9 DoF Planar 4 Wheeled Car Model With Active Chassis Control

Vehicle System Dynamics 2023 Final project

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Workflow

Dynamic system modelling	System verification	Active chassis control	Conclusions

Dynamic system modelling

- General view of SimuLink model
- Construction
- Subsystem development

System verification

Observation of:

- Roll, yaw and pitch angles
- Chassis velocities
- Wheel rotation speeds

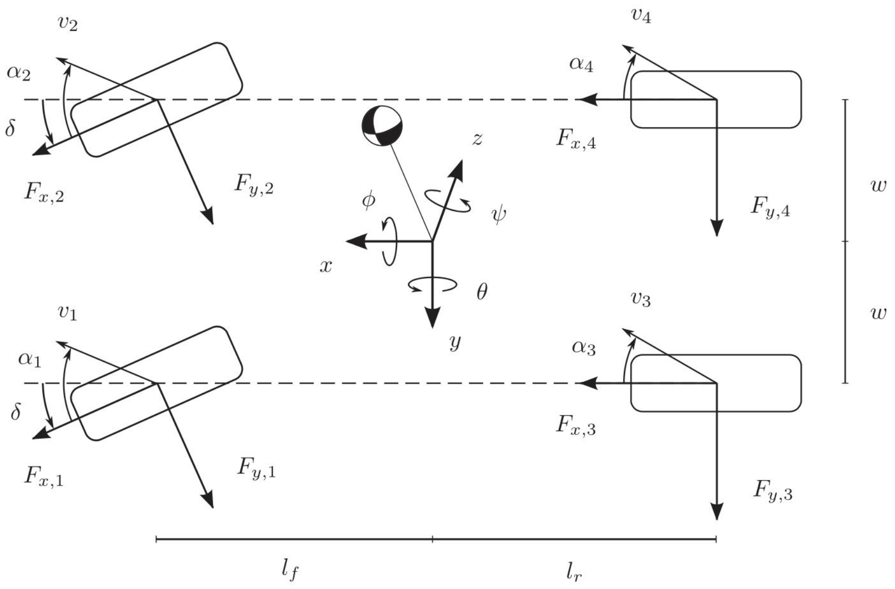
Active chassis control

- Formulation of Control Problem
- Active Chassis Control

Conclusions

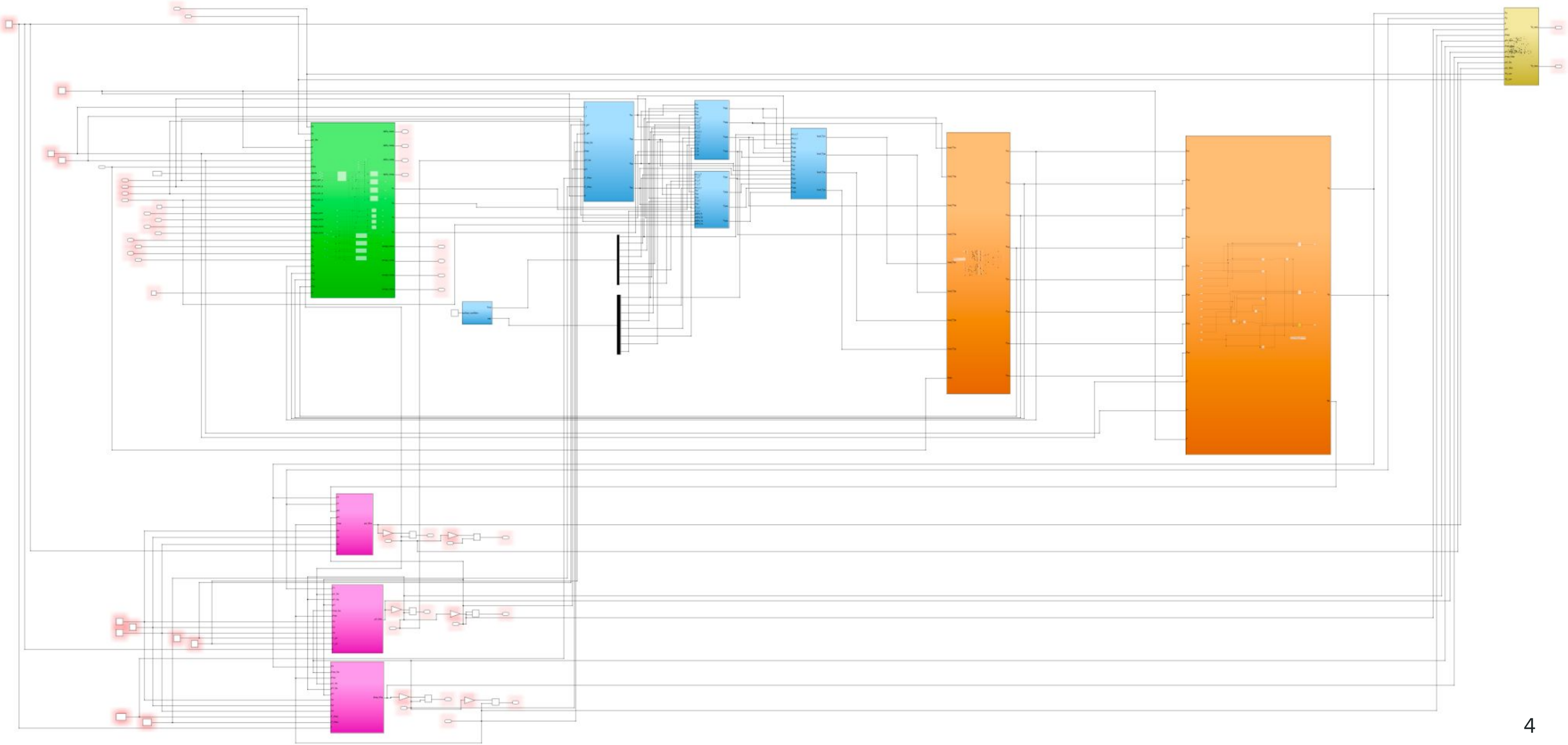
- Comparison And Discussion

Vehicle dynamic system modeling

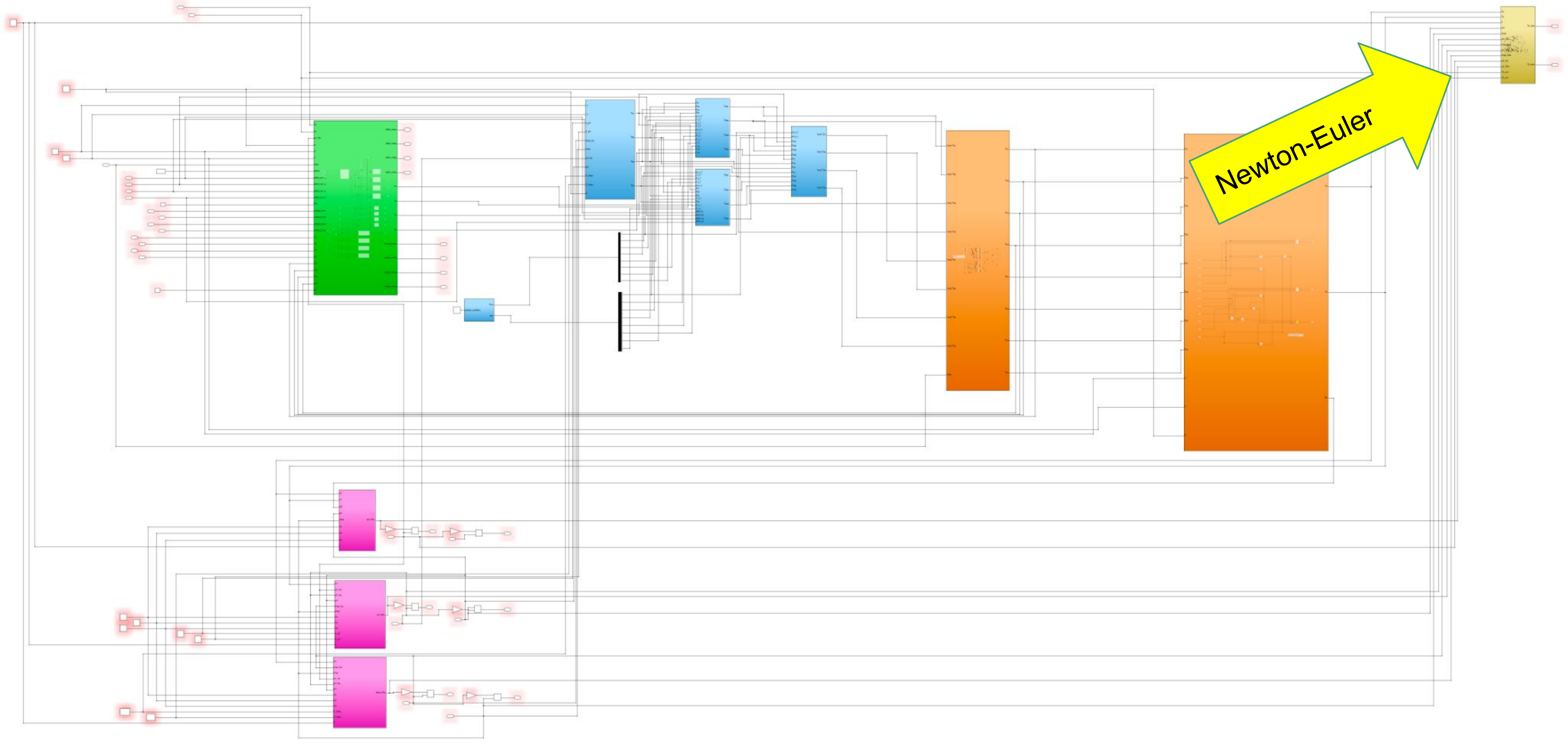


- Newton - Euler equations and reference systems
- Wheel and tyre dynamics
- Reference frame transfers
- Forces transfer from wheel to chassis
  - Pacejka
- Roll, pitch and yaw

General view



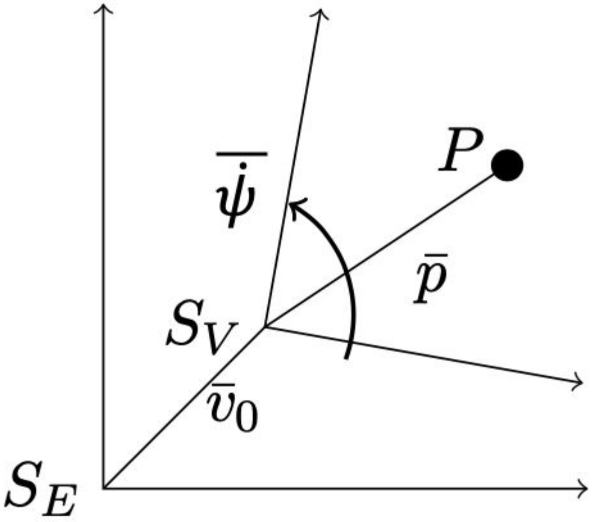
Newton - Euler



Newton - Euler equations, reference systems

The reference system to be used primarily is the body reference system.

The global reference system is only utilized in order to express the yaw rate, which is represented by the letter psi.



Newton - Euler equations

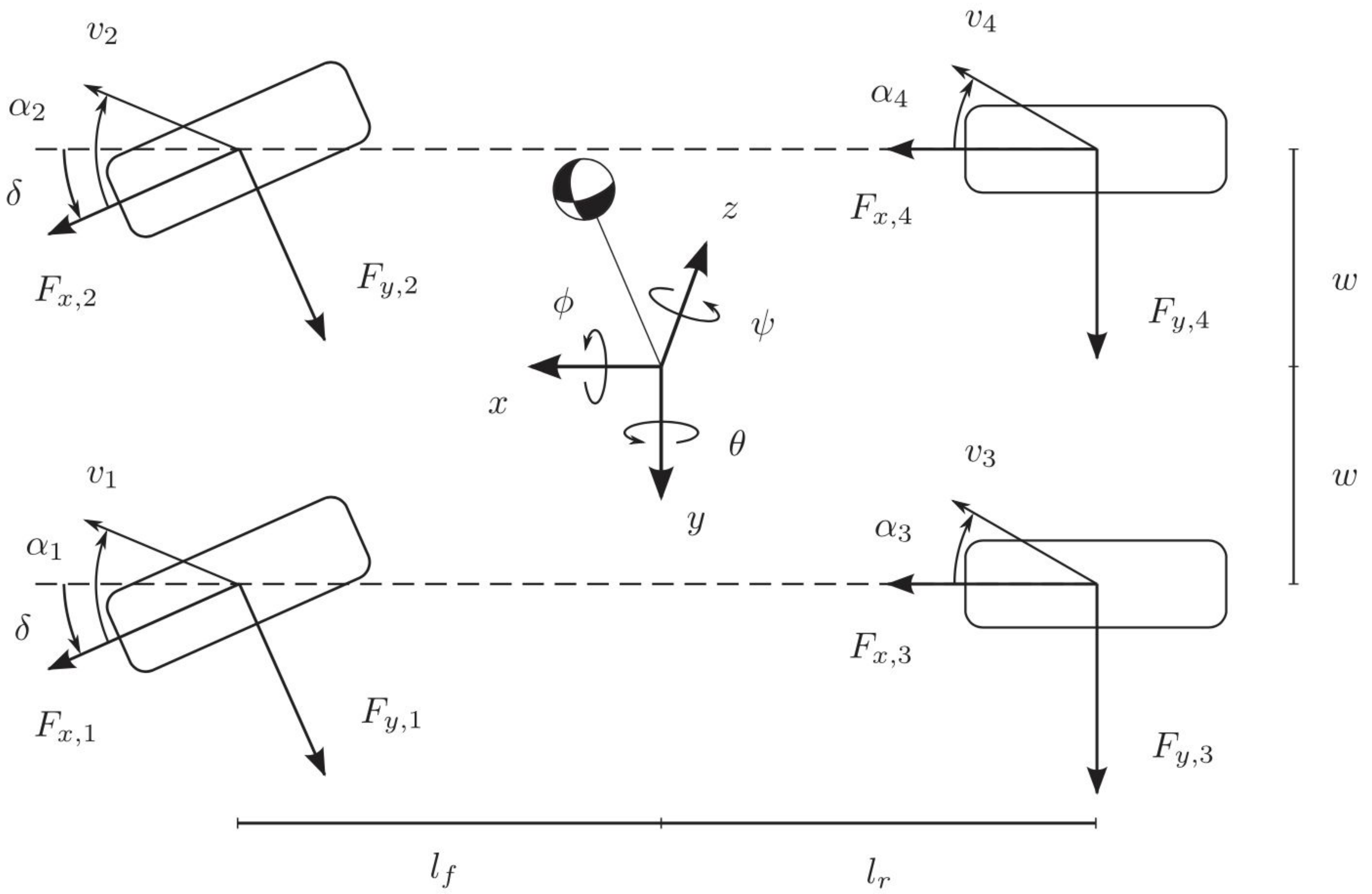
These are the basis of the transitional and rotational dynamics of the model.

With them, we can calculate the effect the forces on the wheels have on the car by coupling them with a tyre model.

$$\boldsymbol{F} = m\boldsymbol{a}_{cm}$$

$$\boldsymbol{M} = \boldsymbol{I}_{cm}\boldsymbol{\alpha} + \boldsymbol{\omega} \times \boldsymbol{I}_{cm}\boldsymbol{\omega}$$

$$\begin{pmatrix} \mathbf{F} \\ \boldsymbol{\tau} \end{pmatrix} = \begin{pmatrix} m\mathbf{I}_3 & 0 \\ 0 & \mathbf{I}_{\mathrm{cm}} \end{pmatrix} \begin{pmatrix} \mathbf{a}_{\mathrm{cm}} \\ \boldsymbol{\alpha} \end{pmatrix} + \begin{pmatrix} 0 \\ \boldsymbol{\omega} \times \mathbf{I}_{\mathrm{cm}}\,\boldsymbol{\omega} \end{pmatrix}$$


v_2
α_2
δ
$F_{x,2}$
$F_{y,2}$
v_4
α_4
$F_{x,4}$
$F_{y,4}$
z
ϕ
ψ
x
θ
y
w
w
v_1
α_1
δ
$F_{x,1}$
$F_{y,1}$
v_3
α_3
$F_{x,3}$
$F_{y,3}$
l_f
l_r

Newton - Euler

$$V_x(t+dt) = V_x(t) + \dot{V}_x(t) * dt$$

$$V_y(t+dt) = V_y(t) + \dot{V}_y(t) * dt$$

$$\begin{aligned}
\dot{v}_x - v_y\dot{\psi} &= h\Big(\sin(\theta)\cos(\phi)(\dot{\psi}^2 + \dot{\phi}^2 + \dot{\theta}^2) - \sin(\phi)\ddot{\psi} - 2\cos(\phi)\dot{\phi}\dot{\psi} \\
&\quad - \cos(\theta)\cos(\phi)\ddot{\theta} + 2\cos(\theta)\sin(\phi)\dot{\theta}\dot{\phi} + \sin(\theta)\sin(\phi)\ddot{\phi}\Big) + \frac{F_X}{m}, \\
\dot{v}_y + v_x\dot{\psi} &= h\Big(-\sin(\theta)\cos(\phi)\ddot{\psi} - \sin(\phi)\dot{\psi}^2 - 2\cos(\theta)\cos(\phi)\dot{\theta}\dot{\psi} \\
&\quad + \sin(\theta)\sin(\phi)\dot{\phi}\dot{\psi} - \sin(\phi)\dot{\phi}^2 + \cos(\phi)\ddot{\phi}\Big) + \frac{F_Y}{m},
\end{aligned}$$

Newton - Euler, Variables and parameters

Inputs

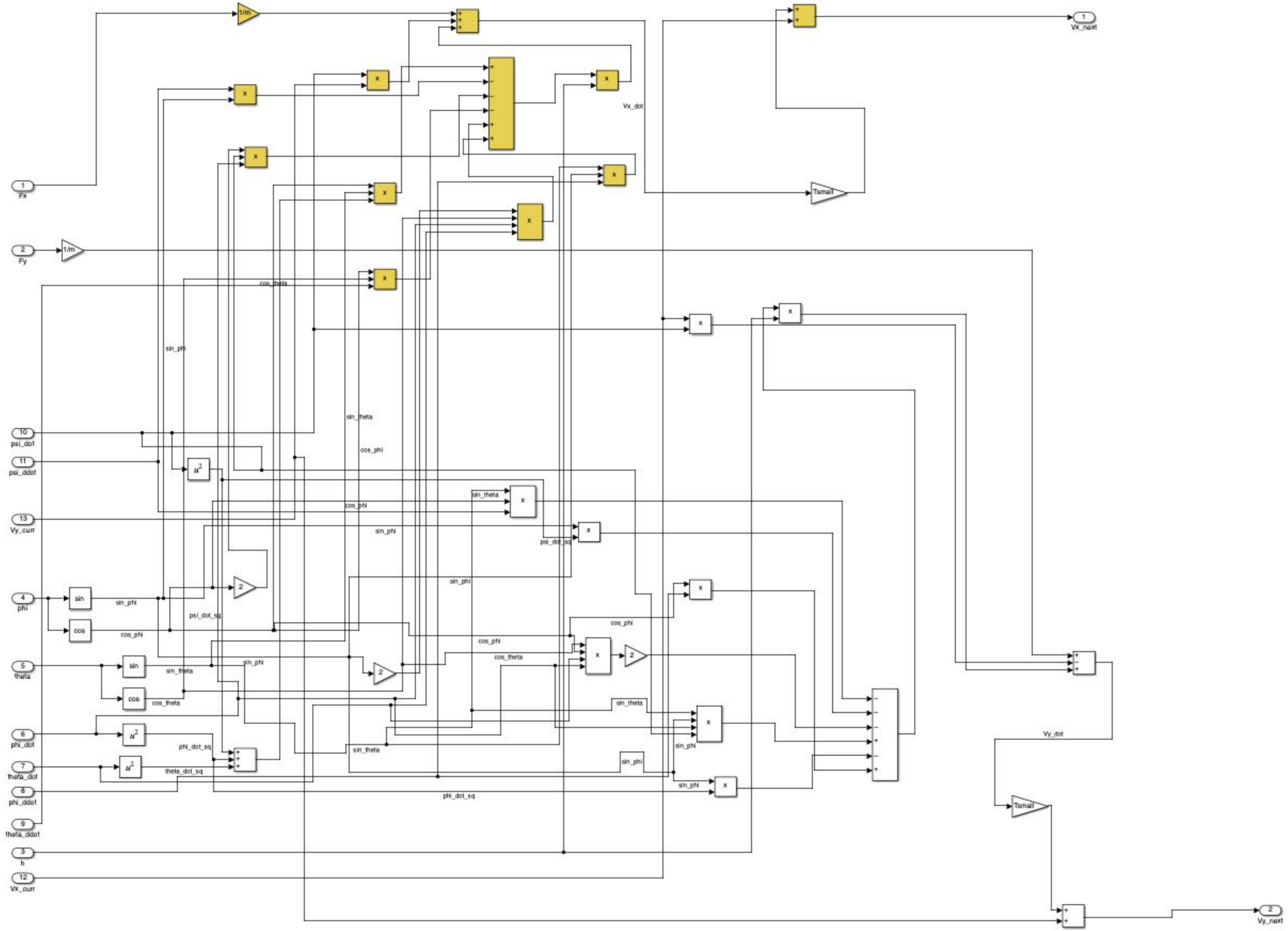
Variables:

- Fx, Fy
- Vx_curr, Vy_curr
- phi, phi_dot, phi_ddot
- theta, theta_dot, theta_ddot
- psi_dot, psi_ddot

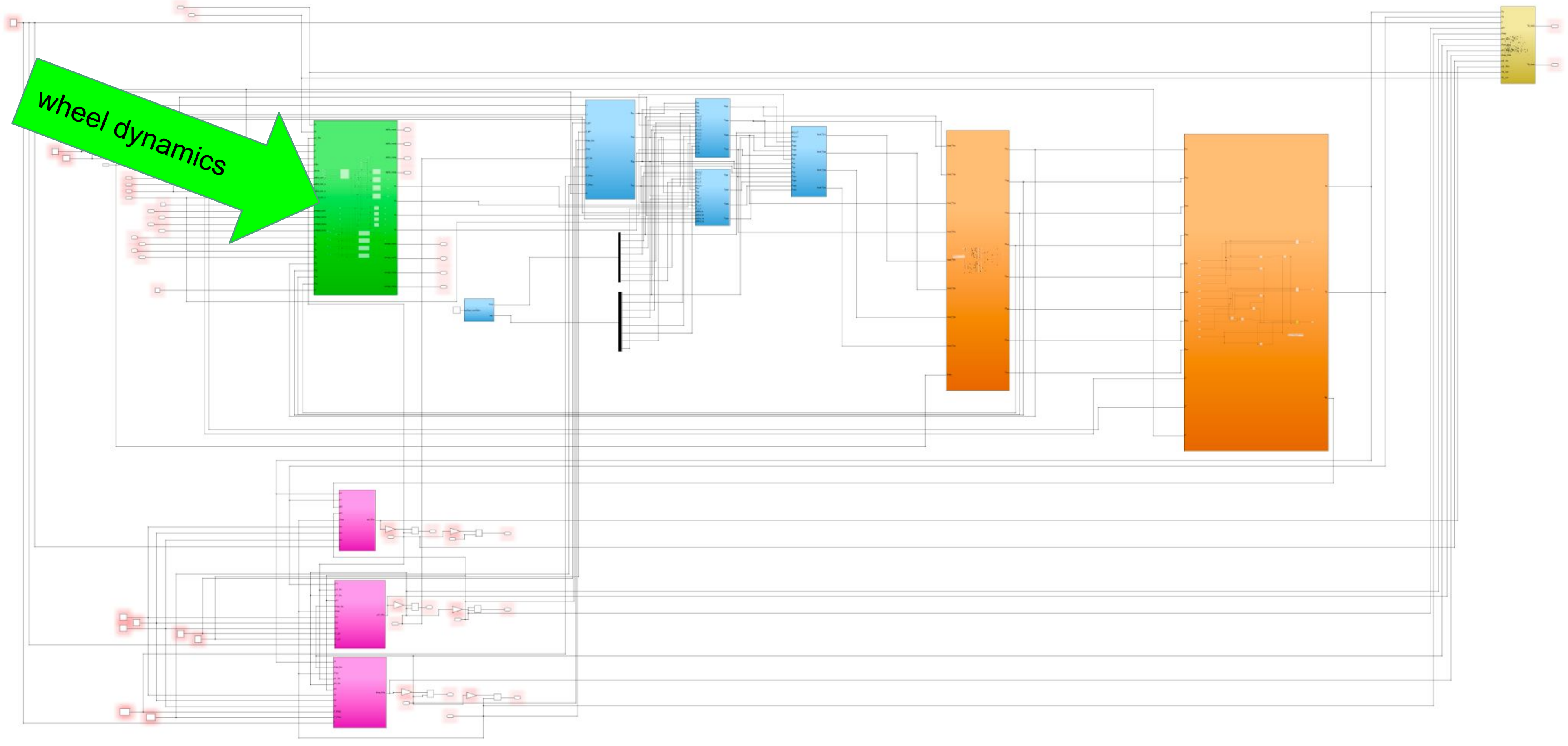
Constants: h

Outputs

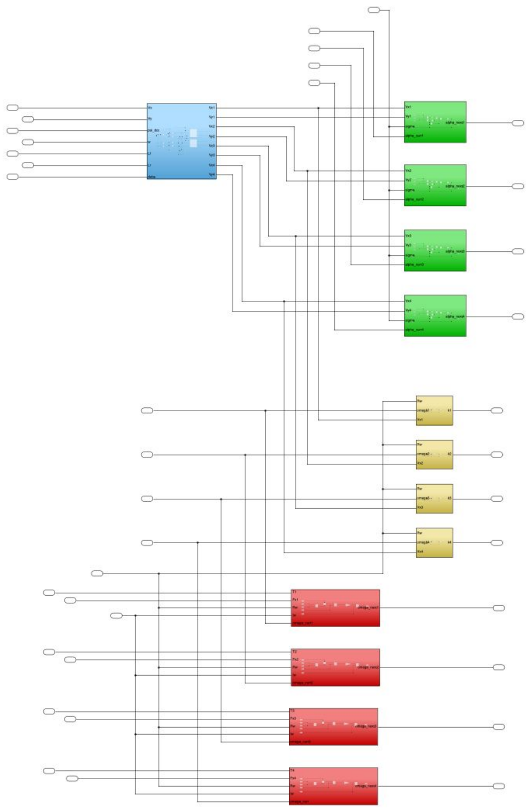
- Vx_next
- Vy_next


1
Fx
2
Fy
1/m
1/m
Vx_dot
Tsmall
1
Vx_next
10
psi_dot
11
psi_ddot
13
Vy_curr
4
phi
5
theta
6
phi_dot
7
theta_dot
8
phi_ddot
9
theta_ddot
3
h
12
Vx_curr
sin
cos
sin
cos
sin_phi
cos_phi
sin_theta
cos_theta
psi_dot_sq
phi_dot_sq
theta_dot_sq
Vy_dot
Tsmall
2
Vy_next

Wheel and tyre dynamics



Wheel dynamics - Omega, alpha and k subsystem



Contains these other subsystems:

- Frame of reference transfer from car to wheels
    - Transfer matrix
- Slip angles
- Wheel dynamics
- Slip ratios

Wheel dynamics - Variables and parameters

Inputs

Variables:

- alpha_curr1...2,3,4
- omega_curr1...2,3,4
- Vx,Vy
- psi_dot
- delta
- T1...2,3,4
- Fx1...2,3,4
-

Constants:

w, Lf, Lr, sigma, Iw

Outputs

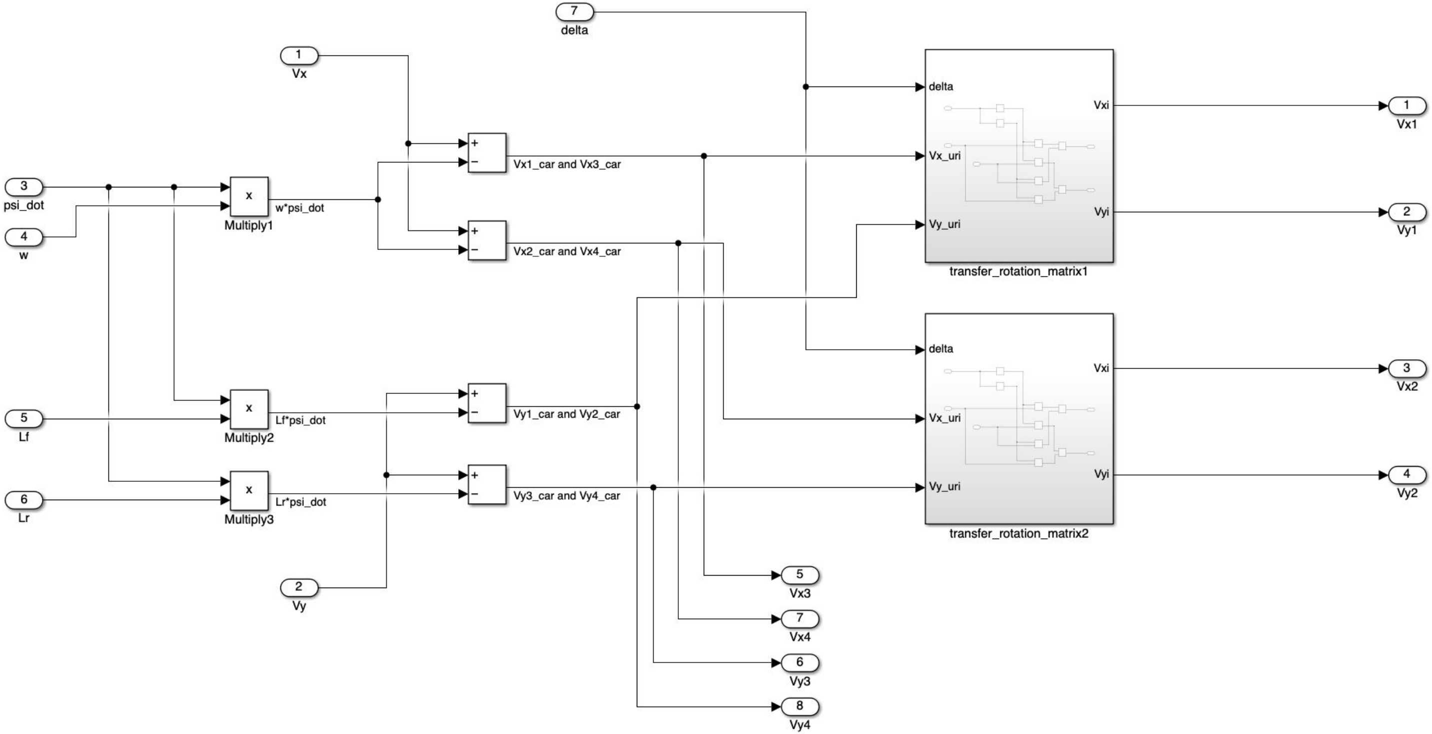
- alpha_next1...2,3,4
- omega_next1...2,3,4
- k1...2,3,4

Frame of reference transfer from car to wheels

From the solid body motion equation of velocity, we calculate the velocities of the car on the point pertaining to the wheels.

$$\boldsymbol{v}_{i,car} = \boldsymbol{v}_o + \begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix} \wedge \overline{OI}$$

Frame of reference transfer from car to wheels

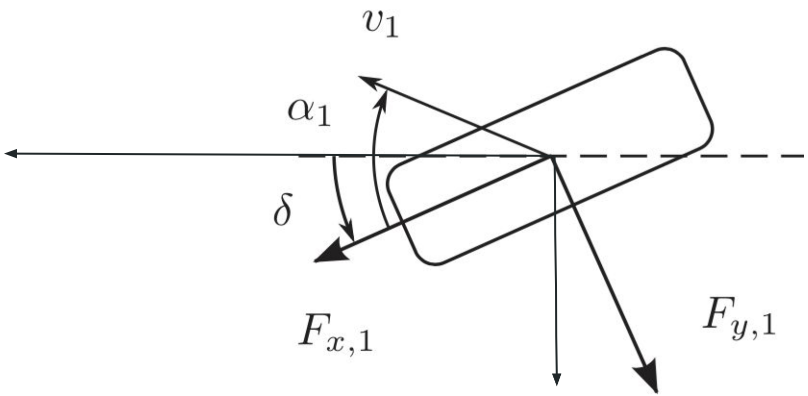


Frame of reference transfer from car to wheels

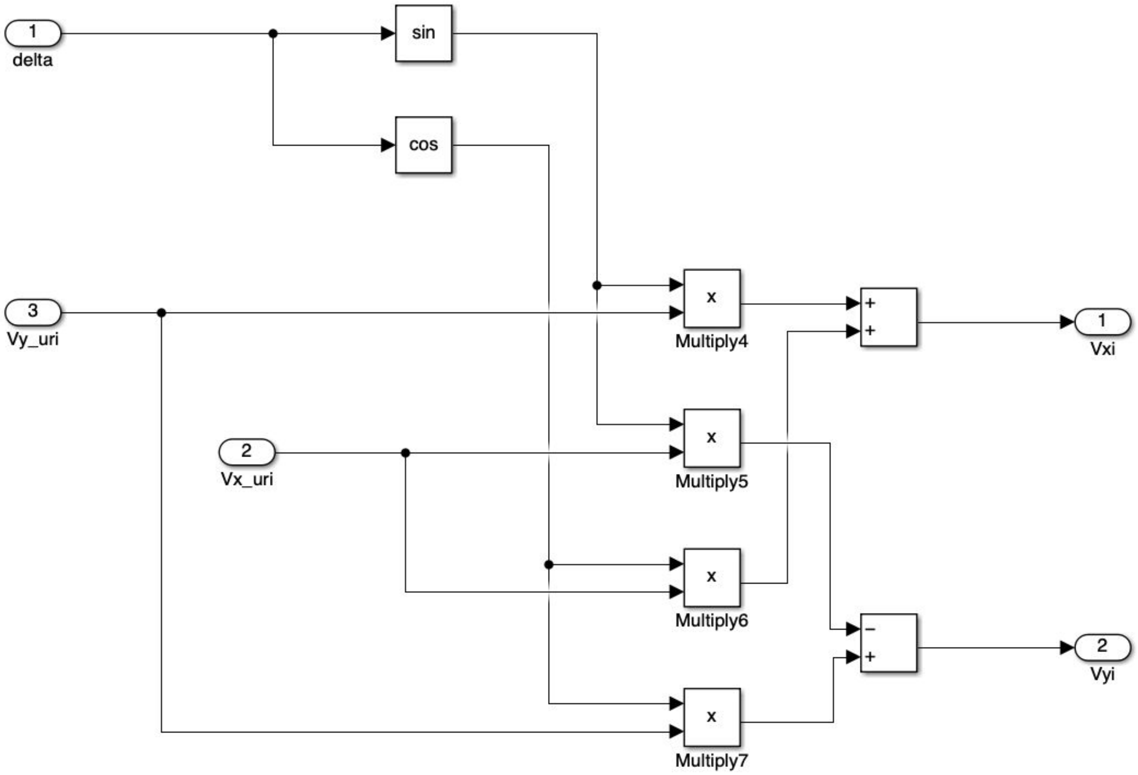
The velocities in the car reference system, calculated on the wheels as part of the same solid, are transferred to each wheel's reference system.

$$v_x = v_{x,car} \cos\delta + v_{y,car} \sin\delta$$

$$v_y = v_{y,car} \cos\delta - v_{x,car} \sin\delta$$



Frame of reference transfer from car to wheels



Slip angles

i: wheel (1 to 4)

σ: relaxation length

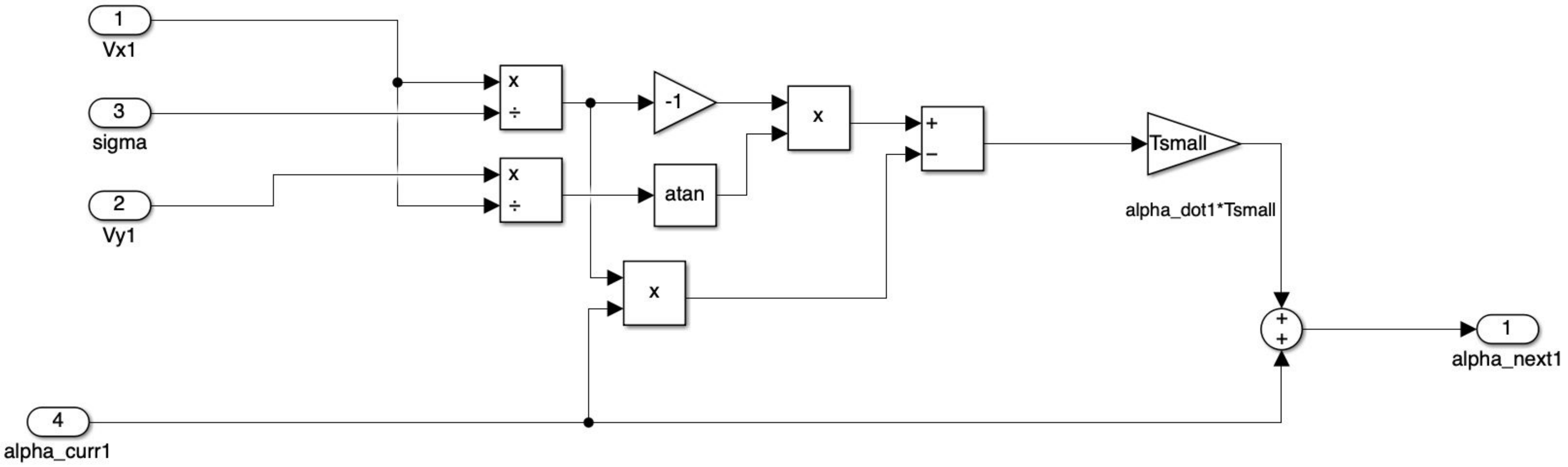
α: slip angle

v: wheel velocity

$$\boxed{\alpha_i(t + dt) = \alpha_i(t) + \dot{\alpha}_i(t) * dt}$$

$$\boxed{\dot{\alpha}_i = -\left(\operatorname{atan}\left(\frac{v_{y,i}}{v_{x,i}}\right) + \alpha_i\right) * \frac{v_{x,i}}{\sigma}}$$

Slip angles



1
Vx1
3
sigma
2
Vy1
4
alpha_curr1
x
÷
x
÷
-1
atan
x
x
+
–
Tsmall
alpha_dot1*Tsmall
+
+
1
alpha_next1

Slip ratios

κ: slip ratio

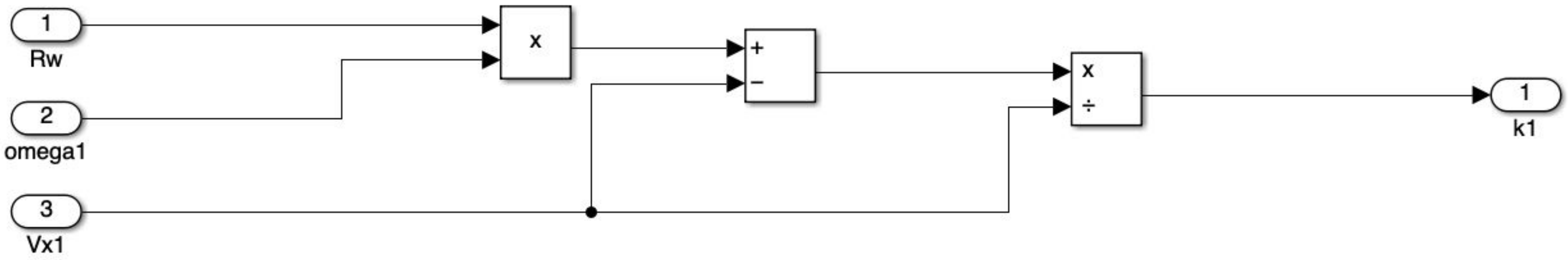
Rw: radius of wheel

ω: wheel rotation speed

v: wheel velocity

$$\kappa_i = \frac{R_w \omega_i - v_{x,i}}{v_{x,i}}$$

Slip ratios



Wheel dynamics

ω: wheel rotation speed

T: wheel torque

F: wheel force against the ground

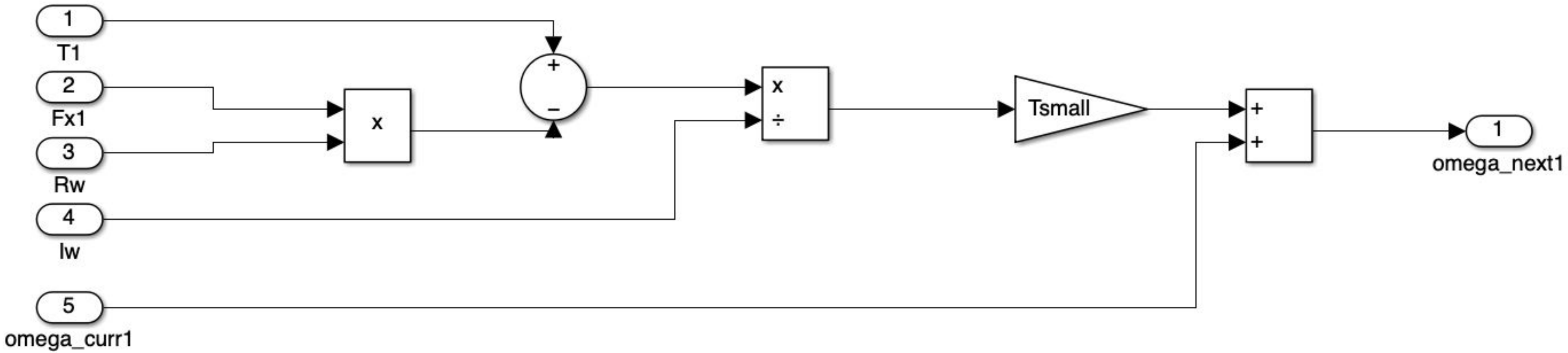
Rw: wheel radius

Iw: wheel moment of inertia

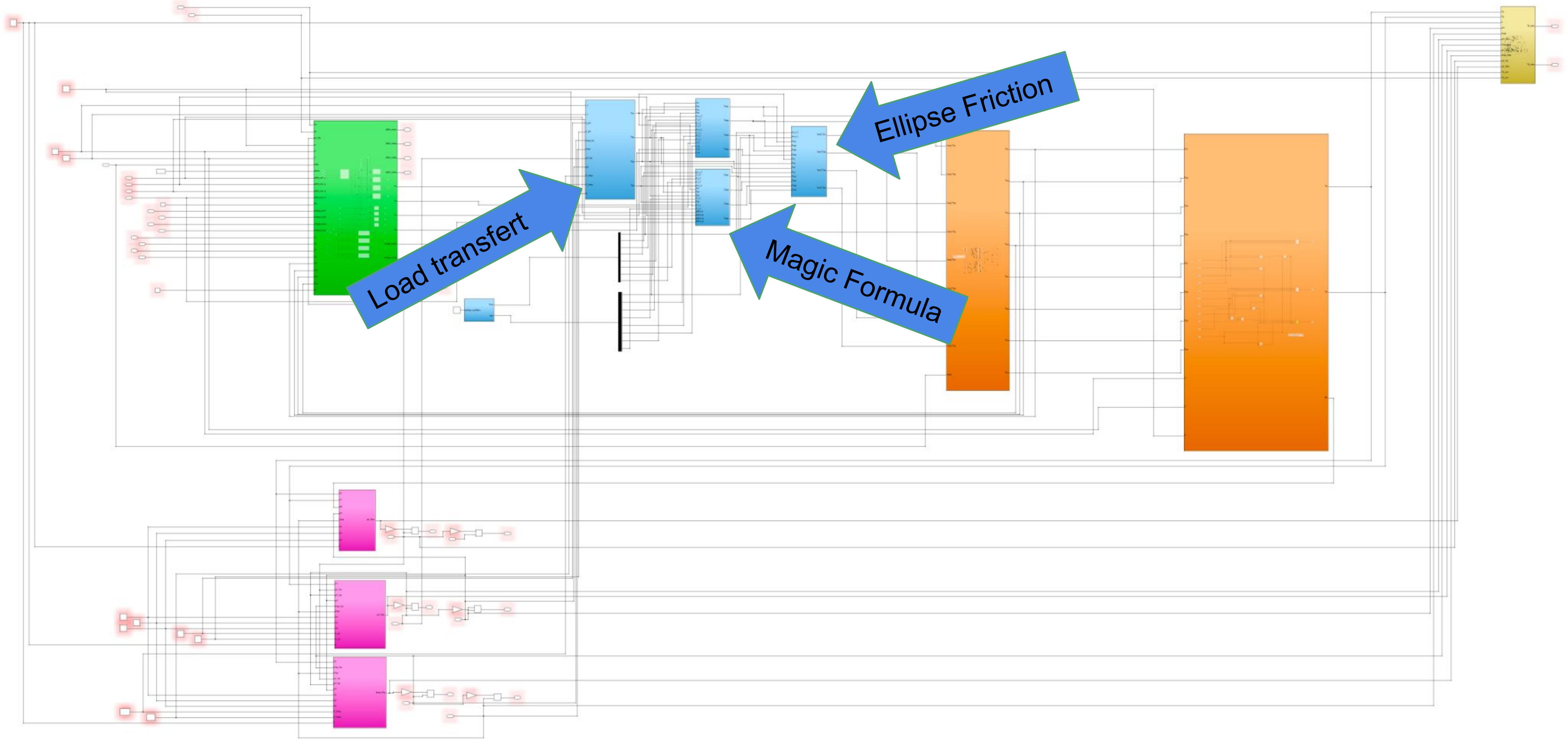
$$\boxed{\omega_i(t + dt) = \omega_i(t) + \dot{\omega}_i(t) * dt}$$

$$\boxed{\dot{\omega}_i = \frac{T_i - F_{x,i} R_w}{I_w}}$$

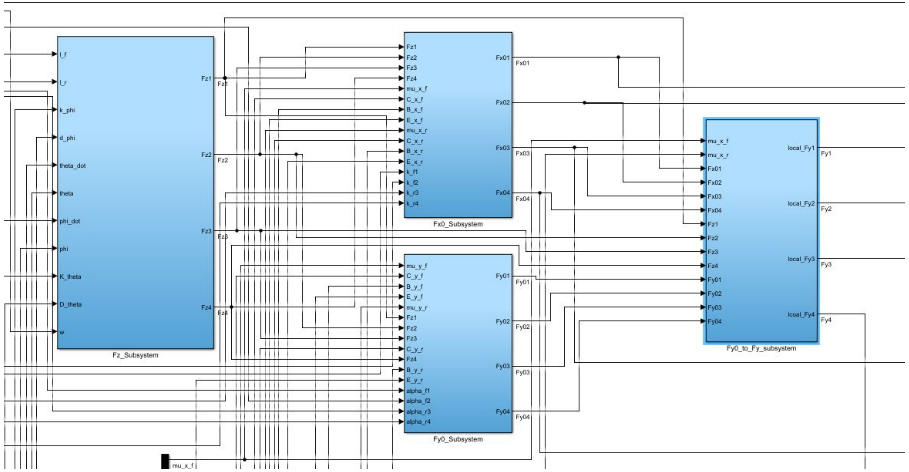
Wheel dynamics



Chassis & Magic formula model



Chassis & Magic formula model



Contains these other subsystems:

- Forces under slip condition subsystem
  - Fz
  - -> Fx0
  - -> Fy0
- Transform to F (Ellipse friction)
- Pacjeka Scenario

Chassis & Magic formula model - Variables and parameters

Inputs

Variables:

- theta_dot
- theta
- k_theta
- d_theta
- phi_dot
- phi
- k_phi
- d_phi

Constants:

w, Lf, Lr, pacjeka parameters

Outputs

- Fy1, Fy2, Fy3, Fy4 (from friction ellipse)

Load transfert - Fz

$$(F_{z,1} + F_{z,2})l_{\mathrm{f}} - (F_{z,3} + F_{z,4})l_{\mathrm{r}} = K_{\theta}\theta + D_{\theta}\dot{\theta}, \quad \sum_{i=1}^{4} F_{z,i} = mg,$$

$$-w(F_{z,1} - F_{z,2}) = K_{\phi,\mathrm{f}}\phi + D_{\phi,\mathrm{f}}\dot{\phi},$$

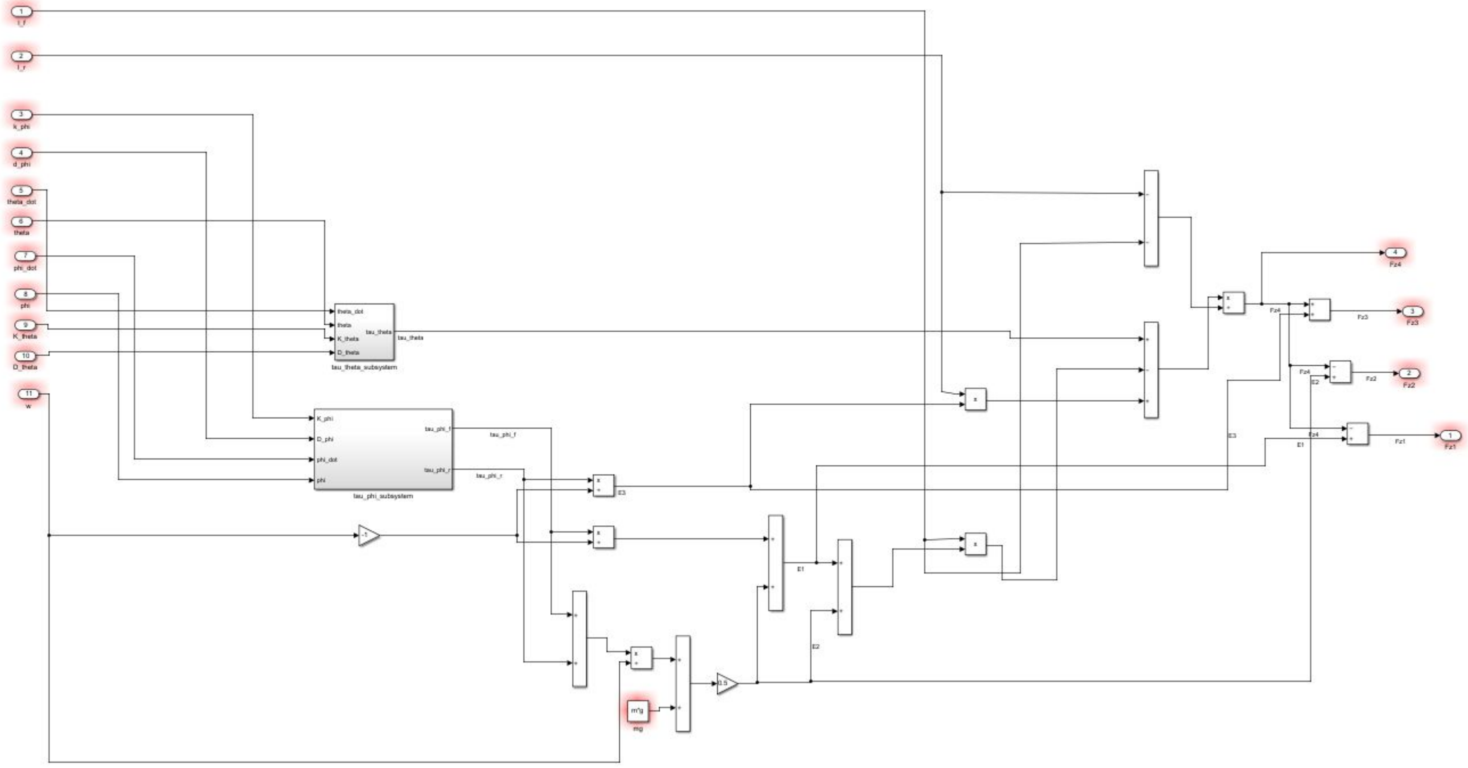
$$-w(F_{z,3} - F_{z,4}) = K_{\phi,\mathrm{r}}\phi + D_{\phi,\mathrm{r}}\dot{\phi},$$

lf, lr : front length et rear length from center of mass

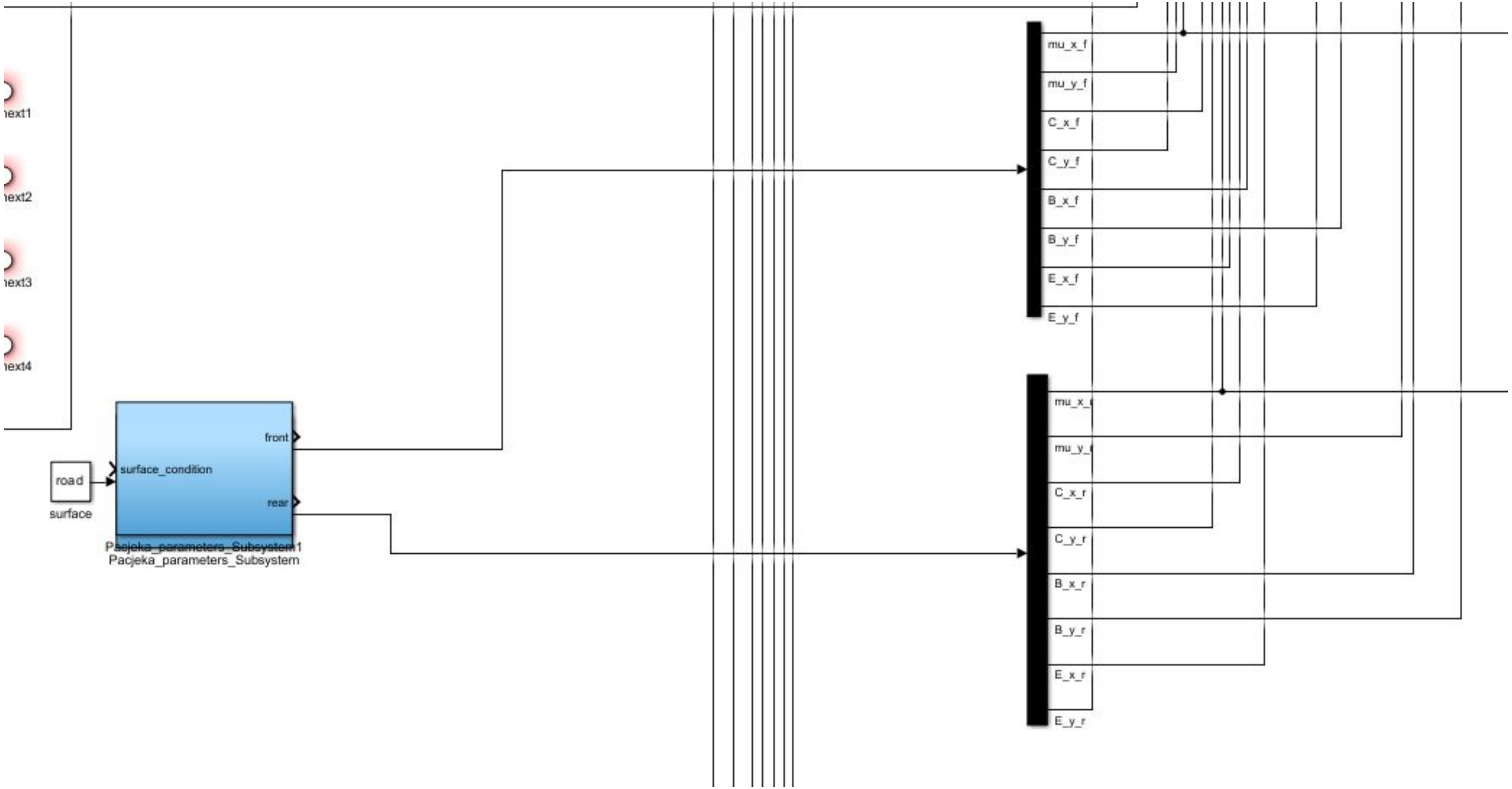
w : width

K : spring characteristic ; D : damper characteristic

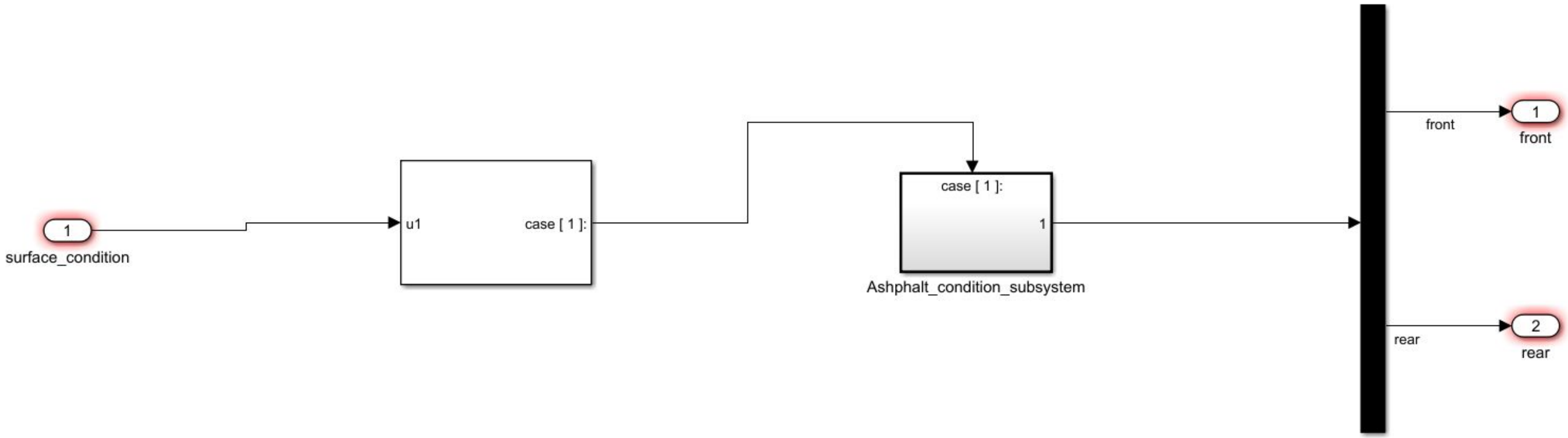
Load transfert - Fz



Magic formula model - Pacjeka parameters



Magic formula model - Pacjeka parameter



Magic formula model - Fy0 & Fx0

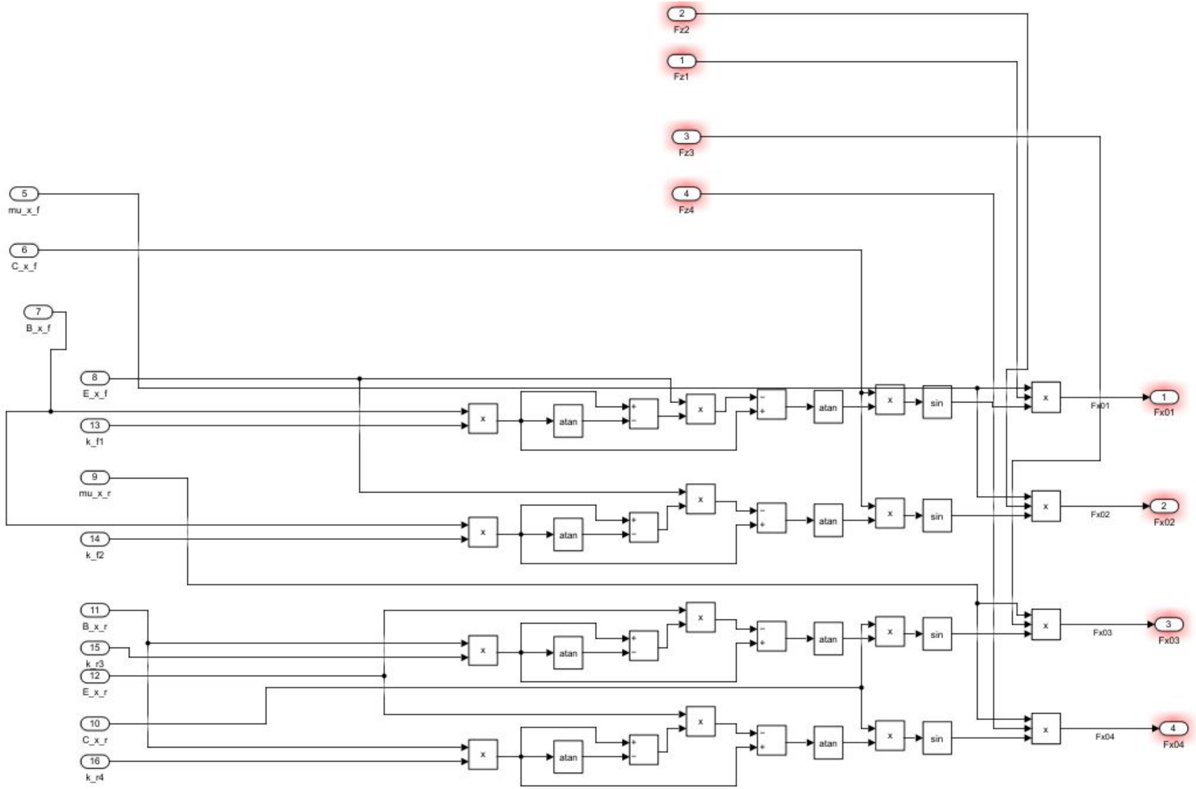
$$F_{x0,i} = \mu_{x,i} F_{z,i} \sin(C_{x,i} \arctan(B_{x,i}\kappa_i - E_{x,i}(B_{x,i}\kappa_i - \arctan B_{x,i}\kappa_i))),$$

$$F_{y0,i} = \mu_{y,i} F_{z,i} \sin(C_{y,i} \arctan(B_{y,i}\alpha_i - E_{y,i}(B_{y,i}\alpha_i - \arctan B_{y,i}\alpha_i))),$$

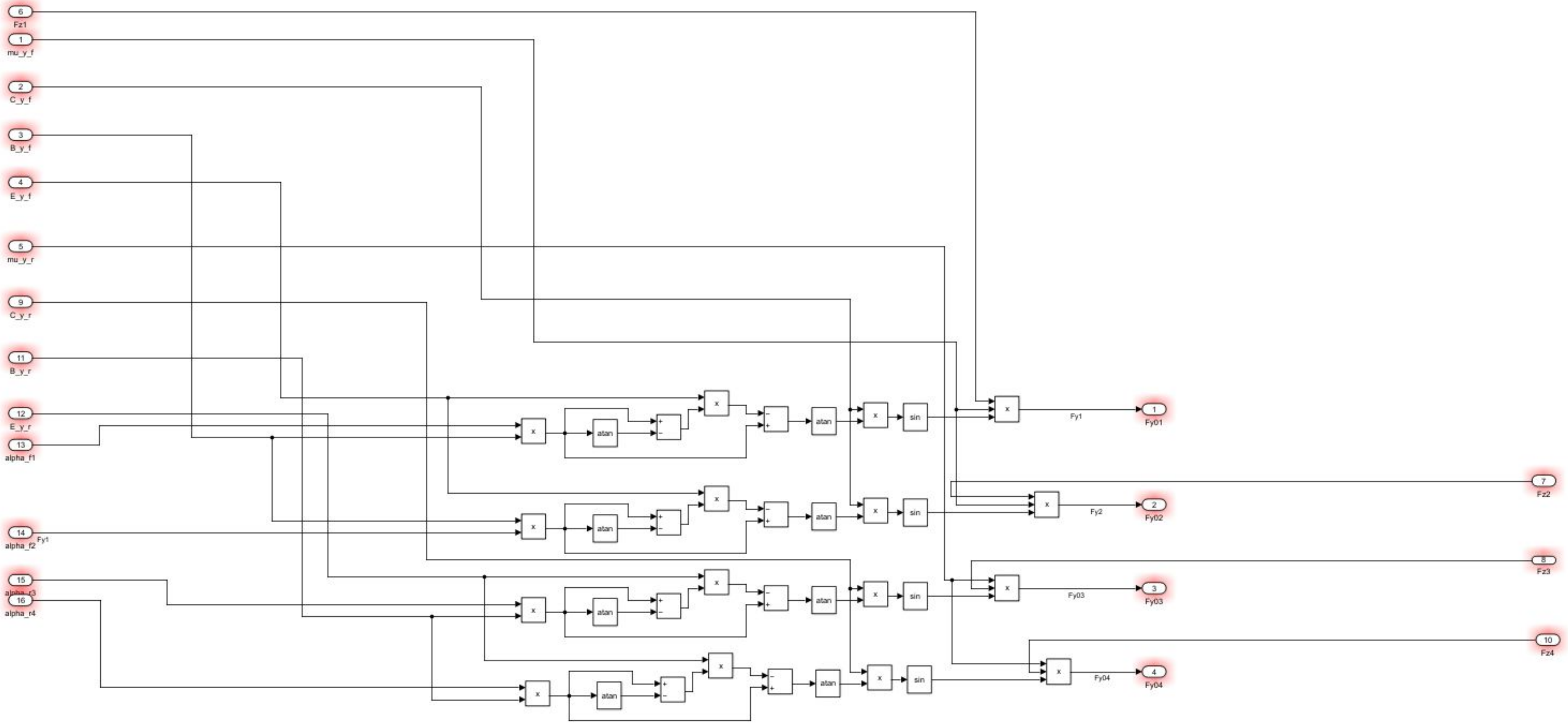
μ : friction coefficient

B, C, E : parameters of pacjeka model.

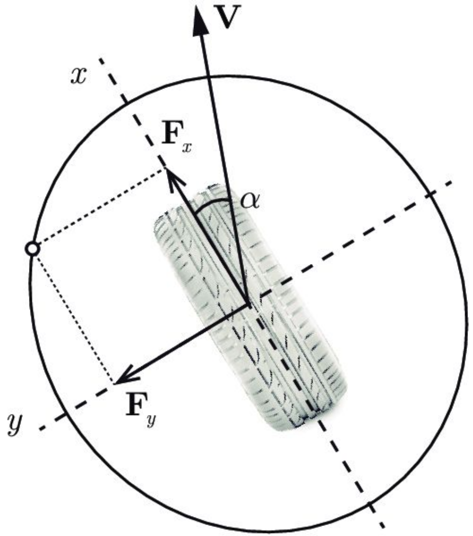
Magic formula model - Fy0 & Fx0



Magic formula model - Fy0 & Fx0

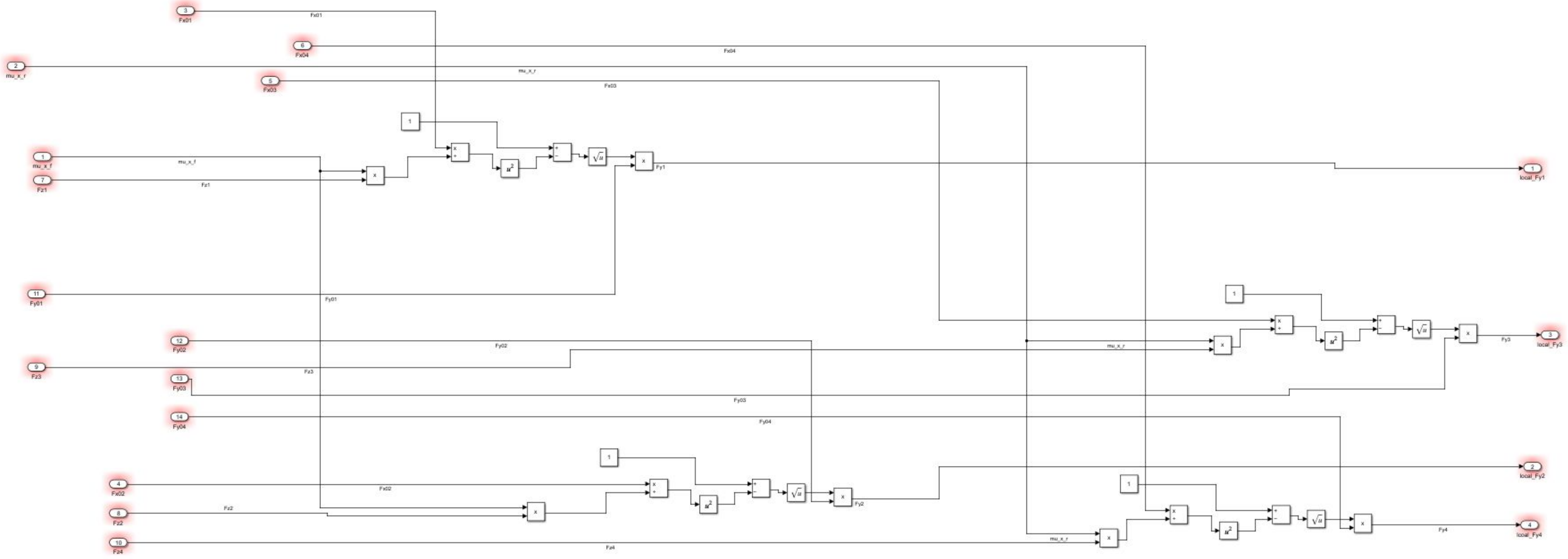


Magic formula model - Combined forces on Fy projection

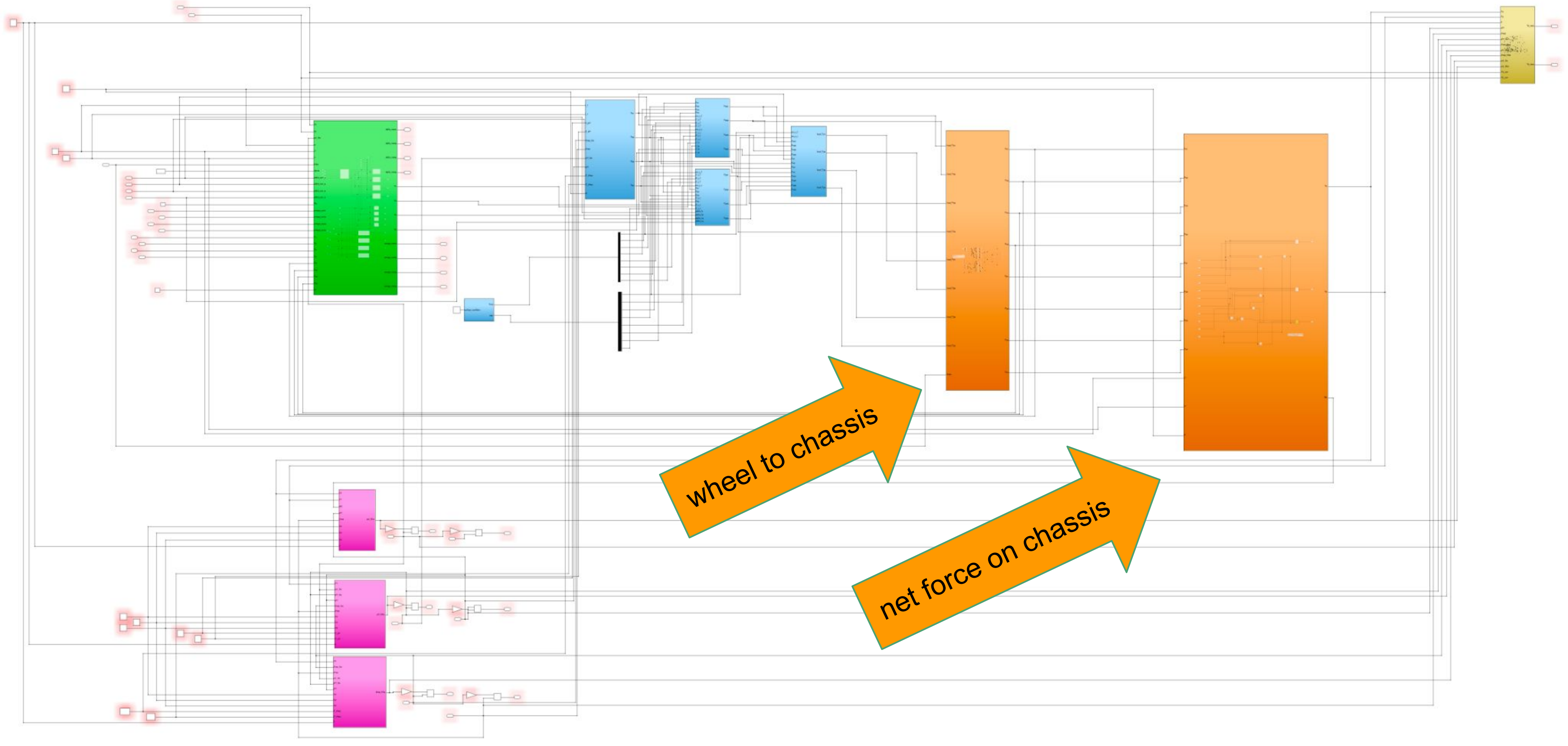


$$F_{y,i} = F_{y0,i}\sqrt{1 - \left(\frac{F_{x0,i}}{\mu_{x,i}F_{z,i}}\right)^2}, \quad i \in \{f, r\} \text{ or } \{1, 2, 3, 4\},$$

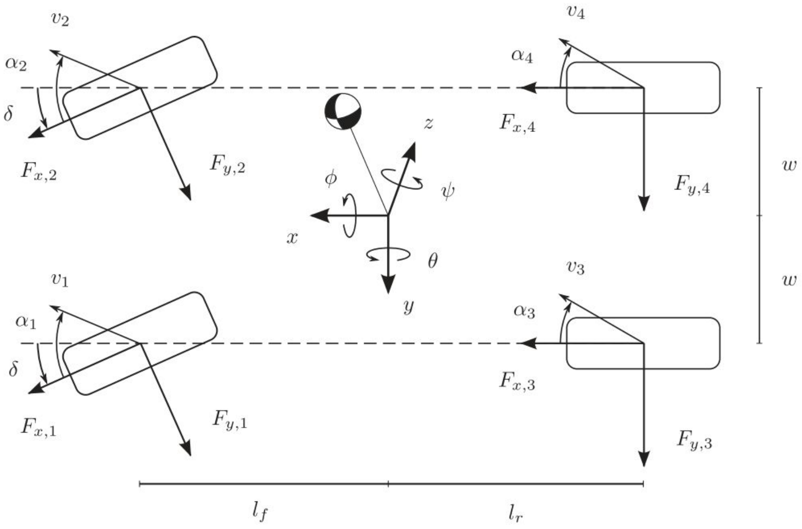
Magic formula model - Combined forces on Fy projection



Wheel to chassis, net forces and moments



External Forces and Moments



Total Forces

- Longitudinal

$$F_X = F_{x1}\cos(\delta_1) - F_{y1}\sin(\delta_1) + F_{x2}\cos(\delta_2) - F_{y2}\sin(\delta_2) + F_{x3} + F_{x4}$$

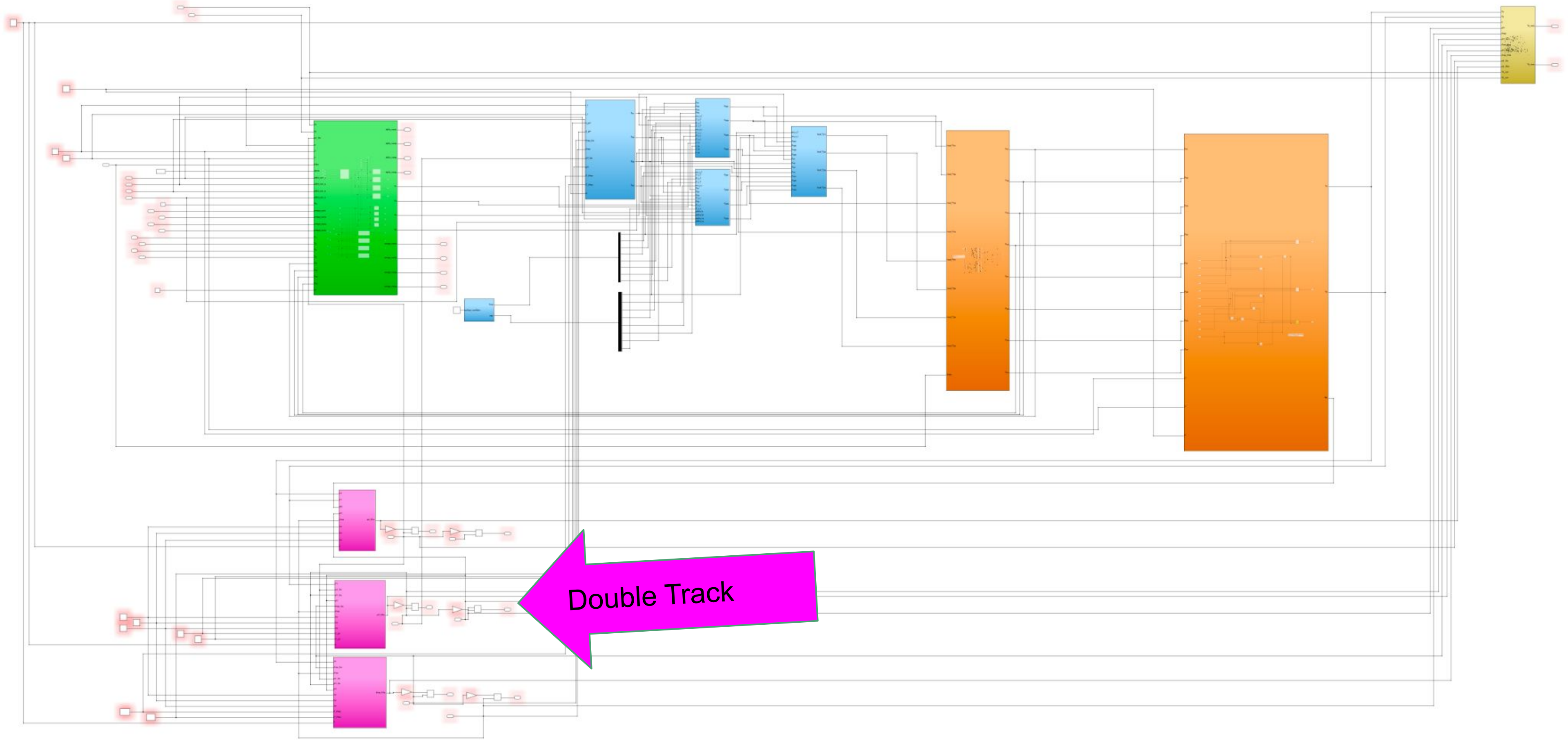
- Lateral

$$F_Y = F_{x1}\sin(\delta_1) + F_{y1}\cos(\delta_1) + F_{x2}\sin(\delta_2) + F_{y2}\cos(\delta_2) + F_{x3} + F_{x4}$$

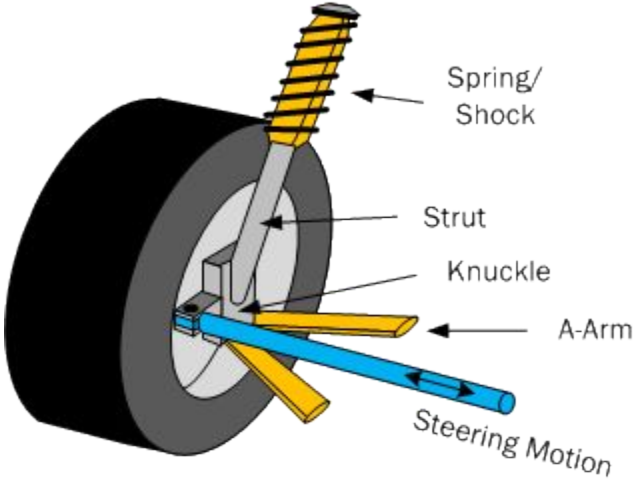
Total Moment

$$\begin{aligned} M_Z = {} & l_f\Big(F_{x1}\sin(\delta_1) + F_{x2}\sin(\delta_2) + F_{y1}\cos(\delta_1) + F_{y2}\cos(\delta_2)\Big) \\ & + w_f\Big(- F_{x1}\cos(\delta_1) + F_{x2}\cos(\delta_2) + F_{y1}\sin(\delta_1) - F_{y2}\sin(\delta_2)\Big) \\ & - l_r(F_{y3} + F_{y4}) - w_r(F_{x3} + F_{x4}) \end{aligned}$$

Chassis Model



Suspension System



Moment of Rotational Spring damper system.

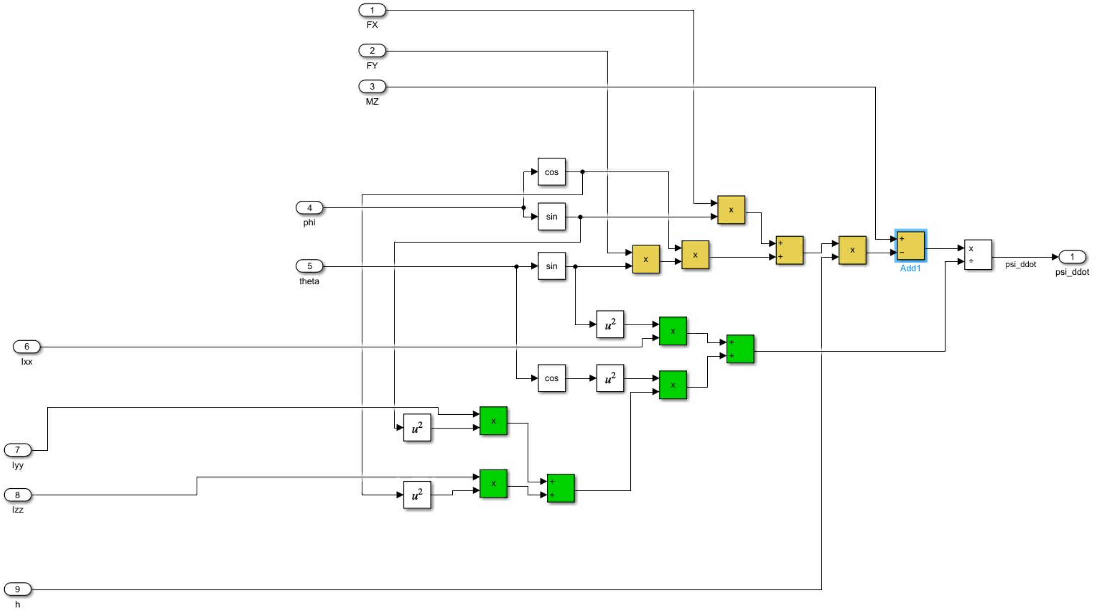
- Roll direction

$$\tau_\phi = (K_{\phi,\mathrm{f}} + K_{\phi,\mathrm{r}})\phi + (D_{\phi,\mathrm{f}} + D_{\phi,\mathrm{r}})\dot{\phi},$$

- Pitch Direction

$$\tau_\theta = K_\theta \theta + D_\theta \dot{\theta},$$

Yaw Dynamics



Colour Marking

- Moment of Inertia
- Moment of force

$$\ddot{\psi}(I_{xx}\sin(\theta)^2 + \cos(\theta)^2(I_{yy}\sin(\phi)^2 + I_{zz}\cos(\phi)^2)) = M_Z - h\Big(F_X\sin(\phi) + F_Y\sin(\theta)\cos(\phi)\Big)$$

Yaw dynamics - Variables and parameters

Inputs

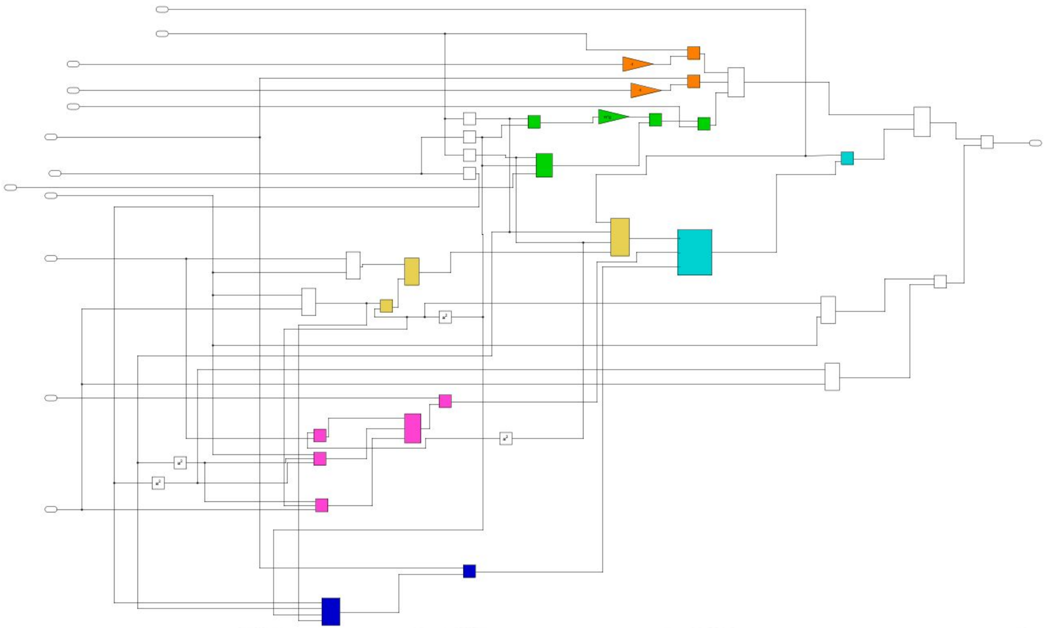
Variables:

- Fx
- Fy
- Mz
- phi
- theta
- Ixx
- Izz
- h

Output

- psi_ddot

Pitch Dynamics



$$\ddot{\theta}(I_{yy}\cos(\phi)^2 + I_{zz}\sin(\phi)^2) = -K_\theta\theta - D_\theta\dot{\theta}$$
$$+ h\Big(mg\sin(\theta)\cos(\phi) - F_X\cos(\theta)\cos(\phi)\Big) + \dot{\psi}\Big(\dot{\psi}\sin(\theta)\cos(\theta)(\Delta I_{xy}$$
$$+ \cos(\phi)^2\Delta I_{yz}) - \dot{\phi}\cos(\theta)^2 I_{xx} + \sin(\phi)^2\sin(\theta)^2 I_{yy}$$
$$+ \sin(\theta)^2\cos(\phi)^2 I_{zz}) - \dot{\theta}\big(\sin(\theta)\sin(\phi)\cos(\phi)\Delta I_{yz}\big)\Big)$$

Colour Marking

- Moment changing by roll and mass center
- Angular Momentum by roll
- Angular Momentum by yaw
- Angular Momentum by pitch

Pitch dynamics - Variables and parameters

Inputs

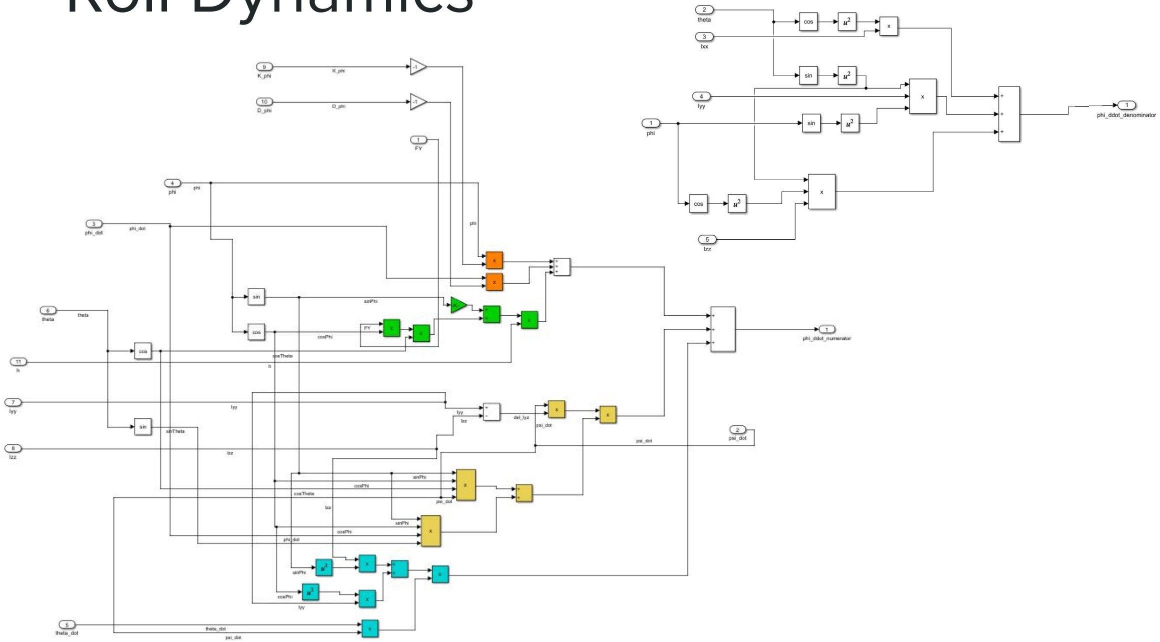
Variables:

- Phi_dot
- theta
- K_theta
- D_theta
- h
- theta_dot
- phi
- Fx
- Iyy
- Ixx
- phi_dot
- Izz

Output

- theta_ddot

Roll Dynamics



Colour Marking

- ○ Rotational terms
- ● Angular Momentum (same inertial forces)
- ● Angular Momentum (different inertial force)
- ● External Force
- ● Moment changing by roll and mass center

$$\ddot{\theta}(I_{yy}\cos(\phi)^2 + I_{zz}\sin(\phi)^2) = -K_\theta\theta - D_\theta\dot{\theta}$$
$$+ h\Big(mg\sin(\theta)\cos(\phi) - F_X\cos(\theta)\cos(\phi)\Big) + \dot{\psi}\Big(\dot{\psi}\sin(\theta)\cos(\theta)(\Delta I_{xy}$$
$$+ \cos(\phi)^2\Delta I_{yz}) - \dot{\phi}\cos(\theta)^2 I_{xx} + \sin(\phi)^2\sin(\theta)^2 I_{yy}$$
$$+ \sin(\theta)^2\cos(\phi)^2 I_{zz}) - \dot{\theta}\big(\sin(\theta)\sin(\phi)\cos(\phi)\Delta I_{yz}\big)\Big)$$

Roll dynamics - Variables and parameters

Inputs

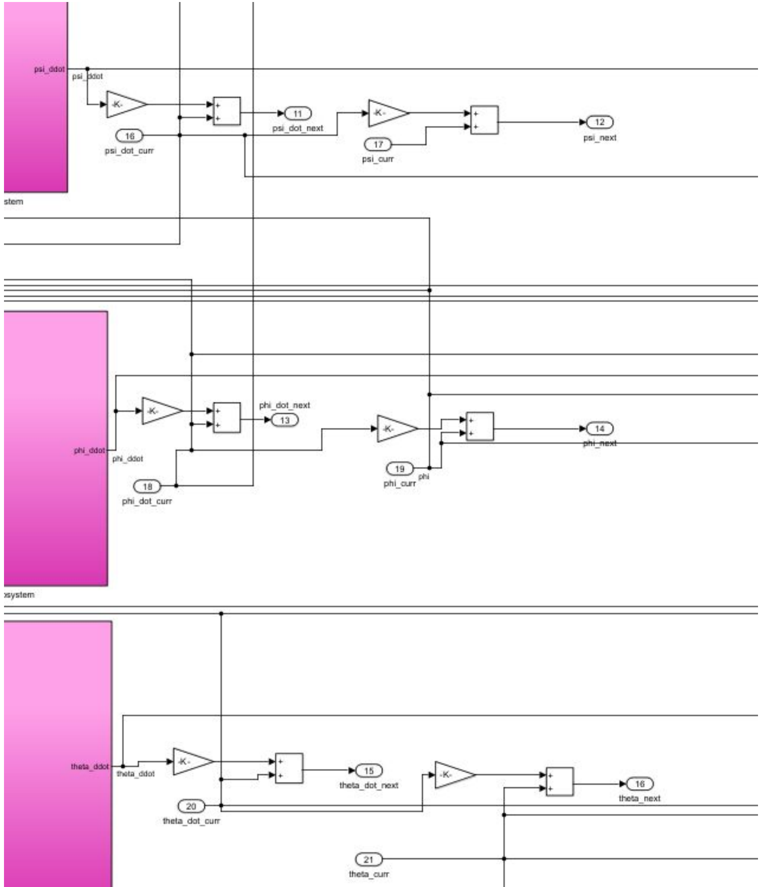
Variables:

- Fy
- psi_dot
- phi
- phi_dot
- theta
- theta_dot
- K_phi
- D_phi
- h

Output

- phi_ddot

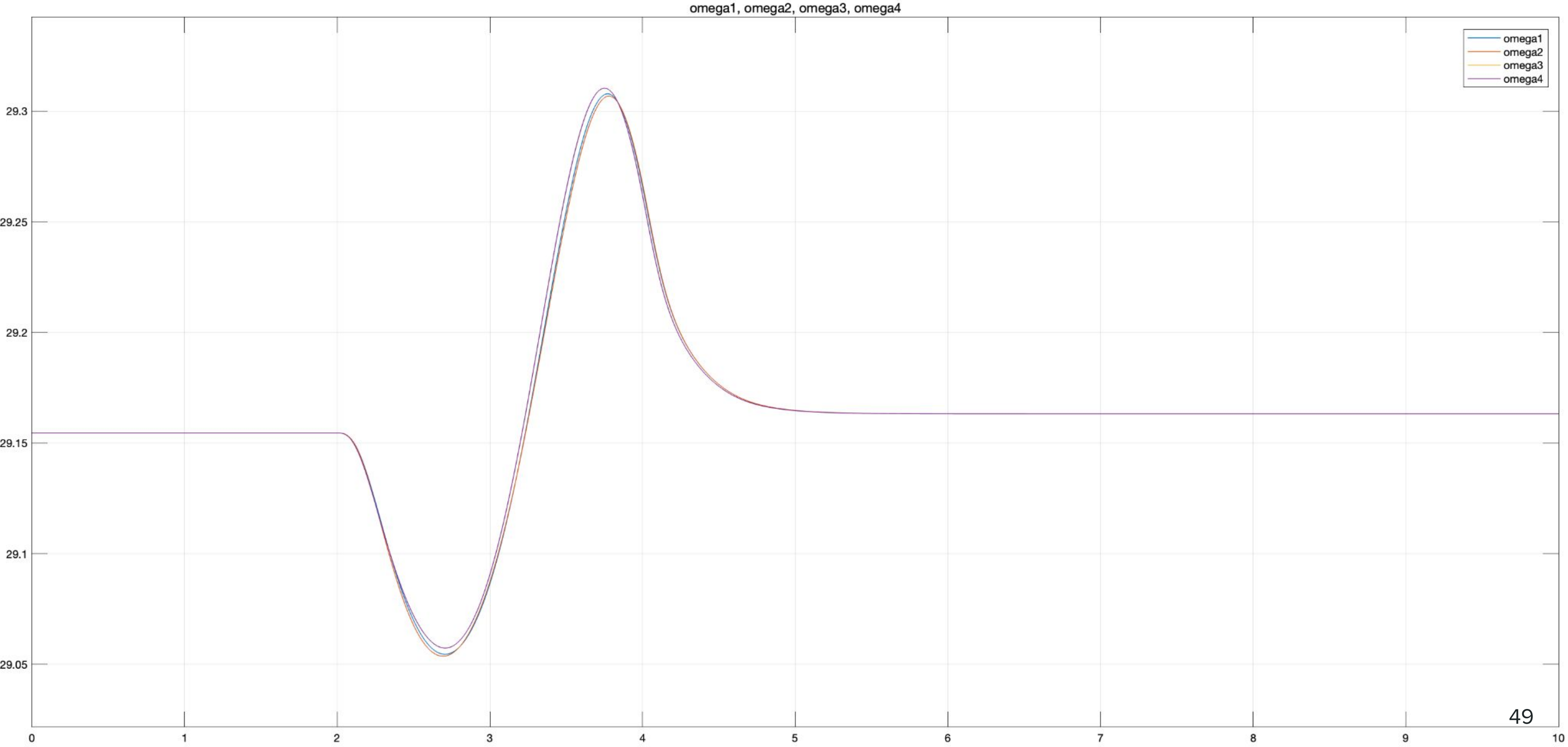
Transforming ddot to _next



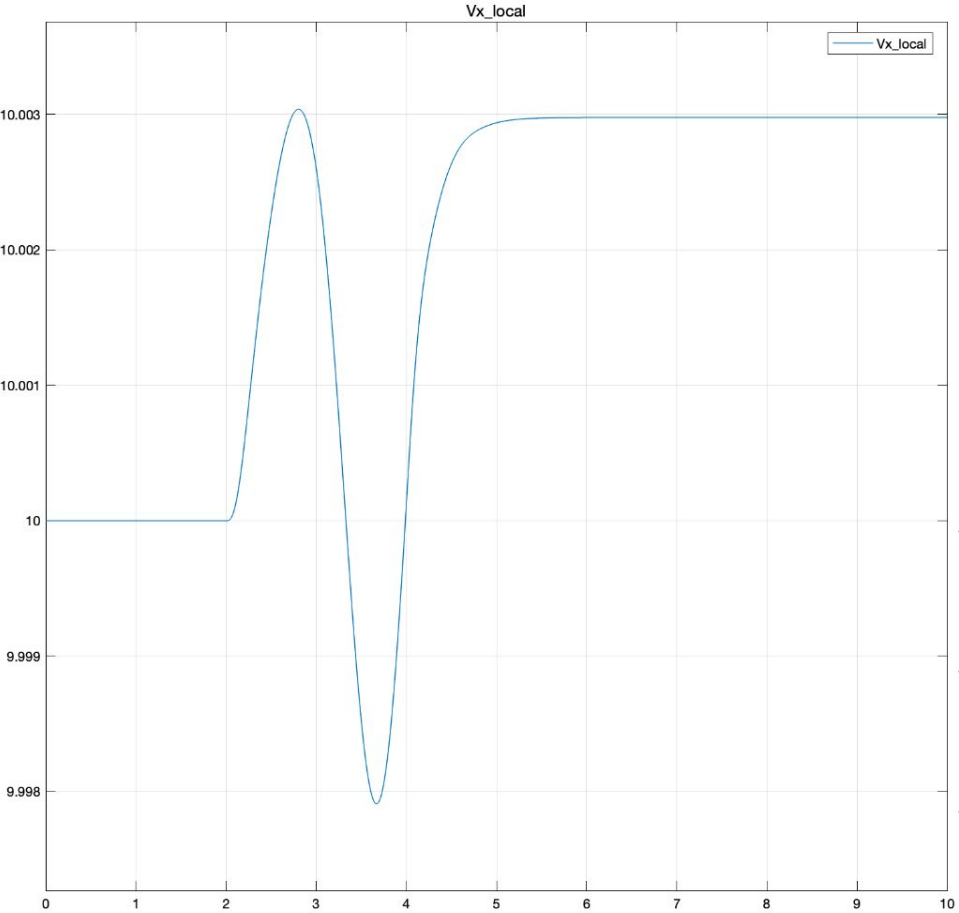
System verification

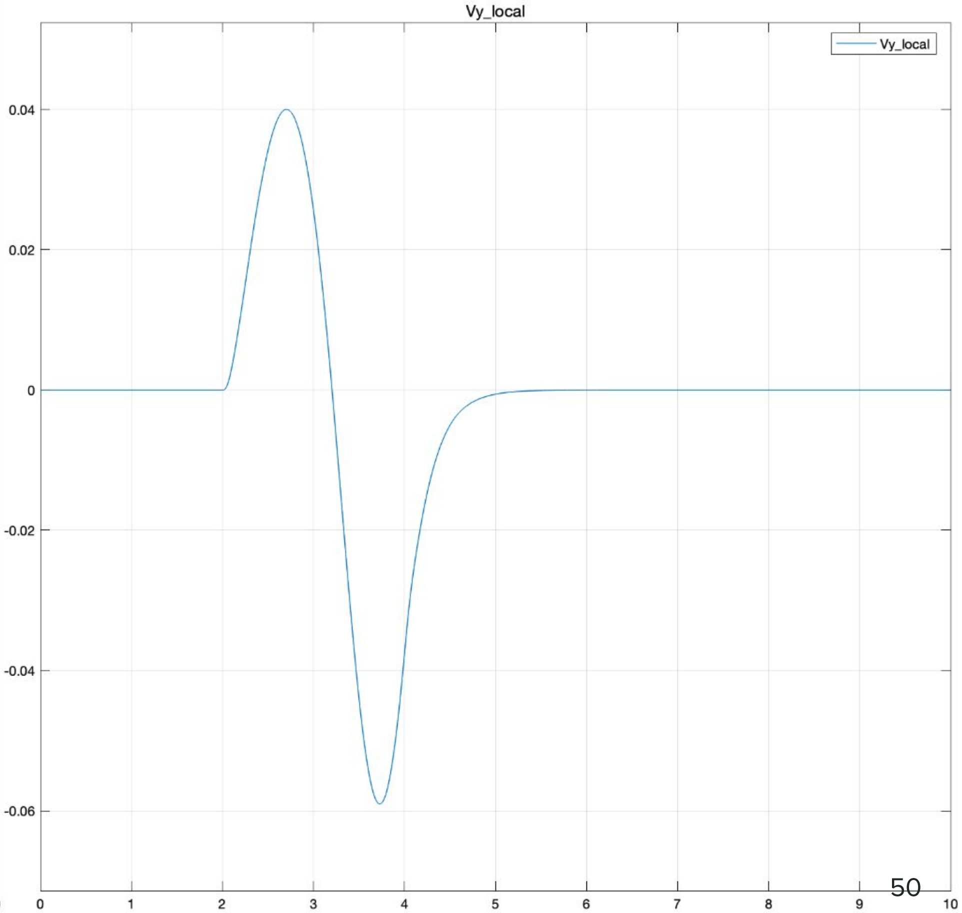
- Roll, yaw and pitch angles and rates
- Chassis velocities
- Wheel rotation speeds

Wheel rotation speeds

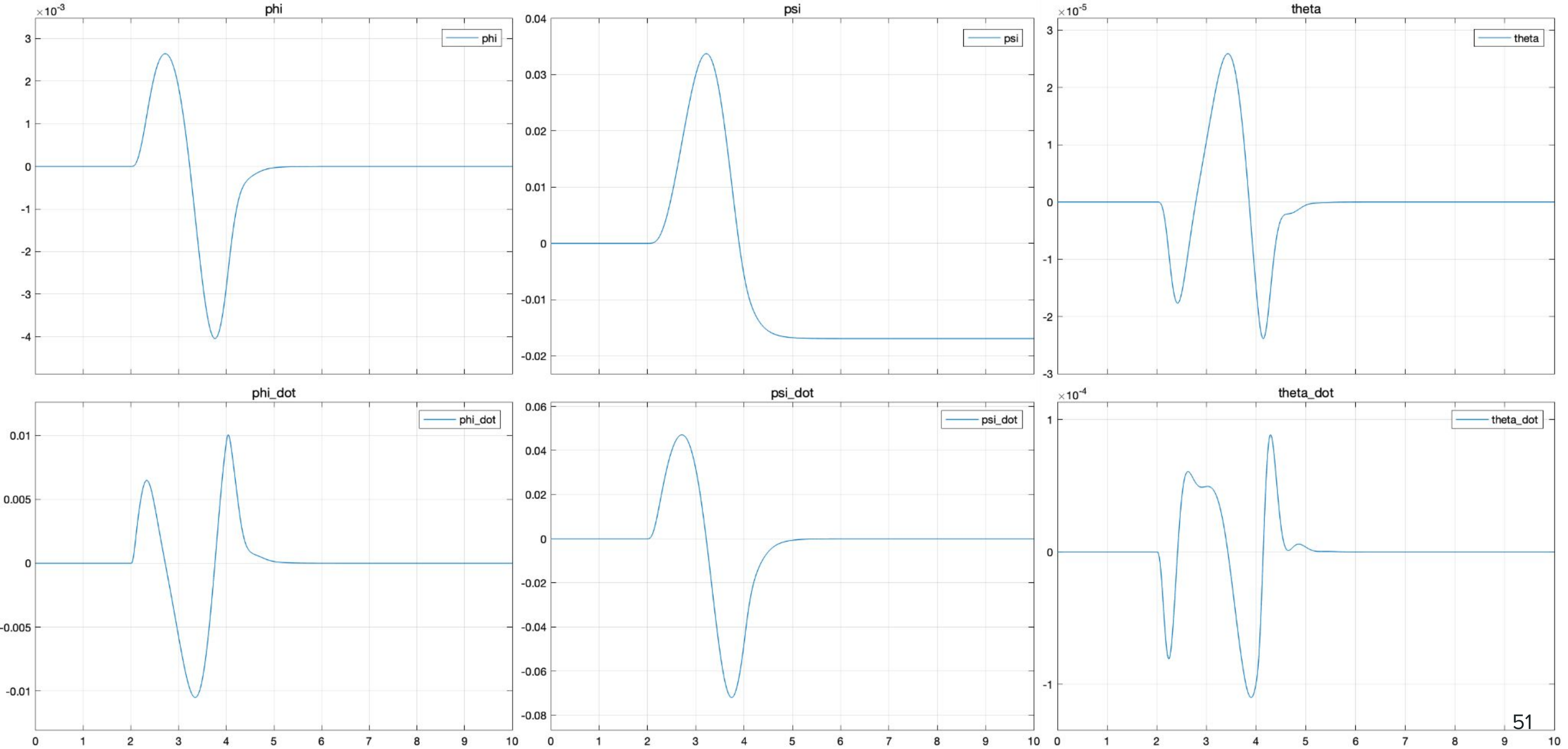


Chassis velocities





Roll, yaw and pitch angles and rates



Active Chassis Control

- Formulation of Control Problem
- Active Chassis Control
  - Direct yaw moment control
  - Active steering control
  - Integrated active steering and direct yaw moment control
- Comparison And Discussion

Formulation of Control Problem

Aim: Yaw stability control, which is a tracking problem (desired psi dot , desired slip angles)

xdot = G(x, y, u) Vehicle model

h(x, y, u) = 0 tyre-force model

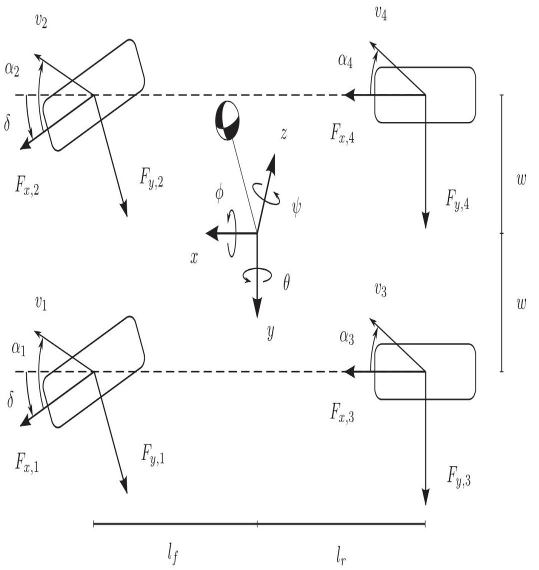
inputs: steering angle, braking torques T=(Tr, Tf)

variables to be controlled: yaw rate and slip angles

How we can get desired psi dot , desired slip angles?

Formulation of Control Problem

Vehicle Model for Simulation



9 DOF nonlinear vehicle dynamic model

Vehicle Model for Controller Design

Assumptions:

(i) Tires forces operate in the linear region.

(ii) The vehicle moves on plane surface/flat road (planar motion).

(iii) Left and right wheels at the front and rear axle are lumped in single wheel at the centre line of the vehicle.

(iv) Constant vehicle speed i.e. the longitudinal acceleration equal to zero (a_x=0)

(v) Steering angle and sideslip angle are assumed small (≈ 0).

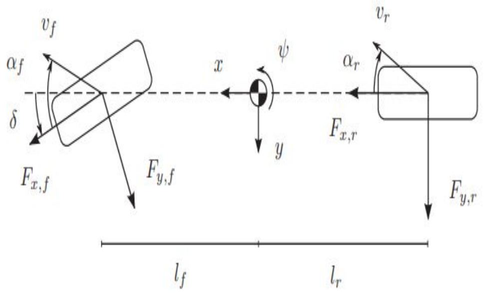
(vi) No braking is applied at all wheels.

(vii) Centre of gravity (CG) is not shifted as vehicle mass is changing.

(viii) 2 front wheels have the same steering angle.

(ix) Desired vehicle sideslip is assumed to be zero in steady state.

vx=10m/s



2 DOF linear bicycle model

Aripin, M.K. *et al.* (2014) ‘A review of active yaw control system for vehicle handling and Stability Enhancement’, *International Journal of Vehicular Technology*, 2014, pp. 1–15. doi:10.1155/2014/437515.

Formulation of Control Problem

2 DOF linear bicycle model

$$\dot{x} = Ax + Bu,$$

$$\begin{bmatrix} \dot{\beta} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \beta \\ r \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} u,$$

$$\begin{bmatrix} \dot{\beta} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \frac{-C_f - C_r}{mv} & -1 + \frac{C_r l_r - C_f l_f}{mv^2} \\ \frac{C_r l_r - C_f l_f}{I_z} & \frac{-C_f l_f^2 - C_r l_r^2}{I_z v} \end{bmatrix} \begin{bmatrix} \beta \\ r \end{bmatrix} + \begin{bmatrix} \frac{C_f}{mv} \\ \frac{C_f l_f}{I_z} \end{bmatrix} \delta_f,$$

where cornering stiffness is in the following when slip angle is small enough (<=5°)

$$C_\alpha = \lim_{\alpha \to 0} \frac{\partial(-F_y)}{\partial \alpha} = \left| \lim_{\alpha \to 0} \right| \frac{\partial F_y}{\partial \alpha}.$$

Aripin, M.K. *et al.* (2014) ‘A review of active yaw control system for vehicle handling and Stability Enhancement’, *International Journal of Vehicular Technology*, 2014, pp. 1–15. doi:10.1155/2014/437515.

Pereira, C.L., da Costa Neto, R.T. and Loiola, B.R. (2021) ‘Cornering stiffness estimation using Levenberg–Marquardt approach’, *Inverse Problems in Science and Engineering*, 29(12), pp. 2207–2238. doi:10.1080/17415977.2021.1910683.

Formulation of Control Problem

Therefore, In the steady state condition, the desired yaw rate response *rd* can be obtained by using the following equation:

$$r_d = \frac{v}{(l_f + l_r) + k_{us} v^2} \cdot \delta_f,$$

rd_upperbound=0.85µg/vdot

where stability factor *kus* is depending on the vehicle parameters and defined as follows:

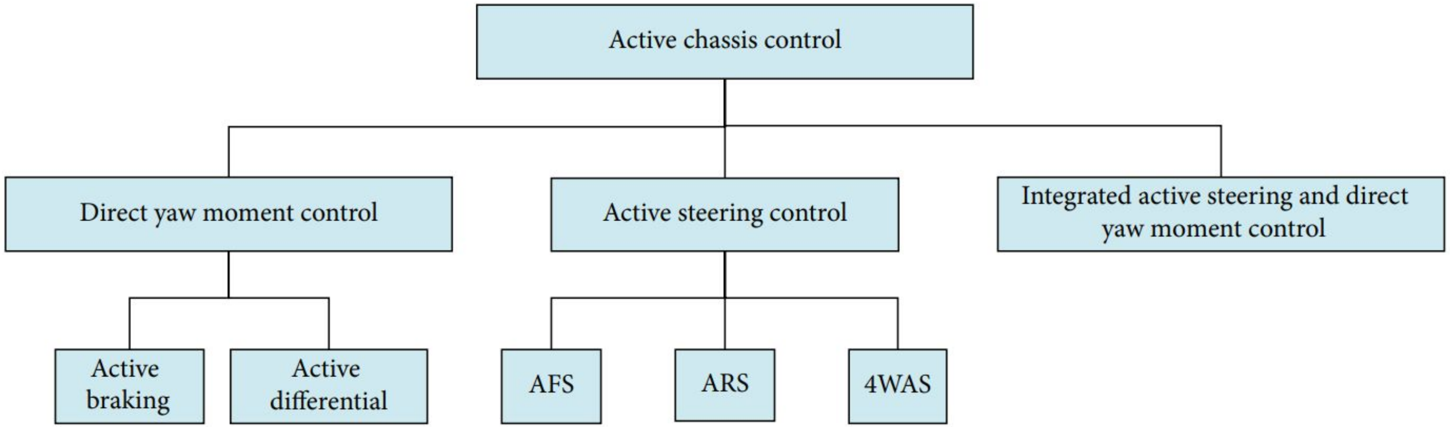
$$k_{us} = \frac{m(l_r C_r - l_f C_f)}{(l_f + l_r) C_f C_r}.$$

For the steady state condition, the desired sideslip is always zero

$$\beta_d = 0$$

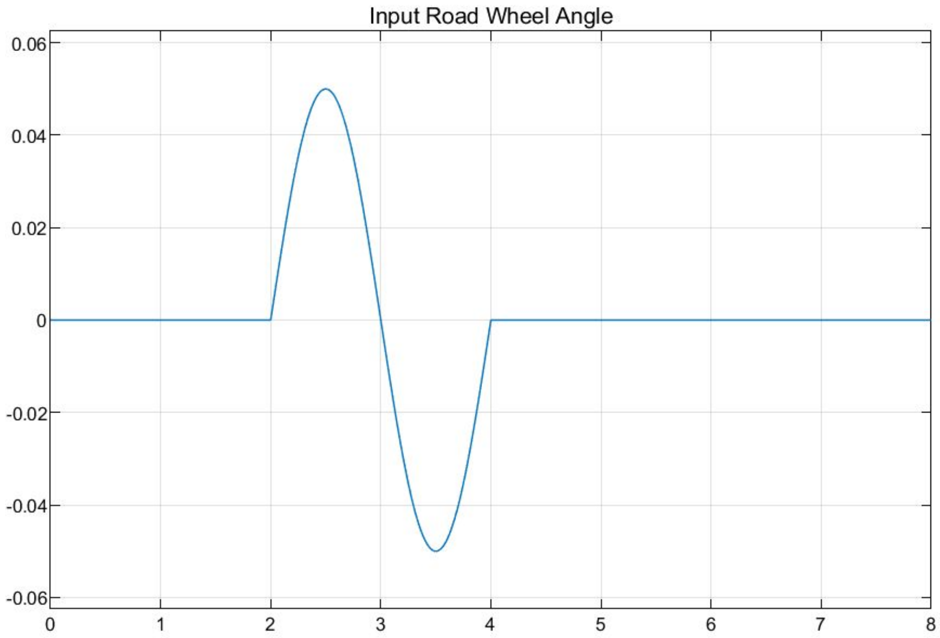
β_upperbound=tan-1(0.02µg)

Active Chassis Control



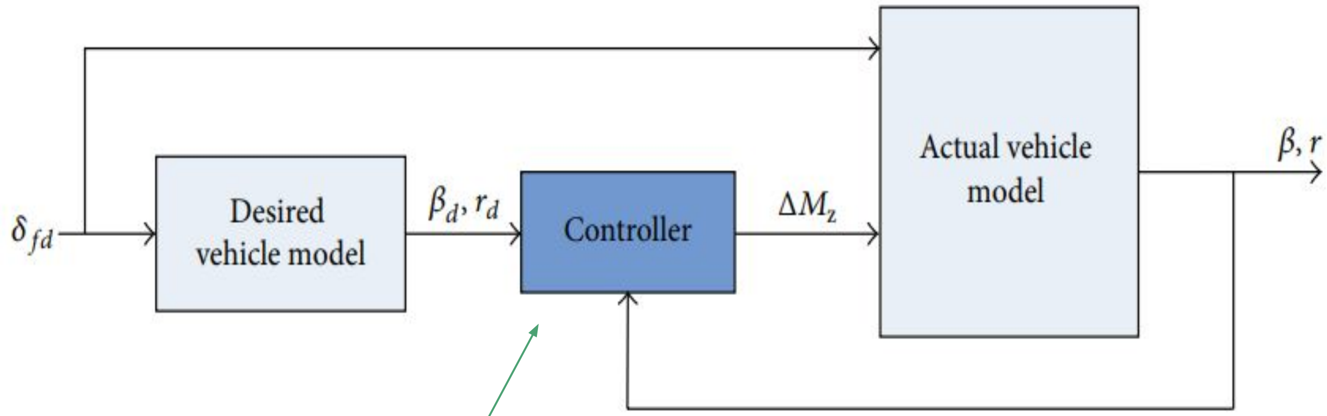
Active Chassis Control

Double-Lane Change Maneuver



Active Chassis Control

Direct Yaw Moment Control



Upper controller + Lower controller

Active Chassis Control

Direct Yaw Moment Control

Upper controller — desired yaw moment

$$\begin{bmatrix} \dot{\beta} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \dfrac{-C_f - C_r}{mv} & -1 + \dfrac{C_r l_r - C_f l_f}{mv^2} \\ \dfrac{C_r l_r - C_f l_f}{I_z} & \dfrac{-C_f l_f^2 - C_r l_r^2}{I_z v} \end{bmatrix} \begin{bmatrix} \beta \\ r \end{bmatrix} + \begin{bmatrix} \dfrac{C_f}{mv} \\ \dfrac{C_f l_f}{I_z} \end{bmatrix} \delta_{fd} + \begin{bmatrix} 0 \\ \dfrac{1}{I_z} \end{bmatrix} M_z.$$

disturbance

check the controllability

rank(B AB) full rank

Active Chassis Control

Direct Yaw Moment Control

Upper controller — desired yaw moment

The steady state solution of the deterministic linear optimal regulator problem

Theorem: Let us consider the previous problem with the additive hypotheses :

- The matrices A, B, Q, R are constant
- The matrix Q is positive definite

Then there exists a unique optimal solution:

$$u^o(t) = -R^{-1}B^T K_r x^o$$
$$\dot{x}^o(t) = \left[A - BR^{-1}B^T K_r\right]x^o(t), \qquad x^o(t_i) = x^i$$

where:

K_r is the constant matrix, unique solution definite positive of the **algebraic Riccati equation:**

$$\boxed{K_r BR^{-1}B^T K_r - K_r A - A^T K_r - Q = 0}$$

The minimum value for the cost index is:

$$\boxed{J\left(x^o, u^o\right) = \frac{1}{2} x^{iT} K_r x^i}$$

ΔM=u(t)

https://sites.google.com/a/uniroma1.it/danielaiacoviello/

Active Chassis Control

Direct Yaw Moment Control

Lower controller — braking torque distribution

$\Delta M = w^*(Fxfr - Fxfl) = w^*\Delta Fxf$

$\Delta Fxf = 2\Delta M/w$

Recall wheel dynamics

$$T_i - I_w\dot{\omega}_i - F_{x,i}R_w = 0, \quad i \in \{f, r\} \text{ or } \{1, 2, 3, 4\}.$$

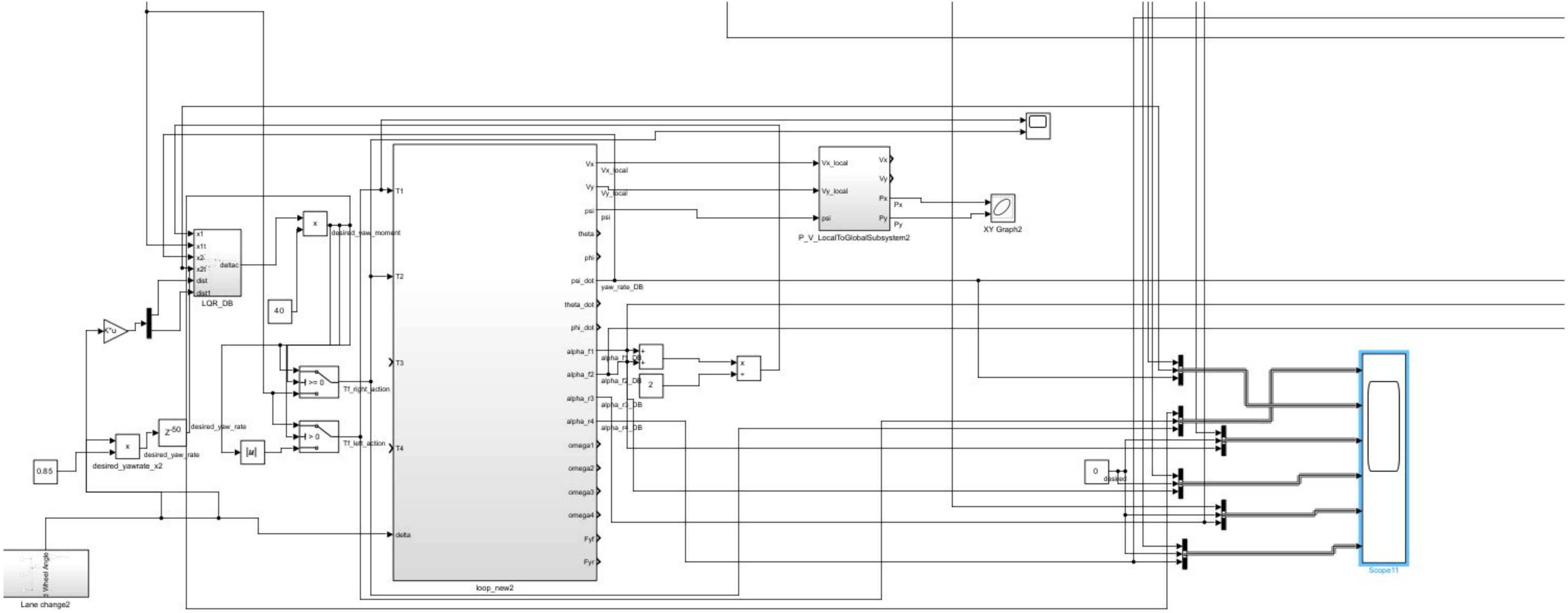
we assume that we can control the brake torque directly and during the driver is not braking

$Tr = \Delta Fxf^*Rw \quad \Delta M > 0$

$Tf = \Delta Fxf^*Rw \quad \Delta M < 0$

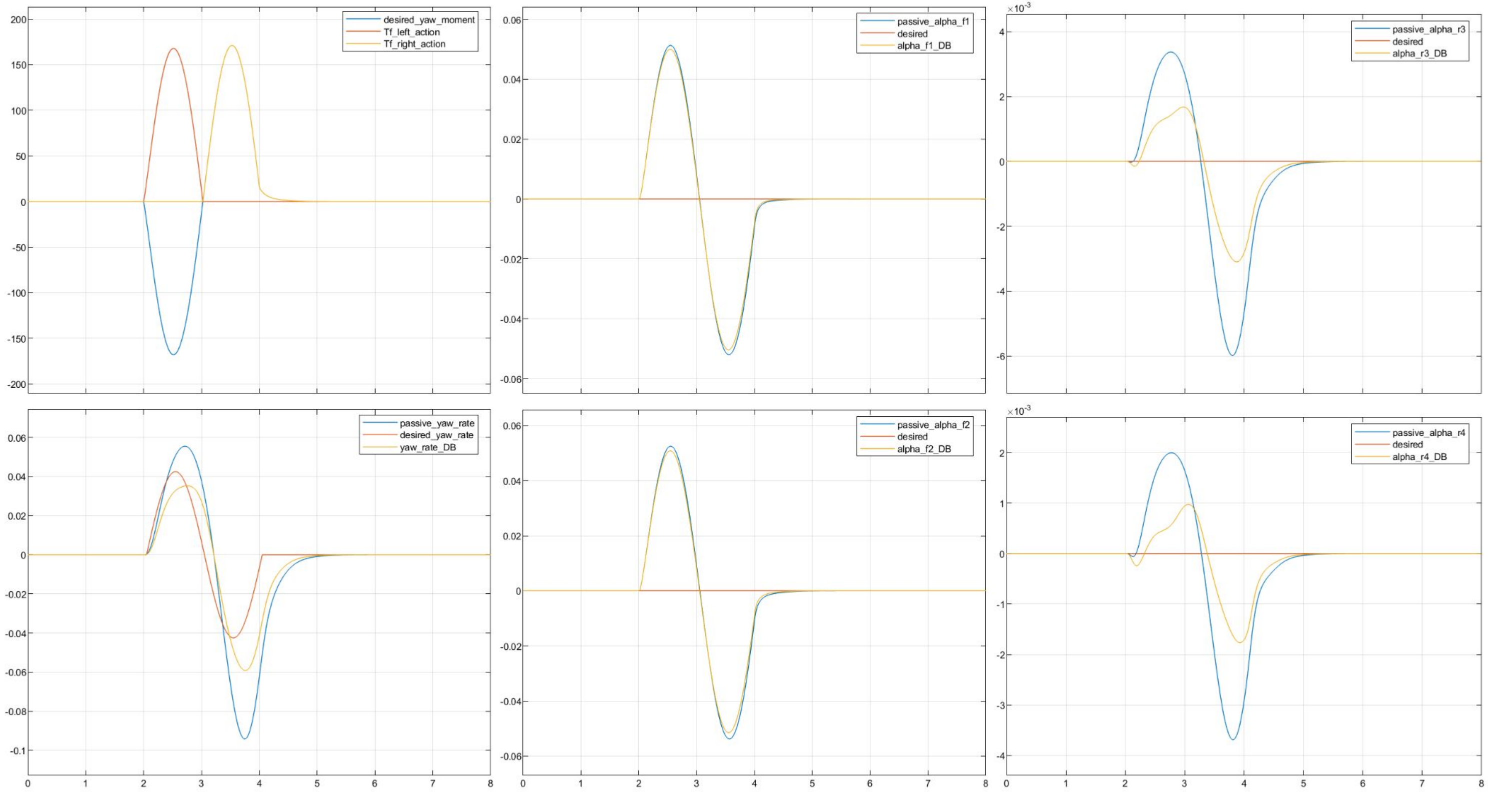
Active Chassis Control

Direct Yaw Moment Control



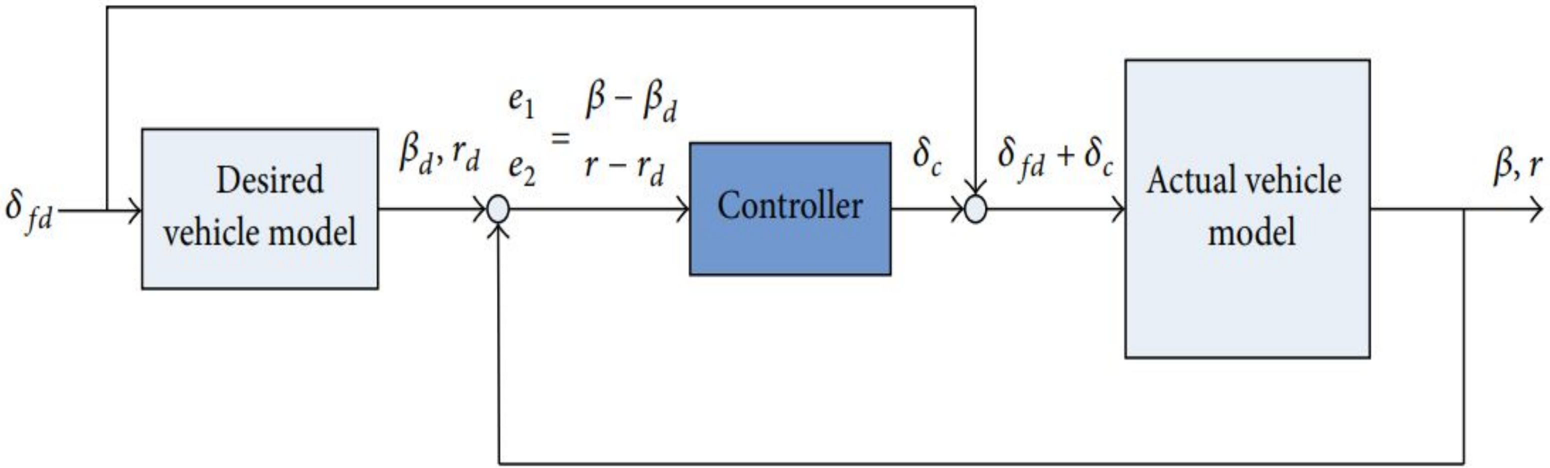
Active Chassis Control

Direct Yaw Moment Control



Active Steering Control (AFS)

Although direct yaw moment control could enhance the vehicle stability for critical driving conditions, it may be less effective for emergency braking on split road surface. At high vehicle speed steady state cornering, direct yaw moment control could decrease the yaw rate and increase a burden to the driver. To overcome this disadvantage, active steering control is proposed



Active Chassis Control

Active Steering Control (AFS)

$$\begin{bmatrix} \dot{\beta} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \dfrac{-C_f - C_r}{mv} & -1 + \dfrac{C_r l_r - C_f l_f}{mv^2} \\ \dfrac{C_r l_r - C_f l_f}{I_z} & \dfrac{-C_f l_f^2 - C_r l_r^2}{I_z v} \end{bmatrix} \begin{bmatrix} \beta \\ r \end{bmatrix} + \begin{bmatrix} \dfrac{C_f}{mv} \\ \dfrac{C_f l_f}{I_z} \end{bmatrix} \left(\delta_{fd} + \delta_c\right).$$

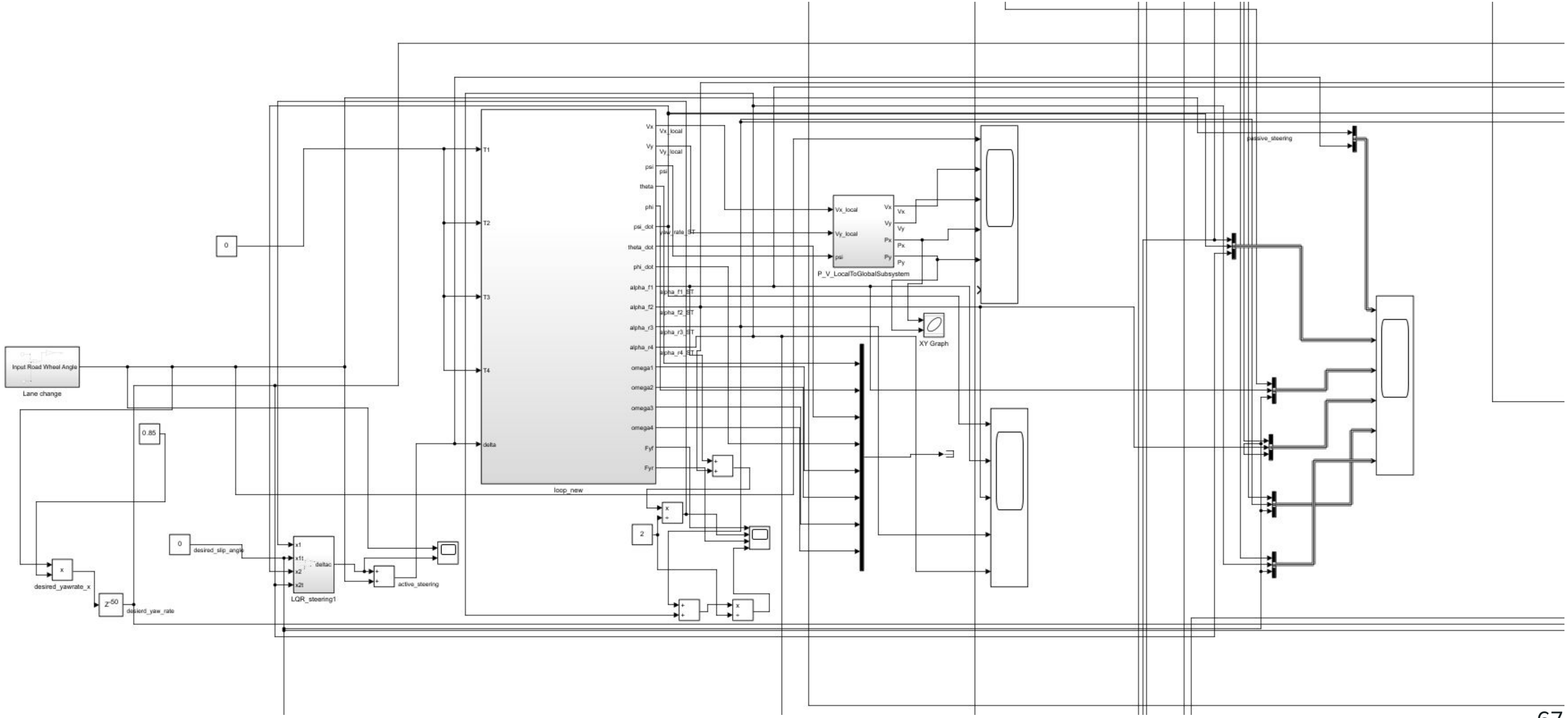
check the controllability rank(B AB)

full rank

LQR controller with upper bound constraints hold

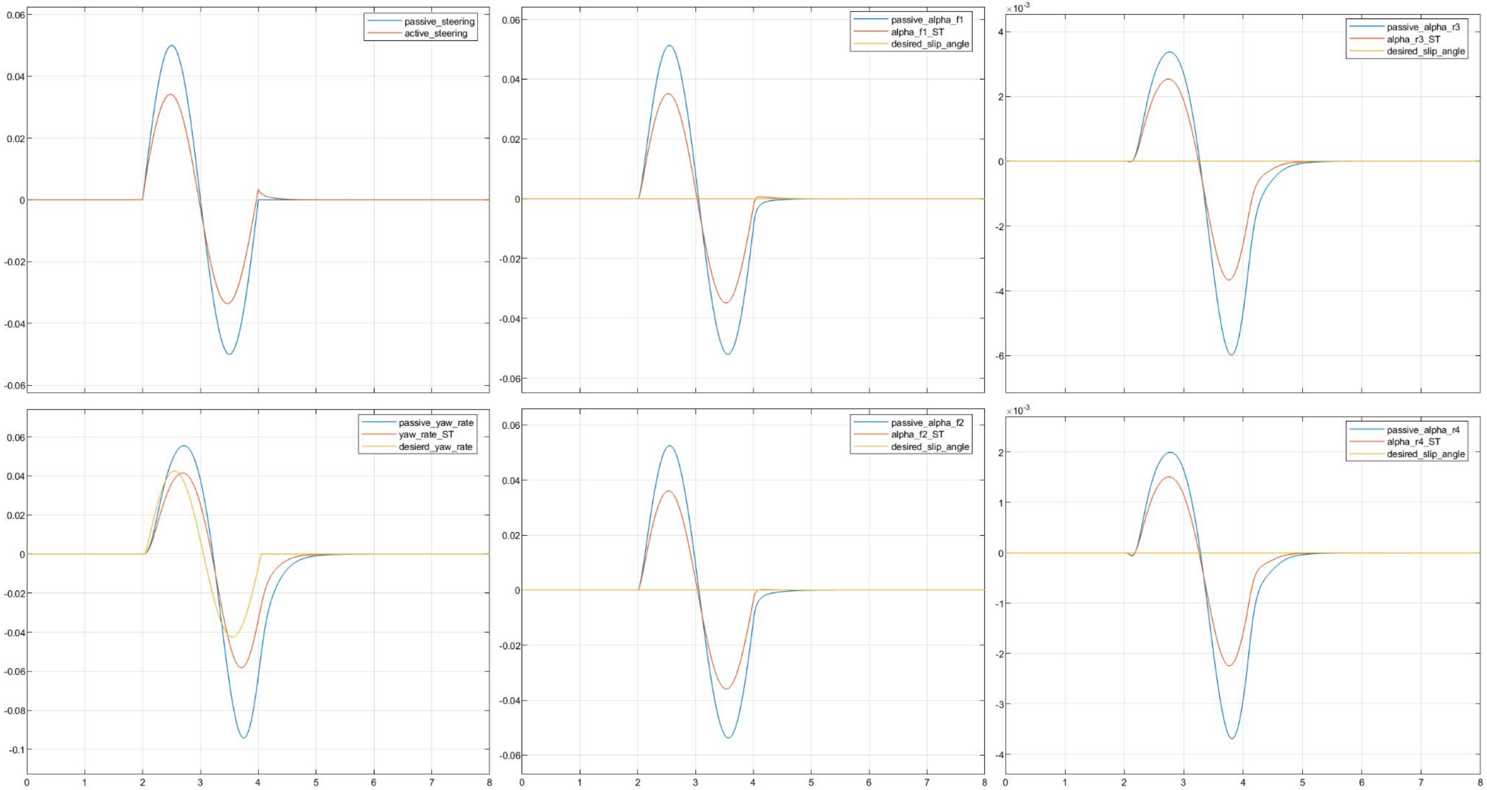
Active Chassis Control

Active Steering Control (AFS)



Active Chassis Control

Active Steering Control (AFS)

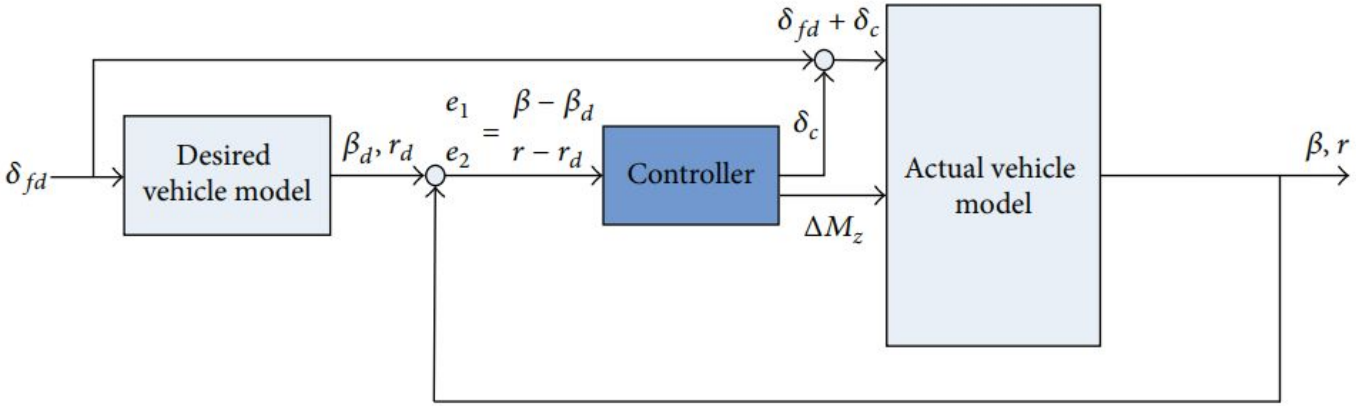


Active Chassis Control

Integrated Active Chassis Control

Disadvantages of Active Steering Control (AFS)

Less effective during critical driving condition



Active Chassis Control

Integrated Active Chassis Control

$$\begin{bmatrix}\dot{\beta}\\ \dot{r}\end{bmatrix} = \begin{bmatrix}\dfrac{-C_f - C_r}{mv} & -1 + \dfrac{C_r l_r - C_f l_f}{mv^2}\\ \dfrac{C_r l_r - C_f l_f}{I_z} & \dfrac{-C_f l_f^2 - C_r l_r^2}{I_z v}\end{bmatrix}\begin{bmatrix}\beta\\ r\end{bmatrix} + \begin{bmatrix}\dfrac{C_f}{mv} & 0\\ \dfrac{C_f l_f}{I_z} & \dfrac{1}{I_z}\end{bmatrix}\begin{bmatrix}\delta_c\\ \Delta M_z\end{bmatrix} + \begin{bmatrix}\dfrac{C_f}{mv}\\ \dfrac{C_f l_f}{I_z}\end{bmatrix}\delta_{fd}.$$

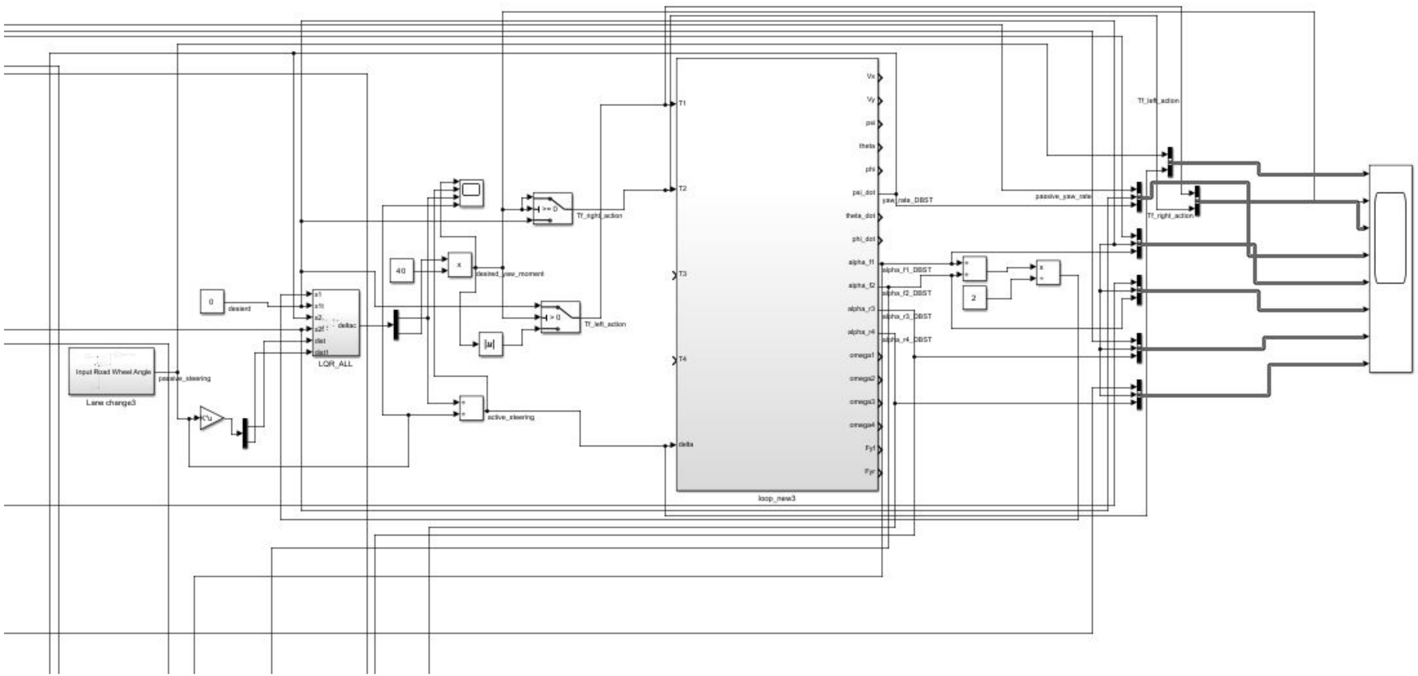
check the controllability rank(B AB)

full rank

LQR controller with upper bound constraints hold

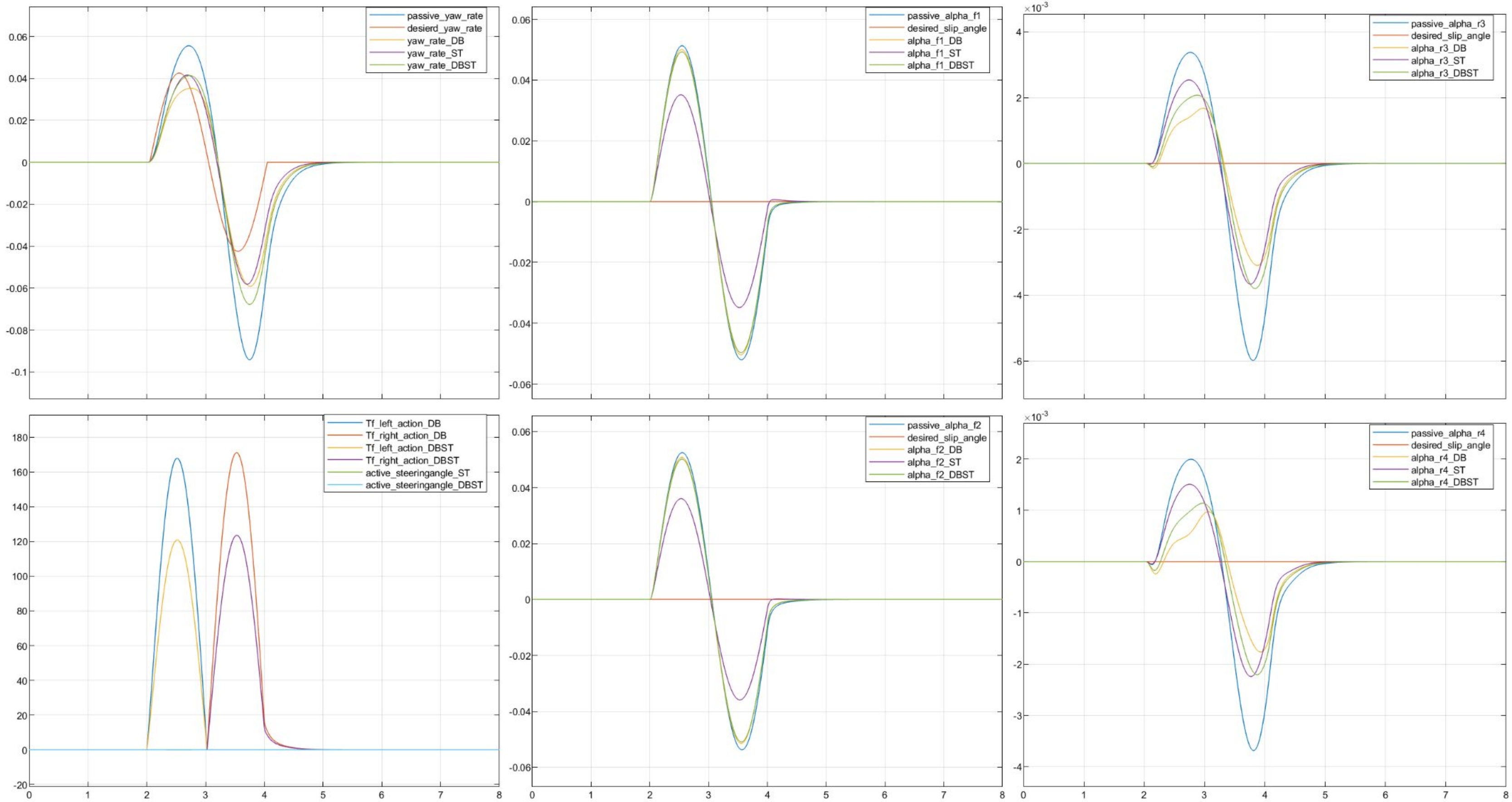
Active Chassis Control

Integrated Active Chassis Control



Comparison and Discussion

Comparison and Discussion



Comparison and Discussion

Vehicle actuator	Active chassis control		Advantages	Disadvantages
Brakes	Direct yaw moment control (DYC)	Active braking active differential	(i) Effective for critical driving condition (ii) Good for sideslip/wheelslip control	(i) Less effective for braking on split road surface (ii) Decrease yaw rate during steady state driving condition (iii) Active differential need extra devices
Steering	Active steering control (ASC)	Active front steering (AFS) control	(i) Effective for steady state driving condition (ii) Ease to integrate with braking control (iii) Good for yaw rate control	Less effective during critical driving condition
		Active rear steering (ARS) control	(i) Rear wheel steer angle can be controlled (ii) Good for yaw rate control	Less effective during critical driving condition
		4 wheels active steering (4WAS) control	(i) Two different steer inputs (ii) Goof for yaw rate control	Less effective during critical driving condition
Steering and brake	Integrated AFS-DYC control		(i) Two different inputs from two different actuator (steering and braking) (ii) Good for yaw rate and sideslip control	Effective for critical and steady state driving condition

also energy saving

9 DoF Planar 4 Wheeled Car Model With Active Chassis Control

Thank you!