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Paper Name:- Signals & System
Paper Code ESC501
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Group - A

1) i) Signal is a function that conveys information about a phenomenon. In electronics and telecommunications, it refers to any time varying voltage, current or electromagnetic wave that carries information.

v) Static System :- It is a system in which output at any instant of time depends on the input sample at the same time.

Dynamic System :- It is a system in which output at any instant of time depends on the input sample at the same time as well as other times.

ix) A function derived from a given function and representing it by a series of sinusoidal functions. Fourier Transform is a mathematical tool used for frequency analysis of signals.

Group - A

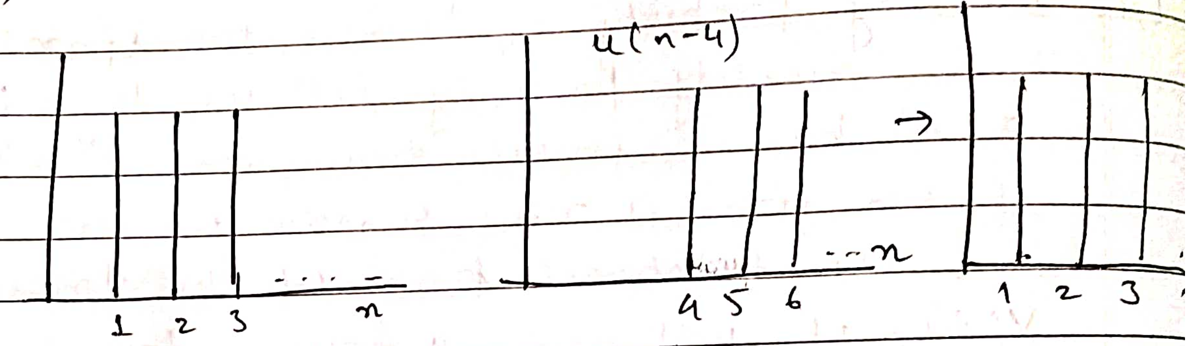
1) iii) $\sum_{n=-\infty}^{\infty} n^2 \delta(n+2)$

$\delta(n+2)$
$\begin{cases} 1, & \text{for } n=-2 \\ 0, & \text{for } n \neq -2 \end{cases}$

$n^2 = -2 \times -2$

$= 4$ Ans

iv) $x(n) = u(n-1) - u(n-4)$



vi) $\frac{dy}{dx}(t) + 2ty(t) = x(t)$

$x(t) = 2ty + \frac{dy}{dx} t$

$x(t, T) = 2ty + \frac{dy}{dx}(t-T)$ —(1)

$x(t-T) = 2(t-T) \int^{(t-T)} t \frac{dy}{dt} (t-T)$ —(2)

eq (1) $\neq$ eq (2)

So, it is variant

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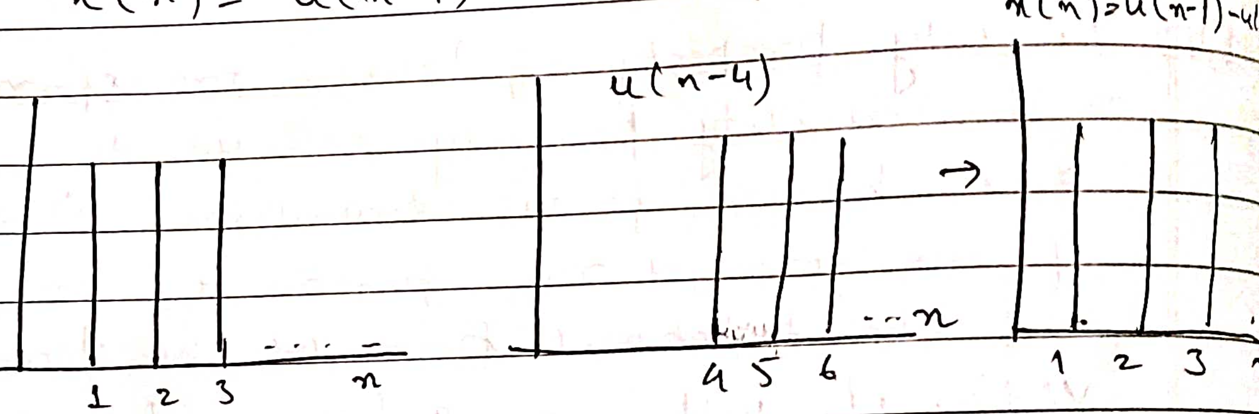

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vii) $x(t) = 2ty(t) + \frac{dy}{dx}(t)$

$x(t, T) = 2ty(t-T) + \frac{dy}{dx}(t-T)$ — ①

$x(t-T) = 2(t-T)y(t-T) + \frac{dy}{dx}(t-T)$ — ②

eq ① $\neq$ eq ②

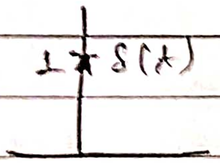
Time variant.

x) Periodic Function is a function that repeat its value at regular intervals, For example, the trignometric function which repeat intervals of 2π radians.

viii) In general, the most common criteria for pointwise convergence of a periodic function f are as follows:- o) If f satisfies a Holder condition, then its Fourier series converges uniformly. If f is bounded variation, then its Fourier series converges everywhere.

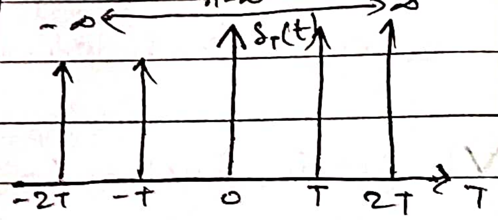
ii) Impulse Function

$\int_{-\infty}^{\infty} \delta(t) = 1$, for $t=0$; $\delta(t)=0$, for $t \neq 0$.



Impulse Train Function.

$$\delta_T(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT)$$



~~Section~~ Group - B

3) Let $f(t)$ be a function defined for $t \geq 0$. Then, the integral

$$F(s) = \int_0^{\infty} e^{-st} f(t)\, dt \qquad (1)$$

is called Laplace transform of $f(t)$, provided the integral exists. The Laplace transform of $f(t)$ is usually denoted by $L\{f(t)\}$, L is called Laplace transform operator.

That is:- $L\{f(t)\} = F(s)$ --- (2)

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The original function $f(t)$ is called inverse Laplace transform and we can write

$$L^{-1}\{F(s)\} = f(t) \qquad (3)$$

The convenient notation to evaluate the improper integral is

$$\int_0^{\infty} e^{-st} f(t)\, dt = \lim_{T \to \infty} \int_0^{T} e^{-st} f(t)\, dt$$

A function $f(t)$ has a Laplace transform if it's of exponential order. Theorem is piecewise continous function on the interval $[0, \infty]$ and is of exponential order α for $t \geq 0$.

2) First, let's determine $x(-t)$:

$$x(-t) = \cos(-t) + \sin(-t) + \cos(-t)\sin(-t)$$
$$= \cos(t) - \sin(t) - \sin(t)\cos(t)$$

Now, we can evaluate what are the even and odd components of $x(t)$ using the formulas
$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$ and

$$x_o(t) = \frac{1}{2}[x(t) - x(-t)].$$

$$x_e(t) = \frac{1}{2}[\cos(t) + \sin(t)\cos(t) + \cos(t) - \sin(t)\cos(t)$$
$$= \cos(t)$$

$x_o(t) = \frac{1}{2}[\cos(t) + \sin(t) + \sin(t)\cos(t) - \cos(t) + \sin t(t)$
$- \sin(t)\cos(t)]$

$= \sin(t) + \sin(t)\cos(t)$

5 B) $y(n) = x(n+2)$
for $y(n, k)$ i.e output for delayed input
$x(n-k)$
$x_2(n) = x(n-k)$
so
$y_2(n) = x_2(n) = x((n-k)2)$
and for delayed output signal $y_1(n)$,
replaced n by $n-k$ in equation(1), so we
get
$y_1(n) = x((n-k)2)$
$\therefore$ Invariant

6 A) In order for a function to be expanded
properly, it must satisfy the following
Dirichelet's conditions in the interval
$(-T/2, +T/2)$, if,

i) f is bounded on the interval $(-T/2, +T/2)$

ii) The interval $(-T/2, +T/2)$ may be sub divided
into a finite number of sub-interval in
each of which the derivative f' exists
throughout and does not change sign.

If these conditions are fulfilled then it is
certainly the case that this equation is valid

at each point of continuity of f in (−T/2, +T/2). Moreover, at any point t at which f is discontinous, both the one-sided limits f(t+) and f(t−) will necessarily exist and we will have-

$$\frac{1}{2}\left[f(t+) + f(t-)\right] = \frac{a_0}{2} + \sum_{n=1}^{+\infty} \left[a_n \cos(n \ldots \right.$$

so that Fourier series converges to mean value of function at such points.

4) $x(n) = \{-2, 1, -2, 1, 3\}$, $\uparrow$ $n=0$

$$x(n) = \sum_{k=0}^{4} x(k)\, \delta(n-k)$$

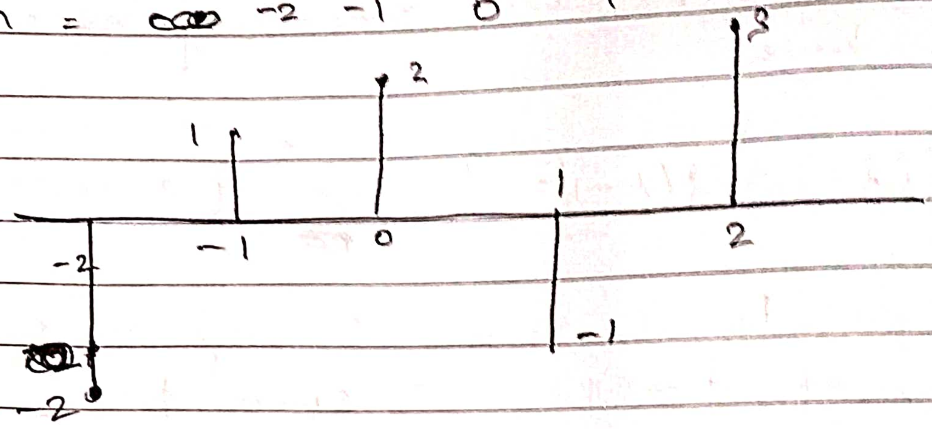
$$= x(0)\,\delta(n-0) + x(1)\,\delta(n-1) + x(2)\,\delta(n-2) + x(3)\,\delta(n-3) + x(4)\,\delta(n-4)$$

$$= 2\delta(n) + \delta(n-1) - 2\delta(n-2) + \delta(n-3) + 3\delta(n-4)$$

Group-C

8) $x(n) = \{-2, 1, 2, -1, 3\}$

$n = \quad -2 \quad -1 \quad 0 \quad 1 \quad 2$



$$x_e(n) = \tfrac{1}{2}\left[x(n) + x(-n)\right]$$

$$x_e(1) = \frac{1}{2}\left[x(n) + x(-n)\right] = \frac{-1+1}{2} = 0$$

$$x_e(2) = \frac{1}{2}\left[3-2\right] = \frac{1}{2} = 0.5$$

$$x_e(0) = \frac{1}{2}\left[2+2\right] = 2$$

$$x_e(n) = \{0.5, \; 0, \; 2, \; 0, \; 0.5\}$$

$n=-2 \quad n=-1 \quad n=0 \quad n=1 \quad n=2$

when we are calculating for $x_e(-2)$, $x_e(-1)$, we are getting value of $x_e(2)$, $x_e(1)$ respectively.

$x_e(-1) = 0$, $\quad x_e(-2) = 0.5$

Now for odd :-

$$x_o(n) = \frac{x(n) - x(-n)}{2}$$

$$x_o(0) = \frac{x(0) - x(-0)}{2} = \frac{2-2}{2} = 0$$

$$x_o(1) = \frac{x(1) - x(-1)}{2} = \frac{-1-1}{2} = -1$$

$$x_o(2) = \frac{x(2) - x(-2)}{2} = \frac{3-(-2)}{2} = \frac{5}{2} = 2.5$$

$$x(-1) = \frac{1-(-1)}{2} = \frac{2}{2} = 1$$

$$x(-2) = \frac{-2-3}{2} = -5/2$$

$$x_o(n) = \{-2.5,\ 1,\ 0,\ -1,\ 2.5\}$$

9) $x(t) = t \cdot u(t)$

$$E = \int_{-\infty}^{\infty} x^2(t)\, dt$$

$$\int_{-\infty}^{0} x^2(t)\, dt + \int_{0}^{\infty} x^2(t)\, dt$$

$$\downarrow$$

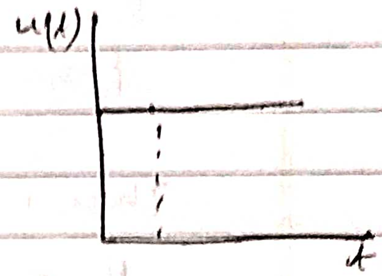
$$0 + \int_{0}^{\infty} t^2 u(t)\, dt$$

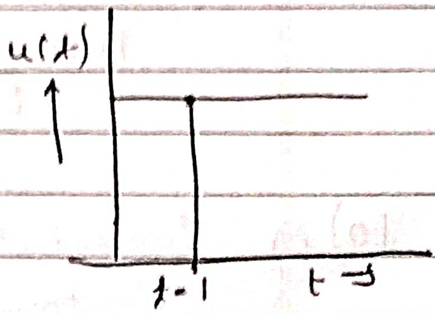
$$0 + \frac{t^3}{3}\Big|_0^{\infty}$$

$$= \infty$$

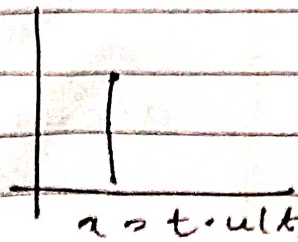
[It is not an energy signal]

Graph





$u(t) = 1, \ t \geq 0$
$\quad\quad\ \ 0, \ t < 0$



$x = t \cdot u(t)$

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\, dt$$

$$\lim_{T\to\infty} \left[\frac{1}{T} \int_{-T/2}^{T/2} t^2 u(t)\, dt + \int_{0}^{T/2} t^2 u(t)\, dt \right.$$

$$0 + \lim_{T\to\infty} \frac{1}{T} \left[\frac{t^3}{3} \right]^{T/2}$$

$$\lim_{T\to\infty} \frac{1}{T} \left[\frac{\left(\frac{T}{2}\right)^3}{3} \right]$$

$$\lim_{T\to\infty} \frac{1}{T} \left[\frac{T^3}{24} \right]$$

$$\lim_{T\to\infty} \frac{T^2}{24} = \infty$$

This is not a power signal

$P = \infty$.

This is not a energy signal

$E = \infty$.

10) i) Convolution :-

Continous

$$y(t) = x(t) * h(t)$$

$$= \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau.$$

discrete

$$y(n) = \sum_{k=-\infty}^{\infty} x(k) - h(n-k)$$

(ii) $x(t) = 2e^{-2t}u(t) + 4e^{-4}u(t)$

$L\,x(t) = X(s)$

$L(2e^{-2t}u(t)) + L(4e^{-4}u(t))$

$\left[e^{at}u(t) = \frac{1}{s-a} \right.$

$= 2\left(\frac{1}{s-(-2)}\right) + 4\left(\frac{1}{s-(-4)}\right)$

$= \frac{2}{s+2} + \frac{4}{s+4}$

12B) (i) Duality property of Fourier transform:-

→ The duality property tells us that if $x(t)$ has a fourier transform $X(\omega)$, then if we form a new function of time that has the functional form of the transform, $X(t)$, it will have a fourier transform $x(\omega)$ that has the functional form of the original time function.

(ii) $x(t) = e^{at}u(-t)$; $a > 0$

$\rightarrow x(\omega) = \int_{-\infty}^{\infty} e^{at}u(-t)\cdot e^{-j\omega t}\,dt$

$= \int_{-\infty}^{0} e^{at}\cdot 1 \cdot e^{-j\omega t}\,dt$

$= \int_{-\infty} e^{(a-j\omega)t}\,dt$

$= \frac{\left[e^{(a-j\omega)t}\right]_{-\infty}^{0}}{a-j\omega}$

$= \frac{1}{a-j\omega}\left[\underset{1}{e^{0}} - \underset{0}{e^{-\infty}}\right]$

$x(\omega) = \frac{1}{a-j\omega}$

$\therefore e^{at}u(-t) \rightleftharpoons \frac{1}{a-j\omega}$

when $a > 0$