

EUCLID WEAK LENSING ENCYCLOPAEDIA

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ABSTRACT

Weak lensing is an effect where the images of distant galaxies are distorted by the gravitational potential caused by matter perturbations along the line of sight. It is a method that majority of wide-field imaging surveys are, or will, use to determine cosmological parameters through the dependency of this effect on the geometry of the Universe and the growth of structure. It is also an effect that can be observed around galaxy clusters and individual galaxies, and used to make maps of the gravitational potential and inferred matter density. Weak lensing studies, as a sub-field of cosmology, contain significant specialised terms and definitions that are used commonly throughout the weak lensing literature. In this document we provide a reference for some of those specialised terms in an easily referenceable manner, as well as commonly used terms that have a special meaning in weak lensing studies. This document is a live document that will be updated in response to community input that can be submitted here <http://github.com/Weak-Lensing/Encyclopedia>.

1. PURPOSE

The purpose of this document is to create a glossary of terms for the study of weak lensing. In particular, this document can provide a basis for common definitions of previously undefined or ambiguously defined terms in the literature, to elucidate terms that are in common parlance in the weak lensing community but have not yet been defined formally, and to highlight special use of common phrases (i.e. jargon). Throughout we have attempted to provide as concise a definition as possible, and reference applicable papers where the reader can find more detailed definitions.

To reflect the continuing development of the field of weak lensing, and to allow for alternative and additional definitions to be submitted from the community, this document is a live document, with a repository here: <http://github.com/Weak-Lensing/Encyclopedia>. Any suggested changes, clarifications, new or amended definitions can be raised as issues on this site, and a new version will be submitted periodically.

This document covers only those terms in common usage in the weak lensing literature covering aspects in that field of gravitational lensing theory, cosmology, statistical inference and survey-specific definitions. In this dictionary we will not redefine words that are already defined in commonly dictionaries, except in the cases that the definition is changed or supplemented in its use in the field of weak lensing. We will use the following documents as reference documents for definitions: Oxford Dictionary of Astronomy (Ridpath, 2012), Oxford Companion to Cosmology (Liddle & Loveday, 2014), Oxford Dictionary of Physics (2015).

2. FORMAT

This document is not a review article of the field. To serve the purposes described above, it is organised in two parts. Part I is in a dictionary-style, where a list of terms with short/concise definitions are provided. Where appropriate, a list of references to relevant papers/review articles that are useful in clarifying the definitions are also provided. This list of definitions is provided in an alphabetical order so that phrases are easy to lookup and reference. Part II contains fewer entries, but each have longer and more in-depth definitions of the terms which may also have an entry in Part I. Part II has a *review* style, where general understating of the field of weak lensing can be found.

Short Definitions

1-point Statistics: Statistical term referring to 1-point statistical properties of a continuous or discrete stochastic process, such as the matter density field or galaxy distribution, using values of the field at a single point or wavenumber. Examples are moments of the field or the distribution

2-point Statistics: Statistical term referring to 2-point statistical properties of a continuous or discrete stochastic process, such as the matter density field or galaxy distribution, using values of the field at two distinct points or wavenumbers. Examples are the 2-point correlation function or the power spectrum SEE [CORRELATION FUNCTIONS AND POWER SPECTRA](#)

2D Cosmic Shear Analysis: Analysis of the cosmic shear (SEE [COSMIC SHEAR](#)) signal without employing individual redshift information of the lensed source galaxies.

3-point Statistics: Statistical term referring to 3-point statistical properties of a continuous or discrete stochastic process, such as the matter density field or galaxy distribution, using values of the field at three distinct points or wavenumbers. Examples are the 3-point correlation function or the bispectrum SEE [CORRELATION FUNCTIONS AND POWER SPECTRA](#); [BISPECTRUM](#)

3D: Term usually applied to cosmic shear analysis when the radial information is explicitly made use of, and usually with moderate to high resolution.

3D Cosmic Shear: The large-scale shear lensing signal, using the photometric redshift positions of source galaxies as radial information, either per galaxy or in fine radial bins. Also refers to a spherical-Bessel transform of a photometric weak lensing data set see e.g. [Kitching et al. \(2014\)](#).

3D Fast: A software suite¹ used to

1. generate 3D cosmic shear power spectra,
2. create Fisher matrix predictions for cosmic shear surveys,
3. search cosmological parameter likelihood using an MCMC Metropolis-Hastings algorithm.

3D Mass Map: Reconstruction of the total matter distribution in 3D i.e. celestial angle and redshift, or comoving distance (assuming a cosmology). SEE [MASS MAP](#); [KAPPA](#); [POTENTIAL MAP](#).

Absolute Photometry: Absolute photometry ties the measured count rate of an observation in a specific photometric band to an absolute, calibrated flux scale. This is normally done by observing the object through multiple filters and also observing a number of photometric standard stars.

Accuracy: The difference between the results of a measurement, or a combination of a set of measurements, and the true value. The differences are associated with systematic effects, rather than statistical errors. The statistical literature commonly uses the term “bias” instead of accuracy; meaning a highly accurate result has a low bias SEE. [BIAS](#).

Active Galactic Nucleus: An active galactic nucleus (AGN) is a compact region in a galaxy, mostly situated at its centre, where a substantial amount of radiation over a wide range of the electromagnetic spectrum is emitted. A galaxy hosting an AGN at its nucleus is called an ‘active’ galaxy. Observationally, an AGN is often difficult to distinguish from a central star burst. Physically, the radiation from an AGN is not due to the stars or hot gas in the galaxy, but powered by accretion of matter onto a supermassive black hole at the centre of its host galaxy. AGNs are classified into different categories: Seyfert galaxies, radio galaxies, quasars, QSOs, and BL Lac Objects ([Perlman 2013](#)). AGNs are relevant for weak lensing because they affect the distribution of matter through baryonic feedback processes SEE [BARYONIC FEEDBACK](#) that occur during the formation and evolution of galaxies. Furthermore, if the luminosity of an AGN is sufficiently large, it can outshine the radiation of its host galaxy and may thus be misidentified as a point-like source on images, thereby biasing the determination of the point-spread function. However, spectral information or multi-band imaging can be used to remedy this.

Additive Bias of Shear: SEE [MULTIPLICATIVE AND ADDITIVE BIAS OF SHEAR](#).

AGN: Acronym for Active Galactic Nucleus; SEE [ACTIVE GALACTIC NUCLEUS](#).

Alignment: In weak lensing this can refer to several different physical effects depending on the context

1. Galaxy alignment refers to the 3D orientation of the semi-major axis of a galaxy – characterised by its stellar distribution – with respect to some direction.

¹ <https://tomkitching.wordpress.com/2014/01/27/3dfast/>

2. In weak lensing images we observe the projected shapes of galaxies and in this context galaxy alignment refers to the observed apparent 2D projected alignment of a galaxy's observed surface brightness distribution with respect to some direction.
3. Intrinsic alignment refers to the phenomenon that the semi-major axes of a galaxy's stellar distribution may exhibit a tendency to be aligned with the local dark matter over-densities. This is a physical effect, distinct from the previous item which is an observed alignment.
4. In particular contexts alignment can also refer to the weak lensing shear. For example, around galaxy clusters background galaxies tend to be tangentially aligned, due to the weak lensing effect of the cluster.
5. Alignment can also refer to instrumental properties where a particular aspect of a physical object in an instrument or telescope has a degree of freedom which is correlated, or points towards, a particular coordinate. For example a mirror may be aligned with the optical axes of a telescope

Angle-Averaging: Taking the average value of a 2D-function over a circle of equal angular separation in real space or equal wavenumber in Fourier space. Also referred to as azimuthal averaging.

Angular-Diameter Distance: The angular-diameter distance D_A to a source is defined as the square root of the ratio of its physical surface area dA and the solid angle $d\omega$ it subtends on the observer's sky, $D_A = \sqrt{dA/d\omega}$. In general, the angular-diameter distance is affected by the tidal gravitational field along the line-of-sight, leading to (isotropic) focusing or anisotropic shearing of the light bundle. For a homogeneous, isotropic universe, the angular-diameter distance is a function of the cosmological redshift of a source – SEE **FRIEDMAN–ROBERTSON–WALKER MODELS**.

AOCS: An Attitude and Orientation Control System (AOCS) is a system in a satellite which controls the satellite pointing direction, the stability of this pointing, and the rotation of the satellite to point in a different direction. The AOCS is critical in ensuring the safety of the satellite consequent to an equipment failure or commanding error. The system contains sensors, such as Sun sensor, star trackers, fine guidance sensors and gyroscopes, and actuators such as reaction wheels, magnetorquers and thrusters of various powers and controllability. The AOCS operational lifetime is generally limited by consumables, such as hydrazine; SEE **SLEW**.

Aperture Mass: A smoothed version of the convergence κ (SEE **KAPPA**) that can be directly estimated from the ellipticities of sources. Therefore the aperture mass is basically the projected matter density field $\kappa(\boldsymbol{\theta})$ after application of a smoothing kernel $u(x)$. The aperture mass at $\boldsymbol{\theta}_0$ for an aperture with smoothing scale Θ is

$$M_{\text{ap}}(\boldsymbol{\theta}_0; \Theta) := \int \frac{d\boldsymbol{\vartheta}}{\Theta^2} u(|\boldsymbol{\vartheta} - \boldsymbol{\theta}_0| \Theta^{-1}) \kappa(\boldsymbol{\vartheta}). \quad (\text{AM.1})$$

In the case that a compensated filter with $\int_0^\infty dx \, x \, u(x) = 0$ is chosen, we equivalently find

$$M_{\text{ap}}(\boldsymbol{\theta}_0; \Theta) := \int \frac{d\boldsymbol{\vartheta}}{\Theta^2} q(|\boldsymbol{\vartheta} - \boldsymbol{\theta}_0| \Theta^{-1}) \gamma_t(\boldsymbol{\vartheta}; \boldsymbol{\theta}_0), \quad (\text{AM.2})$$

where $\gamma_t(\boldsymbol{\vartheta}; \boldsymbol{\theta}_0)$ denotes the tangential shear component of $\gamma(\boldsymbol{\vartheta})$ relative to the direction of $\boldsymbol{\vartheta} - \boldsymbol{\theta}_0$, and

$$q(x) = \left(\frac{2}{x^2} \int_0^x dy \, y \, (y) \right) - u(x), \quad (\text{AM.3})$$

is the corresponding smoothing filter of the shear field. As galaxy ellipticities are estimators of shear, M_{ap} can be directly estimated from source ellipticities; up to shot noise and possibly a bias due to gaps in the data that overlap with the aperture. The aperture mass is traditionally employed in cosmological studies, in searches for matter concentrations in the (projected) LSS, and in studies of galaxy bias. A transformation of the shear field on the sky with a compensated filter, which corresponds to a weighting of the lensing convergence field on the same patch of sky

Arc: A strongly distorted image of a background object (i.e. the source) that appears as an arc in an observation caused by strong lensing.

Arclet: A single distorted image of a background object (i.e. the source) with significant lensing-induced ellipticity. For a discussion of arclet definitions and detection steps see e.g. [Lenzen et al. \(2004\)](#).

Astrometry: The branch of astronomy involving precise measurements of the positions and movements of stars and other celestial bodies.

Athena: This can refer to:

1. The Advanced Telescope for High-ENergy Astrophysics (ATHENA)² is an X-ray telescope selected as the second L-class mission in ESA’s Cosmic Vision 2015-25 plan, with a launch foreseen in 2028. Athena aims to unravel mysteries of two major components of the Cosmos; The Hot Universe – revealing the properties of the hot gas and relating its evolution to large-scale structure and the cool components in galaxies and stars; The Energetic Universe – unravel the physics of black hole growth, energy output and its evolution to the highest redshifts.
2. A 2D-tree code³ for estimating second-order correlation functions from galaxy catalogues. It computes the shear-shear correlations (cosmic shear), position-shear correlation (galaxy-galaxy lensing) and position-position (galaxy correlation).
3. A grid-based code⁴ for astrophysical magnetohydrodynamics (MHD), which was developed primarily for studies of the interstellar medium, star formation, and accretion flows.

The reference is usually obvious in context.

Atmospheric (effects): The blurring and ‘twinkling’ in the images of astronomical objects caused by turbulence in the Earth’s atmosphere. Commonly known as “astronomical seeing”.

Auto-Correlation Function: A mathematical tool for measuring the similarity between observations of the *same* quantity as a function of their spatial and/or temporal separation. SEE **CORRELATION FUNCTIONS AND POWER SPECTRA**.

B-mode: The “magnetic” or odd-parity component of a spin-2 field, such as the weak lensing shear field SEE **EB-MODE DECOMPOSITION**

Background: In weak lensing, background can mean either

1. The emission or radiation which originates at a distance from the observer which is greater than a source in question e.g. a “background galaxy”, “background radiation”, “background galaxies”.
2. Unresolved sources of radiation that originate between the source and an observer. For example sub-mm observations of high- z QSOs are affected by the the Cosmic Infrared Background (CIB), that originates from smaller redshift.
3. Used to refer to approximately isotropic sources of radiation. For example the Cosmic Microwave Background.
4. A synonym for ‘noise’ in an image

Band: The characterisation of a photometric system in terms of its sensitivity to incident radiation as a function of wavelength/frequency. Hence bands are typically wavelength filters used in a survey over a particular range. The sensitivity usually depends on the optical system, detectors and filters. SEE **NARROWBAND**; **BROADBAND**.

Baryonic Feedback: The transfer of energy and momentum from a non-dark matter-related physical process (or, more generally, physical process beyond those that can attributed to local gravitational interactions) to the surrounding matter (both dark matter and baryonic matter). The subsequent changes in the matter environment may lead to further changes in the non-dark matter physics, hence a feedback loop may be caused. Sources of such feedback processes include the outflows from supernovae, AGNs and star formation. The investigation of the impact of these effects on weak lensing observations is an active area of research. The physics of these feedback loops at the relevant scales are unknown and, therefore, baryonic feedback is a major source of systematic errors in weak lensing measurements e.g. [White \(2004\)](#), [Semboloni et al. \(2011\)](#) and [Osato et al. \(2015\)](#).

Beat Coupling: The coupling of the density contrast to super-survey modes and the effect that this induces on the power spectrum. It is also known as “super-sample covariance”, SEE **SUPER-SAMPLE COVARIANCE**.

Bias: Bias is the deviation of the expectation value of an estimator from the quantity the estimator was designed to estimate. Biases occur in many places in weak lensing for example

1. Shape measurement of galaxies may be biased, SEE **MULTIPLICATIVE AND ADDITIVE BIAS OF SHEAR**
2. Cross-correlations of weak lensing measurement with galaxy positions can measure the ‘galaxy bias’ ([Jullo et al. 2012](#); [Bahé et al. 2012](#); [Samuroff et al. 2016](#); [Chang et al. 2016](#)). Galaxy bias parameterises the hypothesised tendency for galaxies to not be accurate tracers of underlying dark matter distribution.
3. Cosmological parameters can be offset, or biased, with respect to another experiment. For a recent example of this see [MacCrann et al. \(2015\)](#); [Dossett et al. \(2015\)](#); [Kitching et al. \(2016\)](#); [Hildebrandt et al. \(2016\)](#)

² <http://www.the-athena-x-ray-observatory.eu>

³ <http://www.cosmostat.org/software/athena/>

⁴ <https://trac.princeton.edu/Athena/>

The usage of this word is usually obvious depending on the context.

Bispectrum: The 2-nd order polyspectra $P^{(2)}(\equiv B)$, as explained in Equation CFS.5. The bispectrum, gives more information with respect to the power spectrum and is normally used to search for non-linear (non-Gaussian) effects. For an application in weak lensing see e.g. Munshi et al. (2011b,a). SEE CORRELATION FUNCTIONS AND POWER SPECTRA; THREE-POINT; 3-POINT STATISTICS.

Blue and Red Galaxies: The colour-absolute magnitude diagram of low-redshift galaxies shows a distinctive pattern. If the colour is measured in two bands, located on either side of the 4000\AA break, then a concentration of red galaxies at high luminosities, and one of blue galaxies at lower luminosities is seen. Most of the luminous galaxies belong to one of these two concentrations, constituting the *red sequence* and *blue cloud* of galaxies; the space between these two concentration in the colour-magnitude diagram is called *green valley*, but it is populated with only a small fraction of galaxies (see, e.g., Blanton and Moustakas 2009). There is a strong correlation between the galaxy colours and their morphological classification, in that most blue-cloud galaxies are spirals, whereas the majority of red-sequence galaxies are early-type galaxies (ellipticals and S0's), or early-type spirals. The blue and red populations of galaxies appears to also extend to higher redshift. The importance of the distinction between red and blue galaxies for weak lensing comes from their different intrinsic alignment properties and the difficulty to perform a morphological classification for galaxies with the apparent magnitudes typical for weak lensing source galaxies: red-sequence galaxies seem to have considerably higher intrinsic alignment amplitudes than blue-cloud galaxies (see, e.g., Hilbert et al. 2016, and references therein).

Blue Galaxies: SEE BLUE AND RED GALAXIES.

Born Approximation: When considering a weak scattering process, the Born approximation assumes that the total driving field, that is causing the scattering, can be approximated as the incident field at each point in the scatter. This approximation is used in weak lensing for simplicity (Refsdal 1970; Schneider and Weiss 1988). It approximates a lensing mass as being on a 2D plane along the line of sight where the deflection of light occurs instantaneously when the light ray intersects the plane. This is assumed to be a good approximation if the extent of the lensing mass in comoving distance is much smaller than the comoving distance between the observer and the lens, and the lens and source – which is the case in the majority of gravitational lens systems. Several studies have tested the validity of this approximation for example Schäfer et al. (2012) and Marozzi et al. (2016).

Brighter-Fatter Effect: This is an effect in which the profile of point sources (SEE PSF) as recorded on the detector is flattened and broadened to an extent depending on the flux recorded by a CCD detector (the effect is negligible in HeCdTe detectors used in the infrared).

Within a CCD pixel, as absorbed photons with sufficient energy elevate electrons into the conduction band of the Silicon, they collect in the potential well created by the voltage applied to the electrodes. In back-illuminated CCDs such as the CCD273 in Euclid, photons are absorbed in a region between the back surface and the electrodes (the pixel boundaries are set by the field structure extending from the electrodes to the back surface). The accumulation of charge changes the field structure, with the consequence that it modifies the effective boundaries of the pixel. If its neighbouring pixels are collecting fewer photons, this effectively reduces its size as the charge accumulates. When remapped to a regular grid in which all pixel areas are the same, as in an image, this pixel appears to be slightly deficient in charge, and the neighbours appear to have excess charge. The point-source profile is therefore flattened at the peak and broadened, with a dependence on the overall photon flux captured in the exposure.

The impact of this effect on weak lensing has been studied in Niemi et al. (2015); Gruen et al. (2015); Walter (2015).

Broadband: Broadband photometry refers to photometric observations and data for which the wavelength interval $\Delta\lambda$ over which the telescope/instrument/filter/detector system has appreciable sensitivity is not small compared to the central (or mean) wavelength λ_0 . Typically, $\Delta\lambda/\lambda \sim 0.1$ for broadband observations. Typical examples are the Sloan filters u, g, r, i, z, or the Johnson-Cousins filters U, B, V, R, I, J, H, K. SEE BAND; NARROWBAND. Weak lensing literature typically refers to the u, g, r, i, z filters as ‘broadbands’.

Bullet Cluster (-like): The bullet cluster (1E 0657–588), Clowe et al. (2006), consists of two galaxy clusters, that are at a redshift of 0.296. It is of particular interest in weak lensing because the mass distribution inferred from weak and strong lensing measurements is offset from the distribution of the hot intracluster gas; and spatially coincident with the distribution of cluster galaxies. This offset is thought to be due to the collision between the clusters where the dark matter did not interact but the X-ray emitting gas did – causing a shock ‘bullet’-like feature in the X-ray emitting gas distribution. This offset can be used place a limit on the dark matter self-interaction cross-section. ‘Bullet-like’ is used to refer to any cluster, or cluster merging events, in which similar offsets between the inferred dark matter distribution and the X-ray emitting gas in the clusters is observed. These systems provide empirical evidence for the existence of weakly interacting dark matter.

Cadence: The cadence is the frequency of sampling of the data. In a space-based observatory, this is approximately the time interval between spacecraft slews divided by the number of dithers; SEE [SLEW](#).

CFHT: The acronym for the Canada-France-Hawaii Telescope⁵. The CFH observatory hosts a 3.6 meter optical/infrared telescope, which is located on the island of Hawaii.

CFHTLenS: The Canada France Hawaii Lensing Survey (CFHTLenS)⁶. Using data accumulated over five years by the CFHT Legacy Survey (CFHTLS), the CFHTLenS team has analysed the images of over 10 million galaxies for weak lensing statistics. CFHTLenS is a 154 square degree multi-colour optical survey in ugriz incorporating all five years worth of data from the Wide, Deep and Pre-survey components on the CFHT Legacy Survey.

CFHTLS: SEE [CFHT](#); [CFHTLENS](#).

Chromatic Effects: Chromatic effects are those resulting from dispersion in which different colours have different convergence points in an optical system, and hence different wavelengths are focused in different positions. It occurs when the medium through which photons pass has a wavelength-dependent refractive index. The refractive index of transparent materials decreases with increasing wavelength, which is unique to each material. In weak lensing, this is important because;

1. Any finite aperture has a diffraction effect that depends on wavelength.
2. The material of the CCD produces PSF chromaticity; see e.g. [Niemi et al. \(2015\)](#).
3. Atmosphere produces atmospheric differential chromatic refraction; see e.g. [Meyers and Burchat \(2015\)](#).
4. Atmospheric seeing is wavelength dependent; see e.g. [Heymans et al. \(2012a\)](#).

For space-based telescopes, such as Euclid, only the first two elements are relevant.

CLASH: Cluster Lensing And Supernova survey with Hubble (CLASH)⁷ is a survey aiming to observe 25 massive galaxy clusters over a 3 year period using the Hubble Space Telescope (HST).

Colour Gradient: A colour gradient is a spatially varying spectral energy distribution within a galaxy profile SEE [LIGHT PROFILE](#), manifest as a change in the mean colour as a function of radius of the galaxy light profile. This can cause an important potential systematic effect in weak lensing, because it is necessary to measure galaxy shapes with great accuracy, which in turn requires a detailed model of the point spread function (PSF). In general, the PSF varies with wavelength and therefore the PSF integrated over an observing filter depends on the spectrum of the object. For a typical galaxy the spectrum varies across the galaxy image, thus the PSF depends on the position within the image. Therefore colour gradients within galaxies can necessitate a position-dependent PSF correction to be applied when measuring the shapes of galaxies. This effect has been investigated in [Voigt et al. \(2012\)](#) and [Semboloni et al. \(2013\)](#).

Comoving Distance: Comoving distance is the distance to an object where the expansion of the Universe is factored out. This gives a distance to a comoving observer that does not change in time due to the expansion of space-time. Comoving distance is normalised so that it is a distance between two events in space at the present cosmological time. For objects moving with the Hubble flow, it remains constant in time. The comoving distance from an observer to a distant object (e.g. galaxy) can be computed by

$$\chi = \int_{t_e}^t c \frac{dt'}{a(t')}, \quad (\text{CD.1})$$

where $a(t')$ is the scale factor, t_e is the time of emission of the photons detected by the observer, t is the present time, and c is the speed of light in vacuum. SEE [FRIEDMAN-ROBERTSON-WALKER MODELS](#)

Convergence Map: SEE [MASS MAP](#); [3D MASS MAP](#); [KAPPA](#); [POTENTIAL MAP](#).

Convolution: A convolution (sometimes known as ‘folding’) is an integral that expresses the amount of overlap of one function as it is shifted over another function. It therefore ‘blends’ one function with another. Convolution of two functions f and g is given by

$$[f * g](t) = \int f(\tau) g(t - \tau) d\tau. \quad (\text{CV.1})$$

A convolution in real space becomes a multiplication in Fourier space and vice versa. This is a particularly important mathematical concept in weak lensing because the finite aperture of a telescope, and the atmosphere, act to convolve an image with a Point Spread Function (PSF) that blurs images of galaxies. This blurring needs to be accounted for when measuring the weak lensing effect. SEE [CONVOLVED/CONVOLUTIONAL](#); [NON-CONVOLUTIONAL](#).

⁵ <http://www.cfht.hawaii.edu/en/>

⁶ <http://www.cfhtlens.org/>

⁷ <http://www.stsci.edu/~postman/CLASH/Home.html>

Convolved/Convolutional: This refers to any effect that can be represented mathematically as a convolution of an image with some function. For example the diffraction effects of a telescopes aperture can be represented as a convolution of the original image with the Fourier transforms of impulse response function of the telescope (the Point Spread Function (PSF)). SEE [CONVOLUTION](#); [NON-CONVOLUTIONAL](#).

Correlation Function: Correlation functions describe how two variables co-vary with one another on average; it is a measure of the excess probability for any random sample (e.g. distribution of galaxies) within some given separation. In cosmology the random variables are usually position dependent fields.

Correlation function is a general term for any two variables, however there is some nomenclature referring to auto-correlation and cross-correlations that is clarified in SEE [CORRELATION FUNCTIONS AND POWER SPECTRA](#); [AUTO-CORRELATION FUNCTION](#) [CROSS-CORRELATION FUNCTION](#).

Correlation is often used as a synonym for covariance; SEE [COVARIANCE](#).

Correlation Functions and Power Spectra: Let $f_i(\mathbf{x})$, $i = 1, 2$, be homogeneous and isotropic random fields defined on \mathbb{R}^n . We assume that the expectation value of the fields vanishes, $\langle f_i(\mathbf{x}) \rangle = 0$ for all \mathbf{x} .⁸

The *two-point correlation function* ξ of f_i (often simply called ‘correlation function’) is defined through

$$\langle f_i(\mathbf{x}) f_j(\mathbf{y}) \rangle =: \xi_{ij}(|\mathbf{x} - \mathbf{y}|) , \quad (\text{CFS.1})$$

where the angular brackets denote the ensemble average (or expectation value). Statistical homogeneity ensures that the correlator depends only on the separation vector $\mathbf{x} - \mathbf{y}$, statistical isotropy yields a dependence only on the absolute value of this separation vector. If $i = j$ in (CFS.1), ξ is called the *auto-correlation function*, for $i \neq j$ it is the *cross-correlation function* of the two fields.

The *power spectrum* P of f is defined as the correlator in Fourier space (SEE [FOURIER TRANSFORM](#)),

$$\langle \tilde{f}_i(\mathbf{k}) \tilde{f}_j(\mathbf{k}') \rangle = (2\pi)^n \delta_D(\mathbf{k} + \mathbf{k}') P_{ij}(|\mathbf{k}|) . \quad (\text{CFS.2})$$

The Dirac delta ‘function’ is a consequence of the statistical homogeneity, the dependence of P only on $|\mathbf{k}|$ is due to statistical isotropy. For $i = j$, P is the *auto-power spectrum* (or simply power spectrum), whereas for $i \neq j$ it is called *cross-power spectrum*. Correlation function and power spectrum are Fourier transform pairs, i.e.,

$$P_{ij}(|\mathbf{k}|) = \int_{\mathbb{R}^n} d^n x \xi_{ij}(|\mathbf{x}|) \exp(i\mathbf{x} \cdot \mathbf{k}) . \quad (\text{CFS.3})$$

Higher-order correlation functions and spectra are defined analogously. The general m -point correlation function $\xi^{(m)}$ of homogeneous and isotropic real fields $f_i(\mathbf{x})$ with zero mean is defined through

$$\langle f_i(\mathbf{x}_1) f_j(\mathbf{x}_2) \dots f_p(\mathbf{x}_m) \rangle =: \xi_{ij\dots p}^{(m)} . \quad (\text{CFS.4})$$

Due to statistical homogeneity, $\xi^{(m)}$ depends only on the separation vectors $\mathbf{x}_i - \mathbf{x}_1$, $2 \leq i \leq m$. Furthermore, statistical isotropy implies that $\xi^{(m)}$ is invariant under rotations of these separation vectors in \mathbb{R}^n . Note that $\xi^{(2)} \equiv \xi$.

The m -th order polyspectra $P^{(m)}$ are defined as the correlators of the Fourier transforms of the fields,

$$\langle \tilde{f}_i(\mathbf{k}_1) \tilde{f}_j(\mathbf{k}_2) \dots \tilde{f}_p(\mathbf{k}_m) \rangle =: (2\pi)^n \delta_D\left(\sum_{i=1}^m \mathbf{k}_i\right) P_{ij\dots p}^{(m)} , \quad (\text{CFS.5})$$

where again the Dirac delta ‘function’ appears due to homogeneity, and the polyspectrum $P^{(m)}$ is invariant under spatial rotations of the \mathbf{k}_j .

COSEBI: Acronym for Complete Orthogonal Sets of E-/B-mode integrals that were introduced and investigated in [Schneider et al. \(2010\)](#), [Asgari et al. \(2012\)](#) and [Asgari et al. \(2016\)](#). For a measurement of the shear correlation functions $\xi_{\pm}(\theta)$ over the range $\theta_{\min} \leq \theta \leq \theta_{\max}$, one defines

$$E_n = \frac{1}{2} \int_{\theta_{\min}}^{\theta_{\max}} d\theta \theta [T_{+n}(\theta) \xi_{+}(\theta) + T_{-n}(\theta) \xi_{-}(\theta)] ; \quad B_n = \frac{1}{2} \int_{\theta_{\min}}^{\theta_{\max}} d\theta \theta [T_{+n}(\theta) \xi_{+}(\theta) - T_{-n}(\theta) \xi_{-}(\theta)] . \quad (\text{COS.1})$$

⁸ This is not a strong restriction, as one can consider the modified field $f'(\mathbf{x}) = f(\mathbf{x}) - \langle f(\mathbf{x}) \rangle$, whose expectation value vanishes at each point by construction.

The functions $T_{-n}(\theta)$ are related to the $T_{+n}(\theta)$ through

$$T_{-n}(\vartheta) = T_{+n}(\vartheta) + \int_{\theta_{\min}}^{\vartheta} d\theta \theta T_{+n}(\theta) \left(\frac{4}{\vartheta^2} - \frac{12\theta^2}{\vartheta^4} \right). \quad (\text{COS.2})$$

Provided that the $T_{+n}(\theta)$ satisfy the conditions

$$\int_{\theta_{\min}}^{\theta_{\max}} d\theta \theta T_{+n}(\theta) = 0 = \int_{\theta_{\min}}^{\theta_{\max}} d\theta \theta^3 T_{+n}(\theta), \quad (\text{COS.3})$$

E_n is sensitive only to E-modes, whereas B_n is sensitive only to B-modes of the shear. One can now construct a complete set of functions T_{+n} on the interval $\theta_{\min} \leq \theta \leq \theta_{\max}$ which satisfy the conditions (COS.3); this then yields a set of COSEBIs E_n, B_n which contain all E-/B-mode information of the correlation functions on that interval (see Schneider et al. 2010).

Cosmetic Maps: Cosmetic maps are maps of the imperfections in the detectors. These may be “dead pixels” with no sensitivity, or pixels which always have a large charge in them whatever the incident flux (known as “hot pixels”). They may record the presence of dust particles. In some detectors, such as CCDs, the defect can extend to columns of pixels. The maps are used to flag these pixels for the subsequent data processing, at which time their information content is discarded. SEE **FLAT-FIELD (FF)**.

Cosmic Rays: Cosmic Rays (CRs) are high-energy particles (mainly protons and atomic nuclei) originating from outside the Solar System, but of unknown origin. They can saturate pixels in CCDs and even damage the lattice structure of the CCDs.

Cosmic Shear: Cosmic shear is the gravitational lensing effect caused by the large-scale structure along and around the line of sight to distant objects. This large-scale structure can induce both a shear and a magnification in the images of distant sources.

Cosmology-sensitive: A result that is dependent on the cosmological model, or values of cosmological parameters, that have been assumed in deriving the results. If results are solely data-driven without any assumptions made about the Universe we live in, the results are cosmology-insensitive.

Covariance: Covariance is a statistical measure that indicates the extent to which two or more variables co-vary with one another on average.

In the community, *correlation* is often used as a synonym for *covariance*. SEE **CORRELATION FUNCTION**.

Critical Curve: Critical curves, for any general lens mass distribution are closed curves in the lens plane where the determinant of the distortion matrix for the lens is singular, i.e within which there is no unique single solution of the lens equation, resulting in the possibility of multiply-imaged background sources within the critical curve. Caustics are the corresponding curves in the image plane. The Einstein radius is a particular example of a critical curve for a circularly symmetric lens. SEE **CRITICAL SURFACE DENSITY**; **DISTORTION MATRIX**.

Critical Surface Density: SEE **SURFACE DENSITY**.

Cross-component Shear: Cross-component shear (or ellipticity) is the amplitude of the shear which at 45° to the tangential shear (which is that which is perpendicularly aligned with a lensing source) SEE **TANGENTIAL SHEAR**.

Cross-Correlation Function: In weak lensing studies a cross-correlation *between* two quantities A and B refers to the finding the inter-variable variance or covariance of the quantities i.e $\langle AB^* \rangle$ where angle brackets refer to an ensemble average, and $*$ is a complex conjugate. Particular and common examples are the cross-correlation between variables relating to the PSF and inferred shear measurements from galaxies, and the shear cross-correlation function, or power spectrum, between two redshift bins (known as tomography). Correlation can refer to both the intra-variable correlation, or ‘auto’ correlation, whereas cross-correlation refers to inter-variable correlation, SEE **CORRELATION FUNCTIONS AND POWER SPECTRA**; **AUTO-CORRELATION FUNCTION**; **CORRELATION FUNCTION**; **COVARIANCE**

CTE: Charge Transfer Efficiency (CTE) measures the fraction of the charge in a given pixel in a CCD that is transferred to the next row during the readout. The CTE is always less than unity because some of the charge is caught in ‘traps’ in the pixel and is left behind during transfer of that pixel to the readout amplifier. The fraction of charge lost depends on the signal levels of both the source and the background. Since in a CCD, charge must be transferred many times before reaching the readout register, even a small decrease in CTE can have a large effect on the measured count rate near the centre of the detector. For n transfers, the fraction of detected charge will be CTE^n . Charge Transfer Inefficiency (CTI) measures the loss; i.e. $\text{CTI} = 1 - \text{CTE}$. Most of the charge lost during a transfer will reappear during a later transfer, which causes ‘charge tails’ away from the sources towards the read-out register. These trails impact photometry, noise, shape and astrometry of sources. CTI trails result in the following problems;

1. They remove flux from the central pixel, hence degrading the expected S/N for an observation;
2. They bias measurements of sources along the trail, impacting astrometry;
3. CTI from warm pixels and cosmic rays introduce noise in observations.

For a space-based telescope, CTE degrades over time, as the bombardment by energetic particles generate more defects in the lattice structure of the CCDs. The impact of CTE, or CTI, on weak lensing has been investigated in Rhodes et al. (2007, 2010); Massey et al. (2010); Massey (2010); Massey et al. (2014); Israel et al. (2015b).

CTI: SEE CTE.

Current (survey): In weak lensing literature “current” is used to refer to an experiment that is in operations at the time of the writing of paper, and is used as a shorthand for circa (or “around about”). For example “ABC is a current survey...” in a paper in 2015 is shorthand for “The ABC survey (circa 2015)...”.

Curved Sky: In opposition to flat sky (SEE FLAT SKY). If an operation, or statistic, can work in the ‘curved sky’ regime this refers to the operation in question being able to correctly work in the spherical coordinate system relevant for the celestial sphere. An example of a curved-sky approach would be to perform a spherical harmonic transform of function on the sphere which involves spherical harmonic functions $Y_\ell^m(\theta, \phi)$ (where θ, ϕ are co-latitude and co-longitude and ℓ and m are the spherical harmonic transforms of these variables); in the flat-sky case the equivalent transform would be a Fourier transform on a projected flat (tangent) plane.

Deflection Angle: The angle between an incoming and outgoing ray, and deflection by lens, for a geometrically-thin lens is called the deflection angle, which is defined as

$$\hat{\alpha} = -\frac{2}{c^2} \int \nabla_\perp \Phi dr, \quad (\text{DA.1})$$

i.e. gradient of the potential Φ is taken perpendicular to the light path. The deflection angle is twice the classical prediction in Newtonian dynamics if photons were massive particles with velocity c SEE SURFACE DENSITY; WEAK LENSING EQUATIONS.

DEIMOS: DEIMOS Melchior et al. (2011) is a moment-based method for weak lensing shear estimation from galaxies, that does not make some of the assumptions or approximations that are made in applying the KSB algorithm Kaiser et al. (1995). DEIMOS directly estimates the lensed moments from the measured moments, which are affected by PSF convolution and the application of a weighting function. One improvement over KSB is that an exact deconvolution from the PSF is performed that requires only the knowledge of PSF moments of the same order as the galaxy moments to be corrected.

Density Parameters: Density parameters in cosmology are the ratio of the density of component i to the critical density ρ_c , i.e. the density at which the Universe expands asymptotically; $\Omega_i = \rho/\rho_c$. This could be cold dark matter Ω_c , baryonic matter Ω_b , cosmological constant Ω_Λ and etc. SEE FRIEDMAN-ROBERTSON-WALKER MODELS.

Density Profile: A density profile describes how the density of the distribution of matter changes as a function of radial separation from the centre of mass of the matter distribution. There exists three commonly used models for dark matter density profiles that are used in weak lensing. All assume a spherically symmetric mass distribution with the follow functional forms for the density of dark matter as a function of radius:

1. Pseudo-isothermal halo (Gunn and Gott 1972) is defined as

$$\rho(r) = \rho_o \left[1 + \left(\frac{r}{r_c} \right)^2 \right]^{-1}, \quad (\text{DP.1})$$

where ρ_o is the central density and r_c is the core radius. This model is a good fit to most rotation curve data, but is only an approximation.

2. NFW (Navarro-Frenk-White) profile (Navarro et al. 1996) is defined as

$$\rho(r) = \frac{\rho_c \delta_c}{(1/r_s)(1 + r/r_s)^2}, \quad (\text{DP.2})$$

where r_s is a scale radius, δ_c is a characteristic density, and ρ_c is the critical over density defined as $\rho_c = 3H^2/8\pi G$. The NFW profile works for a large range of halo masses and sizes, from individual galaxies to the halos of galaxy clusters.

3. Einasto profile (investigated in [Merritt et al. \(2006\)](#)) is defined as

$$\rho(r) = \rho_e \exp \left[-d_n \left((r/r_e)^{1/n} - 1 \right) \right] , \quad (\text{DP.3})$$

where r is the spatial (i.e., not projected) radius. The term d_n is a function of n such that ρ_e is the density at the radius r_e that defines a volume containing half of the total mass. High resolution computer simulations lead to this model.

For a review on dark matter halo models see e.g. [Taylor \(2011\)](#).

DES: The Dark Energy Survey (DES⁹) will image 5000 square degrees of the southern sky in 5 optical filters using the DECam instrument on the CTIO. DES started taking data in August of 2013 and will continue for five years to record information from 300 million galaxies for a redshift range of $0.2 < z < 1.3$.

DETF: The Dark Energy Task Force (DETF) was a subcommittee established in February 2005 by the Astronomy and Astrophysics Advisory Committee (AAAC) and the High Energy Physics Advisory Panel (HEPAP) in USA to advise NSF (National Science Federation), NASA and DOE (Department of Energy) on the future of dark energy research. The DETF had a remit to help the agencies to identify actions for a dark energy program and understanding the nature of dark energy. The report of DETF can be found in [Albrecht et al. \(2006\)](#), and a second report from the same team was performed in 2009 ([Albrecht et al. 2009](#)). SEE [STAGE \(DARK ENERGY\)](#).

Distortion Matrix: The distortion matrix \mathcal{A}_{ij} (also known as the linear distortion matrix) is the Jacobian of the lens equation (SEE [LENS EQUATION](#)). If the source is much smaller than the angular size upon which the properties of the lens change (SEE [LIGHT BUNDLE](#)) then the local distortion of its image can be described by a linear mapping between the source and image planes. The Jacobian matrix of a lens is defined as

$$\mathcal{A}_{ij} = \frac{\partial \beta_i}{\partial \theta_j} = \frac{\partial}{\partial \theta_j} (\theta_i - \alpha_i) , \quad (\text{DM.1})$$

or related to the Hessian matrix ψ_{ij} of the deflection potential ψ using the relation between the deflection angle and the deflection potential (SEE [DEFLECTION ANGLE](#))

$$A_{ij} = \delta_{ij} - \frac{\partial \alpha_i}{\partial \theta_j} = \delta_{ij} - \frac{\partial^2 \psi}{\partial \theta_i \partial \theta_j} = [M^{-1}]_{ij} , \quad (\text{DM.2})$$

where M_{ij} is the magnification matrix, and the Hessian matrix $\psi_{ij} = \partial^2 \psi / \partial \theta_i \partial \theta_j$ represents the deviation of the lens mapping from the identity mapping (that which describes no distortion). SEE [WEAK LENSING EQUATIONS](#).

Dither: A technique that consists of slightly moving the telescope between ‘exposures’, i.e. within each slew. One purpose of ‘dithering’ is to reduce some types of CCD sensor noise such as pattern noise. A relatively standard method is to randomly move the telescope, known as random dithering. Dithering causes the resultant image to land on a different sets of pixels from one dither to the next.

The advantage of dithering is chip artefacts can be minimised when the different ‘sub’ exposures are combined. The best signal-to-noise ratio improvement that can be achieved, without data smoothing, is the mean combine. For n frames, a mean combine, improves signal-to-noise by \sqrt{n} . However a mean combine does nothing to eliminate artefacts, except by reducing their visibility by $1/n$. A median combine improves signal-to-noise by $\sqrt{2n/3}$, or about 20% less than the mean but does reduce some artefacts. Another combine method is MMClip¹⁰ which improves signal-to-noise by $\sqrt{n-2}$.

Down-sizing: First suggested by [Cowie et al. \(1996\)](#) to refer to a scenario of galaxy formation where massive galaxies were formed first in the history of the Universe, and completed their star formation process more rapidly than the low-mass galaxies. This scenario is in contrast with other scenarios in which a simple hierarchical structure formation is assumed: where large galaxies are formed from the merging of smaller galaxies and hence are formed later in the history of the Universe.

E-mode: SEE [EB-MODE DECOMPOSITION](#).

E/B-mode Shear: A general spin-2 field, such as shear $\gamma(\boldsymbol{\theta})$, can be decomposed into E-modes, B-modes, and ambiguous modes. If the shear field is caused by a (geometrically-thin) gravitational lens, its two Cartesian components are given as second-order partial derivatives of the deflection potential ψ – see Eq. [\(LE.10\)](#); hence, these two components are mutually related. Introducing the complex nabla operator $\nabla_c := \partial/\partial\theta_1 + i\partial/\partial\theta_2$, the shear produced by a lens can be written as $\gamma = \nabla_c \nabla_c \psi / 2$, and the Poisson equation [\(LE.6\)](#) becomes $\kappa = \nabla_c^* \nabla_c \psi / 2$. The local relation [\(LE.18\)](#) between the gradient of κ and the derivatives of the shear reads $\nabla_c \kappa = \nabla_c^* \gamma$. Taking another derivative, we find $\nabla_c^* \nabla_c \kappa = \nabla_c^* \nabla_c^* \gamma$. Since the Laplacian $\nabla_c^* \nabla_c$ and κ are both real, the imaginary

⁹ <http://www.darkenergysurvey.org>

¹⁰ <http://www.hiddenloft.com/notes/dithering.htm>

component of $\nabla_c^* \nabla_c^* \gamma$ vanishes for a shear field due to a gravitational lens.

A general shear field $\gamma(\theta)$ does not have this property. If a shear field is caused by a lens, meaning that there exists a real scalar function ψ^E such that $\gamma = \nabla_c \nabla_c \psi^E / 2$, it is called an ‘E-mode’ shear field. In particular, for an E-mode field, the imaginary part of $\nabla_c^* \nabla_c^* \gamma$ vanishes identically. A pure B-mode shear field is obtained from a real scalar function ψ^B through $\gamma = i \nabla_c \nabla_c \psi^B / 2$, for which the real component of $\nabla_c^* \nabla_c^* \gamma$ vanishes identically. A general shear field is the superposition of both, and can be written as $\gamma = \nabla_c \nabla_c (\psi^E + i\psi^B) / 2$. There also exist ambiguous modes: For a shear field which is a linear function of position, $\nabla_c^* \nabla_c^* \gamma \equiv 0$; hence, such a linear shear field can be either attributed to E-modes or to B-modes. If a shear field has a B-mode contribution, the result of the mass reconstruction relation (LE.17) yields an imaginary component for κ .

A division of the shear into E-, B-, and ambiguous modes also must be considered for second- and higher-order shear statistics (SEE COSMIC SHEAR).

Early and Late Type (Galaxies): According to the Hubble sequence, galaxies can be divided into two main populations. These are referred to in weak lensing for several reasons, some of these are: their intrinsic alignment properties may be different which is important for cosmic shear studies, their dark matter environments may be different which can be inferred using galaxy-galaxy lensing studies.

1. Early-type galaxies are elliptical galaxies which have approximately spheroidal systems. They usually have red colour in optical wavelengths, and are hypothesised to have pressure-supported stellar systems.
2. Late-type galaxies are spiral galaxies which have disc systems. They usually have blue colours optical wavelengths and are hypothesised to have rotation-supported stellar systems. Irregular galaxies also fall into this category.

The origin of the terms “early-type” and “late-type” lies with the historical interpretation of the Hubble tuning-fork diagram, which assumed galaxies evolved *from* ellipticals (early-type) galaxies *to* spiral (late-type) galaxies. The names have no particular basis in the modern picture of formation or evolution of galaxies.

Early Type (Galaxies): SEE EARLY AND LATE TYPE (GALAXIES).

EB-mode Decomposition: Any spin-2 field (such as gravitational shear) can be decomposed into curl-free modes (called E-modes) and divergence-free modes (called B-modes). However such a decomposition can be non-unique because there are also ambiguous modes – for example, a constant shear is neither E- nor B-mode. A pure E-mode shear field displays radial or tangential alignment around matter over- or under-densities. Whereas a pure B-mode shear field features curl patterns, where orientations are at 45 degrees relative to the line between the lensed image and the matter over- or under-densities. Since cosmological signals only generate E-modes to good approximation, the analysis of B-modes is considered a valuable test for systematic effects, such as intrinsic galaxy alignments or errors in the modelling of PSFs. Note that some ambiguous modes exist and are usually discarded after an EB-mode decomposition (see e.g. Leistedt et al. (2016)). For quantitative details on the EB-mode decomposition Schneider et al. (2002) presented a flat-sky approach and Castro et al. (2005) for a curved-sky approach.

Einstein Radius: The angular radius of an Einstein ring. For a point mass lens, this is given by

$$\theta_E = \sqrt{\frac{4G_N M}{c^2} \frac{D_{ls}}{D_l D_s}}, \quad (\text{ER.1})$$

where M is the mass of the lens, D_l , D_s and D_{ls} are the observer-to-lens, observer-to-source and lens-to-source angular diameter distances respectively, and G_N and c are Newton’s gravitational constant and the speed of light in a vacuum.

Einstein Ring: A ring-like image of a gravitationally lensed background galaxy that is produced when the background galaxy lies along the same line of sight to the observer as the lens. In order to observe an Einstein ring, the mass distribution of the lens needs to be axially symmetric, as seen from the observer, and the source must lie on top of the resulting (point-like) caustic. However, note that none of the lenses that produce observed Einstein rings are axi-symmetric, nor are the sources *exactly* aligned; such an explanation only applies to infinitesimally small sources. Such images can be used to determine cosmological parameters through measurement of the angular diameter distances, if one has an independent determination of the mass (and note that in general angular diameter distances are known to better precision than galaxy masses). Several such objects have been studied to date e.g. Langston et al. (1989), Courbin et al. (2002), Gavazzi et al. (2008).

Ellipticity: Ellipticity defines the 2D shape of an object, hence it is a spin-2 quantity which is usually defined as a complex quantity. The symbol e is generically used, which could be ellipticity, polarisation or shear. In a two-dimensional (flat-sky) Cartesian coordinate system the real (imaginary) part of e can be identified with the

first (second) coordinate in that system, usually written as $e = e_1 + ie_2$; where e_1 is called *Ellipticity-1* and e_2 is called *Ellipticity-2*.

The ellipticity of an object in an image can be defined in several ways, which can be based on

1. Quadrupole moments Q_{ij} (SEE QUADRUPOLE MOMENTS);

- (a) Third Eccentricity

$$\chi \equiv \chi_1 + i\chi_2 \equiv \frac{Q_{11} - Q_{22} + 2iQ_{12}}{Q_{11} + Q_{22}}. \quad (\text{EL.1})$$

- (b) Third Flattening

$$\epsilon \equiv \epsilon_1 + i\epsilon_2 \equiv \frac{Q_{11} - Q_{22} + 2iQ_{12}}{Q_{11} + Q_{22} + 2(Q_{11}Q_{22} - Q_{12}^2)^{1/2}}. \quad (\text{EL.2})$$

2. Semi-major a and semi-minor b axes of the ellipse;

- (a) Third Eccentricity

$$|\chi| \equiv \frac{a^2 - b^2}{a^2 + b^2}. \quad (\text{EL.3})$$

- (b) Third Flattening

$$|\epsilon| \equiv \frac{a - b}{a + b}. \quad (\text{EL.4})$$

This definition of the third flattening is sometimes referred to as ϵ -ellipticity.

Please refer to [Viola et al. \(2014\)](#) for more discussions.

Ellipticity (observed): There are different ways to measure the ellipticity of a galaxy/star in an image. Based on 2nd order moments Q_{ij} (SEE QUADRUPOLE MOMENTS) we can define

1. third eccentricity

$$\chi \equiv \frac{Q_{11} - Q_{22} + 2iQ_{12}}{Q_{11} + Q_{22}}, \quad (\text{EO.1})$$

SEE **THIRD ECCENTRICITY** for this definition in terms of semi-major and semi-minor axes.

2. third flattening

$$\epsilon \equiv \frac{Q_{11} - Q_{22} + 2iQ_{12}}{Q_{11} + Q_{22} + 2(Q_{11}Q_{22} - Q_{12}^2)^{1/2}}. \quad (\text{EO.2})$$

SEE **THIRD FLATTENING** for this definition in terms of semi-major and semi-minor axes.

Both of these are measurements of ellipticity. In the case of weak lensing, when we compute the average ellipticity over an ensemble of galaxies whose orientations are assumed to be random, we find that:

$$\langle \epsilon \rangle \simeq \frac{\langle \chi \rangle}{2} \simeq g \simeq \gamma, \quad (\text{EO.3})$$

where $g = \gamma/(1 - \kappa)$ is the reduced shear. Therefore, either definition of ellipticity, under the assumptions outlined above, is taken to be an unbiased noisy estimate of the gravitational shear at the location of the galaxy. In practice, the observed, measured ellipticity will be convolved with a telescope (and possibly atmospheric) point spread function (PSF) which convolves the image and has the effect of smearing the galaxy images. This impacts the measured ellipticity:

$$\chi^o = \frac{\chi + T\chi^{\text{PSF}}}{1 + T}, \quad (\text{EO.4})$$

where χ is the lensed galaxy ellipticity prior to convolution with the PSF, χ^{PSF} is the ellipticity of the point spread function, χ^o is the measured, observed ellipticity of the galaxy after convolution with the PSF and

$$T = \frac{P_{11} + P_{22}}{Q_{11} + Q_{22}}. \quad (\text{EO.5})$$

In the above equation, P_{ij} refers to the 2nd order moments of the PSF and Q_{ij} refers to the 2nd order moments of the galaxy's brightness distribution, as before.

Ellipticity 1 and Ellipticity 2: SEE **ELLIPTICITY**.

Environment (-al dependence): This refers to the dependency of measured galaxy, or inferred dark matter properties, on the properties that can be used to characterise a particular spatial location. For example local gravitational potential, matter density or morphology. This is particularly used in galaxy-galaxy lensing and galaxy clustering studies to characterise the dependency of galaxy properties on local dark matter properties e.g. [Gillis et al. \(2013\)](#) and [Brouwer et al. \(2016\)](#).

Epsilon-ellipticity: SEE [ELLIPTICITY](#).

eth Differential Operator: This is a differential operator acting (sometimes written as *edth*) on the surface of a sphere, relating quantities of different spin. It is usually denoted by the symbol \eth (`\eth` in latex). Combinations of \eth form raising and lowering operators for spin spherical harmonics. For more information, please refer to [Castro et al. \(2005\)](#) and references therein.

Euclid: Euclid¹¹ [Laureijs et al. \(2011\)](#) is an M2-class (medium-class) ESA mission, which is part of ESA’s ‘Cosmic Vision’ (2015–2025) scientific program. Euclid was chosen in October 2011 by ESA as one of the leading probes of cosmology. Euclid will map the geometry of the dark Universe and aims to shed light on dark energy and dark matter by accurately measuring the acceleration of the universe. The mission will investigate the distance-redshift relationship and the evolution of cosmic structures by measuring shapes and redshifts of galaxies and clusters of galaxies out to redshifts $z \simeq 2$ — there are two main surveys in Euclid; weak lensing and galaxy clustering. Euclid aims to observe about 1.5 billion galaxy images. It is scheduled to be launched in 2020. Euclid is named after 3rd century BC Greek mathematician Euclid of Alexandria, the ‘Father of Geometry’.

Exposure: A period of observation on specific part of the sky using a telescope. One exposure in Euclid refers to observation in one dither position. SEE [DITHER](#).

Fermat’s principle: This principle states that a light ray must traverse a path between two points which takes the least time. In gravitational lensing, this is used in field equations describing the light deflection and was described in [Perlick \(1990\)](#) and [Giammoni et al. \(1999\)](#).

Fiducial: The word fiducial indicates that an entity or a set of parameters is being used as a basis/standard of reference.

Fiducial Cosmology: A well established standard of cosmology, established by an ensemble of previous observations/surveys, which is usually specified by a set of cosmological parameters with set values (and their confidence intervals). In the context of cosmological simulations, this may imply the input cosmology (e.g. a six-parameter Λ CDM), that was used to simulate the cosmological data.

Field: Field has several definitions that are dependent on context in which it is used:

1. In physics in general, a field is a mathematical construct for analysis of remote effects, such as the effect of the gravitational force of an object on its surrounding objects.
2. It could also refer to a spatially distributed set with certain properties. For example, a scalar field has scalar value to every point in a space and a vector field has a vector assigned to every point in a space.
3. In astronomical imaging, a field refers to a region of the sky where data is being collected, SEE [FoV](#).
4. ‘The field’ in cosmology is used to refer to areas of the sky that do not contain a large overdensity of galaxies, such as a galaxy cluster, SEE [FIELD STAR/GALAXY](#).
5. Field is also used to refer to a subject, for example ‘weak lensing is a field of cosmology’.

The usage of this words is usually obvious depending on the context.

Field of View: Field of View (FoV) is a solid angle through which a detector is sensitive to electromagnetic radiation.

Field Star/Galaxy: Loosely defined expression for a star/galaxy that is either in the field of view of star/galaxy cluster, but it is not a member of that cluster as it is radially at a different (larger) distance with respect to the cluster; or is not part of a star cluster or a galaxy group of cluster member, see e.g. [Brainerd \(2004\)](#).

Figure of Merit: A figure of merit (FoM) is a quantity that is used to characterize the performance of a survey relative to its alternatives. FoMs should allow for a determination of the ability of any given cosmological probe or individual experiment to measure the properties of interest e.g. parameters for the dark energy equation of state. It should, ideally, be represented by one number or metric. In weak lensing studies and cosmology, FoMs have been defined as some function of the Fisher matrix of the parameters of interest. Some example are:

- A-optimality = $\log(\text{trace}(\mathbf{F}))$
trace of the Fisher matrix (or its log) and is proportional to sum of the variances. This prefers a spherical error region, but may not necessarily select the smallest volume.

¹¹ <http://euclid-ec.org>

- D-optimality = $\log(|\mathbf{F}|)$
determinant of the Fisher matrix (or its log), which measures the inverse of the square of the parameter volume enclosed by the posterior. This is a good indicator of the overall size of the error over all parameter space, but is not sensitive to any degeneracies amongst the parameters.
- Entropy (also called the Kullback-Leibler divergence)

$$E = \int d\theta P(\theta|\hat{\theta}, e, o) \log \frac{P(\theta|\hat{\theta}, e, o)}{P(\theta|o)} \\ = \frac{1}{2} [\log |\mathbf{F}| - \log |\mathbf{\Pi}| - \text{trace}(\mathbf{I} - \mathbf{\Pi}\mathbf{F}^{-1})] , \quad (\text{FOM.1})$$

where $P(\theta|\hat{\theta}, e, o)$ is the posterior distribution with Fisher matrix \mathbf{F} and $P(\theta|o)$ is the prior distribution with Fisher matrix $\mathbf{\Pi}$.

For dark energy surveys there was a particular recommendation from the Dark Energy Task Force (Albrecht et al. 2006), SEE STAGE (DARK ENERGY).

Filter Function: A function or procedure which removes unwanted parts of a signal or transforms the signal into another form. For example, one can take Fourier transform of a signal into frequency space, perform the filtering operation there (i.e. down-weighting or removing particular frequencies), then transform back into the original space.

Fisher Matrix: The Fisher (information) matrix is a way of measuring the amount of information that an observable, or random variable, carries about an unknown parameter upon which the probability of the variable depends. In cosmology and weak lensing, the Fisher matrix is generally used to determine the sensitivity of a particular survey to a set of parameters and has been used largely for forecasting and optimisation. It is defined as the ensemble average of the *curvature* of the likelihood function

$$F_{\alpha\beta} = \left\langle -\frac{\partial^2 \ln \mathcal{L}}{\partial \theta_\alpha \partial \theta_\beta} \right\rangle , \quad (\text{FM.1})$$

where \mathcal{L} is the likelihood function and θ_α and θ_β are cosmological parameters. Its inverse is an approximation of the covariance matrix of the parameters, by analogy with a Gaussian distribution in the θ_α , for which is exact — this was investigated for CMB in (Tegmark et al. 1997). Therefore, using the assumption of Gaussianity in the data, it allows us to estimate the errors on parameters without having to cover the whole parameter space. The authors of Bond et al. (1998) have compared the Fisher matrix analysis with the full likelihood function analysis and found there was great agreement between the two methods if the likelihood function is approximately Gaussian near the peak.

Flat-Field (FF): A flat field (FF) is the response of a telescope to a uniform light across its area. Hence a flat field frame is an image of a field that has a uniform illumination. Flat field correction (flat fielding) is then applied to astronomical images to correct for artefacts in the optical path of the telescopes; these include inter-pixel responsivity in the CCD, vignetting etc. Flat fielding is particularly needed when photometry is required. See e.g. Erben et al. (2013) for an application of flat-fielding in a data analysis chain used for weak lensing.

Flat-Sky: Considered when the area of sky under consideration is small enough that geometric effects caused by observing on the celestial sphere are negligible and can be ignored. The limit and area of the sky that can be considered flat may change depending on the function that is being calculated.

Flexion: Flexion is used to refer to effects caused by second-order derivatives of the deflection angle, or third-order derivatives of the lens potential in the lens equation. While shear gives rise to elliptical image distortions, the combined effect of shear and flexion gives rise to an arc-like (also referred to as arclet, or ‘bannana’-like) image distortions Goldberg and Natarajan (2002); Goldberg and Bacon (2005). Flexion statistics are in principle sensitive to local variations of the potential field and hence can in principle probe small and non-linear scales. In application there are several systematics that need to be overcome Bacon et al. (2006), for example Poisson noise may be higher for flexion, and flexion may be more sensitive to photon pixel noise than shear Rowe et al. (2013).

Focal Plane: For a lens, or a spherical/parabolic mirror, the focal plane is the plane of points onto which light, parallel to the axis, is focused. The distance between the lens or mirror and the focus is called the focal length. In telescopes cameras are placed at the focal plane to collect data from the telescope.

FoM: Acronym for Figure of Merit; SEE FIGURE OF MERIT.

Footprint (of survey): The specific area of the sky that is observed by a survey. See e.g. <http://lambda.gsfc.nasa.gov/toolbox/footprint/aladin/aladinLAMBDA.cfm> to see the footprint of some of the cosmological surveys such as DECam, HSC and BOSS.

Foreground: Part of a view that is radially closer to the observer relative to the desired source. Normally ‘foregrounds’ are used to refer to unwanted non-cosmological signals, and need to be removed from the data, in particular in CMB analysis see e.g. [Dickinson \(2016\)](#). In weak lensing the most dominant foreground contaminant is zodiacal dust at low ecliptic latitudes. Intrinsic alignments are occasionally referred to as a ‘foreground’ contaminant. Foreground estimation and characterisation and removal is an essential step in the data analysis pipeline.

Fourier Space (in reference to cosmology): This refers to the Fourier, spherical harmonic, or spherical Bessel transforms of a field in cosmology, for example the matter overdensity, interpolated galaxy overdensity, ellipticity or CMB temperature or polarisation fields. In the angular direction the Fourier wavenumber is denoted ℓ (which is a dimensionless quantity) and in the radial (comoving distance) direction as k (with units of $h\text{Mpc}^{-1}$). The article may also refer to ℓ -space, or k -space to refer to the coordinate systems after such a transform. SEE **FOURIER TRANSFORM**.

Fourier Space (in reference to PSF): In reference to a Point-Spread-Function estimation, Fourier space refers to the 2D Fourier transform of the optical wavefront, see e.g. [Jee and Tyson \(2011\)](#). SEE **FOURIER TRANSFORM**.

Fourier Transform: Let $f(\mathbf{x})$ be a function on \mathbb{R}^n . We define its Fourier transform as

$$\tilde{f}(\mathbf{k}) := \int_{\mathbb{R}^n} d^n x f(\mathbf{x}) \exp(i \mathbf{x} \cdot \mathbf{k}) . \quad (\text{FT.1})$$

The inverse Fourier transform then reads

$$f(\mathbf{x}) = \frac{1}{(2\pi)^n} \int_{\mathbb{R}^n} d^n k \tilde{f}(\mathbf{k}) \exp(-i \mathbf{x} \cdot \mathbf{k}) . \quad (\text{FT.2})$$

The Fourier transform of a function yields a decomposition of this function into plane waves.

FoV: Acronym for Field of View; SEE **FIELD OF VIEW**.

Friends of Friends: Friends of Friends (FoF) is one of the many halo finder techniques used in cosmological simulations and for finding clusters in data. It uses particle positions to group spatially close particles – $\Delta x < b$, where b is called the linking parameter – to detect halos, sub-haloes in the simulation. See e.g. [Marian et al. \(2010\)](#); [Pastor Mira et al. \(2011\)](#); [More et al. \(2015\)](#).

Fsky: f_{sky} is the fraction of the sky observed by a survey. For example, Euclid will observe 15000 square degrees, so that it has $f_{\text{sky}} \simeq 35\%$. In practice the fraction of usable data less than this value as there will be areas any observed region that need to be masked due to saturated pixels for example.).

Future (Survey): This phrase is used to refer to a (weak lensing) survey that has not yet started observations. In literature from c. 2000 to 2005 such a phrase refers to Stage II like surveys such as CFHTLS; up until c. 2010 in reference to DES, HSC, and KiDS; and up until the present time (c. 2016) to LSST, Euclid, SKA etc.

FWHM: Full width at half maximum (FWHM) is the distance between points on a curve where the function has half its maximum value. This parameter is commonly used to describe the width of the curve or width of a function.

Gaia: Gaia¹² is an ESA mission and the successor to the Hipparcos mission. It is an all-sky astrometric survey telescope with the aim of constructing a 3D map of one billion astronomical objects in the Milky Way (including stars, planets, comets, asteroids, and etc.), providing information about their motion, luminosity, effective temperature, gravity and elemental composition. This will provide information about the origin, structure and evolution of the Milky Way. Gaia was launched in December 2013 and currently operates around the L2 Lagrangian point. The expected completion date of 2021.

Galaxy: A gravitationally bound system of stars, interstellar gas and dust which is hypothesised to live in a dark matter halo. Galaxies can have from a few thousand stars to one hundred trillion stars. Galaxies have historically been categorised according to their morphology and include elliptical, spiral, and irregular. Many galaxies are hypothesised to have blackholes at their centres.

Galaxy Size: Galaxy size can refer to several different properties of a galaxy surface brightness distribution, all of which have some relation to the projected angular extent of a galaxy as observed. Commonly used definitions include

- The sum of the Cartesian axis quadrupole moments $R^2 = Q_{11} + Q_{22}$ SEE **QUADRUPOLE MOMENTS; R-SQUARED**, see e.g. [Massey et al. \(2013\)](#).
- The FWHM (full width half maximum) of the surface brightness distribution,

¹² <http://sci.esa.int/gaia/>

- A measure of the size through a model fitting procedure that has a free parameter controlling the radial width of the model, such as an exponential profile with a particular scale length, see e.g. [Miller et al. \(2007\)](#).

These definitions are also used in PSF fitting to refer to PSF, or stellar, size.

Galaxy Useful for Weak Lensing: A galaxy ‘useful for weak lensing’ is one which has been deemed by a particular study, using observational data, to have passed any selection criteria used to remove galaxies that may have systematic effects present in their derived quantities. Selection criteria typically include signal-to-noise, galaxy size, galaxy colours, the quality of a model fit (e.g. a χ^2 measurement), the size of the galaxy relative to the size of the estimated PSF, or similar. There is no universally agreed criteria for this definition, and each study typically justifies its own selection based on the quality of the data and the measurement techniques it uses.

Galaxy-Galaxy Lensing: Galaxy-galaxy lensing (GGL) can be weak or, occasionally, strong gravitational lensing, where the lens is an individual galaxy. GGL, therefore, correlates shapes of source galaxies with positions of the lens galaxies, and hence gives information about the mass of the lens galaxy (or sample of galaxies). Hence it probes galaxy halos on scales of about a few kpc to a few Mpc, providing insights about halo masses and density profiles as function of stellar mass, luminosity, type, or environment [Mandelbaum et al. \(2006\)](#). GGL produces shear correlations of about 1%, which is weaker than cluster lensing, but stronger than the signal due to cosmic shear.

Galaxy-Galaxy-Galaxy lensing: Galaxy-galaxy-galaxy lensing (G3L) refers to third-order correlations between mass and galaxy ellipticities, which can involve two source galaxies and one lens galaxy, or one source galaxy and two lens galaxies [Schneider and Watts \(2005\)](#); [Simon et al. \(2013\)](#). This technique is very useful in understanding the mass and environments of close correlated galaxies.

Gaussian Noise: A statistical noise with a probability density function (PDF) equal to Gaussian distribution, [SEE PROBABILITY DISTRIBUTION](#). White Gaussian noise is an identically and independently distributed noise, meaning it is sampled from a distribution with a constant power and zero correlation.

GG term: [SEE INTRINSIC ALIGNMENT](#).

GI term: [SEE INTRINSIC ALIGNMENT](#).

GREAT: Gravitational lEnsing Accuracy Testing challenges are a series of challenges, with a goal of testing and facilitating the development of methods within the lensing community for analysing astronomical data in a blind way to find the best methods to measure weak gravitational lensing. So far there has been GREAT08, GREAT10 and GREAT3 each with different aims and challenges;

- The GREAT8 Challenge set a highly simplified version of the problem, using known PSFs, simple galaxy models, and a constant applied gravitational shear ([Bridle et al. 2009, 2010](#)).
- The GREAT10 Challenge increased the realism and complexity of its simulations over GREAT08 by using cosmologically-varying shear fields and greater variation in galaxy model parameters and telescope observing conditions. Since imperfect knowledge of the PSF can also bias shear measurements, GREAT10 tested also PSF modelling in a standalone Star Challenge ([Kitching et al. 2010, 2012, 2013](#)).
- The GREAT3¹³ challenge included effects realistically complex galaxy models based on high-resolution imaging from space; spatially varying and physically-motivated blurring kernel; and combination of multiple different exposures ([Mandelbaum et al. 2014, 2015](#)).

GREAT08 and GREAT10 were preceded by a number of internal challenges within the astrophysics community, known as the Shear Testing Programme, or STEP ([SEE STEP](#)).

Group (of Galaxies): This is a gravitationally bound structure normally with fewer than about 50 galaxies. They are the smallest and the most common aggregates of galaxies in the Universe — comprising at least 50% of the galaxies in the local universe. The Milky Way galaxy is a member of the Local Group. Larger collection of galaxies are called galaxy clusters and these clusters can themselves be clustered, into superclusters of galaxies. Also [SEE GROUP-SCALE](#).

Group-Scale: This corresponds to the typical virial mass M or virial radius R of galaxy groups, i.e. $M \sim 10^{13} M_{\odot}$ and $R \sim 1 \text{ Mpc } h^{-1}$. Groups are typically identified as a distinct class of object, between individual galaxies and galaxy clusters, in galaxy-galaxy lensing studies where the dark matter environment and galaxy formation in groups can be studied e.g. [Viola et al. \(2015\)](#). Group-scale also can pertain to the physical distances that encapsulate the Local Group i.e. that refers to the Milky Way and its immediate neighbouring galaxies.

Halo: This is a commonly used term in weak lensing and generally refers to any hypothetical component that surrounds or encapsulates a galaxy, galaxy group or galaxy cluster. Typically it is used in two contexts:

¹³ <http://www.great3challenge.info>

1. A dark matter halo; which is a dark matter structure within which luminous objects such as a galaxy, or galaxies, are embedded. A dark matter halo typically extends beyond the edge of the visible galaxy/galaxies and its mass dominates the total mass of the luminous plus dark matter structure. Dark matter halos may have substructures within them that are referred to as ‘subhalo’ or ‘satellite’ halos. Dark matter halos are one of the ingredients that influence galaxy formation and evolution models by providing a dark matter environment within which galaxy formation occurs SEE **ENVIRONMENT (-AL DEPENDENCE)**.
2. A galactic halo; which is a roughly spherical component of a galaxy which extends beyond the main, visible component. It can refer to old population-II stars (or globular clusters) which have small or no mean rotation around the galactic centre. It can also refer to high temperature gas around the galaxy

Halo Model: A Halo Model is one that describes the of distribution of dark matter around galaxies on large scales. The main assumption of a halo model is that all matter resides in halos i.e. distinct clouds of dark matter. The main ingredients of a halo model are an average halo density profile $\rho(r)$, a halo mass function $n(M, z)$ that describes the number of halos as a function of mass and redshifts, a halo bias $b_h(M, z)$ function that relates the number of galaxies to the number of dark matter haloes as a function of mass and redshift, and how halos cluster together in the form of the two-point correlation functions $\xi_h(r)$ or power spectra as a function of separation; as well as higher-order correlations and correction. The halo model has been applied to describe the distribution of dark matter, e.g. [Smith et al. \(2003\)](#) and [Takahashi et al. \(2012\)](#), and also has been extended to include the distribution of the baryonic component in the Universe and the interaction between this baryonic component and the dark matter e.g. [Fedeli \(2014\)](#), [Semboloni et al. \(2011\)](#) and [Mead et al. \(2015\)](#).

Halo Occupation Distribution: This is a prescription of how galaxies populate dark matter halos. They typically indicate the number of galaxies as function of halo mass, $n(M)$. They also describe how galaxies above a mass/luminosity threshold can be biased with respect to the underlying dark matter [Berlind et al. \(2003\)](#).

Hankel Transform: The Hankel transform, and inverse Hankel transform, of a quantity $a(k)$ in general are defined as

$$\begin{aligned} a(r) &= \int_0^\infty dk \, r \, (rk)^{-q} J_\mu(kr) \hat{a}(k); \\ \hat{a}(k) &= \int_0^\infty dr \, k \, (rk)^q J_\mu(kr) a(r), \end{aligned} \quad (\text{HT.1})$$

respectively, where J are Bessel functions of the first kind, and r is the Fourier transform variable of k . If the bias $q = 0$, the above indicate the unbiased Hankel transform. In weak lensing a Hankel transform relates cosmic shear power spectra $C(\ell)$ to their respective correlation functions

$$\xi_{+/-}(\theta) = \int_0^\infty d\ell \, J_{0/4}(\ell\theta) C(\ell), \quad (\text{HT.2})$$

see for example [Kilbinger \(2015\)](#), where θ are angular coordinates on the sky and ℓ are angular wavenumbers.

High Mass (clusters): This refers to galaxy clusters with a virial mass M that is large compared to the mean of the observed distribution of cluster masses in the Universe. A typically mass range to warrant this classification would be $M \gtrsim 10^{15} M_\odot$. See for e.g. [Okabe et al. \(2010\)](#).

Higher-order moments: Moments $\langle x^n \rangle$ of a random variable x with $n > 2$; special case of a higher-order statistic, SEE **HIGHER-ORDER STATISTICS**. Higher-order moments explore non-linearities in the data and can be used for estimation of further shape parameters.

Higher-order statistics: Correlations of random variable x that depend on x^n with $n > 2$. Examples are the third and fourth moments (skewness and kurtosis). Higher-order statistics are often used in cosmology to measure deviations from Gaussian distributions – examples are bispectrum and trispectrum estimations, see e.g. [Munshi et al. \(2011b,a\)](#).

HOD: Acronym for Halo Occupation Distribution; SEE **HALO OCCUPATION DISTRIBUTION**.

HSC: The Hyper-Supreme Camera (HSC)¹⁴ is a digital camera for the 8.2 meter Subaru telescope, built by National Astronomical Observatory of Japan. The FoV of the camera is 1.5 degrees diameter.

II term: SEE **INTRINSIC ALIGNMENT**.

¹⁴ <http://subarutelescope.org/Observing/Instruments/HSC/>

IMCAT: The Image and Catalogue (IMCAT)¹⁵ manipulation software was developed for faint galaxy photometry for weak lensing studies (Kaiser 2011).

Impulse Response Function (of telescope): The response of an optical system when observing a point-like object at a distance of infinity. A general term for the Fourier transform of the point spread function (PSF). SEE PSF.

In-band: Light transmission within an optical or IR band.

Inter-pixel Responsivity: The responsivity of a pixel in a CCD varies from pixel-to-pixel across the CCD. This is due to differences in the manufacturing of the pixels, such as pixel area or thickness of pixel surface layers. These inter-pixel variations in a CCD should be kept small for weak lensing measurement to be possible, see e.g. Kannawadi et al. (2016).

Intrinsic: An ‘intrinsic’ property is a property of a system itself, un-affected by external influences. For example, in gravitational lensing, the ellipticity of a galaxy *before* it is lensed is called the intrinsic ellipticity of that galaxy; SEE INTRINSIC ALIGNMENT.

A property that is not inherent is called an extrinsic property. An example in physics is the density of an object, which is an intrinsic property, whereas the weight of the object is an extrinsic property as it depends on the strength of the gravitational field on the object.

Intrinsic Alignment: Intrinsic is a term that is used to denote the individual galaxy or population properties without, or before, gravitational lensing effects are included. Intrinsic alignment refers to hypothesised correlated orientations of galaxies in the Universe, caused by effects other than gravitational lensing.

An observed ellipticity for an individual galaxy is written like

$$\epsilon^{\text{obs}} = \epsilon^{\text{int}} + \gamma, \quad (\text{IA.1})$$

where ϵ^{int} is the unlensed ‘intrinsic’ ellipticity of a galaxy, γ is the weak lensing shear. The intrinsic part is labelled I and the shear part is labelled G . Assuming that the intrinsic ellipticities of galaxies do not have a random orientation or phase, one can take the correlation function of the observed ellipticity

$$\langle \epsilon_i^{\text{obs}} \epsilon_j^{*,\text{obs}} \rangle = \langle \epsilon_i^{\text{int}} \epsilon_j^{*,\text{int}} \rangle + \langle \epsilon_i^{\text{int}} \gamma_j^* \rangle + \langle \gamma_i \epsilon_j^{*,\text{int}} \rangle + \langle \gamma_i \gamma_j^* \rangle, \quad (\text{IA.2})$$

where i and j label different redshifts. The four terms on the right hand side are given the abbreviations II, IG, GI, and GG. In the context of power spectrum this can be written as

$$C_{\ell,ij} = C_{\ell,ij}^{\text{II}} + C_{\ell,ij}^{\text{IG}} + C_{\ell,ij}^{\text{GI}} + C_{\ell,ij}^{\text{GG}}. \quad (\text{IA.3})$$

Assuming that the j redshift bin is more distant than the i redshift bin the correlation IG should be zero, except in the case that galaxy redshifts are incorrect or there are systematic effects in the data. The remaining three terms are described as follows

1. GG is the gravitational-gravitational or shear-shear correlation between two galaxies. This is the cosmic shear that surveys tend to measure, which is the gravitational lensing of a distant source galaxy by the gravitational matter field of the lens.
2. GI is the gravitational-intrinsic or shape-shear correlation, which is the correlation between the intrinsic ellipticity of one galaxy with the shear of another galaxy. This happens when a foreground galaxy ellipticity is correlated via IA to structure of the lens that shears a background galaxy. A lens causes background galaxies to be aligned tangentially while galaxies physically close to the lens are stretched radially towards the lens due to tidal forces. This produces a negative GI correlation.
3. II is the intrinsic-intrinsic or shape-shape correlation of two galaxies. This is only for the galaxies that are physically close and share the same local environment; as the formation of galaxies and their orientation is highly affected by the tidal force of the local environment.

The contamination due to II can be reduced by removing galaxy pairs at the same redshift (Heymans and Heavens 2003; King and Schneider 2002, 2003). The GI contribution is harder to remove since galaxy pairs at the same line-of-sight at all angles are affected. The contribution due to GI can be reduced in a model-independent way within the survey (Joachimi et al. 2008; Joachimi and Schneider 2009), or can be modelled and its effect then removed (King 2005; Joachimi and Bridle 2010). Each of the methods have advantages and disadvantages.

There are various models for the intrinsic alignment:

¹⁵ <http://www.ifa.hawaii.edu/faculty/kaiser/imcat/content.html>

1. Linear alignment model describes the local alignment, or orientation of their semi-major axis, of galaxies due to interactions with their local tidal gravitational field. The model predicts that the ellipticity of galaxy is a linear function of the tidal quadrupole. This is normally applied to elliptical galaxies. The model is applicable on large scales (i.e. $\gtrsim 30$ Mpc). In the model the intrinsic ellipticity of a galaxy is assumed to follow the linear relation

$$e = \frac{C_1}{4\pi G} (\nabla_x^2 - \nabla_y^2, 2\nabla_x \nabla_y) \mathcal{S}[\Psi_P], \quad (\text{IA.4})$$

where Ψ_P is the Newtonian potential at the time of galaxy formation, \mathcal{S} is a smoothing filter that cuts off fluctuations on galactic scales, ∇ is a comoving derivative and C_1 is a normalisation constant. The model is described in [Catelan et al. \(2001\)](#); [Hirata and Seljak \(2004, 2010\)](#)

2. Quadratic alignment model describes the local alignment of galaxies, or orientation of their semi-major axis, due to interactions with their local tidal gravitational field. The model predicts that the ellipticity of a galaxy is a quadratic function of the tidal field. This is normally applied to spiral galaxies because it is hypothesised that the angular momentum of the spiral galaxy during gravitational collapse should influence its orientation. The model requires two tidal quadrupoles to describe the orientation. The model is described in [Hirata and Seljak \(2010\)](#)

For a review of *intrinsic alignments* see e.g. [Troxel and Ishak \(2015\)](#).

Intrinsic Ellipticity Distribution: The intrinsic ellipticity distribution is used to refer to an estimate of the probability density function of galaxy ellipticities, in a population of galaxies, before the the lensing effect. These distributions are referred to in, for example [Miller et al. \(2007\)](#).

K-correction: This is a correction to the magnitude or flux of a source to convert it to its rest frame magnitude or flux. The K-correction can be defined as

$$M = m - 5(\log_{10} D_L - 1) - K_{\text{cor}}, \quad (\text{KC.1})$$

where D_L is the luminosity distance. K-correction is necessary when objects are observed through a single bandpass, as only a fraction of the total spectrum of the source is observed.

Kaiser-Squires Mass Reconstruction: This is used to refer to the inverse method of estimating a map coverage from estimates of galaxy shear, which was first described in [Kaiser and Squires \(1993\)](#); SEE [MASS MAP](#); [3D MASS MAP](#); [KAPPA](#); [POTENTIAL MAP](#).

Kappa: This refers to the ‘convergence’ of the lensing potential, which is denoted using the symbol κ and referred to in shorthand as ‘kappa’, i.e. in ‘kappa map’ means ‘weak lensing coverage map’; SEE [MASS MAP](#); [3D MASS MAP](#); [KAPPA](#); [POTENTIAL MAP](#). The convergence is related to the projected lensing potential by

$$\kappa(\theta) = \frac{1}{2} \bar{\partial} \bar{\partial} \phi(\theta), \quad (\text{KP.1})$$

where θ is an angular coordinate on the sky, ϕ is the projected lensing potential and $\bar{\partial}$ is a complex derivative on the sky (SEE [ETH DIFFERENTIAL OPERATOR](#)) which are simplified covariant derivatives in a Cartesian coordinate system, see e.g. ([Castro et al. 2005](#)). The convergence is a spin-0 quantity. SEE [SURFACE DENSITY](#); [WEAK LENSING EQUATIONS](#).

KiDS: The Kilo-Degree Survey KiDS¹⁶ is one of the public surveys on the VST telescope (operated and maintained by ESO). KiDS will survey two areas of extragalactic sky, which are roughly 750 square degrees each, to a median redshift of 0.7. KiDS observed in four bands (u,g,r,i).

Knot: This is used to refer to a topological feature in a two or three-dimensional field that occurs where several other lower-dimensional feature (e.g. lines, filaments or planes) intersect. This occur in several instances in weak lensing literature, for example:

1. Knots in the matter distribution of the Universe, the cosmic web, can occur and form an approximately spherically symmetric overdensity. Such topological features can be studied using galaxy-galaxy lensing, see e.g. [Brouwer et al. \(2016\)](#).
2. Knots in strong lensing maps can also occur in the observed caustic features of strong lensing.

Knowledge Requirement: Refers to the need to know the error distribution of a desired quantity for use in a weak lensing measurement. For example for a quantity A , a ‘knowledge requirement’ refers to the need to know the value of A to high precision; in the case that A is expected to be Gaussian distributed it would be sufficient to specify the mean and variance of A ; in other cases the full probability distribution. A particular case is that of shape measurement biases where the known error on a methods bias is required. Such requirements are discussed in [Massey et al. \(2013\)](#).

¹⁶ <http://kids.strw.leidenuniv.nl/index.php>

KSB: A perturbative approach for a direct measurement of galaxies ellipticities [Kaiser et al. \(1995\)](#). Improvements of the KSB method have also been achieved by [Luppino and Kaiser \(1997\)](#); [Hoekstra et al. \(1998\)](#); [Kaiser \(2000\)](#); [Bernstein and Jarvis \(2002\)](#), and improvements by extension to higher perturbation order have been achieved by [Okura and Futamase \(2009\)](#).

l-mode: ℓ -mode is the angular wavenumber that corresponds to an angular scale via $\ell \simeq \pi/\theta$. SEE [FOURIER TRANSFORM](#); [FOURIER SPACE \(IN REFERENCE TO COSMOLOGY\)](#).

Large Scales: Refers to a range of physical scales, or Fourier wavenumbers, that are deemed to be ‘large(r)’ than some other set of scales that are being referred to. A common use is in reference to the matter power spectrum where ‘large scales’ refers to the part of the matter power spectrum that can be computed using linear perturbation theory of the matter overdensity distribution, SEE [LINEAR \(MATTER POWER SPECTRUM\)](#). This is also used as a synonym for linear scales

Large-Scale Structure (LSS): Refers to the structure in the distribution of matter — whether baryonic or dark — on cosmological scales of $\gtrsim 10$ Mpc, and its evolution. Matter is not randomly distributed throughout the Universe but rather it is formed into structures, such as filaments, walls, and clusters. There are also regions called voids where distribution of matter is scarce. The evolution of the LSS is primarily driven by gravitational collapse. The distribution of baryonic matter roughly traces the total matter distribution. The mismatch between the baryonic matter distribution and the total matter distribution is called galaxy bias. The weak lensing effect caused by the general LSS is known as Cosmic Shear SEE [COSMIC SHEAR](#).

Late Type (Galaxies): SEE [EARLY AND LATE TYPE \(GALAXIES\)](#).

ΛCDM: Lambda cold dark matter, or ΛCDM, model is currently the standard model of cosmology. This is a parametrisation of the Big Bang cosmological model and in such Universe there is a cosmological constant, Λ and cold dark matter (CDM). The ΛCDM is the favourite model currently as it is the simplest model that can explain many of the observations such as:

1. the cosmic microwave background;
2. the large-scale structure;
3. the abundances of hydrogen, helium, and lithium;
4. the current accelerating expansion of the Universe.

The model assumes that general relativity is the correct theory of gravity on cosmological scales. For a review of the cosmological parameters that describe this model see [Yao et al. \(2006\)](#); [Lahav and Liddle \(2014\)](#).

Lens: In cosmology a lens can be a galaxy, galaxy cluster, matter overdensity, or any other mass perturbation that is causing gravitational lensing of background objects along the line of site.

Lens Equation: An equation that relates the angular position of light ray at which it would have been observed in the absence of gravitational lensing β , to the actual angular observed θ , and the deflection angle in the path of the light ray caused by a gravitational lens α

$$\beta = \theta - \alpha \frac{D_{LS}}{D_S}, \quad (\text{LE.1})$$

where D_{LS} is the angular diameter distance between the source and the lens, and D_S is the angular diameter distance to the source. SEE [DISTORTION MATRIX](#); [WEAK LENSING EQUATIONS](#).

Lens Plane: The transverse 2D projection along the line-of-sight at which it is assumed the change in light paths geodesic is affected by the presence of a massive object causing gravitational lensing; i.e. the plane at which the lens sits.

Lensfit: A likelihood-based shear measurement method proposed by [Miller et al. \(2007\)](#); [Kitching et al. \(2008\)](#). The method is based on creating pixel-based models of the varying point spread function (PSF) in each image exposure. It fits PSF-convolved models to measure the ellipticity of each of the galaxies in the image, while marginalising over galaxy position, size, brightness and bulge fraction. The method can optimally deal with joint measurement of multiple dithered image exposures. It takes into account imaging distortions and possible alignment of the multiple measurements.

Lensing: Used as shorthand to refer to gravitational lensing. It is ambiguous in reference to weak, strong, or micro lensing; but is usually clear within context. In general lensing means ‘gravitational light deflection’.

Lensing Kernel: The lensing kernel, or lensing efficiency, defines the efficiency of lensing for a distribution of sources and lenses with respect to the observers. The kernel is broad and it is most sensitive to structure halfway between the observer and the source. The kernel is defined as the combination of distances that appears in the len equation SEE LENS EQUATION

Lensing Potential: The lensing potential (or deflection potential) $\psi(\theta)$ is a projected gravitational potential that can be related to the observed quantities of weak lensing. If β is the arrival direction of a light ray on the sky in a perfectly homogeneous universe this defines the position of the light ray in the source plane. Gravitational lensing by fluctuations in the matter or energy density shifts β to a new direction θ , which is the position of the deflected light ray in the lens plane. We denote by $\alpha(\theta) := \theta - \beta(\theta)$ (SEE LENS EQUATION [LE.3](#)) as the deflection angle of a light ray observed in the direction of θ . The vector field $\alpha(\theta)$ is curl-free so that it can be expressed as gradient field of a scalar field $\psi(\theta)$. In the flat-field approximation, a possible choice is $\alpha(\theta) = \nabla\psi(\theta) = (\partial_1 + i\partial_2) \psi(\theta)$ with

$$\psi(\theta) = \frac{1}{\pi} \int_{\mathcal{R}^2} d^2\theta \, \kappa(\theta) \ln|\theta - \theta|, \quad (\text{LP.1})$$

where $\kappa(\theta)$ is the convergence in direction of θ . From the previous relation follows the two-dimensional Poisson equation $\nabla\nabla^*\psi(\theta) = 2\kappa(\theta)$. In addition, the local shear $\gamma = \gamma_1 + i \gamma_2$ derives from $\psi(\theta)$ through $\gamma = (1/2) \nabla\nabla\psi(\theta)$.

Light Bundle: A bunch of light rays close to a fiducial light ray. Light bundles are a concept used to study the local differences in light deflections, up to linear order, between the fiducial light ray and rays close-by. In weak gravitational lensing the observation of an entire image of a distant galaxy can be approximated by the observation of a single light bundle. This means that the convergence $\kappa(\theta)$ and shear $\gamma(\theta)$ are normally assumed to be constant over the angular extend of a source (also known as a linear distortion approximation); however in stronger field regions higher-order image distortions can also occur SEE [FLEXION](#).

Light Profile: Galaxies observed in the sky have a finite extent. Their observed distribution of light is described by a surface brightness distribution $I(\theta)$, i.e. a ‘light profile’. These have specific characteristics that define the morphology of galaxies on the sky.

For weak gravitational lensing, the light profile and substructure might need to be taken into account when measuring shapes because the bias in estimators of galaxy shapes usually depend, among other things, on the light profile. A model for the radial light profile of a galaxy can be found by averaging over all angles

$$I(s) := 2\pi s \int_0^{2\pi} d\phi \, s \, I(s \, e^{i\phi}). \quad (\text{LP.1})$$

A commonly used functional form that has been found to be a good approximations to observed galaxy light profiles is known as the ‘Sérsic’ model [Sérsic \(1963\)](#)

$$I_n(s) := \exp\left(-k \left[\frac{s}{s_h}\right]^{\frac{1}{n}}\right), \quad (\text{LP.2})$$

which is determined by the ‘Sérsic index’ n and the half-light radius s_h . [Capaccioli \(1989\)](#) finds $k \sim 1.992n - 0.3271$. Special cases of this function form are the exponential profile that has $n = 1$, and the ‘de Vaucouleur’ ([de Vaucouleurs 1948](#)) profile which has $n = 4$. These are used as approximations for early and late-type galaxies with the exponential profile for late-type galaxies, and the de Vaucouleur profile for early-type galaxies.

Limber Approximation: The Limber approximation ([Limber 1953](#)) is employed for estimates of an angular correlation function (angular domain) or power spectrum (Fourier domain) of spatial random fields $\delta[f_K(\chi)\theta, \chi]$ projected on the celestial sphere. Here χ is the distance of a comoving position $(f_K(\chi)\theta, \chi)$ from the observer, and θ is the line-of-sight direction relative to a fiducial direction. The approximation assumes that the random field quickly decorrelates such that correlations between positions with different radial distances from the observer can effectively be evaluated at the same χ . For example, it assumes for a two-point correlation function that

$$C_{\delta\delta} = \langle \delta[f_K(\chi)\theta, \chi] \delta[f_K(\chi')\theta', \chi'] \rangle \approx \langle \delta[f_K(\chi)\theta, \chi] \delta[f_K(\chi)\theta', \chi] \rangle. \quad (\text{LM.1})$$

Importantly, the Limber approximation assumes small angular separations on the sky, or large angular wave numbers in the case of power spectra ([Simon 2007](#)). Thereby we arrive at the following integral for the angular two-point correlation function $C_{12}(\theta)$ of homogeneous and isotropic random fields δ . For this let

$$g_i(\theta) = \int_0^\infty d\chi \, q_i(\chi) \, \delta[f_K(\chi)\theta, \chi], \quad (\text{LM.2})$$

be projections i of δ on the sky along θ with projection kernel $q_i(\chi)$. The Limber approximation then yields for the cross correlation between $g_i(\theta)$ and $g_j(\theta')$ the integral

$$C_{ij}(\theta) = \int_0^\infty d\chi q_1(\chi) q_2(\chi) \int d(\Delta\chi) C_{\delta\delta}(\sqrt{f_K^2(\chi)\theta^2 + (\Delta\chi)^2}, \chi), \quad (\text{LM.3})$$

for separations $\theta = |\theta - \theta'|$. More frequently employed in gravitational lensing is the Limber approximation of the angular power spectrum at angular wave number ℓ , namely

$$P_{12}(\ell) = \int d\theta C_{12}(\theta) e^{i\ell\cdot\theta} = \int_0^\infty d\chi \frac{q_1(\chi) q_2(\chi)}{f_K^2(\chi)} P_\delta\left(\frac{\ell}{f_K(\chi)}, \chi\right), \quad (\text{LM.4})$$

where $P_\delta(k, \chi)$ is the spatial power spectrum of δ at spatial wave number k and distance χ , and we use the notation of [Joachimi and Bridle \(2010\)](#). For an exposition of the Limber approximation in the case of projected fields see [Loverde and Afshordi \(2008\)](#).

Line-of-Sight: The fiducial light path(s) from, or to, the point of an observer passing through a particular angular coordinate(s) on the celestial sphere θ .

Line-of-Sight Integration: Often, effects in only the transverse direction as function of θ are observed, and effects that take place along the line-of-sight must be integrated to calculate the predicted (gravitational lensing) effects. For example, in the Born approximation of light propagation, the integration is performed along the imaginary path of a unperturbed fiducial light ray in the line-of-sight direction.

Linear (Matter Power Spectrum): The linear power spectrum, or linear part of the matter power spectrum, refers to the part of the power spectrum of matter overdensity fluctuations $P_\delta(k, z)$ (as a function of scale k and redshift z) that is governed by physical effects that can be described using linear equations i.e. that part that can be computed from first order perturbation theory of the matter overdensity field and initial conditions. In the standard cosmological model, the linear power spectrum can be written as

$$P_{\text{lin}}(k, z) = P_0 D_+(z)^2 k^{n_s} T(k)^2, \quad (\text{LMPS.1})$$

where n_s is the power-law index of a primordial matter overdensity power spectrum, P_0 is the spectrum normalisation, $T(k)$ is a transfer function that described the amplitude of power spectrum as a function of scale, and

$$D_+(z) \propto H(a(z)) \int_0^{a(z)} \frac{da}{a^3 H(a)^3}, \quad (\text{LMPS.2})$$

is the linear growth factor to be scaled to $D_+ = 1$ at $z = 0$. The linear growth factor and transfer function depend on the particular set of cosmological parameters, such as the matter density or the parameter of the dark energy equation of state. Commonly, the normalisation P_0 is determined by specifying the variance σ_8 of (linear) matter density fluctuations inside a sphere of comoving radius $R = 8 h^{-1}$ Mpc at a redshift of $z = 0$ as additional cosmological parameter. Hence P_0 is chosen to satisfy

$$\sigma_8^2 = \frac{1}{2\pi^2} \int_0^\infty dk k^2 |W(k R)|^2 P_{\text{lin}}(k, 0), \quad (\text{LMPS.3})$$

for a given σ_8 and for the window function $W(x) = 3 x^{-3} (\sin x - x \cos x)$. The linear matter power spectrum describes fluctuations in the matter overdensity on large scales, where structure on these scales is less affected by late-time gravitational clustering and non-linear gravitational feedback effects or evolution. SEE [CORRELATION FUNCTIONS AND POWER SPECTRA](#).

Linear Alignment (model): SEE [INTRINSIC ALIGNMENT](#).

Low Mass (clusters): This refers to galaxy clusters with a virial mass M that is small compared to the mean of the observed distribution of cluster masses in the Universe. A typically mass range to warrant this classification would be $M \lesssim 10^{13} M_\odot$. See e.g. [Kettula et al. \(2015\)](#).

LSST (Large Synoptic Survey Telescope): The Large Synoptic Survey Telescope (LSST)¹⁷ is a wide-field survey reflecting telescope with an 8.4-meter primary mirror, currently under construction, that will photograph the entire available sky every few nights. The telescope has a very wide 3.5-degree diameter field of view, feeding a 3.2 gigapixel CCD imaging camera. Commissioning is expected to start in 2018.

¹⁷ <http://www.the-athena-x-ray-observatory.eu>

Luminosity Distance: This is defined in terms of the intrinsic luminosity L of an object at redshift z_2 to the flux S that it received by an observer at redshift z_1

$$D_L(z_1, z_2) = \sqrt{\left(\frac{L}{4\pi S}\right)}. \quad (\text{LD.1})$$

In a metric cosmology, such as those described by general relativity, the luminosity distance can be determined by other measures of distance, such as the comoving distance $D_M(z_1, z_2)$

$$D_L(z_1, z_2) = \frac{1 + z_2}{(1 + z_1)^2} D_M(z_1, z_2), \quad (\text{LD.2})$$

and the angular diameter distance $D_A(z_1, z_2)$

$$D_L(z_1, z_2) = \frac{(1 + z_2)^2}{(1 + z_1)^2} D_A(z_1, z_2). \quad (\text{LD.3})$$

As an example, fitting the luminosity of the CMB to a blackbody spectrum has allowed for the determination of the relation between D_L and D_A .

M200 or M500: The mass enclosed within radius R_{200} or R_{500} . SEE [R200](#) OR [R500](#).

Magnification: Gravitational lensing distorts and magnifies the intrinsic light profile of sources. In the weak lensing regime, the effect on a source is described as linear mapping by the distortion matrix \mathcal{A} (SEE [DISTORTION MATRIX](#)) that includes an isotropic focussing by κ and anisotropic focussing by γ . Due to the focussing the observed flux of a source changes relative to the flux of the unlensed source by

$$\mu = \frac{1}{|\det \mathcal{A}|} = \frac{1}{(1 - \kappa)^2 - |\gamma|^2}, \quad (\text{MG.1})$$

where $|\gamma|^2 = \gamma_1^2 + \gamma_2^2$.

In addition, this so-called magnification changes the observed local number density of sources on the sky. If $n_0(> S, z)dz$ is the unlensed number density of galaxies within dz , and flux larger than S , then at an angular position θ , where the magnification is $\mu(\theta, z)$, the observed number counts are modified as

$$n(> S, z)dz = \frac{1}{\mu(\theta, z)} n_0\left(> \frac{S}{\mu(\theta, z)}\right). \quad (\text{MG.2})$$

This effect is prominent for large, extended lenses such as galaxy clusters but can also be studied for galactic lenses by devising cross-correlation techniques ([Bartelmann and Schneider 2001](#)).

Margin: Margin is an allocated fraction of a specific type of error within the maximally allowed error budget, that allows for changes in specification that decrease performance to still enable scientific objectives to be achieved¹⁸.

Mask: Mask is normally a binary (multi-dimensional) matrix, which specifies the regions on the sky that one should, or should not, take into account for further analysis. Normally, the unwanted data has zero elements in the corresponding matrix. The mask need not be binary. In some cases, for e.g. the CMB data, the mask is apodised, meaning there is a smooth function that defines the transition between pixels with 1s and 0s. A mask can cause aliasing for power spectrum measurements that can be accounted for using Pseudo-Cl analyses, see e.g. [Hivon et al. \(2002\)](#) and SEE [PSEUDO-CL](#).

Mass Map: Or kappa map (κ map) is the map of the total matter, which is reconstructed from a shear map in lensing surveys. SEE [KAPPA](#); [POTENTIAL MAP](#); [3D MASS MAP](#).

Matched Filter: Usually obtained by correlating a template with an unknown signal to detect the presence of the template in the unknown signal. The matched filter is the optimal linear filter for maximising the signal to noise ratio (SNR) in the presence of additive stochastic noise. In weak lensing matched filters have been used to identify galaxy clusters, see e.g. [Ford et al. \(2015\)](#).

Minkowski Functional: In mathematics, in the field of functional analysis, a Minkowski functional is a function that recovers a notion of distance on a linear space. In cosmology, they are the main tools to characterise the large-scale galaxy distribution in the Universe. A tutorial on the use of Minkowski Functionals in cosmology is provided by [Schmalzing et al. \(1996\)](#). In weak lensing Minkowski functionals may be used to test modified gravity theories ([Petri et al. 2015](#); [Shirasaki and Yoshida 2014](#); [Petri et al. 2013](#)).

¹⁸ Systems engineering definitions are provided here <http://www.ecss.nl>.

Miscentering: Refers to a situation where the statistics is sensitive to inaccurate estimation of the centre/centre-of-mass/average, of a distribution. Such cases are referred to as a ‘miscentering problems’. Examples include galaxy cluster mass estimates being incorrect due to the estimation of the centre of the cluster mass distribution (e.g. [Hoshino et al. \(2015\)](#)). Another example is if the estimation of galaxy or stellar ellipticity is sensitive to estimates of the central pixel of the objects, then the method involved may be referred to as being ‘sensitive to miscentering’ (see for example [Miller et al. \(2007\)](#) and [Miller et al. \(2013\)](#) for methods to mitigate this issue).

Mixing Matrix: A mixing matrix is a matrix that encodes the correlation introduced by missing data in azimuthal wavenumber space ℓ . Effectively this is a determination of the aliasing between angular modes caused as a result of missing data. The matrix is commonly given the symbol $M_{\ell\ell'}$, where ℓ and ℓ' are angular wavenumber — SEE [L-MODE](#). It is also referred to as the ‘mode-mode coupling matrix’. Referring to [Hikage et al. \(2011\)](#) the harmonic transform of an angular mask $K(\hat{n})$ as a function of angle \hat{n} is

$$K_{\ell m} = \int d\Omega_{\hat{n}} K(\hat{n}) Y_{\ell m}^*(\hat{n}), \quad (\text{MM.1})$$

where $Y_{\ell m}$ are spherical harmonic transforms and the integral is over angle, m and ℓ are angular wavenumbers in latitude and longitude projected onto the sky. The covariance, or correlation, of the mask is given by $\mathcal{K}_{\ell\ell'} \equiv [1/(2\ell+1)] \sum_m K_{\ell m} K_{\ell' m}^*$. For an isotropic spin-2 field the spherical harmonic transforms are spin-2 functions and m is integrated over. In this case aliasing of the E-mode and B-mode correlation functions are both affected by an angular mask, and the resulting E-mode mixing matrix is defined as

$$M_{\ell\ell'}^{EE,EE} = \frac{2\ell'+1}{8\pi} \sum_{\ell''} (2\ell''+1) \mathcal{K}_{\ell\ell''} [1 + (-1)^{\ell+\ell'+\ell''}] \begin{pmatrix} \ell & \ell' & \ell'' \\ 2 & -2 & 0 \end{pmatrix}^2. \quad (\text{MM.2})$$

In the case that a flat-sky approximation is chosen, or in the case of a three dimensional spherical-Bessel representation of the data, this definition changes as described in e.g. [Kitching et al. \(2014\)](#).

Model Bias: Refers to biases in inferred ellipticity measurements of galaxies away from the true ellipticity of an object that would have been inferred if an exact model for the light distribution of the galaxies was used. The hypothetical bias is caused by the use of a non-optimal model in an algorithm that employs some loss function between the data and the model. A particular example, shown in [Voigt and Bridle \(2010\)](#) is the fitting of a 2D Gaussian profile to a 2D exponential disk or 2D Sérsic profile. Related to model biases are biases that are caused as a result of have non-optimal weight functions in a moment-based ellipticity inference method e.g. KSB (e.g. KSB [Kaiser et al. 1995](#)).

Model Fitting (method): In weak lensing ‘a model fitting method’ typically refers to a shape measurement method that uses a model, or set of models, that are fit to some galaxy imaging data with the aim to use the loss function of the fitting method to infer the ellipticity and size of the object in question. Examples include fitting 2D Sérsic or exponential disk profiles to galaxies or a combination (e.g. the lensfit method ([Miller et al. 2007](#); [Kitching et al. 2008](#); [Miller et al. 2013](#)) and im3shape method ([Zuntz et al. 2013](#))), fitting sums of Gaussians (e.g. the NGMIX method ([Sheldon 2015](#))), or fitting more complex 2D functions (e.g. shapelets [Refregier 2003](#); [Refregier and Bacon 2003](#)).

Moment (method): A moment, or ‘moment based’, method is an algorithm that infers the quadrupole moments from an image through a direct integration or sum of the pixel values (typically with some associated weight function). Examples are the KSB [Kaiser et al. \(1995\)](#) and DEIMOS [Melchior et al. \(2011\)](#) algorithms.

Monte Carlo: Refers to a process in which samples are chosen randomly from some distribution. A common example is a Monte-Carlo Markov Chain used to sample a probability distribution where at each step in the chain a sample is drawn randomly from some ‘proposal’ distribution. However this phrase is used more colloquially to mean any random sample, for example “just Monte-Carlo it” would be a suggestion to run the process in question many times where each time a random set of variables in question are chosen.

Monte Carlo methods/experiments/simulations are a broad class of computational algorithms that rely on repeated random sampling to obtain numerical results. They are often used in physical and mathematical problems and are most useful when it is difficult or impossible to use other mathematical methods. Monte Carlo methods are mainly used in three distinct problem classes: optimisation, numerical integration, and generating draws from a probability distribution. In physics-related problems, Monte Carlo methods are quite useful for simulating systems with many coupled degrees of freedom. In principle, Monte Carlo methods can be used to solve any problem having a probabilistic interpretation. By the law of large numbers, integrals described by the expected value of some random variable can be approximated by taking the empirical mean (a.k.a. the sample mean) of independent samples of the variable. When the probability distribution of the variable is too complex, mathematicians often use a Markov Chain Monte Carlo (MCMC) sampler. The central idea is to design a judicious Markov chain model with a prescribed stationary probability distribution. By the ergodic theorem, the stationary probability distribution is approximated by the empirical measures of the random states of the MCMC sampler

Morphology: Galaxy morphology refers to the apparent physical shape of a galaxy as it appears in an image. Morphology is the analysis of those shapes, typically within classification of a shape into a model or category. For example a galaxy may have a spiral, elliptical, barred-spiral, disk, or disk-plus-disk morphology. There are several schemes in use by which galaxies can be classified according to their morphologies, the most famous being the Hubble sequence, devised by Edwin Hubble and later expanded by Gerard de Vaucouleurs and Allan Sandage. SEE [EARLY AND LATE TYPE \(GALAXIES\)](#).

Multi-Bin: A bin is a finite range in a variable of interest over which measurements are averaged. Multi-bin means that more than one bin has been used in an analysis. In cosmic shear “multi-bin” is used as a short-hand prefix to mean that an analysis was done using multiple redshift slices e.g. “multi-bin tomography”, or a “multi-bin analysis”.

Multi-Epoch: Epoch in this context refers to the time at which an observation was performed. Multi-epoch is used as a prefix as in “multi-epoch data” which would refer to a set of data (typically imaging data) that were taken at different real times. This phrase is usually used to refer to times “epochs” that are typically characterised by a qualitative difference in observing conditions. See e.g. [Sluse et al. \(2006\)](#).

Multiplicative and Additive Bias of Shear: The estimated shear $\tilde{\gamma}$ is usually biased with respect to the true shear γ . To a good approximation biases are expected to be linear in γ ([Massey et al. 2013](#); [Paulin-Henriksson et al. 2008, 2009](#)), expressed through the relation

$$\tilde{\gamma} = m \gamma + c. \quad (\text{MC.1})$$

This was first used in [Heymans et al. \(2006\)](#), and has subsequently been extended to more general biases in ellipticity using a similar linear relation. m is referred to as a multiplicative bias and c as an additive bias. Both may depend on characteristics of the source, such as signal-to-noise, apparent size, light profile, or the colour gradient of galaxies. In [Massey et al. \(2007\)](#); [Kitching et al. \(2012\)](#) a quadratic term $q\gamma^2$ was also used.

Multiplicative Bias of Shear: SEE [MULTIPLICATIVE AND ADDITIVE BIAS OF SHEAR](#).

n-of-z $n(z)$: This is a shorthand way to refer to the number density of galaxies, or total number of galaxies, in a survey as a function of redshift. The $n(z)$ can be a functional form, or a binned distribution. It can refer to a surface number density, physical comoving number density (which is less often used), or total number of galaxies. An $n(z)$ can be derived in several different ways, for example, by using one-point redshift estimates or summing posterior redshift distributions in a photometric survey (as well as other methods), or more directly in a spectroscopic survey. Usually pronounced “n-of-z”.

Narrowband: Narrow refers to the width of the wavelength range over which the filter is defined. The definition of narrow vs broad is subjective, but weak lensing literature typically refers to the u, g, r, i, z filters¹⁹ for example as “broadbands”, and anything that has a wavelength interval smaller than these as narrow band. Examples of surveys that use narrow bands are the COMBO-17 survey [Wolf et al. \(2003\)](#), and the PauCAM survey [Benítez et al. \(2009\)](#). The phrase is often used in conjunction as in “narrowband data”, “narrowband survey” etc.. SEE [BAND](#); [BROADBAND](#).

Newtonian Potential: Refers to the gravitational potentials. For example in an expanding universe in the following equation

$$ds^2 = \left(1 + \frac{2\Psi}{c^2}\right) c^2 dt^2 - a^2(t) \left(1 - \frac{2\Phi}{c^2}\right) dl^2, \quad (\text{NP.1})$$

where ds is the line element, a is the scale factor, c is the speed of light, Ψ is the Newtonian potential and Φ is the Newtonian curvature. SEE [FRIEDMAN-ROBERTSON-WALKER MODELS](#).

NFW: The Navarro-Frenk-White (NFW) profile describes the spatial distribution of dark matter halos that was fitted to N-body simulations by [Navarro et al. \(1996\)](#). The NFW profile is one of the most commonly used models in cosmology and it defines the density of dark matter as a function of radius as

$$\rho(r) = \frac{\rho_c \delta_c}{(1/r_s)(1 + r/r_s)^2}, \quad (\text{NFW.1})$$

where r_s is a scale radius, δ_c is a characteristic density, and ρ_c is the critical over density defined as $\rho_c = 3H^2/8\pi G$. The NFW profile works for a large range of halo masses and sizes, from individual galaxies to the halos of galaxy clusters. SEE [HALO MODEL](#).

¹⁹ https://en.wikipedia.org/wiki/Photometric_system

NGMIX: A method for estimating galaxy shapes for weak lensing. It was first used in DES in [Becker et al. \(2015\)](#). Both the galaxy and the PSF profile are modelled using mixtures of Gaussians for 2D images. Convolutions are analytically dealt with, which makes the method faster than some other methods. For the galaxy model, exponential disks, de Vaucouleurs and Sérsic profiles are also supported. For more information please refer to [Sheldon \(2015\)](#) and visit the Github repository <https://github.com/esheldon/ngmix>.

Nicaea: Refers to a public code²⁰ that calculates theoretical weak lensing correlation functions.

Noise Bias: Noise bias can mean one of two things in the weak lensing context

1. In weak lensing noise bias refers to biases in the measured ellipticity of galaxies caused by the influence of random noise in images. It is one of the main sources of systematic errors in weak lensing measurements. It arises from high-order noise terms in the measurement of the shapes of galaxies. It increases in magnitude as a galaxy’s signal-to-noise decrease, and this is due to relative the increase of noise in the galaxy images, see e.g. [Hirata et al. \(2004\)](#), [Refregier et al. \(2012\)](#), [Miller et al. \(2013\)](#), [Viola et al. \(2014\)](#).
2. Noise bias can also, much less commonly in the weak lensing literature, but a standard usage in CMB literature, refer to the bias in a power spectrum measured away from what would have been measured in the absence of shot (Poisson) noise. See e.g. [Namikawa et al. \(2013\)](#)

. SEE [BIAS](#); [ACCURACY](#).

Non-convolutive: This refers to any process that may have an impact on an image that cannot be represented as a convolution of an original image with a kernel that characterises the process as described in [Massey et al. \(2013\)](#) and [Cropper et al. \(2013\)](#). An example would be that the effect of a telescopes aperture is convolutive, as its effect can be represented as a convolution of an original image with a point spread function (PSF). But the effect of CCD charge transfer inefficiency (CTI) and the brighter-fatter effect ([Niemi et al. 2015](#)) on an image cannot be represented in this way, and are therefore “non-convolutive”. SEE [CONVOLVED/CONVULATIVE](#).

Non-linear (Matter Power Spectrum): This refers to the part of the matter power spectrum $P_\delta(k, z)$ (as a function of scale k and redshift z) that is governed by physics that involves the solution of equations that are non-linear. In contrast the linear part of the matter power spectrum can be computed from first order perturbation theory of the matter over-density field and initial conditions SEE [LINEAR \(MATTER POWER SPECTRUM\)](#). The non-linear physics primarily involves gravitational collapse of dark matter structures, but also non-dark matter (baryonic) physics. There is no exact and widely accepted demarkation between the linear and non-linear parts of the function however $k \simeq 1h\text{Mpc}^{-1}$ is an approximate scale at a redshift of $z = 0$. There are various phenomenological models for this range of scales, for example SEE [HALO MODEL](#), and various algorithms that have been calibrated on simulations to model this range, the most recent (c. 2016) of these being from [Mead et al. \(2015\)](#)²¹.

Number Density: The number density may refer to several different quantities depending on the context

1. The number density is the three-dimensional volume number density $n = N/V$, 2D area number density $n = N/A$, or 1D line number density $n = N/L$, where N is the total number of objects, V is volume, A is area and L is length.
2. In the lensing community, it is most commonly used as a short-hand expression to mean the number density of galaxies as a function of redshift; e.g. as in “the number densities of red and blue galaxies are different” SEE [N-OF-Z N\(Z\)](#).
3. It can also be used as a short-hand to refer to the effect that weak lensing magnification can change the observed number density of galaxies as a function of redshift. For example as in “does that statistic include the effect of number density?” may, in context, refer to impact of a statistic on the weak lensing magnification effect

Out-of-Band: This refers to any electromagnetic flux that has wavelengths that are shorter or longer than a pre-determined range that labels the band in question. For example if a CCD is required to be sensitive over the range 500 – 800 nm then any sensitivity of the CCD to photons with wavelengths outside of this range would be referred to as out-of-band. SEE [BAND](#); [IN-BAND](#).

Over-density: This refers in general to an excess over the mean density of a general quantity α and can be defined as

$$\delta_\alpha = \frac{\alpha - \bar{\alpha}}{\bar{\alpha}}, \quad (\text{OD.1})$$

²⁰ <http://www.cosmostat.org/software/nicaea/>

²¹ <https://github.com/alexander-mead/HMcode>

where δ_α is the over-density, and $\bar{\alpha}$ is the mean of some quantity α . Over-density functions appear in the description of the distribution of the matter field in weak lensing,

$$\delta_\rho(x) = \frac{\rho(x) - \bar{\rho}}{\bar{\rho}}, \quad (\text{OD.2})$$

where ρ is the matter density at coordinate x ; the covariance of which leads to the matter power spectrum. However, it can also be used in the context of galaxies or stars, as in “there is a over-density in that part of the image”, or other quantities. It can also be used in an graphical setting for example referring to an over-density of points in a scatter plot. SEE [STRUCTURE FORMATION IN THE UNIVERSE](#).

PanSTARRS: The Panoramic Survey Telescope & Rapid Response System (PanSTARRS)²² is a wide-field imaging facility developed at the University of Hawaii’s Institute for Astronomy. PanSTARRS can observe the entire available sky several times each month, meaning surveying the sky for moving objects on a continual basis. It also has accurate astrometry and photometry of already detected objects. The first Pan-STARRS telescope (PS1) started taking data in May 2010 and was completed in April 2014. As of mid-2014 the second telescope, PS2, was in the process of being commissioned²³.

PCA: Principal component analysis (PCA) is a statistical procedure that uses an orthogonal transformation to convert a set of observations of possibly correlated variables into a set of values of linearly uncorrelated variables called principal components. The number of principal components is less than or equal to the number of original variables. This transformation is defined in such a way that the first principal component has the largest possible variance (that is, accounts for as much of the variability in the data as possible), and each succeeding component in turn has the highest variance possible under the constraint that it is orthogonal to the preceding components. The resulting vectors are an uncorrelated orthogonal basis set. The principal components are orthogonal because they are the eigenvectors of the covariance matrix, which is symmetric. PCA is sensitive to the relative scaling of the original variables. PCA can be done by eigenvalue decomposition of a data covariance (or correlation) matrix or singular value decomposition of a data matrix, usually after mean centering (and normalizing or using Z-scores) the data matrix for each attribute.

The covariance/Fisher matrix is a symmetric $n \times n$ matrix and therefore, can be diagonalised using its eigenvectors. This has the form $\mathbf{C} = \mathbf{E}^T \mathbf{\Lambda} \mathbf{E}$, where \mathbf{C} is the covariance matrix, \mathbf{E} is an orthogonal matrix with the eigenvectors of \mathbf{C} as its rows and $\mathbf{\Lambda}$ is the diagonal matrix with the eigenvalues of \mathbf{C} as its diagonal elements²⁴. This constructs a new set of variables \mathbf{X} that are orthogonal to each other and are a linear combination of the original parameters \mathbf{O} , through the eigenvectors

$$\mathbf{X} = \mathbf{E} \mathbf{O}. \quad (\text{PCA.1})$$

The X_i are called the *principal components* of the experiment and are ordered so that so that X_1 has the smallest eigenvalue and X_n the largest. In this construction, the eigenvalues are the variances of the new parameters, so X_1 and X_n are the best- and worst-measured components respectively. The eigenvectors have been normalised so that $\sum_j e_j^2 = 1$, where e_j are the elements of E_i . We list some properties of PCA below:

1. The main point of PCA is to assess the degeneracies (correlations) amongst the parameters that are not resolved by the experiments, be they fundamental as from cosmic variance or due to the noise and coverage of the experiment. In our case, it will especially help us to see the correlation amongst the bins of the primordial PS, and between the bins and the cosmological parameters.
2. The eigenvalues obtained measure the performance of the experiment — a larger number of small eigenvalues means a better experiment. Another measure of the performance of the experiments is to see how they mix physically independent parameters such as, say, n_s , the spectral index, and Ω_b . This sort of mixture may be improved by improving the experiment’s noise properties or increasing its area or volume.²⁵

PCA is a special case of Singular Value Decomposition (SVD).

Peak (counts and statistics): A peak can refer to a maxima in any function, however specifically in weak lensing ‘peak counts’ and ‘peak statistics’ refer to the maxima identified in maps of the projected mass or projected gravitational potential – colloquially referred to a ‘dark matter maps’ – and the statistics derived from those maxima. ‘Peak statistics’ is a general term that refers to the n-point statistics that can be done with such maxima. ‘Peak counts’ refers to the statistic that can be done with the number density distribution of the maxima. Such statistics can be sensitive probes of cosmology as shown in [Kratochvil et al. \(2010\)](#); [Cardone et al. \(2013\)](#); [Lin and Kilbinger \(2015\)](#).

²² <http://ps1sc.org/KeyScience.shtml>

²³ <http://pan-starrs.ifa.hawaii.edu/public/>

²⁴ It is common to construct the covariance matrix for PCA. However, the Fisher matrix can be used instead; the eigenvectors stay the same, but eigenvalues are reciprocals.

²⁵ However, the so-called ‘geometrical degeneracy’ cannot be improved by improving the experiments; two models with same primordial PS, the same matter content, and the same comoving distance to the surface of last scattering produce identical CMB PS.

Photometric Redshifts (Photo-z; photo-zee; photo-zed): Photometric redshifts are estimates of the redshift of an object derived from imaging data from which measurements of object fluxes (photometry — that is when the object is observed through various standard broadband filters) have been derived — i.e. not spectroscopy. Redshifts are then inferred from a set of such observations in different wavelengths. There are many methods and algorithms that can perform this task with varying degrees of accuracy and precision, see [Banerji et al. \(2008\)](#); [Hildebrandt et al. \(2010\)](#); [Abdalla et al. \(2011\)](#); [Dahlen et al. \(2013\)](#); [Sánchez et al. \(2014\)](#) for comparisons of different photometric redshift estimation methods. Photo-z, can be pronounced “photo-zee” or “photo-zed” and is a short-hand expression used to refer to such photometric redshifts estimates.

Photoz: Shorthand for ‘a photometric redshift’. SEE [PHOTOMETRIC REDSHIFTS \(PHOTO-Z; PHOTO-ZEE; PHOTO-ZED\)](#).

Pixel: Pixel can refer to a physical object or be used in a grammatic sense in weak lensing literature

1. This is a defined area on the surface of a CCD within which the flux of photons is summed over some time-period (defined by the electronics and software of the CCDs and imaging system of a telescope). Pixels are typically square or rectangular – although there sensitivity can become slightly non-square due to electrons and electric fields leaking or interfering between neighbouring pixels. In a rendered image pixels are represented by square or rectangular areas of brightness.
2. Pixel can also be used as a prefix in weak lensing, for example “pixel effect” would refer to an effect caused by the fact that images are rendered in pixels; “pixel noise” ([Melchior and Viola 2012](#)) refers to the noise in images that have been rendered in pixels; from a combination of sources. Pixelisation refers to the process of an objects image being rendered in pixels

Pixel Clamping Bounce: Pixel clamping bounce is the difference between the measured and expected signal in a detector pixel after a sharp transition. Generally this refers to an overshoot, so that the first few faint pixels after bright pixels are measured to be fainter than they in reality are, or the first few bright pixels after faint pixels are measured to be brighter. The effect can arise in various parts of the detection chain electronics, including within the detector and from parasitic capacitance in the circuits connecting them to the electronics.

Pocket Pumping: Pocket pumping is a technique which is used to identify damage in the Si lattice in CCD detectors. Ions, generally Solar protons, but also heavier ions from the Sun and of cosmic origin, incident on the detector may displace atoms in the lattice, especially in areas where there impurities and the lattice is less elastic. These regions are called traps, because as the CCD is being read out at the end of an exposure, electrons in the conduction band may be trapped in the local irregularity. Several species of trap exist, with different trapping and release time constants. Traps with longer release time constants than the readout rate may release their trapped electrons into a subsequent pixel, giving rise to a trail behind the image. This directly modifies the shape of the image, which is important for measuring its shape, and must be corrected for weak gravitational lensing measurements. When pocket pumping, the charge is shuffled backwards and forwards without being read out during the exposure. If the shuffling frequency is resonant with the trap release time, charges in a pixel will be trapped and released in the adjacent pixel, so that an initially uniform image develops bright-faint pairs. These pairs identify the location of each trap with that associated trap release time constant. The shuffling frequency can then be changed to explore another species of trap in the next pocket pumping exposure. The initial uniform image can be generated by the artificial injection of charge or by optical illumination, for example in the case of Euclid, by a calibration source.

Pointing: A pointing is an area of sky from which observations have been taken in sequence. For example a single pointing — or one pointing — may contain multiple exposures (or only one exposure). The word refers to the physical act of pointing the telescope at an area of sky. If an area of sky is revisited in a non-sequential way, i.e. at a later time between which other observations have been made, this would be referred to as multi-epoch imaging where each pointing contained imaging from several epochs. SEE [MULTI-EPOCH](#).

Poisson Noise: Poisson noise – sometimes referred to as shot noise – is a type of noise that can be modelled by a Poisson process, which is applied when the event in question can independently be counted in whole numbers. For example, in electronics shot noise is due to the discrete nature of electric charge; in photon counting in optical devices, such as CCD, shot noise arises due to the particle nature of light; in galaxy power spectrum estimation, shot noise arises due to the discrete sampling of the galaxy density field.

For large numbers, at the point where the events (photons, electrons, etc.) can no longer be individually observed, the Poisson distribution approaches a normal (Gaussian) distribution.

Polarisability: The weak lensing shear is also referred to as polarisability, see e.g. [Kaiser et al. \(1995\)](#).

Position: The coordinate of object, or data point. In the context of weak lensing 2-point statistics this refers to the set of 3D angular and comoving galaxy positions that can be correlated with other properties, for example shear estimates to create a shear-position 2-point cross-correlation statistic, or position-position to create a 2-point statistic of the 3D galaxy field.

Potential Map: An inferred map of the lensing potential, derived using a kappa map. SEE [MASS MAP](#); [3D MASS MAP](#); [KAPPA](#).

Power Spectrum: The power spectrum is the Fourier counterpart of the two-point correlation function. SEE [CORRELATION FUNCTIONS AND POWER SPECTRA](#).

The power can also be in the harmonic space. Given a harmonic transform of a quantity, the power spectrum is the average of the quadrature of the harmonic transform coefficients. An example in weak lensing is the spherical harmonic transform of the shear field $\gamma(\theta, z)$ on a redshift slice z

$$\gamma_\ell^m(z) = \sum_{g \in z} \gamma(\theta, z)_2 Y_\ell^m(\theta) , \quad (\text{PS.1})$$

that results in a power spectrum

$$\langle \gamma_\ell^m(z) \gamma_{\ell'}^{m'*}(z') \rangle = C_\ell(z, z') \delta_{\ell\ell'} \delta_{mm'} , \quad (\text{PS.2})$$

where $C_\ell(z, z')$ is the power spectrum that assumes homogeneity and isotropy. SEE [SHEAR POWER SPECTRUM](#).

Precision: Precision refers to a statistical error of a measurement. It refers to how close a measurement is to the ‘true’ value (that would have been inferred given no systematic effects and no noise in the data). In terms of confidence limits on parameters a precise measurement is one where the inferred limits on parameters are statistically consistent with the true values of the parameters. The opposite is an imprecise measurement. Also the inverse of a covariance matrix is also known as a precision matrix ([Taylor et al. 2013](#)).

Probability Distribution: Probability distribution or probability distribution function (PDF). Here we list some of the commonly used PDFs in weak lensing.

1. Gaussian (Normal) Distribution

This is the most common continuous probability distribution in cosmology. It is very useful because of the central limit theorem; physical quantities that are sum of many independent processes (such as measurement errors) often have nearly normal distributions. For data x , normal distribution is defined as

$$\mathcal{N}(x; \mu, \sigma^2) = \left(\frac{1}{2\pi\sigma^2} \right)^{1/2} \exp \left[-\frac{1}{2} \frac{(x - \mu)^2}{\sigma^2} \right] , \quad (\text{PD.1})$$

where ‘standard’ normal distribution has $\mu = 0$ and $\sigma^2 = 1$. For a collection of data $\{x_n\}$

$$\mathcal{N}(x_n; \mu, \sigma^2) = \left(\frac{1}{2\pi\sigma^2} \right)^{N/2} \exp \left[-\frac{1}{2} \frac{\sum_i^N (x_i - \mu)^2}{\sigma^2} \right] . \quad (\text{PD.2})$$

For multi-variate case

$$\mathcal{N}(\mathbf{D}_n; \mu_n, \mathbf{C}) = \left(\frac{1}{|2\pi\mathbf{C}|} \right)^{1/2} \exp \left[-\frac{1}{2} (\mathbf{D}_n - \mu_n) \mathbf{C}^{-1} (\mathbf{D}_n - \mu_n)^T \right] , \quad (\text{PD.3})$$

where $C = \langle (\mathbf{D}_n - \mu_n) (\mathbf{D}_n - \mu_n)^T \rangle$.

2. Log-normal Distribution

This is a continuous probability distribution of a random variable whose logarithm is normally distributed — a log-normally distributed random variable can only take positive real values. For data x , where $\log(x)$ has normal distribution

$$\ln \mathcal{N}(x; \mu, \sigma^2) = \left(\frac{1}{2\pi\sigma^2} x^2 \right)^{1/2} \exp \left[-\frac{1}{2} \frac{(\ln x - \mu)^2}{\sigma^2} \right] \quad \text{for } x > 0 . \quad (\text{PD.4})$$

This is useful in weak lensing because the matter overdensity field is hypothesised to have an approximately log-normal distribution ([Coles and Jones 1991](#); [Kayo et al. 2001](#); [Xavier et al. 2016](#)).

3. χ^2 Distribution

For x_n is drawn from ‘standard’ normal distribution, then $X = \sum_n^N x_n^2$ has a χ^2 distribution with N degrees of freedom

$$\chi_N^2(X) = \left(\frac{1}{2^N \Gamma^2(N/2)} \right)^{1/2} X^{N/2-1} \exp \left[-\frac{1}{2} X \right], \quad (\text{PD.5})$$

where $\Gamma(N/2)$ is Gamma function. If x_n has mean μ and variance σ^2 then $X = \frac{1}{\sigma^2} \sum_n^N (x_n - \mu)^2$ has a χ^2 distribution with $(N - 1)$ degrees of freedom.

4. Poisson Distribution

This probability distribution describes the distribution of a discrete independent point process

$$\mathcal{P}_n = \frac{\lambda^n}{n!} \exp(-\lambda), \quad (\text{PD.6})$$

where λ is the average number of events per interval (e.g. in a volume of space), n is the event (e.g. a galaxy). This is useful in weak lensing and cosmology because many processes have Poisson distribution for example photon noise.

5. Wishart Distribution

For x being an $N \times p$ matrix, with each row being independently drawn from a p -variate normal distribution with mean $\mu = 0$, the $p \times p$ random scatter matrix $\mathbf{S} = X^T X$ has a Wishart distribution with N degrees of freedom and covariance \mathbf{C}

$$\mathcal{W}_p(\mathbf{C}, N) = \frac{1}{2^{Np/2} |\mathbf{C}|^{N/2} \Gamma_p(N/2)} |\mathbf{S}|^{(N-p-1)/2} \exp \left[-\frac{1}{2} \text{tr}(\mathbf{C}^{-1} \mathbf{S}) \right], \quad (\text{PD.7})$$

where Γ_p is the multivariate Gamma function. In cosmology and weak lensing, this distribution is useful because parameters covariance matrices have Wishart distributions. The Wishart distribution is the multivariate extension of the gamma distribution — in case of integer degrees of freedom, it is a multivariate generalisation of the χ^2 distribution; i.e. for a single data point ($p = 1$), the Wishart distribution is the reduced χ^2 -distribution. As the χ^2 distribution describes the sums of squares of N draws from a univariate normal distribution, the Wishart distribution represents the sums of squares (and cross-products) of N draws from a multivariate normal distribution. For examples of the use of this distribution in weak lensing see [Taylor et al. \(2013\)](#); [Joachimi and Taylor \(2014\)](#).

6. Inverse Wishart Distribution

For x being an $N \times p$ matrix, with each row being independently drawn from a p -variate normal distribution with mean $\mu = 0$, the $p \times p$ random scatter matrix $\mathbf{S} = X^T X$ (note that this is the inverse of \mathbf{S} in Wishart distribution) has a inverse Wishart distribution with N degrees of freedom and covariance \mathbf{C}

$$\mathcal{W}_p^{-1}(\mathbf{C}, N) = \frac{1}{2^{Np/2} |\mathbf{S}|^{N+p+1/2} \Gamma_p(N/2)} |\mathbf{C}|^{N/2} \exp \left[-\frac{1}{2} \text{tr}(\mathbf{C} \mathbf{S}^{-1}) \right], \quad (\text{PD.8})$$

where Γ_p is the multivariate Gamma function. In cosmology, they are useful as normally parameters precision matrices (inverse of covariance matrices) have inverse Wishart distributions. The Inverse-Wishart distribution is the multivariate extension of the inverse-gamma distribution — in case of integer degrees of freedom, it is a multivariate generalisation of the inverse χ^2 -distribution; i.e. for a single data point ($p = 1$), the inverse Wishart distribution is the inverse χ^2 distribution. For examples of the use of this distribution in weak lensing see [Taylor et al. \(2013\)](#); [Joachimi and Taylor \(2014\)](#).

7. Cauchy (Lorentz) Distribution

Far data x , Cauchy distribution is defined as

$$f(x; x_0, \gamma) = \frac{1}{\pi \gamma} \left[\frac{\gamma^2}{(x - x_0)^2 + \gamma^2} \right], \quad (\text{PD.9})$$

where x_0 specifies the location of the peak of the distribution and γ is the HWHM (half width half maximum). This distribution describes the resonance behaviour and the distribution of horizontal distances at which a line segment tilted at a random angle cuts the x -axis.

8. Marsaglia-Tin Distribution

A probability density function (PDF) for ratios of two random Gaussian variables with arbitrary means and correlation. This was first discussed in [Marsaglia \(1965\)](#) and [Tin \(1965\)](#). This distribution is encountered

in weak lensing, as estimating the ellipticity of a galaxy involves a ratio of such variables. For example, the ellipticity can be estimated using the semi-major and semi-minor axis of the galaxy shape $\epsilon = [(a-b)/(a+b)] \exp(2i\phi)$. The Marsaglia-Tin distribution was further generalised by [Viola et al. \(2014\)](#) to the case of two ratios constructed from three correlated random variables, which is relevant for ellipticity measurements.

Projected Density: The projected density refers to the integrated mass density along a line of sight.

Pseudo-Cl: The shear field can be expanded in terms of spherical harmonic functions $Y_{\ell m}$ as

$$\gamma = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m} , \quad (\text{PCL.1})$$

with $a_{\ell m}$ being the spherical harmonic coefficients. For a Gaussian γ with zero mean, $\langle a_{\ell m} \rangle = 0$, the power spectrum is the variance

$$\langle a_{\ell m} a_{\ell' m'}^* \rangle = \delta_{\ell \ell'} \delta_{m m'} C_{\ell}^{\text{th}} , \quad (\text{PCL.2})$$

where C_{ℓ}^{th} is the shear angular power spectrum. We only observe a realisation of this underlying power spectrum on our sky, which we can estimate using the *empirical power spectrum estimator* defined as

$$\hat{C}_{\ell}^{\text{th}} = \frac{1}{2\ell+1} \sum_{m=-\ell}^{\ell} |a_{\ell m}|^2 , \quad (\text{PCL.3})$$

where $\hat{C}_{\ell}^{\text{th}}$ is an unbiased estimator of the true underlying power spectrum; $\langle \hat{C}_{\ell}^{\text{th}} \rangle = C_{\ell}^{\text{th}}$, in the case of noiseless data over full sky. Applying a mask on the sky results in the following modification of the spherical harmonic coefficients:

$$\tilde{a}_{\ell m} = \int d\Omega \gamma(\Omega) W(\Omega) Y_{\ell m}^*(\Omega) , \quad (\text{PCL.4})$$

where $W(\Gamma)$ is the window function applied to the data. The presence of the window function induces correlations between the $a_{\ell m}$ coefficients at different ℓ and different m . One can define the *pseudo power spectrum* \tilde{C}_{ℓ} as the application of the empirical power spectrum estimator on the spherical harmonic coefficients of the masked sky. In case of data contaminated with additive Gaussian stationary noise, the pseudo power spectrum is

$$\tilde{C}_{\ell} = \frac{1}{2\ell+1} \sum_{m=-\ell}^{\ell} |\tilde{a}_{\ell m} + \tilde{n}_{\ell m}|^2 , \quad (\text{PCL.5})$$

where $\tilde{n}_{\ell m}$ are the spherical harmonic coefficients of the masked instrumental noise.

Following the MASTER method from [Hivon et al. \(2002\)](#), the pseudo power spectrum \tilde{C}_{ℓ} and the empirical power spectrum $\hat{C}_{\ell}^{\text{th}}$ can be related through their ensemble averages:

$$\langle \tilde{C}_{\ell} \rangle = \sum_{\ell'} M_{\ell \ell'} C_{\ell'}^{\text{th}} + \langle \tilde{N}_{\ell} \rangle , \quad (\text{PCL.6})$$

where $M_{\ell \ell'}$ describes the mode-mode coupling between modes ℓ and ℓ' resulting from computing the transform on the masked sky. SEE **POWER SPECTRUM**.

PSF: Acronym for Point Spread Function of telescope, which describes the response of the telescope to a point source SEE **IMPULSE RESPONSE FUNCTION (OF TELESCOPE)**. PSF is a convolutive function, which distorts sizes and shapes of galaxies. The observed source has a larger size of

$$R_{\text{obs}}^2 = R_{\text{gal}}^2 + R_{\text{PSF}}^2 , \quad (\text{PSF.1})$$

and a perturbed ellipticity given by

$$\epsilon_{\text{obs}} = \epsilon_{\text{gal}} + \frac{R_{\text{PSF}}^2}{R_{\text{gal}}^2 + R_{\text{PSF}}^2} (\epsilon_{\text{PSF}} - \epsilon_{\text{gal}}) . \quad (\text{PSF.2})$$

Galaxy shapes have to be deconvolved for the effects of PSF, using point objects, such as stellar images. [Kitching et al. \(2013\)](#) review several methods for estimating the PSF from images for weak lensing.

q: SEE **MULTIPLICATIVE AND ADDITIVE BIAS OF SHEAR**.

QE: Acronym for Quantum Efficiency; SEE **QUANTUM EFFICIENCY**.

Quadratic Alignment (model): SEE **INTRINSIC ALIGNMENT**.

Quadrupole Moments: The quadrupole moment is a second-rank tensor. It represents the ellipsoidal shape of an object. In weak lensing it is used to quantify the observed ellipticity of an object, such as a galaxy or a star, and is defined as

$$Q_{ij} = \frac{\int d^2x (x_i - \bar{x}_i)(x_j - \bar{x}_j) I(\mathbf{x}) W(\mathbf{x})}{\int d^2x I(\mathbf{x}) W(\mathbf{x})}, \quad (\text{QM.1})$$

where $\{i, j\} = \{1, 2\}$ and $\mathbf{x} = (x_i, x_j)$ is the 2D position of the object in the image. The weight function $W(\mathbf{x})$ is typically assumed to be a multivariate Gaussian and $I(\mathbf{x})$ is the flux of the image centred at (\bar{x}_i, \bar{x}_j) . The three quadrupole moments $\mathbf{Q} = (Q_{11}, Q_{22}, Q_{12})$ can be related to the ellipticity of an object in two ways;

1. third eccentricity

$$\chi \equiv \chi_1 + i\chi_2 \equiv \frac{Q_{11} - Q_{22} + 2iQ_{12}}{Q_{11} + Q_{22}}. \quad (\text{QM.2})$$

SEE **THIRD ECCENTRICITY**.

2. third flattening

$$\epsilon \equiv \epsilon_1 + i\epsilon_2 \equiv \frac{Q_{11} - Q_{22} + 2iQ_{12}}{Q_{11} + Q_{22} + 2R}, \quad (\text{QM.3})$$

where $R = (Q_{11}Q_{22} - Q_{12}^2)^{1/2}$ is a measure of the size of the image (in some weak lensing literature this is referred to as R^2 ; SEE **R-SQUARED**). SEE **THIRD FLATTENING**.

There is a simple relation between third flattening and third eccentricity $\chi = 2\epsilon/(1 + |\epsilon|^2)$. The definitions above reflect ellipticity as a complex quantity, defined by an amplitude $|\epsilon|$ or $|\chi|$ and a position angle θ such that, e.g., $\epsilon = |\epsilon|e^{2i\theta}$, reflecting the fact that ellipses are symmetrical under a rotation of π . SEE **ELLIPTICITY (OBSERVED)**; **GALAXY SIZE**.

Quantum Efficiency: Quantum efficiency (QE) measures the effectiveness of a CCD to produce electronic charge when hit by incident photons; the percentage of photons that are actually detected (due to photoelectrons being produced) is known as the Quantum Efficiency (QE). For example, the human eye only has QE $\simeq 20\%$, a photographic film has QE $\simeq 10\%$. QE is wavelength dependent and it directly effects the S/N of the CCD. Hence a CCD is generally chosen which has the highest QE in the desired wavelength range. The key factor in determining the QE of a CCD is the actual structure of the CCD. In Euclid CCDs, the QE range from QE $\simeq 50\%$ at wavelength $\lambda = 900$ to QE $\simeq 90\%$ at wavelength $\lambda = 750$.

Quasi-linear (Matter Power Spectrum): The part of the matter power spectrum at k -modes that are slightly larger than those that can be computed using linear perturbation theory of the matter over-density field. These regimes have approximately equal contributions from the linear and non-linear parts of the matter power spectrum. At a redshift of zero these are typically $0.1 \lesssim k \lesssim 1 h\text{Mpc}^{-1}$; SEE **LINEAR (MATTER POWER SPECTRUM)**; **NON-LINEAR (MATTER POWER SPECTRUM)**

R-squared: A particular estimate for the size of a galaxy, obtained from galaxy images, using quadrupole moments. It is defined as

$$R^2 = (Q_{11}Q_{22} - Q_{12}^2). \quad (\text{RS.1})$$

In some weak lensing literature this is referred to as A , since it is also related to the projected area of the galaxy in question (Israel et al. 2015a). SEE **ELLIPTICITY (OBSERVED)**; **QUADRUPOLE MOMENTS**; **GALAXY SIZE**.

R200 or R500: R_{200} or R_{500} are the radii at which the enclosed mean density of a cluster is either 200 or 500 times the critical density of the Universe.

Radial Profile (of a Galaxy): The radial profile of galaxy refers to the projected intensity of light as a function of radius from the projected centre of a galaxy.

Raw Data: Raw data, or primary data, is collected data from a source that has not been processed or reduced.

Ray Tracing: A technique of calculating the light propagation through large-scale structure in N-body simulations. The path along the line of sight is divided into several planes and light rays are followed from one plane to next plane along the deflected direction, which is calculated on the current lens plane. This approach takes into account non-linear couplings between lens planes, so there is no one-to-one mapping between the light cone of emitted rays from an object at high redshift and the observer's field-of-view. Light rays are traced backwards from the observer to the emitting object plane to make sure each photon reaches the observer; see e.g. Blandford and Narayan (1986); Jain et al. (2000).

Red Galaxies: SEE **BLUE AND RED GALAXIES**.

Redshift z : Redshift quantifies the amount by which the light from a distant object, such as a galaxy, is moved to the redder end of the spectrum. Galaxies have redshifts caused by the expansion of the Universe. The redshift is defined by

$$z = \frac{\lambda_{\text{obs}} - \lambda_{\text{emit}}}{\lambda_{\text{emit}}} , \quad (\text{ZZ.1})$$

where λ_{obs} is the observed wavelength and λ_{emit} is the emitted/absorbed wavelength. The redshift can be related to the dimensionless scale factor in the following way

$$z = \frac{a(t_{\text{obs}})}{a(t_{\text{emit}})} - 1 \quad (\text{ZZ.2})$$

where $a(t)$ are the scale factors at the cosmic time t (the proper time measured by an observer at rest to the local matter distribution, or ‘substratum’). Because, by convention, $a(t_{\text{obs}}) = 1$ the redshift is usually related to the scale factor simply by $a = (1 + z)^{-1}$. For local galaxies the redshift is related to the apparent recessional velocity by $z \approx v/c$, where v is the recessional velocity and c is the speed of light in vacuum. Note that local galaxies could have a redshift (or a blueshift) due to their local peculiar velocity.

Reduced Shear g : Weak lensing affects the *shapes* of distant objects, inducing a change in the image of a galaxy that is a combination of the shear and convergence. In a measurement of the ellipticity, or inference of shear, it is the ‘reduced shear’ that is measured. This is defined as

$$g = \frac{\gamma}{1 - \kappa} . \quad (\text{RS.1})$$

Where γ is the (not reduced) shear, and κ is the convergence. This has the same properties as shear such as spin-2 transformation properties. Weak lensing is the regime where the effect of gravitational lensing is very small, with both the convergence and the shear much smaller than unity. Therefore, γ is a good approximation of g to linear order. The total reduced shear g_{tot} contains contributions from both the gravitational reduced shear and intrinsic reduced shear due to the intrinsic alignments of galaxies.

Scattered Light (Gegenschein): This is the sunlight scattered by interplanetary dust in the ecliptic plane of the Solar system, in the anti-solar direction — mostly at the L2 Earth-Sun Lagrangian point, due to the concentration of particles at L2. It forms an oval-shaped glow directly opposite the Sun within the band of zodiacal light. This is strong enough to be taken into account in the observations of surveys. SEE **ZODIACAL LIGHT**.

SED: The Spectral Energy Distribution (SED) is the energy (or wavelength) spectrum of a given astronomical object.

Shapelets: Shapelets²⁶ are a complete and orthonormal set of 2D basis functions (like Fourier basis), which can be used to model galaxy image; i.e. galaxy images are decomposed into several shape components, where each of them provides independent estimates of the local shear. Their use in astronomy was first introduced in Refregier (2003); Refregier and Bacon (2003).

Shear Power Spectrum: Tomographic cosmic shear power spectrum is the auto- and cross-correlation of shear of galaxies at different redshift bins

$$C_{ij}(\ell) = \int_0^{r_H} dr W_{ij}^{GG}(r) P_{\delta\delta} \left(k = \frac{\ell}{S_k(r)}; r \right) , \quad (\text{SP.1})$$

where $P_{\delta\delta}(k = \ell/S_k(r); r)$ is the 3D density matter power spectrum, with $S_k(r)$ being the comoving angular diameter distance and r being the comoving distance. The lensing weight function $W_{ij}^{GG}(r)$ is expressed as

$$W_{ij}^{GG}(r) = \frac{q_i(r)q_j(r)}{S_k^2(r)} , \quad (\text{SP.2})$$

with kernel

$$q_i(r) = \frac{3H_0^2\Omega_m S_k(r)}{2a(r)} \int_r^{r_H} dr' p_i(r') \frac{S_k(r' - r)}{S_k(r')} , \quad (\text{SP.3})$$

where ij subscripts refer to redshift bins, r_H is the horizon distance and a is the scale factor. The comoving source galaxy probability distribution $p_i(r)$ is given by $p_i(z) \propto z^2 \exp(-1.4z/z_m)^{1.5}$, where z_m is the median redshift of the survey and $p_i(r)dr = p_i(z)dz$. SEE **POWER SPECTRUM**.

²⁶ See e.g. <http://community.dur.ac.uk/r.j.massey/shapelets/>.

Simulation: This is the imitation of the operation of a process or system over time, such as the Universe. Numerical simulations play a significant role in cosmology. The first simulations started in 1960s and 1970s (Aarseth 1963; Peebles 1970; Press and Schechter 1974). The important factors that should be taken into account for simulations are 1) setting the initial conditions, 2) setting the equations of evolution of fluctuations that were set in the initial conditions. Some of the simulations of the Universe include

1. Millennium <https://wwwmpa.mpa-garching.mpg.de/galform/virgo/millennium/>
2. Illustris <http://www.illustris-project.org>
3. EAGLE <http://icc.dur.ac.uk/Eagle/>

Simulations are also extensively used to capture the complexity of different steps in the 'end-to-end' pipelines in different experiments. For example, to estimate cosmological parameters, simulations are used to estimate the covariance matrix. Simulations are also used to test different methods and algorithms available before taking data in experiments. See e.g. Klypin (2000) for a review. Simulations of the imaging process in weak lensing can also be performed. SEE **STEP**; **GREAT**.

Single-epoch: Weak lensing observations that were taken a single time; the singular case of 'multi-epoch'. See SEE **MULTI-EPOCH**.

Singular Isothermal Sphere: Singular Isothermal Sphere (SIS) is the simplest symmetrical parameterisation of the matter distribution in astronomical systems, such as galaxies. The density distribution is defined as

$$\rho(r) = \frac{\sigma_V^2}{2\pi G r^2}, \quad (\text{SIS.1})$$

where σ_V^2 is the velocity dispersion. The SIS profile is unphysical because of the singularity at zero radius and the fact that the total mass calculated by integrating the function out to infinite radius does not converge (i.e., it is infinite). However, it is commonly utilised in the literature due to the simplicity of its form. In lensing, SIS is one of the simplest axially symmetric analytical models to describe the matter distribution of extended lenses. The main advantage of using axially symmetric lenses is that their surface density is independent on the position angle with respect to lens centre and this reduces the lensing equations a one-dimensional form.

Size Magnification: Gravitational lensing conserves surface brightness, a consequence from Liouville's theorem which holds in any passive optical system. Since the apparent size of resolved background objects change, their flux changes as well. These two effects are manifestations of gravitational magnification, and can be used as weak-lensing observables in addition to the deformation (shear) of galaxy shapes. See e.g. Casaponsa et al. (2013) for a recent study on this. SEE **MAGNIFICATION**.

SKA: The Square Kilometre Array (SKA)²⁷ is an international radio telescope project that will be built in Australia and South Africa, with a total collecting area of approximately one square kilometre, with receiving stations extending out to distance of at least 3,000 kilometres from a central core. SKA will be 50 times more sensitive than any other radio instrument. It will cover a wide range of frequencies, using thousands of dishes and up to a million antennas. It will be able to survey the sky more than ten thousand times faster than ever before. Construction of the SKA is scheduled to begin in 2018 for initial observations by 2020.

Slew: A slew is the rotation of a spacecraft from one pointing direction to another. Generally this is a relatively large rotation with acceleration, constant speed and deceleration phases. In the weak lensing this may refer to HST or Euclid missions. For Euclid this refers to a rotation of 0.7 times the field-of-view, i.e. approximately the field of view of the instruments or more, and it entails the reacquisition of the knowledge of the pointing direction at its termination using an on-satellite star catalog.

A dither is a smaller rotation, with pointing directions within-field, and sometimes as small as arc seconds. Knowledge of the pointing direction is generally maintained during the dither. Dithers are used to maximise the utility of multiple exposures, by moving the image with respect to the detector pixel grids, to recover spatial resolution in slightly under-sampled images. In the Euclid context the multiple exposures themselves (as opposed to the re-pointings) are often also called dithers, but the meaning is generally clear from the context. SEE **DITHER**.

Small-Scales: Scales that are not defined, within context, as 'large scales' (SEE **LARGE SCALES**). Usually used in reference to the part of the matter power spectrum that cannot be computed using linear perturbation theory of the matter over-density field, and requires N-body simulations to determine its properties and behaviour; SEE **LINEAR (MATTER POWER SPECTRUM)**.

Source: In lensing source galaxies are the ones whose images are lensed.

Source Clustering: When the source galaxies have intrinsic clustering due to their local environment and gravitational attraction between the local galaxies. Bernardeau (1998) have studied the effect of source clustering on weak lensing statistics.

²⁷ <https://www.skatelescope.org/project/>

Source Plane: In lensing a hypothetical surface at the comoving distance upon which source galaxies lie. SEE [LENS PLANE](#); [IMAGE PLANE](#).

Source Population: Population of source galaxies on a source plane, or series of source planes.

Specz: Shorthand for ‘a spectroscopic redshift’, where a prism is used to measure the intensity of light across frequency (or wavelength) of characteristic spectral lines. The shift of these lines with respect to their laboratory positions gives the redshift of the object.

Specz-Photz Plot: A scatter plot of spectroscopic redshift estimates, for a population of galaxies, versus an estimator for the photometric redshifts of the same galaxies. Typically used to show graphically the ability of a photometric redshift estimation code to determine the spectroscopic redshifts of galaxies, see e.g. [Hildebrandt et al. \(2010\)](#) for use of this representation to determine photometric redshift accuracy and precision; SEE [PHOTOMETRIC REDSHIFTS \(PHOTO-Z; PHOTO-ZEE; PHOTO-ZED\)](#).

Spherical Harmonics: A series of functions defined on the surface of a sphere used to solve some kinds of differential equations. They are similar to Fourier transform, but operate on the surface of a sphere. Spherical harmonics are defined as the angular portion of a set of solutions to Laplace’s equation in three dimensions. Represented in a system of spherical coordinates with θ and ϕ , Laplace’s spherical harmonics Y_ℓ^m are a specific set of spherical harmonics that forms an orthogonal system, first introduced by Pierre Simon de Laplace in 1782. They form a complete orthonormal set on the unit sphere and are defined as

$$Y_\ell^m = \sqrt{\frac{2\ell+1}{4\pi} \frac{(l-m)!}{(l+m)!}} P_\ell^m(\cos \theta) e^{im\phi}, \quad (\text{SH.1})$$

where ℓ is the multipole representing angular scale α as $\alpha \simeq \pi/\ell$, with $\ell = 0, \dots, \infty$. The order m ranges as $-\ell \leq m \leq \ell$ and P_ℓ^m are the Legendre polynomials. Spherical harmonics are important in cosmology for representation of gravitational fields, planetary bodies and stars, and characterisation of the cosmic microwave background radiation. In weak lensing the cosmic shear field can be described using spherical harmonics supplemented by a radial Bessel function transform, see e.g. [Castro et al. \(2005\)](#).

Spin: In quantum mechanics and particle physics, spin is an intrinsic form of angular momentum carried by elementary particles, composite particles (hadrons), and atomic nuclei. The conventional definition of the spin quantum number is $s = n/2$, where n can be any non-negative integer. Hence the allowed values of s are 0, 1/2, 1, 3/2, 2, etc.

Parallels are used in weak lensing where the shearing of images of objects is a spin-2 field, meaning under a rotation of π in the coordinate system the field is left unchanged. A rotation by $\pi/2$ changes γ_1 to γ_2 and γ_2 to $-\gamma_1$. A rotation of counterclockwise ϕ changes $(\gamma_1 + i\gamma_2) \rightarrow (\gamma_1 + i\gamma_2) \exp(-2i\phi)$; see e.g. [Kilbinger \(2015\)](#). The weak lensing ellipticity field is therefore a spin-2 field, which is locally symmetric under 180 degree transformations, whereas the convergence field is a spin-0 field (i.e. a scalar).

Spin Weight: This is the type of weighting in a statistic that the spin of the field in question has been implied. For example, ellipticity is a spin-2 ($s = 2$) quantity and whenever the term $\exp(-2i\phi)$ appears in some statistics it would result in a spin-weighted quantity. As a specific example spherical harmonics needs to be modified for fields with a general spin, resulting in spin-weighted spherical harmonics (see [Castro et al. 2005](#)).

Stage (dark energy): In [Albrecht et al. \(2006\)](#) the Dark Energy Task Force defined several different levels of dark energy experiments in order to rank their relative capabilities. The ranking was done using their defined Figure of Merit (FoM) which is the inverse of the area of the predicted 68% confidence region for an experiment in the (w_0, w_a) plane. This is commonly approximated in the majority of the literature by using a Fisher matrix where

$$\text{FoM} = \frac{1}{\det(S_{w_0 w_a}[F])}, \quad (\text{SDE.1})$$

where the denominator is the determinant of the (w_0, w_a) Schur compliment of a Fisher matrix containing cosmological parameters. The dark energy stages were divided into four categories, where a Stage I experiment was the least constraining experiment and Stage IV was the most constraining. As of 2006, these stages were defined in [Albrecht et al. \(2006\)](#) as

1. Stage I represents what is now known.
2. Stage II represents the anticipated state of knowledge upon completion of ongoing projects that are relevant to dark energy.
3. Stage III comprises near-term, medium-cost, currently proposed projects.
4. Stage IV comprises a Large Survey Telescope (LST), and/or the Square Kilometer Array (SKA), and/or a Joint Dark Energy (Space) Mission (JDEM).

The acronyms used in the Stage IV definitions reflect proposed experiments at that time. The projects that will satisfy the Stage IV capabilities and are now (c. 2016) in construction are Euclid, LSST, SKA, DESI and WFIRST.

Standard Cosmological Model: SEE [LCDM](#).

Star: In weak lensing the word ‘star’ can either refer to the physical object ‘a star’ (see [Ridpath 2013](#)), or in a different context to mean a ‘point-like object’ in an image which is an indicator of the Point-Spread-Function (PSF) of the instrument, telescope and atmosphere (if present). For example ‘the stars in this image look broad’ would mean the PSF inferred from the images of stars (which are point-like objects) has a large width, not the stars themselves are physically larger. Derivatives of this term are ‘stellar’; for example, the ‘stellar ellipticity’ is a reference not to the ellipticity of the physical object, the star, but rather to the ellipticity of the observed PSF in an image.

Star-Galaxy: Star-galaxy is used in reference to statistics that involve the multiplication of quantities derived from stars (point-like objects) in an image, that are a measure the PSF, and quantities derived from galaxy observations in the same image. A common statistic used is the star-galaxy cross-correlation which is the 2-point configuration space correlation function of the derived stellar ellipticity and the galaxy ellipticities. In [Heymans et al. \(2012b\)](#) this statistic is described by defining an observed ellipticity ϵ^{obs} as

$$\epsilon^{\text{obs}} = \epsilon^{\text{int}} + \gamma + \eta + A_{\text{sys}}^T \epsilon^*, \quad (\text{SG.1})$$

where ϵ^{int} is the unlensed ‘intrinsic’ ellipticity of a galaxy, γ is the weak lensing shear, η is a random component caused by noise in the images and ϵ^* is the ellipticity of the PSF measured using images of stars (A_{sys}^T is an amplitude that characterises the propagation of PSF ellipticity into observed galaxy ellipticity). By taking the 2-point correlation function of this equation with ϵ^* a statistic can be define

$$\langle \xi_{\text{sg}} \rangle = \langle \epsilon^{\text{int}} \epsilon^* \rangle + \langle \gamma \epsilon^* \rangle + \langle \eta \epsilon^* \rangle + C A_{\text{sys}}^T, \quad (\text{SG.2})$$

where $C_{ij} = \langle \epsilon_i^* \epsilon_j^* \rangle$ is the correlation of the PSF ellipticity between several exposures (i and j). In the absence of systematic effects caused by the PSF this quantity should have a mean of zero and a distribution characterised by intrinsic ellipticity and shear distributions, and has hence been used a test for systematics in weak lensing data.

Stellar Calibration Field: An observed field of galactic stars that will be visited at intervals throughout the lifetime of a weak lensing survey to calibrate the optical system and PSF properties; SEE [PSF](#). These periodic observations enable tracking and characterisation of changes in instrument response.

STEP: The Shear Testing Program (STEP) was a series of weak lensing community challenges [Heymans et al. \(2006\)](#); [Massey et al. \(2007\)](#), which started in 2004 (http://www.roe.ac.uk/~heyman/step/cosmic_shear_test.html). STEP1 involved analysis of ground based images, STEP2 added complex morphologies, STEP3 (SpaceSTEP) analysed space-based images and STEP4 was an abbreviated attempt at a gradual approach to adding increased complexity to simulated images. STEP naturally led into the GRavitational lEnsing Accuracy Testing (GREAT) program; SEE [GREAT](#).

Strong Gravitational Lensing: Strong gravitational lensing is a classification of gravitational lensing effects which is characterised by multiple lensed images of a single source, an d arcs. Strong lensing can occur when the mass density of the lens is greater than a critical density; SEE [SURFACE DENSITY](#), and when the light rays from a source pass within with the critical curve of the lens.

SUNGLASS: Simulated UNiverses for Gravitational Lensing Analysis and Shear Surveys (SUNGLASS) is a pipeline for generating simulated universes for weak lensing and cosmic shear analysis. SUNGLASS performs tomographic cosmic shear analysis using line-of-sight integration through a suite of N-body simulations which can be used for shear analysis, power spectrum estimation and cosmological parameters estimation ([Kiessling et al. 2011a,b](#)).

Super-Sample Covariance: Large-scale modes that exist outside of the survey area are known as super-survey modes. Nonlinear evolution couples smaller-scale modes with the super-survey modes, which produces off-diagonal terms in the power spectrum covariance, known as the super-sample covariance. Super-sample covariance has also been known as “beat coupling” in the literature. For more information see for example [Takada and Hu \(2013\)](#). SEE [BEAT COUPLING](#).

Surface Density: The three dimensional density $\rho(\bar{r})$ of an extended lensing mass can be projected onto a 2D plane, known as the ‘lens plane’ (SEE [BORN APPROXIMATION](#)), to obtain a surface mass density distribution defined as

$$\Sigma(\bar{\xi}) = \int_0^{D_s} \rho(\bar{r}) dr_z, \quad (\text{SD.1})$$

with \bar{r} being a 3D vector in space and $\bar{\xi}$ a 2D vector on the lens plane. The surface density can then be related to the deflection angle for constant surface mass density within a finite azimuthal aperture

$$\alpha(\theta) = \frac{D_{LS}D_L}{D_S} \frac{4\pi G\Sigma}{c^2} \theta, \quad (\text{SD.2})$$

where D_L is the distance to the lens, D_S is the distance to the source and D_{LS} is the lens-source distance. Also, $\xi = D_L\theta$, and θ is the angle of deflection seen by the observer. The critical surface mass density can then be defined as

$$\Sigma_{\text{critical}} = \frac{D_S}{D_{LS}D_L} \frac{c^2}{4\pi G\Sigma}. \quad (\text{SD.3})$$

When the condition $\Sigma > \Sigma_{\text{critical}}$ is satisfied multiple images are produced (i.e. strong lensing).

Systematic Error (Effect): Systematics are effects that bias or contaminate the true astrophysical or cosmological signal. Systematics can originate from instruments, calibration, measurements, inaccurate modelling or even astrophysics (e.g. intrinsic alignments). SEE [ACCURACY](#); [BIAS](#).

Tangential Shear: The components of complex shear γ_1 and γ_2 can be defined relative to a local Cartesian coordinate frame on the sky. However it is often apt to consider the projected shear components in a rotated frame, particularly in the case of galaxy clusters where the centre of the polar coordinate frame can be defined as the centre of the cluster. For a lensing cluster the image distortions are aligned tangentially about the cluster. If ϕ_C specifies the angular position about the centre of the coordinate frame then the tangential and cross-component shears (aligned respectively perpendicular and parallel to the radius vector) are:

$$\gamma_t = -\text{Re}[\gamma e^{-2i\phi_C}] \quad (\text{TS.1})$$

and $\gamma_x = \text{Im}[\gamma e^{-2i\phi_C}]$; where $\gamma = \gamma_1 + i\gamma_2$. An axisymmetric lensing mass should only produce a tangential signal in the shear so the cross-component shear (which should be $\gamma_x = 0$) can be used to estimate the noise on the measurement of the tangential shear.

Tension: When measurements of parameters from different probes or experiments disagree with each other at a statistical level that is considered by the person using this word to be significant. For a recent examples of this in weak lensing see [MacCrann et al. \(2015\)](#); [Dossett et al. \(2015\)](#); [Kitching et al. \(2016\)](#); [Hildebrandt et al. \(2016\)](#).

Theta: In weak lensing this is used to refer to the the great circle distance between two galaxies on the celestial sphere. As given in, for two galaxies $i = 1, 2$ at right ascension and declination (α_i, δ_i) , their great circle distance can be calculated using,

$$\cos \theta = \cos(\alpha_2 - \alpha_1) \cos \delta_1 \cos \delta_2 + \sin \delta_1 \sin \delta_2. \quad (\text{TH.1})$$

See for example [Kilbinger \(2015\)](#).

Third Eccentricity: SEE [ELLIPTICITY](#); [ELLIPTICITY \(OBSERVED\)](#); [QUADRUPOLE MOMENTS](#).

Third Flattening: SEE [ELLIPTICITY](#); [ELLIPTICITY \(OBSERVED\)](#); [QUADRUPOLE MOMENTS](#).

Three-point: The correlation between some variable at three points in space, or with in transformed space such as Fourier space. SEE [CORRELATION FUNCTIONS AND POWER SPECTRA](#); [TWO-POINT](#); [BISPECTRUM](#).

Tomographic Bin: In tomography, the observable is divided into spatial sections in the radial (redshift) direction. The sections are referred to as tomographic bins.

Tomography: The division of some observable into spatial sections. In cosmology and weak lensing, this refers to dividing a population of galaxies into (possibly overlapping) redshift bin. In weak lensing ‘tomographic’ bins are defined by the position of the source background galaxies, but due to the line-of-sight integrated nature of weak lensing, probe projected slices through the Universe.

Top-Hat (correlation function): Refers to an ellipticity correlation function with top-hat functional form as a weight. This is defined in [Kaiser \(1992\)](#) and equations A3 and A4 in [Kilbinger et al. \(2013\)](#).

Total matter: This refers to the sum of dark matter and baryonic matter mass or mass density in a galaxy, galaxy group, galaxy cluster or for the Universe. On cosmological scales this is used to refer to the sum of the dimensionless matter densities: $\Omega_m = \Omega_{\text{dm}} + \Omega_b$.

Tree Rings: Tree rings are an effect in CCDs (resulting from the manufacturing of CCDs) that cause photoelectrons to slightly shift from the position the photon hit the detector to the point the electron was registered. This results in chip-position-dependant systematic bias, which need to be corrected for. See e.g. [Plazas et al. \(2014\)](#).

Two-point: The correlation between some variable at two points in space, or with in transformed space such as Fourier space. SEE **CORRELATION FUNCTIONS AND POWER SPECTRA**.

ugriz: A photometric system with filters ranging from the UV ($u \sim 350\text{nm}$) to the near-infrared ($z \sim 900\text{nm}$). This system was used in the Sloan Digital Sky Survey (SDSS)²⁸.

Vignetting: This is the spatial variation of the transmission of an optical system, generally (but not necessarily) with a reduction towards the perimeter, so that images appear darker in those regions. Vignetting may be caused by inadequate sizes of optical elements intermediate in the optical train, so that they do not intercept the full bundle of rays for objects far from centre of the field-of-view. Alternatively it could be the consequence of mechanical structures such as optical supports and baffles in the ray path blocking the rays. Finally, as the angle from the field centre increases the projected area of the optical system to the ray bundle decreases, but this is a small effect in most astronomical optical systems.

VIKING: VIKING²⁹ is one of the near-infrared public surveys planned on the VISTA telescope. VIKING complements KiDS, SEE KiDS, by observing the same area at five different infrared bands (Z,Y,J,H,K).

Weight Function: A weight function is a mathematical device used when performing a sum, integral, or average to give some elements more weight or influence on the result than other elements in the same set. The result of this application of a weight function is a weighted sum or weighted average. In weak lensing commonly specifically refers to the weight function used in a moment-based measurement of ellipticity; SEE **QUADRUPOLE MOMENTS**.

Weight Map: A weight map is a map which attaches a weight to each pixel on the map. See e.g. [Erben et al. \(2013\)](#) for details of a weight map to describe masking effects in a data analysis chain used for weak lensing.

Weyl Tensor: The Weyle tenor is a measure of the curvature of spacetime (a pseudo-Riemannian manifold). The Weyl tensor expresses the tidal force that a body feels when moving along a geodesic and is used to quantify how the shape of the body is distorted by the tidal force — i.e. a measure image distortions effects in weak and strong lensing (it is the trace-free, anti-symmetric part of the decomposition of the Riemann tensor).

Window Function: A mathematical function with zero values outside a desired range/interval. For example, a top-hat window function is a function that equals 1 inside an interval and zero elsewhere. Window functions are used to zero-value a function in the unwanted range.

Wings (of the PSF): The ‘wings’ of a PSF refer to the extended (and most commonly smoothly varying) component of the point spread function (PSF), i.e. away from the mean (center) of the intensity distribution. For example [Heymans et al. \(2005\)](#) for a reference to this in weak lensing where the wings of the Hubble Space Telescope PSF are investigated.

Zodiacal Light: This is the sunlight scattered by interplanetary dust in the zodiacal cloud in the Solar system, extending up from the vicinity of the Sun along the ecliptic/zodiac. Its intensity covers the whole sky, but decreases with distance from the Sun. This is strong enough to be taken into account in the observations of surveys. SEE **SCATTERED LIGHT (GEGENSCHHEIN)**.

²⁸ <http://skyserver.sdss.org/dr1/en/proj/advanced/color/sdssfilters.asp>

²⁹ <https://www.eso.org/sci/observing/PublicSurveys/sciencePublicSurveys.html>

Long Definitions

Friedman–Robertson–Walker Models: Homogeneous and isotropic models of the universe are described by the metric

$$ds^2 = c^2 dt^2 - a^2(t) [d\chi^2 + f_K^2(\chi) (d\theta^2 + \sin^2 \theta d\varphi^2)] , \quad (\text{FRW.1})$$

where t is the cosmic time, $a(t)$ the (dimensionless) cosmic scale factor, normalized that today, i.e., for $t = t_0$, $a(t_0) = 1$, χ the comoving radial coordinate, θ and φ are the angular coordinates on a unit sphere, and $f_K(\chi)$ the comoving angular diameter distance, which depends on the curvature parameter K in the following way:

$$f_K(\chi) = |K|^{-1/2} \text{sinn}(|K|^{1/2} \chi) := \begin{cases} K^{-1/2} \sin(K^{1/2} \chi) & (K > 0) \\ \chi & (K = 0) \\ (-K)^{-1/2} \sinh[(-K)^{1/2} \chi] & (K < 0) \end{cases} . \quad (\text{FRW.2})$$

The term in brackets in Eq. (FRW.1) describes a homogeneous, isotropic three-dimensional space of constant curvature K . The spatial coordinates χ, θ, φ in Eq. (FRW.1) are called *comoving coordinates*. Object or observers whose worldlines are characterized by constant values of χ, θ and φ are called *comoving sources* or *comoving observers*. Comoving observers all experience the same history of the universe and observe the universe to be isotropic. Their proper time coincides with the cosmic time. If we choose coordinates such that we are at $\chi = 0$, then light rays we receive follow spatially radial paths with $c dt = -a(t) d\chi$, yielding

$$\chi = \int_t^{t_0} \frac{c dt'}{a(t')} . \quad (\text{FRW.3})$$

Hence, the light from a source at comoving distance χ that we receive today was emitted at cosmic time t . From Einstein's field equations follow the Friedman equations, relating the scale factor $a(t)$ to the matter/energy contents of the universe,³⁰

$$H^2(a) := \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_N}{3} \rho(a) - \frac{K c^2}{a^2} ; \quad \frac{\ddot{a}}{a} = -\frac{4\pi G_N}{3} \left(\rho + \frac{3p}{c^2}\right) , \quad (\text{FRW.4})$$

with the *Hubble function* $H(a)$, energy density $\rho(a) c^2$, and isotropic pressure $p(a)$, as measured by comoving observers. The current value of the Hubble function is the *Hubble constant* H_0 . The Hubble constant yields the overall scale of the Universe, and all lengths scale like $c H_0^{-1}$ when redshifts are used as a measure of distance. The uncertainty in the value of the Hubble constant is typically parametrized by writing

$$H_0 = h_x x \text{ km s}^{-1} \text{ Mpc}^{-1} , \quad (\text{FRW.5})$$

where x is a number. For example, traditionally one used h_{100} , so that $H_0 = h_{100} 100 \text{ km s}^{-1} \text{ Mpc}^{-1}$.

Density ρ and pressure p contain the various energy components in the universe, characterized by an *equation-of-state* $p_i = w_i \rho_i c^2$. Pressureless matter (or ‘dust’; after just called ‘matter’) has $w_m = 0$, $p_m = 0$; radiation is characterized by $w_r = 1/3$, $p_r = \rho_r c^2/3$. For dark energy, we write $w_{\text{DE}} \equiv w$, and $p_{\text{DE}} = w \rho_{\text{DE}} c^2$. In the special case that dark energy is described by a constant vacuum energy density, or equivalently, a cosmological constant, $w = -1$, and $p_\Lambda = -\rho_\Lambda c^2$. The adiabatic equation,

$$d(\rho c^2 a^3) = -p da^3 , \quad (\text{FRW.6})$$

then yields for non-interacting energy components

$$\rho_m(a) = \rho_{m,0} a^{-3} , \quad \rho_r(a) = \rho_{r,0} a^{-4} , \quad \rho_{\text{DE}}(a) = \rho_{\text{DE},0} \exp\left(-3 \int_1^a da' \frac{1+w(a')}{a'}\right) , \quad (\text{FRW.7})$$

where the additional subscript ‘0’ indicates densities taken at the current epoch. These are converted into the dimensionless *density parameters* Ω_i by dividing them by the *critical density*

$$\rho_{\text{cr}} = \frac{3H_0^2}{8\pi G_N} , \quad (\text{FRW.8})$$

to obtain

$$\Omega_r = \frac{\rho_{r,0}}{\rho_{\text{cr}}} , \quad \Omega_m = \frac{\rho_{m,0}}{\rho_{\text{cr}}} , \quad \Omega_{\text{DE}} = \frac{\rho_{\text{DE},0}}{\rho_{\text{cr}}} . \quad (\text{FRW.9})$$

Analogous expressions hold for baryons (subscript ‘b’), cold dark matter (subscript ‘c’), hot dark matter (subscript ‘HDM’ or ‘ ν ’, if it refers to neutrinos only), etc. The total density parameter is $\Omega_{\text{tot}} = \sum_i \Omega_i$, and is related

³⁰ Note that the Friedman equations are frequently written by explicitly including a term from the cosmological constant Λ . In this case, ρ and p refer to the density and pressure of matter and radiation only. Here, the cosmological constant is included as a special case of dark energy, with an equivalent density ρ_Λ and pressure $p_\Lambda = -\rho_\Lambda c^2$.

to the curvature through $K = (\Omega_{\text{tot}} - 1)H_0^2/c^2$. The Hubble function $H(a)$ in terms of the density parameters is given by

$$E^2(a) \equiv \left(\frac{H(a)}{H_0} \right)^2 = \frac{\Omega_r}{a^4} + \frac{\Omega_m}{a^3} + \frac{(1 - \Omega_{\text{tot}})}{a^2} + \Omega_{\text{DE}} \exp \left(-3 \int_1^a da' \frac{1 + w(a')}{a'} \right), \quad (\text{FRW.10})$$

The Friedman equation (FRW.10) is a first-order differential equation for $a(t)$, and its solution is specified by the current expansion rate H_0 . Classifying the general properties of its solutions in terms of the density parameters, together with lower limits on Ω_m obtained from cosmological observations, yields the result that $a \rightarrow 0$ at a finite time in the past. That epoch is called *Big Bang*. Relics of the Big Bang, such as the helium abundance in the Universe and the existence of the cosmic microwave background, confirm this prediction of the cosmological model.

The *cosmological redshift* z of a comoving source seen by a comoving observer today is related to the scale factor through $(1 + z) = 1/a$. The scale factor at which the energy density of matter and radiation are equal is $a_{\text{eq}} = \Omega_r/\Omega_m$, corresponding to the *redshift of matter-radiation equality* $z_{\text{eq}} = a_{\text{eq}}^{-1} - 1$.³¹

From the metric (FRW.1) we see that the comoving angular-diameter distance f_K is the ratio of the comoving transverse extent of an object, $f_K(\chi) d\theta$ and the angle $d\theta$ as observed by an observer at $\chi = 0$. The *angular diameter distance* $D_A(z)$ out to redshift z relates the physical size $dR = a(t) f_K(\chi) d\theta$ of a comoving object to its angular size $d\theta$ on the sky, $D_A = dR/d\theta$. It is thus related to the comoving angular diameter f_K though $D_A(z) = a f_K(\chi) = (1 + z)^{-1} f_K(\chi)$.

The angular-diameter distance of a source at redshift z_2 seen by an observer along the same line-of-sight at redshift $z_1 < z_2$ is given as

$$D_A(z_1, z_2) = \frac{1}{1 + z_2} f_K(\chi_2 - \chi_1), \quad (\text{FRW.11})$$

where χ_i is the comoving distance out to redshift z_i .

The luminosity distance $D_L(z)$ yields the relation between the bolometric luminosity L of an isotropically radiating comoving source at redshift z to the observed bolometric flux S ,

$$D_L(z) \equiv \sqrt{\frac{L}{4\pi S}}. \quad (\text{FRW.12})$$

For any metric theory of gravity, $D_L(z) = (1 + z)^2 D_A(z)$, sometimes called the ‘duality relation’ of distances. The comoving distance χ of a source as a function of its cosmological redshift is obtained from (FRW.3) and (FRW.4),

$$\chi = \int_{(1+z)^{-1}}^1 \frac{c da}{H(a) a^2}. \quad (\text{FRW.13})$$

The proper and comoving volume elements corresponding to a solid angle $d\omega$ and a redshift interval dz at redshift z are

$$dV_{\text{prop}} = D_A^2(z) d\omega \frac{dD_{\text{prop}}}{dz} dz; \quad dV_{\text{com}} = a^{-3} dV_{\text{prop}} = f_K^2(\chi(z)) d\omega \frac{d\chi}{dz} dz, \quad (\text{FRW.14})$$

where $dD_{\text{prop}} = c dt = a(z) d\chi$. Finite volumes are obtained from these expression by integration.

Since the speed of light is finite, light can only propagate a finite distance from the Big Bang until a given epoch. This distance is called *horizon*. The comoving horizon size at redshift z is given as

$$r_{\text{H,com}}(z) = \int_0^{(1+z)^{-1}} \frac{c da}{a^2 H(a)}. \quad (\text{FRW.15})$$

Geodesic Deviation Equation: In any metric theory of gravity, the transverse separation vector ξ of two infinitesimally close light rays evolves according to the *equation of geodesic deviation*,

$$\frac{d^2 \xi}{d\lambda^2} = \mathcal{T} \xi, \quad (\text{GDE.1})$$

where λ is the affine parameter along the ray, and $\mathcal{T}(\lambda)$ is the *optical tidal matrix* which depends on the Riemann curvature tensor and the wave-vector of the rays.

³¹ Health warning: Cosmologists often use the same symbol for meaning totally different mathematical functions. For example, they write $H(a)$ to mean the function given in (FRW.10), but also write $H(z)$ to imply the function obtained from $H(a)$ by replacing a by $(1 + z)^{-1}$. of course, this yields a *mathematically* very different function, but usually no confusion arises by this practice.

We consider a light bundle with vertex at λ_0 around a fiducial ray. Each ray of the bundle is specified by the angle θ it encloses with the fiducial ray at the vertex. The linearity of (GDE.1) allows us to write $\xi = \mathcal{D}\theta$, where the distance matrix $\mathcal{D}(\lambda)$ obeys

$$\frac{d^2 \mathcal{D}}{d\lambda^2} = \mathcal{T} \mathcal{D}, \quad (\text{GDE.2})$$

with $\mathcal{D}(\lambda_0) = 0$ and $(d\mathcal{D}/d\lambda)(\lambda_0) = \mathcal{I}$, with \mathcal{I} being the 2×2 unit matrix, if λ is chosen such that it locally agrees with proper distance. The *angular-diameter distance* to a point λ as measured from the vertex at λ_0 is defined as the square root of the ratio of the cross-sectional area of the light bundle and its solid opening angle; hence, $D_A(\lambda) = \sqrt{\det \mathcal{D}(\lambda)}$. For an isotropic space-time, such as the Friedman–Robertson–Walker model, the tidal matrix is proportional to the unit matrix, and the matrix equation (GDE.2) reduces to a scalar equation. In this case, the angular-diameter as defined here reduces to the one given in [Friedman–Robertson–Walker Models](#). A point along the light bundle where $\det \mathcal{D}(\lambda) = 0$ (for $\lambda \neq \lambda_0$) is called a *caustic point* or *conjugate point*. The caustics defined for a gravitational lens system in LENS EQUATION is a special case of this general definition.

Structure Formation in the Universe: The formation of structure in the Universe is a complex research field that cannot be described in the framework of this document. Here, we provide the basic quantities for the simplest case, namely the growth of density perturbations of pressureless matter on scales smaller than the horizon scale.

In the framework of an inhomogeneous universe, the mean density of matter as function of epoch is denoted by $\bar{\rho}_m(t)$ (or with scale factor a , redshift z , or comoving distance χ as argument). The fractional matter density contrast is defined as

$$\delta_m(\mathbf{x}, t) := \frac{\rho_m(\mathbf{x}, t) - \bar{\rho}_m(t)}{\bar{\rho}_m(t)}, \quad (\text{LSS.1})$$

where \mathbf{x} is the comoving spatial position, and the peculiar velocity is denoted by $\mathbf{v}(\mathbf{x}, t)$. Describing the behaviour of the matter field using the fluid approximation, and neglecting pressure (which is a valid approximation for describing (cold) dark matter in the epochs of the Universe for which $a \gg a_{\text{eq}} = \Omega_r/\Omega_m \ll 1$, as long as the density contrast $\delta_m \lesssim 1$), one obtains the coupled set of continuity equation, pressure-less Euler equation, and Poisson equation,

$$\frac{\partial \delta_m}{\partial t} + \frac{1}{a} \nabla \cdot [(1 + \delta_m) \mathbf{v}] = 0; \quad \frac{\partial \mathbf{v}}{\partial t} + \frac{\dot{a}}{a} \mathbf{v} + \frac{1}{a} (\mathbf{v} \cdot \nabla) \mathbf{v} = -\frac{1}{a} \nabla \Phi_N; \quad \nabla^2 \Phi_N = \frac{3H_0^2 \Omega_m}{2a} \delta_m, \quad (\text{LSS.2})$$

where the spatial derivatives are with respect to comoving coordinates \mathbf{x} , and $\dot{a} \equiv da/dt$. Assuming the fraction density contrast to be small, $|\delta_m| \ll 1$, this set of equations can be linearized in δ_m and $|\mathbf{v}|/c$, and then combined into a single equation,

$$\frac{\partial^2 \delta_m}{\partial t^2} + \frac{2\dot{a}}{a} \frac{\partial \delta_m}{\partial t} - \frac{3H_0^2 \Omega_m}{2a^3} \delta_m = 0. \quad (\text{LSS.3})$$

Since the coefficients of this partial differential equation have no dependence on \mathbf{x} , its solution can then be factorized, $\delta_m(\mathbf{x}, t) = D_+(t) \Delta_+(\mathbf{x}) + D_-(t) \Delta_-(\mathbf{x})$, where the $\Delta_{\pm}(\mathbf{x})$ are arbitrary functions of \mathbf{x} , to be determined from the initial conditions, and $D_{\pm}(t)$ satisfy the ordinary differential equation

$$\ddot{D} + \frac{2\dot{a}}{a} \dot{D} = \frac{3H_0^2 \Omega_m}{2a^3} D. \quad (\text{LSS.4})$$

D_- is a decreasing function of time, thus these modes decay during cosmic evolution. The solution D_+ , called the *growth factor*, increases in time and hence describes the growth of density fluctuations. It is normalized to unity today, $D_+(t_0) = 1$. If the dark energy is described by a cosmological constant, then

$$D_+(t) \propto H(t) H_0^2 \int_0^t \frac{dt'}{a^2(t') H^2(t')}, \quad \text{or} \quad D_+(a) \propto \frac{H(a)}{H_0} \int_0^a \frac{da'}{[\Omega_m/a' + \Omega_\Lambda a'^2 - (\Omega_m + \Omega_\Lambda - 1)]^{3/2}}. \quad (\text{LSS.5})$$

The solution $\delta_m(\mathbf{x}, t) = D_+(t) \delta_{m,0}(\mathbf{x})$, where $\delta_{m,0}(\mathbf{x})$ is the *linearly extrapolated density contrast today*, with the function $D_+(t)$ being the growing solution of Eq. (LSS.4), is valid only for subhorizon perturbations in the matter-dominated epoch ($a \gg a_{\text{eq}}$). Superhorizon perturbations grow $\propto a$ for $a \gg a_{\text{eq}}$, which is the same behavior as D_+ for epochs where matter dominates over the dark energy and the curvature term in the Friedman equation (FRW.10). In the radiation-dominated epoch ($a \ll a_{\text{eq}}$), superhorizon fluctuations grow $\propto a^2$, whereas subhorizon matter fluctuations are stalled: their amplitude stays constant. In fact, for sub-horizon fluctuations one finds $\delta_m \propto (1 + a_{\text{eq}}^{-1} a)$. This different growth behavior of super- and subhorizon scale fluctuations in the radiation-dominated epoch suppresses density fluctuations on scales smaller than the comoving horizon scale at matter-radiation equality, $r_{\text{H,com}}(a_{\text{eq}})$ (see Eq. FRW.15), called Mészáros effect. Furthermore, the growth of structure is affected by the presence of baryons in the epoch before recombination ($z \gtrsim 1000$), since the ionized

baryon fluid is strongly coupled to the photons and together form a pressure-dominated fluid in which *baryonic acoustic oscillations* develop. In addition, the presence of neutinos (or other forms of hot dark matter) can suppress structure growth due to free-streaming of fast particles.

If $\tilde{\delta}_m(\mathbf{k}, t)$ denotes the Fourier transform of $\delta_m(\mathbf{x}, t)$, where \mathbf{k} is the comoving wave vector, then

$$\langle \tilde{\delta}_m(\mathbf{k}, t) \tilde{\delta}_m(\mathbf{k}', t) \rangle = (2\pi)^3 \delta_D(\mathbf{k} + \mathbf{k}') P_m(|\mathbf{k}|, t), \quad (\text{LSS.6})$$

where the Dirac delta ‘function’ is due to the assumed statistical homogeneity of the density field, and the *matter power spectrum* P_m depends only on the absolute value of \mathbf{k} , owing to the assumed statistical isotropy.

The primordial power spectrum as predicted from simple inflationary models follows a power law, $P_m \propto k^{n_s}$, where the spectral index of primordial fluctuations n_s is predicted to be slightly less than unity. In the framework of linear perturbation theory, the matter power spectrum then becomes

$$P(k, t) = A k^{n_s} T_k^2 D_+^2(t), \quad (\text{LSS.7})$$

valid for the matter-dominated epoch. The *transfer function* T_k accounts for the Mészáros effect, the presence of baryonic acoustic oscillations and the effects of hot dark matter and can be obtained from solving the linearized coupled Boltzmann equations for the various components (cold dark matter, baryons, photons, neutrinos) in the perturbed Friedman–Robertson–Walker metric (LE.21); there are publicly available codes for obtaining accurate transfer functions (see e.g. (Eisenstein and Hu 1999)).

Weak Lensing Equations: Consider first the case of a localized matter distribution located at an angular-diameter distance D_d , and a source at angular-diameter distance D_s from us. We assume that the localised matter distribution is embedded in an otherwise homogeneous and isotropic background universe, described by the Friedman–Robertson–Walker metric. Assume that the extent of the matter distribution along the line-of-sight is much smaller than D_d and D_{ds} , the angular-diameter distance of the source from the lens; such a lens is called *geometrically thin*. The lensing properties of such a deflector is determined solely by its *surface mass density* $\Sigma(\xi)$

$$\Sigma(\xi) \equiv \int dr_3 \rho(\xi_1, \xi_2, r_3), \quad (\text{LE.1})$$

the projection of its volume density ρ along the line-of-sight, where ξ is a two-dimensional vector perpendicular to the line-of-sight, locating a point in the *lens plane* (a plane at distance D_d perpendicular to the line-of-sight) relative to some conveniently chosen origin. A light ray traversing this matter distribution is deflected, where the *deflection angle* $\hat{\alpha}(\xi)$, the angle between the incoming and outgoing light ray at the lens plane, is given by

$$\hat{\alpha}(\xi) = \frac{4G_N}{c^2} \int d^2\xi' \Sigma(\xi') \frac{\xi - \xi'}{|\xi - \xi'|^2}, \quad (\text{LE.2})$$

provided that the gravitational potential of the deflector is small everywhere, $|\phi_N c^{-2}| \ll 1$ ³² and that the matter distribution causing the gravitational field of the deflector is moving at velocities $\ll c$. The condition $|\phi_N c^{-2}| \ll 1$ also guarantees that the deflection angle is small, $|\hat{\alpha}| \ll 1$, and thus we can safely use the approximations $\tan(|\hat{\alpha}|) \approx |\hat{\alpha}| \approx \sin(|\hat{\alpha}|)$ in the following.

The *lens equation* relates the direction θ of a light ray on the observer’s sky, measured relative to a conveniently chosen origin, to the angular position β the corresponding source would have in the absence of lensing,

$$\beta = \theta - \frac{D_{ds}}{D_s} \hat{\alpha}(D_d \theta) =: \theta - \alpha(\theta), \quad (\text{LE.3})$$

where in the final step the *scaled deflection angle* $\alpha(\theta)$ was defined. Equation (LE.3) is called the *lens equation*,³³ it yields a mapping of the lens plane onto the source plane. Defining the *convergence* or dimensionless surface mass density

$$\kappa(\theta) := \frac{\Sigma(D_d \theta)}{\Sigma_{\text{cr}}} \quad \text{with} \quad \Sigma_{\text{cr}} = \frac{c^2}{4\pi G_N} \frac{D_s}{D_d D_{ds}}, \quad (\text{LE.4})$$

where Σ_{cr} is the *critical surface mass density*, the scaled deflection angle can be expressed as

$$\alpha(\theta) = \frac{1}{\pi} \int_{\mathbb{R}^n} d^2\theta' \kappa(\theta') \frac{\theta - \theta'}{|\theta - \theta'|^2}. \quad (\text{LE.5})$$

³² This condition is satisfied everywhere, except in the immediate vicinity of black holes and neutron stars; additionally, the very small fraction of light rays that traverse strong gravitational fields are irrelevant in a gravitational lens situation owing to their extremely small magnification – see below.

³³ Note that, strictly speaking, the lens equation is specified only up to an irrelevant (since unobservable) translation in the source plane.

A mass distribution which has $\kappa \geq 1$ somewhere, i.e. $\Sigma \geq \Sigma_{\text{cr}}$, produces multiple images for some source positions β . Hence, Σ_{cr} is a characteristic value for the surface mass density which is the dividing line between *weak and strong lensing*.

The identities $\nabla \ln |\theta| = \theta/|\theta|^2$ and $\nabla^2 \ln |\theta| = 2\pi\delta_{\text{D}}(\theta)$ allow us to write the scaled deflection angle as the gradient of the *deflection potential*

$$\psi(\theta) = \frac{1}{\pi} \int_{\mathbb{R}^n} d^2\theta' \kappa(\theta') \ln |\theta - \theta'|, \quad \text{with} \quad \nabla^2 \psi = 2\kappa, \quad \text{as} \quad \alpha = \nabla \psi(\theta). \quad (\text{LE.6})$$

Hence, $\psi(\theta)$ satisfies the two-dimensional Poisson equation with source $\kappa(\theta)$. The *Fermat potential*

$$\tau_{\text{F}}(\theta; \beta) = \frac{1}{2} (\theta - \beta)^2 - \psi(\theta), \quad (\text{LE.7})$$

is proportional to the difference of the light travel times of a light ray from the source β via the lens at θ to the observer and that of a straight ray ($\theta = \beta$) in the absence of a deflector. Fermat's principle in conformally stationary space-times, specialised to the Friedman–Robertson–Walker model, states that actual light rays are those with stationary light travel times. Indeed, $\nabla \tau_{\text{F}}(\theta; \beta) = 0$ is equivalent to the lens equation (LE.3).

Gravitational light deflection preserves the surface brightness; hence, the surface brightness of an image $I(\theta)$ is given by the surface brightness I^s at the corresponding source position,

$$I(\theta) = I^s[\beta(\theta)]. \quad (\text{LE.8})$$

For lensed images that are much smaller than the angular scale on which the lens properties change, the lens mapping can be linearised locally. The distortion of images is then described by the *Jacobi matrix of the lens equation* (or *lens distortion matrix*)

$$\mathcal{A}(\theta) = \frac{\partial \beta}{\partial \theta} = \left(\delta_{ij} - \frac{\partial^2 \psi(\theta)}{\partial \theta_i \partial \theta_j} \right) = \begin{pmatrix} 1 - \kappa - \gamma_1 & -\gamma_2 \\ -\gamma_2 & 1 - \kappa + \gamma_1 \end{pmatrix} =: (1 - \kappa) \begin{pmatrix} 1 - g_1 & -g_2 \\ -g_2 & 1 + g_1 \end{pmatrix}, \quad (\text{LE.9})$$

where the (Cartesian) components of the *shear*

$$\gamma \equiv \gamma_1 + i\gamma_2 = \frac{1}{2}(\psi_{,11} - \psi_{,22}) + i\psi_{,12} = |\gamma|e^{2i\varphi} \quad (\text{LE.10})$$

have been defined, where indices separated by a comma denote partial derivatives w.r.t. the coordinates θ_i . Here and below, the shear is written as a complex quantity to emphasise its mathematical property as a spin-2 quantity: Rotating the coordinate system by an angle ϕ transforms the shear as $\gamma \rightarrow \gamma e^{-2i\phi}$. Since the shear represents the traceless part of \mathcal{A} , this transformation behaviour follows from that of matrices under the action of rotations. In the final expression of Eq. (LE.10), the shear is written as product of its magnitude $|\gamma| = \sqrt{\gamma_1^2 + \gamma_2^2}$ and its phase factor $e^{2i\varphi}$. The final expression in Eq. (LE.9) defines the *reduced shear*

$$g = g_1 + ig_2 = \gamma(1 - \kappa)^{-1}, \quad (\text{LE.11})$$

which describes the deviation of \mathcal{A} from an isotropic matrix, responsible for changing the shape of images compared to the shape of sources (SEE **ELLIPTICITY**). Instead of the Cartesian components of the shear, one is frequently interested in the shear components relative to a specific orientation, characterised by an angle φ ; for example, φ could be the direction of the separation vector to the center of a lens (relevant in galaxy-galaxy lensing – SEE **GALAXY-GALAXY LENSING**) or the orientation of the separation vector between two sources (relevant in cosmic shear – SEE **COSMIC SHEAR**). One therefore defines the *tangential and cross-components of the shear* at θ relative to the given orientation φ as

$$\gamma_{\text{t}}(\theta, \varphi) + i\gamma_{\text{x}}(\theta, \varphi) := -e^{-2i\varphi} \gamma(\theta); \quad g_{\text{t}}(\theta, \varphi) + ig_{\text{x}}(\theta, \varphi) := -e^{-2i\varphi} g(\theta), \quad (\text{LE.12})$$

where the corresponding definition is also applied to the reduced shear. For an axially-symmetric matter distribution $\kappa(|\theta|)$, the tangential shear is constant on circles around the centre, given by

$$\gamma_{\text{t}}(|\theta|) = \bar{\kappa}(|\theta|) - \kappa(|\theta|), \quad \text{where} \quad \bar{\kappa}(\theta) = \frac{2}{\theta^2} \int_0^\theta d\theta' \theta' \kappa(|\theta'|) \quad (\text{LE.13})$$

is the mean convergence inside radius θ , and the cross shear vanishes identically, $\gamma_{\text{x}} \equiv 0$. A similar result holds for a general mass distribution: The mean of the tangential shear on a circle of radius θ around a point θ_0 can be expressed as

$$\langle \gamma_{\text{t}} \rangle(\theta; \theta_0) := \int_0^{2\pi} \frac{d\varphi}{2\pi} \gamma_{\text{t}}(\theta_0 + \theta, \varphi) = \bar{\kappa}(\theta; \theta_0) - \langle \kappa \rangle(\theta; \theta_0), \quad (\text{LE.14})$$

where φ is the polar angle of $\boldsymbol{\theta}$, and we defined

$$\langle \kappa \rangle(\boldsymbol{\theta}; \boldsymbol{\theta}_0) := \int_0^{2\pi} \frac{d\varphi}{2\pi} \kappa(\boldsymbol{\theta}_0 + \boldsymbol{\theta}) ; \quad \bar{\kappa}(\boldsymbol{\theta}; \boldsymbol{\theta}_0) := \frac{2}{\theta^2} \int_0^\theta d\theta' \theta' \langle \kappa \rangle(\boldsymbol{\theta}'; \boldsymbol{\theta}_0) , \quad (\text{LE.15})$$

i.e., the *mean convergence on a circle* and the *mean convergence inside a circle* of radius θ around $\boldsymbol{\theta}_0$. Correspondingly, mean cross component of the shear on a circle vanishes identically. The relation (LE.14) forms the basis of galaxy-galaxy lensing (SEE [GALAXY-GALAXY LENSING](#)). The shear is linearly related to the surface mass density; combining Eqs. (LE.9) and (LE.6), one finds

$$\gamma(\boldsymbol{\theta}) = \frac{1}{\pi} \int_{\mathbb{R}^2} d^2\theta' \mathcal{D}(\boldsymbol{\theta} - \boldsymbol{\theta}') \kappa(\boldsymbol{\theta}') , \quad \text{with} \quad \mathcal{D}(\boldsymbol{\theta}) \equiv \frac{\theta_2^2 - \theta_1^2 - 2i\theta_1\theta_2}{|\boldsymbol{\theta}|^4} = \frac{-1}{(\theta_1 - i\theta_2)^2} . \quad (\text{LE.16})$$

In Fourier space, the relation between shear and convergence reads $\tilde{\gamma}(\boldsymbol{\ell}) = \tilde{\kappa}(\boldsymbol{\ell})e^{2i\varphi(\boldsymbol{\ell})}$, where $\varphi(\boldsymbol{\ell})$ is the polar angle of $\boldsymbol{\ell}$, valid for $\boldsymbol{\ell} \neq \mathbf{0}$. For $\boldsymbol{\ell} = \mathbf{0}$, the relation between $\tilde{\gamma}$ and $\tilde{\kappa}$ is undefined, related to the fact that a uniform convergence yields no shear. These relations can be inverted, $\tilde{\kappa}(\boldsymbol{\ell}) = \tilde{\gamma}(\boldsymbol{\ell})e^{-2i\varphi(\boldsymbol{\ell})}$, yielding

$$\kappa(\boldsymbol{\theta}) = \kappa_0 + \int_{\mathbb{R}^2} d^2\theta' \mathcal{D}^*(\boldsymbol{\theta} - \boldsymbol{\theta}') \gamma(\boldsymbol{\theta}') , \quad (\text{LE.17})$$

which forms the basis of mass reconstructions from weak lensing data; here, κ_0 is an undetermined additive constant. For a general shear field, e.g., if the shear field is estimated from noisy data, the result of the integral in (LE.17) may have an imaginary component. This is related to the E/B-modes of a general shear field (SEE [E/B-MODE SHEAR](#)). Therefore, for practical purposes, one takes the real part of the integral. A local relation between shear (and reduced shear) and convergence is provided by

$$\nabla \kappa = \begin{pmatrix} \gamma_{1,1} + \gamma_{2,2} \\ \gamma_{2,1} - \gamma_{1,2} \end{pmatrix} \equiv \mathbf{u}_\gamma(\boldsymbol{\theta}) ; \quad \nabla K(\boldsymbol{\theta}) = \frac{1}{1 - g_1^2 - g_2^2} \begin{pmatrix} 1 - g_1 & -g_2 \\ -g_2 & 1 + g_1 \end{pmatrix} \begin{pmatrix} g_{1,1} + g_{2,2} \\ g_{2,1} - g_{1,2} \end{pmatrix} \equiv \mathbf{u}_g(\boldsymbol{\theta}) , \quad (\text{LE.18})$$

where $K(\boldsymbol{\theta}) = -\ln[1 - \kappa(\boldsymbol{\theta})]$. The latter equation shows that from an estimate of the reduced shear, $(1 - \kappa)$ can be determined only up to a multiplicative constant.

Assuming the applicability of the linearised lens equation, the surface brightness of the image follows from (LE.8),

$$I(\boldsymbol{\theta}) = I^{(s)} [\boldsymbol{\beta}_0 + \mathcal{A}(\boldsymbol{\theta}_0) \cdot (\boldsymbol{\theta} - \boldsymbol{\theta}_0)] . \quad (\text{LE.19})$$

For larger images for which the linearised lens equation no longer provides an accurate approximation, higher-order expansions of the lens equation need to be considered (SEE [FLEXION](#)). The flux of an image S is obtained by integrating the surface brightness $I(\boldsymbol{\theta})$ over $\boldsymbol{\theta}$, and is related to the flux S_0 of the unlensed source through $S = |\mu|S_0$. The *magnification* (for an infinitesimally small source) μ_p is obtained from the distortion matrix (LE.9),

$$\mu_p = \frac{1}{\det \mathcal{A}} = \frac{1}{(1 - \kappa)^2 - |\gamma|^2} = \frac{1}{(1 - \kappa)^2(1 - |g|^2)} . \quad (\text{LE.20})$$

The sign of μ_p is called the *parity of an image*. Strong lenses can have regions with negative parity. Curves in the lens plane on which $\mu_p = 0$ are called *critical curves*; they form in general closed, smooth curves. Mapping critical curves onto the source plane through the lens equation yields the *caustics*; these are in general also closed curves which are smooth except for a finite number of *cusps* where the tangent vector of the caustic vanishes. The divergence of μ_p on a critical curve is just a formal one; for sources of finite extent, the linearised lens equation fails at critical curves, and the magnification of real (finite) sources, obtained by averaging the point-source magnification $|\mu_p|$ weighted by the surface brightness, remains finite. In weak lensing, where $\kappa \ll 1$ and $|\gamma| \ll 1$, μ_p is positive and close to unity.

If the lensing effect of a generic 3-dimensional matter distribution is considered (as in the case of lensing by the large-scale matter distribution in the Universe – SEE [COSMIC SHEAR](#)), one needs to refer to the laws of light propagation in a curved space-time. This is governed by the *equation of geodesic deviation* (SEE [GEODESIC DEVIATION EQUATION](#)). We now specialize Eq. (GDE.2) to a perturbed Friedman–Robertson–Walker metric of the form

$$ds^2 = a^2(\tau) \left[\left(1 + \frac{2\Phi_N}{c^2} \right) c^2 d\tau^2 - \left(1 - \frac{2\Phi_N}{c^2} \right) (d\chi^2 + f_K^2(\chi) d\omega^2) \right] , \quad (\text{LE.21})$$

where χ is the comoving radial distance (SEE [FRIEDMAN–ROBERTSON–WALKER MODELS](#)), $a = (1 + z)^{-1}$ the scale factor, τ is the conformal time, related to the cosmic time t through $dt = a(t) d\tau$, $f_K(\chi)$ is the comoving angular diameter distance, and $\Phi_N(\mathbf{x}, \chi)$ denotes the Newtonian peculiar gravitational potential which depends on the comoving position \mathbf{x} and cosmic time (or comoving distance χ). In this metric, the tidal matrix \mathcal{T} can be calculated in terms of the Newtonian potential Φ_N , and correspondingly, the equation of geodesic deviation

(GDE.1) yields the evolution equation for the comoving separation vector $\mathbf{x}(\boldsymbol{\theta}, \chi)$ of a ray in a ray bundle, specified by the angle $\boldsymbol{\theta}$ from the fiducial ray at the vertex, located at the observer,

$$\frac{d^2 \mathbf{x}}{d\chi^2} + K \mathbf{x} = -\frac{2}{c^2} \left[\nabla_{\perp} \Phi_N(\mathbf{x}(\boldsymbol{\theta}, \chi), \chi) - \nabla_{\perp} \Phi_N^{(0)}(\chi) \right], \quad (\text{LE.22})$$

with $\mathbf{x}(\boldsymbol{\theta}, 0) = \mathbf{0}$ and $(d\mathbf{x}/d\chi)(\boldsymbol{\theta}, 0) = \boldsymbol{\theta}$, where $K = (H_0/c)^2 (\Omega_{\text{tot}} - 1)$ is the spatial curvature of the Universe, $\nabla_{\perp} = (\partial/\partial x_1, \partial/\partial x_2)$ is the transverse *comoving* gradient operator, and $\Phi_N^{(0)}(\chi)$ is the potential along the fiducial ray. The solution of the homogeneous version of Eq. (LE.22) yields the comoving angular-diameter distance $f_K(\chi)$ (see Eq. FRW.2). The formal solution of Eq. (LE.22) is obtained by the method of Green's function, to yield

$$\mathbf{x}(\boldsymbol{\theta}, \chi) = f_K(\chi) \boldsymbol{\theta} - \frac{2}{c^2} \int_0^{\chi} d\chi' f_K(\chi - \chi') \left[\nabla_{\perp} \Phi_N(\mathbf{x}(\boldsymbol{\theta}, \chi'), \chi') - \nabla_{\perp} \Phi_N^{(0)}(\chi') \right]. \quad (\text{LE.23})$$

A source at comoving distance χ with comoving separation \mathbf{x} from the fiducial light ray would be seen, in the absence of light deflection, at the angular separation $\boldsymbol{\beta} = \mathbf{x}/f_K(\chi)$ from the fiducial ray (this statement is nothing but the definition of the comoving angular diameter distance). Hence, $\boldsymbol{\beta}$ is the unlensed angular position in the ‘comoving source plane’ at distance χ , relative to that of the fiducial ray. Therefore, in analogy with standard lens theory, we define the Jacobi matrix

$$\mathcal{A}(\boldsymbol{\theta}, \chi) = \frac{\partial \boldsymbol{\beta}}{\partial \boldsymbol{\theta}} = \frac{1}{f_K(\chi)} \frac{\partial \mathbf{x}}{\partial \boldsymbol{\theta}}, \quad (\text{LE.24})$$

and obtain from (LE.23)

$$\mathcal{A}_{ij}(\boldsymbol{\theta}, \chi) = \delta_{ij} - \frac{2}{c^2} \int_0^{\chi} d\chi' \frac{f_K(\chi - \chi') f_K(\chi')}{f_K(\chi)} \Phi_{N,ik}(\mathbf{x}(\boldsymbol{\theta}, \chi'), \chi') \mathcal{A}_{kj}(\boldsymbol{\theta}, \chi'), \quad (\text{LE.25})$$

which describes the locally linearised mapping introduced by LSS lensing. This equation still is exact in the limit of validity of the weak-field metric, and the pair of Eqs. (LE.23) and (LE.25) are followed typically in ray-tracing simulations (e.g., Jain et al. 2000; Hilbert et al. 2009, and references therein).

Expanding \mathcal{A} in powers of Φ_N/c^2 , and truncating the series after the linear term yields

$$\mathcal{A}_{ij}(\boldsymbol{\theta}, \chi) = \delta_{ij} - \frac{2}{c^2} \int_0^{\chi} d\chi' \frac{f_K(\chi - \chi') f_K(\chi')}{f_K(\chi)} \Phi_{N,ij}(f_K(\chi') \boldsymbol{\theta}, \chi'), \quad (\text{LE.26})$$

and provides two essential simplification compared to Eq. (LE.25): First, the distortion is obtained by integrating along the unperturbed ray $\mathbf{x} = f_K(\chi) \boldsymbol{\theta}$; this is also called the *Born approximation*. Second, in (LE.25), the matrix product in the integrand yields a coupling of tidal field matrices $\Phi_{N,ij}$ at different distances χ' , which disappears in the linear version (LE.26), rendering \mathcal{A} symmetric. This approximation is frequently referred to as ‘neglecting lens-lens coupling’. In the literature, one often sees both approximations as being collectively called ‘Born approximation’. Corrections to the Born approximation are necessarily of order Φ_N^2/c^2 (see Krause and Hirata 2010, and references therein for higher-order terms).

Employing the Born approximation from here on, we now define the deflection potential

$$\psi(\boldsymbol{\theta}, \chi) := \frac{2}{c^2} \int_0^{\chi} d\chi' \frac{f_K(\chi - \chi')}{f_K(\chi) f_K(\chi')} \Phi_N(f_K(\chi') \boldsymbol{\theta}, \chi'), \quad (\text{LE.27})$$

yielding $\mathcal{A}_{ij} = \delta_{ij} - \psi_{,ij}$, just as in geometrically-thin lens theory. In this approximation, lensing by the 3-D matter distribution can be treated as an equivalent lens plane with deflection potential ψ , convergence $\kappa = \nabla^2 \psi/2$, and shear $\gamma = (\psi_{,11} - \psi_{,22})/2 + i\psi_{,12}$.

One can relate κ to fractional density contrast δ of matter fluctuations in the Universe by using the Poisson equation, to yield

$$\kappa(\boldsymbol{\theta}, \chi) = \frac{3H_0^2 \Omega_m}{2c^2} \int_0^{\chi} d\chi' \frac{f_K(\chi') f_K(\chi - \chi')}{f_K(\chi)} \frac{\delta(f_K(\chi') \boldsymbol{\theta}, \chi')}{a(\chi')}. \quad (\text{LE.28})$$

Note that κ is proportional to Ω_m , since lensing is sensitive to $\Delta\rho \propto \Omega_m \delta$, not just to the density contrast $\delta = \Delta\rho/\bar{\rho}$ itself. An equivalent expression for the convergence reads

$$\kappa(\boldsymbol{\theta}, z_s) = \int_0^{z_s} dz \frac{dD_{\text{prop}}}{dz} \frac{4\pi G}{c^2} \frac{D_A(z) D_A(z, z_s)}{D_A(z_s)} \Delta\rho(D_A(z) \boldsymbol{\theta}, z), \quad (\text{LE.29})$$

which has an immediate intuitive interpretation: the density contrast $\Delta\rho = \rho - \bar{\rho}$ (as a function of proper transverse separation $D_A(z) \boldsymbol{\theta}$) as the source of the gravitational field, multiplied by the proper distance interval

dD_{prop} , yields a surface mass density $d\Sigma$ at redshift z , which is multiplied by the inverse of the critical surface density in the integrand, resulting in an infinitesimal convergence $d\kappa$ at redshift z . The convergence is then obtained by line-of-sight integration of this infinitesimal convergence. If $\delta\rho$ is assumed to be non-zero only on a small redshift interval centred on z_d , the convergence of the geometrically-thin lens equation (LE.4) is reobtained.

For a redshift (or distance) distribution of sources with $p_z(z) dz = p_\chi(\chi) d\chi$, the effective surface mass density becomes

$$\kappa(\boldsymbol{\theta}) = \int d\chi p_\chi(\chi) \kappa(\boldsymbol{\theta}, \chi) = \frac{3H_0^2 \Omega_m}{2c^2} \int_0^{\chi_h} d\chi g_{\text{LE}}(\chi) f_K(\chi) \frac{\delta(f_K(\chi)\boldsymbol{\theta}, \chi)}{a(\chi)}, \quad (\text{LE.30})$$

with

$$g_{\text{LE}}(\chi) = \int_\chi^{\chi_h} d\chi' p_\chi(\chi') \frac{f_K(\chi' - \chi)}{f_K(\chi')}, \quad (\text{LE.31})$$

which is the source-redshift weighted lens efficiency factor D_{ds}/D_s for a density fluctuation at distance χ , and χ_h is the comoving horizon distance, obtained from $\chi(a)$ by letting $a \rightarrow 0$.

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