Physics (PHYS2500J), Unit 2 Magnetostatics: 1. Magnetic field

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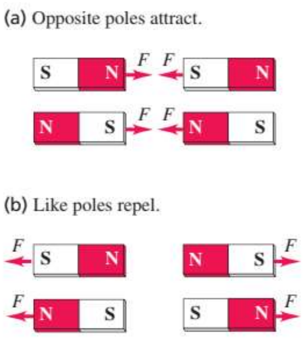
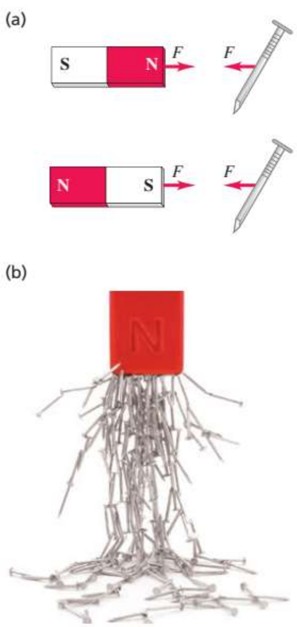
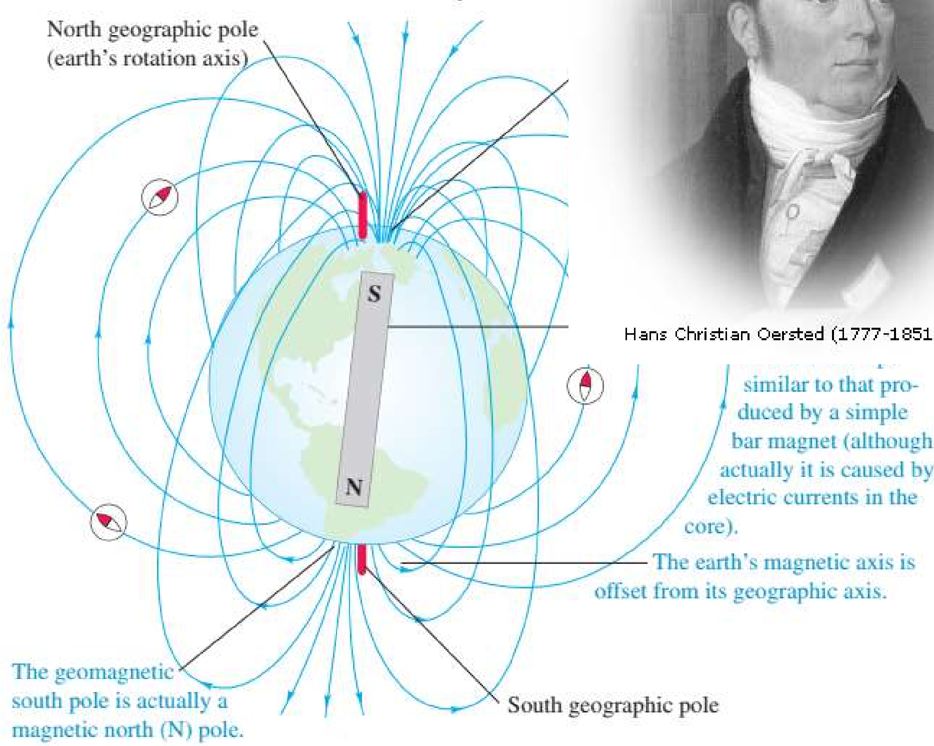
Review of magnetic field



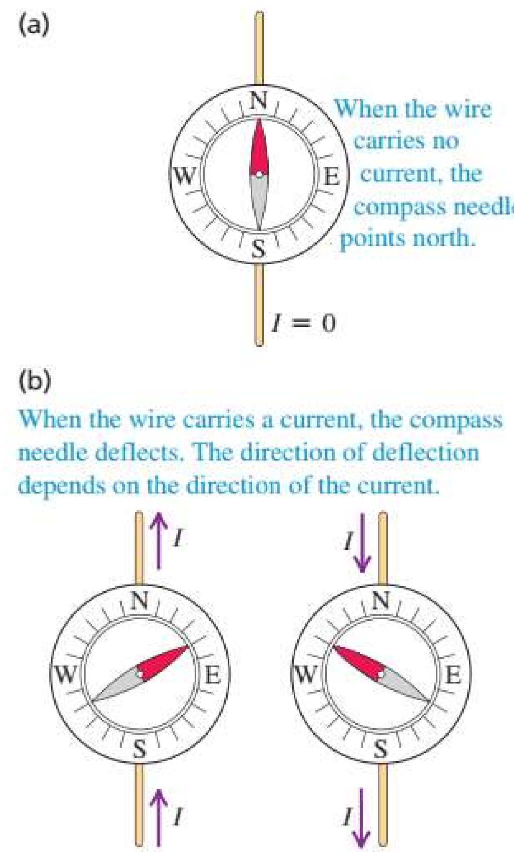


27.3 A sketch of the earth's magnetic field. The field earth's molten core, changes with time; geologic evid entirely at irregular intervals of 10^4 to 10^6 years.



Hans Christian Oersted (1777-1851 Danish)

27.5 In Oersted's experiment, a compass is placed directly over a horizontal wire (here viewed from above).

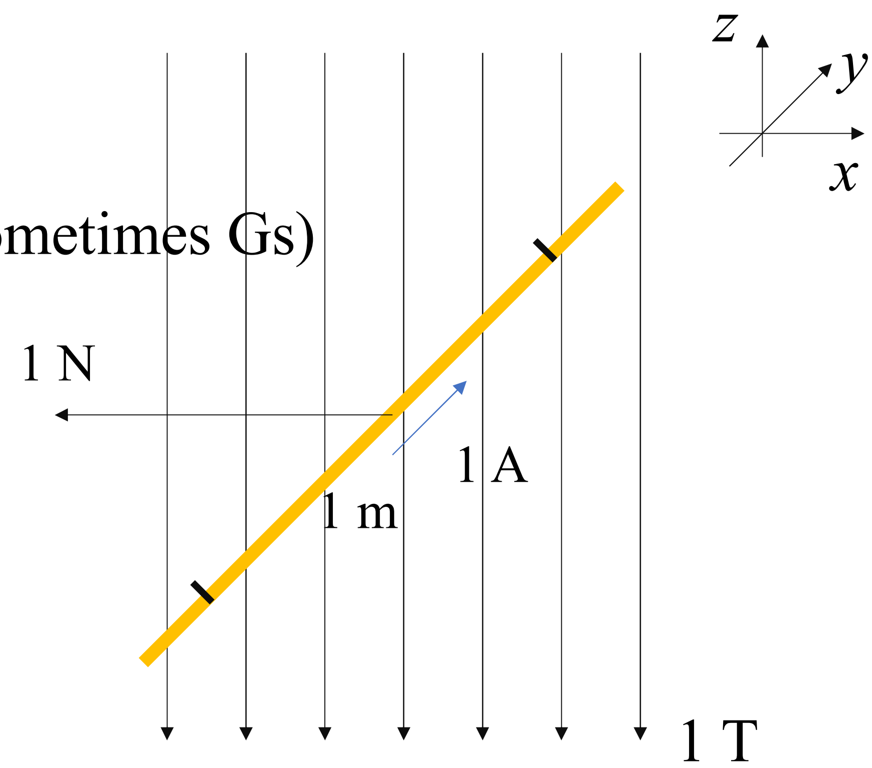


Magnetic flux density *B*

Also called magnetic induction.

Unit: **1 Tesla = 1000 mT = 10,000 G**

(Gauss, magnetic unit system, sometimes Gs)



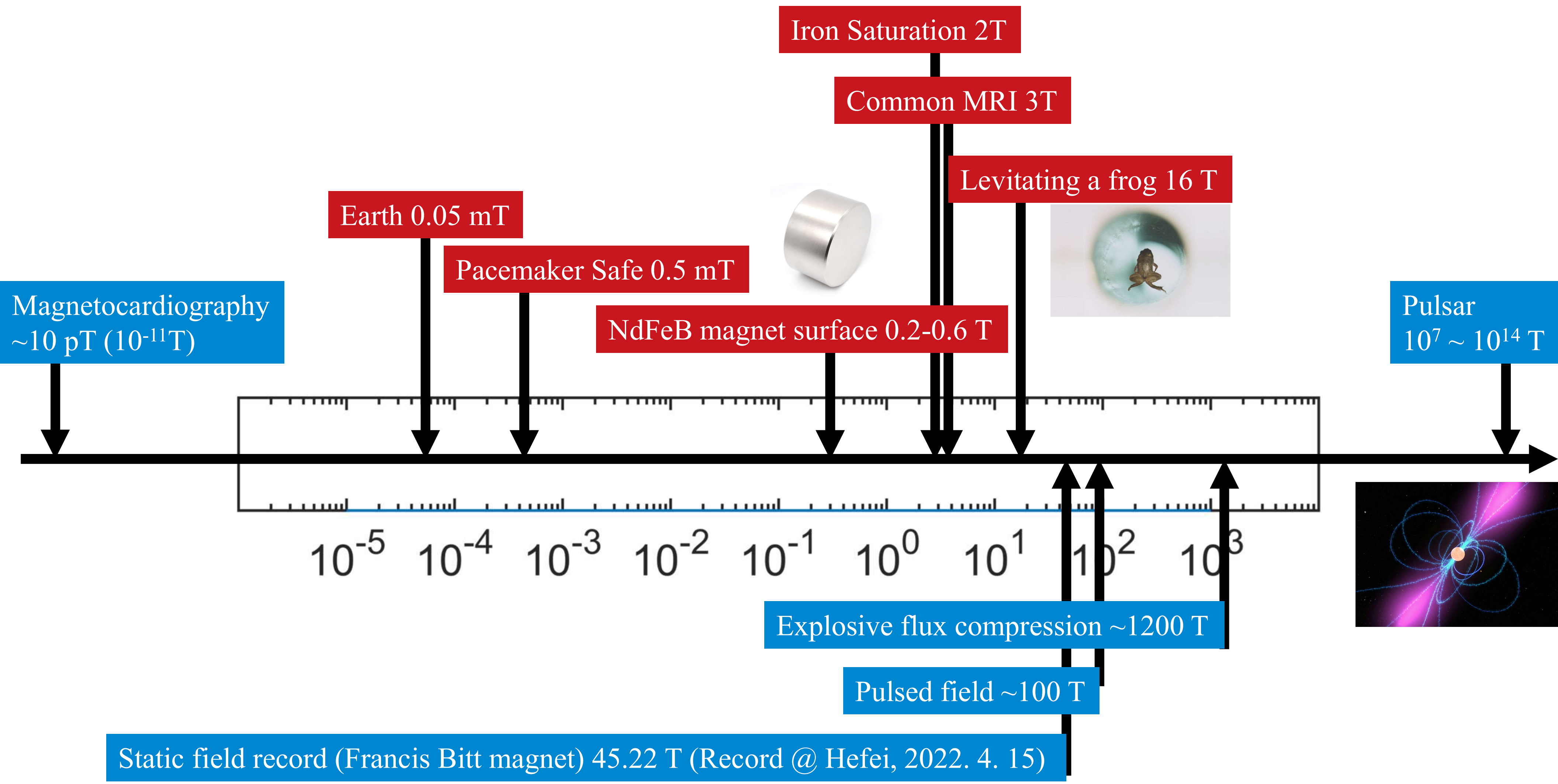
How large is earth magnetic field?

0.5 Gauss = 0.05 mT = 5x10^{-5} T

Historical reasons, *B* is not called magnetic field

Magnetic field: *H*
Described by A/m (a small unit) or Oe.

Magnetic flux density B



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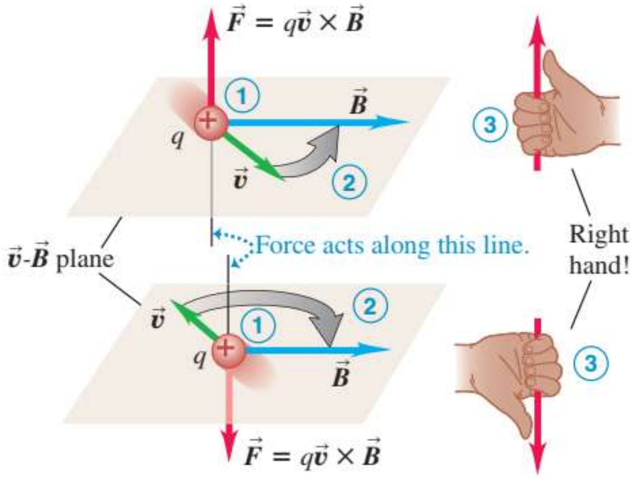
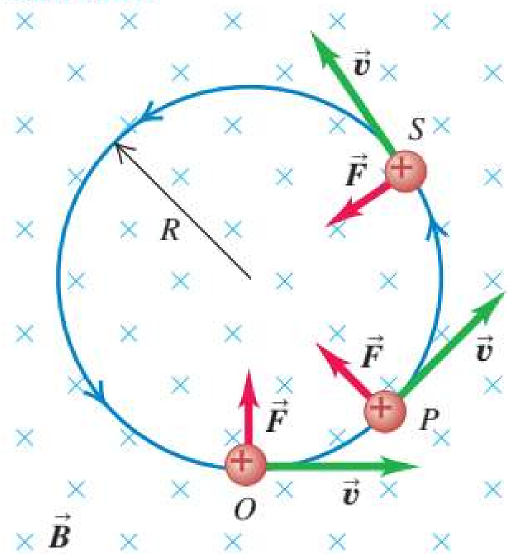
Force on charge with magnetic field present

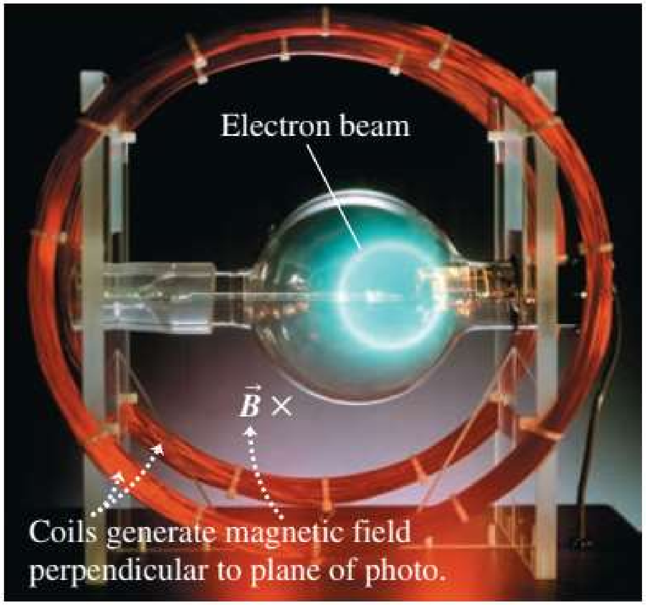
$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$



A charge moving at right angles to a uniform $\vec{B}$ field moves in a circle at constant speed because $\vec{F}$ and $\vec{v}$ are always perpendicular to each other.

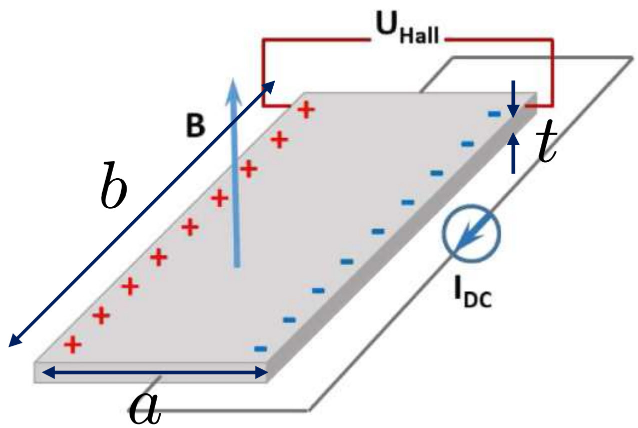


(b) An electron beam (seen as a white arc) curving in a magnetic field



Application of Lorentz force on particles: 1. Hall effect

Hall effect



Most useful application is to make a magnetic field sensor.

A sensor is a device to transfer physical quantities into voltage.

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

The process of equilibrium establishment:

1. Charge particles (current) feels Lorentz force and move to one side.
2. Charge accumulation on the side and an electric field is built.
3. The electric force and magnetic force is canceled for the following charge carriers so that they can pass freely.

$$J = Nqv$$

$$I = atJ = atNqv$$

$$V_H = Ea = qvBa = \frac{IB}{tN}$$

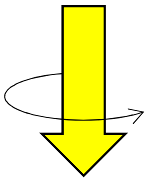
Hall voltage is proportional to B and I

A good Hall sensor: sensitive means high V_H at given B.

Low thickness, low number density of charge carriers but conductivity has to be good at the same time.

Application of Lorentz force on particles: 2. Circular particle accelerator

Dynamics for particles in a circular motion under axial magnetic field



$$q\vec{v} \times \vec{B} = \frac{\mathrm{d}\vec{P}}{\mathrm{d}t}$$

Relativistic case, $v = c$

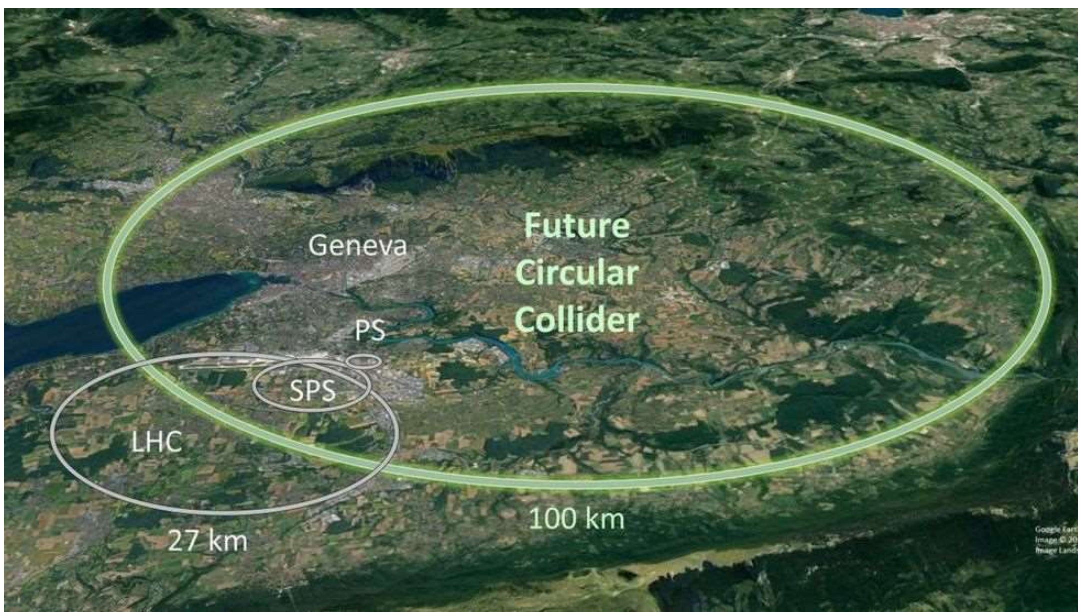
$$qcB = P\omega = \frac{Pc}{R}$$

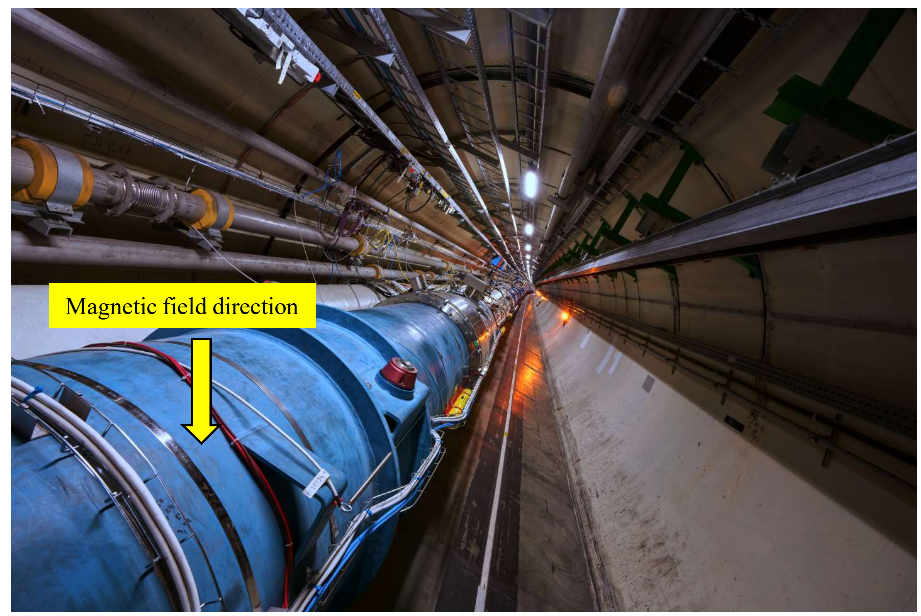
$$P = qBR$$

Please notice the dimension of P

$$\mathcal{E} = mc^2 = \frac{P^2}{m} = \frac{q^2 B^2 R^2}{m}$$

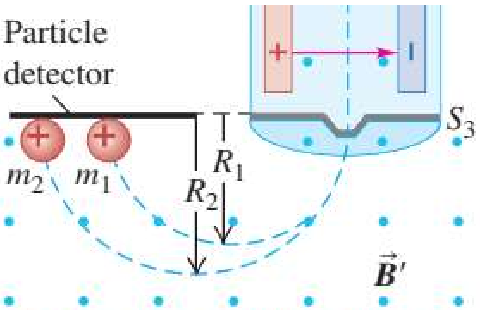
Larger field, larger radius → higher energy





Application of Lorentz force on particles: 3. Mass spectroscopy

Mass spectroscopy: to check what kind of atom (particle) is there



Magnetic field separates particles by mass; the greater a particle's mass, the larger is the radius of its path.

Mass spectroscopy is not only used in accelerator but also chemistry and material science.

$$\frac{mv^2}{R} = qvB$$

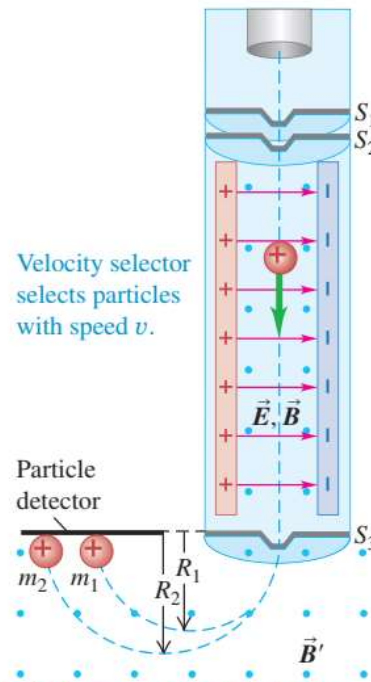
Each particle has a special landing spot on the detector.

$$R = \frac{m}{q}\frac{v}{B}$$

m/*q* is a signature of the particle

What about *v*?

27.24 Bainbridge's mass spectrometer utilizes a velocity selector to produce particles with uniform speed v. In the region of magnetic field B', particles with greater mass ($m_2 > m_1$) travel in paths with larger radius ($R_2 > R_1$).



Magnetic field separates particles by mass; the greater a particle's mass, the larger is the radius of its path.

A velocity selector

Similar to Hall effect sensor:

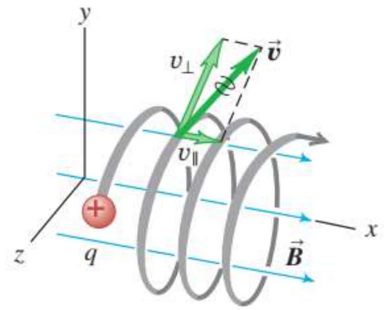
$$\vec{E} = -\vec{v} \times \vec{B}$$

Different part is that a voltage source, instead of a voltmeter is connected to the two plates.

Some other interesting facts about particles in magnetic field

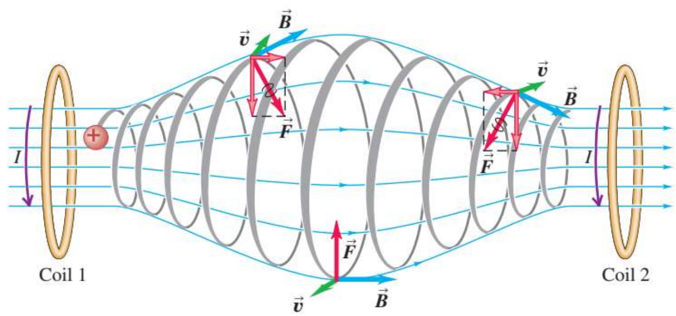
Lorentz force does not do work.

27.18 The general case of a charged particle moving in a uniform magnetic field $\vec{B}$. The magnetic field does no work on the particle, so its speed and kinetic energy remain constant.



Magnetic confinement of charged particles

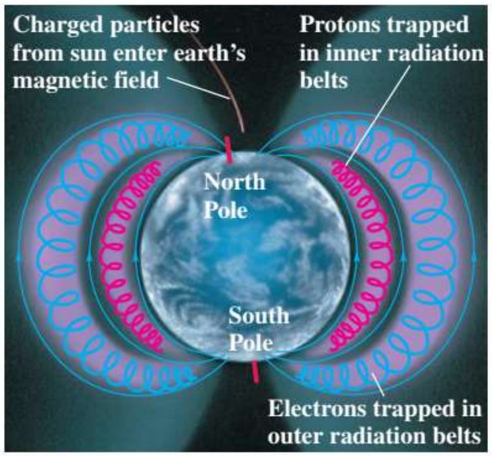
27.19 A magnetic bottle. Particles near either end of the region experience a magnetic force toward the center of the region. This is one way of containing an ionized gas that has a temperature of the order of 10^6 K, which would vaporize any material container.



27.20 (a) The Van Allen radiation belts around the earth. Near the poles, charged particles from these belts can enter the atmosphere, producing the aurora borealis ("northern lights") and aurora australis ("southern lights"). (b) A photograph of the aurora borealis.

Earth magnetic field is protecting the life.

(a)



(b)

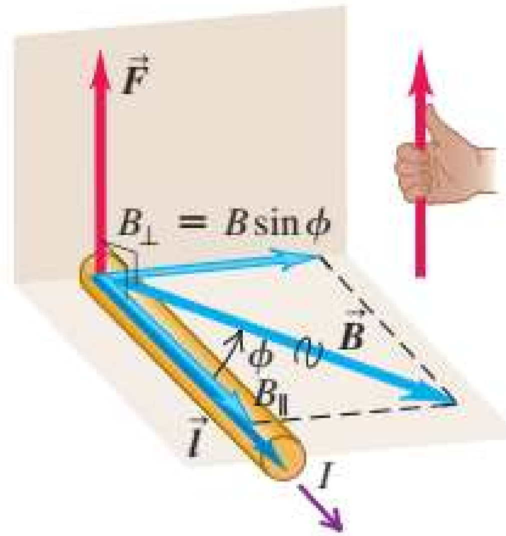
Magnetic force on current carrying conductors

1 Tesla = 1 N/A·m

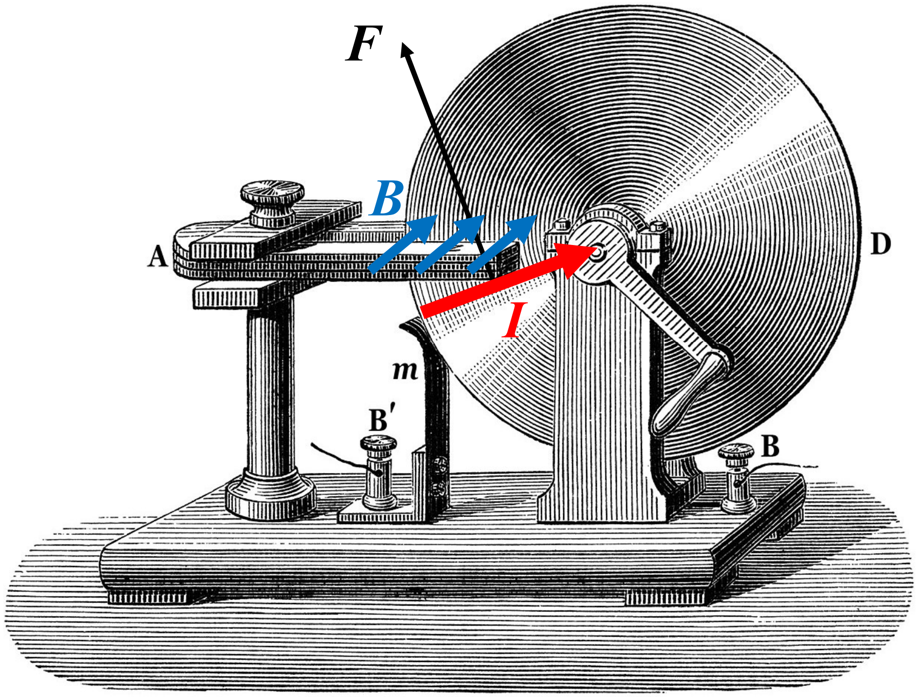
At 1 tesla field, 1 meter wire carrying 1 A (perpendicular) experiences 1 Newton force.

Magnetic force on an infinitesimal wire segment

$$\underbrace{d\vec{F}}_{\text{Magnetic force on an infinitesimal wire segment}} = \overbrace{I}^{\text{Current}}\, \underbrace{d\vec{l}}_{\text{Vector length of segment (points in current direction)}} \times \overbrace{\vec{B}}^{\text{Magnetic field}} \qquad (27.20)$$



Faraday disk



https://en.wikipedia.org/wiki/Homopolar_generator

Lorentz force:

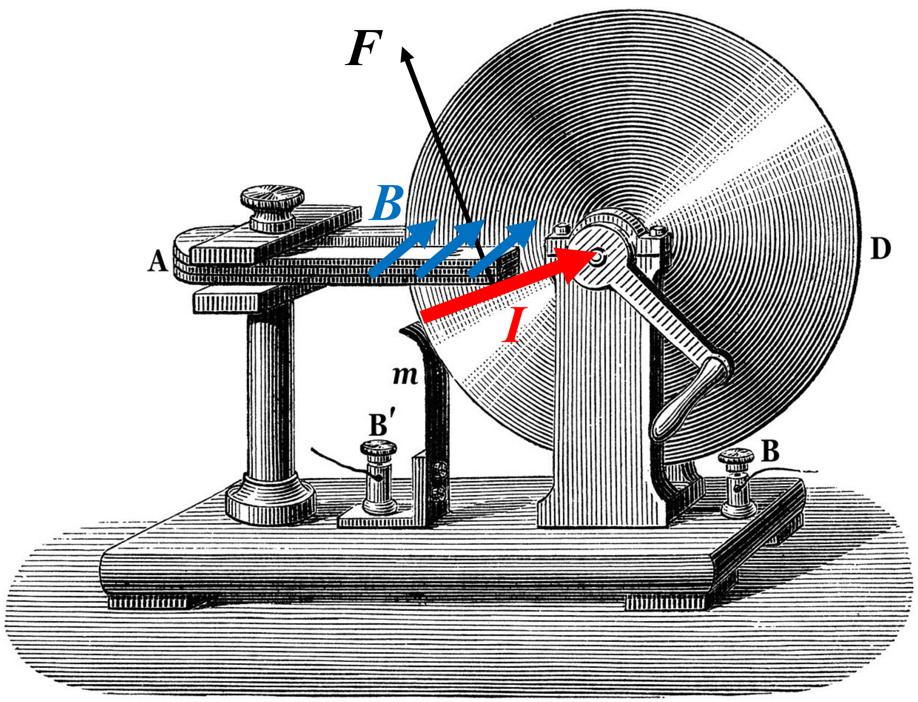
force on moving charged particles in magnetic field

$$\vec{F}_L = q\vec{v} \times \vec{B}$$

qv has the same direction of I

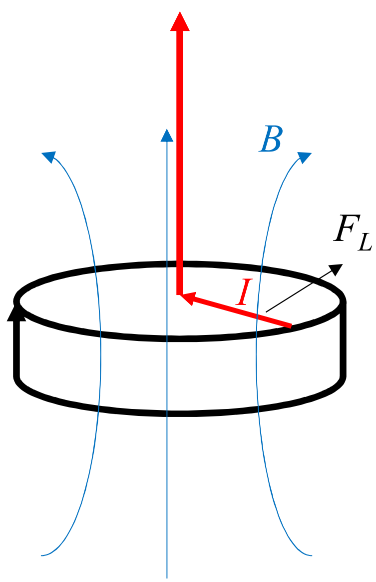
1. Existence of *B* field (axial direction);
2. Radial current;
3. Commutating contact

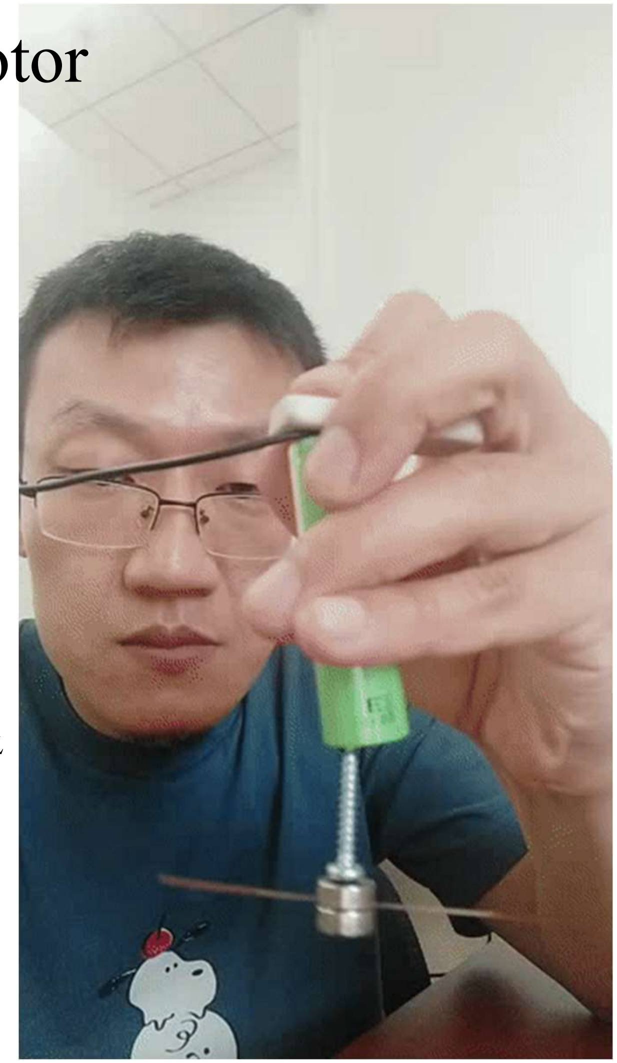
Faraday disk



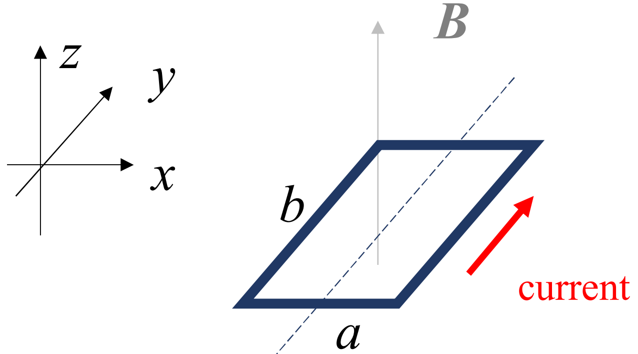
https://en.wikipedia.org/wiki/Homopolar_generator

A homopolar motor



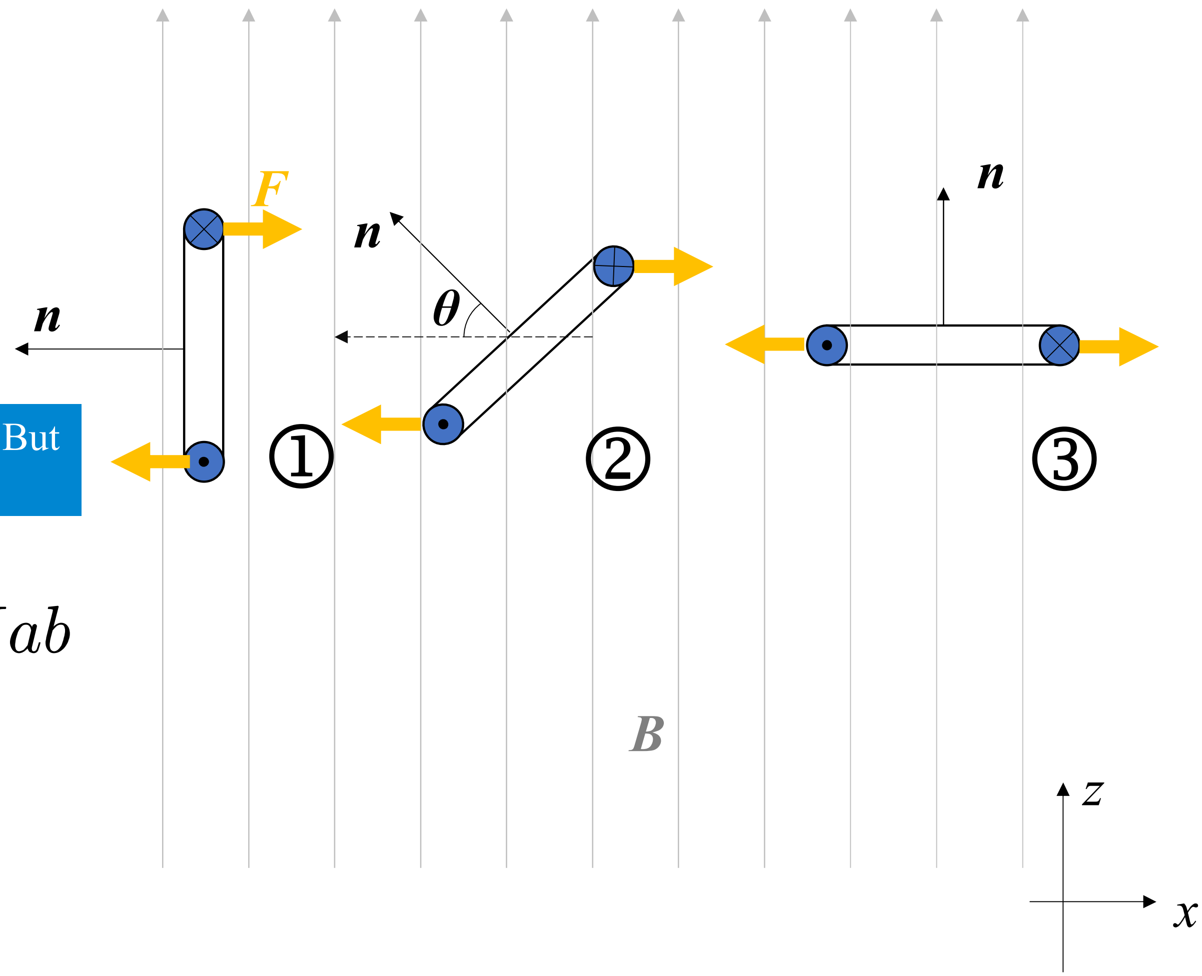


Interaction of magnetic field on a current carrying loop



A current loop rotates about y axis

In homogeneous field, force on a loop is always 0. But not the torque



① $\tau = IBb\frac{a}{2} + IBb\frac{a}{2} = BIab$

② $\tau = BIab\cos\theta$

③ $\tau = 0$

Torque on a loop

$$\tau = BIab\cos\theta$$

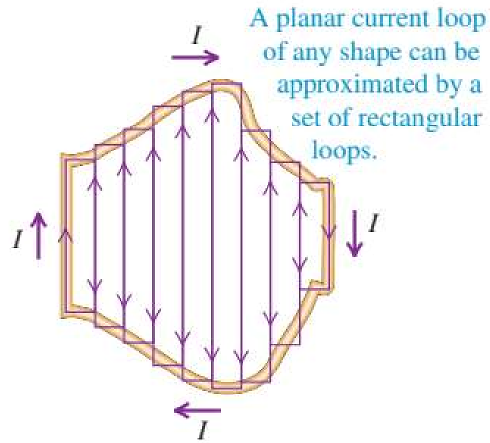
A more formalized form

$$\vec{\tau} = IS\hat{n} \times \vec{B}$$

Define a magnetic moment:

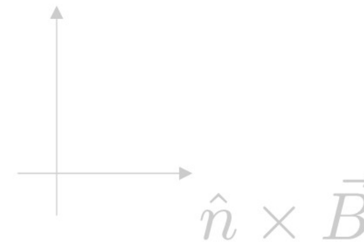
$$\vec{m} \equiv IS\hat{n}$$

27.33 The collection of rectangles exactly matches the irregular plane loop in the limit as the number of rectangles approaches infinity and the width of each rectangle approaches zero.



Direction of rectangle chose as

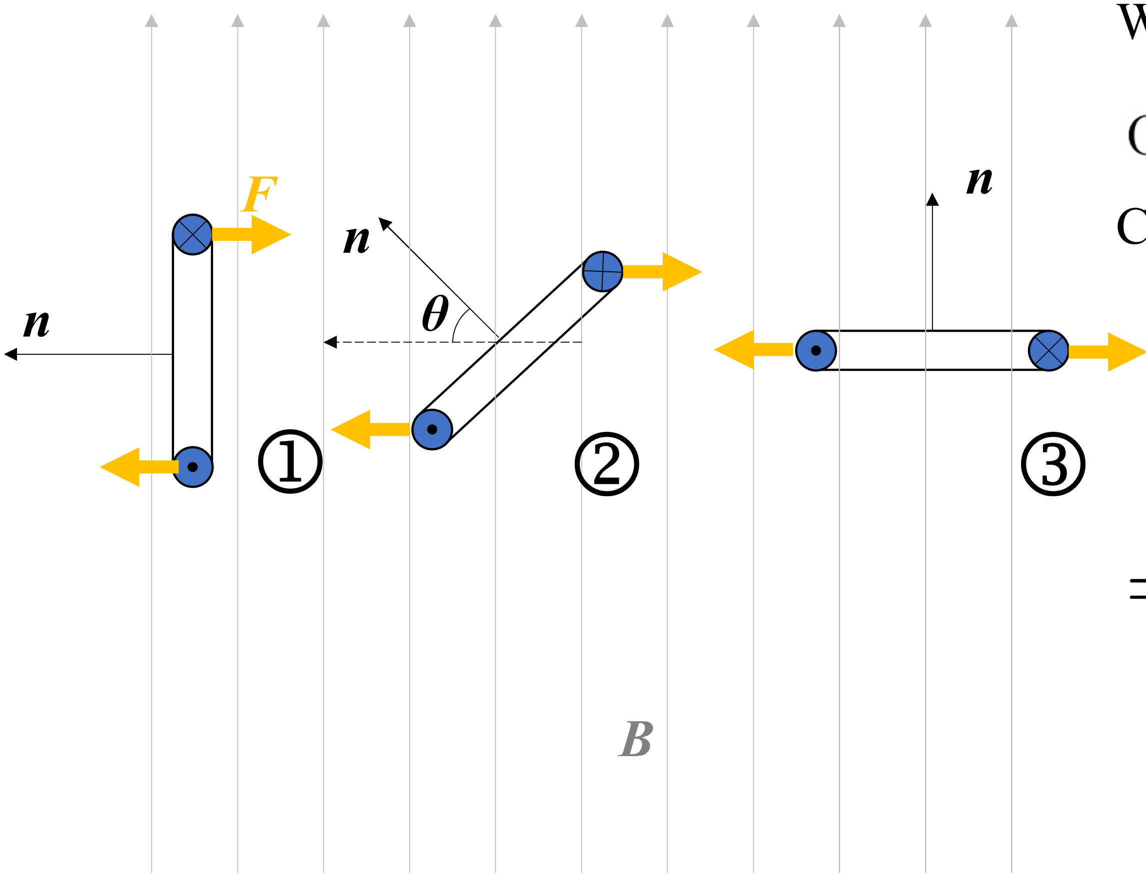
$$\hat{n} \times (\hat{n} \times \vec{B})$$



Generalization: for any loop:

$$\vec{\tau} = \vec{m} \times \vec{B}$$

Energy of the moment



Work done

$$\mathrm{d}W = \tau \mathrm{d}\theta = Bm\cos\theta \mathrm{d}\theta$$

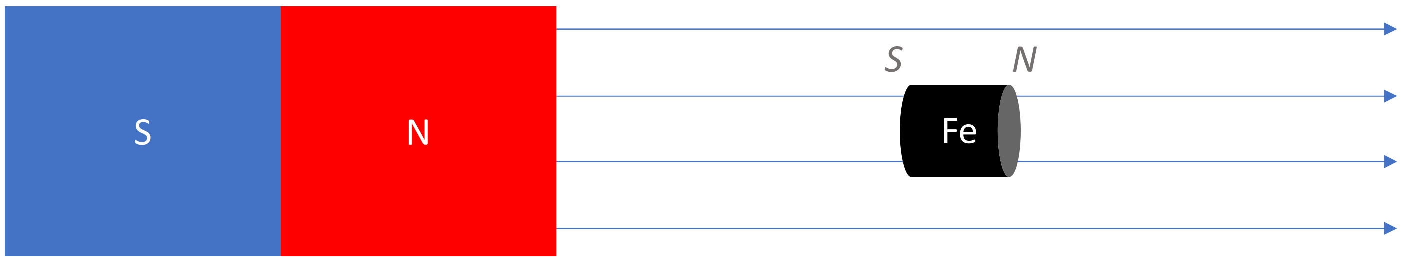
Consider ① to be 0 energy reference:

$$U = -\int_0^{\theta_0} Bm\cos\theta \mathrm{d}\theta$$

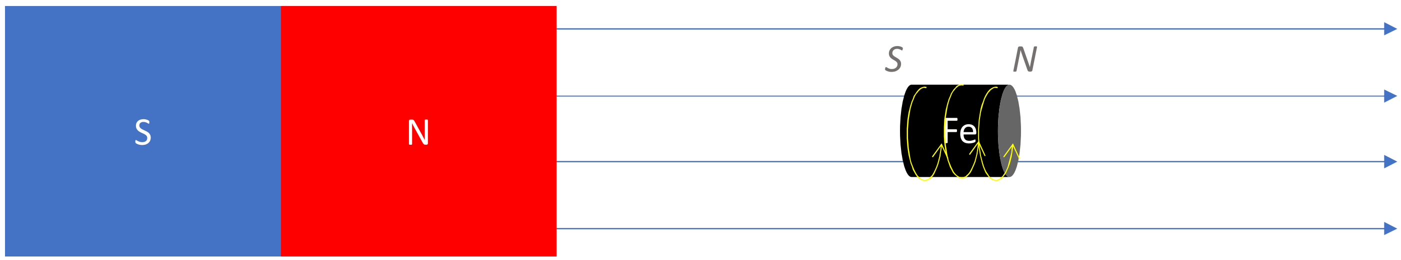
$$= -Bm\sin\theta_0 = -\vec{m}\cdot\vec{B}$$

A moment always tries to align with magnetic field.

Attraction on an iron piece



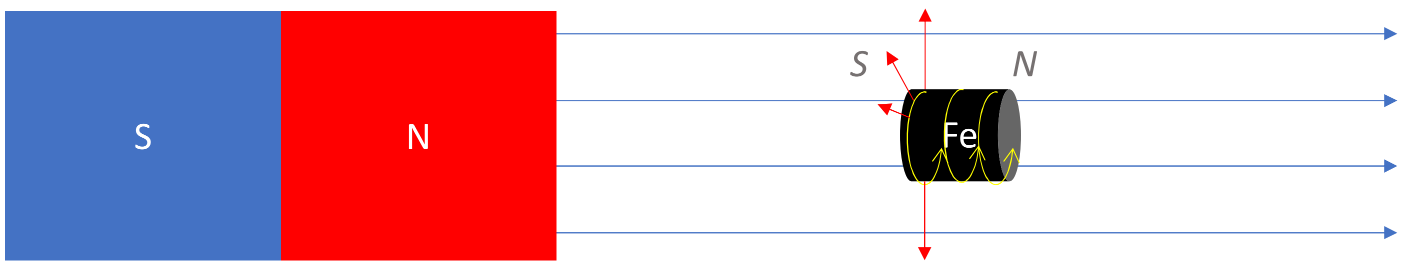
Attraction on an iron piece



magnetization

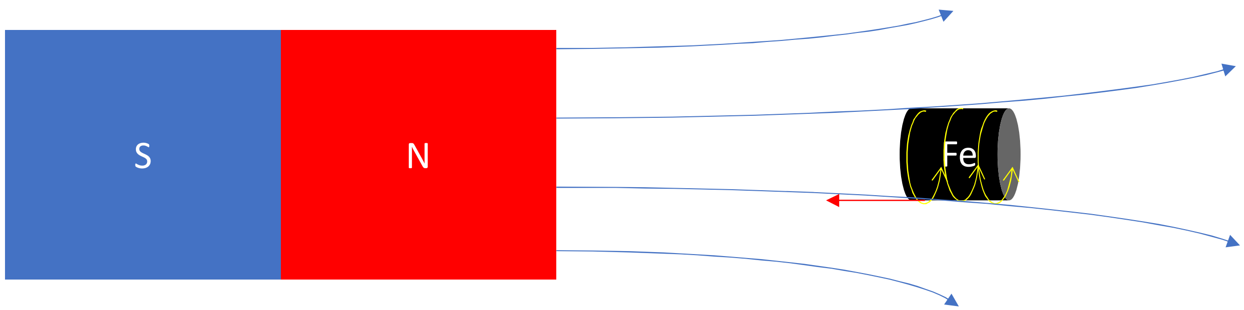
Attraction on an iron piece

No attraction found?



The importance of inhomogeneity

The radial component of the field is the source of attraction.



$$U = -\vec{m} \cdot \vec{B}$$

$$\vec{F} = \nabla(\vec{m} \cdot \vec{B})$$

The magnetic interactions

On a particle

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

On a current carrying conductor

$$\vec{F} = I\vec{l} \times \vec{B}$$

Torque On a planar current loop (*m*=*IS*)

$$\vec{\tau} = \vec{m} \times \vec{B}$$

Force On magnetic dipole moment (*m*=*IS*)

$$\vec{F} = \nabla(\vec{m} \cdot \vec{B})$$

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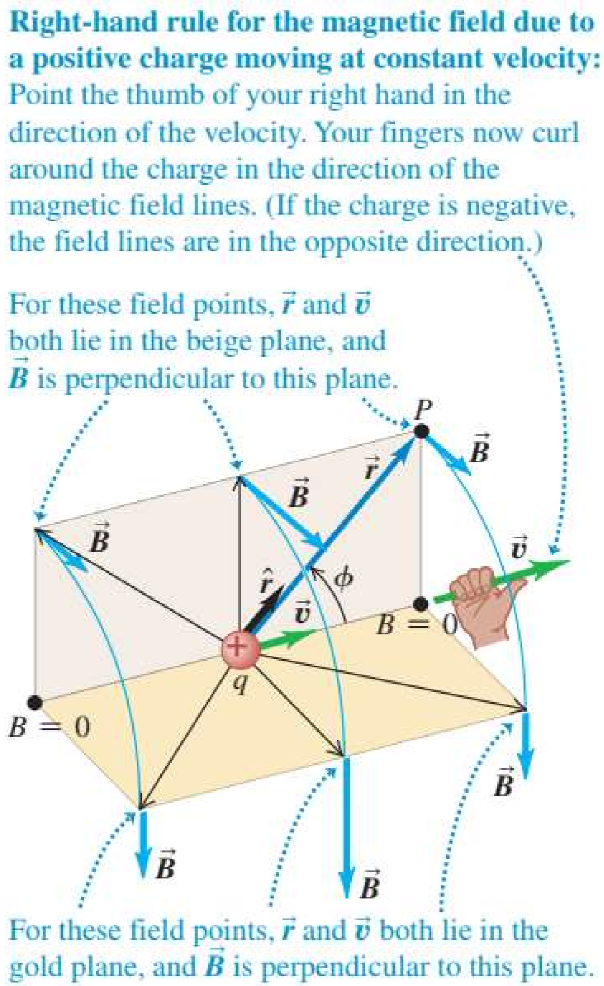
Magnetic field generated by a moving particle



$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\vec{r}}{r^3}$$

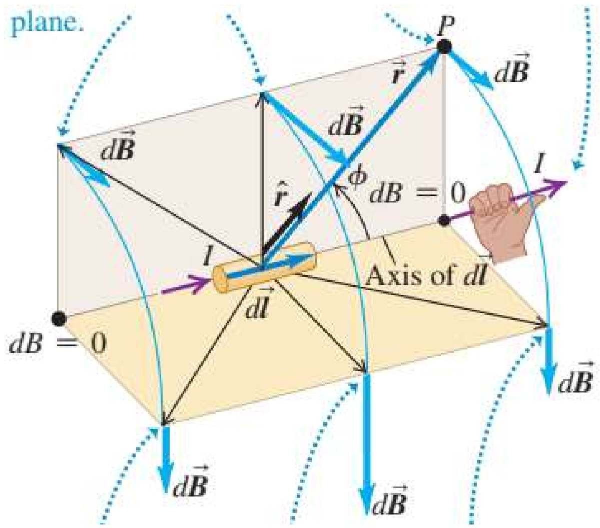
direction of the field

If a positive charge moves along $+z$ direction, field at any position has only $+\theta$ component

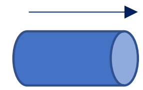
Can also be written as

$$\vec{B}(\vec{r}_f) = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times(\vec{r}_f-\vec{r}_s)}{(\vec{r}_f-\vec{r}_s)^3}$$

Magnetic field generated by a section of wire



$$\sum q_i = \mathrm{d}V N q$$



Current and the wire have the same direction.

$$\sum q_i \vec{v}_i = \mathrm{d}V \vec{J} = A \mathrm{d}l J \hat{J} = I \mathrm{d}\vec{l}$$

Similar picture, just sum of contributions of all charged particles

$$\vec{B}(\vec{r}_f) = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times (\vec{r}_f - \vec{r}_s)}{(\vec{r}_f - \vec{r}_s)^3}$$

$$\vec{B} = \frac{\mu_0}{4\pi} \sum_i q_i \vec{v}_i \times \frac{(\vec{r}_f - \vec{r}_s)}{|\vec{r}_f - \vec{r}_s|^3}$$

Contribution to magnetic field by the current section dl, at the position r_f.

$$\mathrm{d}\vec{B}(\vec{r}_f) = \frac{\mu_0}{4\pi} \frac{I\mathrm{d}\vec{l} \times (\vec{r}_f - \vec{r}_s)}{|\vec{r}_f - \vec{r}_s|^3}$$

Biot-Savart's law

Magnetic field generated by a small current element r_f

current

Cross product

Vector from the source point to the field point.

$$\mathrm{d}\vec{B} = \frac{\mu_0}{4\pi} \frac{\mathrm{d}V \vec{J} \times (\vec{r}_f - \vec{r}_s)}{|\vec{r}_f - \vec{r}_s|^3}$$

Inverse square law

The same as

$$\mathrm{d}\vec{B} = \frac{\mu_0}{4\pi} \frac{I \mathrm{d}\vec{l} \times (\vec{r}_f - \vec{r}_s)}{|\vec{r}_f - \vec{r}_s|^3}$$

Magnetic field is a result of current.

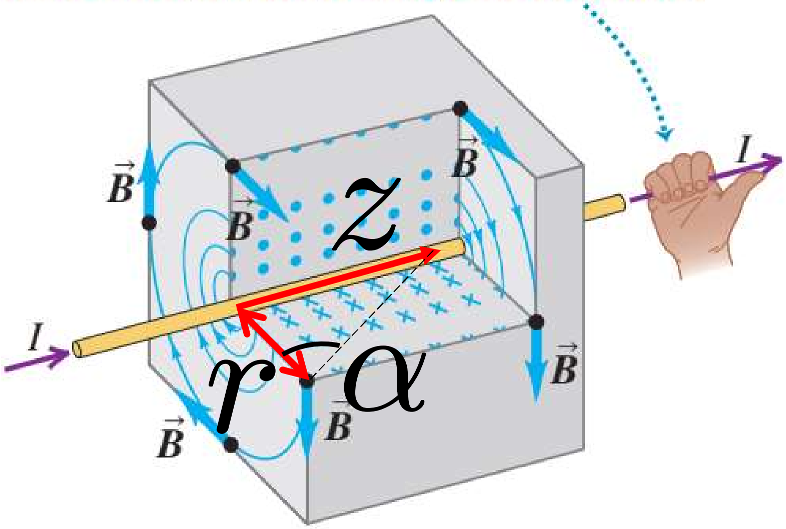
Monopole was never found.

Magnetic field is a relativistic effect of electric field.

The field of a long straight wire

28.6 Magnetic field around a long, straight, current-carrying conductor. The field lines are circles, with directions determined by the right-hand rule.

Right-hand rule for the magnetic field around a current-carrying wire: Point the thumb of your right hand in the direction of the current. Your fingers now curl around the wire in the direction of the magnetic field lines.



$$\frac{\mu_0}{4\pi}\int_{-\infty}^{\infty} I\mathrm{d}z\,\frac{\hat{z}\times(r\hat{r}-z\hat{z})}{(r^2+z^2)^{3/2}}$$
$$=\frac{\mu_0}{4\pi}\int_{-\infty}^{\infty} I\mathrm{d}z\,\frac{r}{(r^2+z^2)^{3/2}}\hat{\theta}$$

$$z=r\tan\alpha$$

$$\int_{-\infty}^{\infty}\frac{\mathrm{d}z}{(r^2+z^2)^{3/2}}=\int_{-\pi/2}^{\pi/2}\frac{\mathrm{d}(r\tan\alpha)}{r^3/\cos^3\alpha}$$
$$=\frac{1}{r^2}\int_{-\pi/2}^{\pi/2}\cos\alpha\,\mathrm{d}\alpha=\frac{2}{r^2}$$

$$\vec{B}=\frac{\mu_0 I}{2\pi r}\hat{\theta}$$

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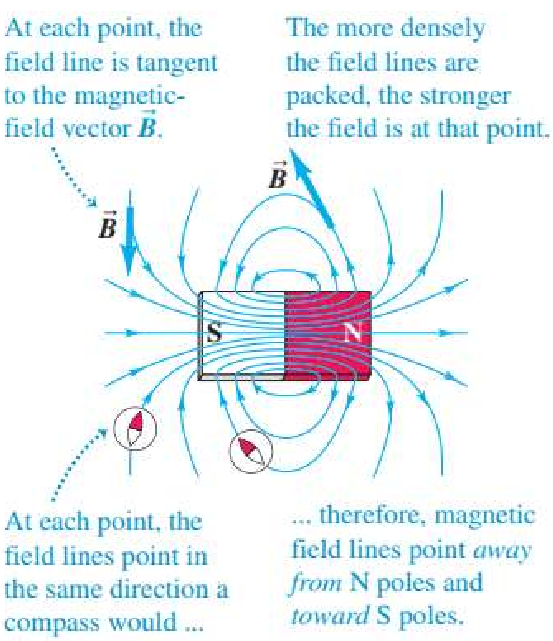
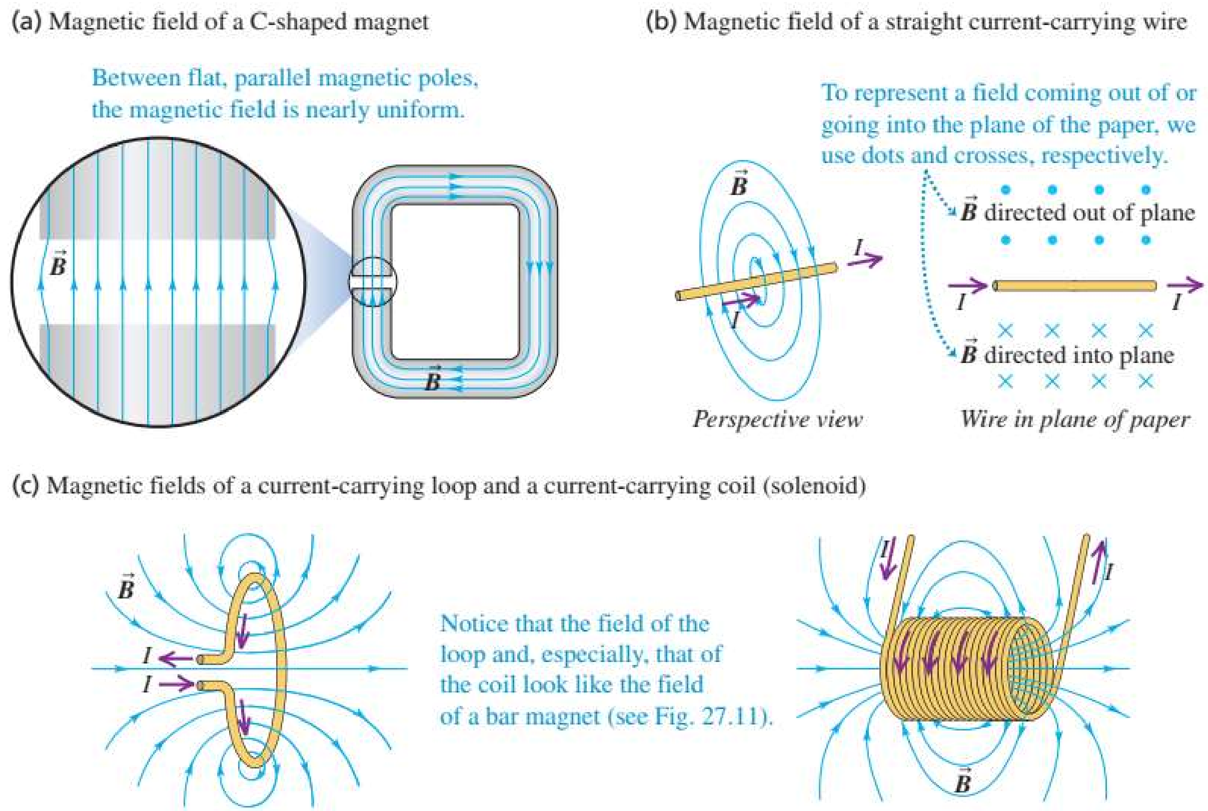
magnetic flux lines

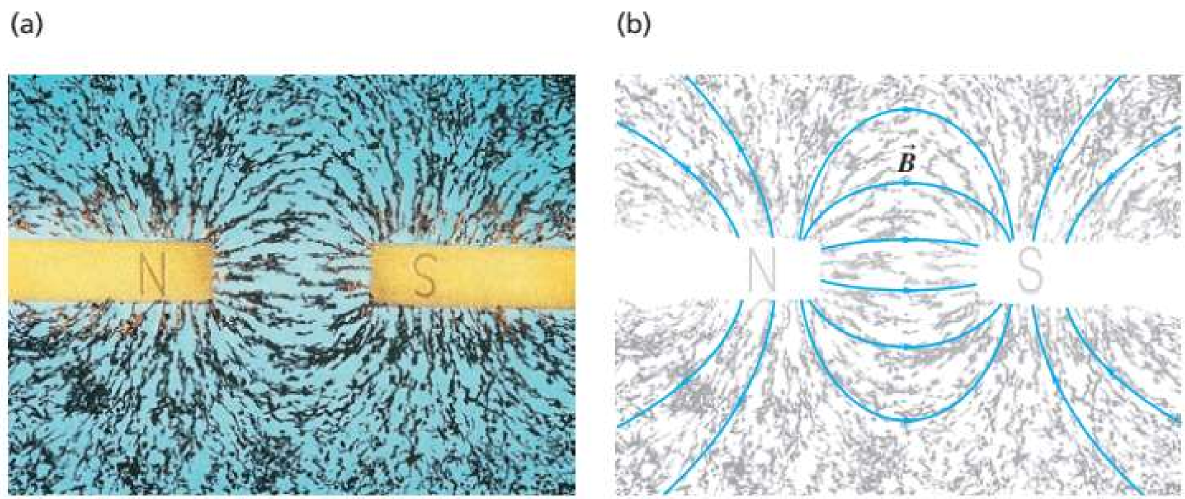
27.11 Magnetic field lines of a permanent magnet. Note that the field lines pass through the interior of the magnet.





Experimental observation of magnetic flux lines

27.14 (a) Like little compass needles, iron filings line up tangent to magnetic field lines. (b) Drawing of field lines for the situation shown in (a).



Magnetic flux

$$\Phi \equiv \int \vec{B} \cdot \mathrm{d}\vec{S}$$

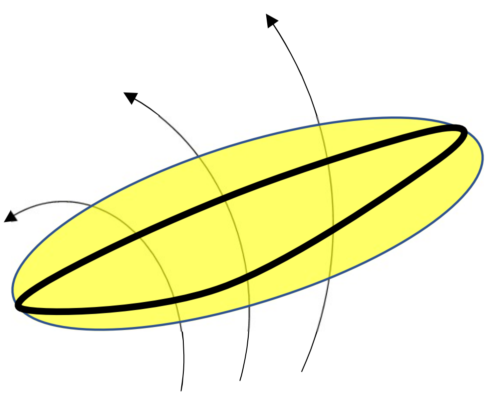
1 Wb = 1 T·m²=1 N·m/A

Geometrical meaning: the number of field lines through a surface

What about the Gaussian flux?

$$\oint_{\partial\Omega} \vec{B} \cdot \mathrm{d}\vec{S} = 0$$

Magnetic Gauss's law



Magnetic flux just depends on the boundary of the surface.

Magnetic field lines are continuous (no start, no end).

$$\nabla \cdot \vec{B} = 0$$

It is simply a fact that the magnetic monopole (magnetic charge) was never found.

Equivalent statements

$$\nabla \cdot \vec{B} = 0$$

$$\iint \vec{B} \cdot \mathrm{d}\vec{S}$$

Depends only on the boundary of the surface.

$$\oiint \vec{B} \cdot \mathrm{d}\vec{S} = 0$$

$$\vec{B} = \nabla \times \vec{A}$$

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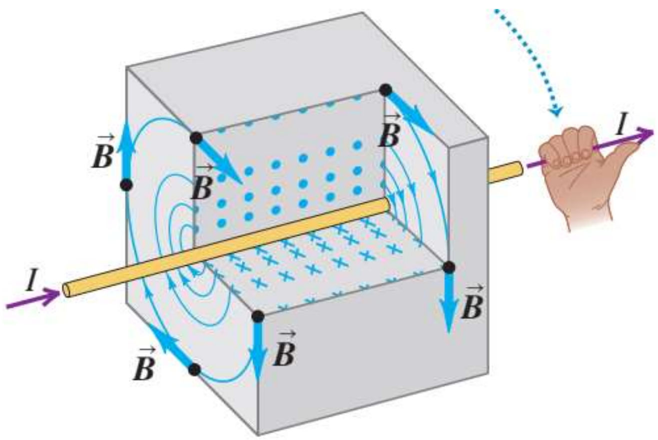
The circulation of magnetic field



$$\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\theta}$$

Around a circular loop, one found

$$\oint \vec{B}\cdot \mathrm{d}\vec{l} = \oint \frac{\mu_0 I}{2\pi r}\hat{\theta}\cdot r\mathrm{d}\theta\hat{\theta} = \mu_0 I$$

The circulation is not depending on the radius of the circle selected.

The result can be generalized to any loop, not only circle.

The result can be generalized to any source current, not only long straight wire current.

Ampere’s circuital law

$$\oint \vec{B} \cdot \mathrm{d}\vec{l} = \mu_0 I$$

He also suggested the molecular current hypothesis during the short period when he studied physics.

André-Marie Ampère

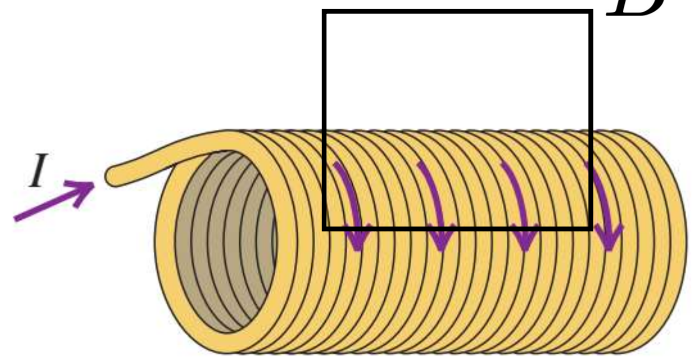
Use ampere's circuital law to solve magnetic field

$$\oint \vec{B} \cdot \mathrm{d}\vec{l} = \mu_0 I$$

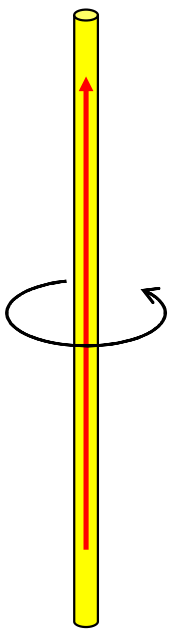
Long solenoid

$$BL = \mu_0 N I$$

$$B = \frac{N\mu_0 I}{L}$$



Long wire

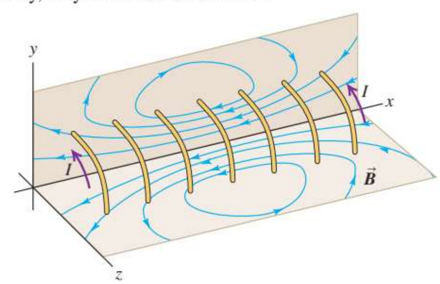


$$2\pi r B = \mu_0 I$$

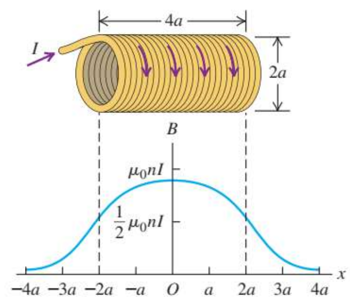
$$B = \frac{\mu_0 I}{2\pi r}$$

Real finite solenoids

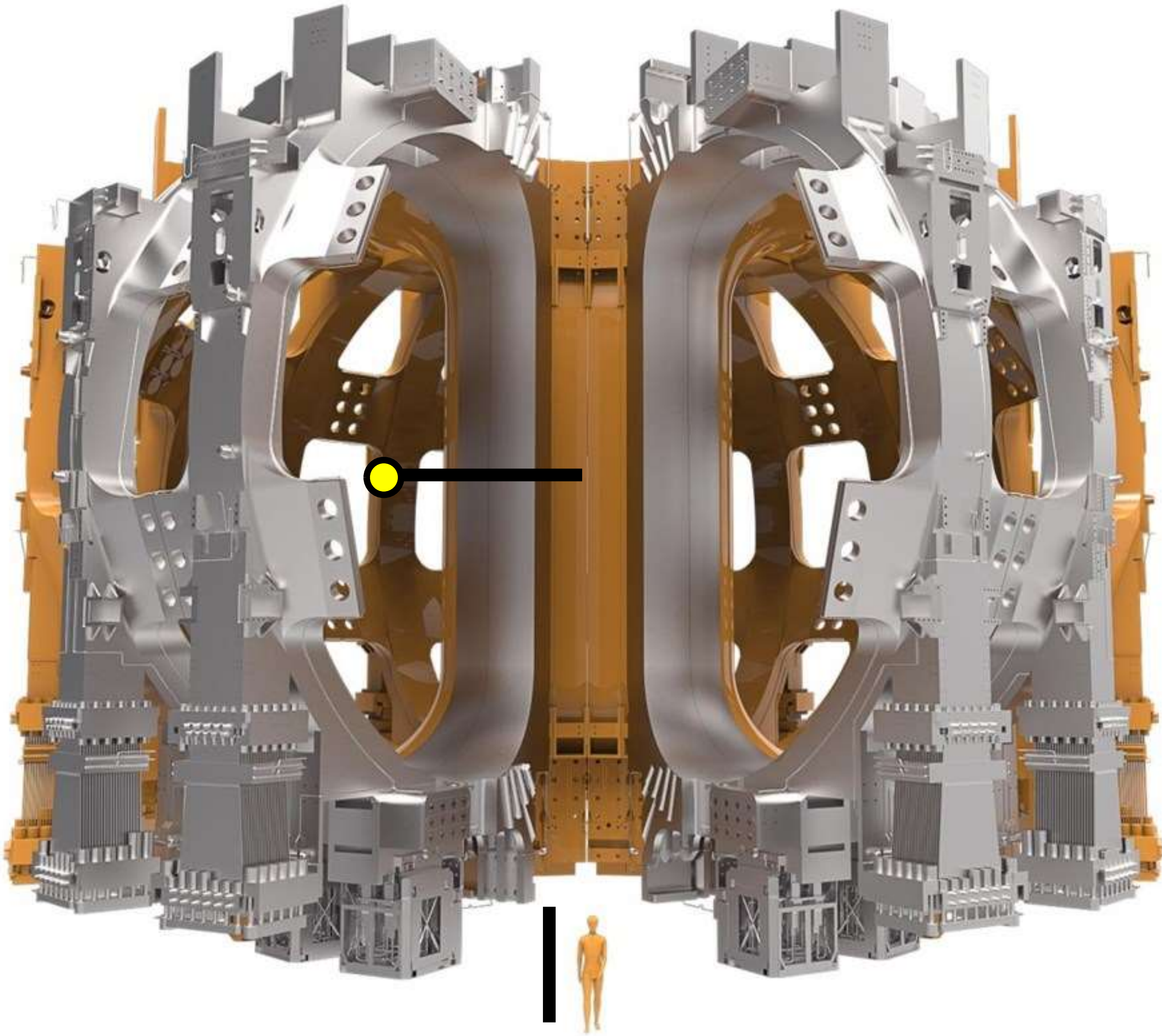
28.22 Magnetic field lines produced by the current in a solenoid. For clarity, only a few turns are shown.



28.24 Magnitude of the magnetic field at points along the axis of a solenoid with length $4a$, equal to four times its radius a. The field magnitude at each end is about half its value at the center. (Compare with Fig. 28.14 for the field of N circular loops.)



Estimate the current in a Tokamak magnet



At the yellow dot, magnetic field is about 11.8 T. Please estimate the total current in one of the 16 magnetic coils (total current is also called magnetomotive force, measured in Ampere turns).

$$2\pi \times 3\mathrm{m} \times B = 16\mu_0 I$$

$$I = \frac{11.8\times 2\pi\times 3}{16\times 4\pi\times 10^{-7}}\mathrm{A} = 11\ \mathrm{MA}$$

Contents

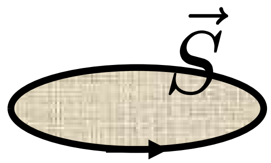
上海交通大学 SHANGHAI JIAO TONG UNIVERSITY

Stokes theorem applied to circulation of *B*

$$\vec{X} \equiv \nabla \times \vec{B}$$

Assume that there is a field ***X***, being the curl of ***B***, what properties does ***X*** have?

$$\int \vec{X} \cdot \mathrm{d}\vec{S}$$



$$\int \vec{X} \cdot \mathrm{d}\vec{S} = \oint \vec{B} \cdot \mathrm{d}\vec{l} = \mu_0 I$$

$$\vec{X} = \mu_0 \vec{J}$$

The differential form of Ampere's circuital law:

$$\nabla \times \vec{B} = \mu_0 \vec{J}$$

Comparison between (static) *E* and *B*

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} \qquad \nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = 0 \qquad \nabla \times \vec{B} = \mu_0 \vec{J}$$