

HW-08

1. 对问题:  $\min x_2 - 3x_1$ , 应用外点罚函数方法。当  $\pi \rightarrow \infty$  时, 求出问题最优解和对应的 lagrange 乘子。  
 s.t.  $\begin{cases} x_1 + x_2 = 1 \\ x_1 - x_2 = 0 \end{cases}$

解: 写出外点罚函数:  $P(x, \pi) = x_2 - 3x_1 + \pi \sum_{i=1}^2 C_i(x) = x_2 - 3x_1 + \pi [(x_1 + x_2 - 1)^2 + (x_1 - x_2)^2]$   
 $= x_2 - 3x_1 + \pi (2x_1^2 + 2x_2^2 - 2x_1 - 2x_2 + 1)$   
 $= 2\pi x_1^2 + (2\pi + 1)x_2^2 - (2\pi + 3)x_1 - 2\pi x_2 + \pi$

则  $\nabla P(x, \pi) = \begin{pmatrix} 4\pi x_1 - 2\pi - 3 \\ 2(2\pi + 1)x_2 - 2\pi \end{pmatrix}$  令  $\nabla P(x, \pi) = 0$ , 得:  $x_\pi = \begin{pmatrix} \frac{2\pi + 3}{4\pi} \\ \frac{\pi}{2\pi + 1} \end{pmatrix}$

则当  $\pi \rightarrow \infty$  时,  $x_\pi = \begin{pmatrix} \frac{2 + \frac{3}{\pi}}{4} \\ \frac{1}{2 + \frac{1}{\pi}} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$  即为所求问题最优解

格 写出拉朗日函数:  $F = x_2 - 3x_1 + u_1(x_1 + x_2 - 1) + u_2(x_1 - x_2)$

令  $\frac{\partial F}{\partial x_1} = -3 + u_1 + u_2 = 0$ , 解得:  $\begin{cases} u_1 = 1 \\ u_2 = 2 \end{cases}$  即对应的 lagrange 乘子  $u = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$\frac{\partial F}{\partial x_2} = 2x_2 + u_1 - u_2 = 0$

$[x_1 = \frac{1}{2}, x_2 = \frac{1}{2}]$

2. 对问题:  $\min 2x_1 + 3x_2$ , 考虑对数罚函数方法。当  $\pi \rightarrow \infty$  时, 求出问题最优解和对应的 lagrange 乘子。  
 s.t.  $1 - 2x_1^2 - x_2^2 \geq 0$  (内点罚函数)

解: 写出对数罚函数  $P(x, \pi) = 2x_1 + 3x_2 - \frac{1}{\pi} \ln(1 - 2x_1^2 - x_2^2)$

则  $\nabla P(x, \pi) = \begin{pmatrix} 2 - \frac{1}{\pi} \cdot \frac{-4x_1}{1 - 2x_1^2 - x_2^2} \\ 3 - \frac{1}{\pi} \cdot \frac{-2x_2}{1 - 2x_1^2 - x_2^2} \end{pmatrix}$  令  $\nabla P(x, \pi) = 0$ , 得:  $x_\pi = \begin{pmatrix} \frac{1}{\sqrt{\pi}} \\ \frac{3}{\sqrt{\pi}} \end{pmatrix}$  或  $\begin{pmatrix} -\frac{1}{\sqrt{\pi}} \\ -\frac{3}{\sqrt{\pi}} \end{pmatrix}$

考虑原问题  $\min 2x_1 + 3x_2$ , 显然取  $x_\pi = \begin{pmatrix} -\frac{1}{\sqrt{\pi}} \\ -\frac{3}{\sqrt{\pi}} \end{pmatrix}$  为最优解, 对应最优解为  $-\sqrt{\pi}$   
 s.t.  $1 - 2x_1^2 - x_2^2 \geq 0$

写出拉格朗日函数  $F = 2x_1 + 3x_2 - \lambda(1 - 2x_1^2 - x_2^2)$   $\lambda \geq 0$

令  $\frac{\partial F}{\partial x_1} = 2 + 4\lambda x_1 = 0$

$\frac{\partial F}{\partial x_2} = 3 + 2\lambda x_2 = 0$

$[x_1 = -\frac{1}{\sqrt{\pi}}, x_2 = -\frac{3}{\sqrt{\pi}}]$

$\Rightarrow \lambda = \frac{\sqrt{\pi}}{2}$ , 即对应的 lagrange 乘子为  $\frac{\sqrt{\pi}}{2}$ .