Physics (PHYS2500J), Unit 5 Electromagnetic wave
2. Sinusoidal wave

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Sinusoidal plane wave

$$\frac{\partial^2 \vec{E}}{\partial x^2} = \epsilon_r \epsilon_0 \mu_r \mu_0 \frac{\partial^2 \vec{E}}{\partial t^2} \qquad \frac{\partial^2 \vec{B}}{\partial x^2} = \epsilon_r \epsilon_0 \mu_r \mu_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

The general solution to these equations?

Sinusoidal electromagnetic plane wave, propagating in +x-direction:

Electric field; Electric-field magnitude; Wave number; Angular frequency

$$\vec{E}(x,t) = \hat{j}E_{\max}\cos(kx - \omega t)$$
$$\vec{B}(x,t) = \hat{k}B_{\max}\cos(kx - \omega t) \qquad (32.17)$$

Magnetic field; Magnetic-field magnitude

The solution is a sinusoidal function of time. It is also a sinusoidal function of space.

Put the solution into the equation and see if the equation is satisfied.

$k = \frac{2\pi}{\lambda}$ Is called the wave number, which describes the spatial frequency of the wave.

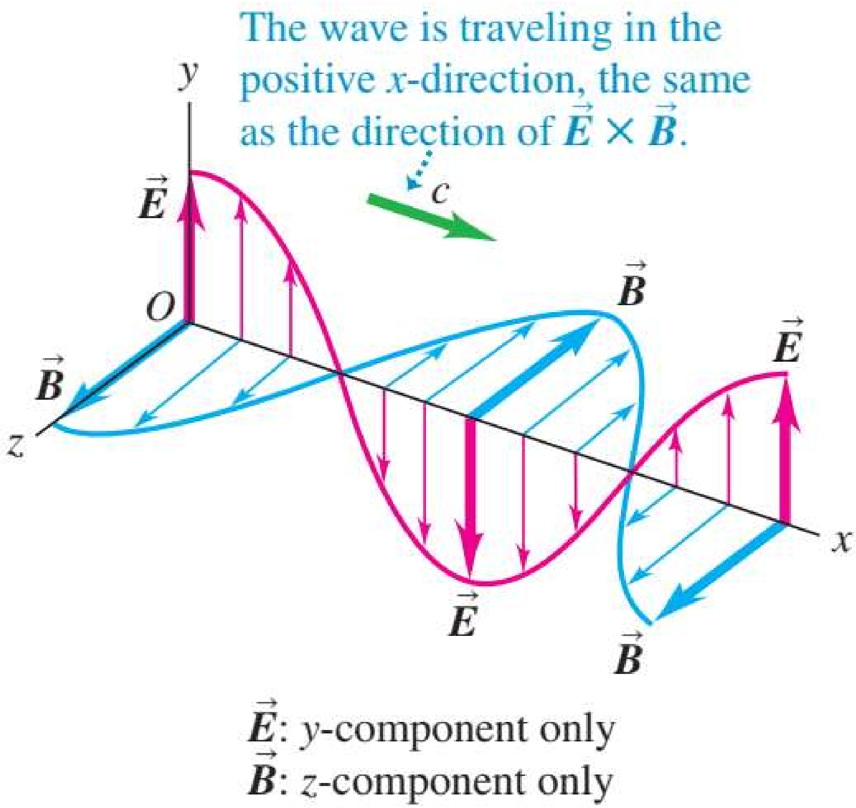
$v = \frac{\omega}{k}$ Information of Speed of EM wave is also included in the solution

Three main features of the sinusoidal function are the amplitude, frequency and the phase

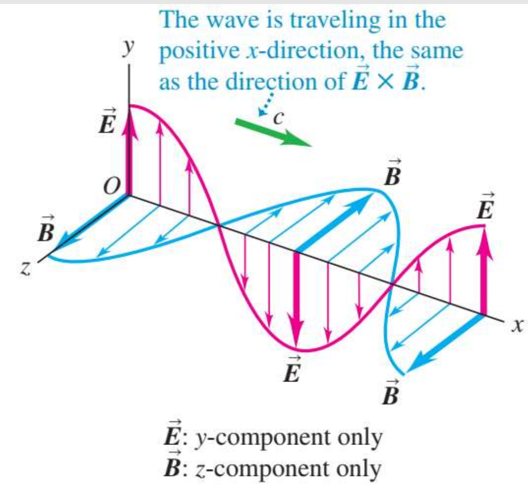
The amplitude determines the intensity of the beam;
The frequency determines the energy of a single photon;
The phase determines the interference properties.

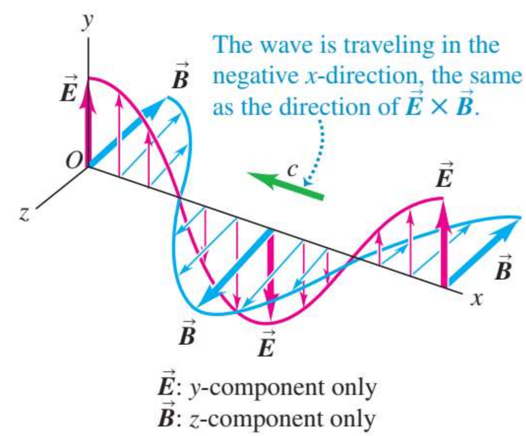
The image of the sinusoidal solution.

1. Imaging the wave start to propagate, what is propagating?



2. The direction of propagation of a sinusoidal wave is also defined as $E \times B$.





The phase of *E* field and *B* field

Sinusoidal electromagnetic plane wave, propagating in +*x*-direction:

$$\vec{E}(x,t) = \hat{\jmath}E_{\max}\cos(kx - \omega t)$$
$$\vec{B}(x,t) = \hat{k}B_{\max}\cos(kx - \omega t) \tag{32.17}$$

Electric field; Electric-field magnitude; Wave number; Angular frequency; Magnetic field; Magnetic-field magnitude

From the wave equation, it is possible that the magnetic field and electric field can have a phase difference. But considering the full Maxwell's equations, the two has to be in the same phase.

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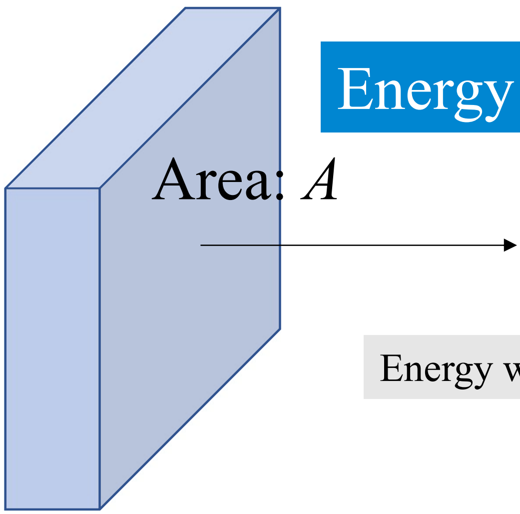
Intensity of EM wave

The intensity of EM wave should be related to the energy per **unit cross section** and per **unit time**

unit: W/m^2



Energy through the surface in the next Δt?

$$u = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0}(\sqrt{\epsilon_0\mu_0}E)^2 = \epsilon_0 E^2$$

Energy written as the product of the energy density and the volume

$$\delta U = \epsilon_0 E^2 A c \delta t = \mu_0 H^2 A c \delta t$$

$$\frac{\delta U}{\delta t}\frac{1}{A} = \epsilon_0 E^2 c = \mu_0 H^2 c = |E \times H|$$

Poynting vector:

Poynting vector in vacuum

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$$

Electric field

Magnetic field

Magnetic constant

A vector describes the flow of energy (能流密度)

EM wave is the time average of Poynting vector magnitude.

$$I = \overline{|\vec{S}|}$$

Alternative proof

From Joule's law

$\int_\Omega \vec{E}\cdot\vec{J}\mathrm{d}V$ is the Joule heating power inside a volume Ω

Maxwell's equation: circuital law of magnetic field: $\vec{E}\cdot\vec{J} = \vec{E}\cdot(\nabla\times\frac{\vec{B}}{\mu_0} - \epsilon_0\frac{\partial\vec{E}}{\partial t})$

Vector differential equality: $\nabla\cdot(\vec{E}\times\vec{B}) = \vec{B}\cdot(\nabla\times\vec{E}) - \vec{E}\cdot(\nabla\times\vec{B})$

$$\int_\Omega \vec{E}\cdot\vec{J}\mathrm{d}V$$

Energy dissipation in the volume

$$= \frac{1}{\mu_0}\int_\Omega \vec{E}\cdot(\nabla\times\vec{B})\mathrm{d}V - \int_\Omega \epsilon_0\vec{E}\cdot\frac{\partial\vec{E}}{\partial t}\mathrm{d}V$$

$$= \underline{-\frac{1}{\mu_0}\int_\Omega \nabla\cdot(\vec{E}\times\vec{B})\mathrm{d}V} - \int_\Omega \underline{\frac{1}{\mu_0}\vec{B}\cdot\frac{\partial\vec{B}}{\partial t}}\mathrm{d}V - \int_\Omega \underline{\epsilon_0\vec{E}\cdot\frac{\partial\vec{E}}{\partial t}}\mathrm{d}V$$

$\oint_{\partial\Omega}(\vec{E}\times\frac{\vec{B}}{\mu_0})\cdot\mathrm{d}\vec{S}$ Energy flow into the volume

Energy storage rate in B field

Energy storage rate in E field

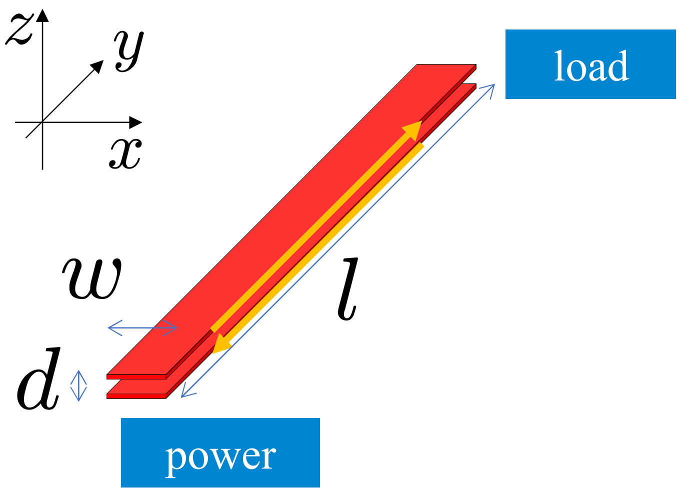
Why are the two terms the energy storage rates?

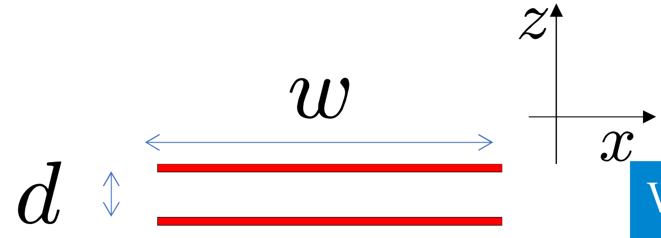
$$\mathcal{E}_e = \frac{\epsilon_0 E^2}{2}$$
$$\frac{\partial \mathcal{E}_e}{\partial t} = \epsilon_0 E \frac{\partial E}{\partial t}$$

Poynting vector describes energy flow in Electro-magnetic fields

1. It is not only valid for EM wave

2. Please draw the electromagnetic field around the parallel long copper strips used as conductors. (consider the conductors have no resistance)





What are the electric and magnetic fields like.

Both fields are confined in the channel. Electric field has only -*z* component and magnetic field has only -*x* component

3. Contribution from both tapes:

$$\vec{E} = -\hat{z}\frac{U}{d}$$

$$\vec{H} = -\hat{x}\frac{I}{w}$$

U being the voltage supplied by the power source

4. The Poynting vector

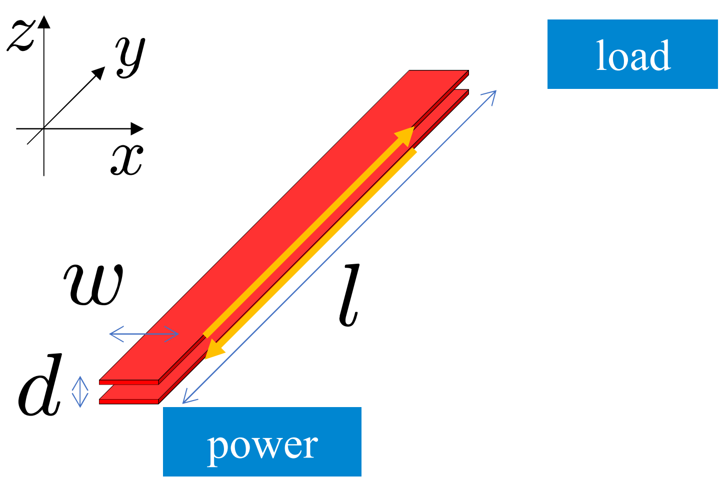
$$\vec{E} \times \vec{H} = \hat{y}\frac{UI}{dw}$$

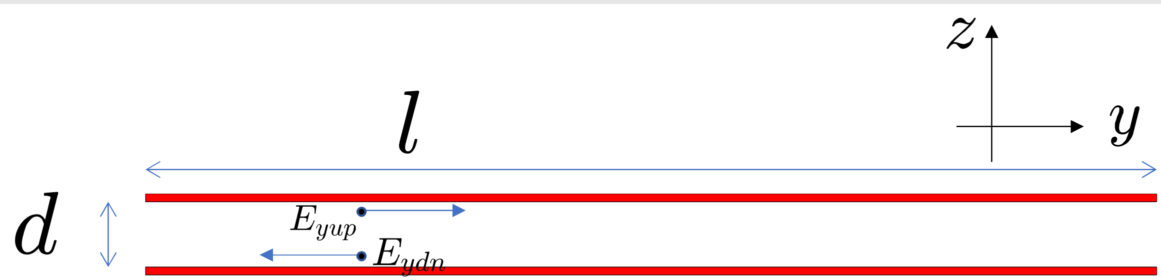
Direction from power source to load.

Poynting vector times cross section, gives exactly the total power.

A real wire with finite resistance

1. What if the wire has finite resistance, say *R* per wire.





2. Electric field contains an *y* component then. Due to the boundary condition of tangential electric field.

$$E_{yup} = \frac{IR}{l}$$

$$E_{ydn} = \frac{-IR}{l}$$

3. Magnetic field is kept unchanged.

$$\vec{H} = -\hat{x}\frac{I}{w}$$

$$\vec{S} = \vec{E} \times \vec{H}$$

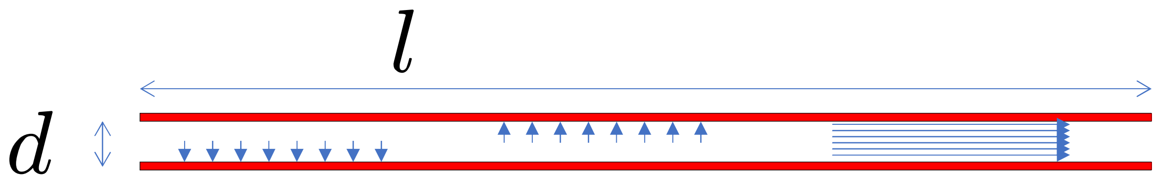
$$S_{zup} = \frac{I^2R}{wl}$$

Direction pointing into the conductor

$$S_{zdn} = -\frac{I^2R}{wl}$$

Poynting vector times cross section, gives exactly the loss power.

Summary

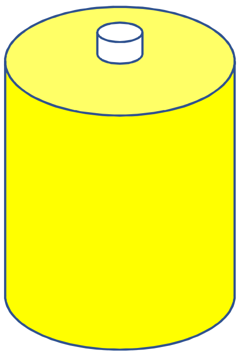


For real conductors, the loss power is also transferred into the conductor through Poynting vector.

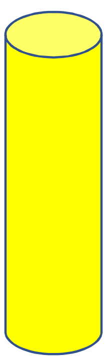
Poynting vector brings energy from power source to the load.

Test

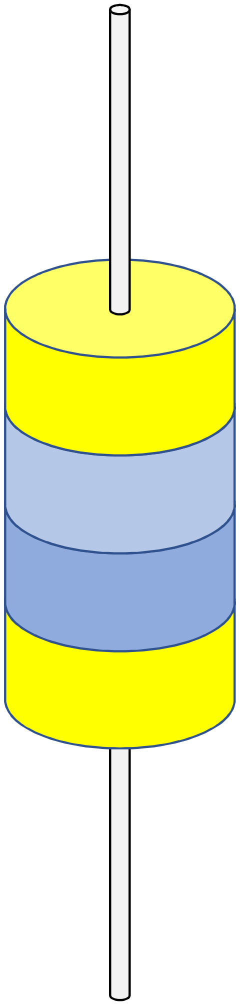
Try to draw the field and Poynting vectors for such devices



A battery



A section of wire



The load

Momentum of EM wave

1. Electromagnetic wave has not only energy but momentum. According to relativity, the relation between energy and momentum of a photon, whose static mass is 0, is:

$$E = mC^2$$

$$\vec{P} = mC\hat{P} = \frac{E}{C}\hat{P}$$

2. The effect can be proved by experiments. Radiation pressure: shine a beam of light to an object, force is exerted as

$$\vec{F} = \frac{\mathrm{d}\vec{P}}{\mathrm{d}t}$$

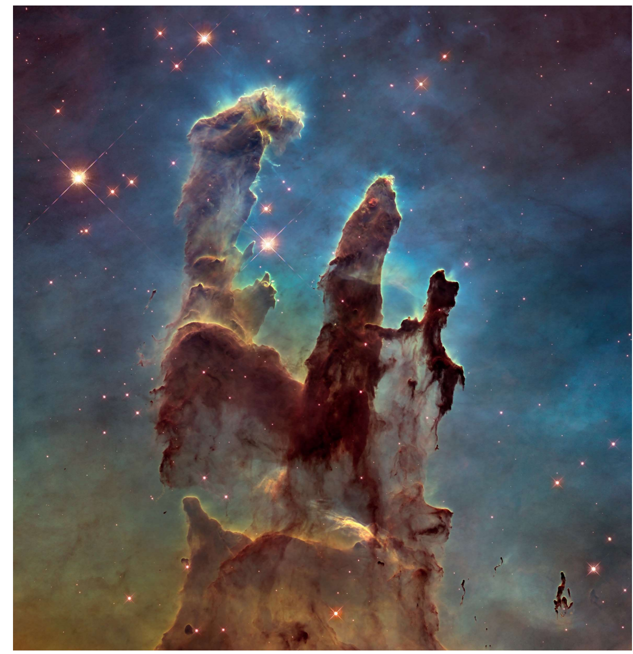
NATIONAL GEOGRAPHIC

Solar sails are made of ultrathin, highly reflective material. When a photon from the sun hits mirror-like surface, it bounces off the sail and transfers its momentum.
PHOTOGRAPH BY NASA/MSFC

Can be used for propulsion (not only in Si-Fi)

New NASA Spacecraft Will Be Propelled By Light

Solar sails could travel to the outermost regions of the solar system fast than ever before.



The Pillars of Creation clouds within the Eagle Nebula shaped by radiation pressure and stellar winds.

Momentum of EM wave: radiation pressure

Direct sun light intensity: 1.4 kW/m^2

$$p_{\text{rad}} = \frac{I}{c} = \frac{1.4 \times 10^3 \text{ W/m}^2}{3.0 \times 10^8 \text{ m/s}} = 4.7 \times 10^{-6} \text{ Pa}$$

If the beam is reflected, What should the radiation pressure be?

doubled

Contents

Boundary for EM wave: boundary conditions

1. If more than one medium is present in the space, one can write down the wave equations with $\mu\varepsilon$ for the left part and with $\mu_0\varepsilon_0$ for the right part.

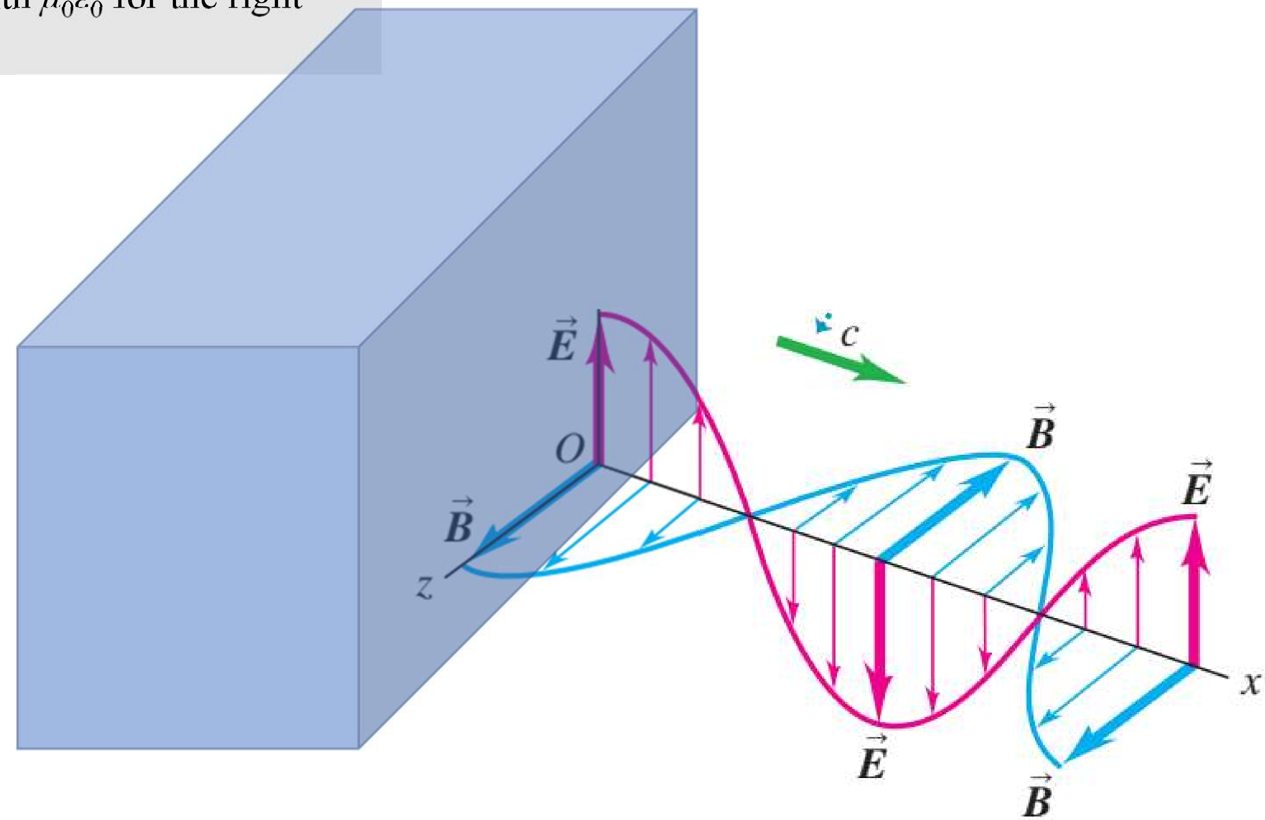
2. But what is the relation between the two wave solutions? Can they be freely changed?

Boundary conditions for electric field

$$E_{t1} = E_{t2}$$
$$\epsilon_1 E_{n1} = \epsilon_2 E_{n2}$$

Similarly, boundary conditions for *B* field

$$B_{n1} = B_{n2}$$
$$\frac{B_{n1}}{\mu_1} = \frac{B_{n2}}{\mu_2}$$

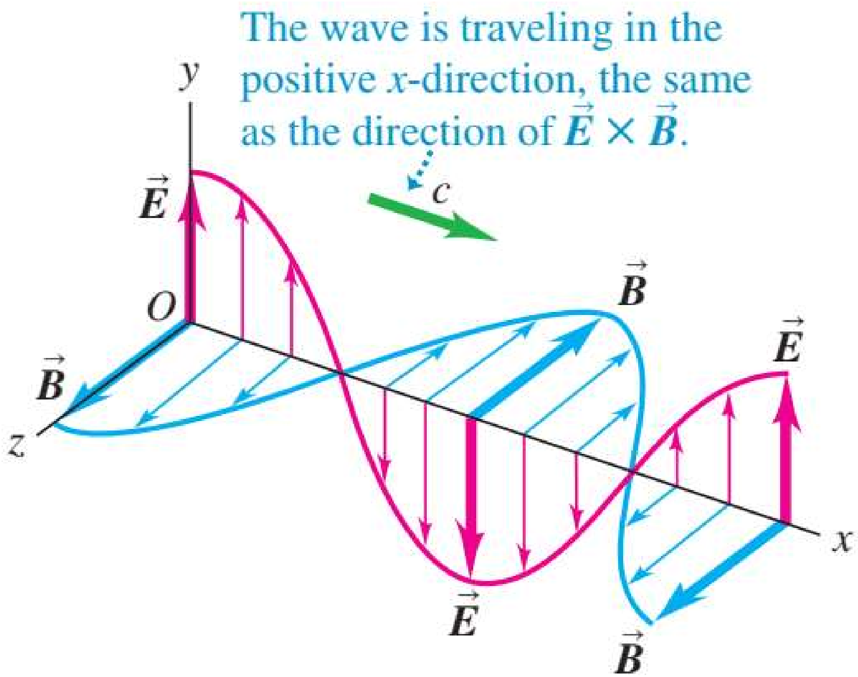


$\vec{E}$: y-component only
$\vec{B}$: z-component only

Metal boundaries

1. Metal is not transparent to EM wave, the wave is absorbed by resistive material with a shallow depth (skin depth)

2. To solve the EM wave, metal can be considered as E=0, so the boundary condition requires that E_t in free space is also 0.



$\vec{E}$: y-component only
$\vec{B}$: z-component only

3. apparently, a plane sinusoidal wave is not the solution for free space, since there is no point with 0 E all the time.

4. What about two plane sinusoidal waves?

$$E_y(x,t) = E_{\max}[\cos(kx + \omega t) - \cos(kx - \omega t)]$$

5. What are the two component waves like? Can you imagine?

Incident beam | Reflected beam

6. Both with E along y, same amplitude, different propagation direction (different E-H direction relation), so what is B_z?

$$B_z(x,t) = B_{\max}[-\cos(kx + \omega t) - \cos(kx - \omega t)]$$

Using

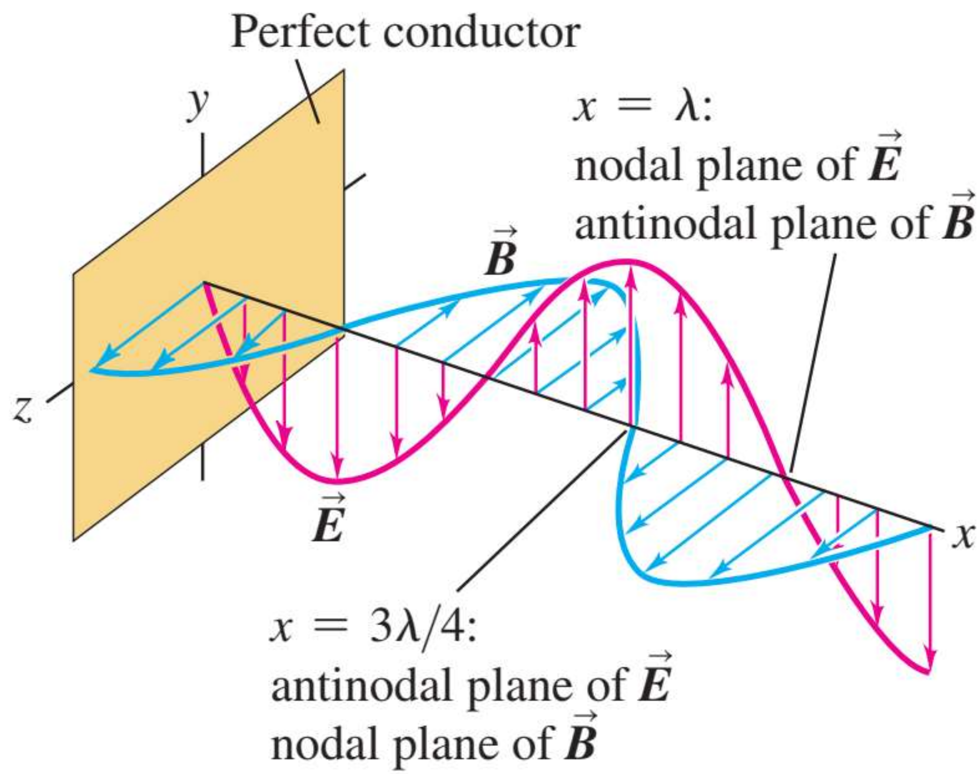
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

get

$$E_y(x, t) = -2E_{max} \sin kx \sin \omega t$$

$$B_z(x, t) = -2B_{max} \cos kx \cos \omega t$$

Standing wave, which contains nodal points (planes) and anti-nodal planes.



1. Nodal plane for E is at the boundary
2. Nodal plane for E is the anti-nodal plane for B, and vise-versa.

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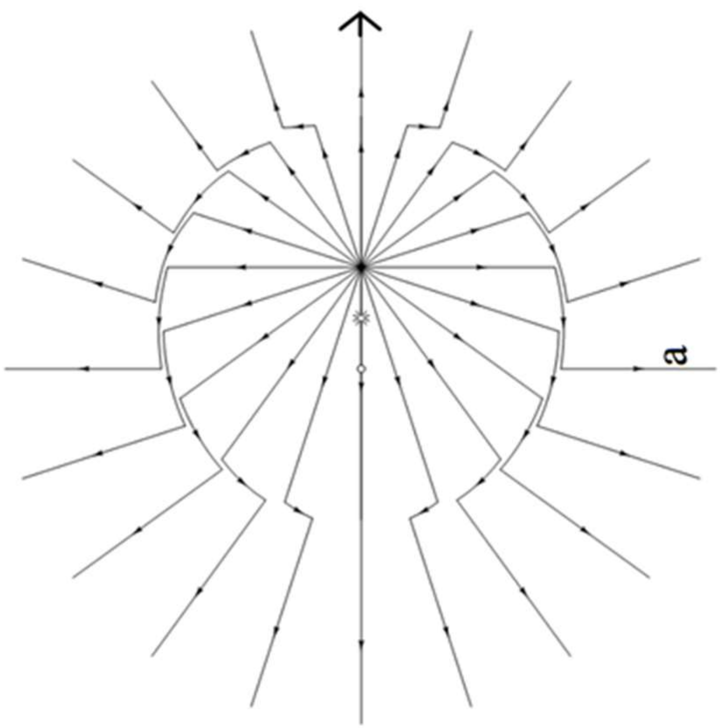
Transverse feature of EM wave requires transverse field.
Relativity gives that constant velocity moving charge will not generate transverse field.

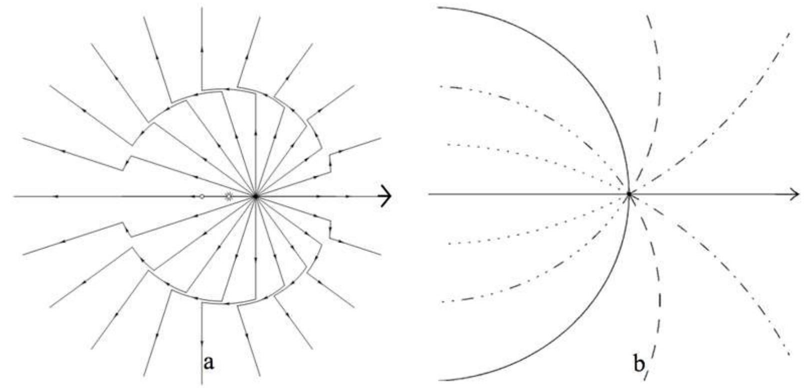


$$S \propto a^2$$

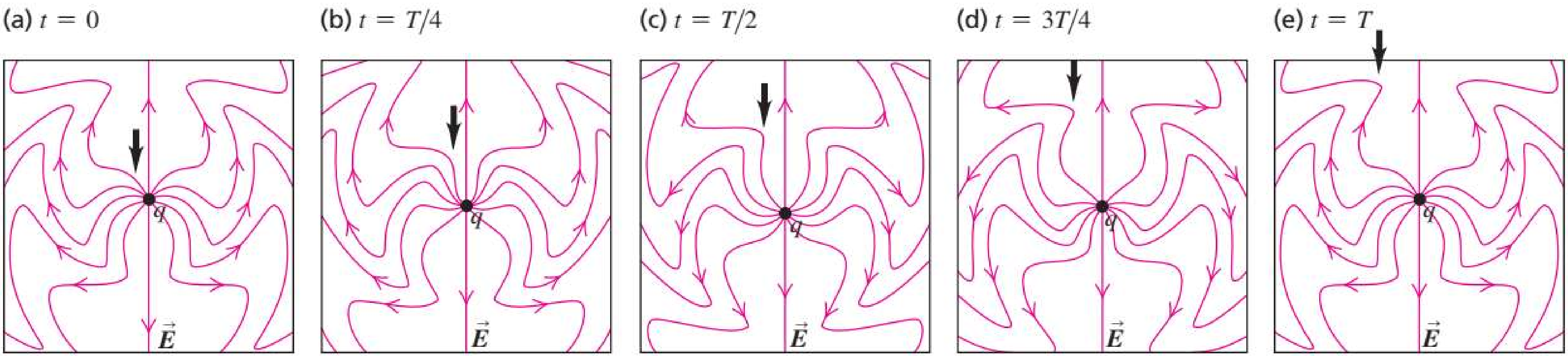
$$S \propto \frac{1}{r^2}$$

$$S \propto \sin^2 \theta$$

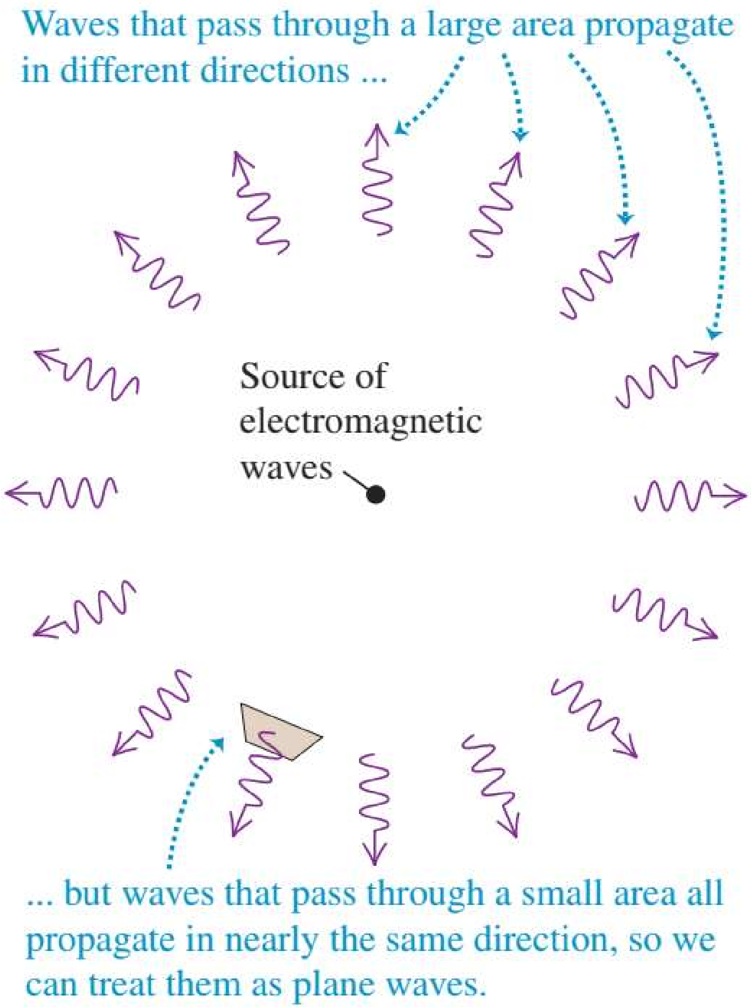
From abrupt acceleration to continuous acceleration

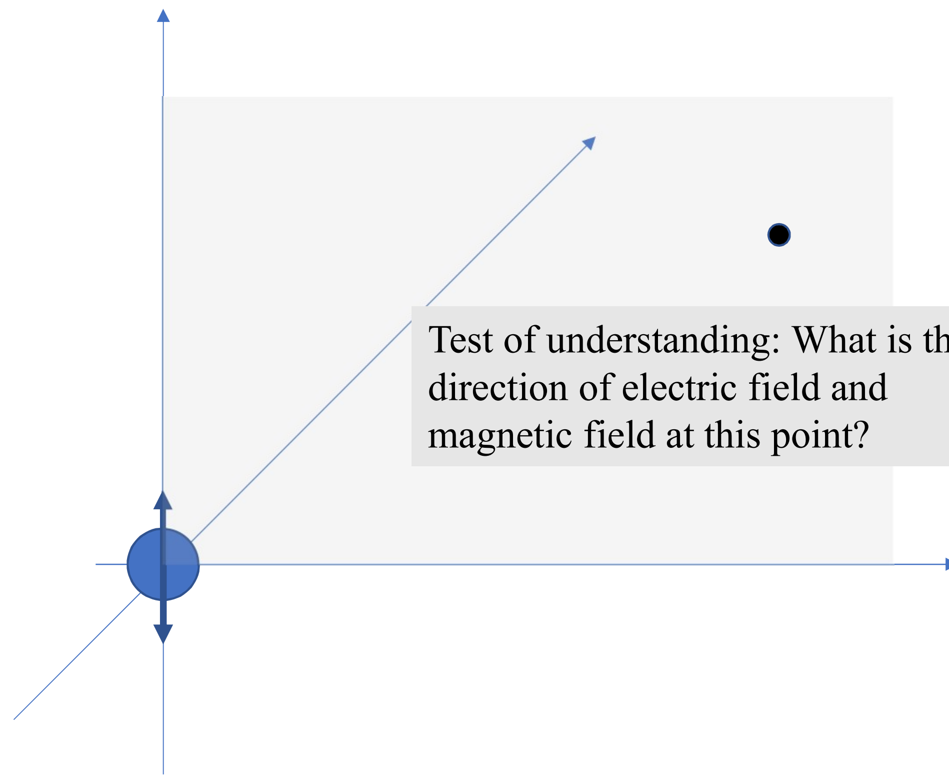


32.3 Electric field lines of a point charge oscillating in simple harmonic motion, seen at five instants during an oscillation period T. The charge's trajectory is in the plane of the drawings. At $t = 0$ the point charge is at its maximum upward displacement. The arrow shows one "kink" in the lines of $\vec{E}$ as it propagates outward from the point charge. The magnetic field (not shown) contains circles that lie in planes perpendicular to these figures and concentric with the axis of oscillation.



Relation between spherical radiation and plane wave

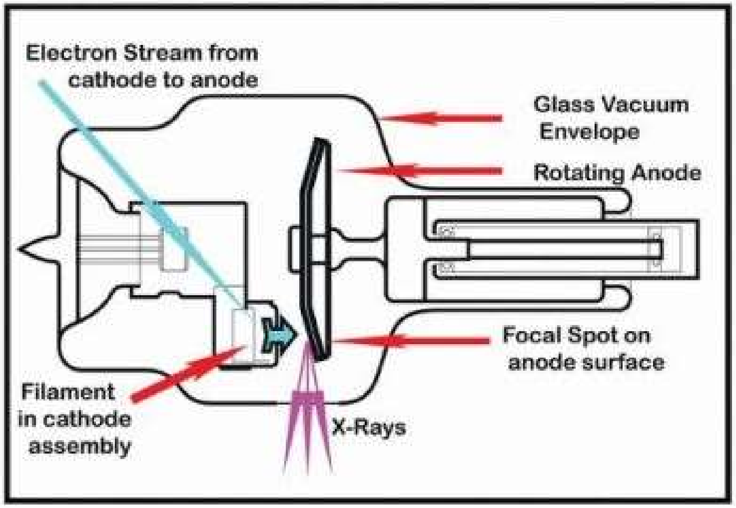


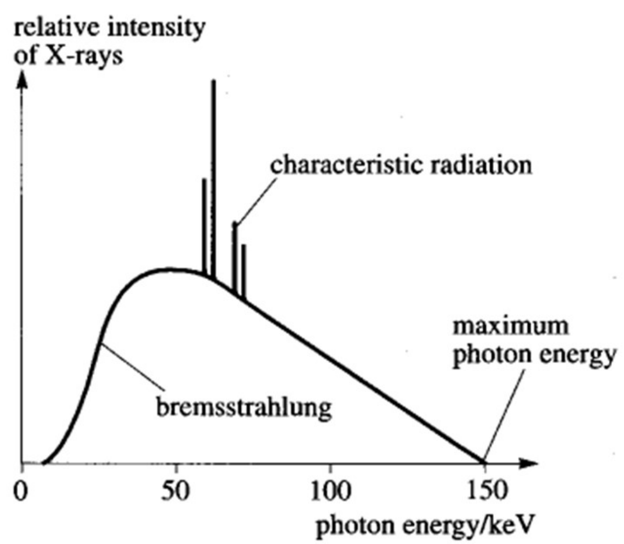


Test of understanding: What is the direction of electric field and magnetic field at this point?

Generation of X-ray: X-ray tube

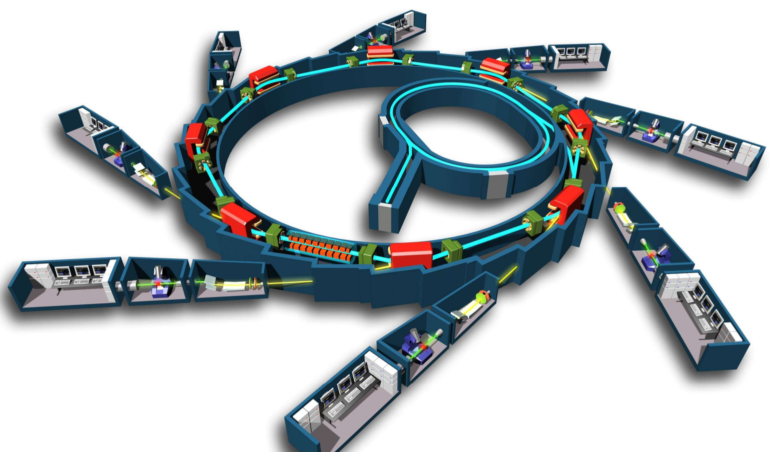
High energy electrons bombarding a "target"



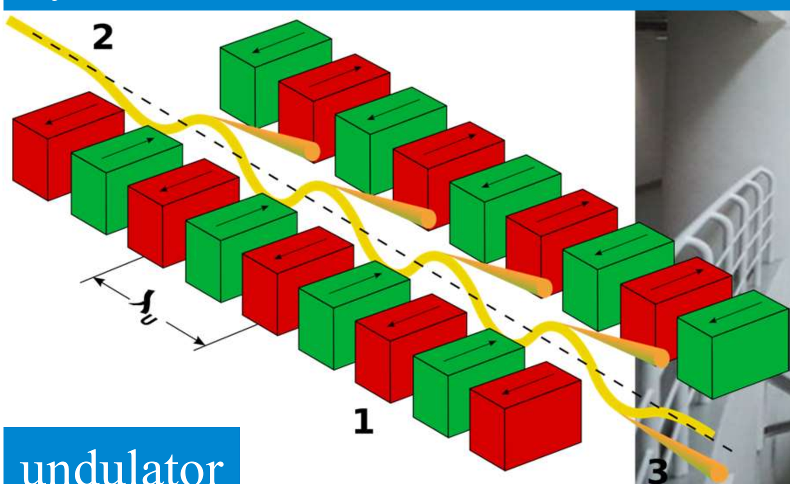


In the spectrum, always a broad background, called Bremsstrahlung (German for break), which is due to the deceleration of electrons.

Generation of X-ray: Synchrotron



Synchrotron: accelerator for electron



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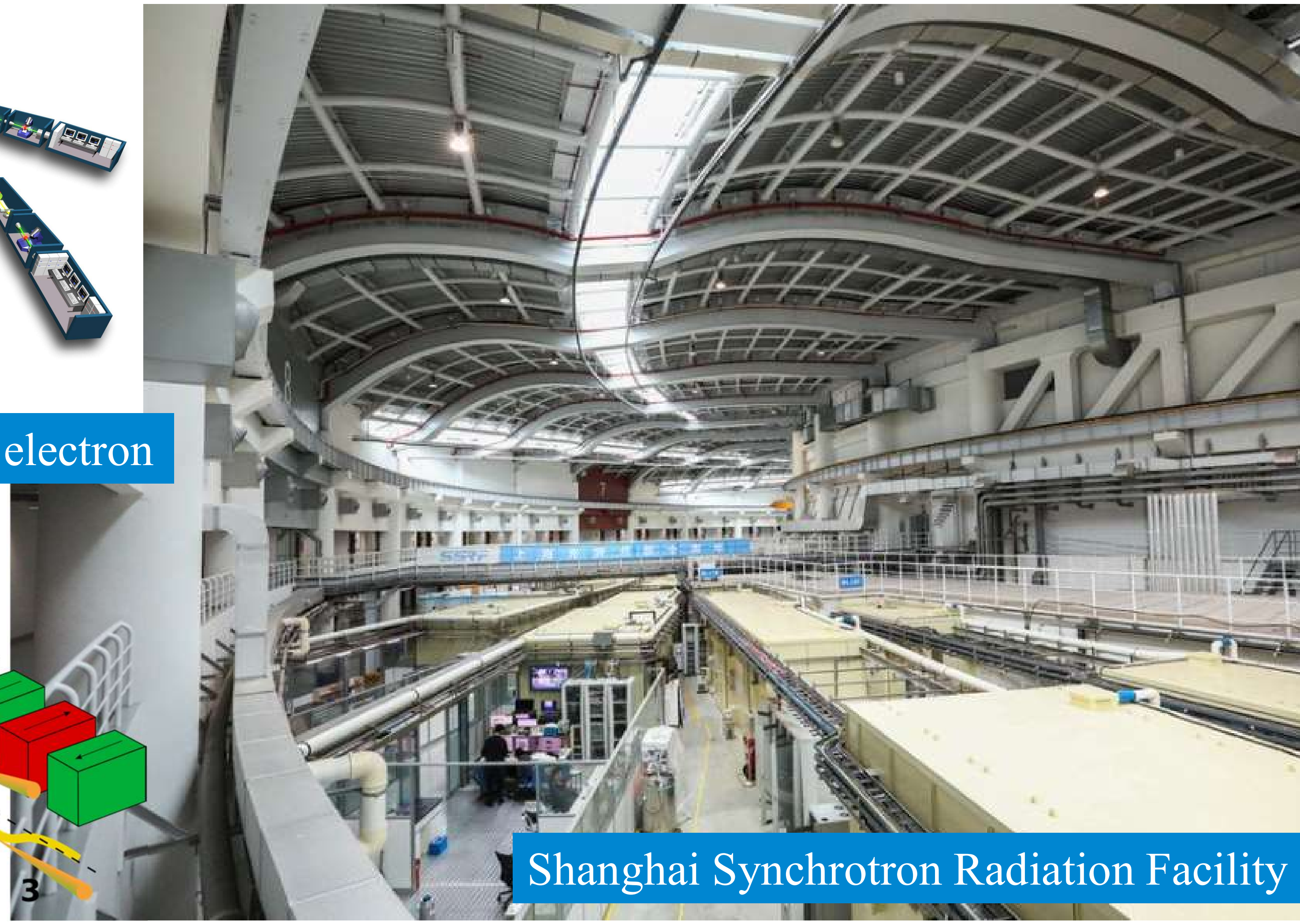

Shanghai Synchrotron Radiation Facility