} - \frac{\partial M_x}{\partial y}$$

Repeat the analysis for the other two component of J

$$\vec{J}_m = \nabla \times \vec{M}$$

Surface and volumetric magnetization current density

1. Volumetric magnetization current density.

$$\vec{J}_m = \nabla \times \vec{M}$$

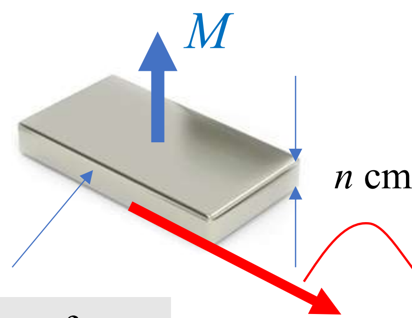
2. Compare the dimension of M and J

M is the volumetric density of moment m, the unit for M is A/m;

The unit for J is A/m^2.

3. Surface magnetization current density.

$$\vec{\sigma}_m = \vec{M} \times \hat{n}$$



4. e.g. on this surface

Direction of surface current

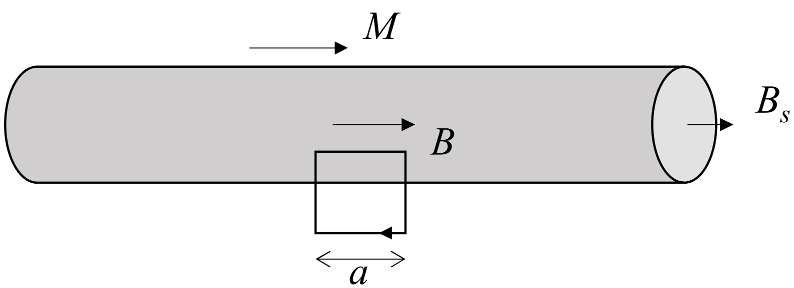
Magnitude of surface current

$$\sigma = M$$

5. When the wall is parallel to M, the surface current density is the same as M. Notice the unit for both is A/m.

Back to the question

A long cylindrical PM is axially magnetized, the magnetization $M = 1.19$MA/m. What is the surface magnetic flux density and flux density deeply inside the magnet?



1. This PM is similar to a solenoid.

$$\oint \vec{B} \cdot \mathrm{d}\vec{l} = \mu_0 I = \mu_0 \sigma a$$

$$B = \mu_0 \sigma = \mu_0 M = 1.50 \text{ T}$$

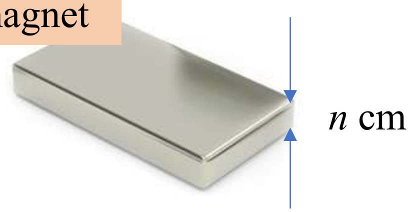
2. B_s can be considered a full solenoid cut by half. The center surface flux density is half of B.

$$B_s = \frac{B}{2} = 0.75 \text{ T}$$

3. The solenoid has a very high current density: 11.9 kA/cm

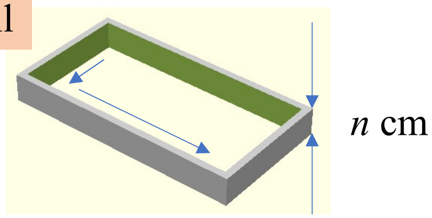
Is a hollow permanent magnet the same as a solenoid?

A permanent magnet



=

A loop coil



What about the hollow magnet below?

One should not forget about the reverse current loop in the hole.

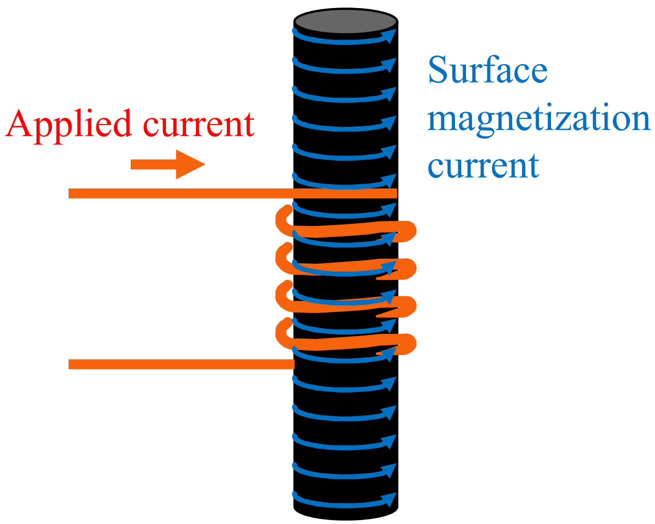
Contents

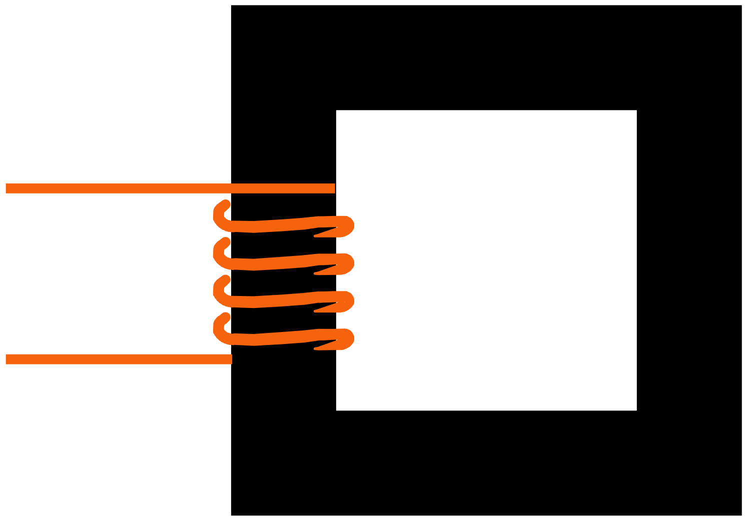
Question: How to study an electromagnet or a transformer?

1. The source of magnetic field is motion of charged particles: free motion (like a free electron in space), drift motion (solution, plasma, metal), microscopic orbital motion (molecular current).

2. A practical problem: what is the source of magnetic field in the left figure (an electromagnet).

3. A more important problem: How to quantitatively describe the behavior of a transformer.





An easy way is to find a vector directly relates to free current

1. Magnetic field is generated by current density (no matter if it is free current or surface magnetization current).

$$\nabla \times \vec{B} = \mu_0 \vec{J} = \mu_0 (\vec{J}_f + \vec{J}_m)$$

2. From geometrical analysis, we found one particular relation between $\boldsymbol{J}_m$ and a (seems irrelevant) vector $\boldsymbol{M}$

$$\nabla \times \vec{M} = \vec{J}_m$$

3. Can you create a vector, say $\boldsymbol{H}$, relates only to the free current $\boldsymbol{J}_f$?

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

$$\nabla \times \vec{H} = \nabla \times \frac{\vec{B}}{\mu_0} - \nabla \times \vec{M} = \vec{J}_f$$

This new vector $\boldsymbol{H}$ has a name: **Magnetic field intensity.** What is its unit?

A/m in SI, Oe in Gauss units.

Reform the relation in a better way

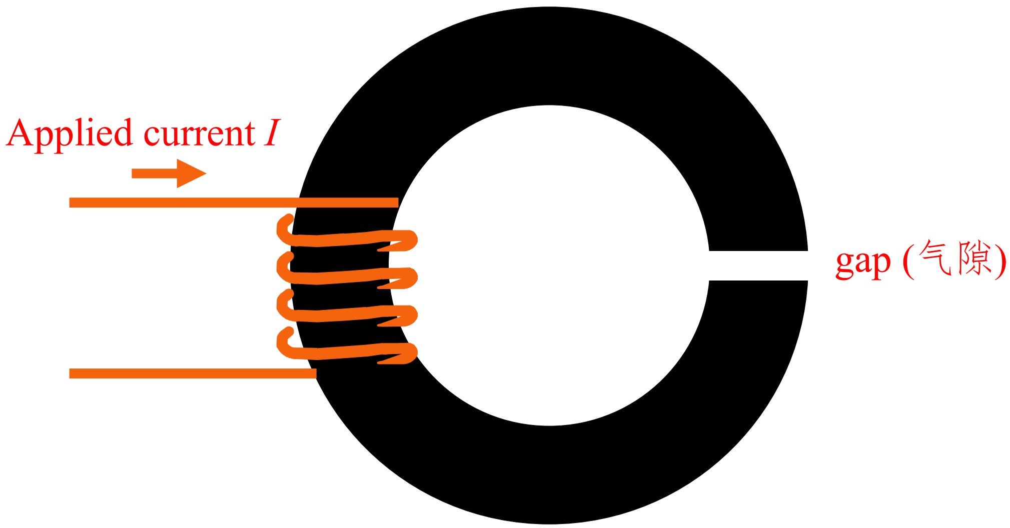
1. Relation between ***B***, ***H*** and ***M*** (important)

$$\vec{B} = \mu_0(\vec{H} + \vec{M})$$

2. Ampere's circuital law for ***B*** and ***H*** (important)

$$\oint \vec{B} \cdot \mathrm{d}\vec{l} = \mu_0 I$$

$$\oint \vec{H} \cdot \mathrm{d}\vec{l} = I_f$$



3. This is a typical design of an electromagnet, utilizing the good properties of iron to create a strong magnetic field in the gap with acceptable current (requires high power, high heat).

4. From the equation of B, we know the magnetization current (in the order of kA/cm) is contributing a lot to the gap field; from the equation of H, we get an easy way to determine the field intensity. (will show in a second)

The differential form of Ampere's circuital law

1. Relation between ***B***, ***H*** and ***M*** (important)

$$\vec{B} = \mu_0(\vec{H} + \vec{M})$$

2. Ampere's circuital law for ***B*** and ***H*** (important)

$$\oint \vec{B} \cdot \mathrm{d}\vec{l} = \mu_0 I$$

$$\oint \vec{H} \cdot \mathrm{d}\vec{l} = I_f$$

3. Differential form of Ampere's circuital law for ***B***, ***M*** and ***H***

$$\nabla \times \vec{B} = \mu_0 \vec{J}$$

$$\nabla \times \vec{H} = \vec{J}_f$$

$$\nabla \times \vec{M} = \vec{J}_m$$

4. Analogy to electric field case:

5. Relation between ***E***, ***D*** and ***P*** (important)

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

6. Gauss's law for ***E*** and ***D*** (important)

$$\oint_{\partial\Omega} \vec{E} \cdot \mathrm{d}\vec{S} = \frac{Q}{\epsilon_0}$$

$$\oint_{\partial\Omega} \vec{D} \cdot \mathrm{d}\vec{S} = Q_f$$

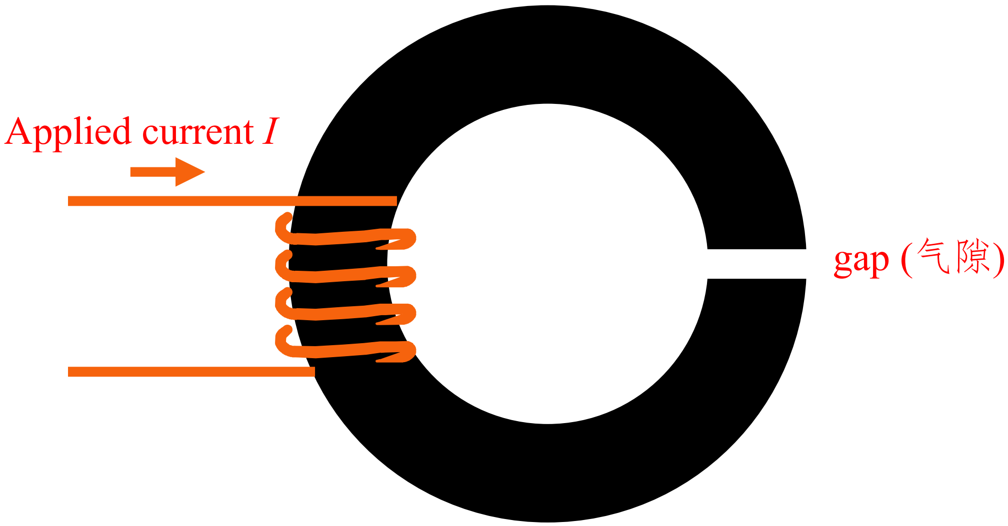
7. Differential form of Ampere's circuital law for ***E***, ***D*** and ***P***

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla \cdot \vec{D} = \rho_f$$

$$\nabla \cdot \vec{P} = -\rho_p$$

Question: What do we still miss?

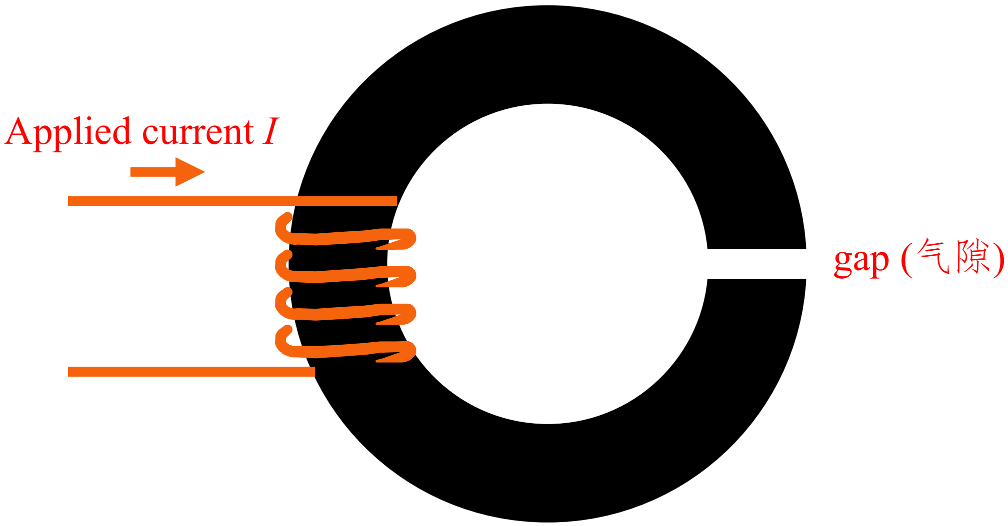


$$\oint \vec{B} \cdot \mathrm{d}\vec{l} = \mu_0 I$$

$$\oint \vec{H} \cdot \mathrm{d}\vec{l} = I_f$$

1. Can we solve this problem now? What do we miss?

Magnetic material properties



$$\oint \vec{B} \cdot \mathrm{d}\vec{l} = \mu_0 I$$

$$\oint \vec{H} \cdot \mathrm{d}\vec{l} = I_f$$

1. Can we solve this problem now? What do we miss?

2. We need the properties of the material.

$$\vec{M} = \chi \vec{H}$$

(Magnetic) susceptibility (磁化率) χ is defined with H instead of B for historical reasons.

Notice this relationship, since H is an auxiliary vector, χ itself does not have clear physical meaning which leads to the existence of a wired term called demagnetization field (not required).

3. Of course, we also have

$$\begin{aligned} B &= \mu_0(H + M) \\ &= \mu_0(H + \chi H) \\ &= \mu_r \mu_0 H \end{aligned}$$

$\mu_r = 1 + \chi$ Relative permeability (相对磁导率)

$\mu = \mu_r \mu_0$ permeability (磁导率)

Typical values of susceptibilities

1. For paramagnetic material, χ is in the order of 10^{-5};

μ_r is ~1.00001

2. For diamagnetic material, χ is in the order of -10^{-5};

μ_r is ~0.99999

3. For good ferromagnetic materials, χ (or μ_r) can reach 10^5;

VAC VACUUMSCHMELZE

Soft Magnetic Materials and Semi-finished Products

TABLE 28.1 Magnetic Susceptibilities of Paramagnetic and Diamagnetic Materials at $T = 20°C$

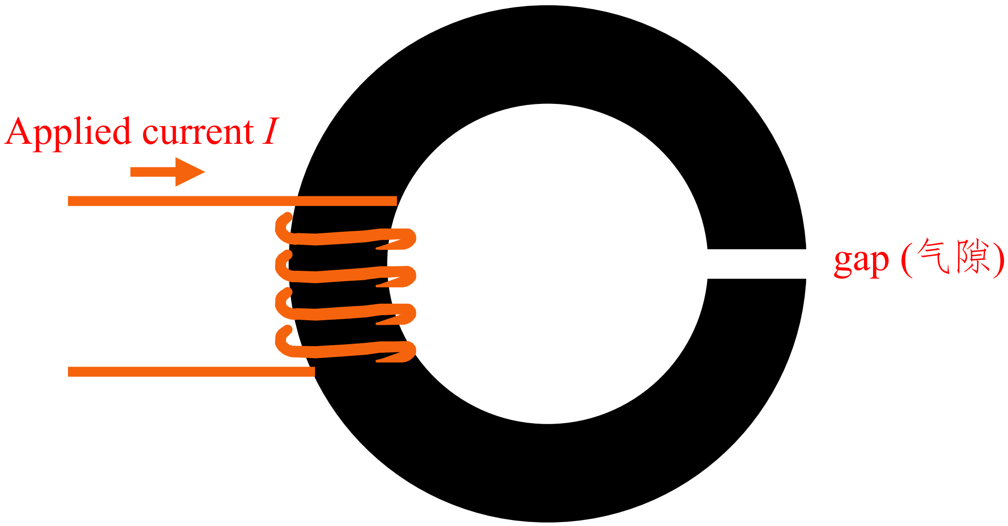
Material	$\chi_m = K_m - 1\ (\times 10^{-5})$
Paramagnetic	
Iron ammonium alum	66
Uranium	40
Platinum	26
Aluminum	2.2
Sodium	0.72
Oxygen gas	0.19
Diamagnetic	
Bismuth	−16.6
Mercury	−2.9
Silver	−2.6
Carbon (diamond)	−2.1
Lead	−1.8
Sodium chloride	−1.4
Copper	−1.0

Table 8: Static Properties of Solid Material. Measured on Stamped Rings, S

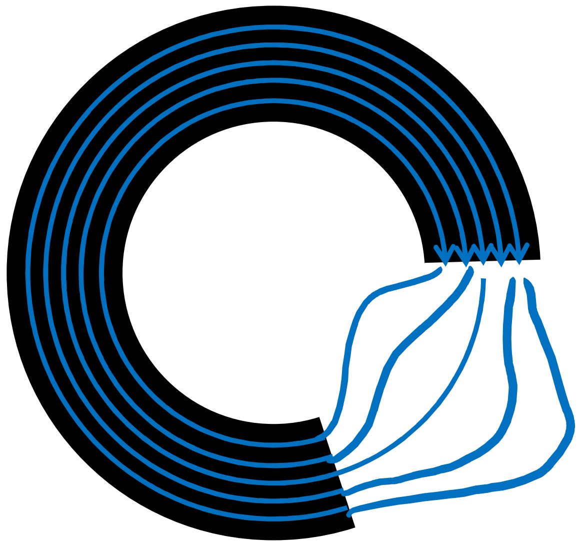
Material	Permeability μ_4	Permeability $\mu_{max.}$
MUMETALL	60000	250000
VACOPERM 100	200000	350000
PERMENORM 5000 H2	7000	120000
PERMENORM 5000 V5	9000	135000
PERMENORM 5000 S4	15000	150000
RECOVAC 50	3500	30000
MEGAPERM 40L	6000	80000
CHRONOPERM 36	6000	50000
PERMENORM 3601 K5	4000	50000
TRAFOPERM N3	1000	30000
VACOFER S1	2000	40000
VACOFLUX 50	1000	9000
VACOFLUX 17	600	4000

Now we can solve the problem of magnet design

Principle 1: constant flux



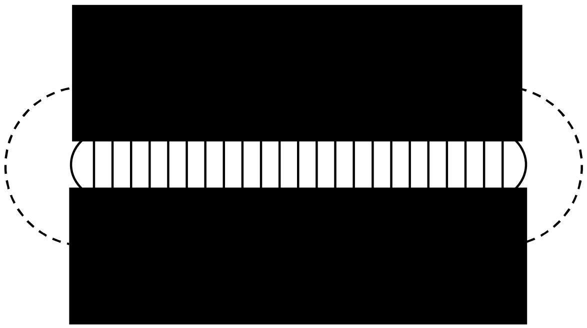
Principle 1: Since flux line is always continuous, this principle is always true (similar to current in a steady state (DC) circuit), even if the gap is huge.



Magnetic flux Φ is playing a role like I in circuit

Principle 2: *B* almost continuous if the gap is small

1. Principle 2: If the gap is small (compared to the cross section of the gap), major part of flux will go through the gap itself.



The dotted line path contains only a little flux.

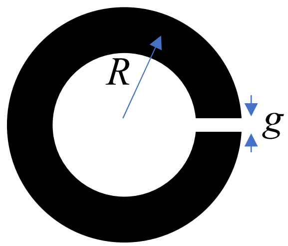
$$B_{gap}A_{gap} = \Phi = B_{iron}A_{iron}$$

2. If the cross section is the same (no special design like field focus),

$$B_{gap} = B_{iron}$$

B is continuous in the whole loop if cross section is not changed.

3. Establish the equation as



$$\oint \vec{H} \cdot \mathrm{d}\vec{l} = nI$$

$$\oint \vec{H} \cdot \mathrm{d}\vec{l} = H_{iron}(2\pi R - g) + H_{gap}g$$

4. Notice that

$$H_{iron} = B/\mu_r\mu_0$$
$$H_{gap} = B/\mu_0$$

B is the same for inside iron or gap.

5. magnetic flux density is obtained

$$B = \frac{n\mu_0 I}{g + \frac{2\pi R - g}{\mu_r}}$$

Several issues

$$B = \frac{n\mu_0 I}{g + \frac{2\pi R - g}{\mu_r}}$$

1. for good material, μ_r=10^5, if g = 1 mm and R = 50 mm.

$$g + \frac{2\pi R - g}{\mu_r} = 1.003 \text{ mm}$$

apparently, the second term contribute very little

$$B \approx \frac{n\mu_0 I}{g}$$

Conclusion 1. Magnetic field at the gap is mainly determined by the opening of the gap.

$$H_{iron} = B/\mu_r\mu_0$$
$$H_{gap} = B/\mu_0$$

Conclusion 2. surprisingly, H_{iron} is very small!

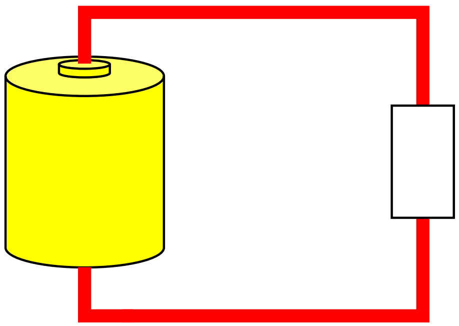
Logically, the reason is that if H_{iron} is not very small, B will be huge.

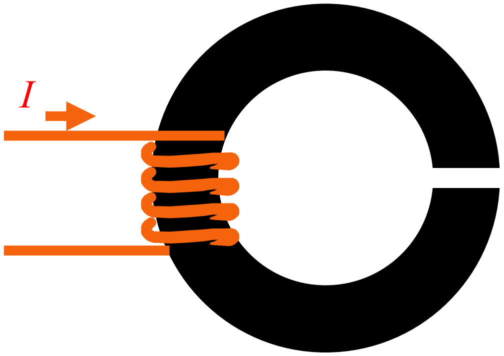
The "uncomfortableness" comes from the fact that H is an auxiliary vector. To explain why H is small, people has to create concepts like "demagnetization field" (退磁场).

Magnetic circuital law

* Not required by the class.

Notice the mathematical similarity between the following two cases, one can define a magnetic circuit:





$$\oint (\vec{K} + \vec{E}) \cdot \mathrm{d}\vec{l} = \mathcal{V}$$

$$\oint \rho \vec{J} \cdot \mathrm{d}\vec{l} = \mathcal{V}$$

$$\oint \vec{H} \cdot \mathrm{d}\vec{l} = nI$$

$$\oint \frac{1}{\mu} \vec{B} \cdot \mathrm{d}\vec{l} = nI$$

μ is playing the role of $1/\rho$.

nI is playing the role of *emf*.

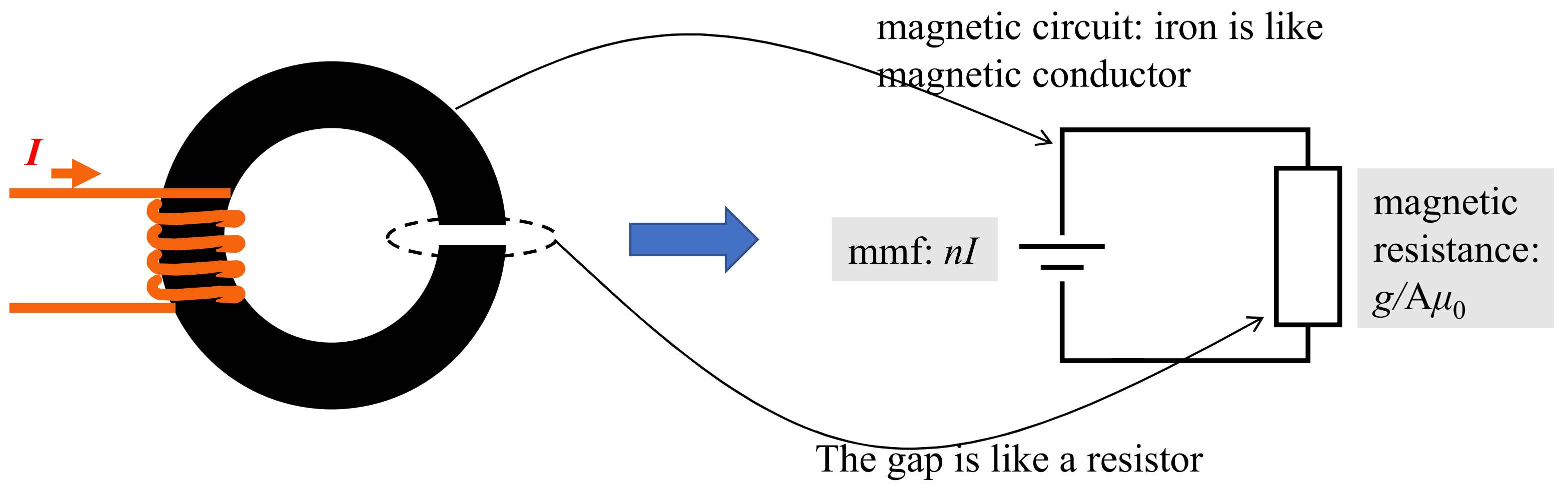
B is playing the role of *J*.

1. "current" in magnetic circuit: magnetic flux Φ, as seen in former slides, flux is a constant through the whole magnetic circuit.

2. "Resistance" in magnetic circuit: magnetic resistance $l\,/\,A\mu$. A better magnetic conductor (ferromagnetic material), a larger cross section, or shorter path allows more magnetic flux to go through under the same driving mmf (magnetomotive force NI).

上海交通大學
SHANGHAI JIAO TONG UNIVERSITY

1. simplify the field analysis into a circuit model



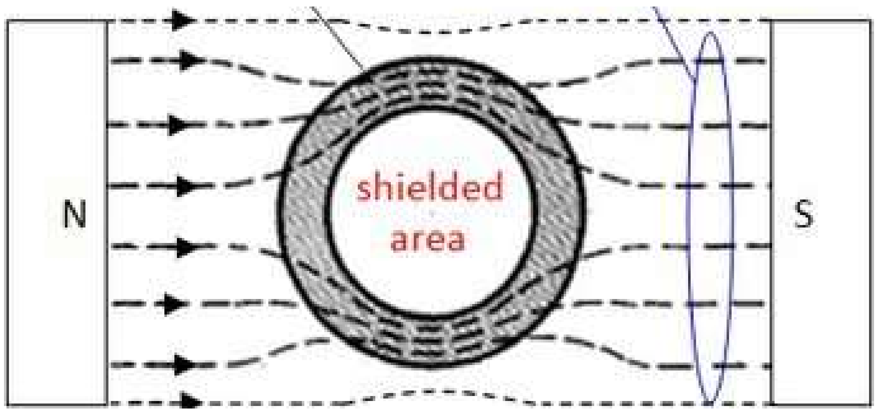
2. Use the familiar circuit method to solve the problem.

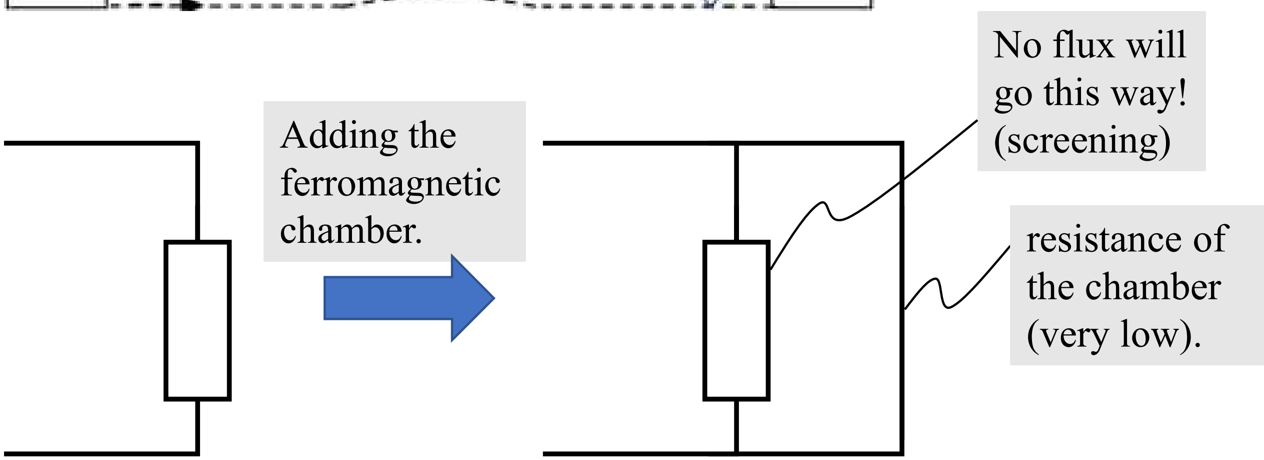
$$I = \frac{V}{R} \Rightarrow \Phi = \frac{nI}{g/\mu_0 A} \qquad B = \frac{n\mu_0 I}{g}$$

3. Similarly, one can actually establish a thermal circuit, gas flow circuit, etc. The major reason that electric circuit is mostly used: a. electric current is most useful; b. the electric conductivity has over 20 orders of magnitude difference, making the circuit method very precise.

Magnetic field screening (magnetic short circuit)

1. enclosed magnetic material chamber can serve as a magnetic field screening. The working principle is like to use a “short circuit” to by-pass the room to be screened.





Terminology

polarizability 极化率	χ_e	susceptibility 磁化率	χ
relative permittivity 相对介电常数	$\varepsilon_r = 1 + \chi_e$	relative permeability 相对磁导率	$\mu_r = 1 + \chi$
permittivity of vacuum 真空介电常数	ε_0	permeability of vacuum 真空磁导率	μ_0
permittivity 介电常数	$\varepsilon = \varepsilon_r \varepsilon_0$	permeability 磁导率	$\mu = \mu_r \mu_0$

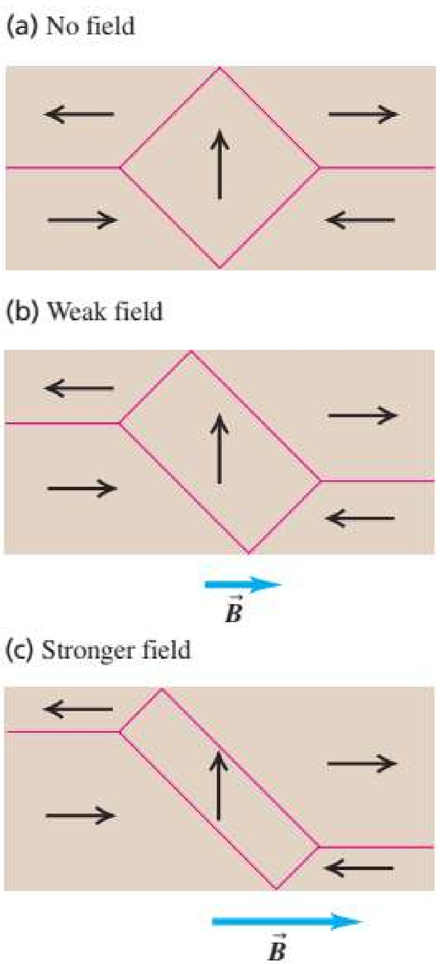
Contents

Ferromagnetic material is not linear

In designing of magnetic circuit or transformer, two aspects are very important:

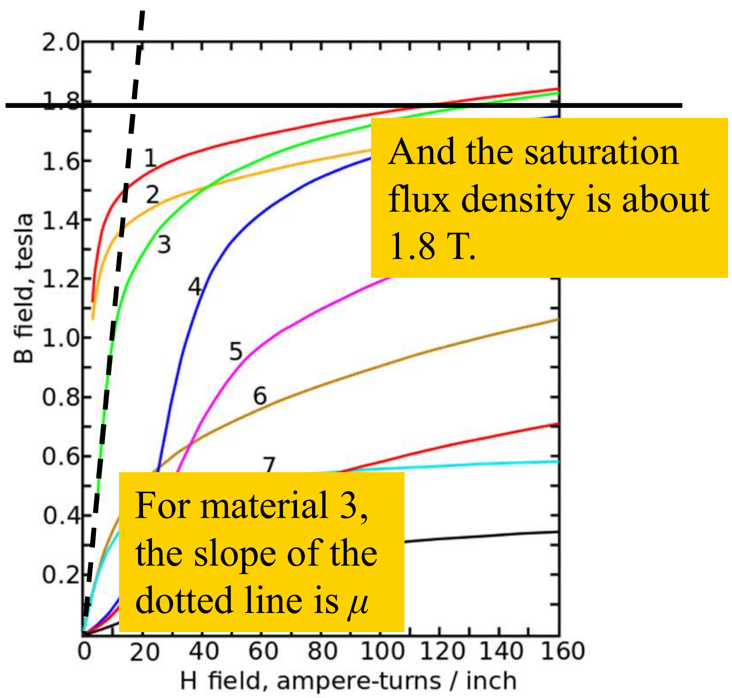
saturation of magnetic material
iron loss (hysteresis loss)

Nonlinear magnetization curve: 1. saturation



1. watching the principle of the ferromagnetism, is there an upper limit for the magnetization ***M*** of iron?

2. Of course there is a limit, when all the existing moment is well aligned, with a tiny thermal disturbance. The question is just how much is that limit.



And the saturation flux density is about 1.8 T.

For material 3, the slope of the dotted line is μ

graph: *https://commons.wikimedia.org/wiki/File:Magnetization_curves.svg*

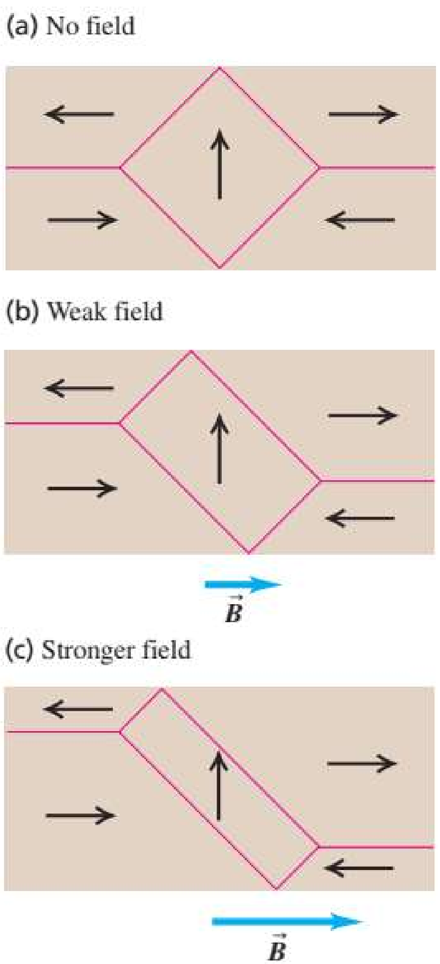
3. It turns out that the saturation magnetization, or saturation flux density for magnetic material is not very high.

In the figure, B increases with H linearly at first, than becomes slower, finally stop fast increasing. Saturation happens.

The saturation is a serious problem to consider when designing any magnetic circuit!

4. Typical values for saturation flux density:
0.5 T to 2.2 T

Nonlinear magnetization curve: 2. hysteresis behavior



1. again, watching the principle of the ferromagnetism, do you think when the applied field is removed, the magnetic domains will recover the initial state?

2. As one can imaging, sometimes, most will recover; sometimes it barely recovers.

Soft ferromagnetic material

Hard ferromagnetic material

3. examples:

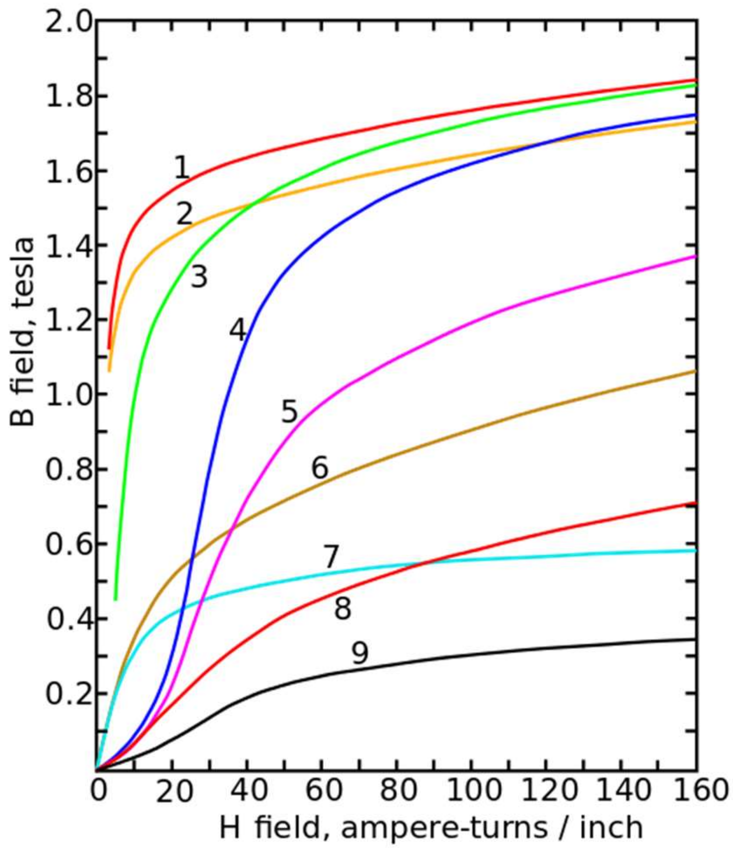
silicon steel, permalloy

permanent magnets

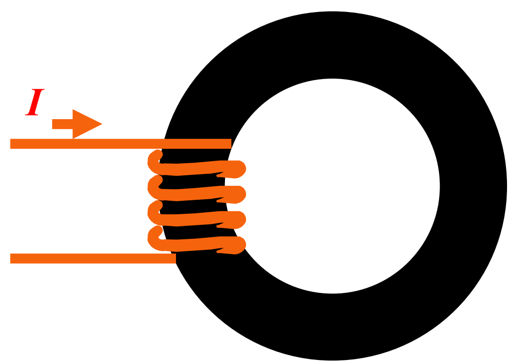
The property that the magnetization is not fully recovering is called hysteresis.

Hysteresis curve

1. The nonlinear properties of material magnetization is usually described by a *B*-*H* curve, or *M*-*H* curve.



2. The curve is measured by applying an increasing *H* to the following structure (a closed loop magnet). How do I control the *H* value I applied?



$$H = \frac{nI}{2\pi R}$$

3. Then the flux density *B* is measured to make the curve on the right. How do you measure *B* in a closed loop?

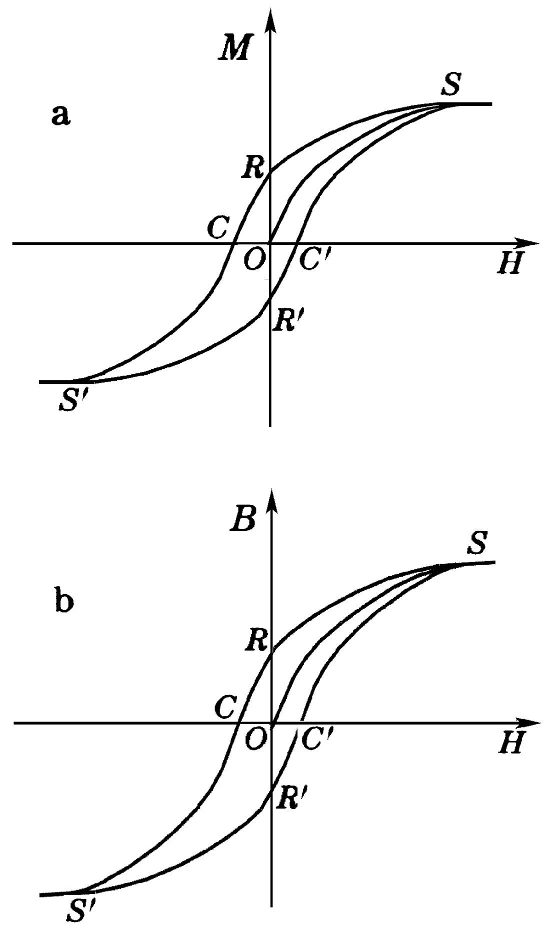
4. Measure the induced voltage in the coil and use Faraday's law (Haven't introduced yet).

Hysteresis curve

1. If the flux density does not recover when the applied *H* is reduced. What will the B-H curve look like?

Hint: for example, a permanent magnet is first magnetized by a strong field after it is manufactured. Then the flux density (and the *M*) is left in the material.

2. The *B*-*H* curve now splits into two branches. The ascending field branch and the descending field branch.



Coercive force and remanent field

* Not required by the class.

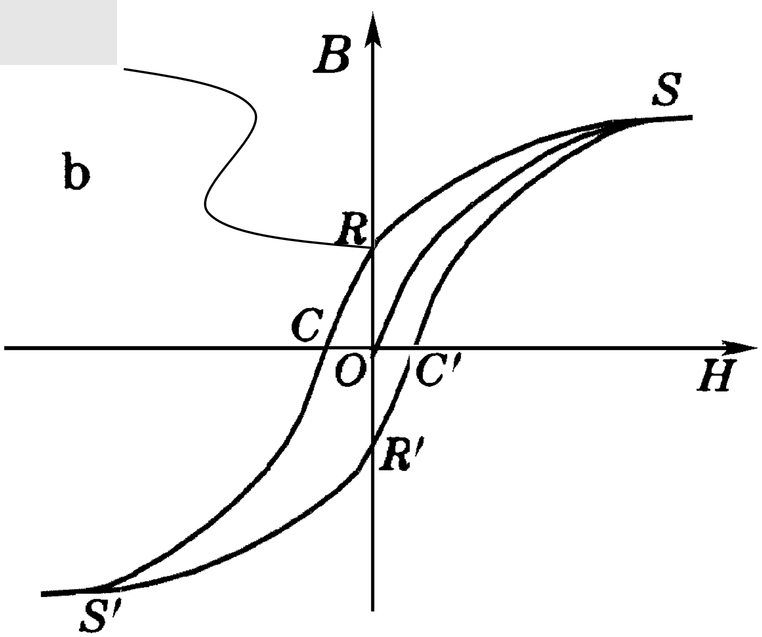
B_r (M_r) Remanent field (Magnetization)

剩余磁感应强度

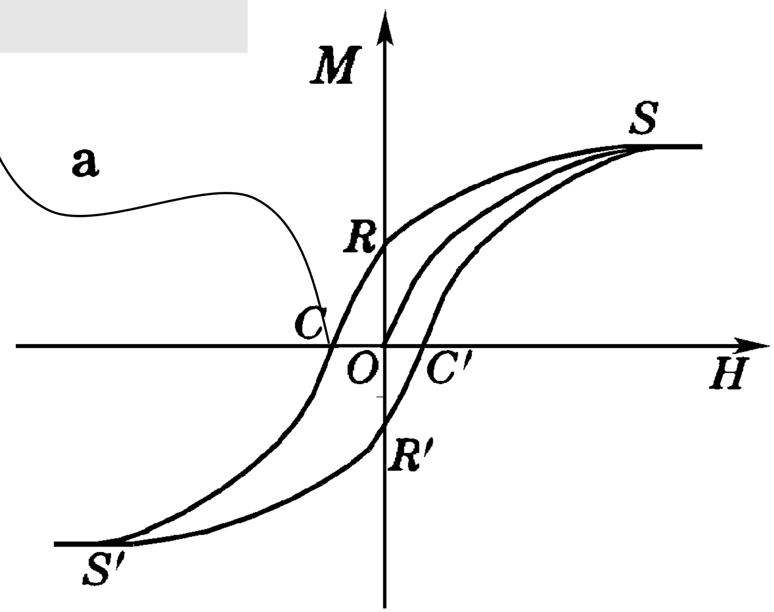
H_{cb} (H_{cm}) coercive force

矫顽力

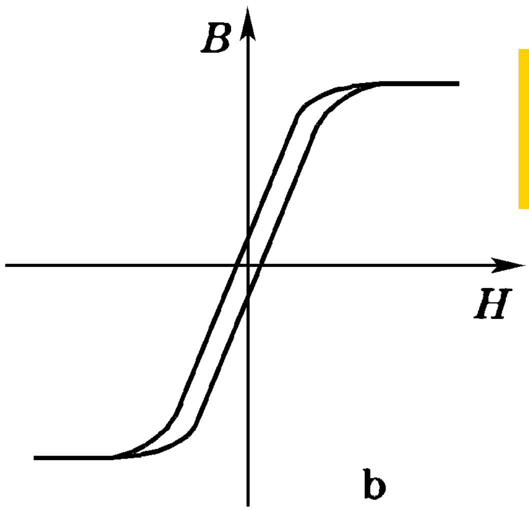
b



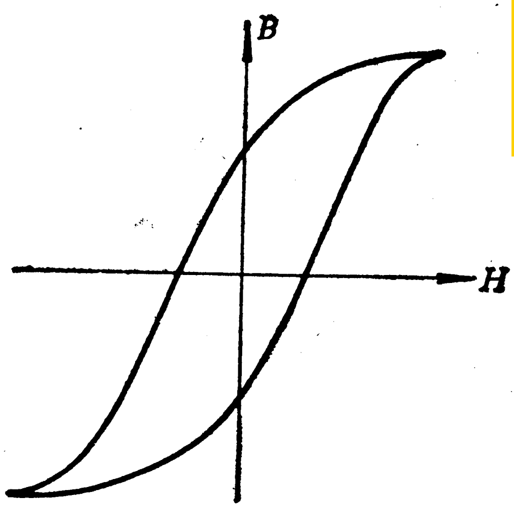
a



Hysteresis loss



Soft ferromagnetic materials are those whose hysteresis curve has a small opening

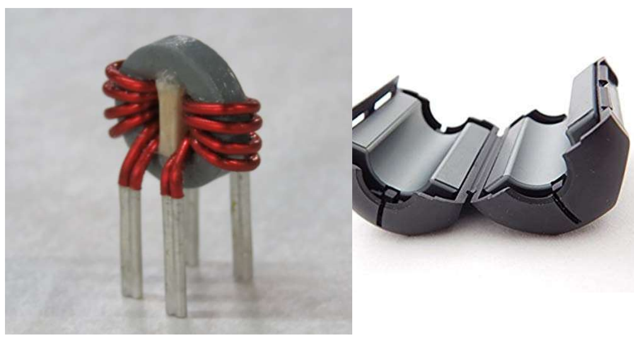


hard ferromagnetic materials are those whose hysteresis curve has a larger opening

1. The beauty of hard magnetic material is to make strong permanent magnets.

2. The opening of the hysteresis curve means its going to have energy dissipation (heat) in AC operations. The following two types of materials are soft magnetic materials which has low magnetic loss (iron loss, hysteresis loss)

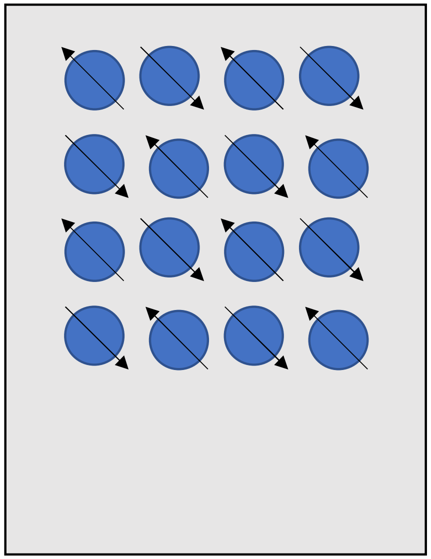
Ferrite



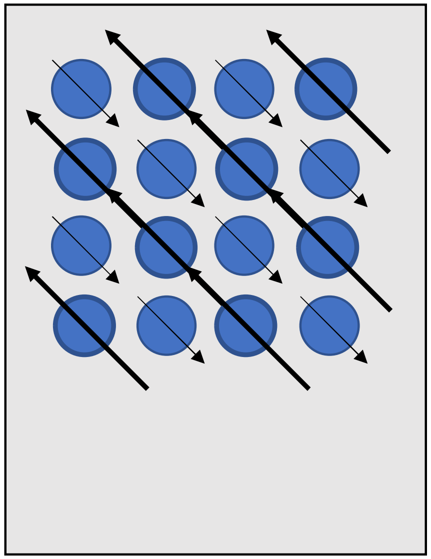
Silicon steel

Other types of magnetism

* Not required by the class.



Anti-ferromagnetism: the microscopic moments are ordered, but aligned reversely. Macroscopically, not showing magnetic.



ferrimagnetism: reversely aligned moments in the magnetic domain. But not the same value, macroscopically, similar to ferromagnetism.

$$\chi = -1$$

Superconductivity: Macroscopic current screening the magnetic field (Type I superconductor is a perfect magnetic insulator). Perfect diamagnetism. (Meissner effect).