

Bottom-Up Parsing

Bottom-Up Parsing

1. Overview
2. LR(0) Parsing and SLR(1) Parsing
3. LR(1) Parsing and LALR(1) Parsing

overview

- *Bottom-up parsing* traces out a **rightmost derivation** of the input string, but the steps of the derivation occur **in reverse order**.
- The most general bottom-up algorithm is LR(k) parsing.
 - LR(0) parsing
 - SLR(1) parsing
 - LR(1) parsing
 - LALR(1) parsing

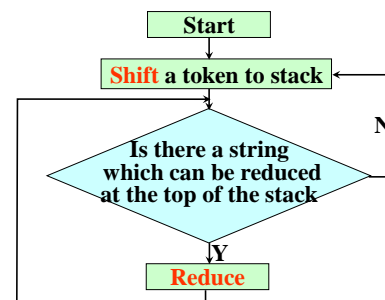
How to Implement

- A bottom-up parser uses an **explicit stack** to perform a parse.
 - contains tokens, nonterminals, **some extra state information**
 - Initial: **empty**
 - Success: **start symbol**
- A **schematic** for bottom-up parsing:

Stack	InputString	Actions
\$	InputString\$	
.....
\$StartSymbol	\$	accept

Two Actions

- Two possible actions (besides "accept"):
 - ✓ **Shift** a terminal from the front of the input to the top of the stack; **移进/移入**
 - ✓ **Reduce** a string α at the top of the stack to a nonterminal A, given the production $A \rightarrow \alpha$; **归约**
- **Shift-reduce** parser
- Grammars are always **augmented with a new start symbol**
增广文法 $S' \rightarrow S$



The LR Parsing Algorithm

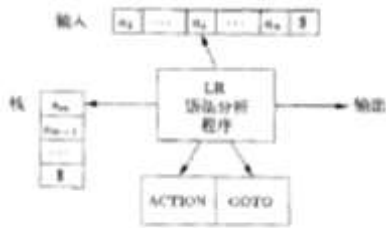


图 4-35 一个 LR 语法分析器的模型

7

The LR Parsing Algorithm

LR分析过程:

假设 i 是当前栈顶状态， a 是下一个输入符号，分析过程如下：

- 初始：符号 $\$$ 和状态 s_0 进栈
- 用状态 i 和输入符号 a 查表，分别执行以下动作：
 - **Shift**移进：若 $ACTION[i, a]=s_j$ ，则将符号 a 和状态 j 进栈（实际上， a 可以不用进栈）
 - **Accept**接受：若 $ACTION[i, a]=acc$ ，则表示语法正确，分析成功完成

8

The LR Parsing Algorithm

– **Reduce**规约：若 $ACTION[i, a]=r(A \rightarrow \gamma)$ ，则执行规约操作：

- ① 弹出栈顶 $|\gamma|$ 个符号和 $|\gamma|$ 个状态，得到目前栈顶状态 k
- ② 若 $m=GOTO[k, A]$ ，将状态 m 和符号 A 压入分析栈（实际上， A 可以不用进栈）

– **Error**出错：若 $ACTION[i, a]$ =出错，则执行错误处理。

9

LR语法分析伪代码

输入串为 $w\$$ ，符号 $\$$ 和初始状态 s_0 压入分析栈：

```

令  $a$  为  $w\$$  的第一个符号；
while(1) { /* 永远重复 */
    令  $s$  是栈顶的状态；
    若 {  $ACTION[s, a] = 移进$  } {
        将  $a$  压入栈中；
        令  $a$  为下一个输入符号；
    } else 若 {  $ACTION[s, a] = 规约$  } {
        从栈中弹出两个符号；
        令  $t$  为当前栈顶状态；
        将  $GOTO[t, A]$  压入栈中；
        输出产生式  $A \rightarrow \gamma$ ；
    } else 若 {  $ACTION[s, a] = 接受$  } break; /* 语法分析完成 */
    else 调用错误恢复例程；
}

```

10

Bottom-Up Parsing

1. Overview
2. LR(0) Parsing and SLR(1) Parsing
3. LR(1) Parsing and LALR(1) Parsing

11

2. LR(0) Parsing and SLR(1) Parsing

- (1) LR(0) ITEMS
- (2) LR(0) Parsing
- (3) SLR(1) Parsing

12

(1) LR(0) ITEMS

- An LR(0) item:
 - A **production choice** with a distinguished **position** in its right-hand side
 - Indicating the distinguished position by a period
- Example:
 - $A \rightarrow \beta\gamma$ is a production choice.
 - $A \rightarrow \beta \cdot \gamma$ is an LR(0) item.
- These are called LR(0) items because they contain **no explicit reference to lookahead**

13

(1) LR(0) ITEMS

Example: The grammar:

$$S' \rightarrow S$$

$$S \rightarrow (S)S$$

$$S \rightarrow \epsilon$$

This grammar has eight items:

$$S' \rightarrow \cdot S \quad S' \rightarrow S \cdot \quad S \rightarrow \cdot (S)S$$

$$S \rightarrow (\cdot S)S \quad S \rightarrow (S \cdot)S \quad S \rightarrow (S) \cdot S$$

$$S \rightarrow (S)S \cdot \quad S \rightarrow \cdot$$

14

(1) LR(0) ITEMS

- An item records **an intermediate step** in the recognition of the right-hand side of a rule choice;
- $A \rightarrow \beta \cdot \gamma$ means that β has **already been seen**, it may be possible to derive the next input tokens from γ ;
- $A \rightarrow \cdot \alpha$ **initial item** 初始项目
- $A \rightarrow \alpha \cdot$ **complete item** 完整项目

15

(2) LR(0) Parsing

项目集的闭包(CLOSURE)

定义: 若I是文法G的一个项目集, 则CLOSURE(I)包括如下项目:

- **Kernal items:** I中所有项目都在CLOSURE(I)中
- **Closure items:** 若 $A \rightarrow \alpha \cdot B\beta$ ($B \in N$) 在I中, 则将B的所有产生式对应的初始项目 (如: $B \rightarrow \cdot \gamma_1$, $B \rightarrow \cdot \gamma_2$) 都加入到CLOSURE(I)中 (实际上就是通过**e-闭包**加入的项目)

例: 文法

$$E' \rightarrow E$$

$$E \rightarrow E+T \mid T$$

$$T \rightarrow T^*F \mid F$$

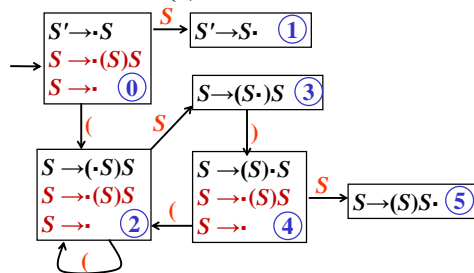
$$F \rightarrow (E) \mid id$$

若项目集 $I = \{[E' \rightarrow \cdot E]\}$, 则CLOSURE(I)

$$= \{[E' \rightarrow \cdot E], [E \rightarrow \cdot E+T], [E \rightarrow \cdot T], [T \rightarrow \cdot T^*F], [T \rightarrow \cdot F], [F \rightarrow \cdot (E)], [F \rightarrow \cdot id]\}$$

16

(2) LR(0) Parsing

Example DFA: $S' \rightarrow S \quad S \rightarrow (S)S \quad S \rightarrow \epsilon$ 

17

(2) LR(0) Parsing

状态间的转换 GOTO函数

- The transitions of items:

– Consider the item

 $A \rightarrow \alpha \cdot X\eta$ and $A \rightarrow \alpha X \cdot \eta$ ($X \in (N \cup T)$)– There has a **transition on the symbol X** from the first item to the second item.

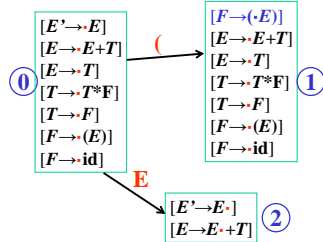
- GOTO(I, X): 项目集I中的项目, 在输入符号X后, 得到的项目集的**闭包**.

18

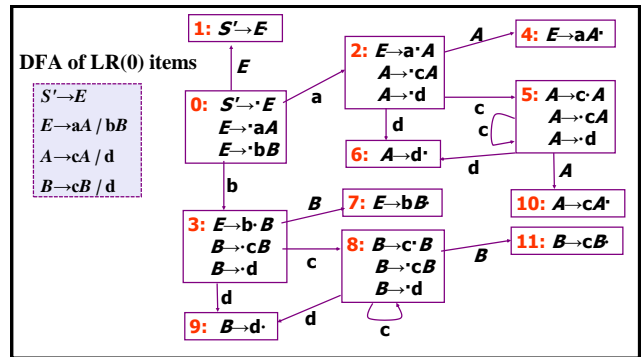
(2) LR(0) Parsing

- Graphical form of GOTO(I, X):

GOTO(I, X): 项目集 I 中的项目, 在输入符号 X 后, 得到的项目集的闭包。



19



20

(2) LR(0) Parsing

构造LR(0)分析表:

- 构造增广文法的LR(0)项目集族的DFA
- 根据各状态中的项目填写分析表:
 - 若从状态i到j有一个基于符号X的转换
 - ✓ $X \in T$ 时, 则 $Action[i, X] = sj$;
 - ✓ $X \in N$ 时, 则 $Goto[i, X] = j$;
 - 若状态i中有完整项目 $[A \rightarrow \gamma \cdot]$, 且 $A \neq S'$, 则对于所有 $X \in T$, $Action[i, X] = r(A \rightarrow \gamma)$;
 - 若状态i中有完整项目 $[S' \rightarrow S \cdot]$, 则 $Action[i, \$] = acc$;

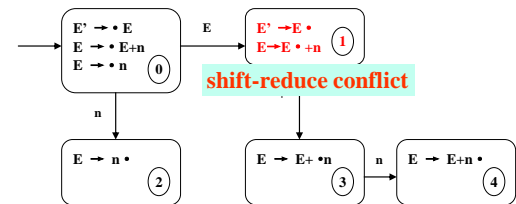
21

Why we need SLR(1) parsing?

- The grammar

$$E' \rightarrow E$$

$$E \rightarrow E+n \mid n$$



22

(3) SLR(1) Parsing

构造SLR(1)分析表:

- 构造增广文法的LR(0)项目集族的DFA
- 根据各状态中的项目填写分析表:
 - 若从状态i到j有一个基于符号X的转换
 - ✓ $X \in T$ 时, 则 $Action[i, X] = sj$;
 - ✓ $X \in N$ 时, 则 $Goto[i, X] = j$;
 - 若状态i中有完整项目 $[A \rightarrow \gamma \cdot]$, 且 $A \neq S'$, 则对于所有 $a \in Follow(A)$, $Action[i, a] = r(A \rightarrow \gamma)$;
 - 若状态i中有完整项目 $[S' \rightarrow S \cdot]$, 则 $Action[i, \$] = acc$;

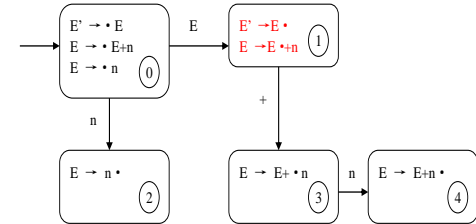
23

- Example: Consider the grammar

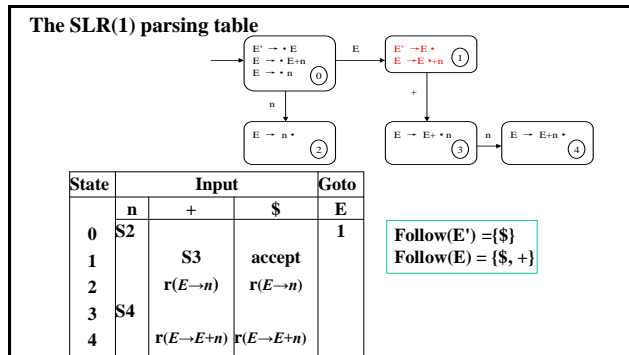
$$E' \rightarrow E \quad E \rightarrow E+n \mid n$$

- Construct the SLR(1) parsing table

$$Follow(E') = \{\$, Follow(E) = \{\$, +\}$$



24

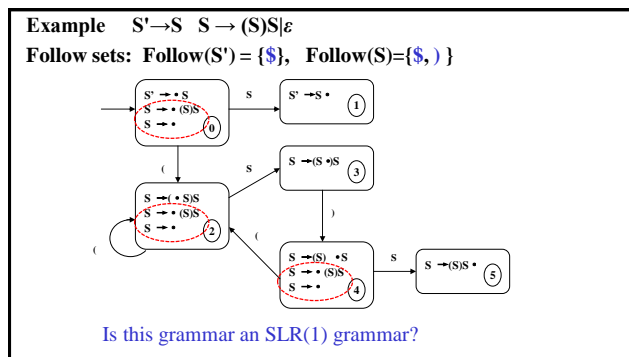


25

(3) SLR(1) Parsing

- A grammar is SLR(1) **if and only if**, for any state s , the following **two conditions are satisfied**:
 - For any item $A \rightarrow \alpha \cdot X \beta$ ($X \in T$) in s , there is **no** complete item $B \rightarrow \gamma \cdot$ in s with $X \in \text{Follow}(B)$.
 - For any two complete items $A \rightarrow \alpha \cdot$ and $B \rightarrow \beta \cdot$ in s , $\text{Follow}(A) \cap \text{Follow}(B)$ is empty.
- Violate the first condition: **shift-reduce conflict**
- Violate the second condition: **reduce-reduce conflict**

26



27

课堂练习

Consider the grammar:

$$S \rightarrow SS+ \mid SS* \mid a$$

- Construct the DFA of LR(0) items for this grammar;
- Construct the SLR(1) parsing table.
- Is this grammar an SLR(1) grammar? Give the reason.
- Show the parsing stack and actions of an SLR(1) parser, given the input string aa^*a+ .

28

Bottom-Up Parsing

- Overview
- LR(0) Parsing and SLR(1) Parsing
- LR(1) Parsing and LALR(1) Parsing

29

3. General LR(1) and LALR(1) Parsing

- Finite Automata of LR(1) Items
- LR(1) parsing table
- LALR(1) parsing

30

(1) Finite Automata of LR(1) Items

- The SLR(1) method:
 - Applies lookaheads **after** the construction of the DFA of LR(0) items
 - The **construction of DFA ignores lookaheads**
- The general LR(1) method:
 - Using a **new DFA with the lookaheads built into its construction**, LR(1) items include a single lookahead token in each item.
 - An LR(1) item is a pair:
 - [an LR(0) item, a lookahead token]
 - LR(1) item example: $[A \rightarrow \alpha \cdot \beta, a]$

31

(1) Finite Automata of LR(1) Items

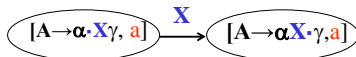
- LR(1)分析中CLOSURE(I)的定义:
 - All items in I are also in CLOSURE(I);
 - If an item $[A \rightarrow \alpha \cdot B \gamma, a]$ $B \in N$ is in I, items $[B \rightarrow \beta, b]$ for every $B \rightarrow \beta$ and **every token** $b \in \text{First}(\gamma a)$ are in CLOSURE(I).

32

(1) Finite Automata of LR(1) Items

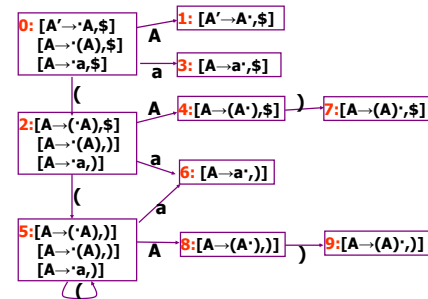
- Start state: $\text{CLOSURE}(\{[S' \rightarrow \cdot S, \$]\})$
- Each CLOSURE(I) is a state of DFA

$[A \rightarrow \alpha \cdot B \gamma, a]$
 $[B \rightarrow \beta, b]$
- GOTO: The transitions between states
 - Similar to LR(0) transitions except keeping track of lookaheads



33

Example G: $A \rightarrow (A) \mid a$ The DFA of sets of LR(1) items



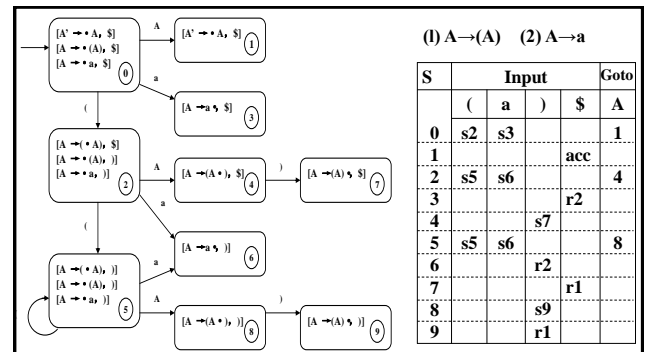
34

(2) LR(1) parsing table

构造LR(1)分析表:

- 构造增广文法的LR(1)项目集族的DFA
- 根据各状态中的项目填写分析表:
 - 若从状态i到j有一个基于符号X的转换
 - $X \in T$ 时, 则 $\text{Action}[i, X] = sj$;
 - $X \in N$ 时, 则 $\text{Goto}[i, X] = j$;
 - 若状态i中有完整项目 $[A \rightarrow \gamma \cdot, a]$, 且 $A \neq S'$, 则 $\text{Action}[i, a] = r(A \rightarrow \gamma)$;
 - 若状态i中有完整项目 $[S' \rightarrow S \cdot, \$]$, 则 $\text{Action}[i, \$] = \text{acc}$;

35



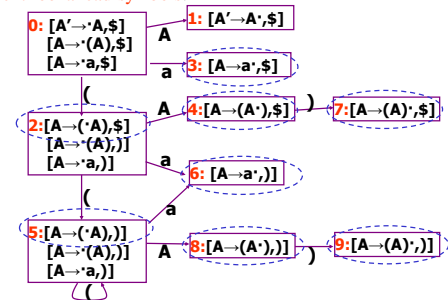
36

(2) LR(1) parsing table

- A grammar is LR(1) **if and only if**, for any state s , the following **two conditions** are satisfied.
 - For any item $[A \rightarrow \alpha \cdot X \beta, a] (X \in T)$ in s , there is no item in s of the form $[B \rightarrow \gamma \cdot, X]$ (otherwise there is a shift-reduce conflict).
 - There are no two items in s of the form $[A \rightarrow \alpha \cdot, a]$ and $[B \rightarrow \beta \cdot, a]$ (otherwise, there is a reduce-reduce conflict).

37

- In the DFA of sets of LR(1) items, many states have the **same LR(0) items**, and **different lookahead symbols**.



38

(3) LALR(1) parsing

- The LALR(1) parsing algorithm:
 - Identify all such states and **combine** their lookaheads;
 - Then, we have a DFA identical to the DFA of LR(0) items, except that each state consists of items with sets of lookaheads.
- In the case of **complete items** these lookahead sets are often **smaller** than the corresponding **Follow sets**.

39

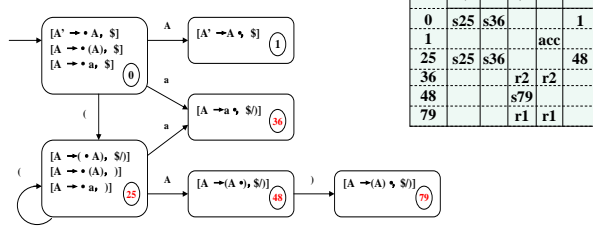
(3) LALR(1) parsing

- Constructing the DFA of LALR(1) items:
 - Constructed from the DFA of LR(1) items by identifying all states that have the same core
 - And forming the union of the lookahead symbols for each LR(0) item
- Each LALR(1) item in this DFA will have an LR(0) item as its first component and a set of lookahead tokens as its second component.
- 用LALR(1)项目的DFA构造分析表的方法与用LR(1)项目的DFA构造分析表完全一样。

40

(3) LALR(1) parsing

- Example**

(1) $A \rightarrow (A)$ (2) $A \rightarrow a$ 

State	Input				Goto
	(a)	\$	A
0	s25	s36			1
1				acc	
25	s25	s36			48
36			r2	r2	
48			s79		
79			r1	r1	

41

(3) LALR(1) parsing

- It is possible** for the LALR(1) parsing table construction to create **parsing conflicts** that do not exist in general LR(1) parsing table, but this **rarely happens** in practice
- Indeed, if a grammar is LR(1), then the LALR(1) parsing table cannot have any shift-reduce conflicts, there **may be reduce-reduce conflicts**.

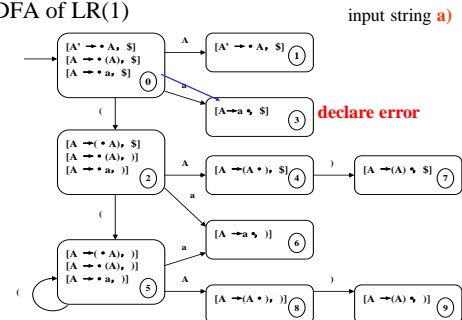
42

(3) LALR(1) parsing

- If a grammar is SLR(1), then it is LALR(1)
- LALR(1) parsers often do as well as general LR(1) parsers in removing typical conflicts that occur in SLR(1) parsing
- If the grammar is already LALR(1), the only **consequence of using LALR(1) parsing** over general LR parsing is that, in the presence of errors, **some spurious reductions** may be made before error is declared
- For example:
Given the erroneous input string **a)**

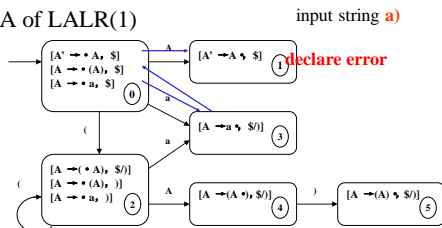
43

The DFA of LR(1)



44

The DFA of LALR(1)



Note:

➤ LALR(1) parser: (1) reduce($A \rightarrow a$) (2) declare error

➤ A general LR(1) parser: declare error immediately after shift **a**

45