

$$W_0 = \frac{2\pi}{6} = \frac{\pi}{3}$$

$$a_0 = \frac{1}{T} \int_T x(t) dt = \frac{1}{6} \left(\int_0^3 1 dt + \int_3^4 3 dt \right) = 1$$

$$\begin{aligned} a_k &= \frac{1}{T} \int_T x(t) e^{-jk\pi t/3} dt = \frac{1}{6} \left(\int_0^3 e^{-jk\pi t/3} dt + \int_3^4 3 e^{-jk\pi t/3} dt \right) \\ &= \frac{1}{6} \left(\frac{e^{-jk\pi t/3}}{-jk\pi/3} \Big|_0^3 + 3 \cdot \frac{e^{-jk\pi t/3}}{-jk\pi/3} \Big|_3^4 \right) \\ &= \frac{1}{6} \left(\frac{e^{-jk\pi}}{-jk\pi/3} + \frac{1}{jk\pi/3} + 3 \cdot \frac{e^{-jk4\pi/3}}{-jk\pi/3} + 3 \cdot \frac{e^{-jk\pi}}{jk\pi/3} \right) \\ &= \frac{j \cdot e^{-jk\pi}}{2k\pi} - \frac{j}{2k\pi} + \frac{3j e^{-jk4\pi/3}}{2k\pi} - \frac{3j e^{-jk\pi}}{2k\pi} \\ &= \frac{1}{2k\pi} (2j \cdot e^{-jk\pi} - j + 3j e^{-jk4\pi/3}) \end{aligned}$$

$$\begin{aligned} x(t) &= 1 + \sum_{k=1}^{\infty} \left[\frac{1}{2k\pi} (3j e^{-jk4\pi/3} - j - 2j e^{-jk\pi}) e^{jk\pi t/3} \right. \\ &\quad \left. - \frac{1}{2k\pi} (3j e^{jk4\pi/3} - j - 2j e^{jk\pi}) e^{-jk\pi t/3} \right] \end{aligned}$$

Method 2:

$$\text{Two rectangular wave: } C_0 = \sum \frac{T X_0}{T_0} = \frac{3 \times 1}{6} + \frac{1 \times 3}{6} = 1$$

$$C_k = \sum \frac{T X_0}{T} \text{sinc}\left(\frac{T k W_0}{2\pi}\right)$$

$$x(t) = \sum_{k=-\infty}^{\infty} \frac{2}{3} \text{sinc}\left(\frac{2k}{3}\right) e^{-jk\frac{2\pi}{3}} e^{jk\frac{\pi}{3}t} + \sum_{k=-\infty}^{\infty} \frac{1}{3} \text{sinc}\left(\frac{k}{6}\right) e^{-jk\pi/6} e^{jk\pi t/3}$$

Sketch:

