Physics (PHYS2500J), Unit 6 Wave Optics:
1. The nature and propagation of Light

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How do we treat the problems in optics?

The two personalities of light

Until Newton:
Particle (straight propagation)

1665 – early 19th century:
Evidence of wave

1873,
Maxwell predicted EM wave.

Newton even explained Snell's law with particle hypothesis.

Most convincing is Thomas Young's double slits interference

Propagation of light

Existence of light

1887,
Hertz experimentally proved EM wave.

Early 20th century, Plank, Einstein,
Modern concept of Photon

Blackbody radiation theory and Photoelectric effect.

Interaction with material world

Light has to be treated like wave (when considering the propagation) and particle (when considering interaction with material)

So is other particles (de Broglie)

Basic properties of light as particles

Energy of a single photon

$$\mathcal{E}_p = h\nu$$

Plank constant: 6.63×10^-37 Js

Frequency of the EM wave

Or, $$\mathcal{E}_p = \hbar\omega$$

$$\hbar = \frac{h}{2\pi} \qquad \omega = 2\pi\nu$$

Also called Plank constant

Angular frequency

Momentum of a photon

$$\vec{P}_p = \frac{\mathcal{E}_p}{c}\hat{p} = \frac{h\nu}{c}\hat{p}$$

How do we treat the problems in optics?

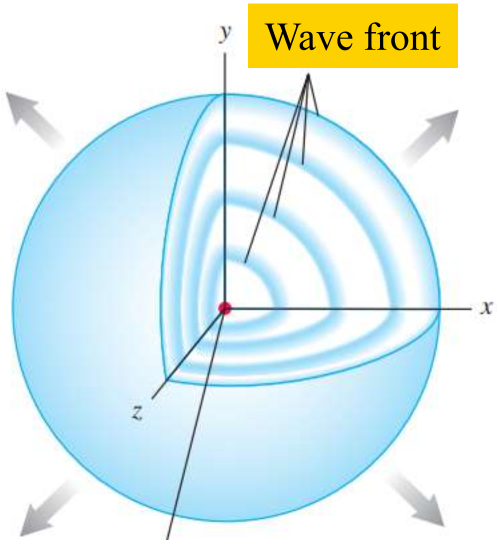
Feynman's lecture:

A lot of photons. When the aperture is large (much larger than the wavelength), light can be treated as straight lines (ray, beam) —— geometrical optics

When the size of apertures (and other optical devices) is comparable to the wavelength, light needs to be treated as wave —— wave optics

Just a few or even one photon (weak light), energy of photon is comparable to the resolution of detector, light needs to be treated as particles —— quantum optics.

Terms for propagation of light

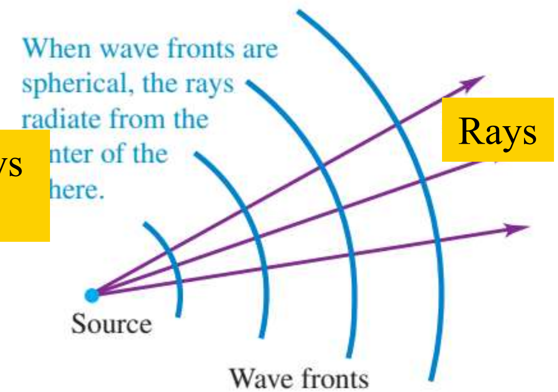


Point sound source producing spherical sound waves (alternating compressions and rarefactions of air)

Wave front is the surface perpendicular to the propagation direction of light (similar to equipotential surface relative to electric field).

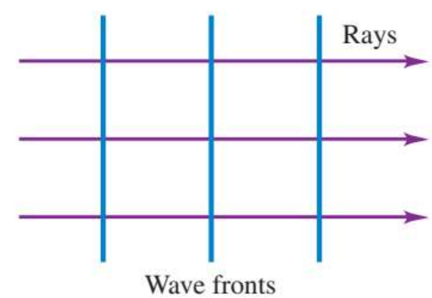
Wave front is actually the “equi-phase” surface.

Wave front and rays of spherical wave.



Wave front and rays of plane wave.

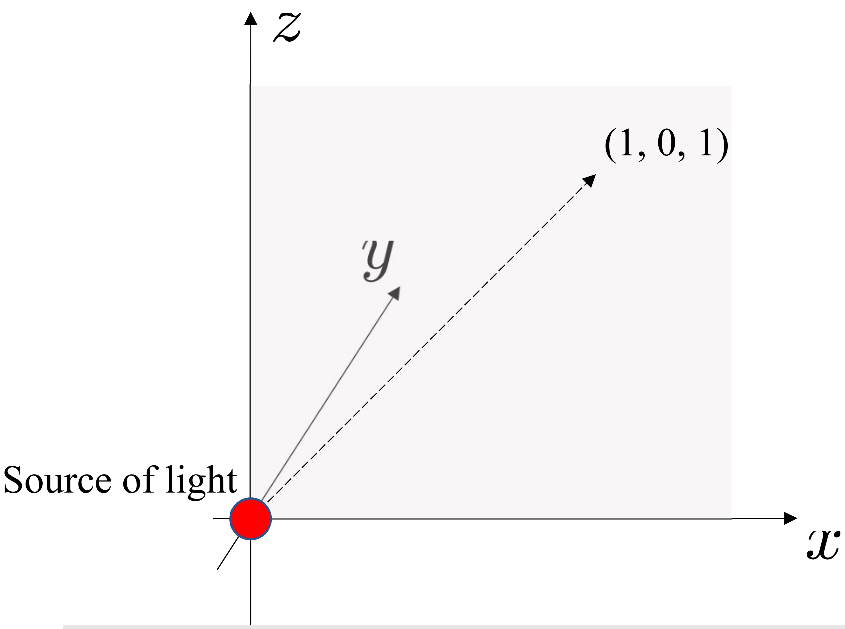
When wave fronts are planar, the rays are perpendicular to the wave fronts and parallel to each other.



Example problem

Look at the picture. If flux of red light photon (760 nm) through a 1 cm^2 square at point (1, 0, 1) (unit m) is 10^{20}/s. What is the electric field at the same point?

In reality, should ask what is the rms value of the light or specify the polarity.



Step 1, get the intensity of the light from photon flow rate.

$$I = \frac{d\mathcal{E}}{dt}\frac{1}{S} = \frac{10^{20}\times h\nu}{1\times 10^{-4}} = 2.6\times 10^{5}\ \mathrm{W/m^2}$$

Step 2, Calculate properties of EM wave from intensity

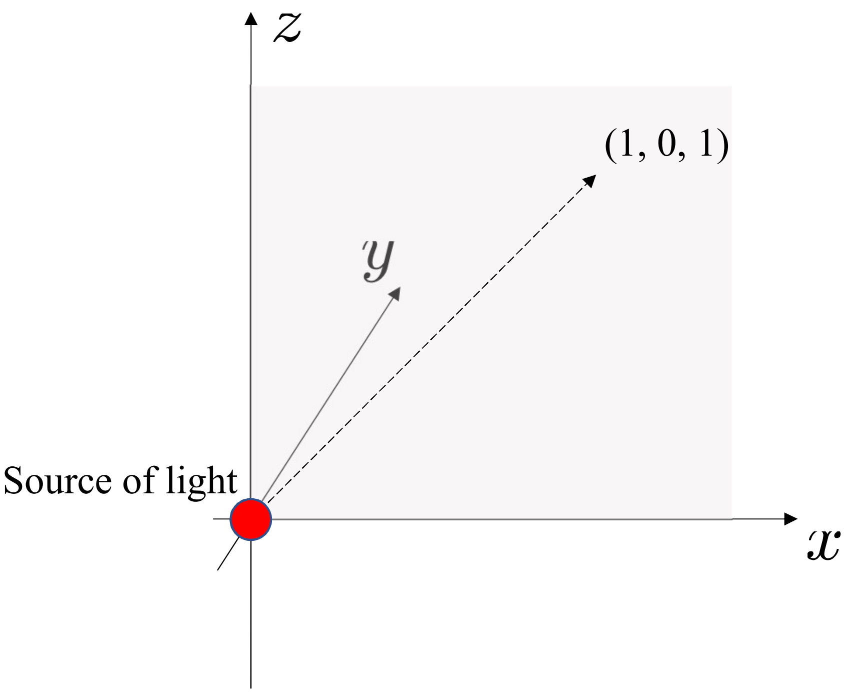
$$I = \overline{S} = \overline{E\times H} = \frac{\overline{E^2}}{c\mu_0}$$

$$E_{rms} = \sqrt{\overline{E^2}} = \sqrt{Ic\mu_0} = 9.9\ \mathrm{kV/m}$$

It can be understood as a circular polarized light, where $|E|$ is a constant.

Example problem

What about at point (2, 0, 2)?



Energy conservation determines that the intensity of light is inverse proportional to distance square.

$$I_{202} = \frac{I_{101}}{4}$$

The relation between electric field and intensity determines the electric field is inverse proportional to distance.

$$\left(\frac{E_{202}}{E_{101}}\right)^2 = \frac{I_{202}}{I_{101}}$$

$$E_{202} = \frac{E_{101}}{2}$$

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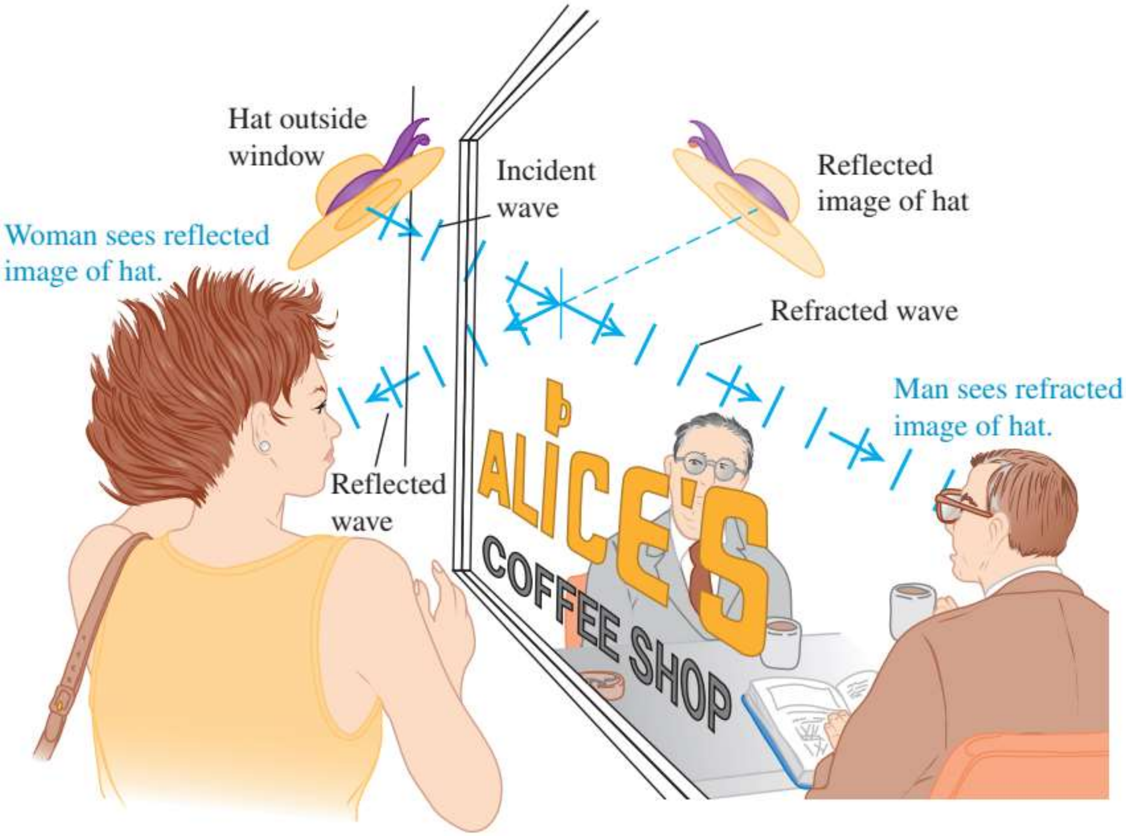
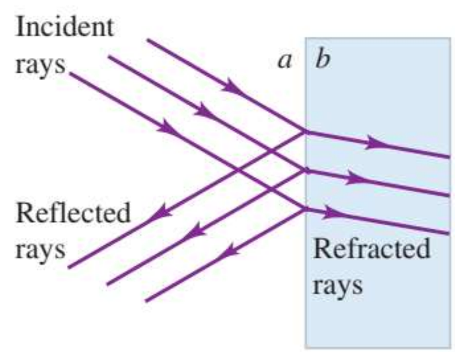
Refraction and reflection

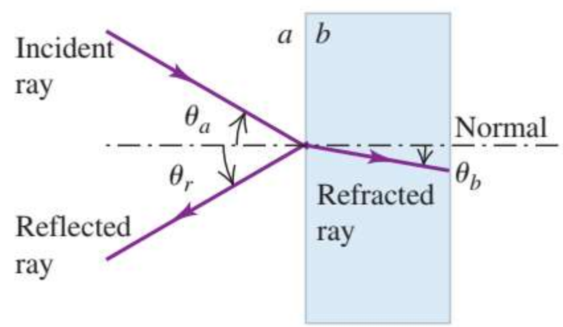
(a) Plane waves reflected and refracted from a window



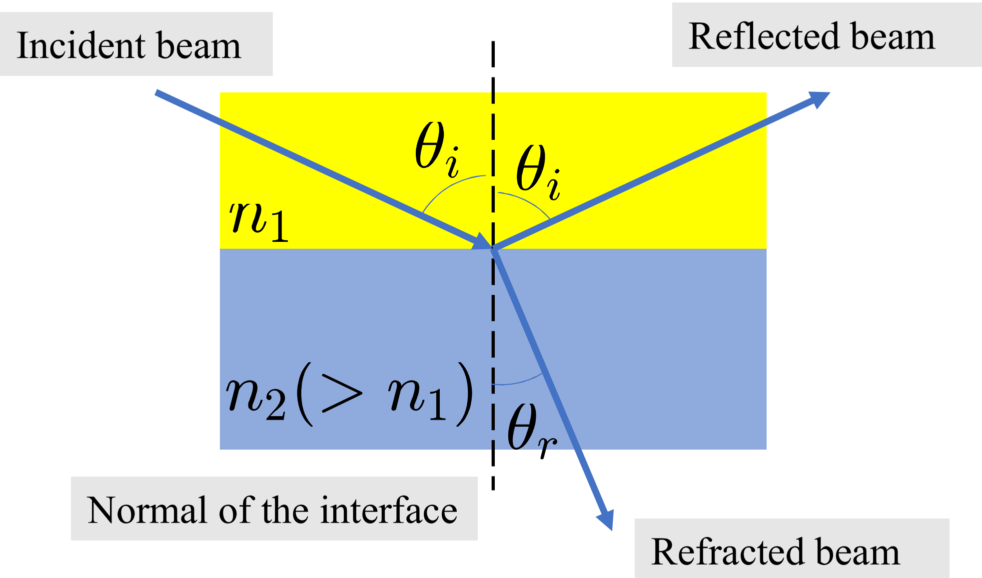
It is natural to have the multiple ray step, which is not shown in a lot of books.



(c) The representation simplified to show just one set of rays



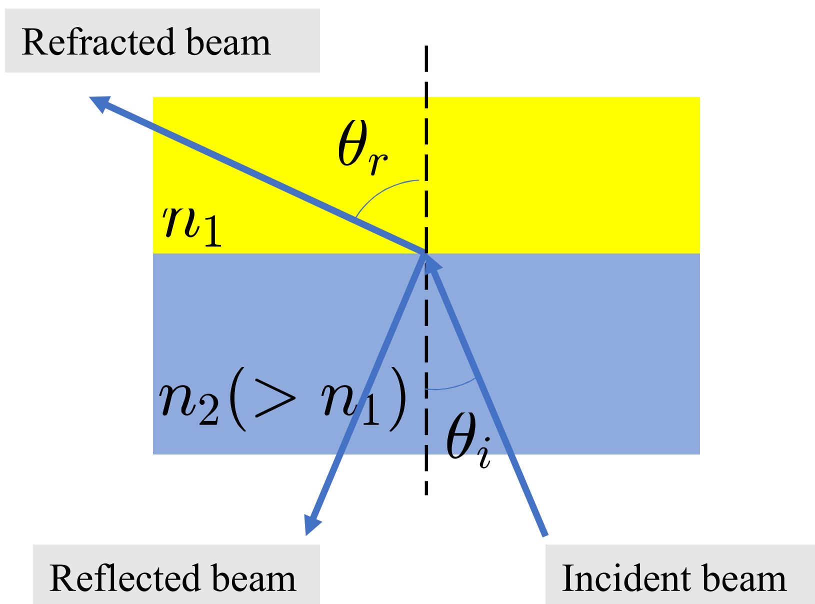
Snell's law



Snell's law

$$n_1 \sin\theta_i = n_2 \sin\theta_r$$

Total internal reflection



1. Reverse the light path in the last page. Light hits the interface from the denser medium.

2. Increase the incident angle, the refracted angle is also going to increase.

$$\sin\theta_r = \frac{n_2}{n_1}\sin\theta_i$$

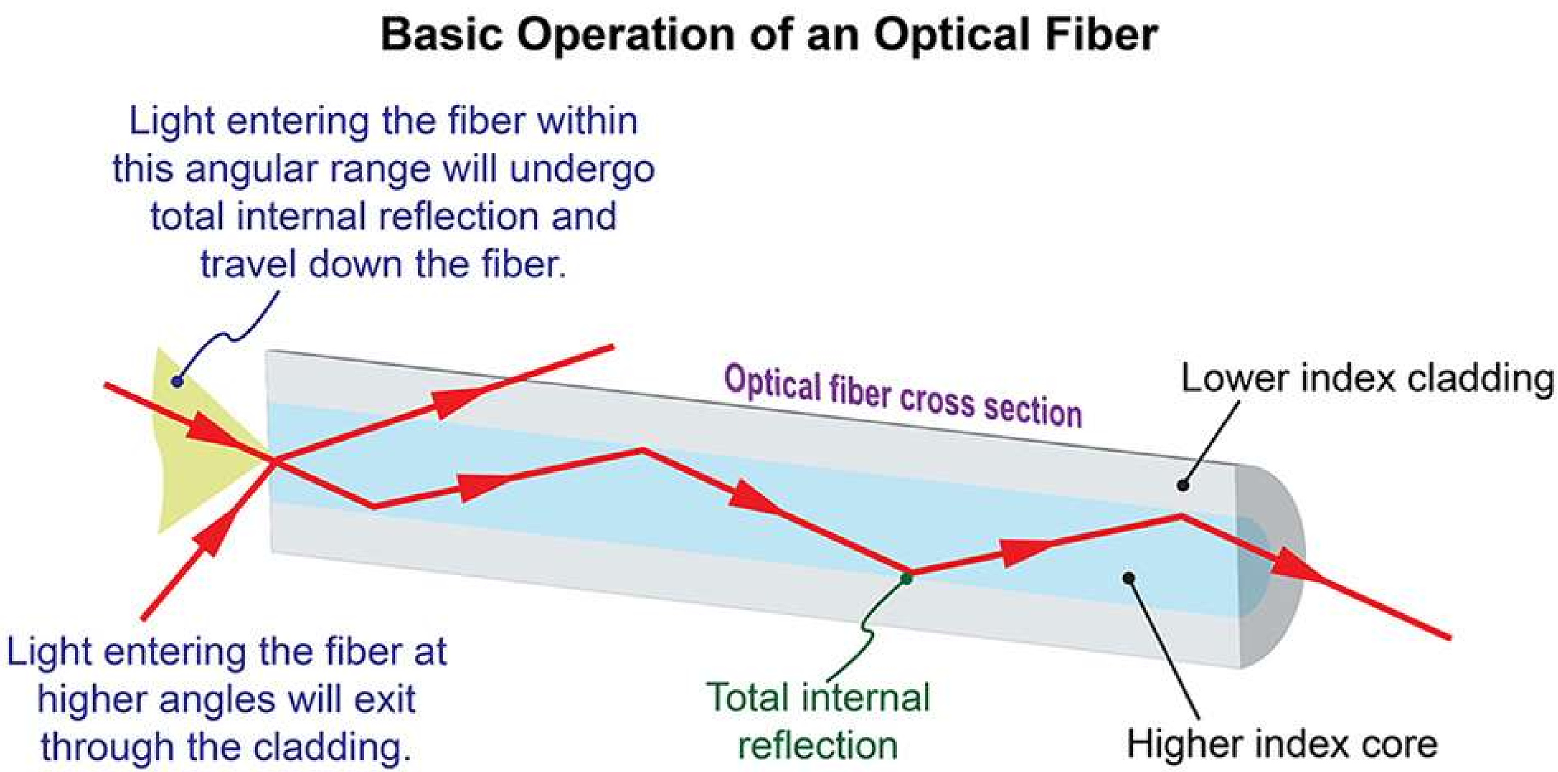
3. Until the incident beam increased to $\mathrm{asin}(\frac{n_1}{n_2})$

$$\theta_r = \frac{\pi}{2}$$

4. Further increase of incident angle, Snell's condition can not be satisfied. No refraction beam. It is called total (internal) reflection.

Optical fiber is based on the principle of total internal reflection

https://www.coherent.com/news/glossary/optical-fibers



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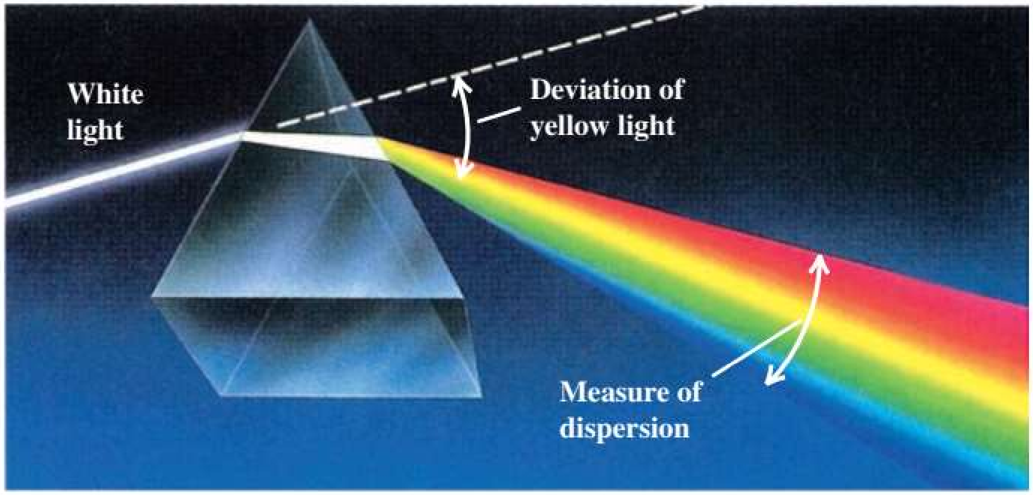
dispersion

33.18 Dispersion of light by a prism. The band of colors is called a spectrum.



1. Ordinary white light contains light of multiple wave lengths.

2. Dispersion is caused by different refraction indices for light of different wavelength.

3. In the left figure, which wavelength has higher refraction index, red or violet?

Violet

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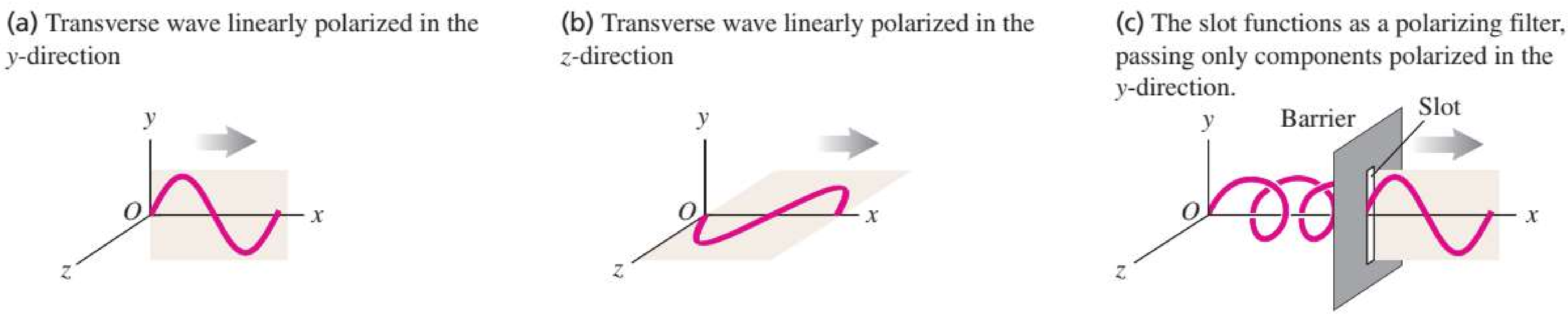
Polarization (偏振)

1. Transverse wave: for an EM wave travelling in *x*-direction, limited the direction of Electric field (as a vector) in the *yoz* plane.

$$\vec{E} = E_y \hat{y} + E_z \hat{z}$$

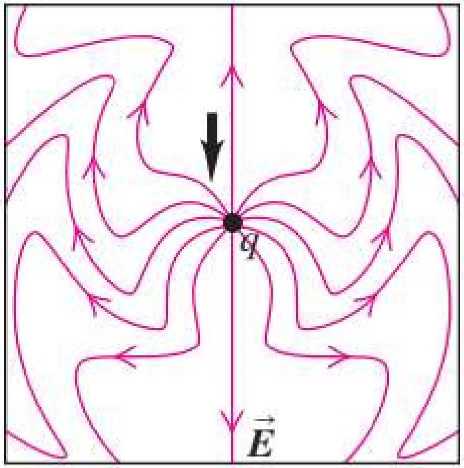
2. Polarization: further requirements on E_y and E_z

3. Example, analogy in mechanical wave, where E_z=0



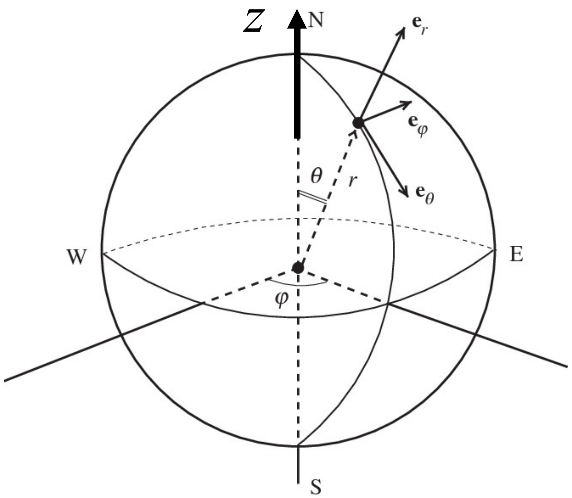
For cases of E_y=0, or E_z =0, or E_y/E_z = constant, which is actually the same if the coordinate system is rotated about *x*-axis, it is called a linear polarization.

Polarization light source



A single oscillator is a (linearly) polarized source.

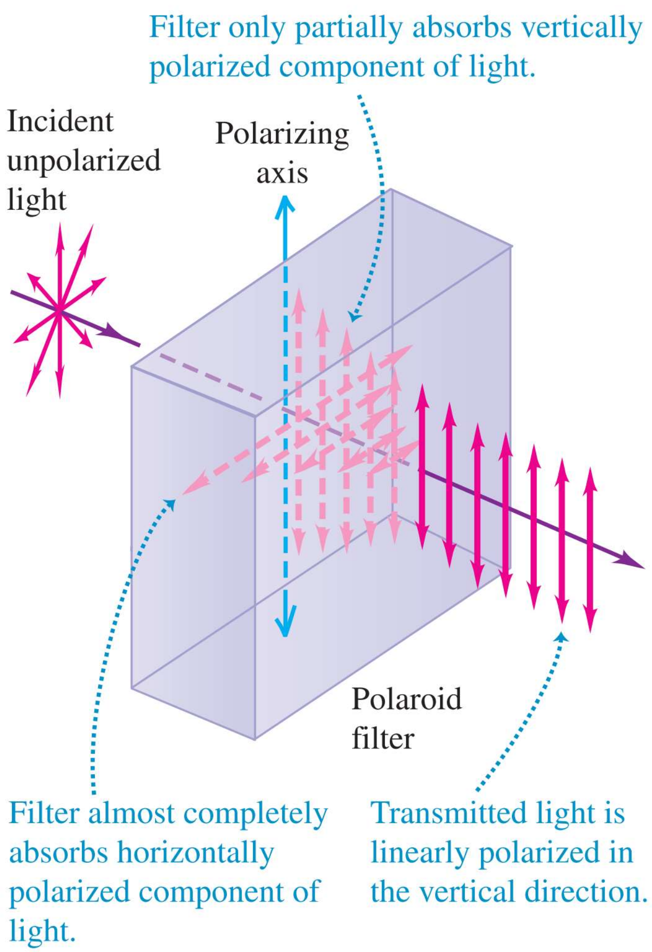
An oscillator vibrates in z direction, the E field of light is in θ direction in spherical coordinate system. No r or φ components.



A light bulb is an unpolarized source.

Many electrons vibrates randomly in any direction. The radiation is symmetric in direction. No polarization found in the light. Similar for **direct** sun light.

Polarizing filters



1. A polarization filter is such a device that only light polarized in one direction can path it. That direction is called

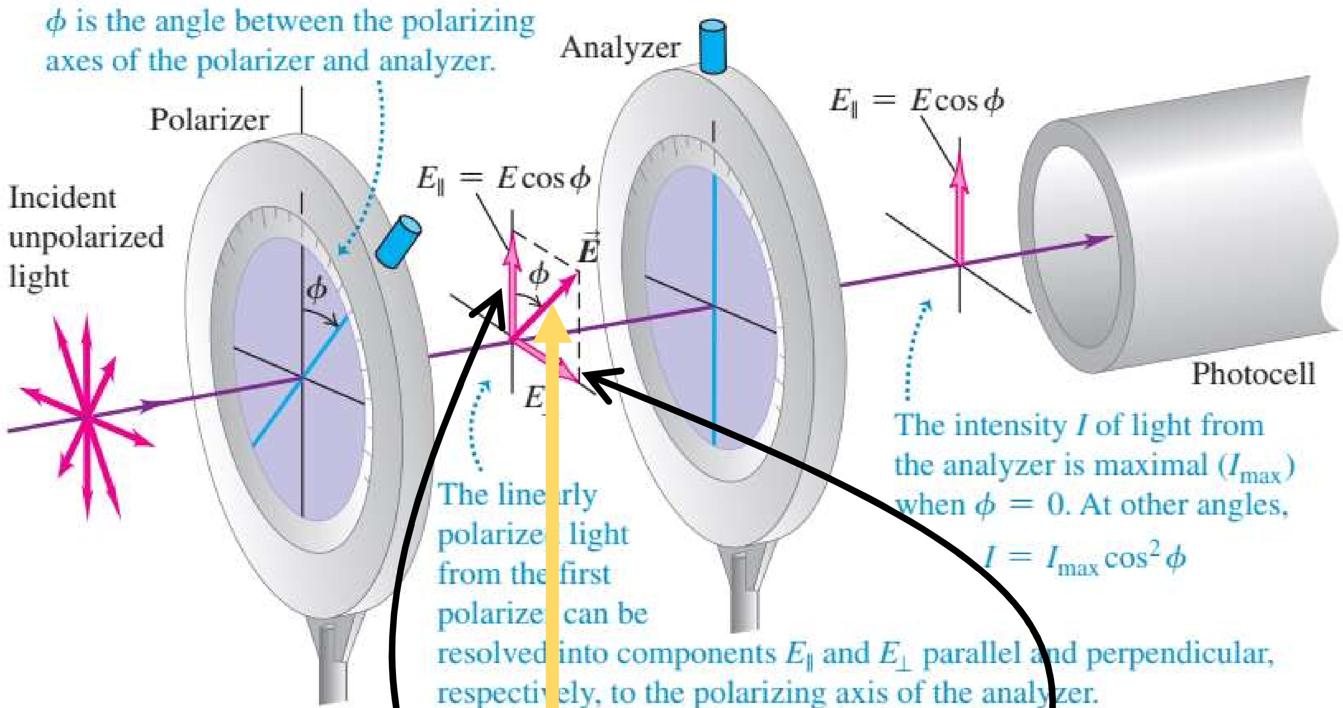
Polarizing axis

2. A natural light can be considered summation of light polarized in any direction (what do I mean by SUMMATION? Instead of superposition, related to the concept of coherence, will be answered in next topic)

3. After the filter only half of the intensity is kept. But the light is polarized now.

Why half?

Using the polarizing filters

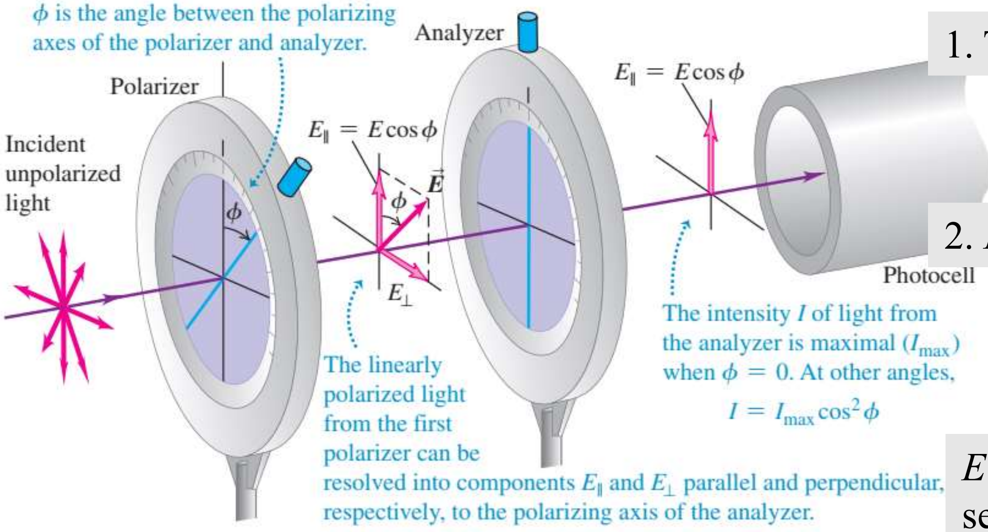


1. Using one polarizer to generated a polarized light, use another one (whose polarizing axis forms a φ angle with the first).

2. The polarized light can be considered superposition of two independent light. (superposition of electric field)

$$\vec{E} = E_y\hat{y} + E_z\hat{z}$$

Malus's law*



1. The intensity measured by the detector is proportional to

$$E \times H$$

2. H is proportional to E, so the intensity is proportional to

$$I \propto E^2 = (E_0 \cos\phi)^2$$

E is the field after the second polarization filter

φ is the angle between the two polarizing axis.

E_0 is the field between the two polarization filters

3. Malus's law (important)

$$I = I_{max} \cos^2 \phi$$

Maximum intensity is reached when φ=0 and intensity drops to 0 when φ=π/2.

Now can you answer why the intensity drops by half after the 1st polarizer?

Understanding Malus's law

Any linear polarized light (in yoz plane, say, angle α to +y axis), can be considered two parts.

1. $E_y = E_0 \cos\alpha$
2. $E_z = E_0 \sin\alpha$

The intensity of which being

1. $I_1 = I_0 \cos^2\alpha$
2. $I_2 = I_0 \sin^2\alpha$

Notice that $I_1 + I_2$ is always I_0

For two analyzers (secondary polarization filter) perpendicular to each other, along y axis and z axis. The two parts will be allowed to pass by one of the filters and rejected by the other, respectively.

Understanding Malus's law

Natural light, composed of polarized light in all directions. After the first polarization filter, the intensity goes down by

Intensity of light after the polarization filter

Integral for polarized light in all directions

$$\frac{\int_0^{\pi/2} \cos^2 \phi \mathrm{d}\phi}{\int_0^{\pi/2} 1 \mathrm{d}\phi} = \frac{1}{2}$$

Original intensity of light in any angle

Applications of polarization: 3D movies

The formation of 3D images involves two photo taken from different angles (left eye and right eye). The two photos can be separately taken and individually delivered to your both eyes, independently.

Some filter is needed so that the left eye image is delivered to the left eye without going into the right eye.

The filter can be an ugly bi-color glasses.

But apparently a polarized filter is better.

A polarized light microscope*

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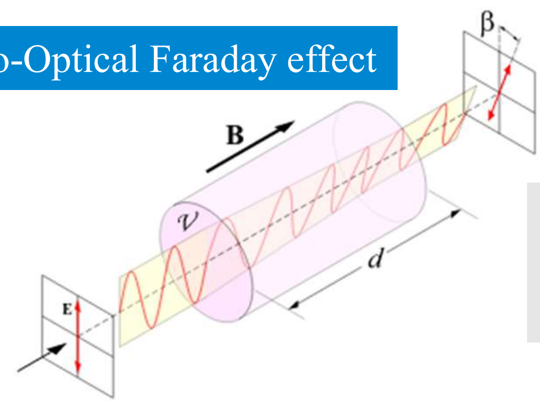
Microscope + Polarized light = Polarized light Microscope

A simple concept but a very useful device.

Example: it can be used to see magnetic field

Magneto-Optical Faraday effect



The polarization direction rotated by MO crystal under longitudinal magnetic field.

Under normal microscope:
Just a boring black thin film

Under a polarized microscope (equipped with MO crystals):
Magnetic field is penetrating into the superconductor

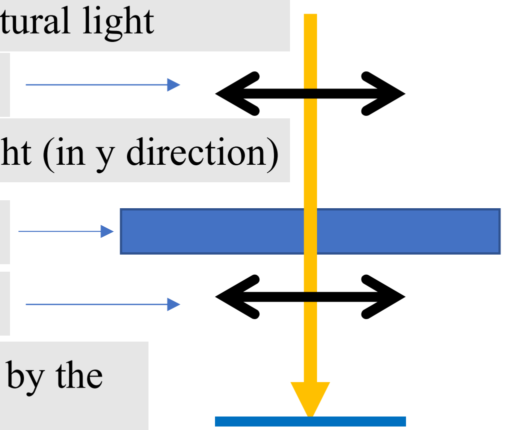
Incoming natural light

Polarizer (in y-direction)

Polarized light (in y direction)

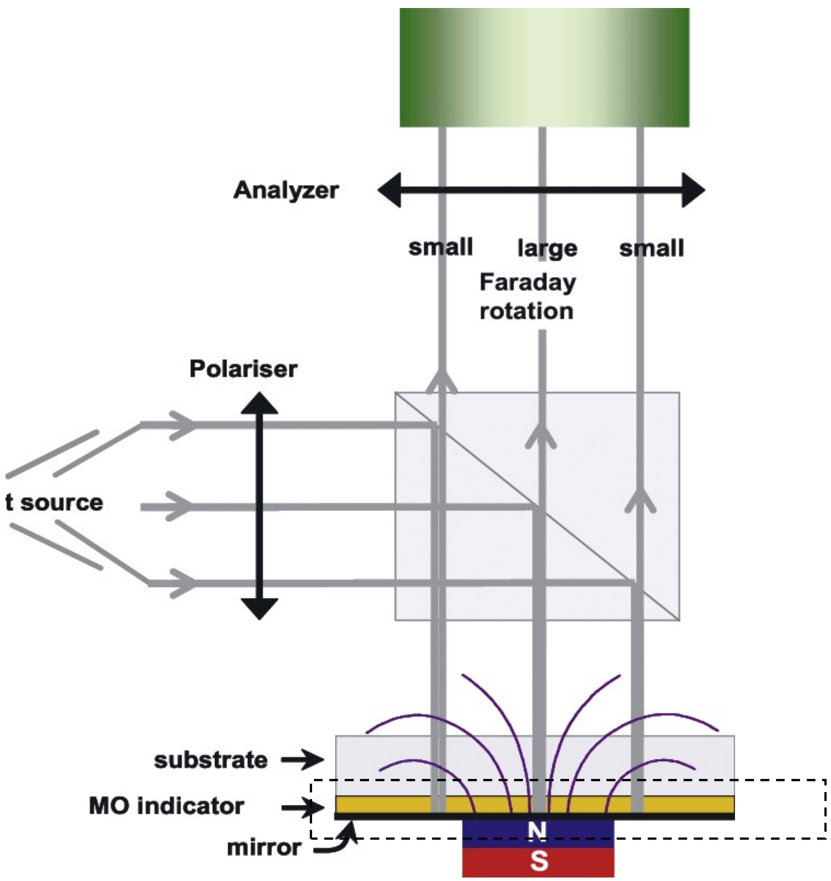
MO crystal

Analyzer (in z-direction)



Detector sees nothing if the polarized light is not rotated by the MO crystal. Intensity increases with magnetic field.

The full setup

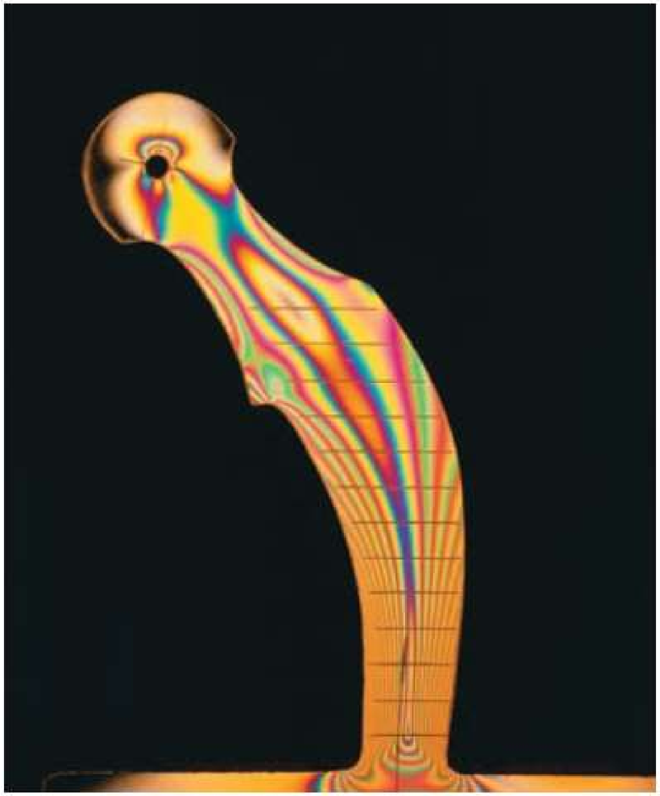


The intensity depends on the rotation of polarization direction, which is achieved by the MO crystal and magnetic field here.

Similarly, polarized light can also see stress

Reflected light changed polarization direction with the material under stress.

33.30 This plastic model of an artificial hip joint was photographed between two polarizing filters (a polarizer and an analyzer) with perpendicular polarizing axes. The colored interference pattern reveals the direction and magnitude of stresses in the model. Engineers use these results to help design the actual hip joint (used in hip replacement surgery), which is made of metal.

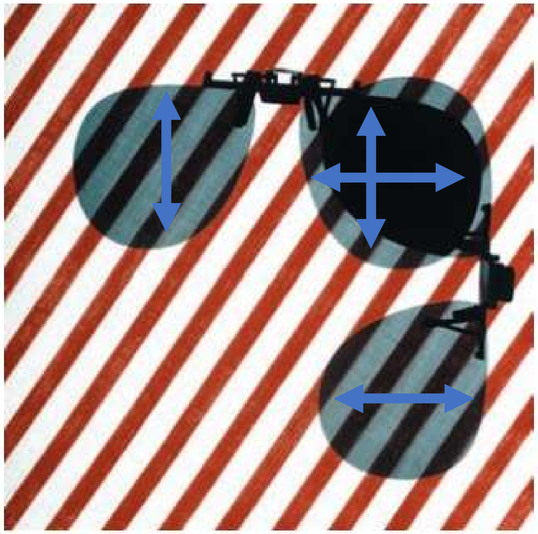


It is not guaranteed any material will behave birefringence under stress

Polarizer and analyzer in life

Polarized sunglasses are made of linear polarizers.

33.25 These photos show the view through Polaroid sunglasses whose polarizing axes are (left) aligned ($\phi = 0$) and (right) perpendicular ($\phi = 90°$). The transmitted intensity is greatest when the axes are aligned; it is zero when the axes are perpendicular.

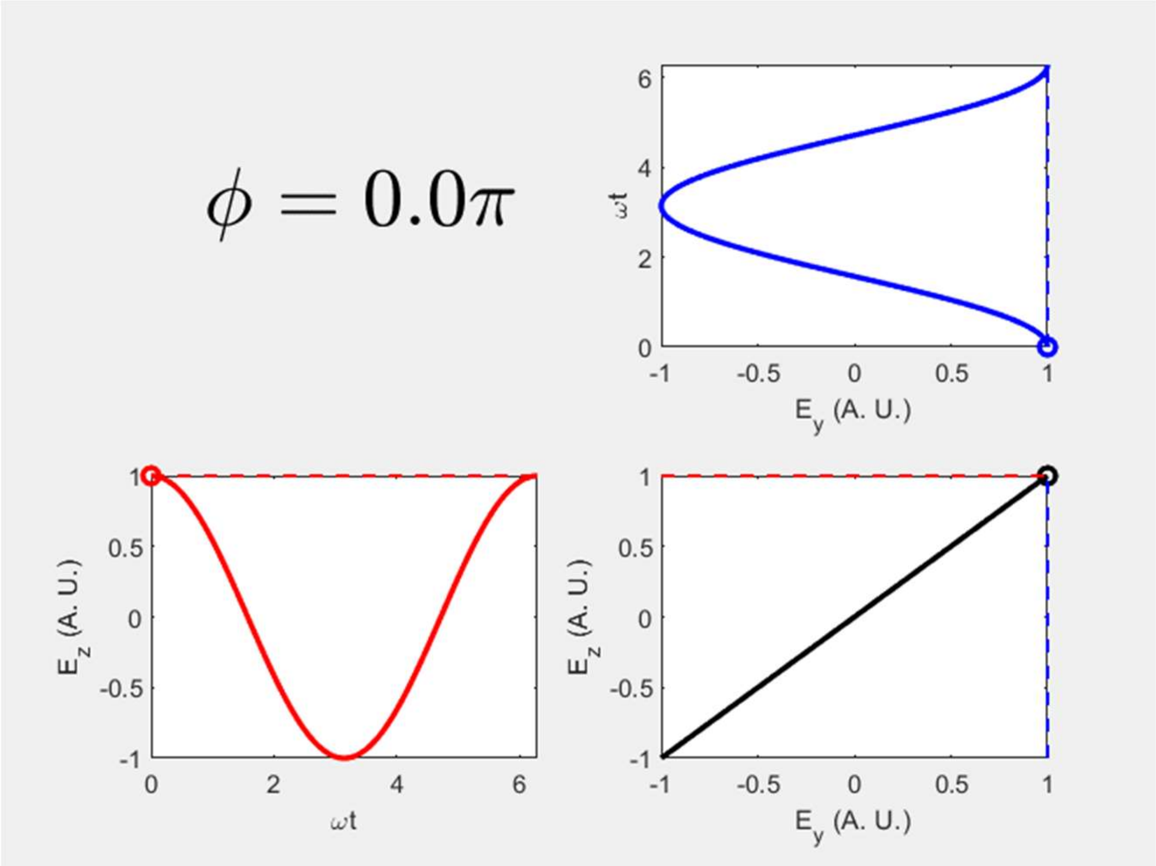


Circular and elliptical polarizations

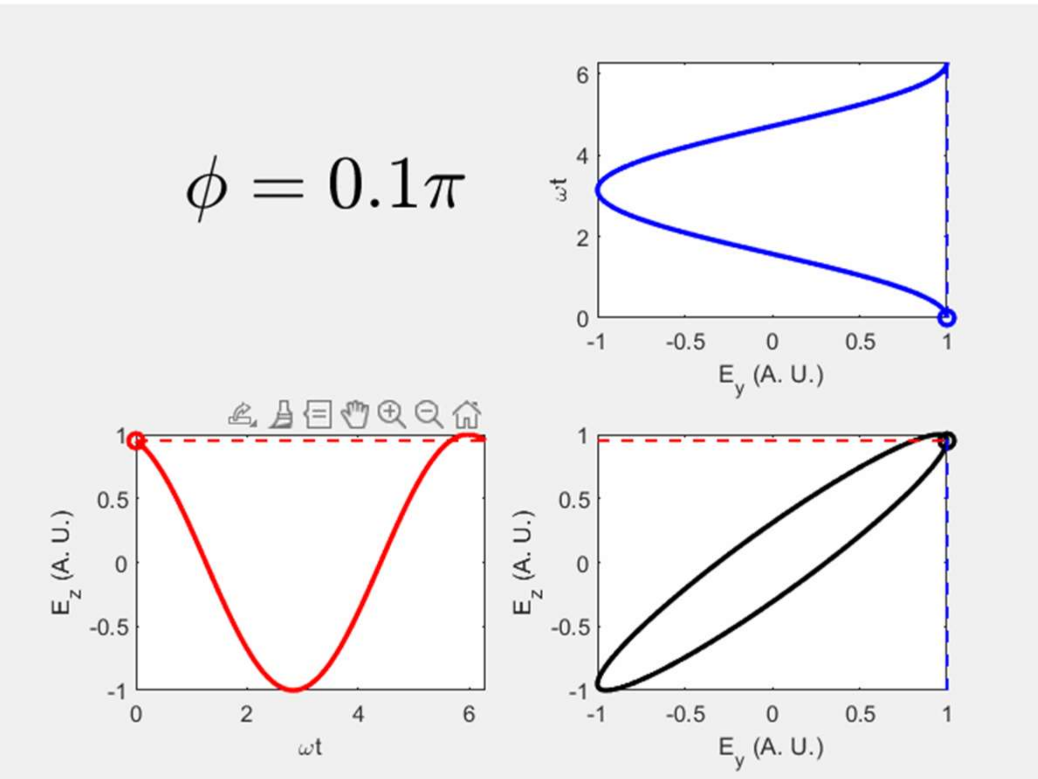
Generally speaking, the two components of electric field may also have a finite phase difference φ.

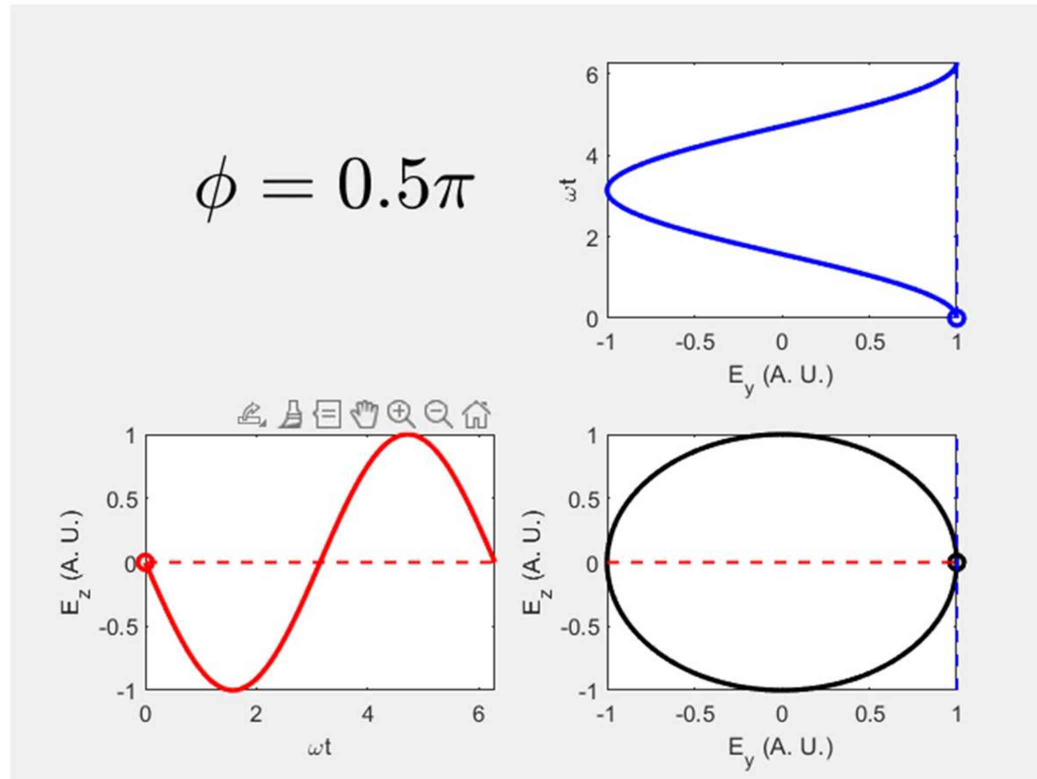
$$\vec{E} = \hat{y}E_{y0}\cos(kx - \omega t) + \hat{z}E_{z0}\cos(kx - \omega t + \phi)$$

When φ=0, the superposition of the two linearly polarized wave gives a linear polarized light.



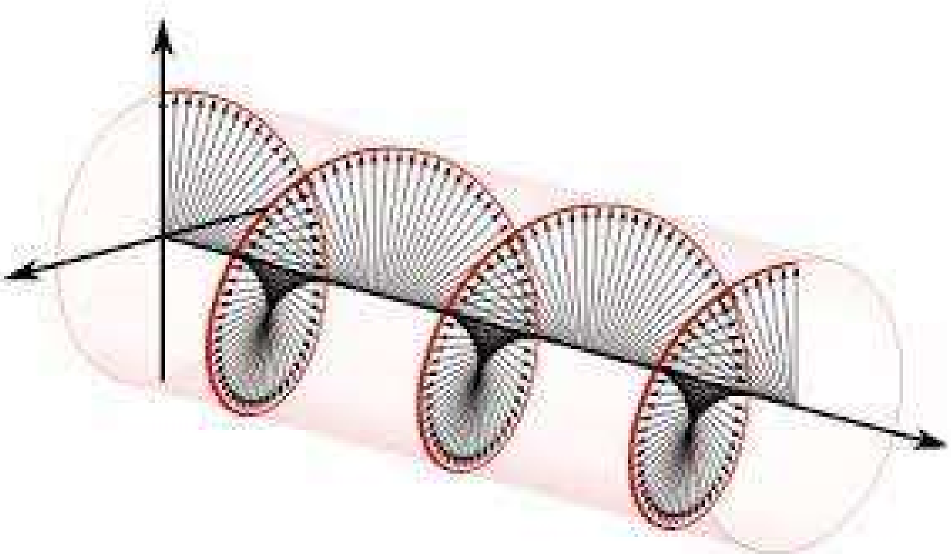
When φ is not 0, the combination of two linear polarized light may give an elliptical or circular polarized light.

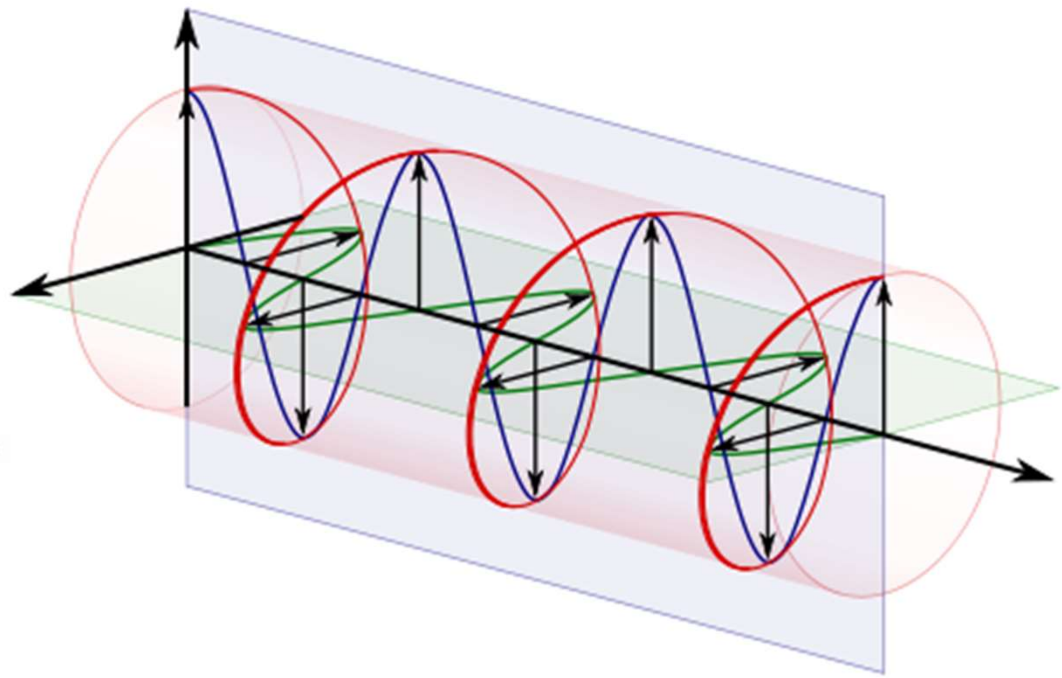




Circular: When $E_{y0}=E_{z0}$, and the relative phase is 0.5 π. The E vector is rotating.

Circular polarized light





Any polarized light can be written as sum of two polarized lights

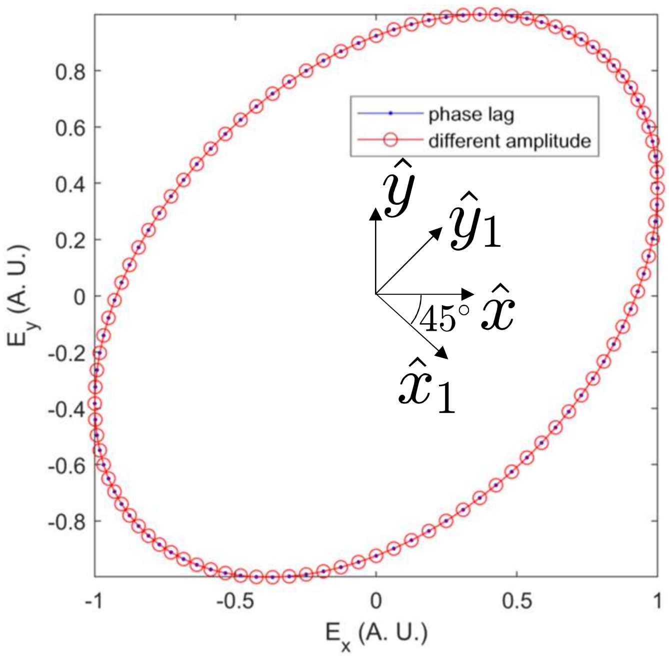
Can be considered the superposition of two linear polarized lights (with perpendicular polarizing directions)

Or, considered as the superposition of two circular polarized lights (one anti-clockwise and one clockwise)

You can just manipulate the amplitude and relative phase of the two basic components to construct any polarized light.

Elliptical polarized light as sum of linear polarized lights

Elliptical polarization can be considered the superposition of two linear polarized light.



1. Of not $\pi/2$ phase lag.

$$\vec{E} = E_0[\hat{x}\cos\omega t + \hat{y}\sin(\omega t + \phi_0)]$$

2. Of different amplitude.

$$\vec{E} = \hat{x}_1 E_x \cos(\omega t + \theta) + \hat{y}_1 E_y \sin(\omega t + \theta)$$

The electric field is along the ellipse: elliptical polarization.

$$E_x = E_0\sqrt{1 - \sin\phi_0}$$
$$E_y = E_0\sqrt{1 + \sin\phi_0}$$

$$\tan\theta = \frac{E_y}{E_x}$$

Elliptical polarized light as sum of linear polarized lights

```
N=100;
t=0:N;
t=t*2*pi/N;

phi0=pi/8;

Ex=sqrt(1-sin(phi0));
Ey=sqrt(1+sin(phi0));
theta=atan(Ey/Ex);

figure
plot(t,cos(t),'bo-');
hold on
plot(t,sin(t+phi0),'ro-');
sr2=sqrt(2)/2;
plot(t,Ex*cos(t+theta)*sr2+Ey*sin(t+theta)*sr2,'b.-');
plot(t,-Ex*cos(t+theta)*sr2+Ey*sin(t+theta)*sr2,'r.-');
xlabel('\omega t')
ylabel('E')
legend('E_x phase lag','E_y phase lag','E_x diff. amp.','E_y diff. amp.')

figure
plot(cos(t),sin(t+phi0),'b.-');
hold on
plot(Ex*cos(t+theta)*sr2+Ey*sin(t+theta)*sr2, ...
-Ex*cos(t+theta)*sr2+Ey*sin(t+theta)*sr2,'ro-');
xlabel('E_x (A. U.)')
ylabel('E_y (A. U.)')
legend('phase lag','different amplitude')
axis equal tight
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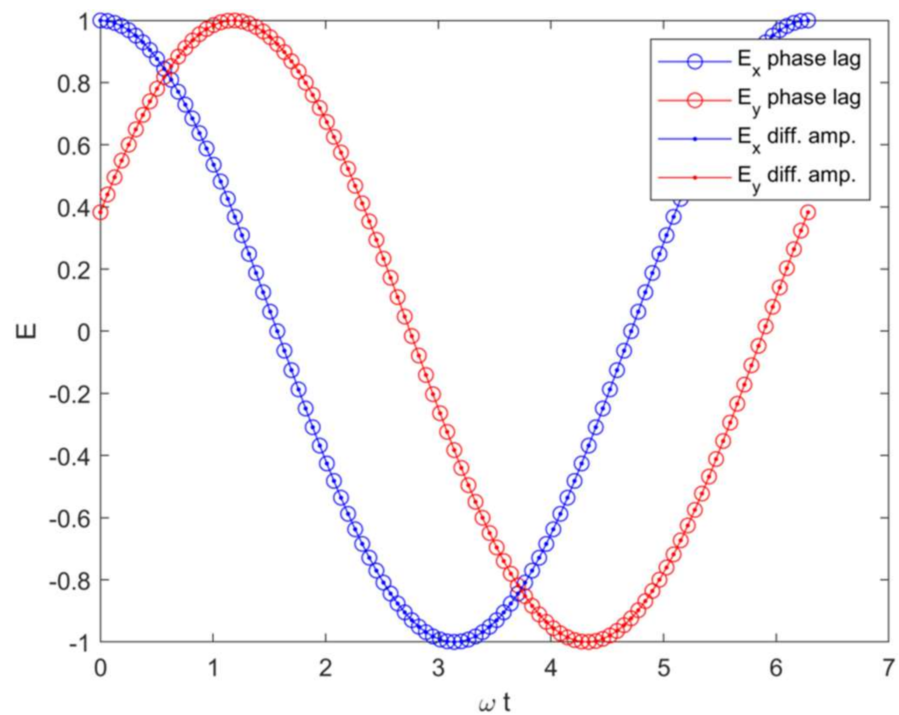

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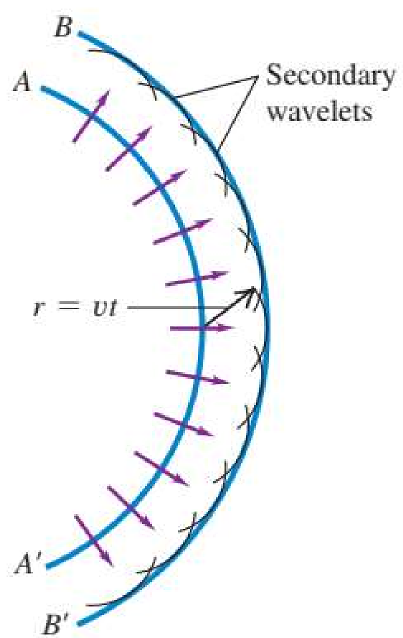
Huygens's principle

Huygens assumed that every point of a wave front may be considered the source of secondary wavelets that spread out in all directions with a speed equal to the speed of propagation of the wave.

33.33 Applying Huygens's principle to wave front AA' to construct a new wave front BB'.

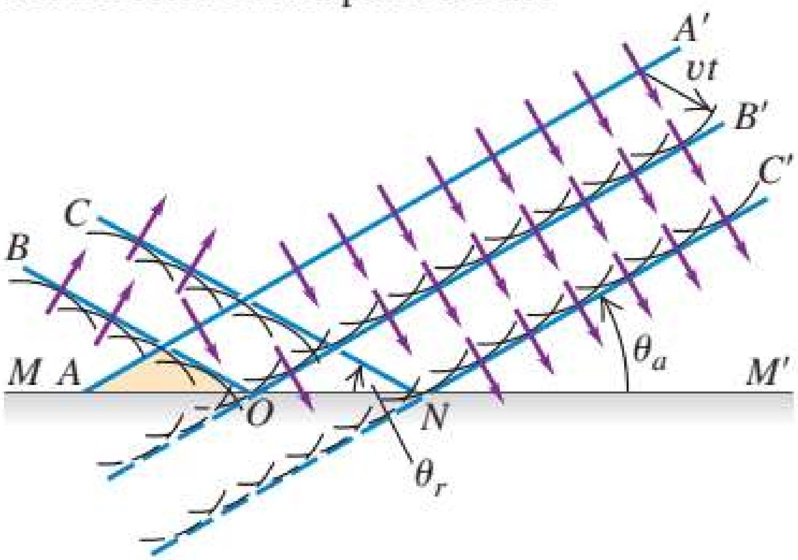


Huygens’s principle and the reflection law

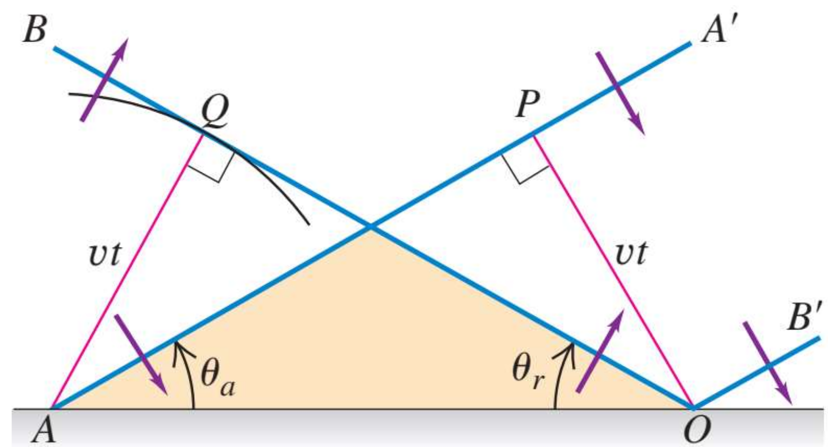
During the analysis, please notice that the new wave front is tangential to all circles originated from the old wave front.

33.34 Using Huygens’s principle to derive the law of reflection.

(a) Successive positions of a plane wave AA' as it is reflected from a plane surface

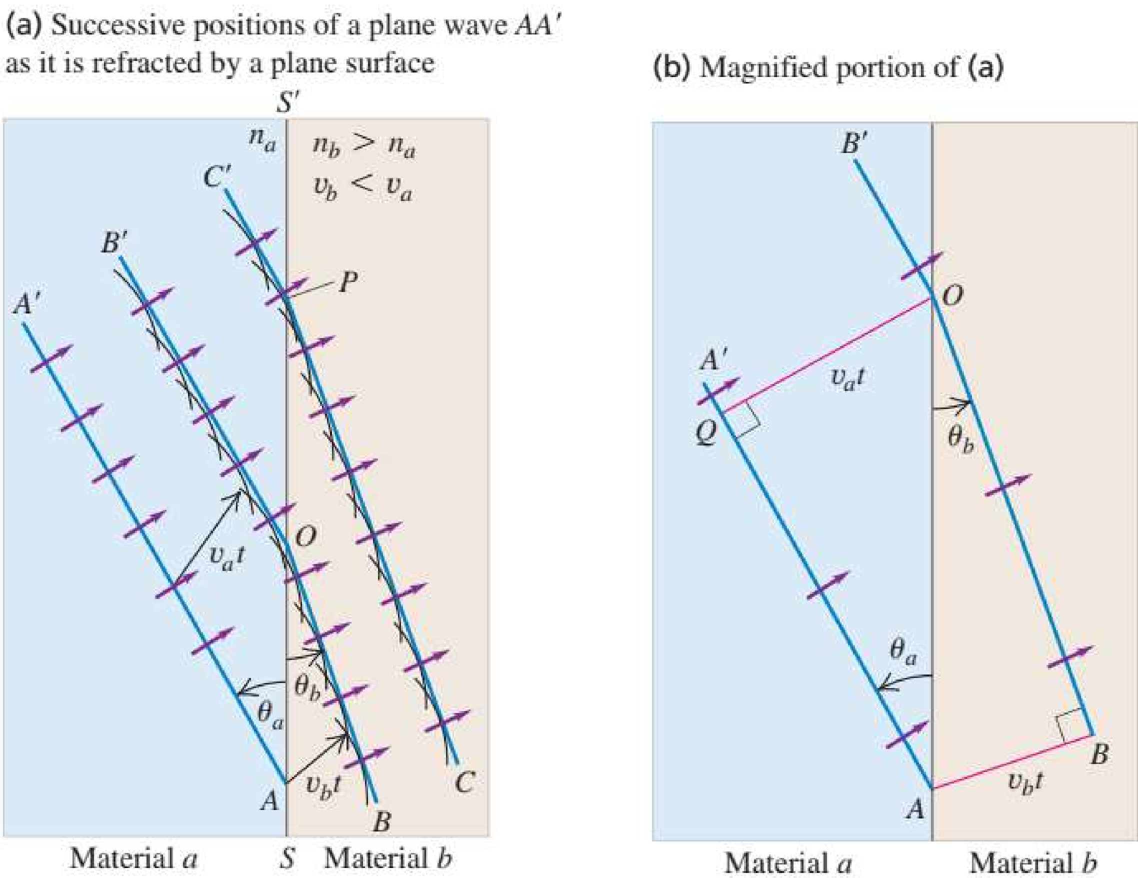


(b) Magnified portion of **(a)**



The reflection law is due to the equality of wave speed of both incident and reflected beam.

Huygens's principle and the refraction law



$$OA \sin\theta_a = v_a t$$
$$OA \sin\theta_b = v_b t$$

$$\frac{\sin\theta_a}{\sin\theta_b} = \frac{v_a}{v_b} = \frac{n_b}{n_a}$$

The refraction law is due to the difference between the wave speeds