

EE6222 Assignment 1

Dimensionality Reduction for Classification

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Abstract

This report studies how dimensionality reduction (DR) affects classification performance on high-dimensional image datasets. The current draft is generated from the available run `ee6222.dr.quick.20260226_161510`, and all sections that depend on not-yet-finished experiments (e.g., full multi-seed run, significance testing) are kept as explicit placeholders.

1 Introduction

High-dimensional visual data usually contain substantial redundancy, strong feature correlation, and noise dimensions that are weakly related to class identity. In this setting, directly training a classifier in the original space can lead to unstable covariance estimation, poor distance geometry, and a high risk of overfitting when the sample size is limited relative to dimensionality. Dimensionality reduction (DR) is therefore not only a compression tool; it is also a way to regularize the representation and shape the bias–variance tradeoff before classification [1–3].

At the same time, DR is not guaranteed to help. A representation that preserves global variance may still discard low-variance but class-discriminative cues, while a supervised projection may fit training labels well but generalize poorly if the model is insufficiently regularized. The key practical question in this assignment is thus empirical: for a given dataset and classifier, how does test accuracy evolve as the retained dimension d increases, and which methods produce a better accuracy–complexity tradeoff.

This report addresses that question on two image datasets with different data characteristics: Fashion-MNIST and Olivetti Faces. I compare linear, nonlinear, and deep DR families under a leakage-free protocol and report both accuracy and error-rate curves against d . The current version is based on the available quick run, which already supports preliminary conclusions and method comparison; sections that depend on full multi-seed experiments are explicitly marked as pending.

2 Methods and Related Work

2.1 DR Methods

Let an input sample be $x \in \mathbb{R}^D$. Each DR model learns a mapping $f_\theta : \mathbb{R}^D \rightarrow \mathbb{R}^d$ with $d \ll D$, and the reduced feature is

$$z = f_\theta(x). \quad (1)$$

The method set is designed to cover complementary principles. For linear DR, PCA projects data onto directions of maximal variance, which can be written as

$$U_d = \arg \max_{U^\top U = I_d} \text{tr}(U^\top S U), \quad z = U_d^\top (x - \mu), \quad (2)$$

where S is the sample covariance matrix [4]. LDA, in contrast, is supervised and seeks maximal between-class separation relative to within-class scatter:

$$W = \arg \max_W \frac{|W^\top S_b W|}{|W^\top S_w W|}, \quad d \leq C - 1, \quad (3)$$

where C is the number of classes [5]. The PCA-LDA pipeline applies PCA first for conditioning and noise suppression, then LDA in the PCA subspace.

To capture nonlinear structure and alternative latent assumptions, this work also evaluates Kernel PCA, NMF, and ICA [6–8]. In Kernel PCA, principal directions are computed in RKHS by solving

$$K\alpha_k = \lambda_k \alpha_k, \quad (4)$$

where K is the kernel Gram matrix. NMF factorizes nonnegative data as

$$X \approx WH, \quad W \geq 0, \quad H \geq 0, \quad (5)$$

which often yields parts-based interpretable components in image tasks. ICA assumes a linear mixing model $x = As$ and estimates independent latent sources s from observed data.

Deep DR is represented by AE and VAE [9, 10]. AE learns a deterministic bottleneck by reconstruction minimization:

$$\min_{\theta} \sum_i \|x_i - g_{\theta}(f_{\theta}(x_i))\|_2^2. \quad (6)$$

VAE uses a probabilistic latent variable model and optimizes the ELBO:

$$\mathcal{L}_{\text{ELBO}} = \mathbb{E}_{q_{\phi}(z|x)} [\log p_{\theta}(x|z)] - D_{\text{KL}}(q_{\phi}(z|x) \| p(z)). \quad (7)$$

Compared with AE, VAE typically gives smoother latent structure but may sacrifice discriminative sharpness under short training schedules.

2.2 Classifiers Used in This Project

According to the project implementation and README, all DR features are evaluated with three classifiers: **1-NN**, **Mahalanobis**, and **Logistic Regression**. For a test feature z , 1-NN predicts

$$\hat{y} = y_{i^*}, \quad i^* = \arg \min_i \|z - z_i\|_2. \quad (8)$$

Mahalanobis classifier uses class means μ_c and a shared covariance estimate:

$$\hat{y} = \arg \min_c (z - \mu_c)^\top \Sigma^{-1} (z - \mu_c), \quad (9)$$

with regularized covariance in practice, e.g. $\Sigma_{\lambda} = (1 - \lambda)\hat{\Sigma} + \lambda I$, to improve numerical stability. Logistic regression models posterior probabilities as

$$p(y = c|z) = \frac{\exp(w_c^\top z + b_c)}{\sum_j \exp(w_j^\top z + b_j)}, \quad \hat{y} = \arg \max_c p(y = c|z). \quad (10)$$

Using the same three classifiers across all DR methods makes the accuracy-vs- d comparison more controlled, because changes can be attributed mainly to representation quality.

2.3 Datasets

The two datasets were selected to provide complementary difficulty profiles. Fashion-MNIST is a large-scale grayscale object dataset with substantial intra-class appearance variation and inter-class visual similarity, making it suitable for observing gradual performance changes across dimensions [11]. Olivetti Faces, in contrast, has far fewer samples but much higher raw dimensionality per image and strong person-specific structure, which is a typical regime where supervised subspace learning can have large impact [12].

Using both datasets helps avoid overfitting the narrative to one data regime. If a method performs well on only one dataset, the report can analyze whether the advantage comes from data geometry, class structure, sample size, or model assumptions. This cross-dataset perspective is central to the assignment objective of understanding when DR helps classification and when it does not.

3 Experimental Setup

3.1 Leakage-Free Protocol

All preprocessing and DR model fitting are performed only on training data, with train-only CV for hyperparameter selection; the test set is used only once for final evaluation. This matches the assignment requirements.

3.2 Current Available Run (Quick)

Table 1: Configuration snapshot of the available quick run.

Item	Value
Run ID	ee6222_dr_quick_20260226_161510
Datasets	Fashion-MNIST, Olivetti
Seeds	1 seed (<code>seed=0</code>)
CV folds	2
Classifiers	1-NN, Mahalanobis, Logistic
Methods	PCA, LDA, PCA-LDA, KPCA, NMF, ICA, AE, VAE

4 Results from Existing Experiments

4.1 Accuracy and Error Curves vs Dimension

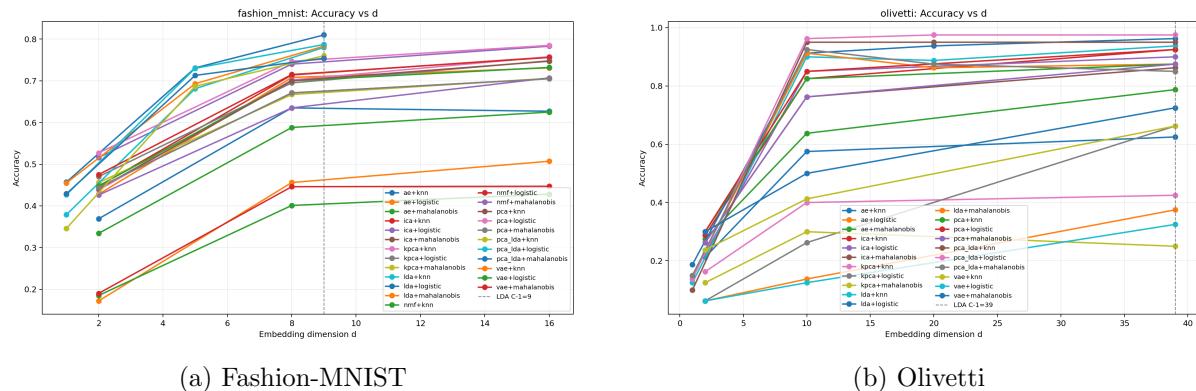


Figure 1: Classification accuracy vs reduced dimension d .

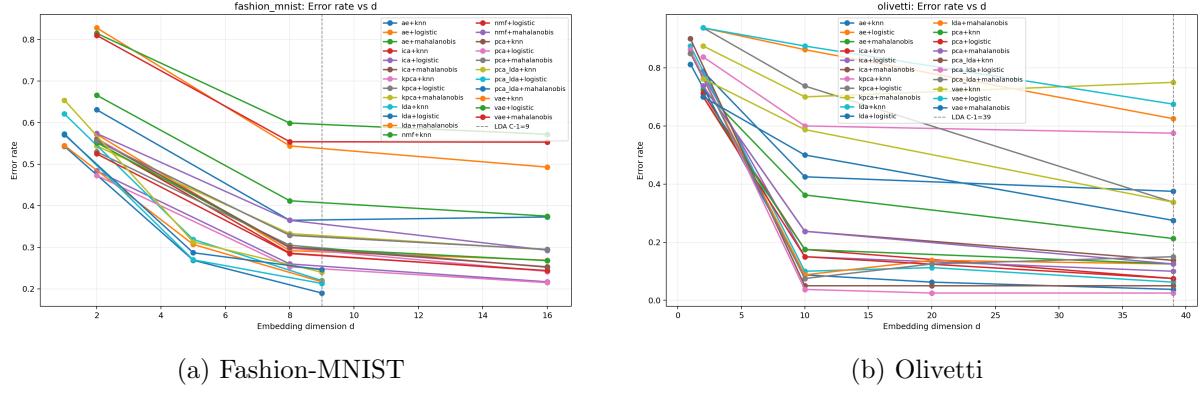


Figure 2: Classification error rate vs reduced dimension d .

4.2 Best Quick Results (Logistic Classifier)

Table 2: Fashion-MNIST: best test accuracy per method (quick run, logistic).

Method	Best d	Accuracy
AE	16	0.7310
ICA	16	0.7830
KPCA	16	0.7480
LDA	9	0.8100
NMF	16	0.7560
PCA	16	0.7850
PCA-LDA	9	0.7870
VAE	16	0.4280

Table 3: Olivetti: best test accuracy per method (quick run, logistic).

Method	Best d	Accuracy
AE	39	0.3750
ICA	39	0.9000
KPCA	39	0.6625
LDA	39	0.9625
PCA	39	0.9250
PCA-LDA	20	0.9750
VAE	39	0.3250

Current key observations. On Fashion-MNIST, the best quick result is LDA + Logistic at $d = 9$ with accuracy 0.8100. On Olivetti, the best quick result is PCA-LDA + Logistic at $d = 20$ with accuracy 0.9750. In this run, NMF failed on Olivetti (no valid hyperparameter setting), so those entries are missing and should be revisited in the full run.

4.3 Interpretability Figures

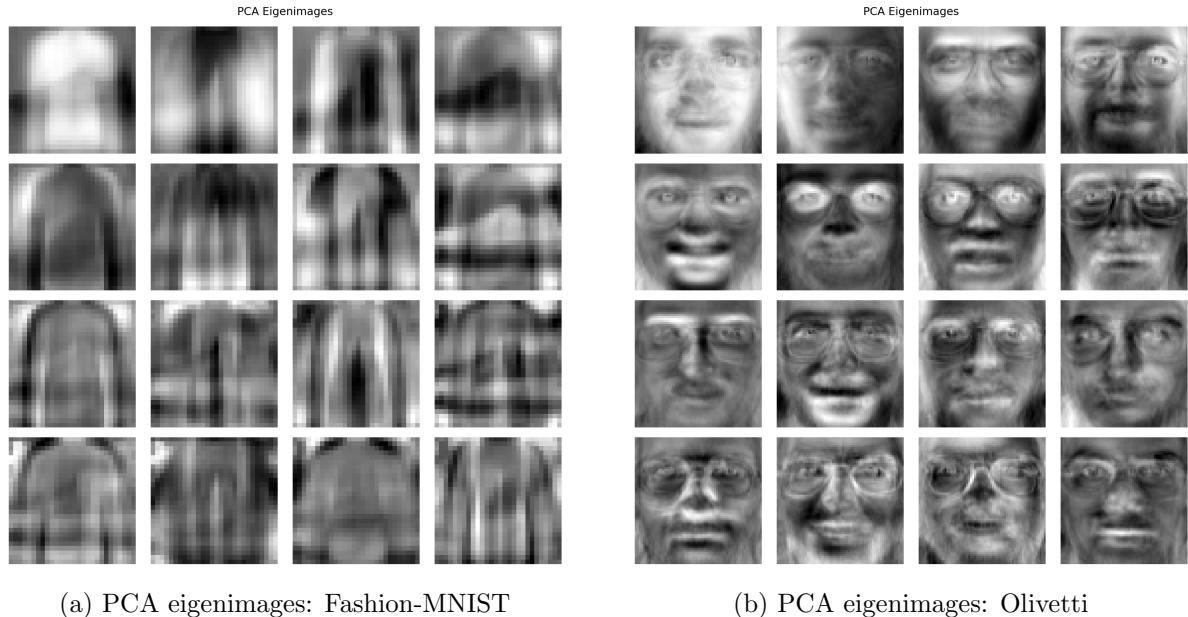


Figure 3: PCA component visualization.

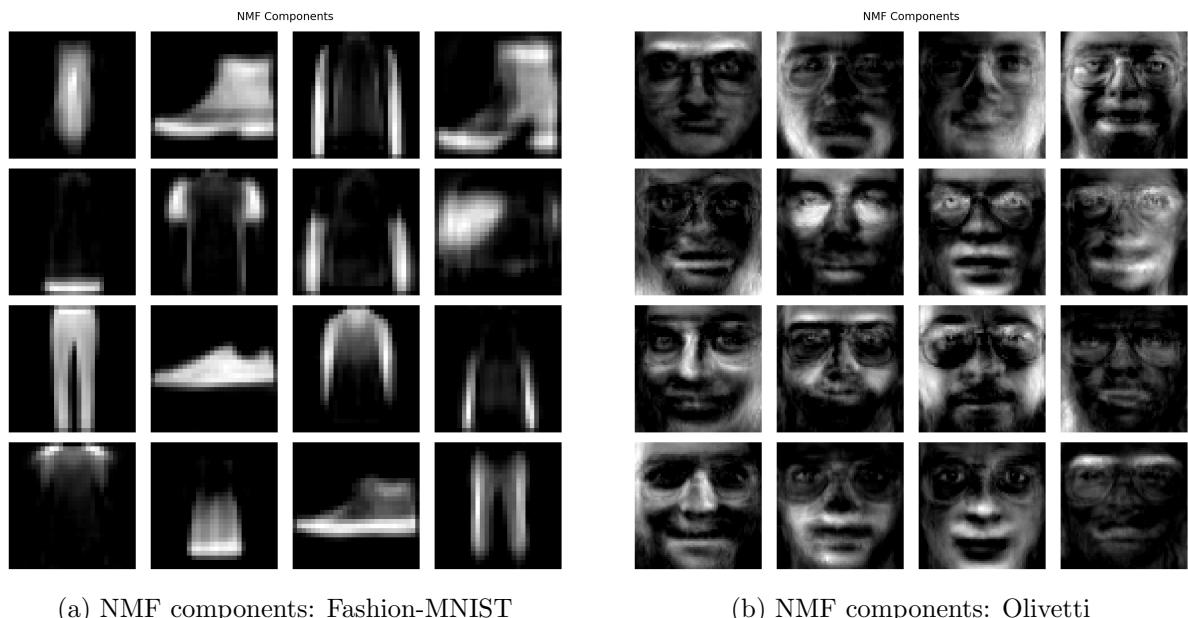


Figure 4: NMF basis visualization.

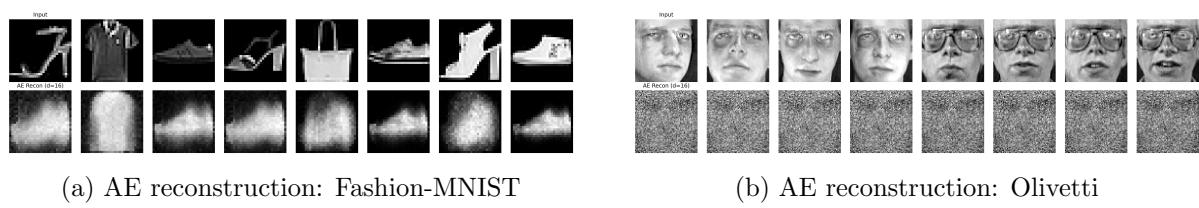


Figure 5: AE reconstruction examples at $d = 16$.

5 Sections Reserved for Pending Results (TBD)

5.1 Full-Config Multi-Seed Summary (TBD)

Table 4: Mean \pm Std accuracy across seeds for full configuration (to be filled).

Dataset	Method	Best d	Mean Accuracy	Std
Fashion-MNIST	[TBD]	[TBD]	[TBD]	[TBD]
Olivetti	[TBD]	[TBD]	[TBD]	[TBD]

5.2 Significance Test (TBD)

Table 5: Paired statistical comparison at matched d (to be filled).

Comparison	Metric	p -value	Conclusion
LDA vs PCA (Fashion-MNIST)	[TBD]	[TBD]	[TBD]
PCA-LDA vs LDA (Olivetti)	[TBD]	[TBD]	[TBD]

5.3 Additional Analysis Text (TBD)

TBD Why some methods plateau early with increasing d .

TBD Why VAE underperforms in quick mode (possible undertraining).

TBD How NMF should be fixed/evaluated on Olivetti.

TBD Robustness across random seeds and split variability.

6 Conclusion (Draft)

The currently available quick run already supports the required core plot-and-analyze workflow for the assignment. Preliminary evidence suggests strong performance from supervised DR (LDA and PCA-LDA), especially on Olivetti. A final high-confidence conclusion still requires the pending full multi-seed run and statistical validation.

A Reproducibility Commands

```
# quick run
python -m ee6222_dr.cli run --config configs/quick.json --mode quick --device auto --output ou

# full run (pending)
python -m ee6222_dr.cli run --config configs/full.json --mode full --device auto --output ou
```

References

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