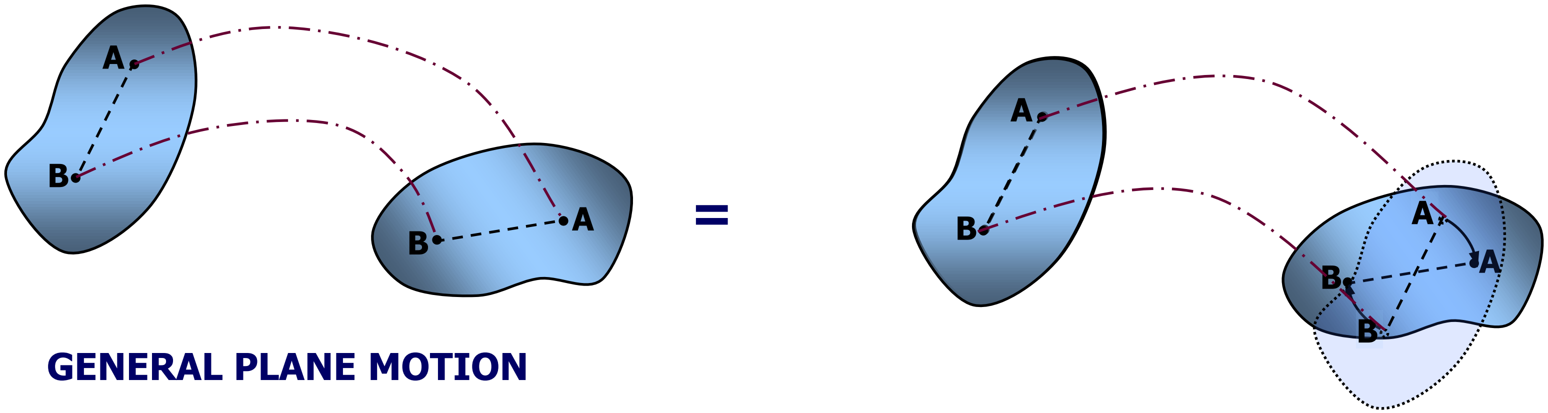
KINEMATICS OF RIGID BODIES

CHASLE’S PRINCIPLE



GENERAL PLANE MOTION

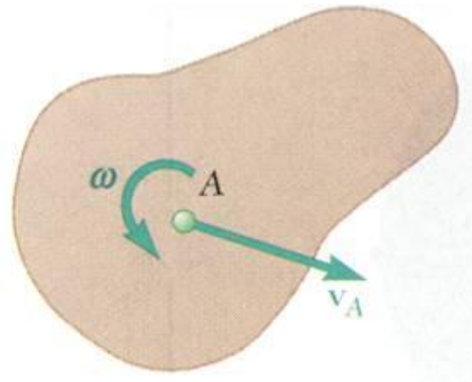
PURE TRANSLATION

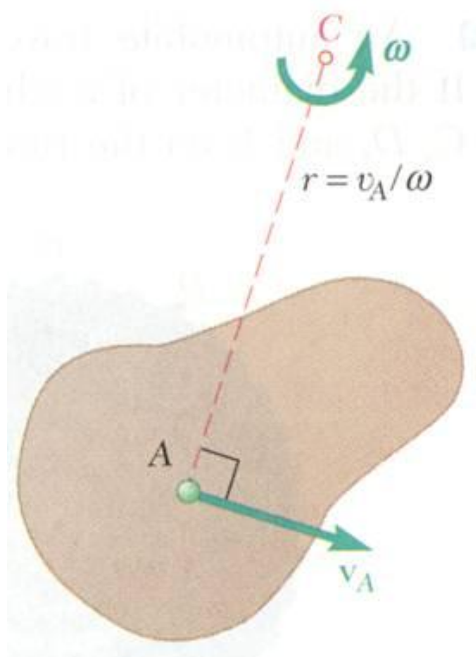
+

PURE ROTATION

ABOUT FIXED AXIS

Instantaneous Center of Rotation in Plane Motion





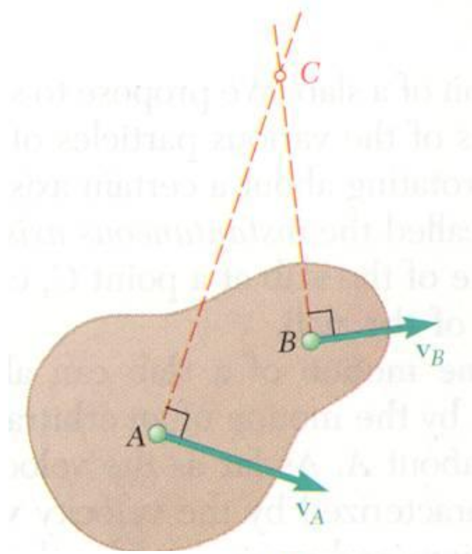
- Plane motion of all particles in a slab can always be replaced by the translation of an arbitrary point *A* and a rotation about *A* with an angular velocity that is independent of the choice of *A.*

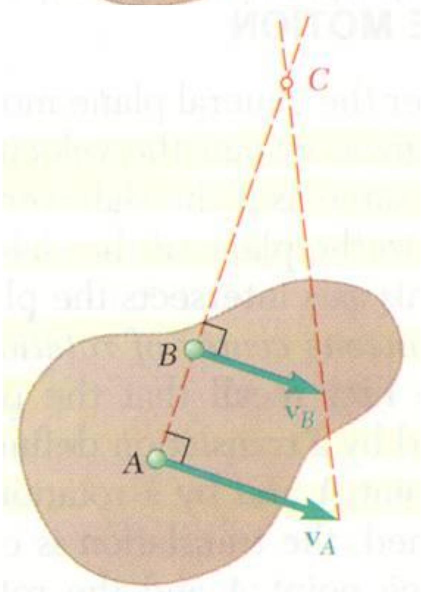
- The same translational and rotational velocities at *A* are obtained by allowing the slab to rotate with the same angular velocity about the point *C* on a perpendicular to the velocity at *A.*

- The velocity of all other particles in the slab are the same as originally defined since the angular velocity and translational velocity at *A* are equivalent.

- As far as the velocities are concerned, the slab seems to rotate about the *instantaneous center of rotation C*.

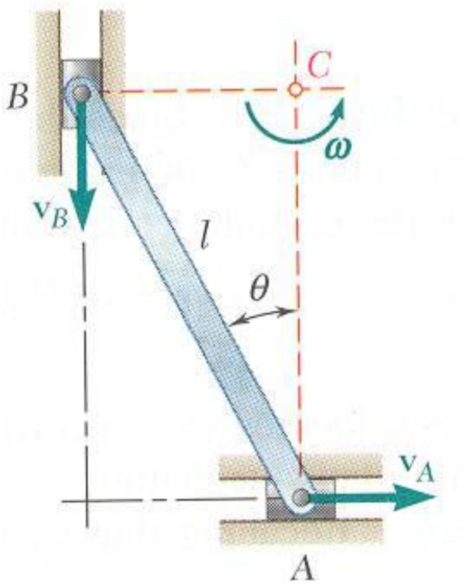
Instantaneous Center of Rotation in Plane Motion





- If the velocity at two points *A* and *B* are known, the instantaneous center of rotation lies at the intersection of the perpendiculars to the velocity vectors through *A* and *B* .
- If the velocity vectors are parallel, the instantaneous center of rotation is at infinity and the angular velocity is zero.
- If the velocity vectors at *A* and *B* are perpendicular to the line *AB*, the instantaneous center of rotation lies at the intersection of the line *AB* with the line joining the extremities of the velocity vectors at *A* and *B*.
- If the velocity magnitudes are equal, the instantaneous center of rotation is at infinity and the angular velocity is zero.

Instantaneous Center of Rotation in Plane Motion



- The instantaneous center of rotation lies at the intersection of the perpendiculars to the velocity vectors through *A* and *B* .

$$\omega = \frac{v_A}{AC} = \frac{v_A}{l\cos\theta}$$

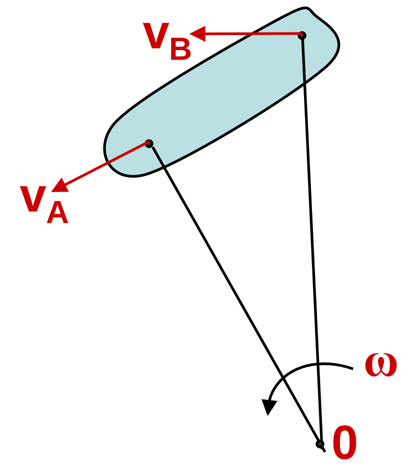
$$v_B = (BC)\omega = (l\sin\theta)\frac{v_A}{l\cos\theta} = v_A\tan\theta$$

- The velocities of all particles on the rod are as if they were rotated about *C.*
- The particle at the center of rotation has zero velocity.
- The particle coinciding with the center of rotation changes with time and the acceleration of the particle at the instantaneous center of rotation is not zero.
- The acceleration of the particles in the slab cannot be determined as if the slab were simply rotating about *C.*

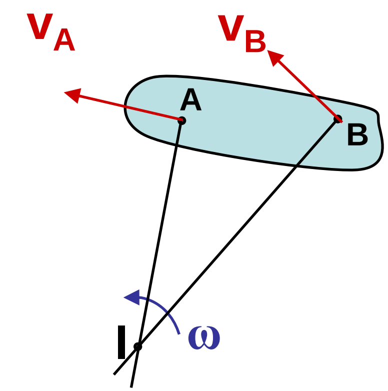
ISTANTANEOUS CENTER OF ZERO VELOCITY (ICZV)

OR

ISTANTANEOUS CENTER OF ROTATION (ICR)



Consider a body rotating about point 'O' with an angular velocity 'ω' as shown. Every point on the body undergo circular motion. Hence the linear velocity of any point on the body will be perpendicular to the radius of rotation as shown in the figure.

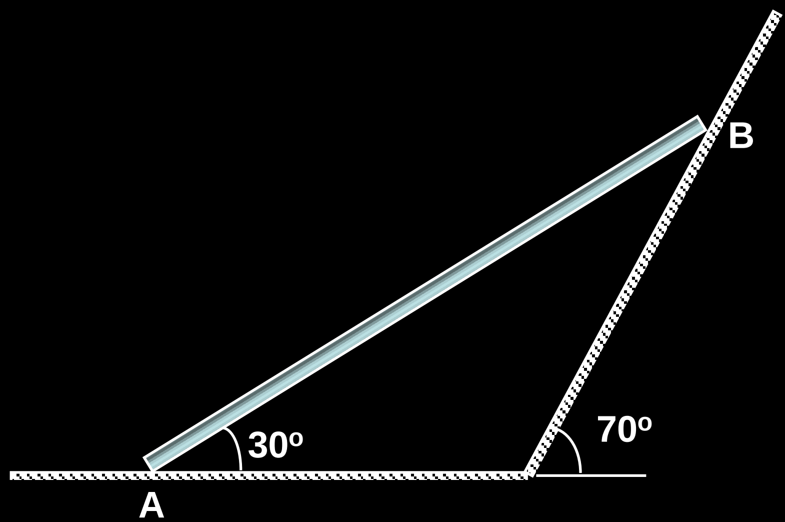


Conversely, if the direction of velocity of any two points on a rigid body are known, then center of rotation of the body may simply be located by drawing perpendiculars to these velocity vectors, as shown.

Once the center of rotation is located, then the velocity of any point on the rigid body can be easily obtained from the following relation -

$$\frac{V_A}{IA} = \frac{V_B}{IB} = \frac{V_C}{IC} = \omega$$

A rod of length 3m slides on two smooth surfaces. In the position shown, end A of the rod has a velocity of 5m/s directed to the left, determine the linear velocities of end B and its mid-point. Also find the angular velocity of the rod.



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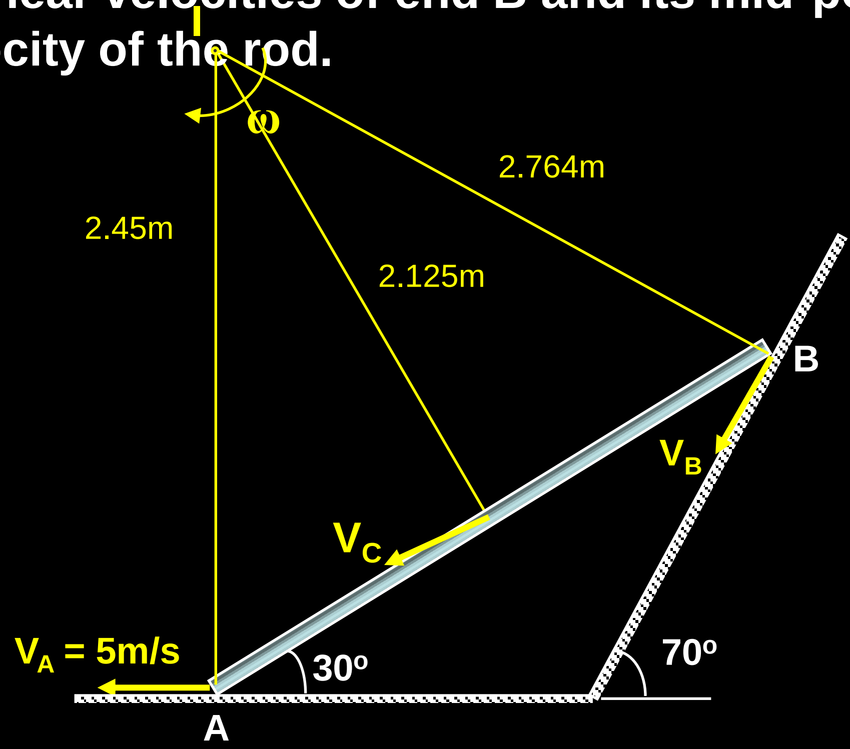


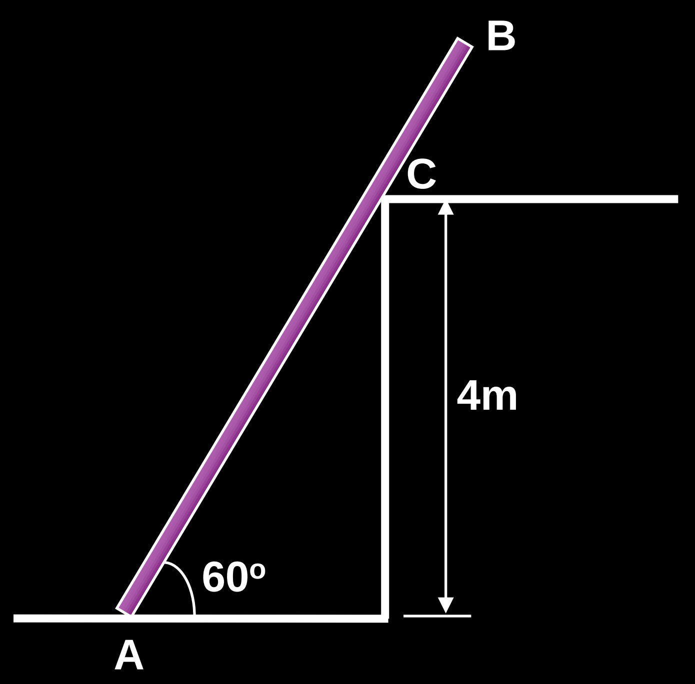
V_B = 5.65m/s

ω = 2.05r/s

V_C = 4.36m/s

$$\frac{V_A}{IA} = \frac{V_B}{IB} = \frac{V_C}{IC} = \omega$$

A rod, 5m long, slides along a surface and a sharp edge as shown. In the position, if the velocity of its end A is 3m/s, determine its angular velocity and the linear velocities of end B & its centre.



A rod, 5m long, slides along a surface and a sharp edge as shown. In the position, if the velocity of its end A is 3m/s, determine its angular velocity and the linear velocities of end B & its centre.

AC = 4/sin 60 = 4.62m
BC = 5-4.62 = 0.38m

Applying Sine Rule for Triangle AIC

IA = 5.334m
IC = 2.667m

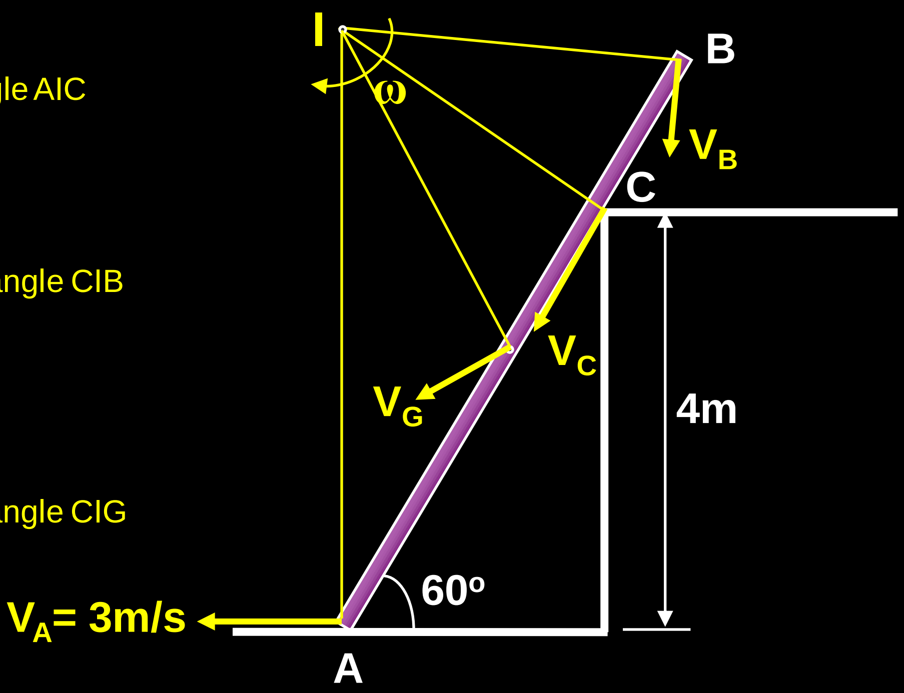
Applying Cosine Rule for Triangle CIB

IB = 2.694m

CG = 2.5-0.38 = 2.12m

Applying Cosine Rule for Triangle CIG

IG = 3.405m



$$\frac{V_A}{IA} = \frac{V_C}{IC} = \omega$$

$$V_C = 1.5\text{m/s}$$

$$\omega = 0.562\text{r/s}$$

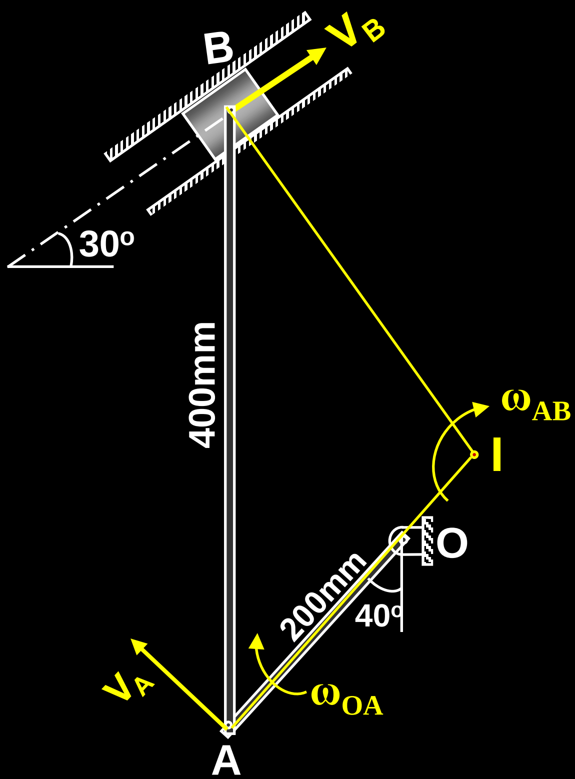
$$\frac{V_A}{IA} = \frac{V_B}{IB} = \frac{V_G}{IG}$$

$$V_B = 1.52\text{m/s}$$

$$V_G = 1.91\text{m/s}$$

Locate the instantaneous center of rotation for the link AB and determine the angular velocity of the link OA. Velocity of the slider B is 2500mm/sec.

IB = 273.6 mm
IA = 212.8 mm



$$\frac{V_B}{IB} = \frac{V_A}{IA} = \omega_{AB}$$

V_A = 1945mm/s

ω_{AB} = 9.13r/s

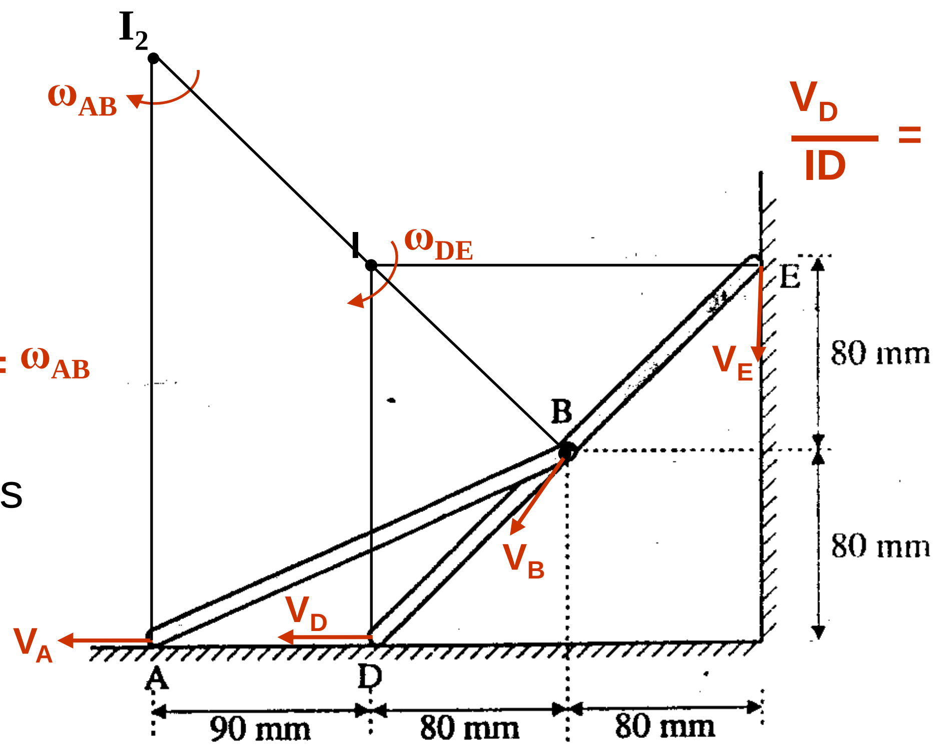
$$\omega_{OA} = \frac{V_A}{OA}$$

ω_{OA} = 9.72r/s

Two rods AB and DE are connected as shown in figure. The end D moves to the left at 500mm/sec. Determine:
(a) angular velocity of each rod
(b) velocity of point A.

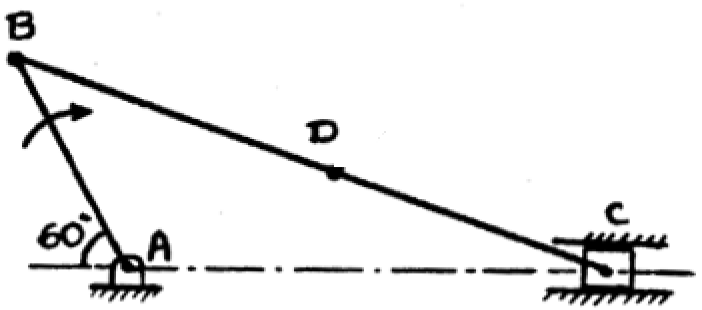
$$\frac{V_D}{ID} = \frac{V_E}{IE} = \frac{V_B}{IB} = \omega_{DE}$$

V_B = 353.5mm/s

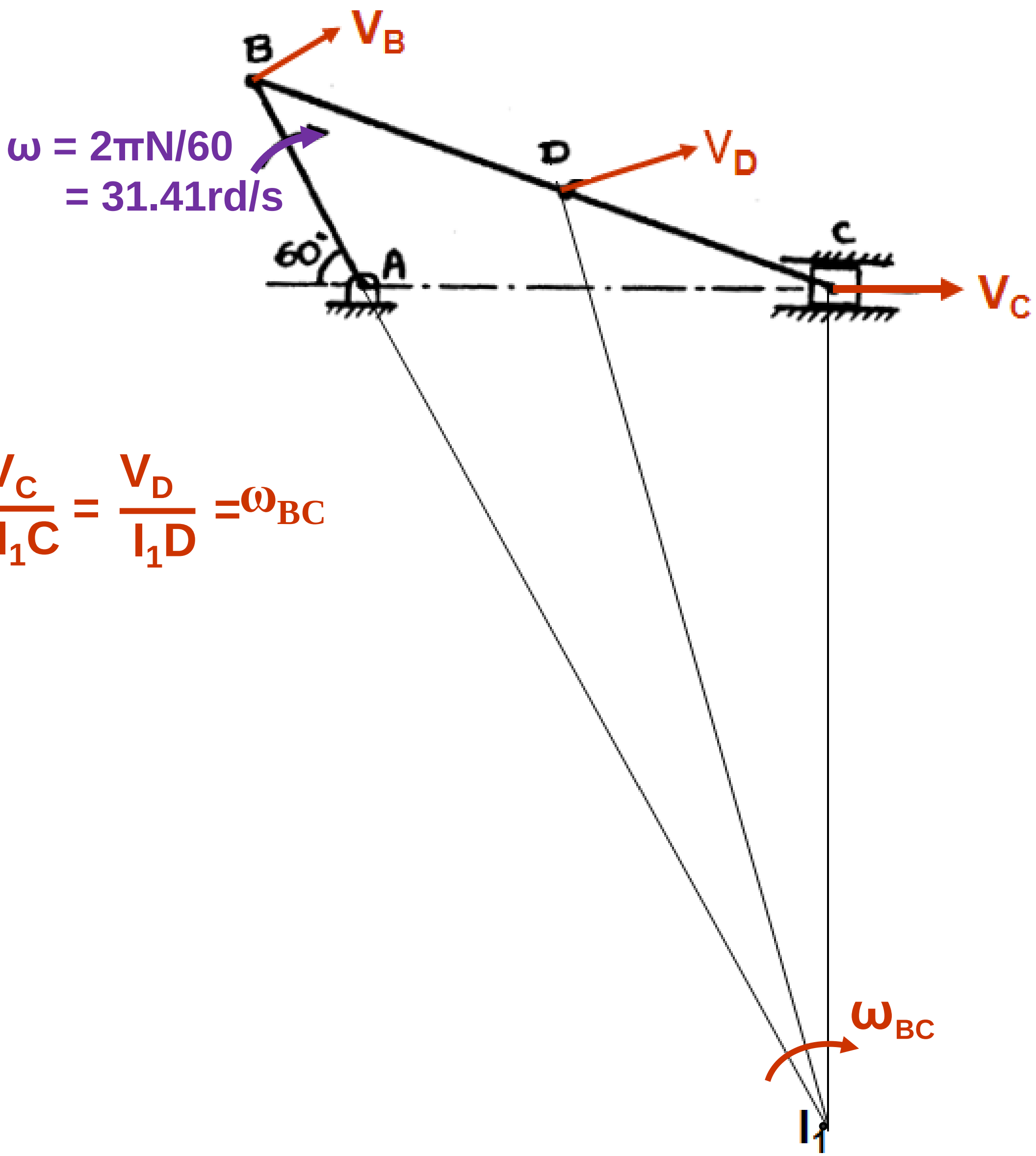


$$\frac{V_B}{I_2B} = \frac{V_A}{I_2A} = \omega_{AB}$$

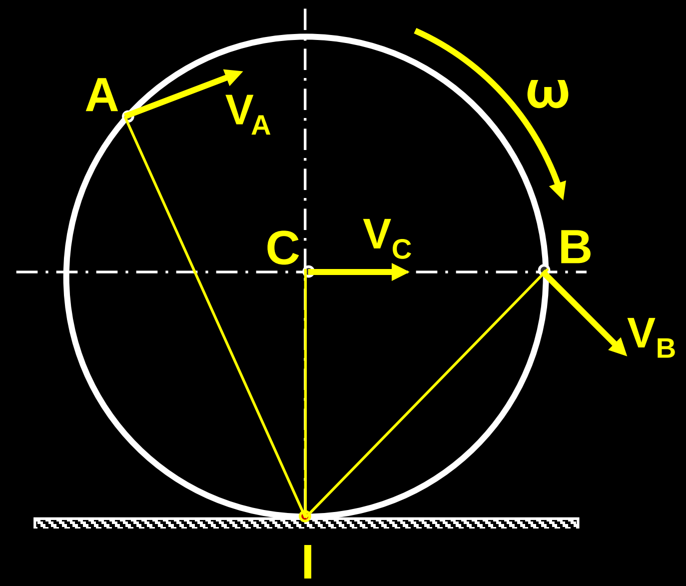
V_A = 367.6mm/s

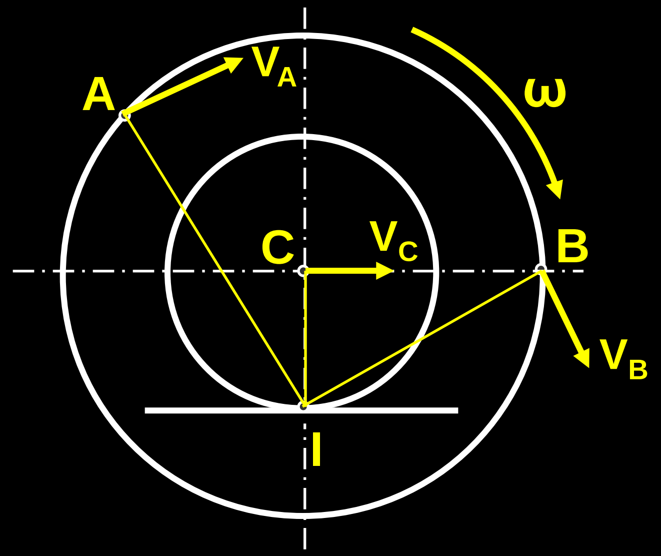


In a slider mechanism shown, the crank AB rotates at a speed of 300rpm clockwise. Determine the linear velocity of the slider C and midpoint of link BC and the angular velocity of link BC.
Given :- AB = 2m, BC = 5m.



$$\frac{V_B}{I_1B} = \frac{V_C}{I_1C} = \frac{V_D}{I_1D} = \omega_{BC}$$





When any circular body purely rolls (without slipping) on a surface, its point of contact with the surface is the instantaneous center.

Then, the linear velocity of any point on the body can be easily obtained as shown in the figure.

$$\frac{V_A}{IA} = \frac{V_B}{IB} = \frac{V_C}{IG} = \omega$$

Two rods AB and CD are attached to a cylinder by pins as shown. The ends A and D of the rods slide along the two surfaces as the cylinder rolls, without slipping, over the horizontal surface. At the given position, if end 'A' of AB has a velocity V_A = 30cm/s, determine the velocity of end 'D' of CD. Given :- Length of AB = 50cm, Length of CD = 35cm , Dia. of the cylinder = 30cm.

$$\frac{V_A}{I_2A} = \frac{V_B}{I_2B} = \omega_{AB}$$

V_B = 42.4cm/s

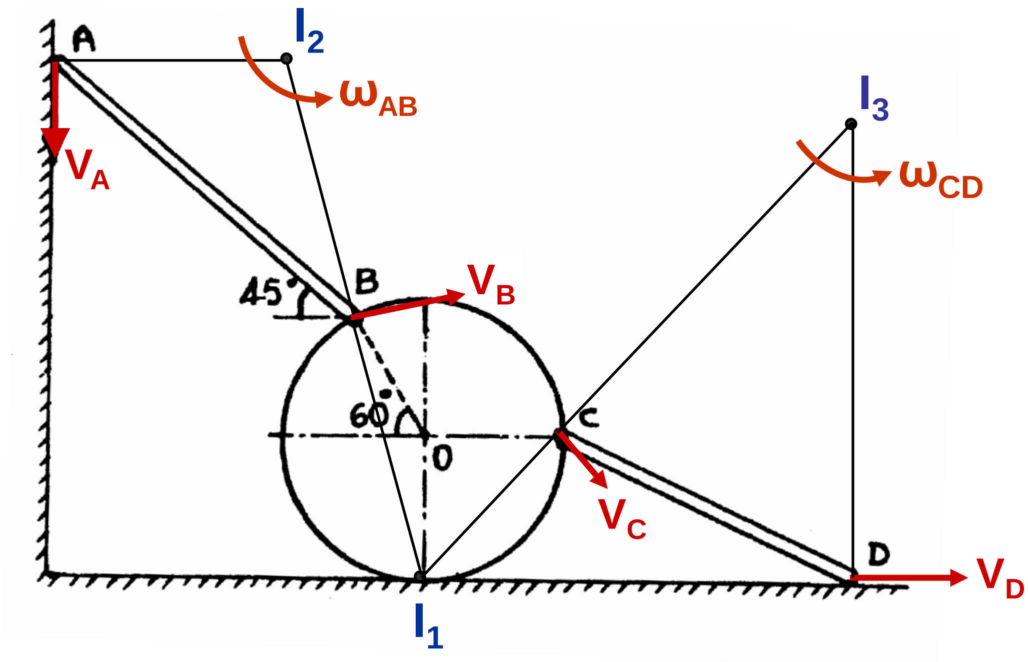
$$\frac{V_B}{I_1B} = \frac{V_C}{I_1C} = \omega_{DISK}$$

ω_{disk} = 1.46r/s

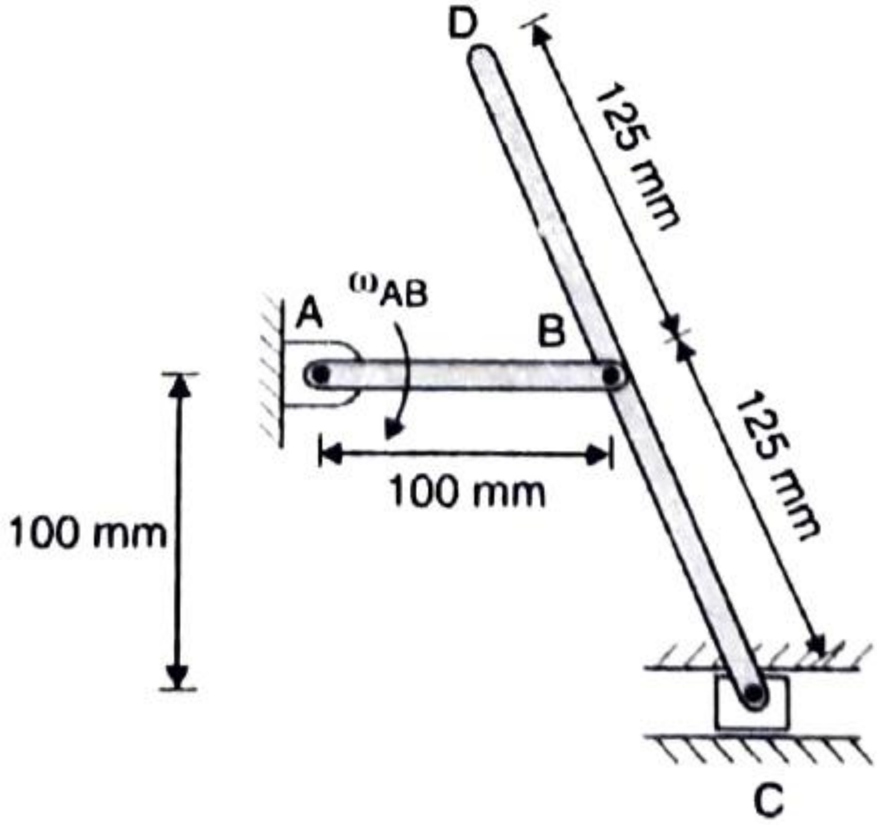
V_C = 30.97cm/s

$$\frac{V_C}{I_3C} = \frac{V_D}{I_3D} = \omega_{CD}$$

V_D = 32.3cm/s



Angular velocity of AB is 3 r/s. Determine linear velocity of slider C & D



Radius of the wheel is 0.24m,
Determine the velocities at C & D for the instance