Physics (PHYS2500J), Unit 2 Magnetostatics:
2. Magnetic materials

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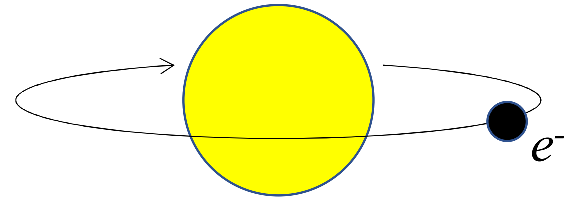
Question: what is the origin of magnetism in materials

We know the magnetic field generation and interaction of moving charged particles, and that of current. What about a simple magnet? A piece of iron needle?

Magnetic properties of materials come from microscopic motion of e^-

1. Ampere suggested the magnetic material are composed of little current loops.

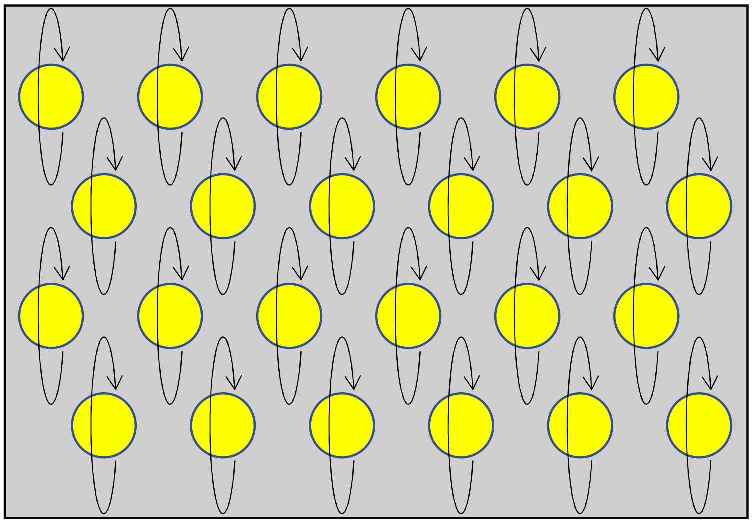
2. Now we know, the current corresponding to the orbital motion of electron is as shown in the figure.



Angular speed ω

$$I = \frac{Q}{t} = \frac{-e\omega t/2\pi}{t} = \frac{-e\omega}{2\pi}$$

3. A permanent magnet looks like the following:



The contribution of each small molecular current is measured by moment

1. The magnetic moment of one atom contains the moment of all its electrons.

$$\vec{m} = \sum (\vec{m}_l + \vec{m}_s)$$

Spin of electrons also generates magnetic moment

Orbital motion of electrons

All electrons involved

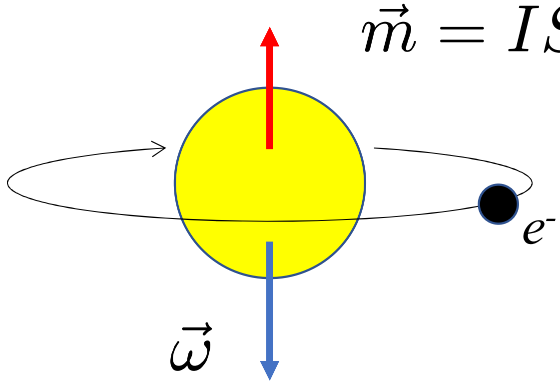
2. Take the orbital motion as an example, moment is calculated using the current and the area of the loop.

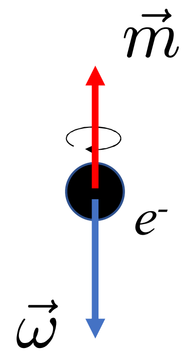
$$\vec{m} = I\vec{S} = \frac{-e\vec{\omega}}{2\pi}\pi R^2 = -\frac{eR^2\vec{\omega}}{2}$$

Current and moment

1. Direction of angular speed is opposite to direction of moment (electron carries negative charge);
2. Spin can not be described in classical picture, but it contributes to the moment, too.

$$\vec{m} = I\vec{S} = -\frac{e\vec{\omega}}{2\pi}S$$

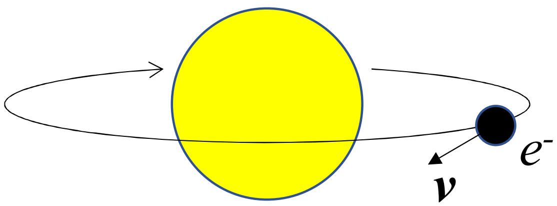




Larmor precession*

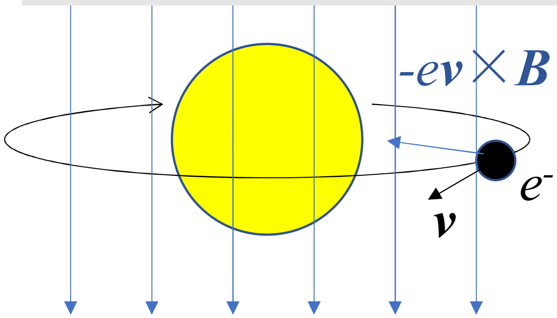
* Not required by the class.

1. Balance established for orbital motion.



$$m_e R\omega_0^2 = \frac{Qe}{4\pi\epsilon_0 R^2}$$

2. When magnetic field is applied to the atom, will ω increase or decrease?



Angular speed increases, an extra moment is induced.

The induced moment is reverse to applied field.

3. The quantitative description:

$$m_e R(\omega_0 + \delta\omega)^2 = \frac{Qe}{4\pi\epsilon_0 R^2} + eR(\omega_0 + \delta\omega)B$$

If $\delta\omega << \omega_0$

$$\delta\omega = \frac{eB}{2m_e}$$

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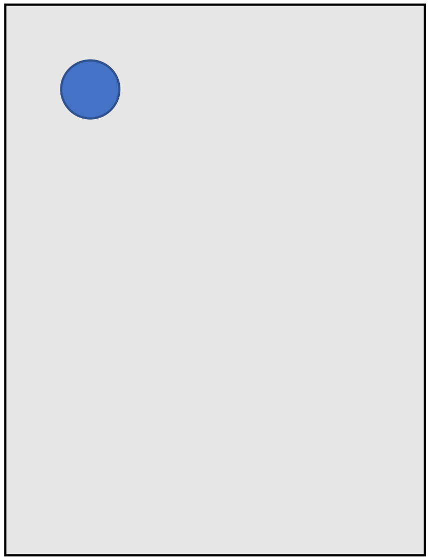
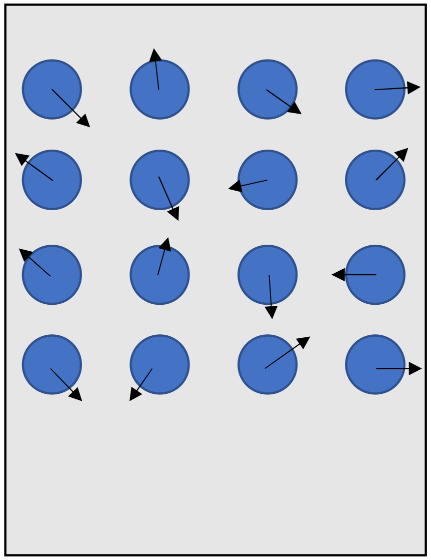
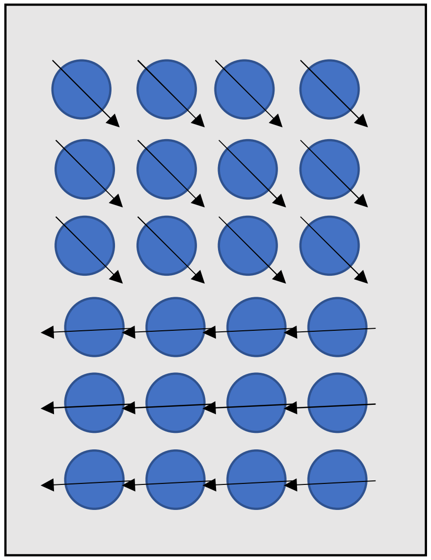
Three types of material



1. The atom contains no net moment, since the contributions from all electrons cancels one another.



2. The atoms contain net moment. But thermal motion makes the direction of moment random. The material shows no magnetic properties.

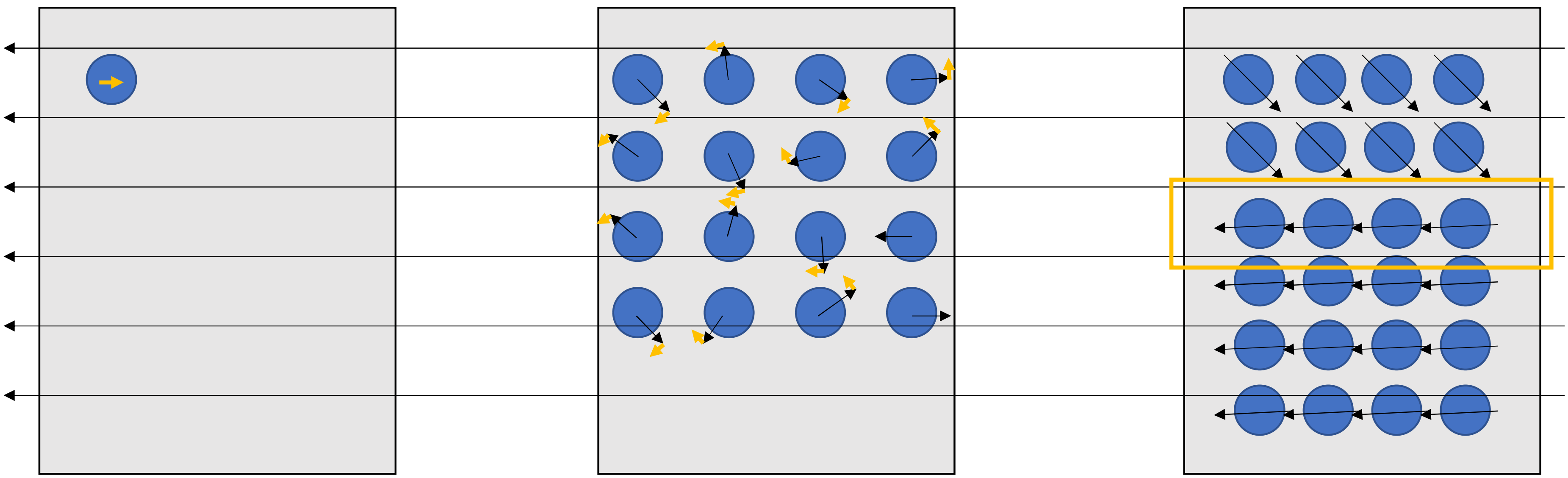


3. The interactions between moments of neighboring atoms are strong, so that domains (inside which moments are aligned) are formed. However, the domains are randomly aligned. No macroscopic moment.

The three types of material shows no macroscopic magnetic properties.

When external magnetic field is applied.

Apply magnetic field to the left

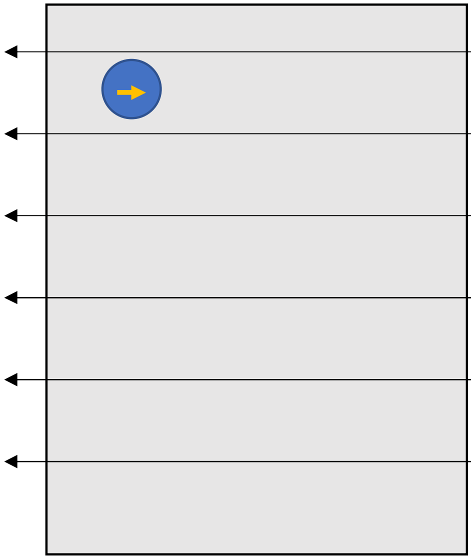


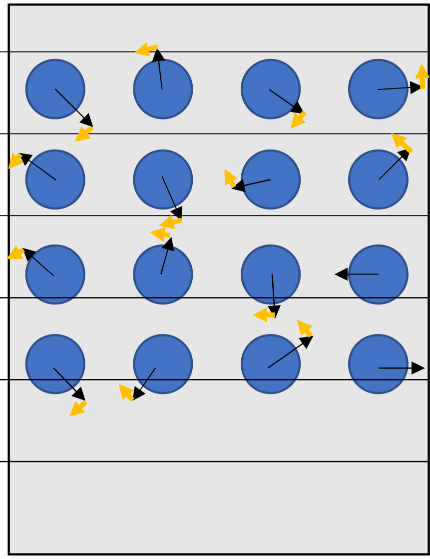
Reverse moment induced. (Larmor precession, universal to any atoms).

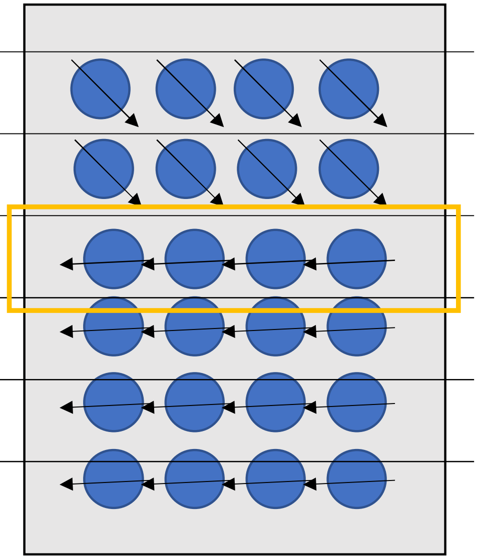
Moments align themselves with externally applied magnetic flux. (Reverse moment also induced but not comparable to the former factor)

The domain which is in the same direction with field is growing bigger.

Three types of materials







Diamagnetism:

E.g. Highly oriented pyrolytic graphite, water, etc.

Paramagnetism:

E.g. O_2

Ferromagnetism:

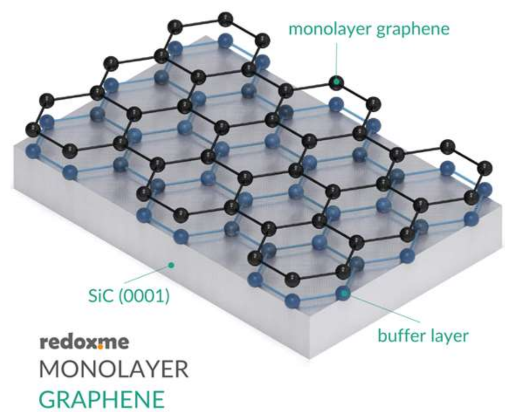
What we usually call magnetic materials. Such as Fe, Co, Ni, Gd, Fe_3O_4, etc.

Diamagnetic Levitation (introduction)

Video:

https://www.kjmagnetics.com/blog.asp?p=diamagnetic-levitation

Andre Geim



Get back to the question

Energy of a magnetic moment in magnetic field (understand)

Formerly, from the work done by Lorentz force torque on magnetic moment, we derived the energy of moment in magnetic field.

$$\mathcal{E} = -\vec{m} \cdot \vec{B}$$

1. If m is not affected by B, the moment will align itself to magnetic field. (paramagnetic and ferromagnetic materials).

$$\mathcal{E} = -mB$$

2. If field is not homogeneous, the moment will move to the location where the magnitude of field is largest. (magnetic attraction happened to a nail).

3. If m is a function of B, and unfortunately, $m = -kB$, which is the case for diamagnetic material, what happens?

The small moment tries its best to go to the place where the magnitude of B is small. (diamagnetic levitation)

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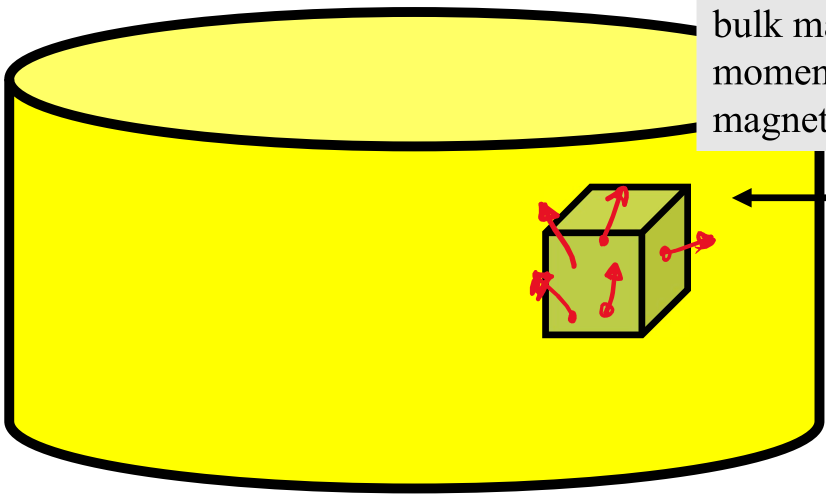
Question: what is the magnetic field generated by a permanent magnet?

A new density variable: magnetization *M*



1. Consider a small volume *Ω* in a bulk magnetic material. The total moment and the density of magnetic moment.

Sum for multiple types of magnetic moments.

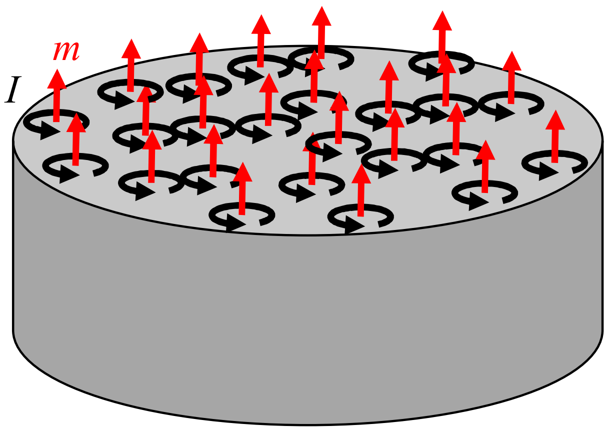
$$\vec{M} = \lim_{V=0} \frac{\sum_i n_i \vec{m}_i}{V}$$

$$\vec{M} = N\vec{m} = NI\vec{S}$$

Analogy to polarization vector *P*

$$\vec{P} = \lim_{V=0} \frac{\sum_i n_i \vec{p}_i}{V}$$

Surface current: the equivalence of a permanent magnet and a coil



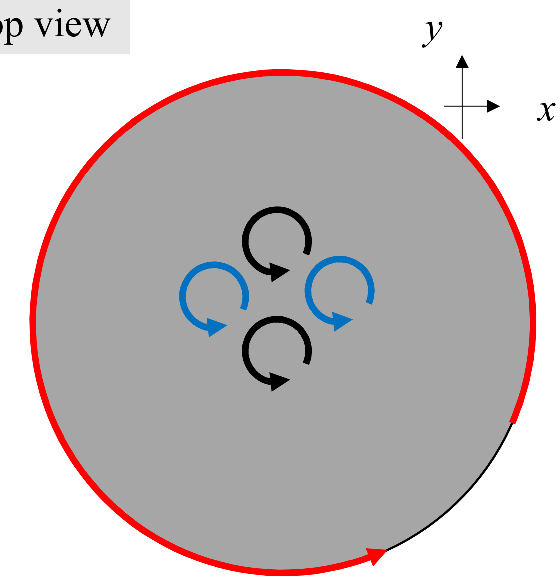
1. Homogeneously magnetized bulk, no macroscopic current density inside the material.

$$\vec{J}(\vec{r}) = 0, \;\; \vec{r} \in \Omega$$

J_x is 0 since the two black current loop cancels each other.

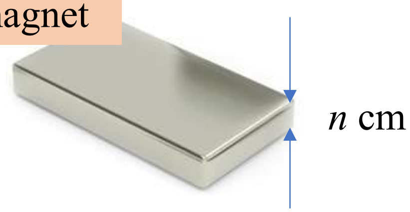
J_y is 0 since the two blue current loop cancels each other.

Top view



2. Only a surface current will appear.

A permanent magnet



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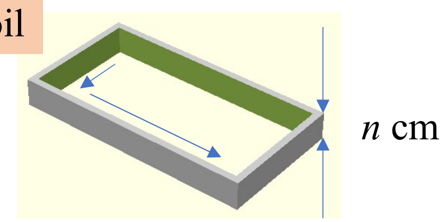
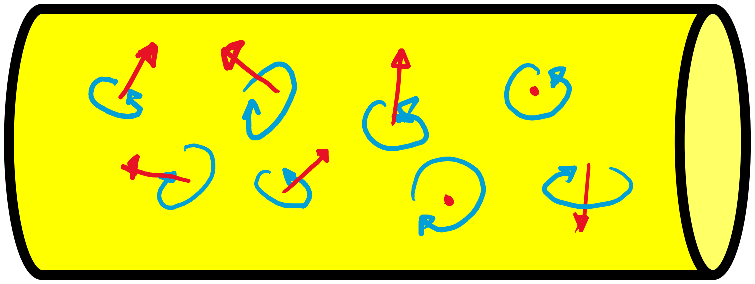
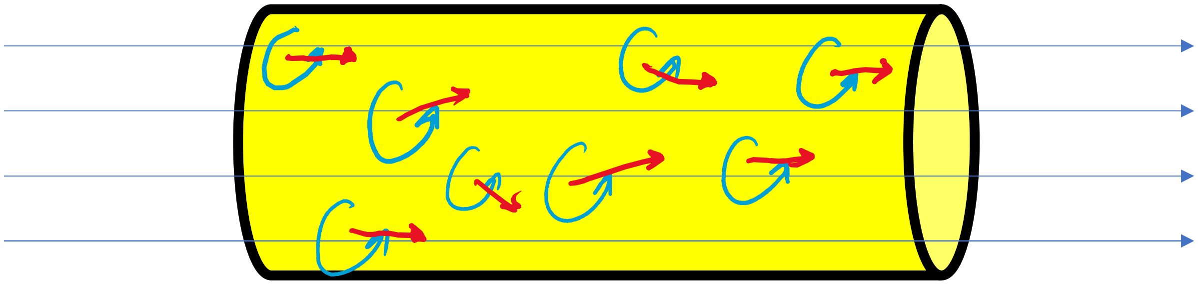
A loop coil



Paramagnetism from molecular current point of view



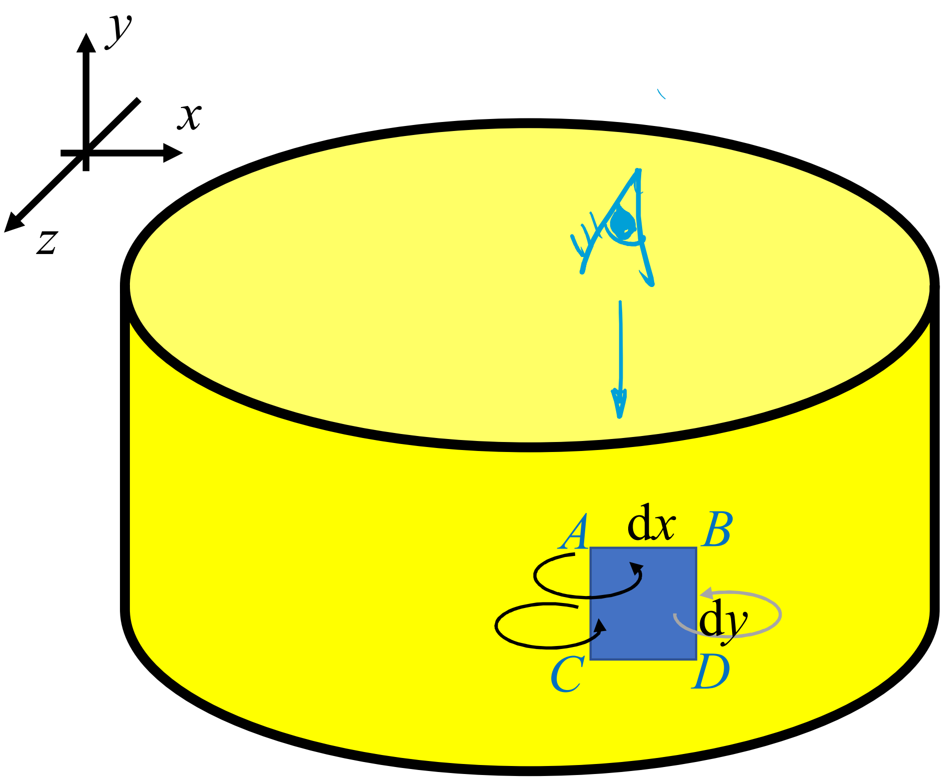
Molecular current, which is random oriented



Alignment under external field (paramagnetism).

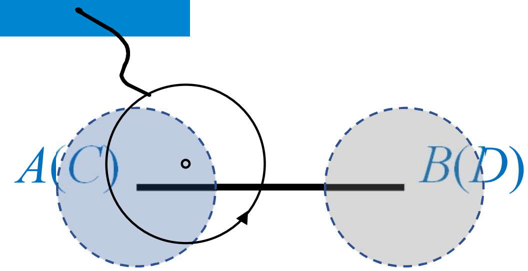
Quantitative relation between *J* and *M* *

* Not required by the class.



Black moments contributes current by *I* each, gray one contribute by -*I*

Example moment
m=*SI*



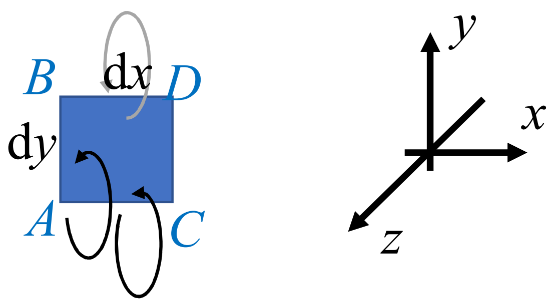
The area of *I* contribution

The area of -*I* contribution

$$J = \frac{(n_+ - n_-)I}{\mathrm{d}x\mathrm{d}y} = \frac{(\mathrm{d}ySN(x) - \mathrm{d}ySN(x+\mathrm{d}x))I}{\mathrm{d}x\mathrm{d}y}$$

$$J = \frac{M(x) - M(x+\mathrm{d}x)}{\mathrm{d}x}$$

Or, $$-J_z = -\frac{\partial M_y}{\partial x}$$



When *M* contains only *y* component,

$$-J_z = -\frac{\partial M_y}{\partial x}$$

When *M* contains only *x* component

$$-J_z = -\frac{\partial(-M_x)}{\partial y}$$

Or,

$$J_z = \frac{\partial M_y}{\partial x} - \frac{\partial M_x}{\partial y}$$

Repeat the analysis for the other two component of *J*

$$\vec{J}_m = \nabla \times \vec{M}$$

Surface and volumetric magnetization current density

1. Volumetric magnetization current density.

$$\vec{J}_m = \nabla \times \vec{M}$$

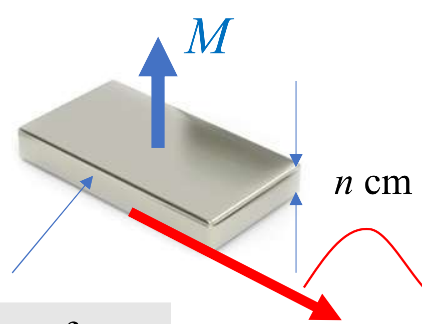
2. Compare the dimension of M and J

M is the volumetric density of moment m, the unit for M is A/m;

The unit for J is A/m^2.

3. Surface magnetization current density.

$$\vec{\sigma}_m = \vec{M} \times \hat{n}$$



Direction of surface current

Magnitude of surface current

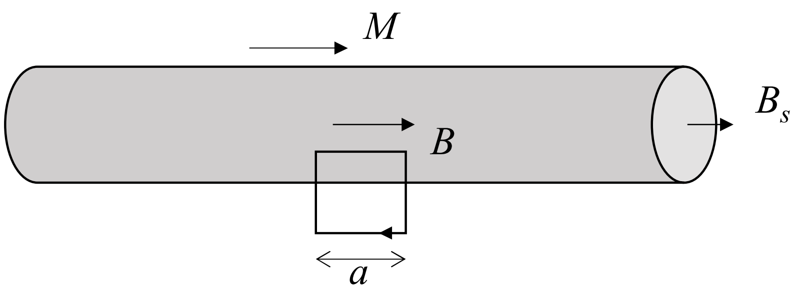
4. e.g. on this surface

$$\sigma = M$$

5. When the wall is parallel to M, the surface current density is the same as M. Notice the unit for both is A/m.

Back to the question

A long cylindrical PM is axially magnetized, the magnetization M = 1.19MA/m. What is the surface magnetic flux density and flux density deeply inside the magnet?



1. This PM is similar to a solenoid.

$$\oint \vec{B} \cdot \mathrm{d}\vec{l} = \mu_0 I = \mu_0 \sigma a$$

$$B = \mu_0 \sigma = \mu_0 M = 1.50 \ \mathrm{T}$$

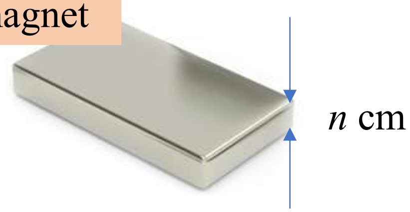
2. B_s can be considered a full solenoid cut by half. The center surface flux density is half of B.

$$B_s = \frac{B}{2} = 0.75 \ \mathrm{T}$$

3. The solenoid has a very high current density: 11.9 kA/cm

Is a hollow permanent magnet the same as a solenoid?

A permanent magnet



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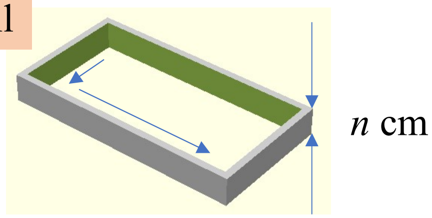
A loop coil



What about the hollow magnet below?

One should not forget about the reverse current loop in the hole.