Physics (PHYS2500J), Unit 6 Wave optics: 3. Diffraction

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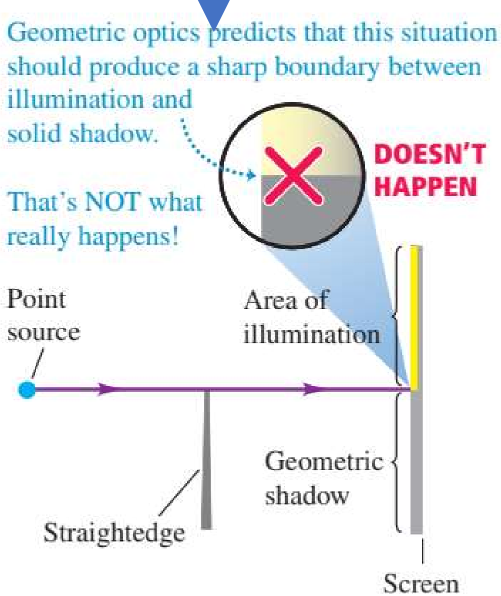
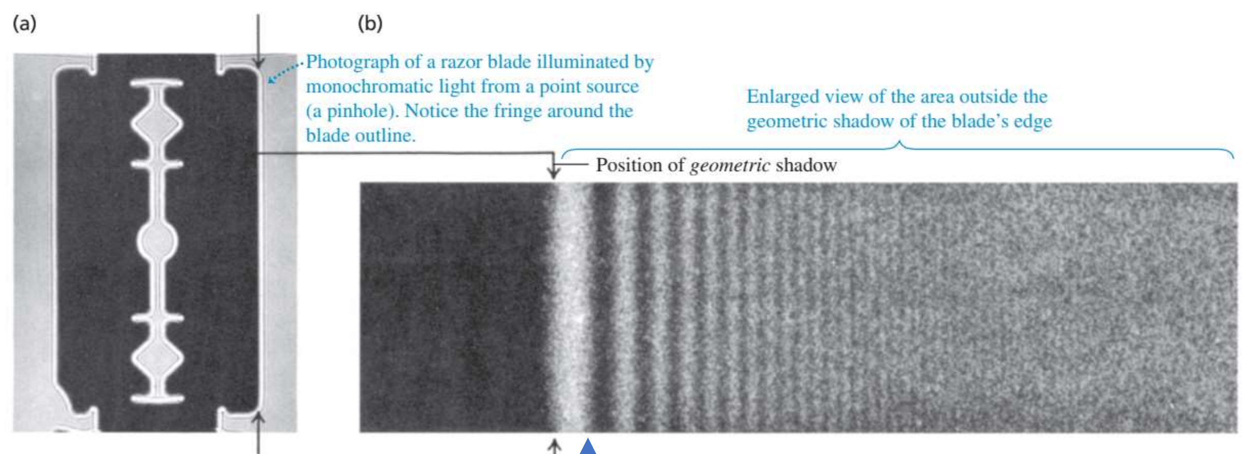
Diffraction (衍射) phenomenon

Geometrical optics: light goes by straight lines, thus producing sharp shadows.



In reality, NOT this way!



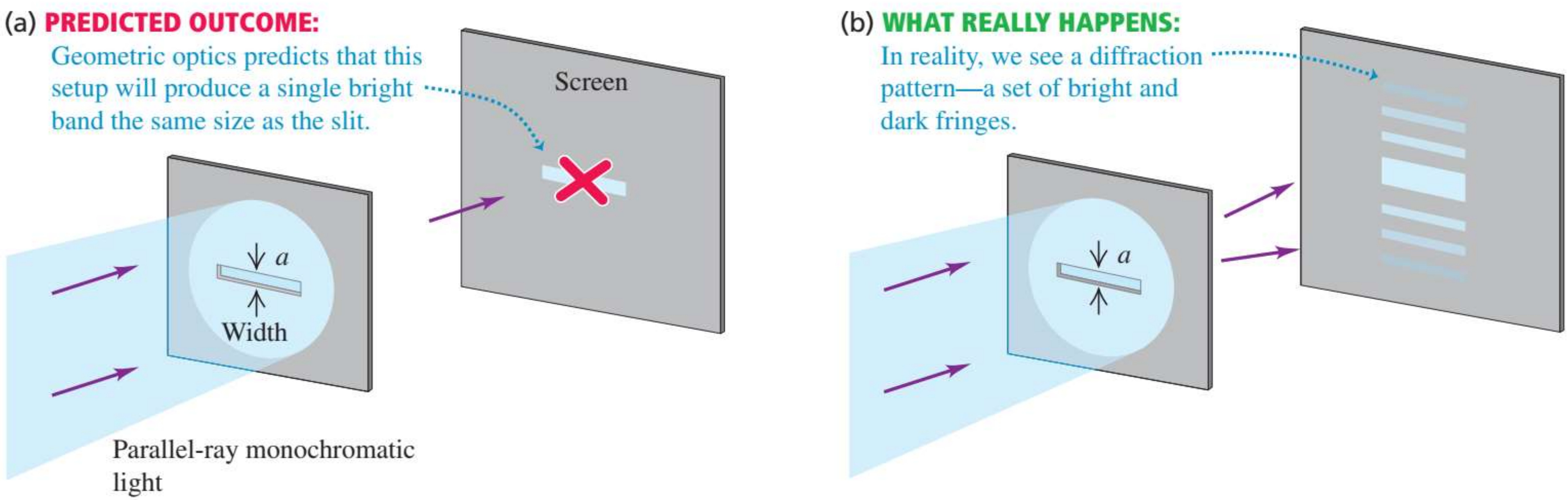
Instead, light goes around sharp edges, the shadow is illuminated at the edge, and the bright part shows fringes.

It is called diffraction. Huygens's principle is the key to understand the diffraction phenomenon.

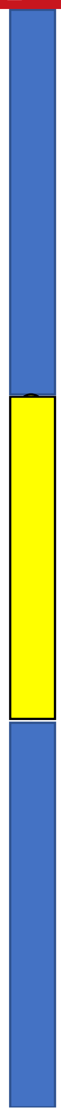
Diffraction from a single slit

If a narrow slit is shine by a parallel monochromatic light, what will happen?

Not the real case | **Instead,**



Explanation of single slit diffraction: 1



Start from a simple question: how do you think a dark stripe forms?

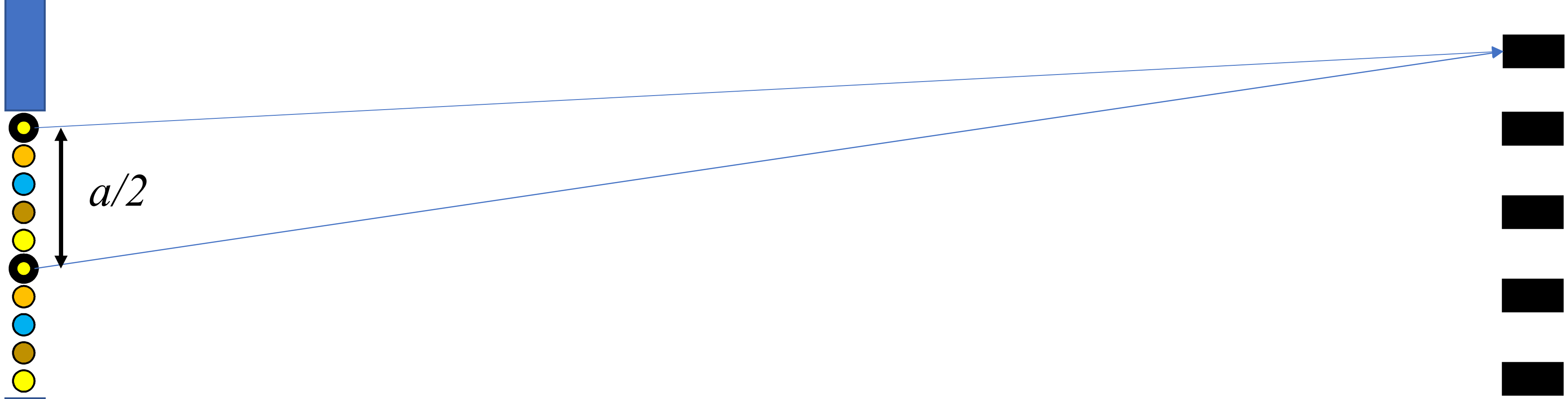
Destructive interference, of course!

Only one slit, who is going to interference with whom?

Explanation of single slit diffraction: 1

Huygens's principle, each point (on the same wave front) can be considered a source for further light wave.

Too many light sources are taking part in the interference (diffraction). Just take a pair, the distance between which is half the slit width $a/2$.



If the pair cancels the electric field at y from each other, the whole slit can be divided into many such pairs and total E will be 0 at y

Explanation of single slit diffraction: 1

When $l >> y$ and a, the phase difference

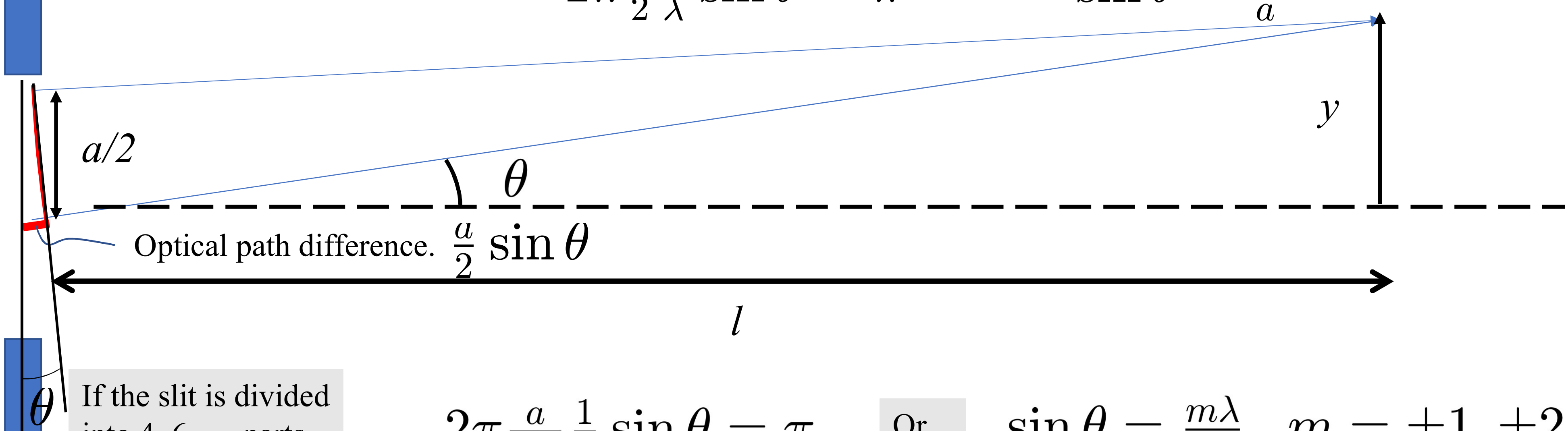
$$2\pi \frac{a}{2} \frac{1}{\lambda} \sin\theta$$

For dark fringes:

$$2\pi \frac{a}{2} \frac{1}{\lambda} \sin\theta = \pi$$

Or,

$$\sin\theta = \frac{\lambda}{a}$$



If the slit is divided into 4, 6, … parts,

$$2\pi \frac{a}{2m} \frac{1}{\lambda} \sin\theta = \pi$$

Or,

$$\sin\theta = \frac{m\lambda}{a}, \quad m = \pm 1, \pm 2, \ldots$$

Hidden assumption: y is much larger than a, so that θ is a constant angle for all points on the slit.

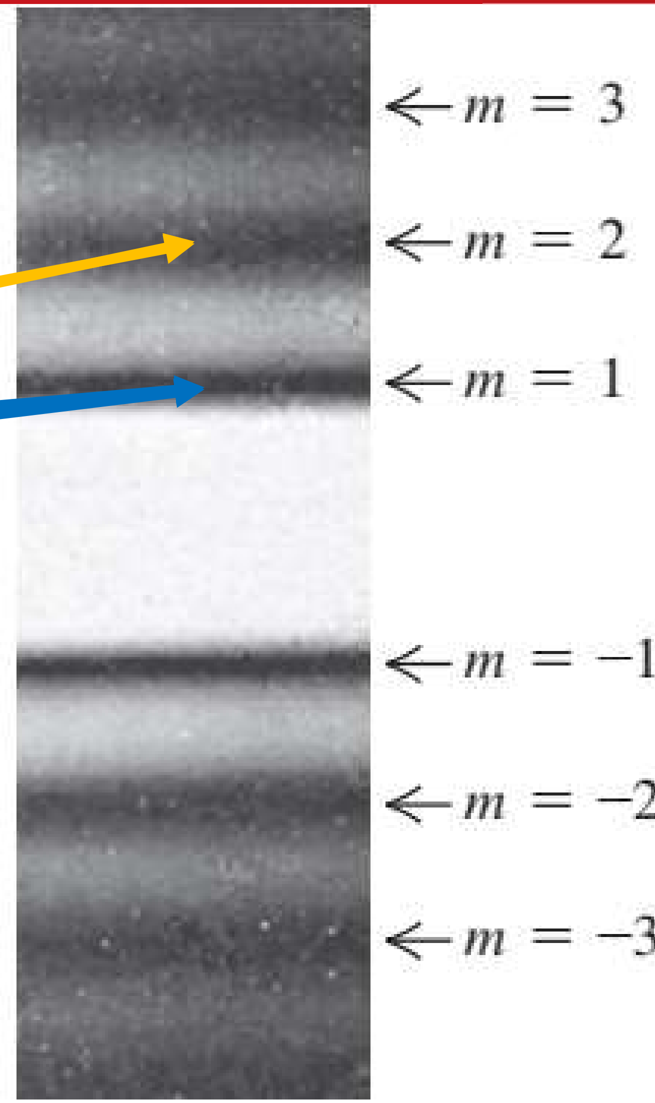
$$\theta = \frac{y}{l}$$

Dark fringes

$$\sin\theta = \frac{m\lambda}{a}, \ \ m = \pm 1, \pm 2, \dots$$

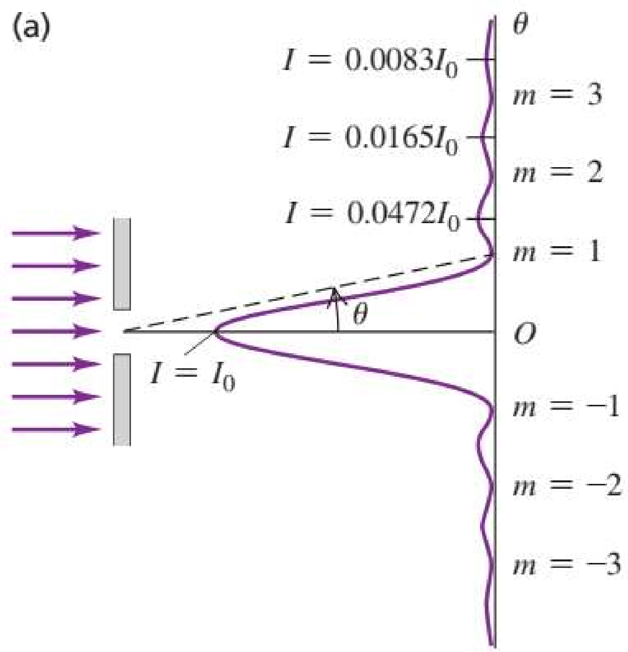
Angle increased so that a quarter of the slit cancels each other now. Similarly, the whole slit destructively interferes.

Half slit cancels each other, so the whole slit interferes destructively.

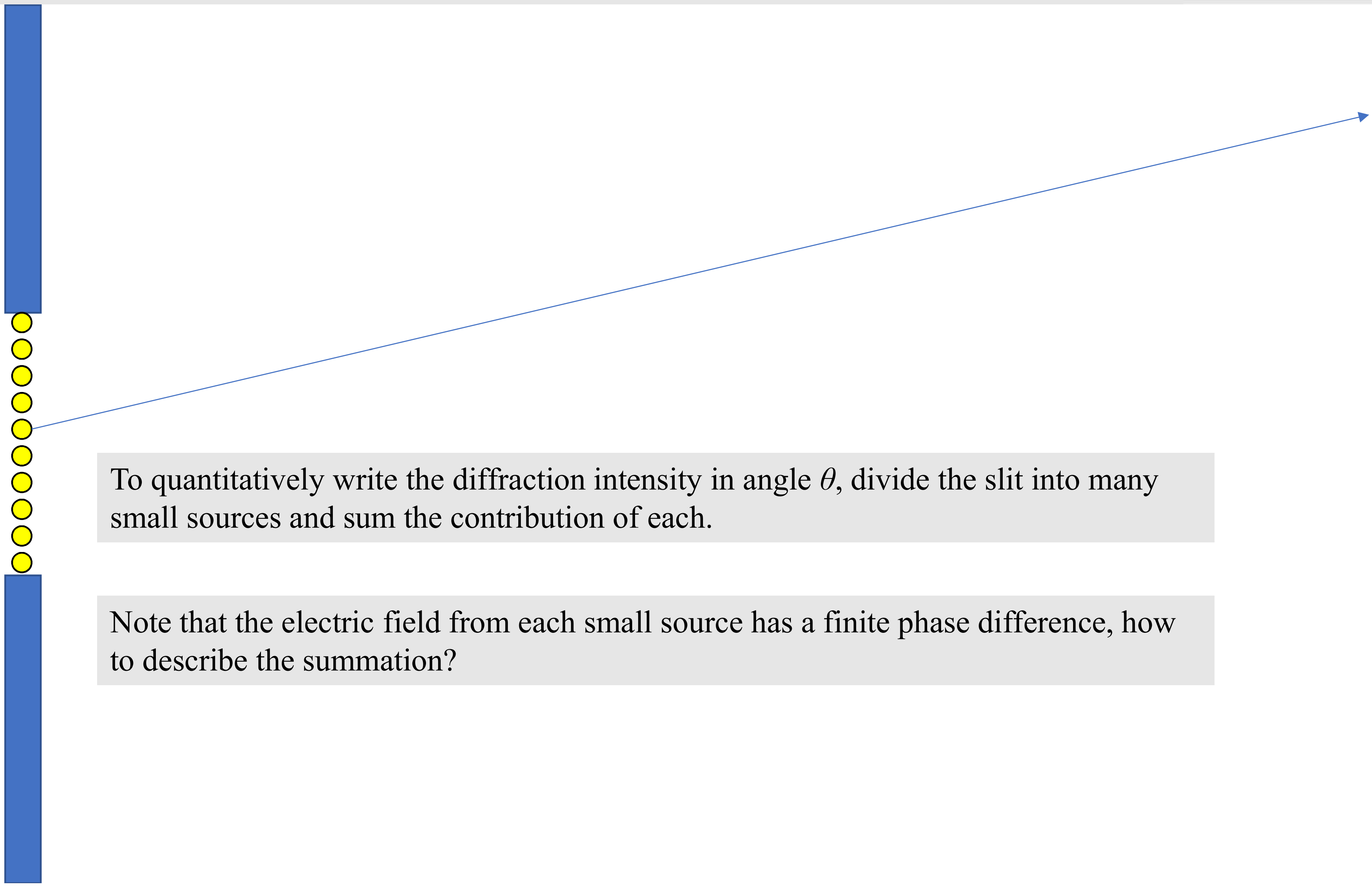


Intensity of single slit diffraction pattern

The intensity of the bright stripes are not the same, but decaying fast from the center stripe. (which is quite different from two slits case)



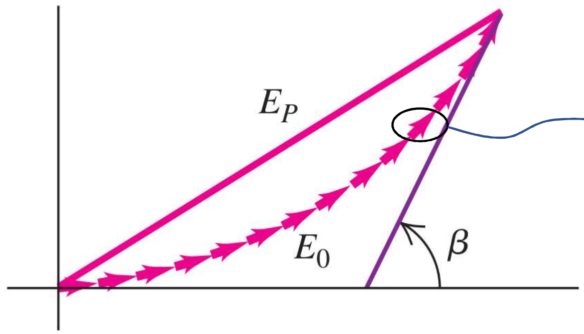
Mathematical description of the bright strips: Intensity



To quantitatively write the diffraction intensity in angle θ, divide the slit into many small sources and sum the contribution of each.

Note that the electric field from each small source has a finite phase difference, how to describe the summation?

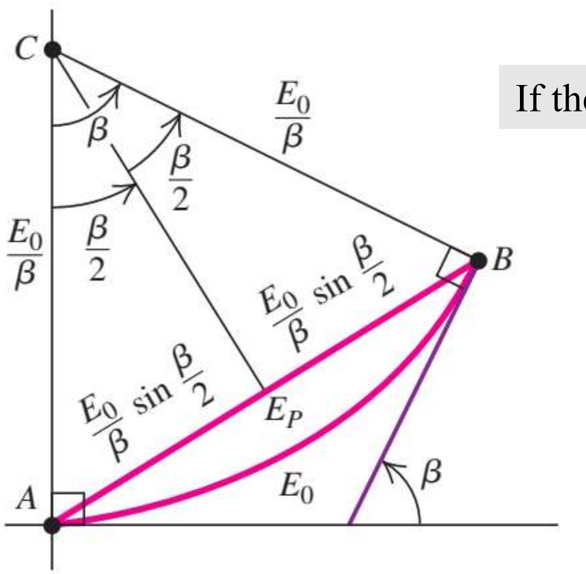
Head to tail summation of the Phasors



Each small source is contributing a small E with phase difference.

The arc length is proportional to the electric field E_0, times n sections.

The distance between the two end points is the magnitude of the total electric field.



If the phase angle from the beginning to the end is β,

$$E = 2\frac{E_0}{\beta}\sin\frac{\beta}{2}$$

The "radius" of the phasor plot.

$$I = CE^2 = CE_0^2\left[\frac{\sin(\beta/2)}{\beta/2}\right]^2 = I_0\left[\frac{\sin(\beta/2)}{\beta/2}\right]^2$$

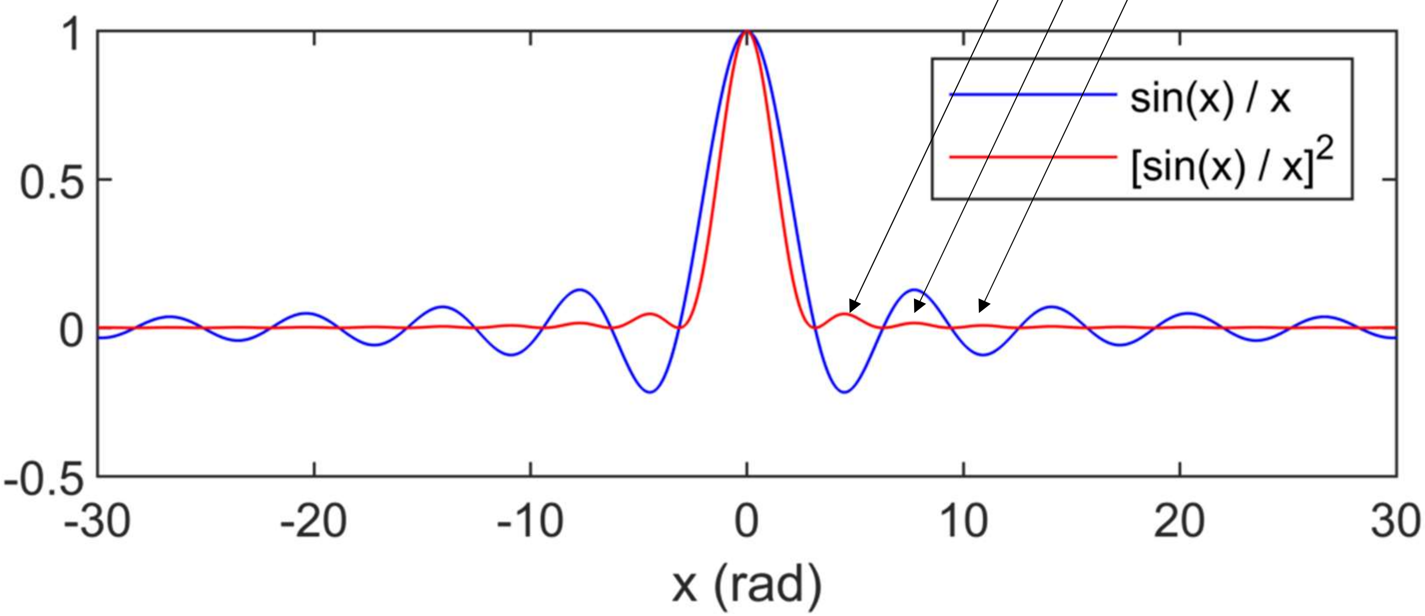
Where C is a constant relating the intensity to the square of E field.

The sin(x)/x function

The electric field can be described by a sin(x)/x function

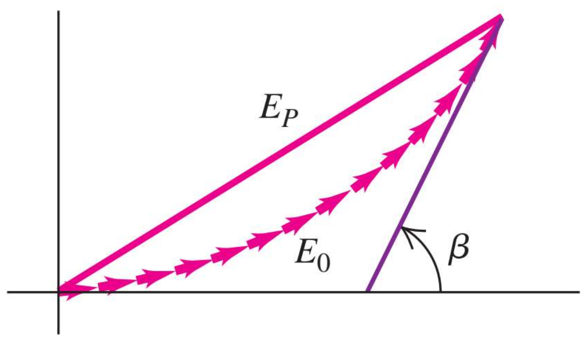
$$I_m \approx \frac{I_0}{\left(m+\frac{1}{2}\right)^2\pi^2}$$

The m^{th} intensity maxima



The function describes an interesting fact that the sources are interfering with each other in a destructive way that the contribution from most components are canceled by each other.

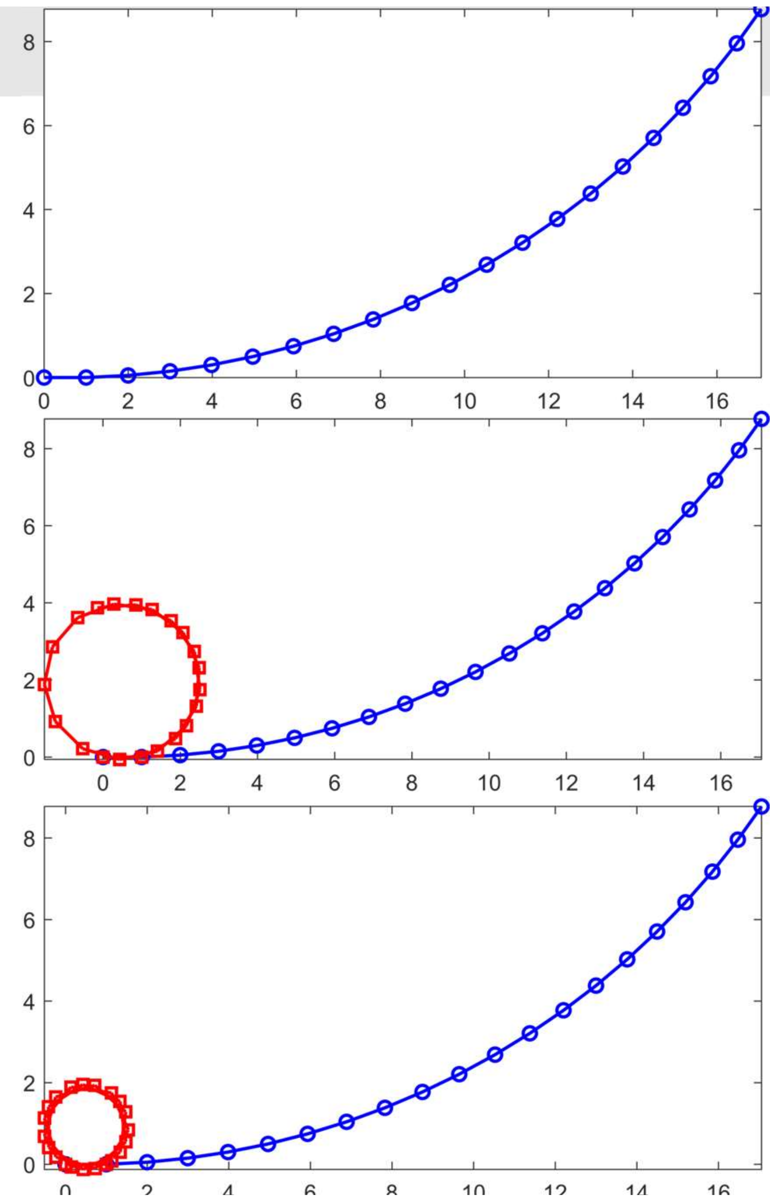
Geometrical meaning of the fast decaying intensities



dφ = 0.05

dφ = 0.5

dφ = 1

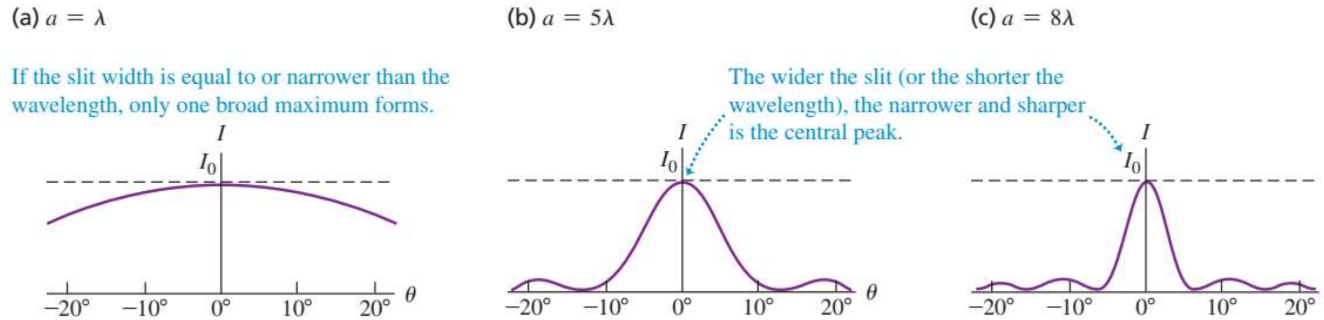


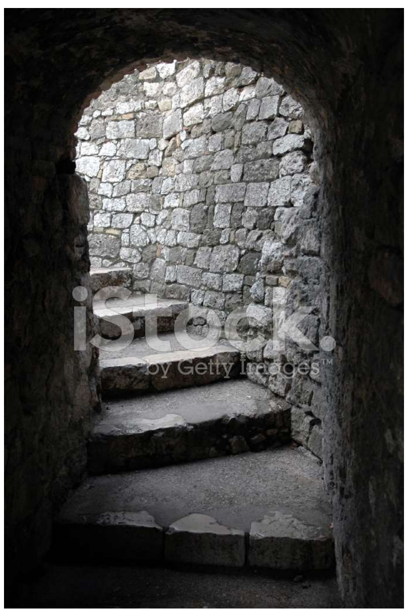
20 phasors with the same length but different phase difference.

The role of slit width

$$I = CE^2 = CE_0^2\left[\frac{\sin(\beta/2)}{\beta/2}\right]^2 = I_0\left[\frac{\sin(\beta/2)}{\beta/2}\right]^2$$

The larger is the aperture, diffraction effect is more close to a straight line propagation.



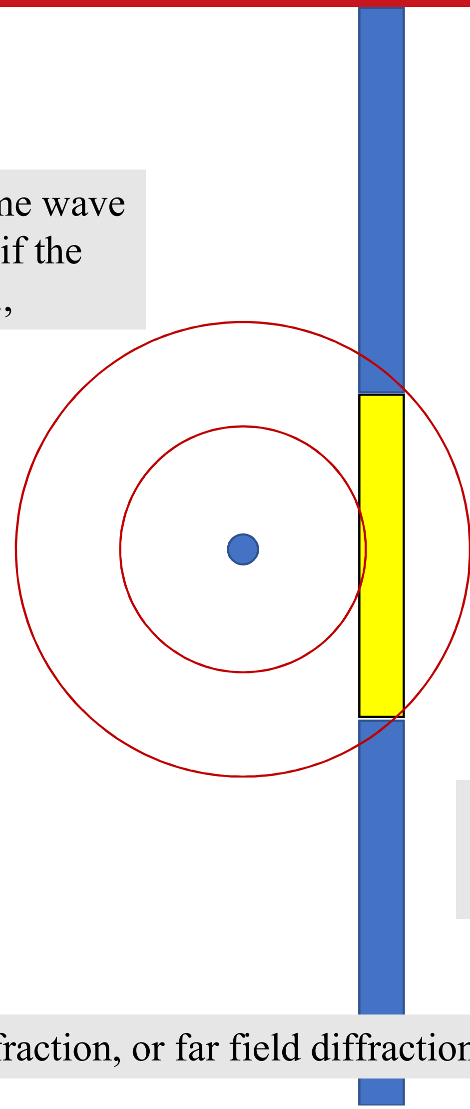


Sound velocity = 340 m/s, Sound frequency = 20 Hz -20 kHz
Sound wavelength ~ 1.7 cm to 17 m, it can go around normal obstacles.

However for light whose wavelength is ~100 nm. It travels in straight lines for normal size apertures and obstacles.

Two assumptions for the former calculations

1. The whole slit is on the same wave front. Which may not be true if the source is close to the slit. E.g.,



2. The detector plane is far from the slit so that angle θ can be used as a variable.

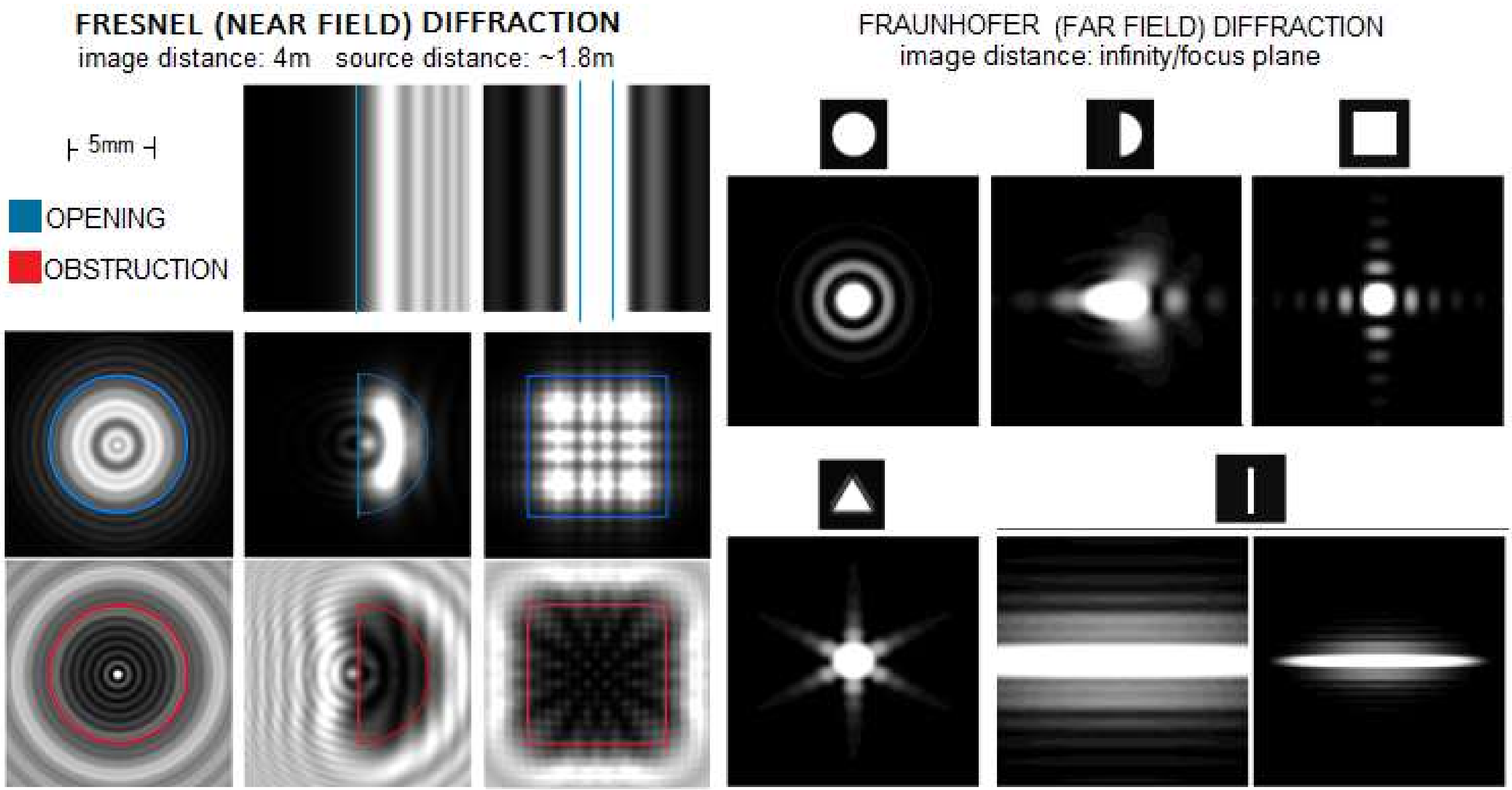
Both the light source and the detector far from the slit:

Fraunhofer diffraction.

Fraunhofer diffraction, or far field diffraction, is the “good” diffraction for its simplicity to deal with.

Fraunhofer diffraction and Fresnel diffraction

Fresnel diffraction, or near field diffraction, is generally much more complicated.



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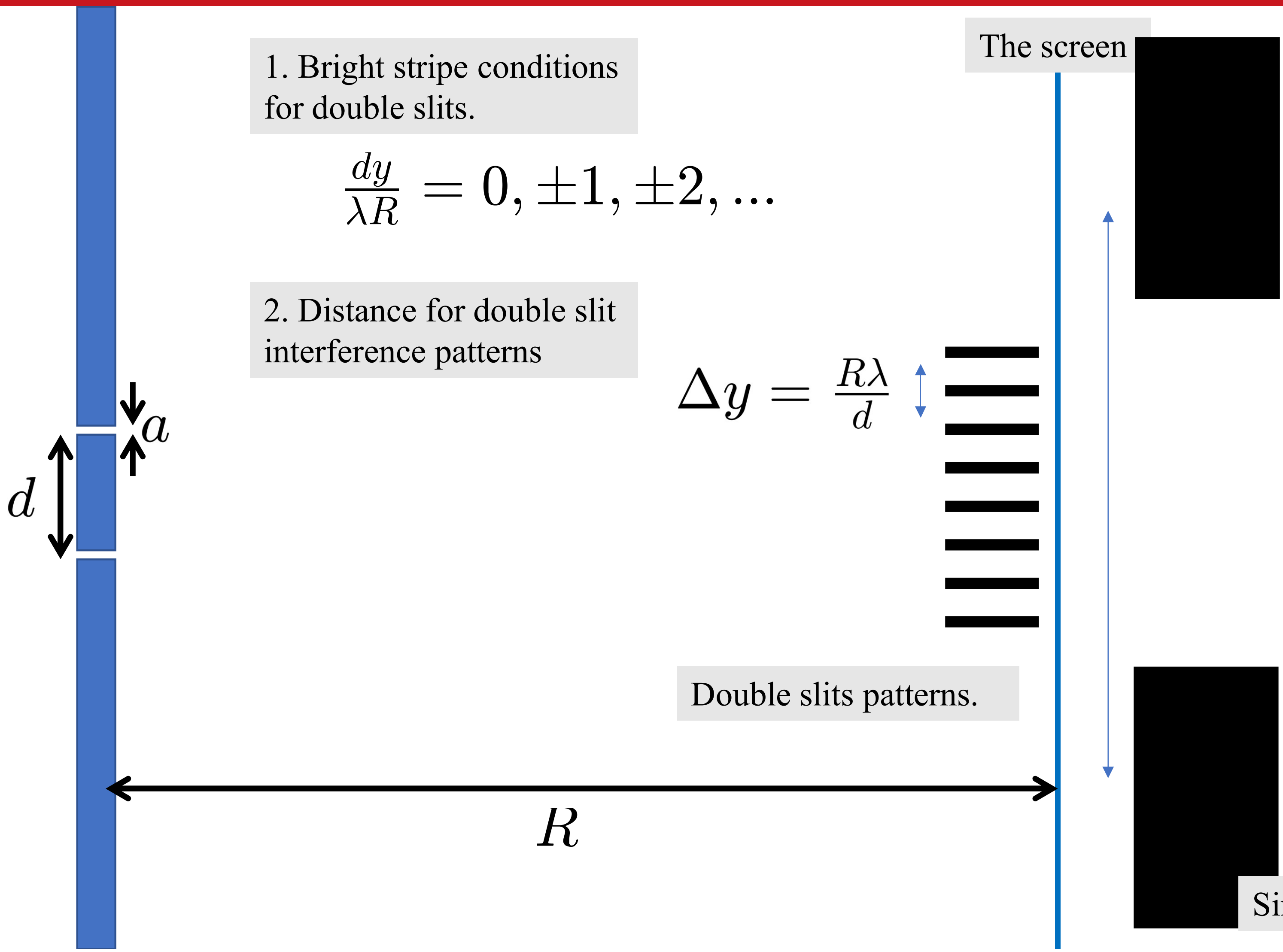
Revisiting the double slit interference pattern



1. Bright stripe conditions for double slits.

$$\frac{dy}{\lambda R} = 0, \pm 1, \pm 2, \ldots$$

2. Distance for double slit interference patterns

$$\Delta y = \frac{R\lambda}{d}$$

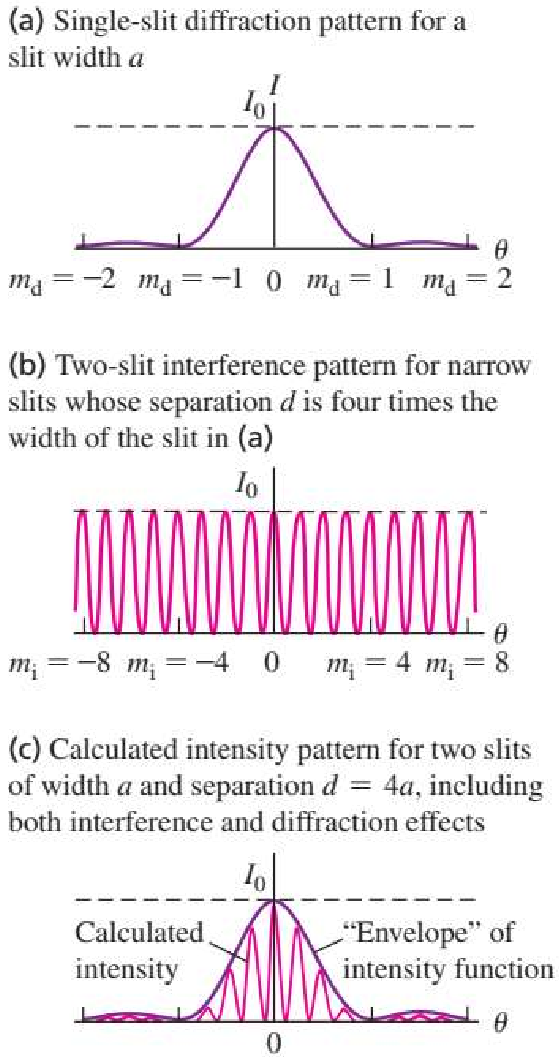
3. Dark strip conditions for single slit.

$$\frac{y}{R} = \sin\theta = \frac{m\lambda}{a},$$
$$m = \pm 1, \pm 2, \ldots$$

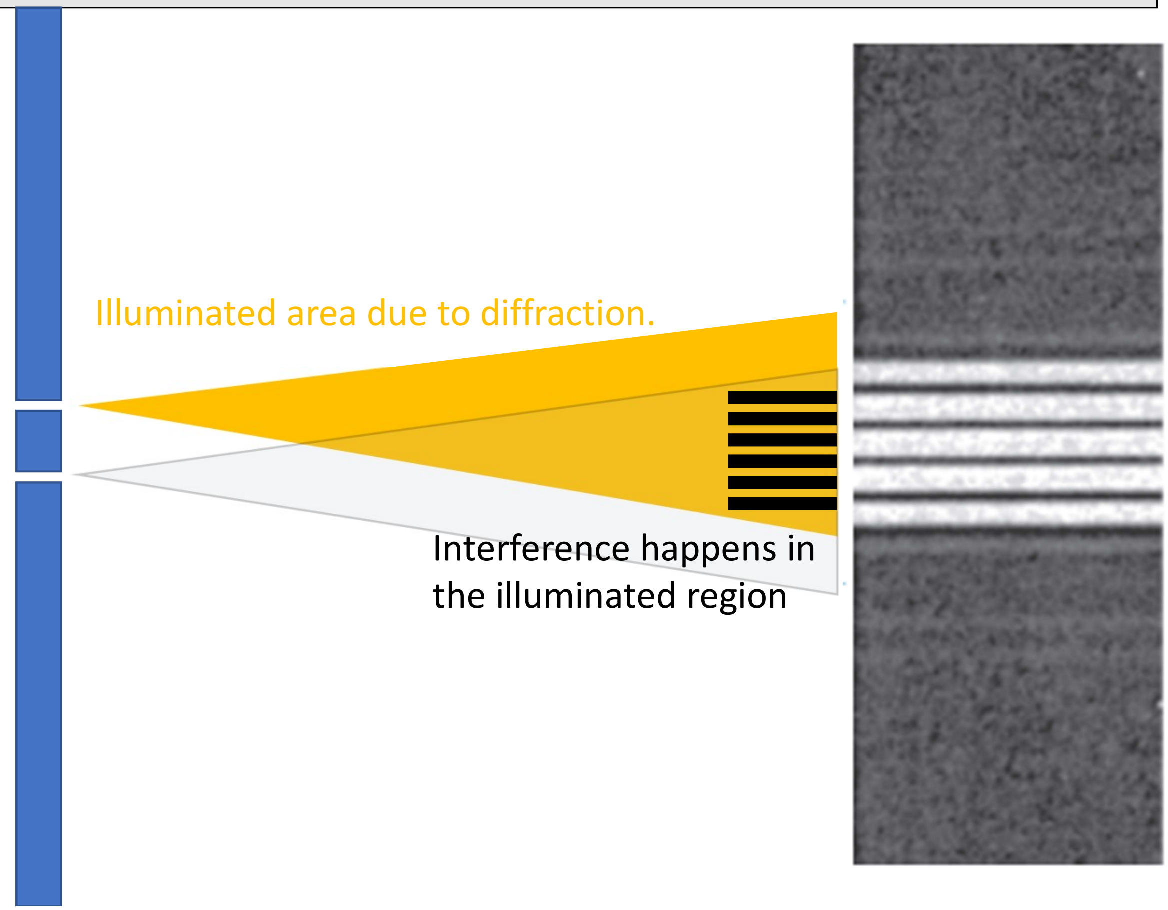
4. Distance for single slit diffraction patterns

$$\Delta y = \frac{R\lambda}{a}$$

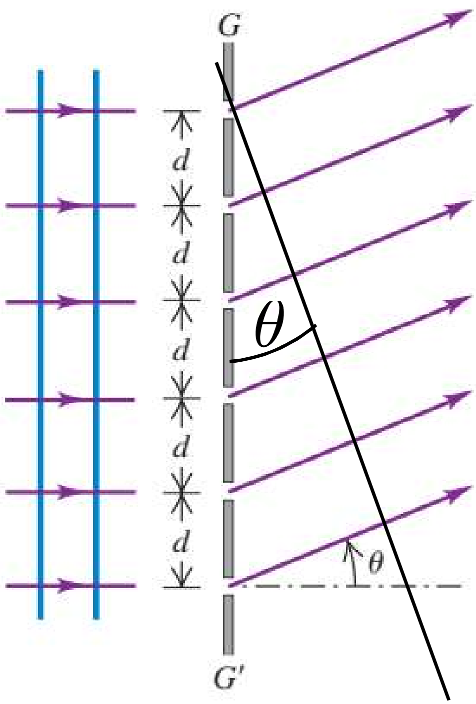
Revisiting the double slit interference pattern



For a double slits experiment, the diffraction and interference happens at the same time. The double slits interference is actually under an "envelope" determined by the single slit diffraction.



Diffraction grating: multiple slits

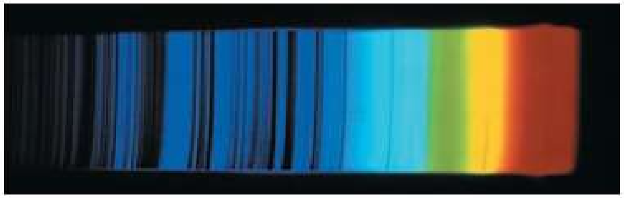


If multiple slits with the same distance co-exist, and constructively interfere at angle θ, what is the condition for d, θ and λ?

$$d\sin\theta = m\lambda,\ \ m = 1, 2, 3, \ldots$$

Discussion 1: for one monochromatic light source, multiple diffraction peaks are possible. The intensity of higher order peaks are not decaying fast as in single slit.

Discussion 2: What happens if white light is shine on the diffraction grating?



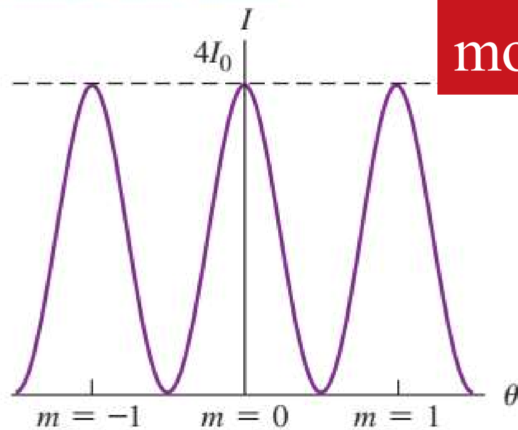
Spectrum will form.

Multiple slits compared with double slits

Case 1: multiple slits

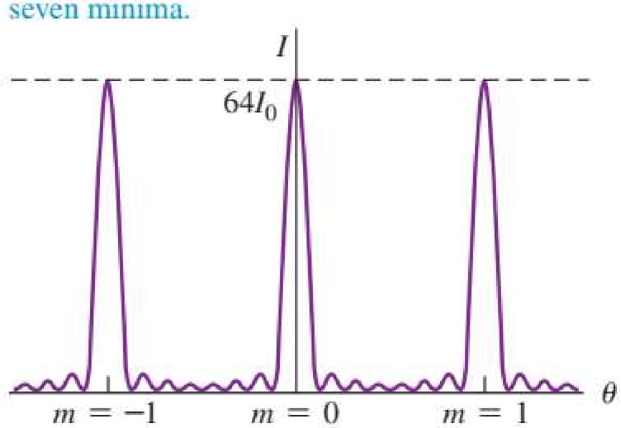
Case 2: two slits (with the same spacing)

(a) $N = 2$: two slits produce one mini
between adjacent maxima.

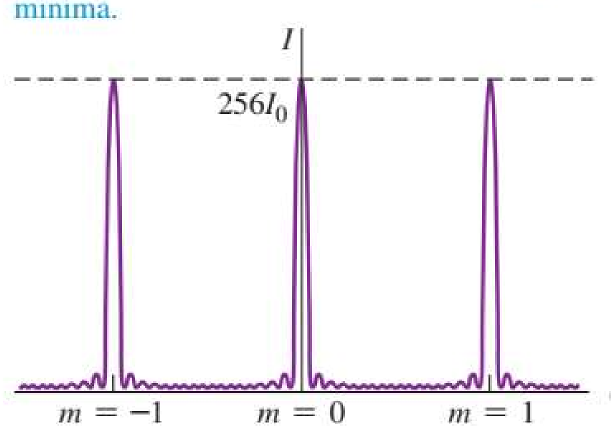


vs. 2 slits: sharper and more intense peaks.

(b) $N = 8$: eight slits produce taller, narrower maxima in the same locations, separated by seven minima.



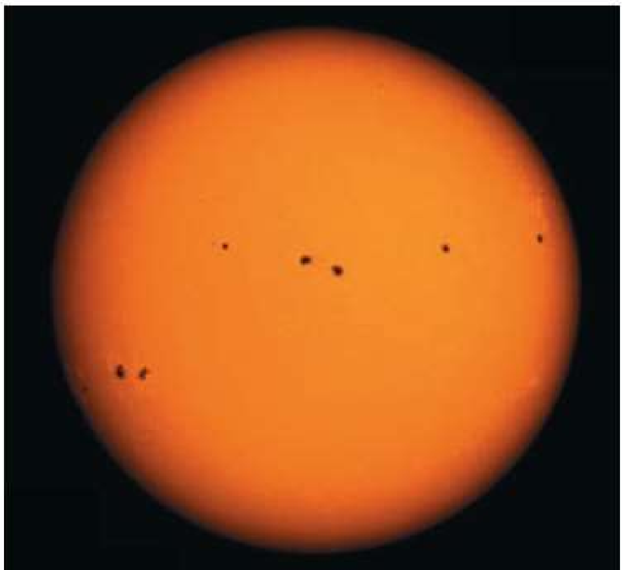
(c) $N = 16$: with 16 slits, the maxima are even taller and narrower, with more intervening minima.



Application of diffraction grating: Spectroscopy

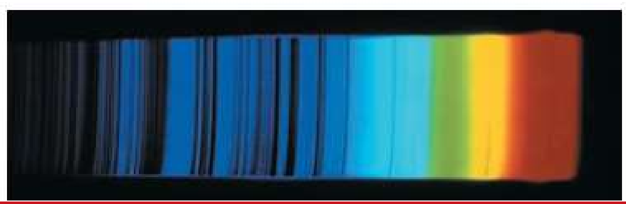
Wavelength information is transferred into angle. Measure the angle of diffraction directly gives the wavelength.

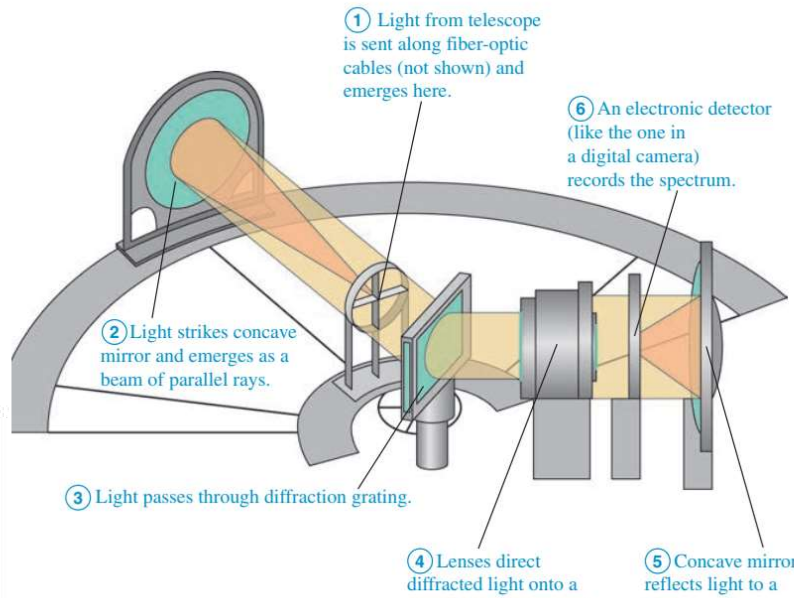
(a)



36.18 (a) A visible-light photograph of the sun. (b) Sunlight is dispersed into a spectrum by a diffraction grating. Specific wavelengths are absorbed as sunlight passes through the sun's atmosphere, leaving dark lines in the spectrum.

(b)





From the specific dark lines (absorption lines) in the spectrum, the composition of the distant stars are determined.

An example of diffraction grating

36.17 Microscopic pits on the surface of this DVD act as a reflection grating, splitting white light into its component colors.

The regular features on a DVD is making it a diffraction grating. The white light is split into multiple monochromatic lights.

Diffraction grating can work in transmitting mode of reflection mode.