unit 1.3 – Diffraction of Light (13 marks)

The phenomenon of bending of light round the corner of an opaque obstacle & spreading of light waves into the region of geometrical shadow of the obstacle is called as Diffraction of light.

The bending of light depends upon the size of the obstacle. It is much pronounced when the dimensions of the obstacle are small w.r.t the wavelength of the light.
As very small obstacles are needed to create the diffraction, it is not evident easily in daily life.

* Types of Diffraction:—

1) Fresnel Diffraction:- In this, the light source and the screen are at finite distance from the diffracting aperture/slit as shown in figure below. The incident wavefront can be either spherical or cylindrical.

2) Fraunhoffer Diffraction :- In this, the light source and the screen are at infinite distance from the diffracting aperture or slit as shown in figure. To see diffraction pattern we need lenses. The incident wavefront is plane. It is very important in optical instruments. The initial phase of secondary wavelets is same at all points in the plane of aperture. Where as in fresnel's diffraction, initial phase of secondary wavelets is different at different points in the plane of aperture.

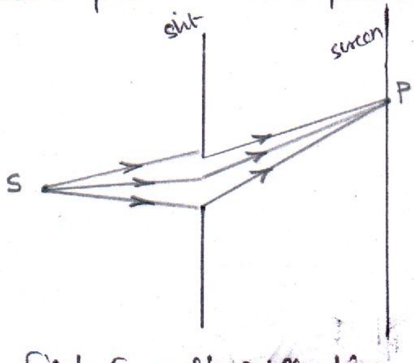


fig 1; fresnel's Diffraction

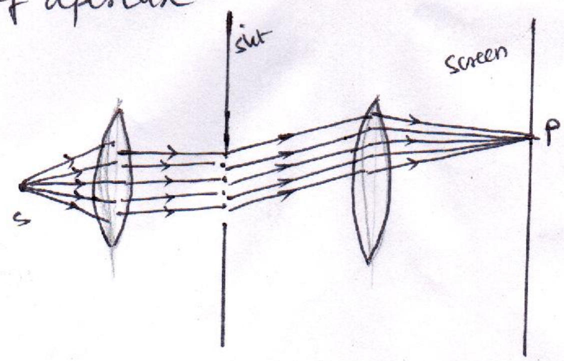


fig 2; fraunhoffer Diffraction

* Difference between Interference & Diffraction :-

Interference of Light	Diffraction of Light
1) It is the result of interaction of light coming from two different wavefronts originating from the same source.	1) It is the result of interaction of light coming from two different parts of the same source.
2) Fringes may or may not be of same width	2) Fringes in a particular pattern are not of same width.
3) Fringes are of same Intensity.	3) Fringes are of decreasing intensity.
4) Contrast between the fringes is good as the points of minimum intensity are perfectly dark and bright fringes are of same intensity.	4) Contrast between the fringes is not good as bright fringes are of decreasing intensity.

* Single Slit Fraunhoffer Diffraction :-

Consider a single slit of width 'a' illuminated by mono-chromatic light of wavelength 'λ' as shown in figure. The incident plane wavefront is diffracted by the slit and is then focussed on screen by lens L.

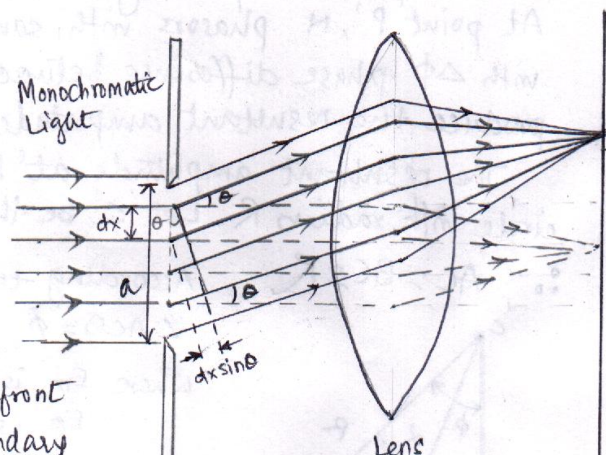


Fig. 1

Every point of the incident wavefront in the plane of slit acts as a secondary source and sends out secondary waves in all directions. The secondary wavelets travelling normally to the slit are brought to focus at point 'P_0' by the lens.

All these secondary waves travel the same distance at $\theta=0$ hence produce maximum intensity of light at P_0.

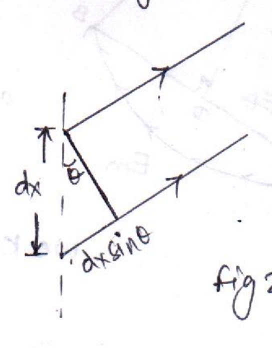


fig 2

Consider at point 'P' secondary wavelets travelling at an angle θ with the normal. The intensity at point 'P' depends upon the path difference between secondary wavelets originating from the corresponding points of wavefront.

Consider the given slit is divided into N parallel slits, each of width 'dx'. $\therefore a = dx_1 + dx_2 + \cdots + dx_N$

$\therefore$ Path difference between the rays diffracted through each slit is 'dx sinθ', $\therefore \Delta\phi = \frac{2\pi}{\lambda} dx \sin\theta$ —① (Phase difference)

Now,
Total path difference between rays diffracted from the top and bottom edges of the slit of width 'a' is 'a sinθ' and the corresponding phase difference between them is

$$\phi = \frac{2\pi}{\lambda} . a\sin\theta \quad \text{—②}$$

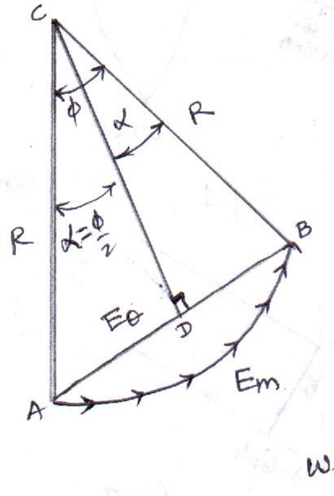
To find out total amplitude of waves at 'P', we have to consider 'N' phasors corresponding to N parallel infinitesimal slits.
At point 'P', N phasors with same amplitude and same frequency with $\Delta\phi$ phase difference between adjacent waves combine to produce the resultant amplitude.

The resultant amplitude at 'P' is represented by arc AB of a circle with radius R. Let 'C' be the centre of arc AB.

$\therefore$ AC = BC = R, According to diagram,

$\angle ACB = \phi$, let arc AB $= E_m$ & chord AB $= E_\theta$

Where E_m is Amplitude at the centre
E_θ is resultant amplitude at point P.



$AD = DB$, $\angle ACD = \angle BCD$

$$\angle ACD = \frac{\phi}{2} = \frac{\pi}{\lambda} a\sin\theta = \alpha \quad \text{—③}$$

also, $AD = \frac{E_\theta}{2} = R . \sin\frac{\phi}{2}$

$$\therefore E_\theta = 2R . \sin\frac{\phi}{2} \quad \text{—④}$$

we know, angle $= \frac{\text{arc}}{\text{radius}}$

$\therefore \phi = E_m/R \quad \therefore R = \frac{E_m}{\phi}$

$\therefore$ eqⁿ (4) can be written as

$$E_\theta = 2\frac{E_m}{\phi}\sin\frac{\phi}{2}$$

$$\boxed{E_\theta = E_m \cdot \frac{\sin\alpha}{\alpha}} \quad \text{—— (5)}$$

This gives resultant amplitude at 'P'.

Intensity can be written as

$$\boxed{I_\theta = I_m \cdot \left(\frac{\sin\alpha}{\alpha}\right)^2} \quad \text{—— (6)} \quad \text{where} \quad \begin{pmatrix} I_\theta = E_\theta^2 \\ I_m = E_m^2 \end{pmatrix}$$

The resultant intensity will be maximum i.e. $I_\theta = E_o^2$ and this implies the central maximum.

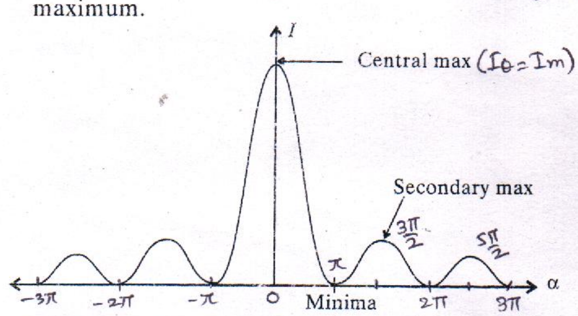


Fig. 2.6(a) : Intensity Plot in Single Slit Diffraction

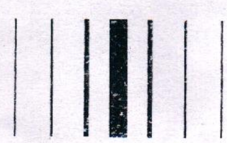


Fig. 2.6(b) : Intensity Pattern

Conditions for Maxima and Minima points :-

1) Principal Maxima : for E_θ to be maximum, All the waves must be in phase i.e; $\theta = 0$ and $\alpha = 0$ $\left(\alpha = \frac{\pi}{\lambda} a\sin\theta\right)$

$$\therefore \alpha = \frac{\pi}{\lambda} a\sin\theta = 0$$

$$\Rightarrow \sin\theta = 0 \quad \text{or} \quad \theta = 0$$

The waves which travels in the direction $\theta = 0$ will reach at point P producing maximum amplitude. $\frac{\sin\alpha}{\alpha}$ term will be equal to 1 as $\alpha \to 0$

$$\therefore \quad I_\theta = I_m$$

2) Minimum Intensity :- Intensity at point P will be zero if $\sin\alpha = 0$

$$\therefore \quad \alpha = \pm m\pi \quad \text{where } m = 1, 2, 3, ---$$

($m \neq 0$ because if $m = 0$, α becomes zero which is the condition of principal maxima.)

$$\therefore \quad \alpha = \frac{\pi}{\lambda} a\sin\theta = m\pi$$

or $\boxed{a\sin\theta = m\lambda}$ where $m = 1, 2, 3, ---$

3) Secondary Maxima (Maxima of decreasing Intensity)

These secondary maxima lie approximately half way between the two minima.

Secondary maxima can be obtained for

$$\alpha = \pm\left(m+\tfrac{1}{2}\right)\pi \qquad , m = 1,2,3,\ldots$$

using this condition in eqn (6), ratio of Intensities can be written as

$$I_\theta / I_m = \left[\frac{\sin\left(m+1/2\right)\pi}{\left(m+1/2\right)\pi}\right]^2$$

Putting $m = 1,2,3,\ldots$

we have $I_\theta / I_m = 0.045, 0.016, 0.008, \ldots$

Thus the successive maxima decreases in intensity rapidly.

*Problems on single slit diffraction:—

(1) Calculate the angular position in the 1^{st} minima in fraunhoffer pattern of a slit 10^{-6}m wide if it is illuminated by light of wavelength 4000 A°.

⇒ Here $m = 1$, $a = 10^{-6}$m, $\lambda = 4000\times10^{-10}$m (a & λ are in metre)

formula; condition for minima is $a\sin\theta = m\lambda$

$\therefore \sin\theta = m\lambda / a$

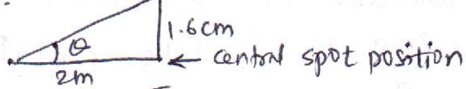
$\therefore \theta = \sin^{-1}\left(\frac{m\lambda}{a}\right) = \sin^{-1}\left(\frac{1\times4000\times10^{-10}}{10^{-6}}\right)$

$\theta = 23.57^\circ$

(2) A single slit of width 0.14 mm is illuminated normally with monochromatic light and diffraction bands are observed on a screen 2m away. If the centre of the second dark band is 1.6 cm from the middle of the central bright band, deduce the wavelength of light.

⇒ formula, $a\sin\theta = m\lambda$ —(1)

Here θ is unknown therefore we will find it first.



∴ from geometry

$\sin\theta \approx \theta \approx \tan\theta = \frac{1.6\text{ cm}}{2\text{ m}} = \frac{1.6\times10^{-2}\text{ m}}{2\text{ m}}$

∴ putting given values in (1)

$a\sin\theta = m\lambda$

$\lambda = \frac{a\sin\theta}{m} = \frac{0.14\times10^{-3}\times1.6\times10^{-2}}{2\times2}$

$\lambda = \frac{0.14\times1.6\times10^{-5}}{4}$

$\lambda = 0.056\times10^{-5}$

$\lambda = 5.6\times10^{-7}$

or $\lambda = 5600\times10^{-10}$ m

or $\boxed{\lambda = 5600\text{ A}^\circ}$

V. IMP

* 'N' Parallel equidistant slit fraunhoffer diffraction / theory of Diffraction Grating :-

Grating or Diffraction grating is an arrangement consist of a large number of parallel slits of same width and separated by equal opaque spaces.

Let N be the number of parallel slits each of width 'a' and separated by opaque space 'b'. Then the distance between the centres of the adjacent slits is $d = (a+b)$ which is known as 'Grating element'.

Let a plane wavefront of monochromatic light of wavelength 'λ' be incident normally on the grating. Every point in each slit acts as a source of secondary wavelets which spread out in all direction. Now consider few of them which moves in the direction of θ with the incident light are focussed at point P on the screen as shown in figure below.

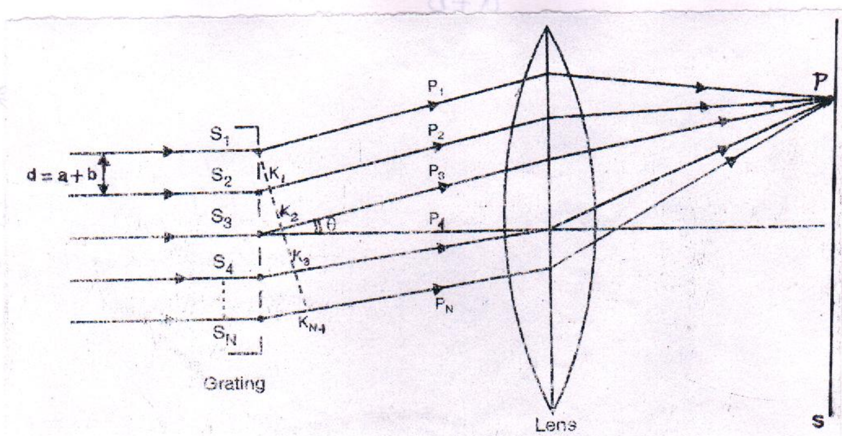


Fig. 3.7.1 : Plane Diffraction Grating

from above figure, the path difference between S_1P_1 & S_2P_2 is given by,

$$S_2K_1 = (a+b)\sin\theta$$

its corresponding phase difference is,

$$\Delta\phi = \frac{2\pi}{\lambda}.(a+b)\sin\theta \quad \text{—①}$$

Similarly the path difference between S_3P_3 and S_1P_1 is given by

$S_3K_2 = 2(a+b)\sin\theta$, its phase difference is,

$$2\Delta\phi = \frac{2\pi}{\lambda}.2(a+b)\sin\theta$$

Thus phase difference between successive waves is

$$\Delta\phi = \frac{2\pi}{\lambda}(a+b)\sin\theta = 2\beta \text{ (say)} \quad \text{--- (2)}$$

The resultant amplitude of these 'N' waves can be found out by vector addition method and is given by

$$\boxed{E_\theta = E_m \frac{\sin\alpha}{\alpha} \cdot \frac{\sin N\beta}{\sin\beta}} \quad \text{--- (3)}$$

The corresponding resultant intensity at point 'P' is given by

$$I_\theta = I_m \left(\frac{\sin\alpha}{\alpha}\right)^2 \left(\frac{\sin^2 N\beta}{\sin^2\beta}\right) \quad \text{--- (4)}$$

where $\alpha = \frac{\pi}{\lambda} a \sin\theta$

$$\beta = \frac{\pi}{\lambda}(a+b)\sin\theta$$

Maxima and Minima Conditions:-

for various principal maxima (like zero order, 1st order, 2nd order etc) when $\sin\beta = 0$, that is $\beta = \pm m\pi$ $(m = 0, 1, 2, 3, \ldots)$ we have $\sin N\beta = 0$ and hence $\frac{\sin N\beta}{\sin\beta}$ becomes indeterminate. We can find its value for $\beta \to m\pi$ using L'Hospital's rule.

$$\lim_{\beta \to m\pi} \frac{\sin N\beta}{\sin\beta} = \pm N$$

$$\therefore \lim_{\beta \to \pm m\pi} \left(\frac{\sin N\beta}{\sin\beta}\right)^2 = N^2$$ substituting this in eq^n (4), we get

$$\boxed{I_\theta = I_m \left(\frac{\sin\alpha}{\alpha}\right)^2 N^2}$$ which is maximum.

The direction of the principal maxima are given by

$$\sin\beta = 0, \text{ that is } \beta = \pm m\pi$$

or $\boxed{d \sin\theta = \pm m\lambda}$ --- (5) where $a + b = d$ = grating element

eq^n (5) is known as the grating equation.

If we put $m = 0$, we get $\theta = 0$. This is the direction in which waves from all slits arrive in phase and produce a bright central image. This maxima is called the zero order principal maxima. If we put $m = 1, 2, 3, \ldots$ we obtain the directions of the first, second third order principal maxima respectively. Therefore, the direction

(continue)

of occurrence of principal maxima is given by,

$$\sin\theta_m = \frac{m\lambda}{d} = \frac{m\lambda}{a+b}$$

or

$$\boxed{\sin\theta_m = mN\lambda}$$

where $N = \frac{1}{a+b}$ is the number of ruled lines per unit width of the grating.

Minima :- The intensity is zero when $\sin N\beta = 0$, or $N\beta = \pm n\pi$

or

$$N\beta = \frac{N\pi d \sin\theta}{\lambda} = \pm n\pi$$

or

$$\boxed{N \cdot d \sin\theta = \pm n\lambda} \quad \text{—— (6)}$$

Here, 'n' can take all integral values except 0, N, 2N, 3N, etc because these values give the positions of principal maxima. It is seen from eq (6) that $n=0$ gives principal maximum of zero order while $n = 1, 2, 3, \ldots (N-1)$ give the minima. Then $n=N$ gives principal maxima of first order. Thus between zero order, and first order principal maxima we have $(N-1)$ minima. Similarly, it can be shown that there are $(N-1)$ minima between first order and second order principal maxima and so on. Between two such consecutive minima, the intensity has to maximum, and these maxima are known as secondary maxima. The secondary maxima are not visible in the grating spectrum, as the number of slits is very large.

Missing Orders / Condition for Absent Spectra :-

Principal maxima in case of a grating are obtained in the given direction $(a+b)\sin\theta = m\lambda$ —(1) where $d = a+b$

Also the minimum in case of a single slit are obtained in the directions is given by $a\sin\theta = n\lambda$ —(2)

$\therefore$ from (1) & (2) $\frac{a+b}{a} = \frac{m}{n}$ $\Rightarrow$ condition for Absent spectra.

If $a=b$, $m = 2n$ where $n = 1, 2, 3, \ldots$

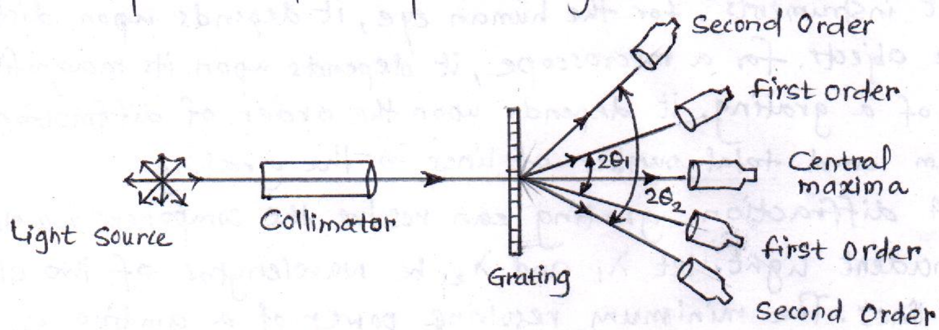
i.e; $2^{nd}, 4^{th}, 6^{th}, \ldots$ etc order spectra will be absent.

i.e; The even order spectra will be absent if the width of ruling is equal to width of the slit.

V.IMP

* Determination of Wavelength of Light using Grating :—

The diffraction grating is often used in the laboratories for the determination of wavelength of light. A monochromatic source of light, spectrometer and diffraction grating are required for the experimental set up. The diagram below shows the arrangement.



The spectrometer is first adjusted for parallel rays. The grating is then placed on the prism table and adjusted for normal incidence. In the same direction as that of the incident light, the direct image of the slit or the zero order spectrum can be seen in the telescope. On either side of this direct image a symmetrical diffraction pattern consisting of different orders can be seen. The angle of diffraction θ for a particular order 'm' of the spectrum is measured. The number of lines per inch of grating are written over it by the manufacturers. The grating element is

$$(a+b) = \frac{1}{\text{No. of Lines/cm}} = \frac{2.54}{\text{No. of lines/inch}}$$

Thus using the equation

$$(a+b)\sin\theta = m\lambda$$

The unknown wavelength λ can be calculated by putting the values of the grating element $(a+b)$, the order 'm' and the angle of diffraction θ.

* Resolving Power of Diffraction Grating : —

Resolving power is an important parameter of any optical instrument. It expresses the ability of an optical instrument to distinguish between two closely spaced objects. It depends on different parameters of different instruments. for the human eye, it depends upon distance from the objects. for a microscope, it depends upon its magnification. In case of a grating, it depends upon the order of diffraction maximum and total number of lines in the grating.

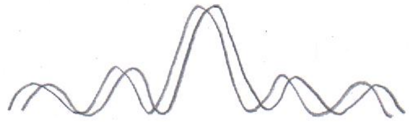
A diffraction grating can resolve the component wavelength from incident light. Let λ_1 and λ_2 be wavelengths of two close spectral lines. The minimum resolving power of a grating is estimated by considering a factor $\lambda/\Delta\lambda$ where, λ is usually taken as the smaller of the two wavelengths and $\Delta\lambda$ is the absolute difference between them.

Thus, $$\text{Resolving Power} = \frac{\lambda}{\Delta\lambda}$$

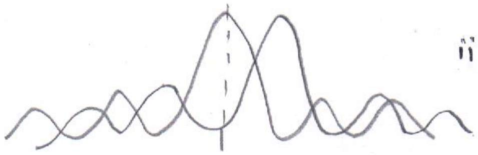
In order to decide weither two closely spaced spectral lines are properly resolved from each other or not, Rayleigh's criterion proves to be useful.

* **Rayleigh's Criterion** :- Any spectral line obtained by a diffraction grating is actually a diffraction pattern whose central maximum is regarded as the spectral line itself. When two spectral lines are sufficiently close to each other, it may happen that their diffraction patterns overlap each other. The Rayleigh's criterion is based on the separation of those diffraction pattern from each other.

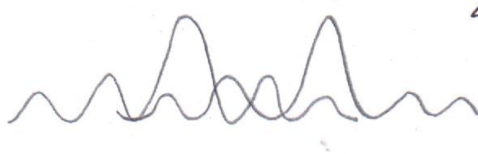
It states that "two closely spaced spectral lines are said to be just resolved from each other if the central maximum in the diffraction pattern of one spectral line coincides with the first minimum in the diffraction pattern of the other spectral line."



i) Unresolved Spectral line
(The two maxima are coinciding)
$\frac{\lambda}{\Delta\lambda} > mN$



ii) Just resolved spectral lines
(Minima of one diffraction pattern coinciding with maxima of other diffraction pattern)
$\frac{\lambda}{\Delta\lambda} = mN$



iii) Well-resolved spectral lines
$\frac{\lambda}{\Delta\lambda} < mN$

Here, 'm' is the order of diffraction maximum and N is the total no. of lines in the grating.

* Dispersive Power of Grating :-

It is defined as "the ratio of the difference in the angle of diffraction of any two neighbouring spectral lines to the difference in the wavelength between the two spectral lines."

According to the theory of diffraction grating, the m^{th} order principal maxima for λ is given by

$$(a+b)\sin\theta = m\lambda$$

On differentiating w.r.t. λ

$$(a+b)\cos\theta \cdot \frac{d\theta}{d\lambda} = m$$

$$\frac{d\theta}{d\lambda} = \frac{m}{(a+b)\cos\theta} \quad \text{or} = \frac{mN}{\cos\theta}$$

$$\therefore \boxed{\text{Dispersive Power} = DP = \frac{d\theta}{d\lambda} = \frac{mN}{\cos\theta}}$$