

1. 对问题:  $\min x_1^2 - 3x_1$ , 应用外点罚函数方法。当 $\pi \rightarrow \infty$ 时, 求出问题最优解

$$\begin{cases} x_1 + x_2 = 1 \\ x_1 - x_2 = 0 \end{cases}$$

和对应的Lagrange乘子。

解: 写出外点罚函数:  $P(x, \pi) = x_1^2 - 3x_1 + \pi \sum_{i=1}^2 C_i^2(x) = x_1^2 - 3x_1 + \pi [(x_1 + x_2 - 1)^2 + (x_1 - x_2)^2]$

$$= x_1^2 - 3x_1 + \pi(2x_1^2 + 2x_2^2 - 2x_1 - 2x_2 + 1)$$

$$= 2\pi x_1^2 + (2\pi + 1)x_2^2 - (2\pi + 3)x_1 - 2\pi x_2 + \pi$$

$$\text{则 } \nabla P(x, \pi) = \begin{pmatrix} 4\pi x_1 - 2\pi - 3 \\ 2(2\pi + 1)x_2 - 2\pi \end{pmatrix} \quad \text{令 } \nabla P(x, \pi) = 0, \text{ 得: } x_\pi = \begin{pmatrix} \frac{2\pi + 3}{4\pi} \\ \frac{\pi}{2\pi + 1} \end{pmatrix}$$

$$\text{则当 } \pi \rightarrow \infty \text{ 时, } x_\pi = \begin{pmatrix} \frac{2 + \frac{3}{\pi}}{4} \\ \frac{1}{2 + \frac{1}{\pi}} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} \quad \text{即求出问题最优解}$$

写出拉格朗日函数:  $F = x_1^2 - 3x_1 + l_1(x_1 + x_2 - 1) + l_2(x_1 - x_2)$

$$\begin{cases} \frac{\partial F}{\partial x_1} = -3 + l_1 + l_2 = 0 \\ \frac{\partial F}{\partial x_2} = 2x_2 + l_1 - l_2 = 0 \end{cases}, \text{ 解得: } \begin{cases} l_1 = 1 \\ l_2 = 2 \end{cases} \quad \text{即对应的Lagrange乘子 } l = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$[x_1 = \frac{1}{2}, x_2 = \frac{1}{2}]$$

2. 对问题:  $\min 2x_1 + 3x_2$ , 考虑对数罚函数方法。当 $\pi \rightarrow \infty$ 时, 求出问题最优解

$$\text{s.t. } 1 - 2x_1^2 - x_2^2 \geq 0 \quad (\text{内点罚函数}) \quad \text{和对应的Lagrange乘子。}$$

解: 写出对数罚函数  $P(x, \pi) = 2x_1 + 3x_2 - \frac{1}{\pi} \ln(1 - 2x_1^2 - x_2^2)$

$$\text{则 } \nabla P(x, \pi) = \begin{pmatrix} 2 - \frac{1}{\pi} \cdot \frac{-4x_1}{1 - 2x_1^2 - x_2^2} \\ 3 - \frac{1}{\pi} \cdot \frac{-2x_2}{1 - 2x_1^2 - x_2^2} \end{pmatrix} \quad \text{令 } \nabla P(x, \pi) = 0, \text{ 得: } x_\pi = \begin{pmatrix} \frac{1}{\sqrt{11}} \\ \frac{3}{\sqrt{11}} \end{pmatrix} \text{ 或 } \begin{pmatrix} -\frac{1}{\sqrt{11}} \\ -\frac{3}{\sqrt{11}} \end{pmatrix}$$

考虑原问题  $\min 2x_1 + 3x_2$ , 显然取  $x_\pi = \begin{pmatrix} -\frac{1}{\sqrt{11}} \\ -\frac{3}{\sqrt{11}} \end{pmatrix}$  为最优解, ~~因为最优解为~~  $-\frac{1}{\sqrt{11}}$

写出拉格朗日函数  $F = 2x_1 + 3x_2 - \lambda(1 - 2x_1^2 - x_2^2) \quad \lambda \geq 0$

$$\begin{cases} \frac{\partial F}{\partial x_1} = 2 + 4\lambda x_1 = 0 \\ \frac{\partial F}{\partial x_2} = 3 + 2\lambda x_2 = 0 \end{cases} \Rightarrow \lambda = \frac{\sqrt{11}}{2}, \text{ 即对应的Lagrange乘子为 } \frac{\sqrt{11}}{2}.$$

$$[x_1 = -\frac{1}{\sqrt{11}}, x_2 = -\frac{3}{\sqrt{11}}]$$