pulsed_interference

July 20, 2023

[ ]:
```
#sympy pour effectuer les calculs
import sympy as smp
from IPython.display import display, Math
```

[ ]:
```
a = smp.symbols(r'\alpha', real=False, complex = True)
b = smp.symbols(r'\beta', real=False, complex = True)
u = smp.symbols(r'\mu', real=False, complex = True)
v = smp.symbols(r'\nu', real=False, complex = True)
t = smp.symbols('t', real=True)
o = smp.symbols(r'\sigma', real=True, positive = True)
b = smp.symbols('b', real=False, complex = True)
d = smp.symbols(r'\delta', real=True)
z = smp.symbols('z', real=True)
k = smp.symbols('k', real=True, positive=True)
w = smp.symbols(r'\omega', real=True, positive=True)
tau = smp.symbols(r'\tau', real=True)
c = smp.symbols('c', real=True, positive=True, constant=True)
#delta_D = smp.symbols(r'\Delta', real=True)
```

[ ]:
```
A = (smp.sqrt(1/((smp.sqrt(2*smp.pi))*o)))*smp.exp(-((t)**2)/(4*o**2))

E_1 = A*smp.exp(smp.I*(k*(z) - w*t))
display(Math(r'|E_{1}(z,t)> = '+smp.latex(E_1)))

A_f = (smp.sqrt(1/((smp.sqrt(2*smp.pi))*o)))*smp.exp(-((t+tau)**2)/(4*o**2))
display(Math('A(t) = '+smp.latex(A)))

E_2 = A_f*smp.exp(smp.I*(k*(z) - w*(t+tau)))
display(Math(r'|E_{2}(z,t)> = '+smp.latex(E_2)))

#champ total initial
E_i = (E_1+E_2)
display(Math(r'|E_{i}(z,t)> = '+smp.latex(E_i)))

#résourd l'opérateur d'interaction
U = smp.symbols(r'\hat{U}')
eq1 = smp.Eq(U*E_1, E_2)
display(Math(smp.latex(eq1)))
```

```python
eq2 = smp.solve(eq1, U)
U = eq2[0].simplify()
display(Math(r'\hat{U} = ' + smp.latex(U)))

eq3 = smp.Eq(U, smp.exp(smp.I*d))
display(Math(smp.latex(eq3)))
eq4 = smp.solve(eq3, d)
delta = eq4[0]
display(Math(r'\delta = ' + smp.latex(delta)))

#postselection sur D = 1/sqrt(2)*(H_faible + V)
E_t = ((1/smp.sqrt(2))*(E_1 + E_2))
#mesure faible sur H
display(Math(r'|E_{t}(z,t)> = '+smp.latex(E_t)))
```

$$|E_1(z,t)> = \frac{2^{\frac{3}{4}} e^{i(-\omega t+kz)} e^{-\frac{t^2}{4\sigma^2}}}{2\sqrt[4]{\pi}\sqrt{\sigma}}$$

$$A(t) = \frac{2^{\frac{3}{4}} e^{-\frac{t^2}{4\sigma^2}}}{2\sqrt[4]{\pi}\sqrt{\sigma}}$$

$$|E_2(z,t)> = \frac{2^{\frac{3}{4}} e^{i(-\omega(\tau+t)+kz)} e^{-\frac{(\tau+t)^2}{4\sigma^2}}}{2\sqrt[4]{\pi}\sqrt{\sigma}}$$

$$|E_i(z,t)> = \frac{2^{\frac{3}{4}} e^{i(-\omega t+kz)} e^{-\frac{t^2}{4\sigma^2}}}{2\sqrt[4]{\pi}\sqrt{\sigma}} + \frac{2^{\frac{3}{4}} e^{i(-\omega(\tau+t)+kz)} e^{-\frac{(\tau+t)^2}{4\sigma^2}}}{2\sqrt[4]{\pi}\sqrt{\sigma}}$$

$$\frac{2^{\frac{3}{4}} \hat{U} e^{i(-\omega t+kz)} e^{-\frac{t^2}{4\sigma^2}}}{2\sqrt[4]{\pi}\sqrt{\sigma}} = \frac{2^{\frac{3}{4}} e^{i(-\omega(\tau+t)+kz)} e^{-\frac{(\tau+t)^2}{4\sigma^2}}}{2\sqrt[4]{\pi}\sqrt{\sigma}}$$

$$\hat{U} = e^{\frac{-4i\omega\sigma^2\tau+t^2-(\tau+t)^2}{4\sigma^2}}$$

$$e^{\frac{-4i\omega\sigma^2\tau+t^2-(\tau+t)^2}{4\sigma^2}} = e^{i\delta}$$

$$\delta = \left(\operatorname{asin}\left(e^{-\frac{\tau(\tau+2t)}{4\sigma^2}} \sin(\omega\tau)\right) + \pi, \right)$$

$$|E_t(z,t)> = \frac{\sqrt{2}\cdot\left(\frac{2^{\frac{3}{4}} e^{i(-\omega t+kz)} e^{-\frac{t^2}{4\sigma^2}}}{2\sqrt[4]{\pi}\sqrt{\sigma}} + \frac{2^{\frac{3}{4}} e^{i(-\omega(\tau+t)+kz)} e^{-\frac{(\tau+t)^2}{4\sigma^2}}}{2\sqrt[4]{\pi}\sqrt{\sigma}}\right)}{2}$$

[ ]:
```python
I = smp.re(smp.conjugate(E_t)*E_t)
display(Math('I(z,t) =' +smp.latex((I.simplify().factor()))))

I_ref = smp.re(smp.conjugate(E_1*(1/smp.sqrt(2)))*(E_1*(1/smp.sqrt(2))))
display(Math(r'I_{ref}(z,t) =' +smp.latex((I_ref.simplify()))))
```

$$I(z,t) = \frac{\sqrt{2}\cdot\left(1 + e^{-\frac{\tau^2}{2\sigma^2}} e^{-\frac{\tau t}{\sigma^2}} + 2e^{-\frac{\tau^2}{4\sigma^2}} e^{-\frac{\tau t}{2\sigma^2}} \cos(\omega\tau)\right) e^{-\frac{t^2}{2\sigma^2}}}{4\sqrt{\pi}\sigma}$$

$$I_{ref}(z,t) = \frac{\sqrt{2}e^{-\frac{t^2}{2\sigma^2}}}{4\sqrt{\pi}\sigma}$$

[ ]:
```
G_1_TAU = (smp.integrate(smp.conjugate(E_1)*E_2, (t, -smp.oo, smp.oo))).
    simplify()
display(Math(r'G(\tau) = ' + smp.latex(G_1_TAU)))

G_1_ZERO = smp.integrate(smp.conjugate(E_1)*E_1, (t, -smp.oo, smp.oo))
display(Math(r'G(0) = ' + smp.latex(G_1_ZERO.simplify())))

g_1 = (G_1_TAU/G_1_ZERO).simplify()
display(Math(r'g^{(1)}(\tau) = ' + smp.latex(g_1)))
```

$$G(\tau) = e^{-\tau\left(i\omega+\frac{\tau}{8\sigma^2}\right)}$$

$$G(0) = 1$$

$$g^{(1)}(\tau) = e^{-\tau\left(i\omega+\frac{\tau}{8\sigma^2}\right)}$$

[ ]:
```
I_g = smp.conjugate(E_1)*E_1 + smp.conjugate(E_2)*E_2 + 2*smp.sqrt(smp.
    conjugate(E_1)*E_1*smp.conjugate(E_2)*E_2)*smp.re(g_1)
display(Math(r'I_g ='+smp.latex((I_g.simplify().factor()))))
```

$$I_g = \frac{\sqrt{2}\cdot\left(2e^{\frac{\tau^2}{8\sigma^2}}e^{\frac{\tau t}{2\sigma^2}}\cos\left(\omega\tau\right)+e^{\frac{\tau^2}{2\sigma^2}}e^{\frac{\tau t}{\sigma^2}}+1\right)e^{-\frac{\tau^2}{2\sigma^2}}e^{-\frac{t^2}{2\sigma^2}}e^{-\frac{\tau t}{\sigma^2}}}{2\sqrt{\pi}\sigma}$$

[ ]:
```
tau_c = smp.integrate(smp.conjugate(g_1)*g_1, (tau, -smp.oo, smp.oo))
display(Math(r'\tau_c = '+smp.latex(tau_c)))
```

$$\tau_c = 2\sqrt{\pi}\sigma$$

[ ]:
```
I_1 = (smp.conjugate(E_1)*E_1)
I_2 = (smp.conjugate(E_2)*E_2)

V = ((2*smp.sqrt(I_1)*smp.sqrt(I_2))/(I_1+I_2))*(g_1)
display(Math(r'V='+smp.latex(V.simplify().doit())))
```

$$V = \frac{2e^{\tau\left(-i\omega+\frac{\tau}{8\sigma^2}+\frac{t}{2\sigma^2}\right)}}{e^{\frac{\tau^2}{2\sigma^2}+\frac{\tau t}{\sigma^2}}+1}$$

[ ]:
```
f = smp.symbols('f', real=True, constante=True, positive=True)
G_1_TAU=G_1_TAU.subs(w, 2*smp.pi*f)
#using the autocorrelation function
S = (smp.integrate(G_1_TAU*smp.exp(-smp.I*2*smp.pi*f*tau), (tau, -smp.oo, smp.
    oo)))*(1)
display(Math(r'S(f) = ' + smp.latex(S.simplify())))
```

$$S(f) = 2\sqrt{2}\sqrt{\pi}\sigma e^{-32\pi^2\sigma^2 f^2}$$

```
[ ]: df_up = (smp.integrate(S, (f, 0, smp.oo))**2).simplify()
     df_down = (smp.integrate(S**2, (f, 0, smp.oo))).simplify()

     df = (df_up/df_down)
     display(Math(r'\Delta f = '+smp.latex(df.simplify())))
```

$$\Delta f = \frac{1}{8\sqrt{\pi}\sigma}$$

```
[ ]: S_w = smp.integrate(E_1.subs(w, 2*smp.pi*f).subs(z, 0)*smp.exp(-smp.I*2*smp.
      ↪pi*f*t), (t, -smp.oo, smp.oo))
     display(Math(r'S(f) ='+smp.latex(S_w.simplify())))
```

$$S(f) = 2^{\frac{3}{4}}\sqrt[4]{\pi}\sqrt{\sigma}e^{-16\pi^2\sigma^2f^2}$$

Mesure faible temporel

```
[ ]: #intiallement
     phi = a + b
     H = smp.symbols('|H>')
     V = smp.symbols('|V>')
     display(Math(r'|\varphi(\theta, \phi)> = ' +smp.latex(a*H + b*V)))
     display(Math(r'|\Psi_i> = |\varphi(\theta, \phi)> \otimes |\xi(z,t)>'))
```

$$|\varphi(\theta, \phi)> = \alpha|H> + b|V>$$

$$|\Psi_i> = |\varphi(\theta, \phi)> \otimes |\xi(z,t)>$$

```
[ ]: xi_i = (smp.sqrt(1/((smp.sqrt(2*smp.pi))*o)))*smp.exp(-((t)**2)/(4*o**2))
     xi_1 = xi_i*smp.exp(-smp.I*w*t)
     display(Math(r'|\xi_1(z,t)> = '+smp.latex(xi_1)))

     xi_2 = (smp.sqrt(1/((smp.sqrt(2*smp.pi))*o)))*smp.exp(-(((t+tau))**2)/
      ↪(4*o**2))*smp.exp(-smp.I*w*(t+tau))
     display(Math(r'|\xi_2(z,t+\tau)> = '+smp.latex(xi_2)))
     display(Math(r'|\Psi_i> = |\varphi(\theta, \phi)> \otimes |\xi(z,t)>'))
     psi_i = (a*xi_1+b*smp.exp(smp.I*tau*w)*xi_2)
     display(Math(r'|\Psi_i> = '+smp.latex(psi_i)))

     psi_f = (smp.conjugate(u)*a*xi_1+smp.conjugate(v)*b*smp.exp(smp.I*tau*w)*xi_2)
     display(Math(r'|\Psi_f> = <J|U|\Psi_i> = '+smp.latex(psi_f)))

     T = (smp.integrate(smp.conjugate(psi_f)*t*psi_f, (t, -smp.oo, smp.oo))).
      ↪simplify()
     display(Math(r'<T> ='+smp.latex(T)))
```

$$|\xi_1(z,t)> = \frac{2^{\frac{3}{4}}e^{-\frac{t^2}{4\sigma^2}}e^{-i\omega t}}{2\sqrt[4]{\pi}\sqrt{\sigma}}$$

$$|\xi_2(z,t+\tau)>=\frac{2^{\frac{3}{4}}e^{-\frac{(\tau+t)^2}{4\sigma^2}}e^{-i\omega(\tau+t)}}{2\sqrt[4]{\pi}\sqrt{\sigma}}$$

$$|\Psi_i>=|\varphi(\theta,\phi)>\otimes|\xi(z,t)>$$

$$|\Psi_i>=\frac{2^{\frac{3}{4}}\alpha e^{-\frac{t^2}{4\sigma^2}}e^{-i\omega t}}{2\sqrt[4]{\pi}\sqrt{\sigma}}+\frac{2^{\frac{3}{4}}be^{-\frac{(\tau+t)^2}{4\sigma^2}}e^{i\omega\tau}e^{-i\omega(\tau+t)}}{2\sqrt[4]{\pi}\sqrt{\sigma}}$$

$$|\Psi_f>=<J|U|\Psi_i>=\frac{2^{\frac{3}{4}}\alpha e^{-\frac{t^2}{4\sigma^2}}e^{-i\omega t}\overline{\mu}}{2\sqrt[4]{\pi}\sqrt{\sigma}}+\frac{2^{\frac{3}{4}}be^{-\frac{(\tau+t)^2}{4\sigma^2}}e^{i\omega\tau}e^{-i\omega(\tau+t)}\overline{\nu}}{2\sqrt[4]{\pi}\sqrt{\sigma}}$$

$$<T>=\frac{\tau\left(-\alpha\nu\overline{\mu}\overline{b}-\mu b\overline{\alpha}\overline{\nu}-2\nu be^{\frac{\tau^2}{8\sigma^2}}\overline{\nu}\overline{b}\right)e^{-\frac{\tau^2}{8\sigma^2}}}{2}$$

[ ]:
```python
F = (smp.integrate(psi_f*smp.exp(-smp.I*w*t), (t, -smp.oo, smp.oo))).simplify()
display(Math(r'|F(\omega)> = ' + smp.latex(F)))
```

$$|F(\omega)>=2^{\frac{3}{4}}\sqrt[4]{\pi}\sqrt{\sigma}\left(\alpha\overline{\mu}+be^{2i\omega\tau}\overline{\nu}\right)e^{-4\omega^2\sigma^2}$$

[ ]:
```python
W = smp.integrate(smp.conjugate(F)*w*F, (w, -smp.oo, smp.oo))
display(Math(r'<\Omega> = ' + smp.latex(W.simplify())))
```

$$<\Omega>=\frac{i\pi\tau\left(-\alpha\nu\overline{\mu}\overline{b}+\mu b\overline{\alpha}\overline{\nu}\right)e^{-\frac{\tau^2}{8\sigma^2}}}{8\sigma^2}$$

[ ]:
```python

```