

Π.Θαυόμενες και Στατιστική

Θέμα 1ο

Έχω: $n=24000$, $\mu=1.75$, $\sigma=0.05$

$$1. P(1.75 < x < 1.78) = P(0 < Z < \frac{0.05}{0.05}) = P(0 < Z < 1) = \Phi(1) - \Phi(0) = 0.8413 - 0.5 = 0.3413 = 34.13\%$$

$$\text{Άρα Φοιτητές} = 24000 \cdot 34.13\% = 8194$$

$$2. P(x > 1.85) = 1 - P(x \leq 1.85) = 1 - P(Z \leq \frac{0.10}{0.05}) = 1 - \Phi(2) = 1 - 0.9772 = 0.0228$$

$$3. P(x < k) = 0.1 \Leftrightarrow P(Z < \frac{k-1.75}{0.05}) = 0.1 \Leftrightarrow \Phi(\frac{k-1.75}{0.05}) = 0.1 \Leftrightarrow \frac{k-1.75}{0.05} = \Phi^{-1}(0.1) \Leftrightarrow \frac{k-1.75}{0.05} = 1 - \Phi^{-1}(0.9) = 1.28 \Leftrightarrow k = 1.686$$

$$4. \text{Ομοία... } P(x \geq k) = 0.15 \Leftrightarrow 1 - \Phi(\frac{k-1.75}{0.05}) = 0.15 \Leftrightarrow 0.85 = \Phi(\frac{k-1.75}{0.05}) \Leftrightarrow 1.04 = \frac{k-1.75}{0.05} \Leftrightarrow k = 1.802$$

Θέμα 2ο

1. Έχω $\bar{x} = \frac{350}{9} = 38.8$, $\alpha = 0.05$

$$x_i - \bar{x} = |-0.88| 4.12 |1.2| 6.2 | -3.8 | 5.2 | 3.2 | -1.8 | -2.8$$

$$\text{Άρα } s^2 = \frac{120.42}{8} = 15.05 \Leftrightarrow s = 3.87$$

$$t_{8,0.02} = 2.896 \quad t_{8,0.02} \cdot \frac{s}{\sqrt{n}} = 33.62$$

$$\text{Τελικά: } [\bar{x} - t_{n-1, \alpha/2} \cdot \frac{s}{\sqrt{n}}, \bar{x} + t_{n-1, \alpha/2} \cdot \frac{s}{\sqrt{n}}] \Leftrightarrow [5.26, 72.42]$$

2. Επιπροσθέτα έχω: $\sigma=4$, $\epsilon=0.03$, $\text{νέο } \alpha=0.02$

$$\text{Άρα } n = \frac{4Z_{\alpha/2}^2 \cdot \sigma^2}{\epsilon^2} = \frac{4 \cdot (2.33)^2 \cdot 4^2}{(0.03)^2} \approx 3190$$

$$\hookrightarrow \text{Άφού } Z_{\alpha/2} = Z_{0.01} = \Phi^{-1}(0.99) = 2.33$$

Θέμα 3ο

Κανονική κατανομή με $E(x) = \mu = 300$, $\sigma = 11$, $\bar{x} = 305$, $\alpha = 5\% = 0.05$

Άρα: H_0 H_1 Απόρρ.

$$\mu_x = 300 \quad \mu_x \neq 300 \quad |Z| > Z_{\alpha/2}$$

$$\text{Όμως: } Z = \frac{305-300}{\frac{11}{\sqrt{10}}} = 1.437, \text{ δηλαδή } |Z| < Z_{\alpha/2} = 1.96, \text{ Άρα δεν απορρ.}$$

Θέμα 4ο

Έχω $B \sim (4, 0.1)$:

$$(a) P(x=1) = \binom{4}{1} (0.1)^1 (0.9)^{4-1} = 0.2816$$

$$\hookrightarrow \binom{4}{1} = \frac{4!}{1!(4-1)!} = \frac{4 \cdot 3!}{1 \cdot 3!} = 4$$

$$(b) P(x \geq 1) = 1 - P(x < 0) = 1 - P(x=0) = 1 - 0.6561 = 0.3439$$

$$\hookrightarrow \binom{4}{0} (0.1)^0 \cdot (0.9)^{4-0} = 0.6561$$

$$\hookrightarrow \binom{4}{0} = \frac{4!}{0!(4)!} = 1$$