RELATIVITY

INTRODUCTION

• Einstein's special theory of relativity deals with the physical law as determined in two references frames moving with constant velocity relative to each other.

• **Event:** An event is something that happens at a particular point in space and at a particular instant of time, independent of the reference frame. Which we may use to described it.

• A collision between two particles, an explosion of bomb or star and a sudden flash of light are the examples of event.

• **Observer:** An observer is a person or equipment meant to observe and take measurement about the event. The observer is supposed to have with him scale, clock and other needful things to observe that event.

FRAME OF REFERENCE (INERTIAL AND NON-INERTIAL FRAMES OF REFERENCE):

• An object, either at rest or in motion, can be located with reference to some co-ordinate system called the frame of reference.

• A frame of reference is any coordinate system. For example in Cartesian coordinate system the frame S is denoted by S[OXYZ] as shown in Figure (a).

• If the coordinates [x, y, z] of all points of a body do not change with respect to time and frame of reference, the body is said to be at rest.

Fig. (a)

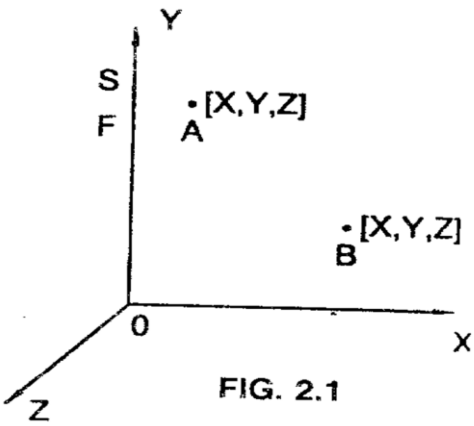


FIG. 2.1

Fig.(b)

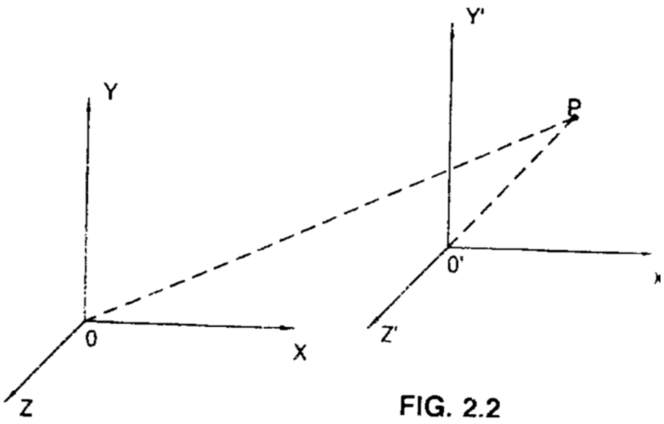


FIG. 2.2

• If the coordinates of a point of the body change with time and with respect to the frame of reference, the body is said to be in motion.

• As shown in the figure (a), consider a body at point A having coordinates [x, y, z] in the frame of reference S. If the body always remains at A, it will be at rest relative to the frame of reference S. But if it moves to point B having coordinates [x1, y1, z1], in certain time duration, then it is said to be in motion relative to the frame of reference S.

• Suppose motion of the particle is observed by observers O and O’ as shown in Figure (b). If O and O’ are at rest with respect to each other, they will observe the same motion of P. But if O and O’ are in relative motion their observation of motion of P would certainly differ.

• Any object can be located or any event can be described using a coordinate system. This coordinate system is called the **Frame of Reference**.

• **Inertial Frame:** An inertial frame is defined as a reference frame in which the law of inertia holds true. i.e. Newton’s first law. Such a frame is also called un-accelerated frame. e.g. a distant star can be selected as slandered inertial frame of reference.

• **Non-inertial Frame:** It is defined a set of coordinates moving with acceleration relative to some other frame in which the law of inertia does not hold true. It is an accelerated frame. e.g., applications of brakes to a moving train make it an accelerated (decelerated) frame. So it becomes a non-inertial frame.

GALILIEAN TRANSFORMATION:

Galilean transformation successfully explained the invariance of laws of the Newtonian mechanics in a different inertial frame. No experiment in Physics carried out in a single inertial frame, can tell us that our frame is at rest or moving with a uniform velocity. As we all know that our earth moves with a velocity of 30 km/sec with respect to the space in a nearly circular path around the Sun. There is no preferred inertial frame for the laws of mechanics to hold true. The assumption that time can be treated as absolute, is at the heart of the Galilean Transformation.

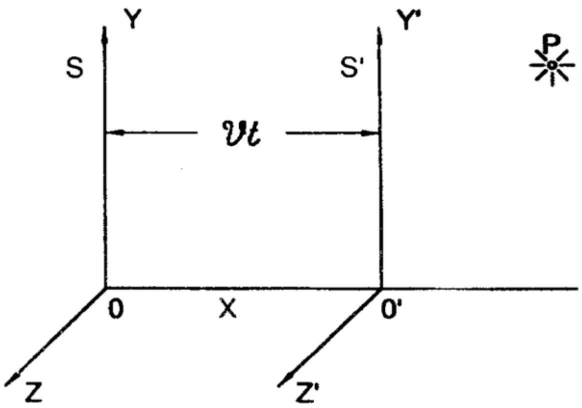
Galilean transformation equations:

• As shown in the figure consider two inertial reference frames S[O-XYZ] and S’[O’-X’Y’Z’].

• At time t = 0, O and O’ coincide with each other.

• Suppose S’ frame with observer O’ moves with the velocity ‘v’ along positive X-axis.

• Some event occurs at a point P whose space coordinates and time coordinates are recorded by both the observers in their respective inertial frames.



• The observer O in S frame of reference records coordinates (x, y, z) and time t for the event P.

• The observer O’ in S’ frame of reference records coordinates (x’, y’, z’) and time t’ for the same event at P.

• According to classical physics, motion does not affect the length, so we have the relationship between the measurements as

$$\boxed{\begin{aligned} x' &= x - vt \\ y' &= y \\ z' &= z \end{aligned}}$$

• Also according to the concept of absolute nature or universal nature of time we have

$$t' = t$$

• Then the following set of equations

$$\boxed{\begin{aligned} x' &= x - vt \\ y' &= y \\ z' &= z \\ t' &= t \end{aligned}}$$

is called the Galilean transformation equations.

The observer O and O' can further calculate the velocity of the particle by assigning three components to it with (u_x, u_y,u_z) and (u'_x, u'_y, u'_z) being velocity components as measured by O'.

The relationship between (u_x, u_y,u_z) and (u'_x, u'_y, u'_z) is obtained from time differentiation of the Galilean coordinate transformation.

Thus from x'=x-vt

$$u'_x = \frac{dx'}{dt'} = \frac{d(x - vt)}{dt'} = (\frac{dx}{dt} - vx1)$$

$$u'_x = u_x - v$$

Altogether, the Galilean velocity transformations are

$$\boxed{u'_x = u_x - v \quad , u'_y = u_y \ \ , u'_z = u_z}$$

The acceleration components are same for both observers moving with uniform relative velocity can also be written as

$$\boxed{a'_x = a_x \quad , \quad a'_y = a_y \ \ , a'_z = a_z}$$

EINSTEIN'S SPECIAL THEORY OF RELATIVITY

• Einstein proposed the special theory of relativity in 1905. This theory deals with the problems of mechanics in which one frame moves with constant velocity relative to the other frame.

• The **two postulates** of the **Special Theory of Relativity** are:

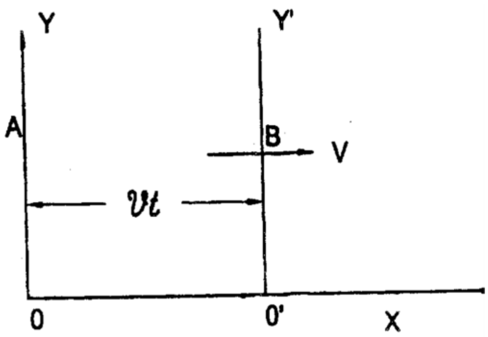
1. The laws of physics are the same in all inertial systems. No preferred inertial system exists.

2. The speed of light(c) in free space has the same value in all the inertial systems.

LORENTZ TRANSFORMATION

Galilean transformation is not valid when the speed of a particle approaches the velocity of light. Newton had considered time as an absolute constant and thus, was only in space. However Lorentz proposed that, in transformations, all the four coordinates (x,y,z) changes into (x',y',z') provided it applies the condition of the constancy of the speed of light. Such transformation is called as Lorentz Transformation.

Consider two frames of references A and B as shown in the Figure (a). As shown in the figure, A is fixed and B is moving along the direction of the X-axis with a constant velocity v.



• After time 't' the frame of reference B has moved a distance OO' = vt.
• For the point P in space the coordinates are (x, y, z) with reference to the frame A and (x', y', z') with reference to the frame B.
• According to Galilean transformation equations,

$$\boxed{\begin{aligned} x' &= x - vt \\ y' &= y \\ z' &= z \\ t' &= t \end{aligned}} \qquad \text{-----------------} \ (1)$$

Differentiating equation 1,

$$\frac{dx'}{dt'} = \frac{dx}{dt} - v$$

$$c' = c - v$$

• This equation says if a person is moving in a spaceship the speed of the passing light will be (c-v)

• But according to the postulates of the special theory of relativity the velocity of light remains constant in free space.

- This suggests that the Galilean transformations are not in accordance with the special theory of relativity. So the need for the new transformation equations is there.
- However, the equation $x' = x - vt$ is in accordance with the ordinary laws of mechanics. So the new transformation for the x coordinates must be similar to this equation. The simplest possible form of this can be

$$x' = k(x - vt) \qquad (2)$$

- Where k depends only on the value of v and doesn't depend upon the values of x and t. The above equation is linear and x' has only one value for given value of x.
- According to the first postulate of the special theory of relativity observation made in the frame of reference B must be identical to those made in A expect for a change in the sign of v and having the same value for the constant of proportionality k.

$$x = k(x' + vt') \qquad (3)$$

- Since the relative motion of A and B is combined to only x

$$y' = y$$

$$z' = z$$

$$t' = t$$

- The value of x' from equation (2) can be substituted in equation (3)

$$x = k\,[k(x - vt) + vt']$$

$$x = k^2(x - vt) + kvt'$$

$$kvt' = x - k^2(x - vt)$$

$$t' = \frac{x - k^2 x + k^2 vt}{kv}$$

$$t' = \frac{x - k^2 x}{kv} + \frac{k^2 vt}{kv} = \frac{x(1-k^2)}{kv} + \frac{k^2 vt}{kv}$$

$$t' = kt + \frac{x(1-k^2)}{kv} \tag{4}$$

- To find the value k, consider two reference frames A and B. The spaceship in reference frame A measures the time t and the spaceship in reference frame B measure the time t'.

$$x = ct \tag{5}$$

$$x' = ct' \tag{6}$$

- Substituting the value of x' and t' from equations (2) and (4) in equation (6)

$$k(x - vt) = c\left[kt + \frac{x(1-k^2)}{kv}\right]$$

$$kx - kvt = c\,kt + cx\frac{(1-k^2)}{kv}$$

$$kx - cx\frac{(1-k^2)}{kv} = c\,kt + kvt$$

$$x\left[k - c\frac{(1-k^2)}{kv}\right] = c\,kt\left[1 + \frac{v}{c}\right]$$

$$x = \frac{c\,kt\left[1+\frac{v}{c}\right]}{\left[k - c\frac{(1-k^2)}{kv}\right]} = \frac{c\,kt\left[1+\frac{v}{c}\right]}{k\left[1-\frac{c}{v}\left(\frac{1}{k^2}-1\right)\right]}$$

$$x = c\,t\frac{\left[1+\frac{v}{c}\right]}{\left[1-\frac{c}{v}\left(\frac{1}{k^2}-1\right)\right]} \tag{7}$$

- Then substituting the value of x from equation (5) into (7) we get

$$ct = c\,t\frac{\left[1+\frac{v}{c}\right]}{\left[1-\frac{c}{v}\left(\frac{1}{k^2}-1\right)\right]}$$

$$1-\left(\frac{c}{v}\right)\left(\frac{1}{k^2}-1\right)=1+\frac{v}{c}$$

$$-\left(\frac{c}{v}\right)\left(\frac{1}{k^2}-1\right)=\frac{v}{c}$$

$$1-\frac{1}{k^2}=\frac{v^2}{c^2}$$

$$1-\frac{v^2}{c^2}=\frac{1}{k^2}$$

$$k^2=\frac{1}{1-\frac{v^2}{c^2}} \tag{8}$$

$$k=\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \tag{9}$$

- The value of k when substituted in equation (2) we get

$$x' = k(x-vt)$$

$$\boldsymbol{x'} = \frac{\boldsymbol{(x-vt)}}{\sqrt{\boldsymbol{1-\frac{v^2}{c^2}}}} \tag{9}$$

$$\boldsymbol{y'} = \boldsymbol{y} \tag{10}$$

$$\boldsymbol{z'} = \boldsymbol{z} \tag{11}$$

- Now we can rewrite the equation (4) as

$$t' = kt+\frac{x(1-k^2)}{kv} = kt+\frac{x}{kv}-k\left(\frac{x}{v}\right)$$

$$t' = kt+\frac{x}{kv}-k\frac{x}{v} = kt+\frac{x}{v}\left[\frac{1}{k}-k\right]$$

$$t' = kt+k\frac{x}{v}\left[\frac{1}{k^2}-1\right]$$

$$t' = k\left[t+\frac{x}{v}\left[\frac{1}{k^2}-1\right]\right] = \frac{t+\frac{x}{v}\left[1-\frac{v^2}{c^2}-1\right]}{\sqrt{1-\frac{v^2}{c^2}}}$$

- Then substituting the value of k and k^2 in above equation from equations (9) and (8), we get

$$t' = k\left[t + \frac{x}{v}\left[\frac{1}{k^2} - 1\right]\right] = \frac{t+\frac{x}{v}\left[1-\frac{v^2}{c^2}-1\right]}{\sqrt{1-\frac{v^2}{c^2}}}$$

t'

$$\boldsymbol{t' = \frac{t-\frac{xv}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}} \qquad (12)$$

- The equations (9), (10), (11) and (12) are called ***Lorentz transformation equations***.
- These equations give the conversions for the measurements of time and space made in the stationary frame A to B.
- The ***Inverse Lorentz transformation*** for measurements of space and time made in frame in B to A**.**

$$\boldsymbol{x = \frac{x'+vt'}{\sqrt{1-\frac{v^2}{c^2}}}} \qquad (13)$$

$$\boldsymbol{y' = y} \qquad (14)$$

$$\boldsymbol{z' = z} \qquad (15)$$

$$\boldsymbol{t = \frac{t'-\frac{vx'}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}} \qquad (16)$$

LORENTZ FITZERALD CONTRACTION (LENGTH CONTRACTION OF OBJECTS):

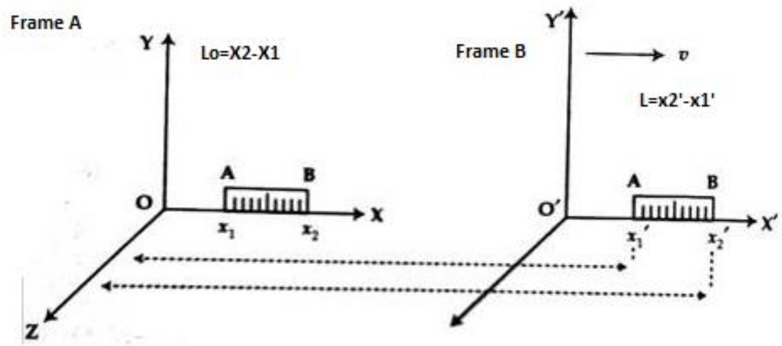
In consequence of Lorentz transformation, any object moving with a constant velocity, when observed by an observer from stationary frame of reference, the length of the object appears to be contracted by factor

$\gamma = \sqrt{1 - \frac{v^2}{c^2}}$. This phenomenon is known as **length contraction.**

- Measurement of space and time are not absolute but depend on the relative motion of the observer and the observed objects.

- Consider a rod of length L_o parallel to the X-axis and having co-ordinates x_1 and x_2 in the frame A.

- An observer in the reference frame A measures the length of the rod as $L_o = x_2 - x_1$.



- Also consider a second reference frame B moving with a velocity v along the x axis with respect to the reference frame A.
- An observer in the reference frame B measures the length of the rod as $L = x_2' - x_1'$.
- The relation between x_1 and x_1' and also between x_2 and x_2 according to the Inverse Lorentz Transformations will be

$$x_1 = \frac{x_1'+vt'}{\sqrt{1-\frac{v^2}{c^2}}} \quad x_1 = \frac{x_1'+vt'}{\sqrt{1-\frac{v^2}{c^2}}} \text{ and } \quad x_1 = \frac{x_2'+vt'}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$L_o = x_2 - x_1$$

$$L_o = \frac{x_2' + vt'}{\sqrt{1-\frac{v^2}{c^2}}} - \frac{x_1' + vt'}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$L_o = \frac{x_2' - x_1'}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{L}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\boxed{L = L_o\sqrt{1-\frac{v^2}{c^2}}}$$

OR

$$\boldsymbol{L = \frac{L_0}{\gamma}}$$

- This equation shows that the length of stationary object with respect to an observer in motion appears to be shorter than length measured by an observer at rest.
- Similarly when an object is in motion with respect to a stationary observer, again the object appears to be shortened in length.

- This relativistic result is true for both the cases, i.e whether object is in motion or the observer is in motion the object appears to be contracted or shortened in length. This phenomenon is called ***Lorentz-Fitzerald Constraction***.

- Lorentz –Fitzerald contraction is appreciable only when the velocity, v is comparable to the velocity of light c.

- Let consider a rod of length L moving with a velocity which is equal to 0.6c. Then its length as measured from another frame is given by L_0. Here we have $v = 0.6\,c$

$$L = L_o\sqrt{1-\frac{v^2}{c^2}}$$

$$L = L_o\sqrt{1-0.36}$$

$$L = 0.8\ L_o$$

The contraction in length = $L - L_o = 0.2\ L_o$

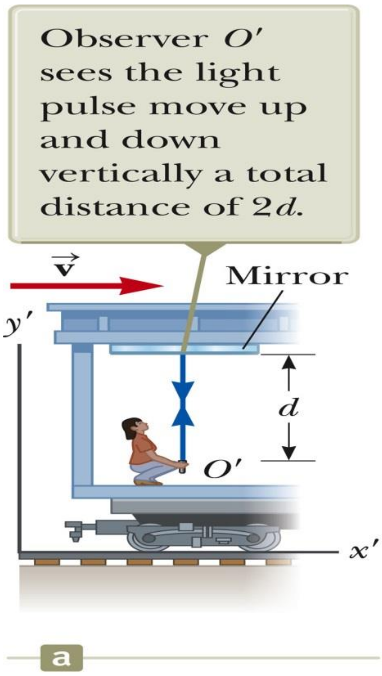
- Instead if the velocity of the body is negligibly small as compared to c, the contraction the length is also negligible i.e., $L = L_o$

- Suppose $v = 0.01\,c$

$$L = L_o\sqrt{1-(10)^{-4}}$$
$$L = 0.9999\,L_o$$

TIME DILATION (INCREASE OF TIME):

When an observer in a stationary frame of reference observes the time interval of any event of moving frame of reference, the time interval appears to be slowed down by factor γ. This effect is known as Time Dilation. That is time interval appears to be longer in the stationary frame of reference.

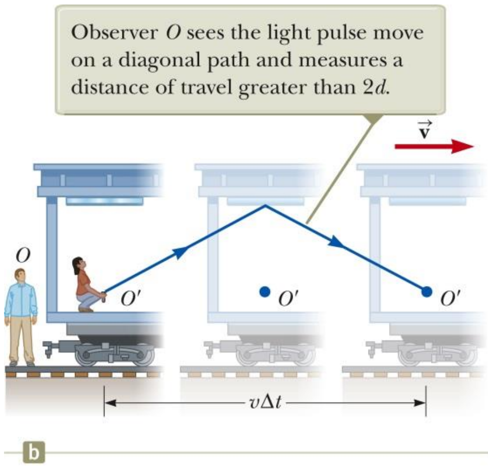


Consider a mirror is fixed to the ceiling of a vehicle as shown in figure below. The vehicle is moving to the right with speed **v**. An observer, ***O'***, at rest in the frame attached to the vehicle holds a flashlight a distance ***d*** below the mirror.

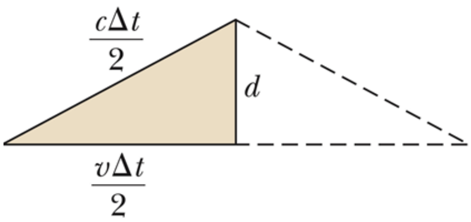
The flashlight emits a pulse of light directed at the mirror (event 1) and the pulse arrives back after being reflected (event 2).
Observer ***O'*** carries a clock.
She uses it to measure the time between the events **(Δt_p)**.
We can think of the pulse of light as a particle under constant speed.

- The observer in vehicle sees the events to occur at the same place.

- Δt_p **= distance/speed = (2*d*)/*c***



- Observer **O** is in a second frame at rest with respect to the ground.
- He observes the mirror and **O'** moving with speed ***v***.
- By the time the light from the flashlight reaches the mirror, the mirror has moved to the right.
- The light must travel farther with respect to **O** than with respect to **O'**.
- Both observers must measure the speed of the light to be *c.*
- The light travels farther for **O**.
- The time interval, Δt, for **O** is longer than the time interval for **O'**, Δt_p.
- The time interval Δt is longer than the time interval Δt_p



From geometry of path travelled by light

$$(\frac{C\Delta t}{2})^2 = (\frac{v\Delta t}{2})^2 + d^2$$

$$(c^2 - v^2)\Delta t^2 = 2d^2$$

$$\Delta t = \frac{2d}{\sqrt{c^2 - v^2}} = \frac{2d/_c}{\sqrt{1 - v^2/_{c^2}}}$$

But $\frac{2d}{c} = \Delta t_p$

$\therefore$

$$\Delta t = \frac{\Delta t_p}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma \Delta t_p$$

$$\text{where } \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

γ is always greater than unity.

The time interval Δt between two events measured by an observer moving with respect to a clock is longer than the time interval Δt_p between the same two events measured by an observer at rest with respect to the clock. This effect is known as Time Dilation.

Solved Problems:

1. A passenger in a train moving at 30 m/s passes a man standing on a station platform at $t = t' = 0$. Twenty seconds after the train passes him, the man on the platform determines that a bird flying along the track in the same direction as the train is 800 m away. What are the coordinates of the bird as determined by the passenger?

Solution : The man standing on the platform assigns coordinates to the bird as

$$(x, y, z, t) = (800 \text{ m}, 0, 0, 20\text{s})$$

The passenger measures the distance x' to the bird as

$$\begin{aligned} x' &= x - \upsilon t \\ &= 800 \text{ m} - 30 \text{ m/s} \times 20\text{s} = 200 \text{ m} \end{aligned}$$

Therefore, the passenger determines the coordinates of the bird as

$$\mathbf{(x', y', z', t) = (200\ m, 0, 0, 20s).}$$

2. A train moving with a velocity of 60 km/hr passes through a railroad station at 12.00. Twenty seconds later a bolt of lightning strikes the railroad tracks one km from the station in the same direction that the train is moving. Find the coordinates of the lightening flash as measured by an observer of the station and by the engineer of the train.

Solution : Both observers measure the time coordinate as

$$t = t' = \left(\frac{1 \text{ hr}}{3600 \text{ s}}\right) = \frac{1}{180} \text{ hr}$$

The observer at the station measures the spatial coordinate to be $\mathbf{x = 1\ km}$.

The spatial coordinate as determined by the engineer of the train is

$$\mathbf{x' = x - \upsilon t = 1\ km - (60\ km/hr)\left(\frac{1}{180}\ hr\right) = \frac{2}{3}\ km}$$

3. A hunter on the ground fires a bullet in the north east direction, which strikes a deer 0.25 km from the hunter. The bullet travels with a speed of 1800 km/hr. At the instant when the bullet is fired, an air plane is directly flying over the hunter at an altitude of one km and is travelling due east with a velocity of 600 km/hr. When the bullet strikes the deer, what are the coordinates as determined by an observer in the airplane?

Solution : Using Galilean transformations

$$t' = t = \frac{0.25 \text{ km}}{1800 \text{ km/hr}} = 1.39 \times 10^{-4} \text{ hr}$$

$$x' = x - \upsilon t = (0.25 \text{ km}) \cos 45° - (600 \text{ km/hr})(1.39 \times 10^{-4} \text{ hr})$$

$$y' = y = (0.25 \text{ km}) \sin 45° = \mathbf{0.177\ km}$$

$$z' = z - h = 0 - 1 \text{ km} = \mathbf{-1\ km}$$

4. An event occurs at $x = 100$ m, $y = 10$ m, $z = 5$m and $t = 1 \times 10^{-4}$ second in a frame S. Find the coordinates of this event in a frame S′ which is moving with velocity 2.7×10^8 meters per second with respect to the frame S along the common XX′ axes using Galilean Transformation and Lorentz Transformation.

Solution : According to Galilean Transformation, we have

$x' = x - \upsilon t, \; y' = y, \; z' = z$ and $t' = t$, we get

$x' = 100 - 2.7 \times 10^8 \times 1 \times 10^{-4} = -26900$ m

$y' = y = 10$ m, $z' = z = 5$ m

$t' = t = 1 \times 10^{-4}$ sec

Thus, coordinates in the frame S′ are

$x' = -26900$ m, $y' = 10$ m, $z' = 5$ m, $t' = 10^{-4}$ sec

5. Use Lorentz transformation to show that $x^2 + y^2 + z^2 - c^2t^2 = x'^2 + y'^2 + z'^2 - c^2t'^2$. Show that the quantity $x^2 + y^2 + z^2 - c^2t^2$ is invariant under Lorentz transformation.

Solution : According to Lorentz transformation, we have

$$x' = \frac{x - \upsilon t}{\sqrt{1 - \frac{\upsilon^2}{c^2}}}, \; y' = y, \; z' = z, \; t' = \frac{t - \upsilon x/c^2}{\sqrt{1 - \frac{\upsilon^2}{c^2}}}$$

∴ Taking the R.H.S. of equation

$$x'^2 + y'^2 + z'^2 - c^2t'^2 = \left(\frac{x - \upsilon t}{\sqrt{1 - \frac{\upsilon^2}{c^2}}}\right)^2 + y^2 + z^2 - c^2\left(\frac{t - \upsilon x/c^2}{\sqrt{1 - \frac{\upsilon^2}{c^2}}}\right)^2$$

$$= \frac{(x - \upsilon t)^2 - c^2\,(t - \upsilon x/c^2)^2}{1 - \frac{\upsilon^2}{c^2}} + y^2 + z^2$$

$$= \frac{x^2 + \upsilon^2t^2 - 2x\upsilon t - c^2\left(t^2 + \frac{\upsilon^2x^2}{c^4} - \frac{2\upsilon xt}{c^2}\right)}{1 - \frac{\upsilon^2}{c^2}} + y^2 + z^2$$

$$= \frac{x^2\left(1 - \frac{\upsilon^2}{c^2}\right) - c^2t^2\left(1 - \frac{\upsilon^2}{c^2}\right)}{1 - \frac{\upsilon^2}{c^2}} + y^2 + z^2$$

$= x^2 - c^2t^2 + y^2 + z^2$
$= x^2 + y^2 + z^2 - c^2t^2$
$=$ L.H.S.

This shows that the quantity $x^2 + y^2 + z^2 - c^2t^2$ is same in both frames of reference. So, this is invariant under Lorentz transformation.

6. A rocket ship is 100 metre long on the ground. When it is in flight, its length is 99 metres to an observer on the ground, what is its speed?

Solution : Given : $l' = 100 \text{ m} = 100 \times 100 = 10^4$ cm
$l = 99 \text{ m} = 99 \times 100 = 99 \times 10^2$ cm

Formula : $l = l' \sqrt{1 - \frac{\upsilon^2}{c^2}}$

$$9900 = 10000 \sqrt{1 - \frac{\upsilon^2}{c^2}}$$

$$\frac{\upsilon^2}{c^2} = 1 - \frac{99^2}{100^2} = \frac{10000 - 9801}{10000} = \frac{199}{10000}$$

$$\therefore \quad \boldsymbol{\upsilon = c\frac{\sqrt{199}}{100} = 3 \times 10^8 \frac{\sqrt{199}}{100} = 4.23 \times 10^9 \text{ cm/sec.}}$$

7. A certain process requires 10^{-6} sec to occur in an atom at rest in laboratory. How much time will this process require to an observer in the laboratory, when the atom is moving with a speed of 5×10^7 m/s.

Solution : Formula : $\Delta t' = \dfrac{\Delta t}{\sqrt{1 - \frac{\upsilon^2}{c^2}}}$

Given : $\Delta t = 10^{-6}$ sec, $\upsilon = 5 \times 10^7$ m/s

$$\therefore \quad \Delta t' = \frac{10^{-6}}{\sqrt{1 - \left(\frac{5 \times 10^7}{3 \times 10^8}\right)^2}}$$

$$= \frac{10^{-6}}{\sqrt{1 - \frac{25}{900}}} = \mathbf{1.013 \times 10^{-6} \text{ sec.}}$$

8. With what velocity should space ship fly so that everyday spent on it may correspond to three days on earth's surface.

Solution : Formula : $\Delta t' = \dfrac{\Delta t}{\sqrt{1 - \frac{\upsilon^2}{c^2}}}$

Given : $t = 1$ day and $t' = 3$ days

$$\therefore \quad 3 = \frac{1}{\sqrt{1 - \frac{\upsilon^2}{(3 \times 10^8)^2}}}$$

$$1 - \frac{\upsilon^2}{9 \times 10^{16}} = \frac{1}{9}$$

$$\text{or } \upsilon^2 = \frac{9 \times 10^{16} \times 8}{9}$$

$$\upsilon = \sqrt{8} \times 10^8 = 2.83 \times 10^8 \text{ m/s}$$

Space ship should fly with a speed of 2.83×10^8 m/s.

9. A rocket ship 100 m long on the earth is moving with a velocity 0.9 c. How much its length will appear to an observer on the earth?

Solution : **Given :** $l' = 100$ m, $\upsilon = 0.9$ c

Formula : $l = l'\sqrt{1 - \frac{\upsilon^2}{c^2}}$

$$l = 100\sqrt{1 - \frac{0.9^2 c^2}{c^2}}$$

$$= 43.58 \text{ m}$$

Length of the rocket will appear as 43.58 m.

10. A metre stick is projected into space and its length appears to be contracted to 50 cm. What is the velocity with which the stick is projected?

Solution : **Formula :** $l = l'\sqrt{1 - \frac{\upsilon^2}{c^2}}$

Given : $l' = 1$ m, $l = 50$ cm $= 0.5$ cm

$$0.5 = 1\sqrt{1 - \frac{\upsilon^2}{c^2}}$$

$$1 - \frac{\upsilon^2}{c^2} = 0.25$$