

Numerical Analysis

Solutions of Equations in One Variable 单变量方程解

- The Root-Finding Problem
- The Bisection Method
  - $O(l/2n)$
- Fixed-pointed Iteration
  - linearly convergent
- Newton's Method(Taylor Polynomial)
  - at least quadratically convergent
  - p0条件
    - f二阶连续
    - p0是p的近似值
    - f'(p0)≠0
- Error Analysis for Iteration Methods
  - $\alpha=1$ , 线性收敛;  $\alpha=2$ , 二次收敛

Solutions of Nonlinear Systems 非线性方程组解

- Vector Norms
- Fixed-points for Functions of Several Variables
  - 分别进行不动点迭代
- Newton's Method for Nonlinear Systems
  - The Jacobian matrix
- Gardient/Steepest Descent Techniques
  - local minimun

Solutions of Linear Systems 线性方程组解

- Gaussian Elimination
  - $O(n^3)$ ,  $O(n^3)$ ,  $O(n^2)$
- Matrix Factorization
  - $O(n^3)A=LU$ ,  $Ly=b$ ,  $Ux=y$
- (Strictly)Diagonally Dominant Matrices
- Norms
  - Frobenius Norm
  - p-Norm
- Eigenvalues and Eigenvectors
  - Spectral Radius  $\rho(A)=\max|\lambda|$
- Convergent Matrix
  - $\rho(A)<1$
- Iteration Methods
  - The Jacobi Iteration Method
  - The Gauss-Seidel Method
    - superior
  - General Iteration Methods
  - Error Bounds
    - condition number

Approximating Eigenvalues 特征值近似

- The Power Method
  - the eigenvalue of maximum
- The Inverse Power Method
  - the eigenvalue of minimum

Interpolation & Polynomial Approximation 插值和多项式近似

- Taylor Polynomial
- Lagrange Interpolating Polynomials
- Nevile's Method
- Newton's Divided Difference Interpolation Polynomial
- Piecewise-linear Interpolation
  - natural cubic spline
  - clamped cubic spline

Approximation Theory 近似理论

- Linear Least Squares
  - $y=ax+a_0$ , 差的平方和
- Polynomial Least Squares

Numerical Differentiation 数值微分

- General Derivation Approximation Formulas
- Three-point Formulas
- Five-point Formulas
- Numerical Approximation to Higher Derivations
- Round-off Error Instability
- Richardson's Extrapolation

Numerical Integration 数值积分

- Trapezoidal Rule
- Simpson's Rule
- Composite Numerical Integration
  - for Trapezoidal Rule
  - for Simpson's Rule
- Round-off Error Stability
- Romberg Integration

Solution of Differential Equations 微分方程解

- Initial-Value Problems for ODEs
- Euler's Method
- Runge-Kutta Methods
  - The Midpoint Method
  - Modified Euler Method