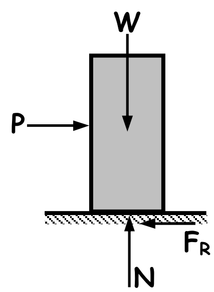
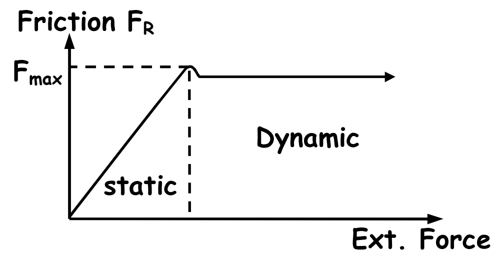
FRICTION

VARIATION OF FRICTIONAL FORCE :





Consider a block of weight 'W' placed on a rough horizontal surface. An external force 'P' is gradually applied to the block. Consider the free body diagram of the block. As the magnitude of the force 'P' keeps increasing, the magnitude of the frictional force also keeps increasing. The block remains in equilibrium till the value of the frictional force reaches its maximum value. Once the external force overcomes the maximum resistance offered by the surface, the block starts moving. The variation of frictional force 'vs' external force is as shown in the figure.

LAWS OF FRICTION :

(i) The frictional resistance acting on a body is directed opposite to the direction of slipping 'or' tendency of slipping.

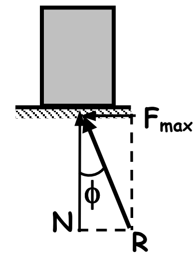
(ii) The value of maximum frictional resistance is always directly proportional to the value of normal reaction at the contact surface.
ie. $F_{max} \; \alpha \; N$
Hence **$F_{max} = \mu N$**
where μ = coefficient of static friction.

(iii) The value of frictional resistance is independent of the area of contact.

(iv) The value of frictional resistance is independent of velocity of motion.

(v) The value of static frictional coefficient is always more than the kinetic frictional coefficient.

ANGLE OF FRICTION :

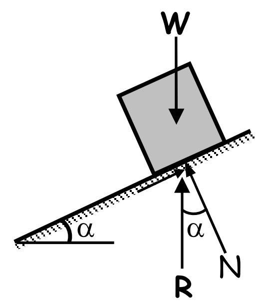


It is the angle made by the resultant reaction with the normal reaction, as shown in the diagram, such that –

$$\tan(\phi) = \frac{F_{max}}{N} = \frac{\mu N}{N} = \mu$$

Hence, $\boldsymbol{\phi = \tan^{-1}(\mu)}$

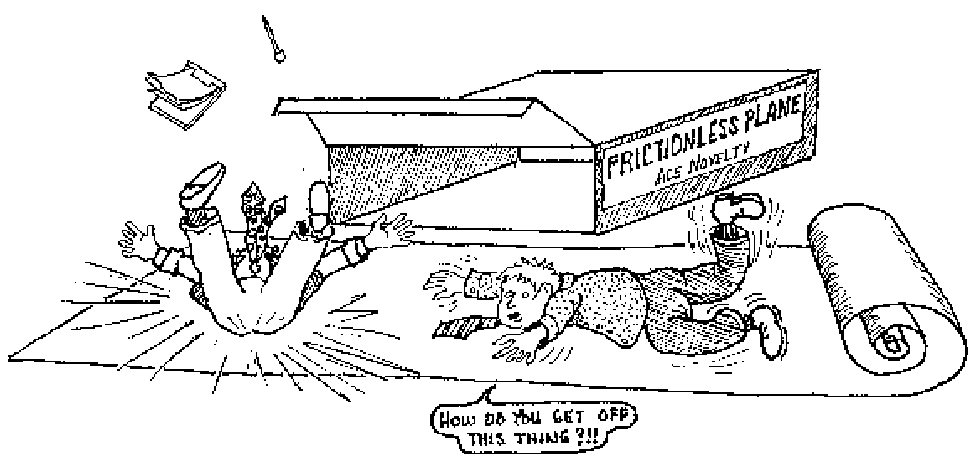
ANGLE OF REPOSE :



Consider a block of weight 'W' placed on a plane inclined at an angle 'α' with the horizontal. As the inclination of the plane is gradually increased, it will be observed that at some critical angle 'α', the block just starts sliding. At this position, the weight of the block just gets balanced by the resultant reaction, as shown.

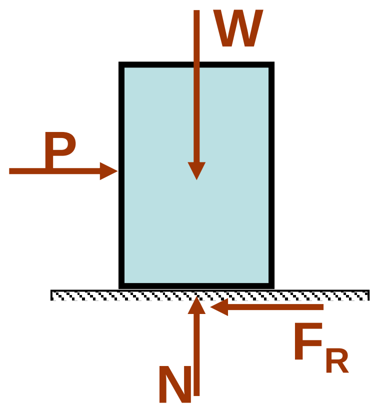
The maximum angle 'α' of the inclined plane upto which the block remains in equilibrium is called the 'angle of repose'.

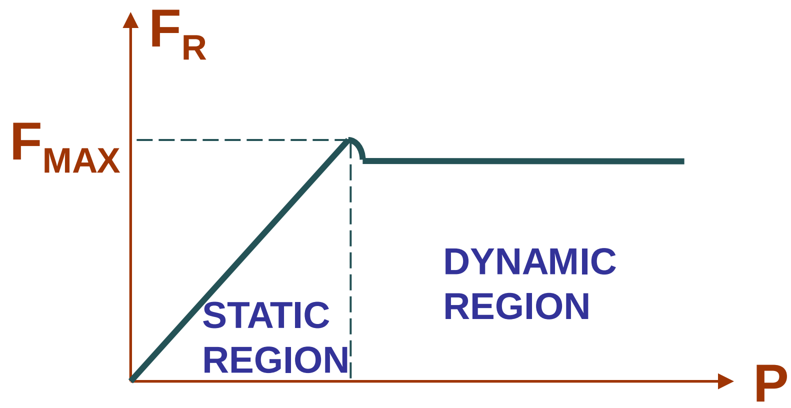
From the above diagram, note that - $\alpha = \phi = \tan^{-1}(\mu)$. **Hence, a block placed on an inclined plane will remain in equilibrium until $\alpha \leq \tan^{-1}(\mu)$.**



FRICTION

VARIATION OF FRICTIONAL FORCE vs VERSUS EXT. FORCE





LAWS OF FRICTION :

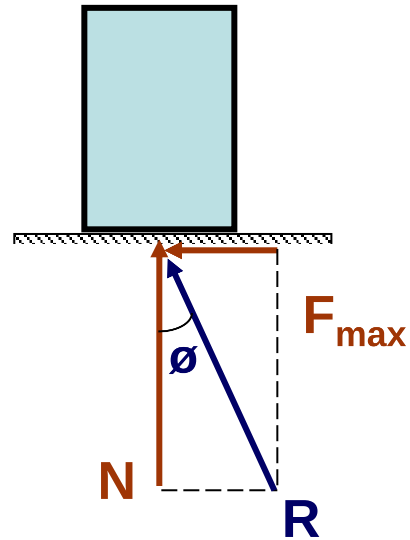
1. THE FRICTIONAL FORCE ACTS IN OPPOSITE DIRECTION TO THAT OF SLIPPING.
2. THE MAXIMUM FRICTIONAL RESISTANCE IS ALWAYS PROPORTIONAL TO THE NORMAL REACTION AT THE SURFACE OF CONTACT. ie. $F_{MAX} \propto N$. HENCE, $F_{MAX} = \mu N$
3. THE FRICTIONAL FORCE IS INDEPENDENT OF AREA OF CONTACT.
4. THE FRICTIONAL FORCE IS INDEPENDENT OF VELOCITY OF MOTION.
5. THE KINETIC FRICTIONAL RESISTANCE IS ALWAYS LESS THAN THE STATIC FRICTIONL RESISTANCE.

PREPARED BY M.B.RAO

ANGLE OF FRICTION :

IT IS THE ANGLE MADE BY THE RESULTANT REACTION WITH THE NORMAL REACTION AT THE SURFACE OF CONTACT , AS SHOWN, SUCH THAT –

$$\tan ø = F_{max} / N = \mu N / N = \mu$$

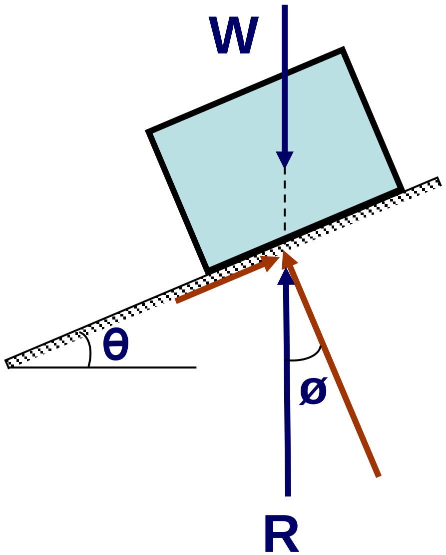


ANGLE OF REPOSE :

IT IS THE MAXIMUM INCLINATION OF THE INCLINED SURFACE ON WHICH A BLOCK KEPT STAYS IN EQUILIBRIUM.

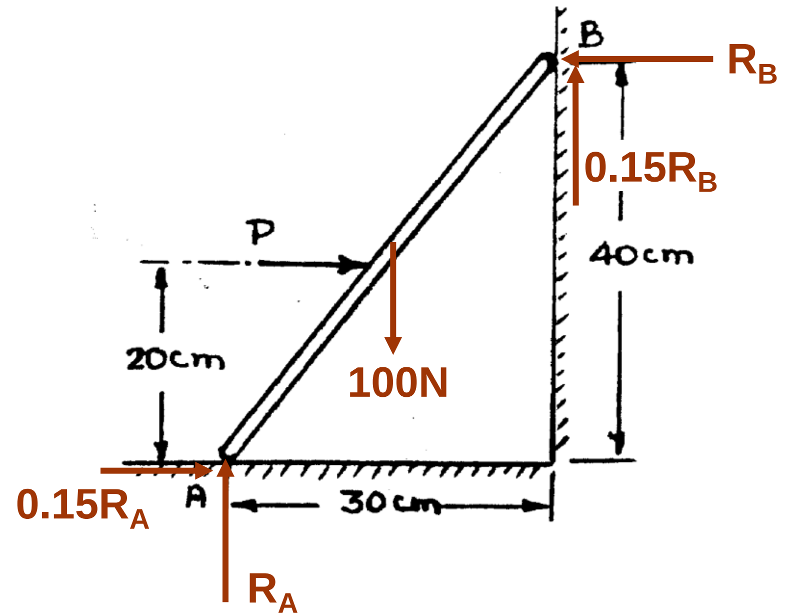
NOTE THAT, IN THIS POSITION, THE WEIGHT OF THE BLOCK EXACTLY GET BALANCED BY THE RESULTANT REACTION.

HENCE, $\theta = ø = \tan^{-1}\mu$



PREPARED BY M.B.RAO

1] A 100N uniform rod AB is held in the position shown. If the coefficient of friction is 0.15 at A and B, calculate the range of values of 'P' for which the equilibrium is maintained.



TO FIND P_{min}:-

$\sum F_x = 0$

ie. $P + 0.15R_A - R_B = 0$ → eq(i)

$\sum F_Y = 0$

ie. $R_A + 0.15R_B = 100$ → eq(ii)

$\sum M_A = 0$

ie. $-(P \times 20) - (100 \times 15) + (R_B \times 40) + (0.15R_B \times 30) = 0$

ie. $20P - 44.5R_B = -1500$ → eq(iii)

ON SOLVING, WE GET $P_{MIN} = \underline{36.1N}$

SIMILARLY, $P_{MAX} = \underline{137.75N}$

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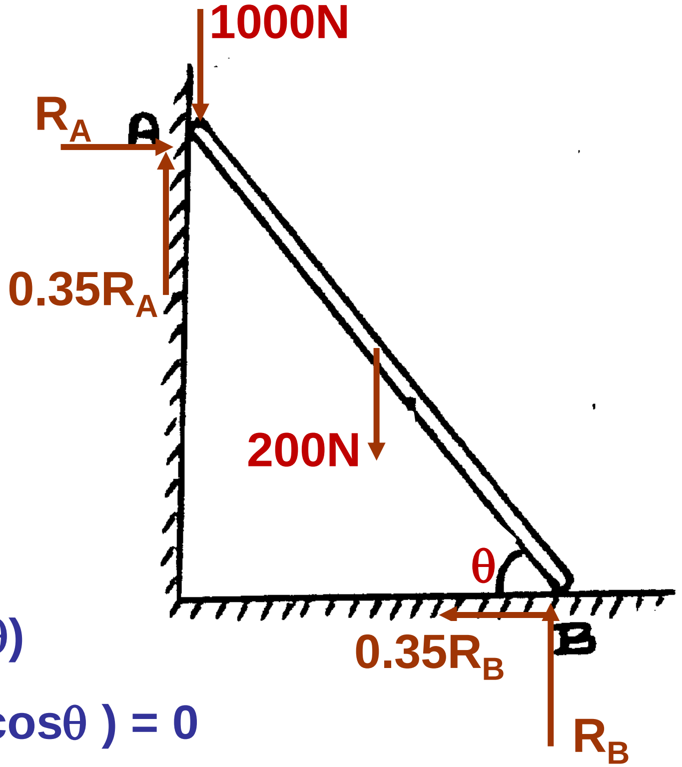
2] A ladder AB, 3m in length and weighing 20Kg, is placed against a wall and resting on the floor. The line AB makes an angle of 'θ' with the floor. The coefficient of friction at both the contact surfaces is 0.35. In addition to the self weight of the ladder, it has to support a man weighing 100Kg at its top end A. Find the least angle θ the ladder needs to be placed to prevent it from slipping.



$$\sum F_x = 0$$

$$R_A - 0.35R_B = 0 \quad \rightarrow \text{eq(i)}$$

$$\sum F_Y = 0$$

$$0.35R_A + R_B = 1200 \quad \rightarrow \text{eq(ii)}$$

ON SOLVING, $R_A = \underline{374.16N}$, $R_B = \underline{1069N}$

$$\sum M_B = 0$$

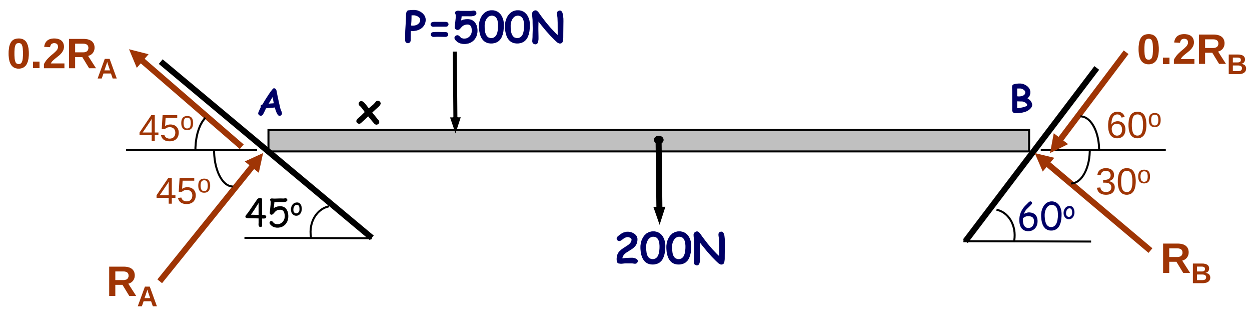
$$(1000 \times 3\cos\theta) + (200 \times 1.5\cos\theta) - (374.16 \times 3\sin\theta)$$

$$- (0.35 \times 374.16 \times 3\cos\theta) = 0$$

$$3075.5 \cos\theta - 1122.5 \sin\theta = 0$$

ON SOLVING, WE GET $\theta = \underline{68.8^o}$

3] An uniform rod AB weighing 200N and length 4m is placed horizontally on two inclined surfaces as shown. Determine the safe distance 'x' from end A where an external force P may be made to act without disturbing the equilibrium of the rod. Take μ = 0.2 for both the contact surfaces.



TO FIND X_{min} :

$\sum F_x = 0$ ie. $R_A\cos45 - 0.2R_A\cos45 - R_B\cos30 - 0.2R_B\cos60 = 0$ → eq(i)

$\sum F_Y = 0$ ie. $R_A\sin45 + 0.2R_A\sin45 + R_B\sin30 - 0.2R_B\sin60 = 700$ → eq(Ii)

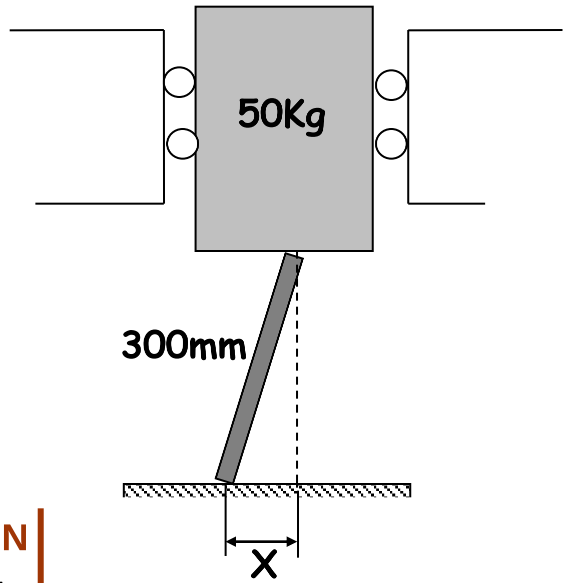
ON SOLVING, WE GET R_A = 673N, R_B = 394.2N

$\sum M_A = 0$ ie. $-(500.X) - (200 \times 2) + (R_B\sin30 \times 4) - (0.2R_B\sin60 \times 4) = 0$

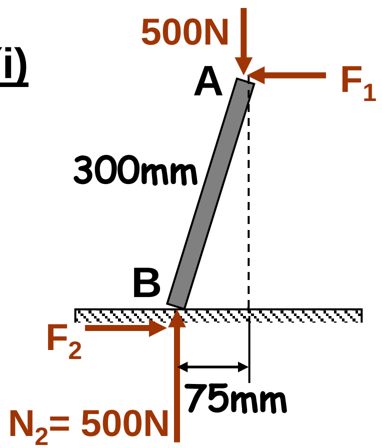
ON SOLVING, WE GET X_{min} = 0.23m

SIMILARLY X_{max} = 2.38m

4] A light bar is used to support a 50Kg block in the vertical guides as shown. The coefficient of static friction is 0.3 between the block & the bar and 0.4 between the floor & the bar. (i) Find the frictional force acting at each end of the bar when x = 75mm. (ii) Also find the maximum value of ‘x’ for which the system remains in equilibrium.



CASE (i)



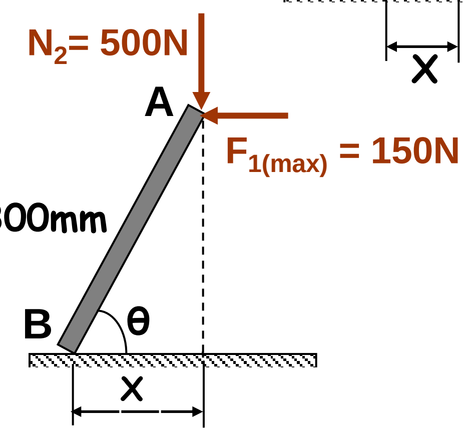
NOTE : $F_{1(max)}$ = 0.3x500 = 150N

$F_{2(max)}$ = 0.4x500 = 200N

BY TAKING MOMENT AT ‘B’ :

WE GET – F_1 = 129.1N = F_2

CASE (ii)



NOTE: SLIPPING HAPPENS AT ‘A’ FIRST. HENCE THE FRICTION IS MAXIMUM AT ‘A’.

BY TAKING MOMENT AT ‘B’ :

500 x 0.3cosθ = 150 x 0.3sinθ

HENCE θ = 73.3°

x = 300cosθ = 86.2mm

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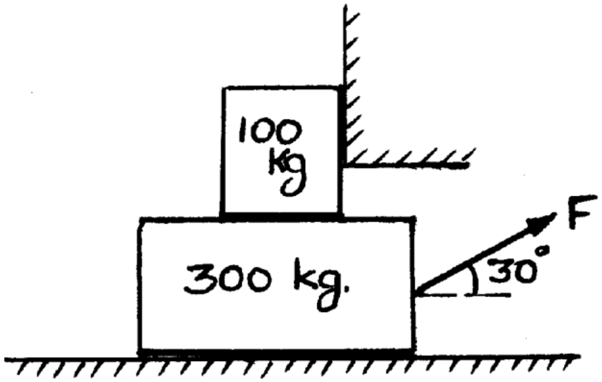
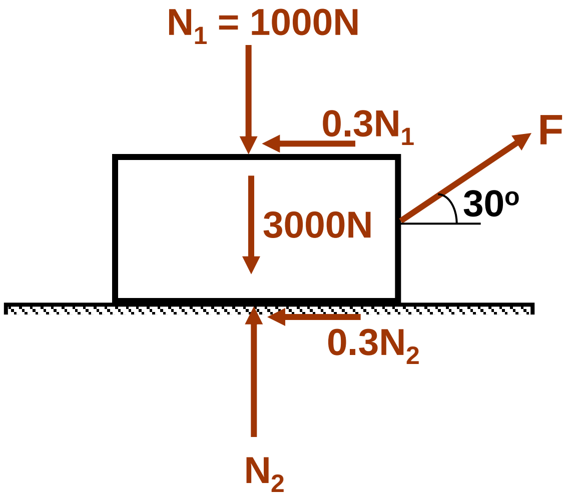
5] What force F is needed to get the 300Kg block to move to the right ? The coefficient of friction for all contact surfaces is 0.3 .





NOTE: As the 300Kg block is pulled to the right, observe that 100Kg block remains static.

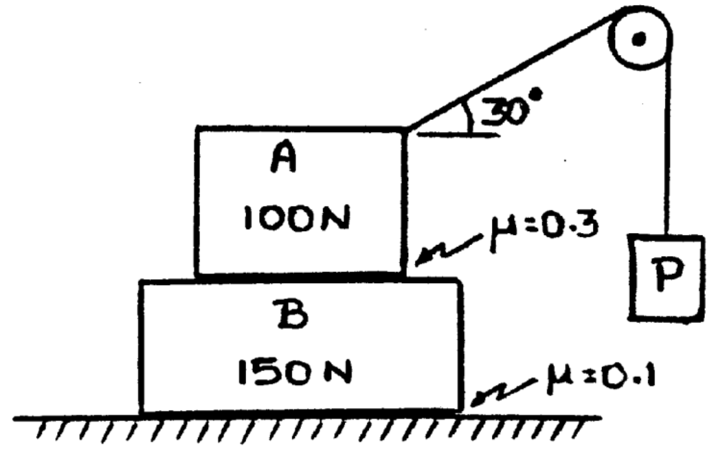
$\sum F_x = 0$ ie. $F\cos30 - 0.3(N_1 + N_2) = 0$ → eq(i)

$\sum F_Y = 0$ ie. $F\sin30 + N_2 - N_1 - 3000 = 0$ → eq(ii)

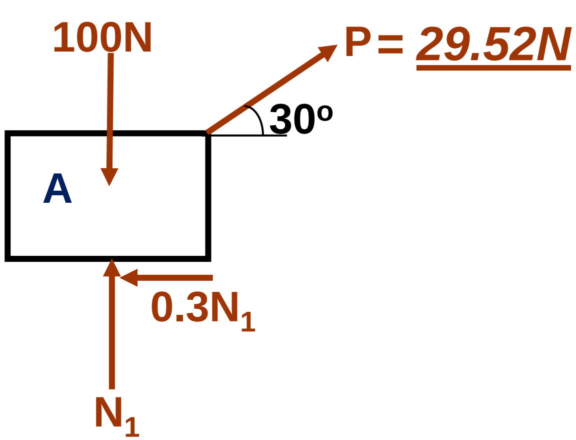
ANS : F = 1476N

PREPARED BY M.B.RAO

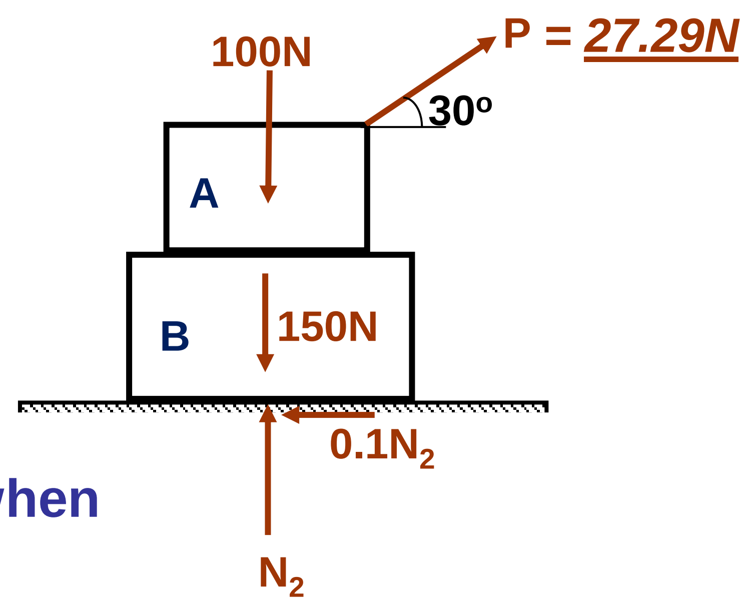
6] Two blocks A(100N) and B(150N) are resting as shown. The cofft. of friction between B and the ground is 0.1 & 0.3 between A and B. Find the minimum value of weight ‘P’ to start the motion.



CASE (i) : Force ‘P’ needed to move 100N block.

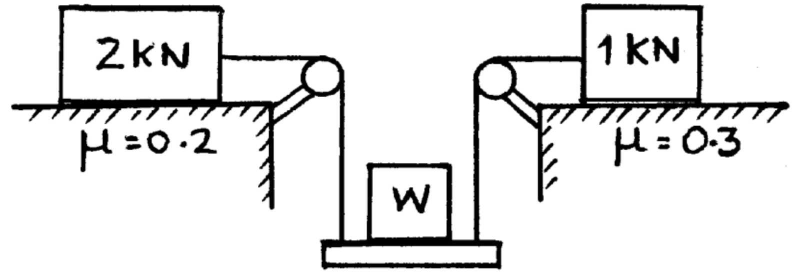


CASE (i) : Force ‘P’ needed to move both blocks together.



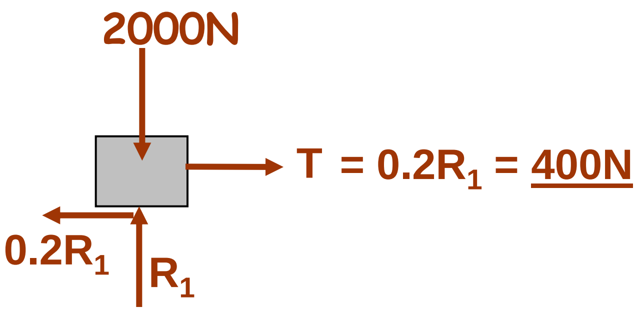
Hence, minimum value of P = 27.29N when both blocks begins to move.

7] A platform is supported by two ropes, which are attached to blocks that can slide horizontally. At what value of W does the platform begin to descend ?

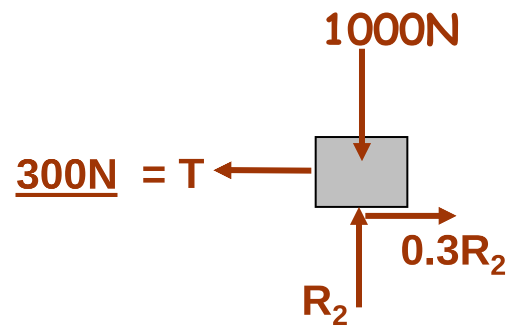


<u>NOTE</u>: As the load placed in the middle of the platform, the tensions in the ropes supporting the platform are same.

<u>CASE (i)</u> : <u>Force 'T' needed to move 2KN block.</u>



<u>CASE (ii)</u> : <u>Force 'T' needed to move 1KN block.</u>

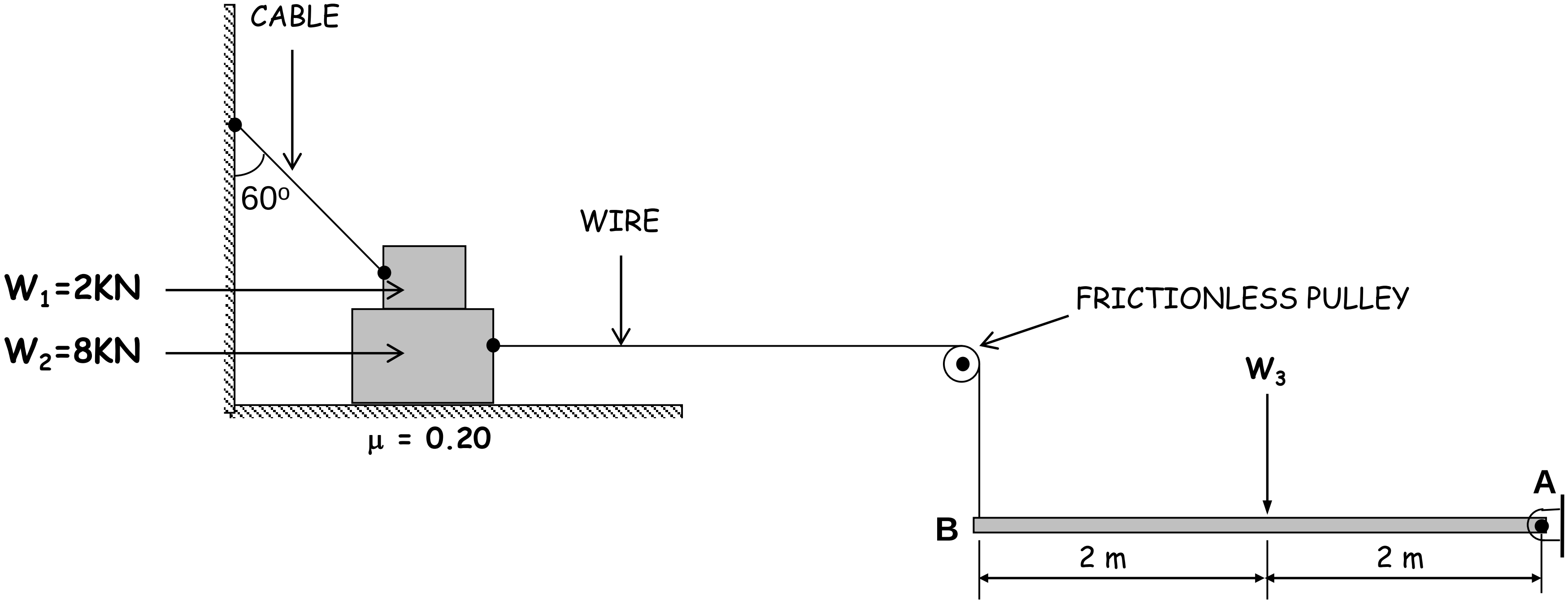


Therefore, the minimum value of 'T' is equal to 300N when 1000N block begins to move.

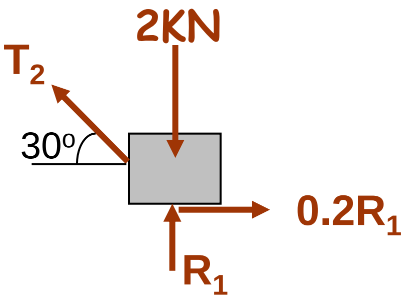
Hence, $W = 2T = \underline{600N}$

<u>*PREPARED BY M.B.RAO*</u>

8] Find the minimum weight W_3 for limiting equilibrium. Take μ = 0.2 for all contact surfaces



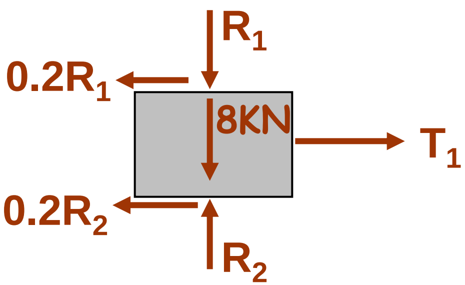
PREPARED BY M.B.RAO



$\sum F_x = 0$ ie. $0.2R_1 - T_2\cos 30 = 0 \rightarrow$ eq(i)

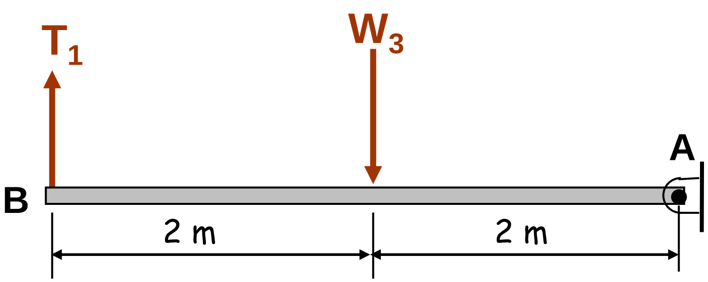
$\sum F_Y = 0$ ie. $R_1 + T_2\sin 30 = 2 \rightarrow$ eq(ii)

ON SOLVING - $R_1 = 1.793KN$ & $T_2 = 0.414N$



$\sum F_Y = 0$ ie. $R_2 = 8 + 1.793 = 9.793KN$

$\sum Fx = 0$ ie. $T_1 = 0.2R_1 + 0.2R_2 = 2.31KN$



TAKING MOMENT AT A :

$2.31 \times 4 = W_3 \times 2$

HENCE $W_3 =$ <u>4.62KN</u>

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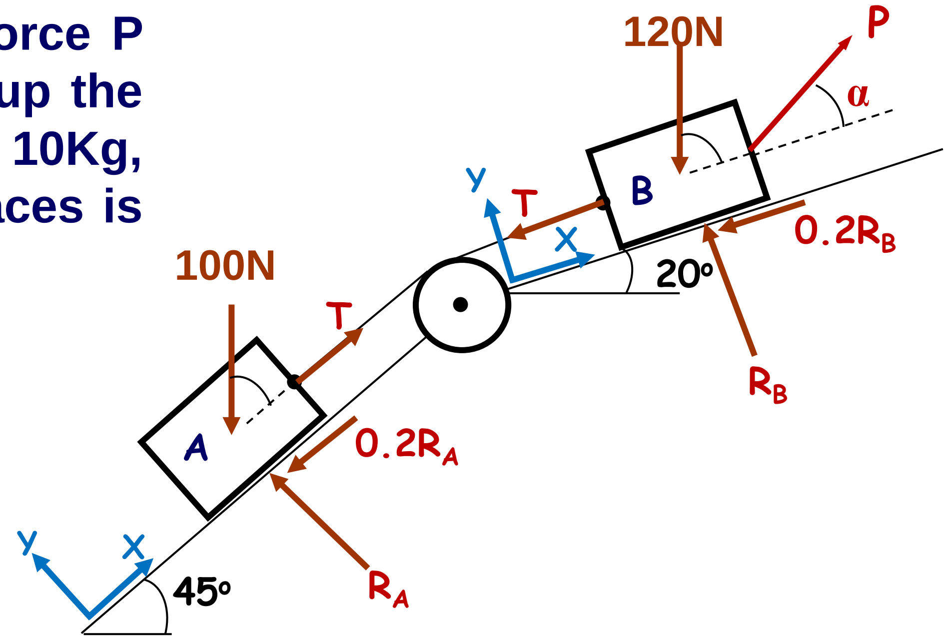
9] Determine the minimum value of force P needed to pull the system of blocks up the inclined surfaces shown. Given: m_A = 10Kg, m_B = 12Kg and 'μ' at all contact surfaces is 0.2 . Assume the pulley to be smooth.



First consider Bock A:

$\sum F_Y = 0$: $R_A = 100 \sin 45 = 70.71N$

$\sum F_X = 0$: $T = 100\cos 45 + (0.2\,R_A)$

$= 84.85N$

Next consider Bock B:

$\sum F_Y = 0$: $R_B = 120 \sin 70 - P \sin\alpha$

$\sum F_X = 0$: $P \cos\alpha = 120\cos 70 + 0.2R_B + 84.85$

$P \cos\alpha - 0.2(120 \sin 70 - P \sin\alpha) = 125.89$

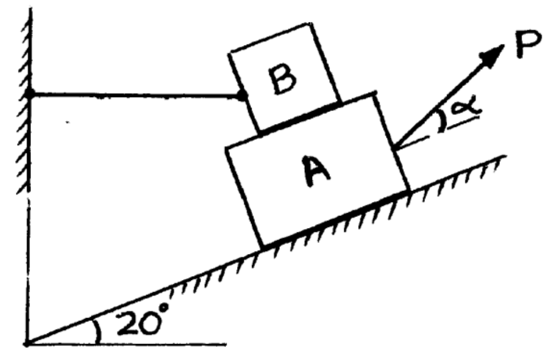
$$\text{Hence, } P = \frac{148.44}{\cos\alpha + 0.2 \sin\alpha}$$

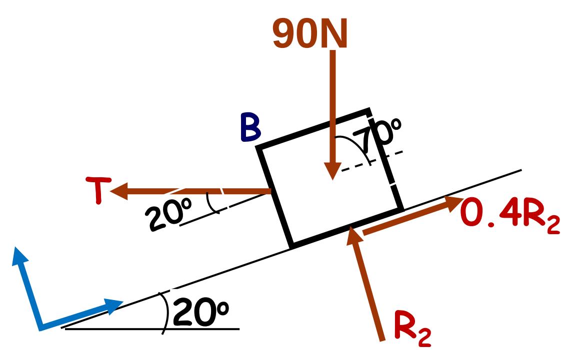
For P to be minimum → $\dfrac{d}{d\theta}(\cos\alpha + 0.2\sin\alpha) = 0$

$\alpha = 11.3^{o}$

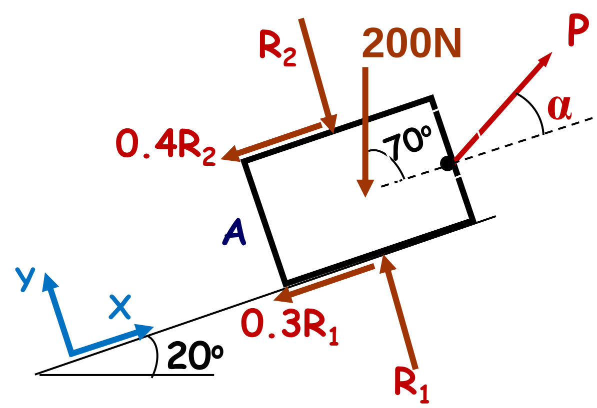
Hence P_{min} = 145.55N

10] Find the minimum value of force ‘P’ required to get the block to move up the plane. Frictional coefficient between A and the surface is 0.3 and that between A & B is 0.4. Given : W_A = 200N, W_B = 90N.





ANS : P_{min} = 175.6N



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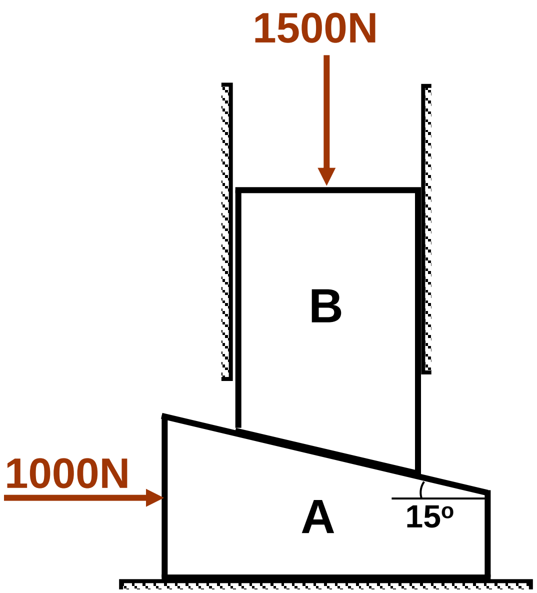
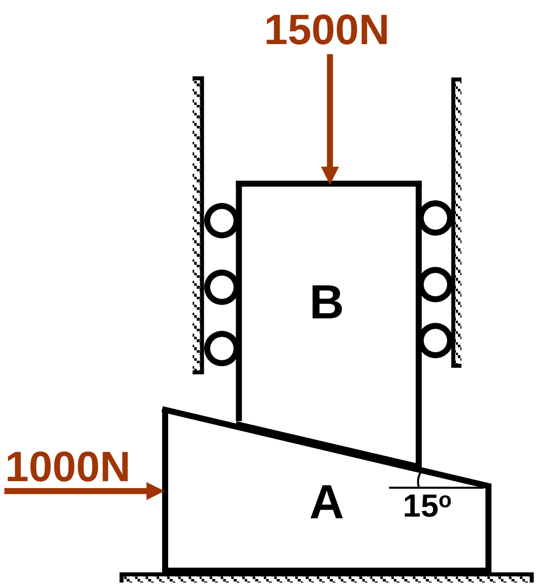
11] Wedge 'A' is used to raise a load of 1500N acting on block 'B' which is guided between two vertical surfaces as shown. Determine whether 1000N force applied to block A is sufficient to raise the load or no in the following two cases. Take μ = 0.2 for all contact planes.





ANS : P = 1155N

ANS : P = 1042N

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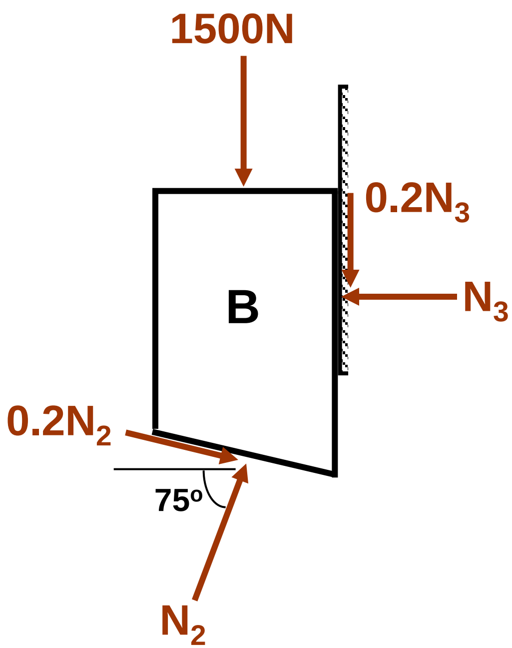
Let 'P' be the actual force applied to 'A' to raise the block 'B'.



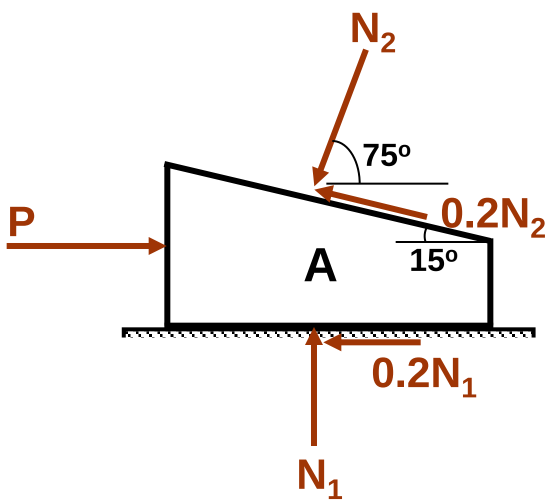
$\sum F_x = 0$

ie. $N_2\cos75 + 0.2N_2\cos15 - N_3 = 0 \rightarrow$ eq.(i)

$\sum F_y = 0$

ie. $N_2\sin75 - 0.2N_2\sin15 - 0.2N_3 = 1500 \rightarrow$ eq.(ii)

On solving, **we get – $N_2 = 1821N$, $N_3 = 823N$**



$\sum F_x = 0$

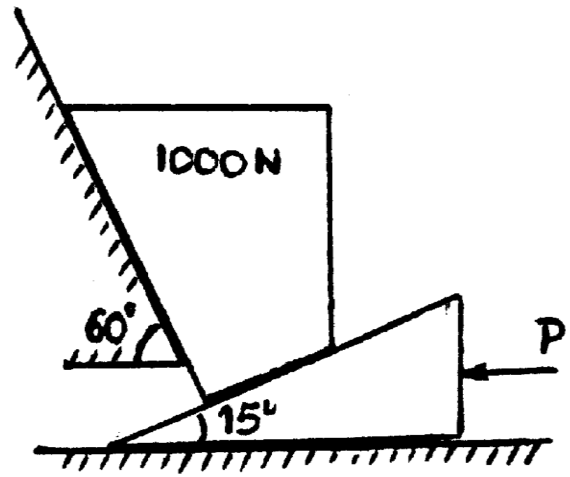
ie. $P - 1821(\cos75 + 0.2x\cos15) - 0.2N_1 = 0 \rightarrow$ eq.(i)

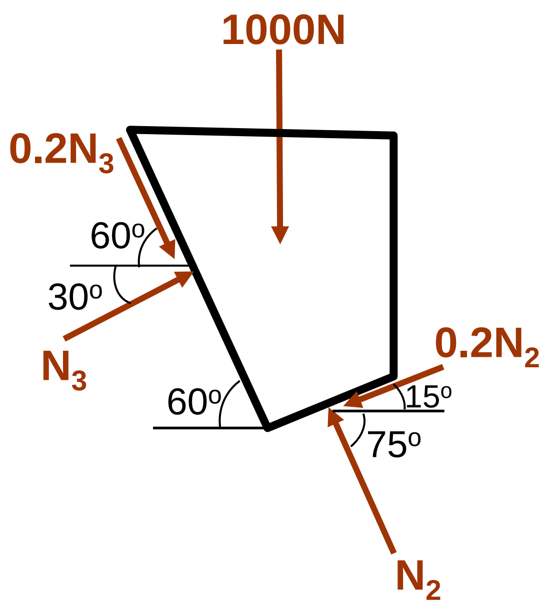
$\sum F_x = 0$

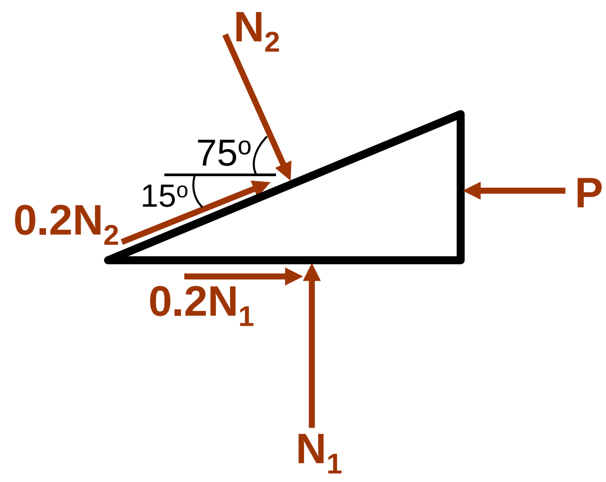
ie. $1821(-\sin75 + 0.2\sin15) + N_1 = 0 \rightarrow$ eq.(ii)

On solving, **we get – $N_1 = 1665N$, $P = 1155N$**

12] A block, weighing 1000N, is to be raised against a surface inclined at 60° by means of a 15° wedge as shown. Find the horizontal force P that will start the motion of the block. Assume frictional co-efficient for all the surfaces as 0.2.







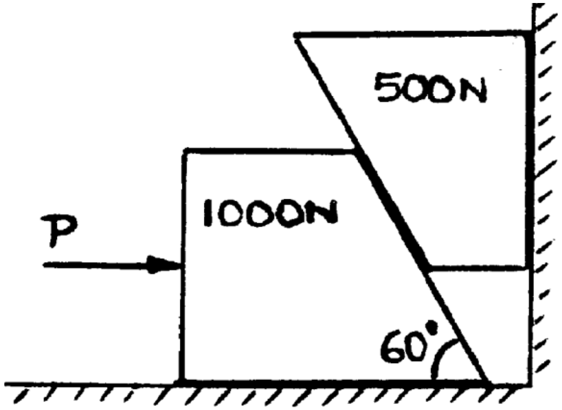
N_2 =

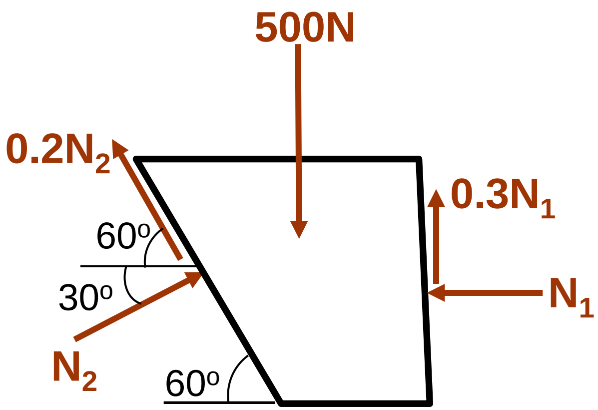
N_3 =

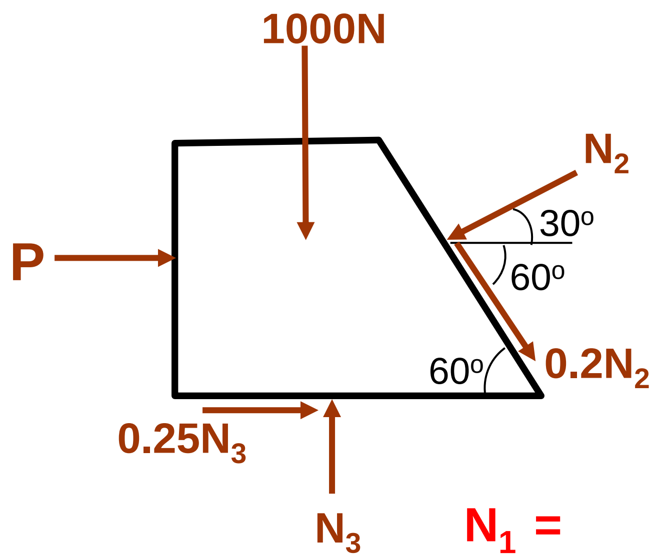
N_1 =

<u>**P = 595N**</u>

13] Referring to the figure, the cofft. of friction are: 0.25 at the floor, 0.3 at the wall & 0.2 between the blocks. Find the minimum value of the horizontal force ‘P’ required for equilibrium.







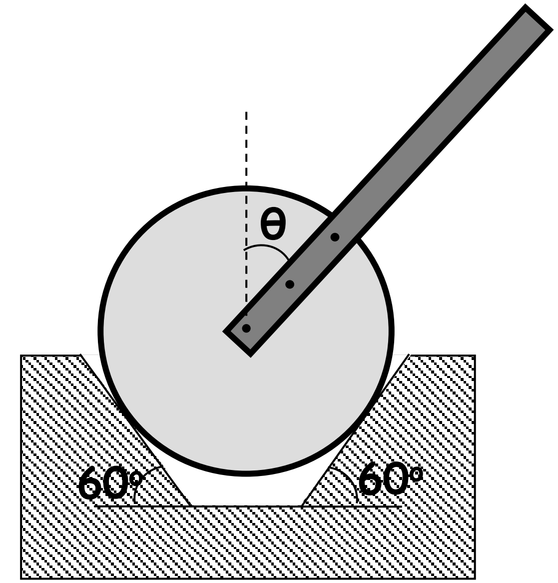
N_1 =

N_2 =

N_1 =

P = 80.96N

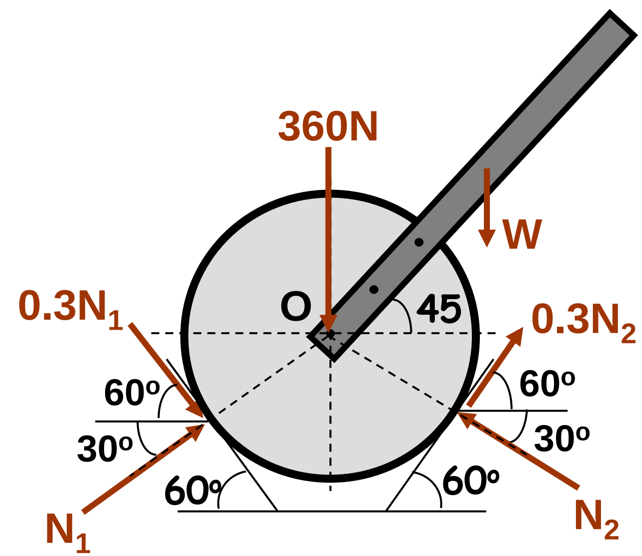
14] The cylinder having a mass of 36Kg and radius 300mm is attached to an uniform slender rod of length 1m having an unknown mass ‘m’. The system remains in equilibrium upto θ = 45^o and starts slipping there after. If the coefficient of static friction is 0.3, determine the value of ‘m’.



$\sum F_x = 0$ ie. $N_1\cos30 + 0.3N_1\cos60 – N_2\cos30 + 0.3N_2\cos60 = 0$ → eq(i)

$\sum F_Y = 0$ ie. $N_1\sin30 – 0.3N_1\sin60 + N_2\sin30 + 0.3N_2\sin60 – W = 360$ → eq(ii)

$\sum M_O = 0$ ie. $(0.3N_1 + 0.3N_2)\text{x } 0.3 – (W \text{ x } 0.5\cos45) = 0$ → eq(iii)



ON SOLVING - W = 315.5N

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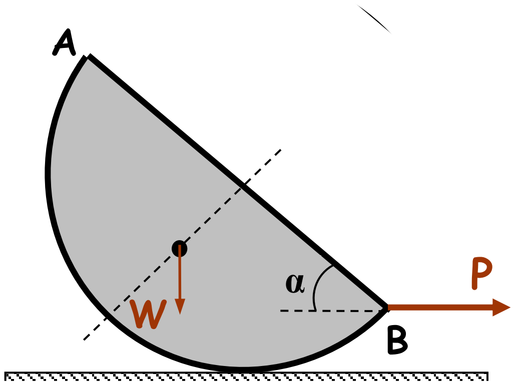
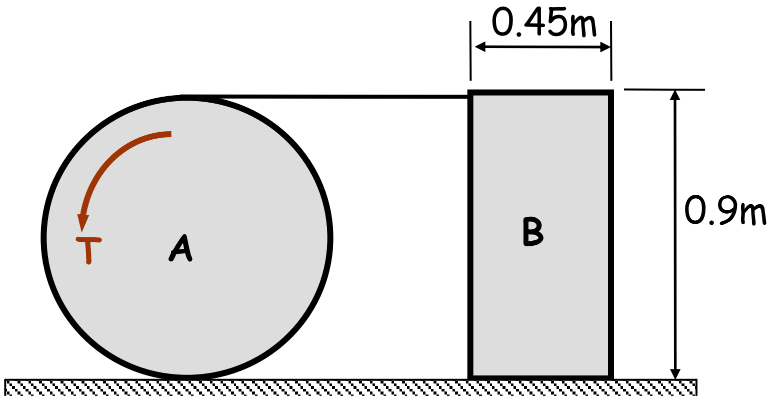
15] A homogeneous semi-circular lamina has a mass of 4Kg and radius 10cm. Determine the value of the horizontal force, applied as shown, to make the lamina move. Take $\mu = 0.2$ between the lamina and the plane. Also find the angle 'α'.



ANS: P = 8N ;   α = 18.68°

16] A wheel weighing 20N is resting on a surface for which μ = 0.2. A cord wrapped around it is attached to a 30N block B as shown. If the coefficient of static friction between the block and the surface is 0.3, determine the smallest value of torque T applied to the cylinder which will cause the motion to impend.



ANS: T = 5.995N- m

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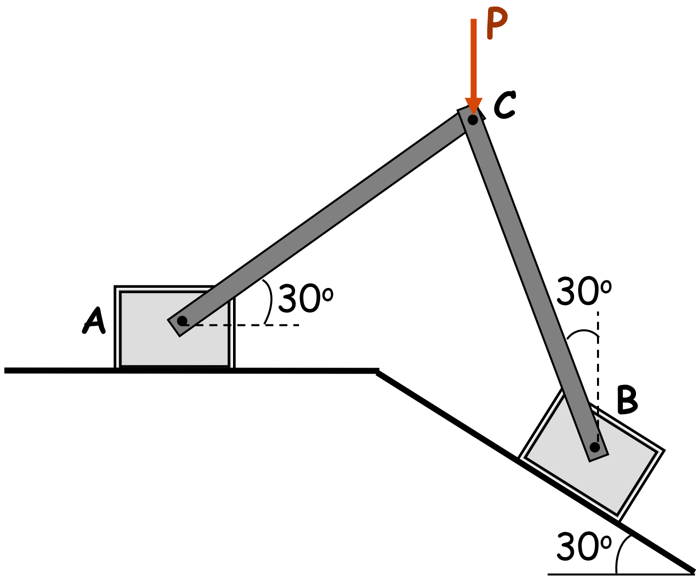
17] Two blocks A and B, each having mass of 6Kg, are connected by two links as shown. If $\mu_A = 0.2$ & $\mu_B = 0.8$, find the largest vertical force that may be applied to pin C without causing the blocks to slip.



ANS: P = 28.67N

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